§ 3.8 Exponential Growth & Decay

Ex y = # people world wide who have been infected w/ novel coronavirus (COVID-19)

listen to news: yesterday 1000 new infections

$$\left.\frac{dy}{dt}\right|_{t=60} = 1000 \quad \text{for } t = \text{days since 1 december}$$

What is $\frac{dy}{dt}$?

$\frac{dy}{dt} = 10$ $\qquad y = 10t$

t = days

$\frac{dy}{dt} = 8t$ $\qquad y = 4t^2$

$\frac{dy}{dt} = 8y$ $\qquad y = e^{8t}$

$y' = e^{8t} \cdot 8 = 8y$

In general: if $\frac{dy}{dt} = ky$

then $y = Ce^{kt} = y(0) \cdot e^{kt}$ ← initial population

Ex Bacteria
- growth is proportional to size of pop. → write expression for population as a fcn. of time
- start w/ 20 cells
- 2 hours later 100 cells.

$P = P_0 e^{kt}$

$P = 20e^{kt}$

$100 = 20e^{k \cdot 2}$

$5 = e^{2k}$

$\ln 5 = 2k$

$k = \frac{1}{2}\ln 5$

ANS: $P(t) = 20e^{\frac{\ln 5}{2}t}$

- Find & interpret $P'(3)$ & $P(3)$
- When does pop reach 1000?

$\frac{1000}{20} = e^{\frac{\ln 5}{2}t}$

$\frac{2 \cdot \ln 50}{\ln 5} = t \approx 4.8614$

$P(3) = 223.6$ ← units bacteria

$P'(t) = \frac{20 \cdot \ln 5}{2} e^{\frac{\ln 5}{2}t}$

$P'(3) = 10\ln 5\, e^{\frac{3}{2}\ln 5} = 179.9$ bacteria/hour