

Homework # 8

Problem List Ch 7 #8, 10, 12, 21, 30, 32, 35; Ch 8 # 4, 10, 14, 22, 25, 26, 30

Problem Directions

- Ch 7: Use any technique you like.
- Ch 8: Use the methods introduced in this section.

Chapter 7

8. Suppose $a, b \in \mathbb{Z}$. Prove that $a \equiv b \pmod{10}$ if and only if $a \equiv b \pmod{2}$ and $a \equiv b \pmod{5}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

10. If $a \in \mathbb{Z}$, then $a^3 \equiv a \pmod{3}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

12. There exists a positive real number x for which $x^2 < \sqrt{x}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 21.** Every real solution to $x^3 + x + 3 = 0$ is irrational.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 30.** Suppose $a, b, p \in \mathbb{Z}$ and p is prime. Prove that if $p \mid ab$ then $p \mid a$ or $p \mid b$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 32.** If $n \in \mathbb{Z}$, then $\gcd(n, n+2) \in \{1, 2\}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 35.** Suppose $a, b \in \mathbb{N}$. Then $a = \gcd(a, b)$ if and only if $a \mid b$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

Chapter 8

- 4.** If $m, n \in \mathbb{Z}$, then $\{x \in \mathbb{Z} : mn \mid x\} \subseteq \{x \in \mathbb{Z} : m \mid x\} \cap \{x \in \mathbb{Z} : n \mid x\}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 10.** If A and B are sets in a universal set U , then $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 14.** If A , B and C are sets, then $(A \cup B) - C = (A - C) \cup (B - C)$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 22.** Let A and B be sets. Prove that $A \subseteq B$ if and only if $A \cap B = A$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

- 25.** Suppose A, B, C , and D are sets. Prove that $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

26. Prove that $\{4k + 5 : k \in \mathbb{Z}\} = \{4k + 1 : k \in \mathbb{Z}\}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.

30. Prove that $(\mathbb{Z} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{Z}) = \mathbb{N} \times \mathbb{N}$.

Proof. YOUR PROOF HERE. □

Your thoughts/concerns/questions here.