Now, let's <u>use</u> the QR-factorization

## Motivating Problem

Solve
$$\begin{aligned} x_1 + x_2 \phantom{{}+x_3} &= 2.3 \\ x_1 \phantom{{}+x_2} + x_3 &= 0.18 \\ x_2 + x_3 &= -8.41 \end{aligned}$$
← call it $S$

**A.** Reframe as a matrix equation

$S$ is $Ax = b$ where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 2.3 \\ 0.18 \\ -8.41 \end{bmatrix}$$

**B.** Find QR-factorization of $A$

$$Q = \begin{bmatrix} 1/\sqrt{2} & \frac{1}{\sqrt{6}} & -1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix} \qquad R = \begin{bmatrix} \sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 3/\sqrt{6} & 1/\sqrt{6} \\ 0 & 0 & 2/\sqrt{3} \end{bmatrix}$$

C. Find $Q^Tb$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 1/\sqrt{2} & 0 \\ \frac{1}{\sqrt{6}} & -1/\sqrt{6} & 2/\sqrt{6} \\ -\frac{1}{\sqrt{3}} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} 2.3 \\ 0.18 \\ -8.41 \end{bmatrix} = \begin{bmatrix} 1.6122034 \\ -6.0828995 \\ -5.9640282 \end{bmatrix}$$

Julia :)

D. Solve $\boxed{Rx = Q^Tb}$ via back-substitution

$$\begin{bmatrix} \sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 3/\sqrt{6} & 1/\sqrt{6} \\ 0 & 0 & 2/\sqrt{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.6122034 \\ -6.0828995 \\ -5.9640282 \end{bmatrix}$$

OR

$$\sqrt{2}x_1 + \frac{1}{\sqrt{2}}x_2 + \frac{1}{\sqrt{2}}x_3 = 1.6122034$$

$$\frac{3}{\sqrt{6}}x_2 + \frac{1}{\sqrt{6}}x_3 = -6.0828995$$

$$\frac{2}{\sqrt{3}}x_3 = -5.9640282$$

$$\sqrt{2}\,x_1 + \frac{1}{\sqrt{2}}x_2 + \frac{1}{\sqrt{2}}x_3 = 1.6122034$$

$$\frac{3}{\sqrt{6}}x_2 + \frac{1}{\sqrt{6}}x_3 = -6.0828995$$

$$\frac{2}{\sqrt{3}}x_3 = -5.9640282$$

So $\quad x_3 = (-5.9640282)\dfrac{\sqrt{3}}{2}$

$$x_2 = \left[(-6.0828995) - \frac{1}{\sqrt{6}}x_3\right]\frac{\sqrt{6}}{3}$$

$$x_1 = \left[1.6122034 - \frac{1}{\sqrt{2}}(x_2+x_3)\right]\cdot\frac{1}{\sqrt{2}}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5.445 \\ -3.145 \\ -5.265 \end{bmatrix}$$

Julia :)

# QR-backsub

October 22, 2024

[13]:
```
sq2=sqrt(2);
sq6=sqrt(6);
sq3=sqrt(3);
QT=[1/sq2 1/sq2 0;
    1/sq6 -1/sq6 2/sq6;
    -1/sq3 1/sq3 1/sq3];
b=[2.3, 0.18,-8.41];
QTB=QT*b
```

[13]:
```
3-element Vector{Float64}:
  1.7536248173426374
 -6.001249869818787
 -6.07949833456676
```

[14]:
```
x3=QTB[3]*sq3/2
```

[14]:
```
-5.265000000000001
```

[15]:
```
x2=(sq6/3)*(QTB[2] - x3/sq6)
```

[15]:
```
-3.145
```

[16]:
```
x1=(1/sq2)*(QTB[1] - (1/sq2)*(x3+x2))
```

[16]:
```
5.4449999999999985
```

[17]:
```
x=[x1,x2,x3]
```

[17]:
```
3-element Vector{Float64}:
  5.4449999999999985
 -3.145
 -5.265000000000001
```

[18]:
```
A=[1 1 0;
    1 0 1;
    0 1 1];
A*x
```

[18]:
```
3-element Vector{Float64}:
  2.2999999999999985
  0.17999999999999794
 -8.41
```

[19]:
```
##alternatively
A*x-b
```

[19]:
```
3-element Vector{Float64}:
 -1.3322676295501878e-15
 -2.0539125955565396e-15
  0.0
```

[ ]:
```

```