- Draw the calculations from ⓒ and ⓓ on the graph of ⓑ
- Write in words what $L(x)$ is saying

$$\underbrace{L(x)}_{\text{A way to approximate y-values of } f(x) \text{ for x-values CLOSE to a. Do this by}} = \underbrace{f(a)}_{\text{Starting at the y-value when } x=a. \text{ (ie } f(a)\text{) then}} + \underbrace{f'(a)(x-a)}_{\text{estimating how much to add (or subtract) from that y-value.}}$$

A way to approximate y-values of $f(x)$ for x-values CLOSE to a.

Do this by

Starting at the y-value when $x = a$. (ie $f(a)$) then

estimating how much to add (or subtract) from that y-value.

Specifically,

$f'(a)$ tells us how much y changes for a 1-unit change in x.

$(x-a)$ tells us how much our x-value actually changed from a. (ie what portion of the 1-unit change our x represents)

What does the differential do?

$$\underbrace{dy}_{\text{A way to estimate how much y will change by}} = \underbrace{f'(x)}_{\text{Knowing how much y changes for a 1-unit change in x then}} \underbrace{dx}_{\text{multiplying by how much x actually changed.}}$$

A way to estimate how much y will change by

Knowing how much y changes for a 1-unit change in x then

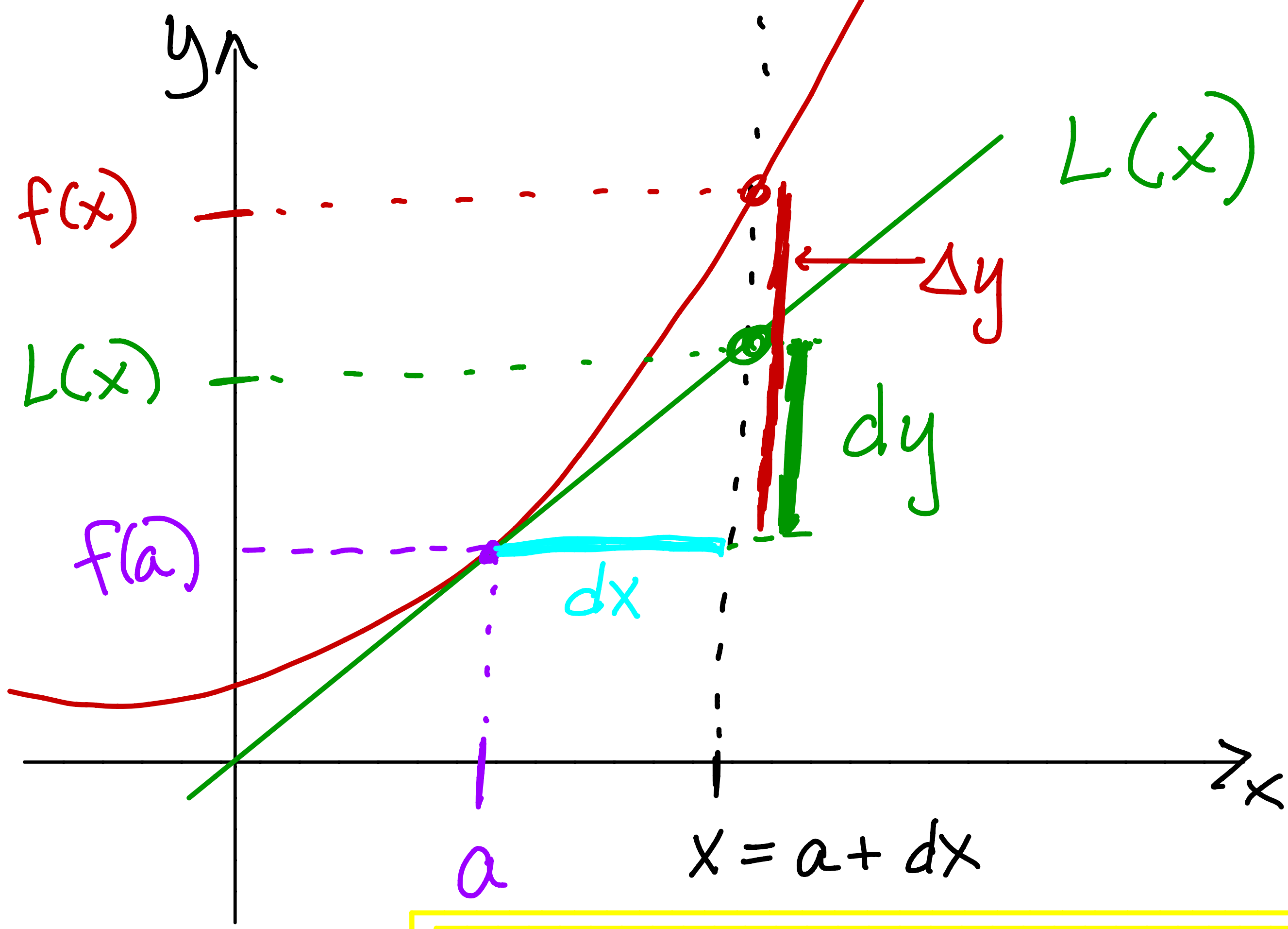
multiplying by how much x actually changed.

Do you see the differential hidden inside $L(x)$?

That is:

$$f(x) \approx L(x) = \underbrace{f(a)}_{\text{y-value at a}} + \overbrace{dy}^{\text{add on how much you estimate y will change}}$$

4.2 in a picture.



dx - how much x changes

Δy - how much $y=f(x)$ changes when x changes

dy - how much $y=L(x)$ changes when x changes.

The point:
For x close to a,

$$f(x) \approx L(x) \quad \text{and}$$

$$\Delta y \approx dy$$