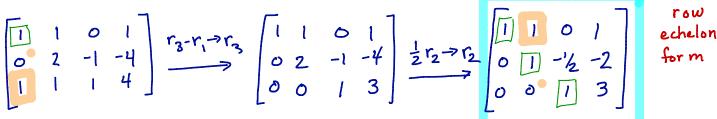
WORKSHEET: GAUSSIAN ELIMINATION (A FIRST LOOK)

1. Use Gaussian elimination to solve the system
$$S_1=\begin{cases} x+y&=1\\ 2y-z=-4\\ x+y+z=4 \end{cases}$$



$$\begin{array}{cccc}
x & = \frac{3}{2} \\
y & = -\frac{1}{2} \\
z & = 3
\end{array}$$
reduced row e chelon form

2. Use Gaussian elimination to solve the system
$$S_2=\begin{cases} x-5y+z=2\\ x-4y+z=2\\ 2x+z=5 \end{cases}$$

$$\begin{bmatrix}
1 & -5 & 1 & 2 \\
1 & -4 & 1 & 2 \\
2 & 0 & 1 & 5
\end{bmatrix}
\xrightarrow{\Gamma_2 - \Gamma_1 \to \Gamma_2}
\xrightarrow{\Gamma_3 - 2\Gamma_1 \to \Gamma_3}
\begin{bmatrix}
1 & -5 & 1 & 2 \\
0 & 11 & 0 & 0 \\
0 & 10 & -1 & 1
\end{bmatrix}
\xrightarrow{\Gamma_3 - 10\Gamma_2 \to \Gamma_3}
\begin{bmatrix}
1 & 0 & 1 & 2 \\
0 & 11 & 0 & 0 \\
\hline{\Gamma_1 + 5\Gamma_2 \to \Gamma_1}
\begin{bmatrix}
0 & 0 & -1 & 1
\end{bmatrix}$$

$$-1r_3 \rightarrow r_3 \qquad \boxed{1} \quad 0 \quad \boxed{1} \quad 2 \\ 0 \quad \boxed{1} \quad 0 \quad 0 \\ 0 \quad \boxed{1} \quad -1 \qquad \boxed{1} \qquad \boxed{1} \quad 0 \quad 0 \quad 3 \\ 0 \quad \boxed{1} \quad 0 \quad 0 \\ 0 \quad \boxed{1} \quad -1 \qquad \boxed{1} \qquad \boxed{1} \qquad 0 \quad \boxed{1} \quad 0 \quad 3 \\ 0 \quad \boxed{1} \quad -1 \qquad \boxed{1} \qquad \boxed{1} \qquad 0 \quad \boxed{1} \quad -1 \qquad \boxed{1} \qquad \boxed{1} \qquad 0 \quad \boxed{1} \quad -1 \qquad \boxed{1} \qquad 0 \quad \boxed{1} \quad -1 \qquad \boxed{1} \qquad \boxed$$

Linear 1

3. Use Gaussian elimination to solve the system $S_3 = \begin{cases} x_1 + & x_2 + 3x_3 = 5 \\ x_1 + 2x_2 + 4x_3 = 6 \end{cases}$

$$\begin{bmatrix}
1 & 1 & 3 & 5 \\
1 & 2 & 4 & 6
\end{bmatrix}
\xrightarrow{\Gamma_2 - \Gamma_1 \rightarrow \Gamma_2}
\begin{bmatrix}
1 & 1 & 3 & 5 \\
0 & 1 & 1 & 1
\end{bmatrix}
\xrightarrow{r_1 - r_2 \rightarrow r_1}
\begin{bmatrix}
1 & 0 & 2 & 4 \\
0 & 1 & 1 & 1
\end{bmatrix}$$

$$x_1 + 2x_3 = 4$$
 $x_1 = 4-2x_3$
 $x_2 + x_3 = 1$ or $x_2 = 1-x_3$

Solution
$$\begin{cases} \begin{bmatrix} 4-2x_3 \\ 1-x_3 \end{bmatrix} & : x_3 \text{ any real number } \end{cases}$$

reduced row echelon form of a matrix

- any row(s) of all zeros at bottom
- any nonzero row leads with a []. (ie. left-most entry is [])
- any column with a leading [1] has all other entries Zero
- leading 1) in a row is to the right of the leading 1) in row about.