

Student Name:

	Problem	Total Points	Score
Part I		27	
Part II			
	1	13	
	2	10	
	3	15	
	4	15	
	5	10	
Total	100		

Guidelines

- You have 1 hour to take the exam.
- The exam will be given in two parts.
- Part I is written without any aids: no notes, no book, no phone. You should spend no more than 15 minutes on Part I.
- Return your completed Part I to the proctor and you will be given Part II. You cannot go back to Part I once you have turned it in.
- For Part II, you may use a calculator and two pages of notes (i.e. two sheets of paper with writing on both sides of each sheet).

Part I

This part is written without notes or aids of any kind. It is worth 27 points out of 100 total points.

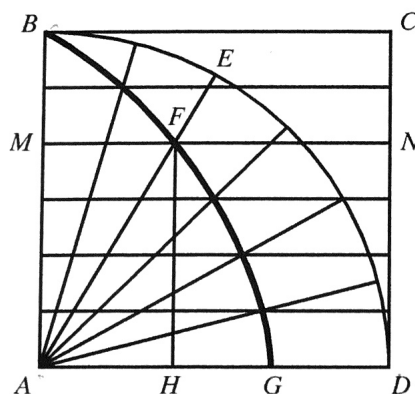
Below is a list of nine mathematicians, listed in alphabetical order. For each name, state whether the he lived before or after Euclid. Next to each name, state the title of a mathematical work the person authored **OR** a mathematical theorem or idea for which this person is given credit.

1. Archimedes (order: _____)
2. Apollonius of Perga (order: _____)
3. Eratosthenes (order: _____)
4. Eudoxus of Cnidos (order: _____)
5. Hippias of Elis (order: _____)
6. Hippocrates of Chios (order: _____)
7. Claudius Ptolemy (order: _____)
8. Thales of Miletos (order: _____)
9. Zeno of Elia (order: _____)

Part II

For this part, you may use a calculator and up to two pages of notes. This part is worth 73 points out of 100 total points. All parts of all questions are either mathematical or **short answer**. Short answer questions do not require more than an appropriately detailed sentence or two.

1. (13 points) The questions below concern Hippias' development of the Quadratrix. The figure of the quadratrix (below) is from our textbook. Recall that the thick arc from B through F to G is the curve.



- (a) Use the *definition* of the quadratrix to give a precise mathematical relationship between $\angle EAD$ and line segment FH .
- (b) Explain why Hippias' definition of the quadratrix technically did not contain point G and explain how we define that point in modern terms.
- (c) Hippias' used the quadratrix for what purpose?
- (d) In the historical development of Greek mathematics, the quadratrix was the first example of a curve with what property?

2. (10 points) The questions below concern Hippocrates' Quadrature of a Lune.

(a) What is a *lune*?

(b) What is meant by *a quadrature of a lune*?

(c) Why was Hippocrates' quadrature of a lune considered to be a significant result at the time? (To be clear, this question is asking why Hippocrates' and his contemporaries viewed this result as important. It is not asking for a modern view of the result.)

3. (15 points) The following questions concern Euclid's *Elements of Geometry*.

(a) Why did the fifth postulate of Book I receive so much attention by so many mathematicians? (Your answer should be limited to the motivation of mathematicians in roughly the first 1000 years after the *Elements* was written.)

(b) With what two propositions does Book I end?

(c) Describe at least two mathematical subjects that appear in the *Elements* other than 2-dimensional plane geometry.

4. (15 points) The following questions concern the mathematics of Archimedes.

(a) Describe the method Archimedes used to estimate the circumference of a circle. You may draw a picture to aid your written description.

(b) Given a circle of diameter 20, assume a mathematician estimates the circumference to be $63\frac{1}{10}$ (i.e. 63.1). What estimate of π does this correspond to?

(c) Describe the method Archimedes used in his quadrature of a parabolic segment. You may draw a picture to aid your written description.

5. (10 points) Below is a translation of Proposition 9 from Book II of Euclid's *Elements* along with an accompanying figure.

Begin Quote

If a straight line is cut into equal and unequal segments, then the sum of the squares on the unequal segments of the whole is double the sum of the square on the half and the square on the straight line between the points of section.

Let a straight line AB be cut into equal segments at C , and into unequal segments at D .

I say that the sum of the squares on AD and DB is double the sum of the squares on AC and CD .

End Quote



If $AC = x$, $CD = y$ and $DB = z$, rewrite the proposition using the symbols x , y , and z and modern algebraic notation. Then show that this algebraic relationship is true.