

Your Name Here  
February 2, 2026

Math 265

Homework #500

due 1/11/2026

**§4.1, #3:** Evaluate  $\int_1^2 \frac{1}{x^2} dx$ .

*Answer:*

$$\int_1^2 \frac{1}{x^2} dx = \int_1^2 x^{-2} dx = -x^{-1} \Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

**§4.2, #17** Prove  $\sqrt{2}$  is irrational.

*Proof.* Suppose, to the contrary, that  $\sqrt{2}$  is rational. Then

$$\sqrt{2} = \frac{a}{b}$$

where  $a, b \in \mathbb{Z}$ ,  $b \neq 0$  with  $a, b$  having no common factors. Squaring yields

$$2 = \frac{a^2}{b^2},$$

so

$$2b^2 = a^2.$$

This shows 2 divides  $a^2$ , and so since 2 is prime by a lemma proved in class, we see 2 divides  $a$ . Letting  $a = 2c$  for some  $c \in \mathbb{Z}$ , this implies

$$2b^2 = 4c^2,$$

so

$$b^2 = 2c^2.$$

Now the same argument as above, but with  $b, a$  replaced by  $c, b$ , shows 2 divides  $b$ . Therefore 2 divides both  $a$  and  $b$ . But this contradicts that  $a, b$  had no common factors.  $\square$

**§4.2, #18** Find the product of  $x$  and  $y$  supposing that both are odd.

This shows you an aligned string of equations.

$$\begin{aligned} xy &= (2a + 1)(2b + 1) \\ &= (2a)(2b) + (2a)(1) + 1(2b) + 1(1) \\ &= 4ab + 2a + 2b + 1 \\ &= 2(2ab + a + b) + 1 \\ &= 2k + 1, \end{aligned}$$

This shows you an aligned string of equations with justifications.

$$\begin{aligned} xy &= (2a + 1)(2b + 1) && a, b \text{ integers, (by definition of odd)} \\ &= (2a)(2b) + (2a)(1) + 1(2b) + 1(1) && (\text{expanding binomial multiplication}) \\ &= 4ab + 2a + 2b + 1 && (\text{simplifying}) \\ &= 2(2ab + a + b) + 1 && (\text{factoring out a } 2) \\ &= 2k + 1, && (k = 2ab + a + b) \end{aligned}$$

**§4.2, #19** Make a table of useful L<sup>A</sup>T<sub>E</sub>X symbols.

words	what you type into L <sup>A</sup> T <sub>E</sub> X	what appears in the PDF	example
is an element of	\in	$\in$	$x \in \mathbb{R}$
is not an element of	\not\in	$\notin$	$x \notin \mathbb{R}$
is a subset of	\subseteq	$\subseteq$	$\mathbb{Q} \subseteq \mathbb{R}$
for all	\forall	$\forall$	$\forall x \in \mathbb{R}$
there exists	\exists	$\exists$	
logical and	\land	$\wedge$	
logical or	\lor	$\vee$	
logical negation	\sim	$\sim$	
subscript	A_2	$A_2$	
superscript	x^2	$x^2$	
number sign	\#	$\#$	
ampersand	\&	$\&$	
new line	\cr or \\		
backslash	\textbackslash	$\backslash$	