Math 265

**YOUR NAME**

February 10, 2026

# Homework # 6

Due: Wednesday 02/18/2026

Problem List   Ch 4 #4,6,11,12,13,16,18,20,21,26,28

Problem Directions   Prove each statement below using a direct proof.

**4.** Suppose $x, y \in \mathbb{Z}$. If $x$ and $y$ are odd, then $xy$ is odd.

*Proof.* YOUR PROOF GOES HERE $\square$

**6.** Suppose $a, b, c \in \mathbb{Z}$. if $a|b$ and $a|c$, then $a|(b+c)$.

*Proof.* YOUR PROOF GOES HERE $\square$

**11.** Suppose $a, b, c, d \in \mathbb{Z}$. If $a|b$ and $c|d$, then $ac \mid bd$.

*Proof.* YOUR PROOF GOES HERE $\square$

**12.** If $x \in \mathbb{R}$ and $0 < x < 4$, then $\dfrac{4}{x(4-x)} \geq 1$.

*Proof.* YOUR PROOF GOES HERE $\square$

**13.** Suppose $x, y \in \mathbb{R}$. If $x^2 + 5y = y^2 + 5x$, then $x = y$ or $x + y = 5$.

*Proof.* YOUR PROOF GOES HERE $\square$

**16.** If two integers have the same parity, then their sum is even.

*Proof.* YOUR PROOF GOES HERE $\square$

**18.** Suppose $x$ and $y$ are positive real numbers. If $x < y$, then $x^2 < y^2$.

*Proof.* YOUR PROOF GOES HERE $\square$

**20.** If $a$ is an integer and $a^2 \mid a$, then $a \in \{-1, 0, 1\}$.

*Proof.* YOUR PROOF GOES HERE $\square$

**21.** If $p$ is prime and $k$ is an integer for which $0 < k < p$, then $p$ divides $\binom{p}{k}$.

**Quick Review:** For positive integers $n$ and $k$, the symbol $\displaystyle\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and it counts the number of $k$-element subsets from a set with $n$-elements. By implication, the output of $\displaystyle\binom{n}{k}$ must be an integer. You can delete this review from your solutions.

*Proof.* YOUR PROOF GOES HERE $\square$

**26.** Every odd integer is the difference of two squares. (Example: $7 = 4^2 - 3^2$.)

*Proof.* YOUR PROOF GOES HERE $\square$

**28.** Let $a, b, c \in \mathbb{Z}$. Suppose $a$ and $b$ are not both zero and $c \neq 0$. Prove that $c \cdot \gcd(a, b) \leq \gcd(ca, cb)$.

*Proof.* YOUR PROOF GOES HERE $\square$