

Student Name:

	Problem	Total Points	Score
	1	15	
	2	15	
	3	15	
	4	10	
	5	15	
	6	30	
Total	100		

Guidelines

- You have 2 hours to take the exam.
- You may use a calculator and two pages of notes (i.e. two sheets of paper with writing on both sides of each sheet).

1. (15 points)

(a) Use the *method of false position* to solve Problem 31 of the Rhind Papyrus stated below:

A quantity, and its $\frac{2}{3}$, its $\frac{1}{2}$, and its $\frac{1}{7}$ add together become 33. What is the quantity?

(b) The method of false position was a common strategy for solving equations though the Middle Ages in Europe. What made this method so durable?

(c) What are some of the limitations of this method and why is it no longer common to find the method taught in school?

2. (15 points)

(a) Use the ancient Egyptian method to find 163 divided by 30.

(b) Demonstrate how ancient Greek mathematicians could *construct* a line segment of length $\sqrt{2}$ even if they didn't feel $\sqrt{2}$ could be a number.

(c) A modern human might answer that $163 \div 30 = 5.433$ or describe $\sqrt{2}$ as 1.4142. Give some examples of how our modern notion of number and notation for number has evolved since the ancient Greeks and Egyptians.

4. (10 points)

(a) Solve Problem 27 from Book I of Diophantus' *Arithmetica*:

Find two numbers such that their sum and product are given numbers: say their sum is 20 and their product is 96.

Hint: Call the numbers $10 + x$ and $10 - x$.

(b) Explain what is meant today by a Diophantine equation and how this differs from what Diophantus meant in his *Arithmetica*.

5. (15 points)

(a) Explain how the algebra of Muhammad al-Khwarizmi differed from the algebra of Francois Vieta.

(b) Explain why algebraic notion was important in the development of analytic geometry.

(c) Explain why the development of analytic geometry was important to the development of calculus.

6. (30 points) Short Answer.

(a) Give two examples illustrating that *how* humans write mathematics influences how we *do* mathematics. One example should predate 1000 AD and one should come after 1000 AD.

(b) Describe the origins of the mathematical subject we now call *Trigonometry*. Who is responsible for the unit circle definition of trigonometric functions taught in modern classrooms?

(c) Describe the origins of the mathematical subject we now call *Probability*.

- (d) State two mathematical accomplishments of Archimedes.
- (e) Describe one of the first appearances of negative numbers and what motivates them. Include names and dates.
- (f) Describe one of the first appearances of imaginary numbers and what motivates them. Include names and dates.