The goal is to practice evaluating limits

- numerically
- graphically

This sheet is a 1.5 class sheet. Day 1: pp1-2 ; Day 2: pg 3.

① Remind students that in §2.1 we used

$$m_{sec} \approx m_{tan}.$$

OR

$$\text{as } Q \to P, \quad m_{PQ} \to m_{\tan @ P}$$

This was a type of limit.

② State the definition + work problem #2.

③ Have the students work #3, 4, 5 + put on board. Ask them how to split the work.

(Get students to split #3 into the jobs of two groups b/c it looks hard.)

- Emphasize
  - #3 answer should feel shakey.
  - limits do not always exist
  - limits do not care what happens @ $x=a$
  - infinite limits do not exist
- Use the last two problems on page 1 to motivate one-sided limits. Go back + write these in.

④ Work #7 + #8. (I put on screen. Get students to tell me answers.) Emphasize that v. asymptotes correspond to infinite limits.

Things to add if there is time on day 1

- What do the graphs of #1-4 look like? Does this confirm our numerical work?
- What is the algebraic limit from the example from 2.1?

On day 2, have students put their answers on board for both problems.

For #9 emphasize that knowing that $x=-5$ makes the denominator 0 is not sufficient: See #'s 2 and 3 from page 1. Checking the limit is infinite is essential.

For #10, emphasize that there are multiple correct answers.

1. DEFINITION: **Two-Sided Limit**

Notation: $\lim_{x \to a} f(x) = L$

Words: the limit of f(x), as x approaches a, is L.

It means: As the x-values get closer + closer to a (larger + smaller than a) the y-values of f(x) get close to L. In fact, the y's can be forced arbitrarily close to L.

Evaluate the limits below numerically. Estimate the limit to 4 decimal places, if possible.

2. $\lim_{x \to 0} \frac{\sin(x)}{x} = \boxed{1}$

x	sin(x)/x
0.1	0.99833
0.01	0.99998
0.001	0.99999
0	DNE
−0.001	0.99999
−0.01	0.99998
−0.1	0.99833

3. $\lim_{x \to 2} \frac{\cos(x)(x-2)}{3x^2-5x-2} = \boxed{0.0594}$

x	cos(x)(x−2)/(3x²−5x−2)
2.1	−0.069157
2.01	−0.060486
2.001	−0.05955
2.00001	−0.05945
2.	
1.99999	−0.059448
1.999	−0.059345
1.99	−0.058397
1.9	−0.04825

4. $\lim_{x \to -1} \frac{|x+1|}{x+1} = \boxed{\text{DNE}}$

x	|x+1|/(x+1)
−0.9	1
−0.99	1
−0.999	1
−0.9999	1
−1	DNE
−1.0001	−1
−1.001	−1
−1.01	−1
−1.1	−1

from page 2

$\lim_{x \to -1^+} \frac{|x+1|}{x+1} = \boxed{1}$

$\lim_{x \to -1^-} \frac{|x+1|}{x+1} = \boxed{-1}$

5. $\lim_{x \to 1} \frac{1}{x-1} = \boxed{\text{DNE}}$

x	1/(x−1)
1.1	10
1.01	100
1.001	1000
1.0001	10,000
1	DNE
0.9999	−10,000
0.999	−1000
0.99	−100
0.9	−10

from page 2

$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$

$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$

6. $\boxed{\text{DEFINITION:}}$ **One-Sided Limits**

Notation:

$\lim_{x \to a^-} f(x) = L$

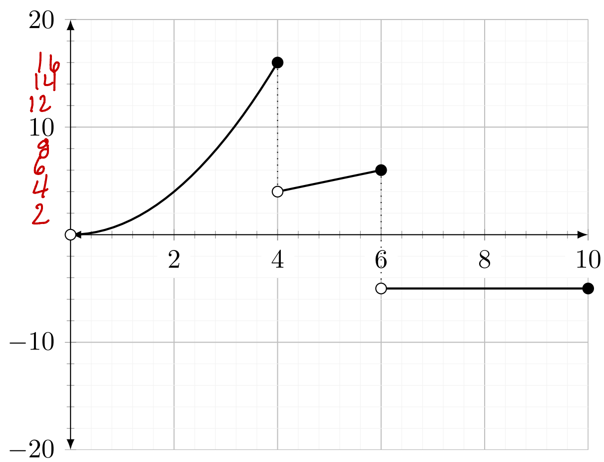
x-values approach a only on left or below or x-values less than $x=a$

$\lim_{x \to a^+} f(x) = L$

x-values approach $x=a$ only on the right or above or from x-values larger than $x=a$.

Limits can also be evaluated graphically.

7. The function $g(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x \to 4^-} g(x) =$ 16

(b) $\lim_{x \to 4^+} g(x) =$ 4

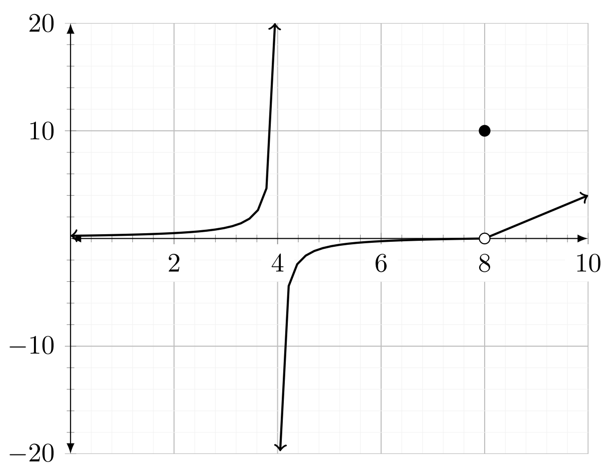
(c) $\lim_{x \to 4} g(x) =$ DNE

(d) $g(4) =$ 16

(e) $\lim_{x \to 8} g(x) =$ −5

(f) $g(8) =$ −5

8. The function $h(x)$ is graphed below. Use the graph to fill in the blanks.



(a) $\lim_{x \to 4^-} h(x) =$ $+\infty$

(b) $\lim_{x \to 4^+} h(x) =$ $-\infty$

(c) $\lim_{x \to 4} h(x) =$ DNE

(d) $h(4) =$ DNE

(e) $\lim_{x \to 8} h(x) =$ 0

(f) $h(8) =$ 10

9. Find any vertical asymptotes of $f(x) = \frac{2}{x+5}$ and *justify* your answer using a limit.

V.a.: $x = -5$ ← Value makes denominator zero.

Justification:

$$\lim_{x \to -5^+} \frac{2}{x+5} = +\infty$$

as $x \to -5^+$ (#'s like $-4.9, -4.99$)

$x+5 \to 0^+$

10. Sketch the graph of an function that satisfies *all* of the given conditions. Compare your answer with that of your neighbor.

$\lim_{x \to 0^-} f(x) = 1$ $\quad \lim_{x \to 0^+} f(x) = -2$ $\quad \lim_{x \to 4^-} f(x) = 3$ $\quad \lim_{x \to 4^+} f(x) = 0$

$f(0) = -2$ $\qquad f(4) = 1$