Observe that the document has spaced out the problems so that there space for comments.

Chapter 2 Problems

1. Use Proof by Induction to prove the statement below.

$$\text{For all } n \in \mathbb{N}, \frac{1}{2} + \frac{1}{6} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

Proof: Your proof goes here.

2. For $a = 23771$ and $b = 19945$, calculate $gcd(a,b)$ and find integers s and t such that

$$as + bt = gcd(a,b).$$

Answer: Your answer goes here.

3. Suppose that a and b are integers such that $gcd(a,b) = 1$. Let s and t be integers such that $as + bt = 1$. Prove that $gcd(a,s) = gcd(r,b) = gcd(s,t) = 1$.

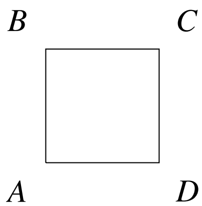
   Proof: Your proof goes here.

4. Let $a, b, c \in \mathbb{N}$. Prove that if $gcd(a,b) = 1$ and $a \mid bc$, then $a \mid c$.

   Proof: Your proof goes here.

Chapter 3 Problems

5. This question is about the symmetries of a **square**, as opposed to the example of the rectangle at the beginning of this section. You may want to use a drawing or permutations to describe your ideas but you don't have to. It's OK to have essay-style answers.



   (a) Describe the symmetries of the square.
   Answer: Your answer goes here.

   (b) How many symmetries of the square are there?
   Answer: Your answer goes here. It's going to be just a number!

   (c) How many permutations of the set $\{A, B, C, D\}$ are there?
   Answer: Your answer goes here. It's going to be just a number! A little explanation of your reasoning here never hurts!

   (d) Give an example of a permutation of the set $\{A, B, C, D\}$ that cannot correspond to a symmetry of the square pictured above?
   Answer: Your answer goes here. Justify your answer

6. Let $S = \mathbb{R} \setminus \{-1\}$ and define a binary operation $*$ by $a * b = a + b + ab$. Prove that $(S, *)$ is an abelian group.

   Proof: Your proof goes here.

7. Give an example of two elements A and B in $GL_2(\mathbb{R})$ with $AB \neq BA$.

   Answer: Your answer goes here.

8. Given the groups $\mathbb{R}^*$ and $\mathbb{Z}$, let $G = \mathbb{R}^* \times \mathbb{Z}$. Define a binary operation $\circ$ on G by $(a,m) \circ (b,n) = (ab, m+n)$. Prove that $(G, \circ)$ is a group.

   Proof: Your proof goes here.

9. Let $(G, \circ)$ be a group. Let $g_1, g_2, \cdots, g_n \in G$. Show that the inverse of $g_1 g_2 \cdots g_n$ is $g_n^{-1} \cdots g_2^{-1} g_1^{-1}$.

Proof: Your proof goes here.