1. Find all of the ideals in each of the following rings. Which of these ideals are maximal and which are prime?
   (a) $\mathbb{Z}_{18}$
   (b) $\mathbb{Z}_{25}$
   (c) $\mathbb{M}_2(\mathbb{R})$
   (d) $\mathbb{Q}$

2. Find all homomorphisms $\phi : \mathbb{Z}/6\mathbb{Z} \to \mathbb{Z}/15\mathbb{Z}$.

3. Prove that $\mathbb{R}$ is not isomorphic to $\mathbb{C}$. Define a map $\phi : \mathbb{C} \to \mathbb{M}_2(\mathbb{R})$ by $\phi(a+bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$. Show that ϕ is an isomorphism of $\mathbb{C}$ to its image under ϕ.

4. Prove that the Gaussian integers, $\mathbb{Z}[i]$, form an integral domain.

5. Prove that if R is a field, the only ideals of R are $\{0\}$ and R itself.

6. Suppose $\phi : R \to S$ is a ring homomorphism. Prove each of the following statements.
   (a) If R is a commutative ring, then $\phi(R)$ is a commutative ring.
   (b) $\phi(0_R) = 0_S$.
   (c) If ϕ is onto, then $\phi(1_R) = 1_S$.
   (d) If R is a field and $\phi(R) \neq \{0\}$, then $\phi(R)$ is a field.

7. Let R be a ring and let $\{R_\alpha\}$ be a collection of subrings of R for $\alpha \in A$. Prove that $\cap_{\alpha \in A} R_\alpha$ is also a subring and show by example that the union of two subrings may not be a subring. Note that A is just an index set.

8. Let R be a ring and let $\{I_\alpha\}$ be a collection of ideals of R for $\alpha \in A$. Prove that $\cap_{\alpha \in A} I_\alpha$ is also a subring and show by example that the union of two subrings may not be a subring. Note that A is just an index set.