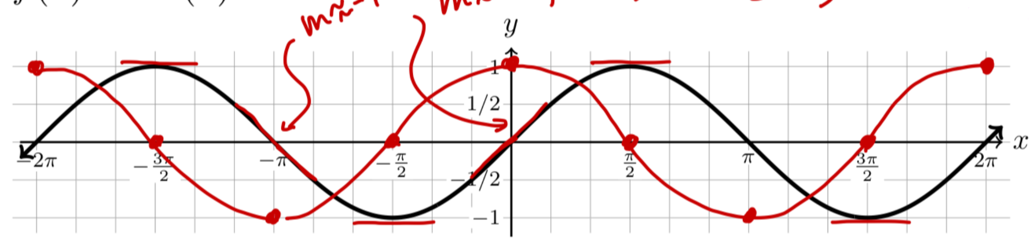
1. Review from Section 3.2.

   (a) State the definition of $f'(x)$ using the "h"-notation and use it to find $f'(x)$ for at least one of the functions in the list: $f_1(x) = C$, $f_2(x) = x$, $f_3(x) = x^2$, $f_4(x) = x^3$, $f_5(x) = x^{-1}$, $f_6(x) = x^{1/2}$.

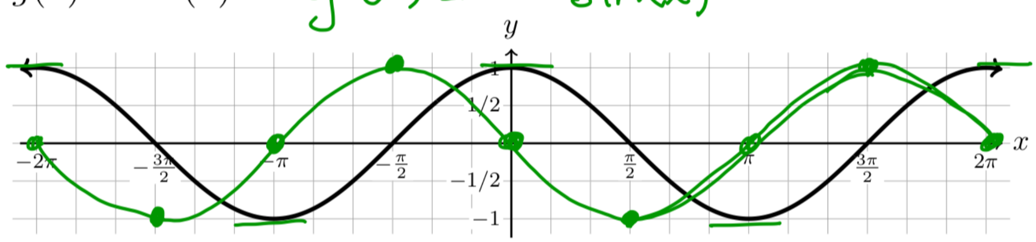
   See all answers on next sheet →

   (b) Use the graphs of $f(x) = \sin(x)$ and $g(x) = \cos(x)$ (below) to sketch the graph of their derivatives $f'(x)$ and $g'(x)$.

$f(x) = \sin(x)$ $\quad$ $f'(x) = \cos(x)$



$g(x) = \cos(x)$ $\quad$ $g'(x) = -\sin(x)$



   (c) Use the work from the previous problems to fill in the blanks below:

   i. $\frac{d}{dx}[C] = 0$

   ii. $\frac{d}{dx}[x^n] = n x^{n-1}$

   iii. $\frac{d}{dx}[\sin(x)] = \cos(x)$

   iv. $\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\boxed{f_1(x) = C}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{C-C}{h} = \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = 0; \text{ So } \boxed{f'(x) = 0}$$

$\boxed{f_2(x) = x}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)-(x)}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1; \text{ So } \boxed{f'(x) = 1}$$

$\boxed{f_3(x) = x^2}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h = 2x; \boxed{f'(x) = 2x}$$

$\boxed{f_4(x) = x^3}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3-x^3}{h} = \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h}$$

$$= \lim_{h\to 0} 3x^2+3xh+h^2 = 3x^2, \quad \boxed{f'(x) = 3x^2}$$

$\boxed{f_5(x) = x^{-1} = \frac{1}{x}}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right) = \lim_{h\to 0} \frac{1}{h}\left(\frac{x-(x+h)}{(x+h)(x)}\right) = \lim_{h\to 0} \frac{1}{h}\left(\frac{-h}{(x+h)(x)}\right)$$

$$= \lim_{h\to 0} \frac{-1}{(x+h)(x)} = \frac{-1}{(x+0)(x)} = \frac{-1}{x^2} = -x^{-2}; \quad \boxed{f'(x) = -x^{-2}}$$

$\boxed{f_6(x) = x^{\frac{1}{2}} = \sqrt{x}}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})}{h} \frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\boxed{f_6'(x) = \frac{1}{2} x^{-1/2}}$$

2. Use your intuition to evaluate the derivatives of the functions below and *ask yourself what assumptions you are making.*

   (a) $S(x) = x^5 + \sin(x)$

   $S'(x) = 5x^4 + \cos(x)$

   Take derivative of each term separately.

   (b) $M(x) = 20\cos(x)$

   $M'(x) = 20(-\sin(x))$

   $= -20\sin(x)$

   Take constant outside of derivative.

3. Summary Rules

   (a) Sum and Difference

   $$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$$

   (b) Constant Multiple

   $$\frac{d}{dx}\left[c f(x)\right] = c \cdot \frac{d}{dx}\left[f(x)\right]$$

   (c) Product

   $$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = \frac{d}{dx}\left[f(x)\right] \cdot g(x) + f(x) \cdot \frac{d}{dx}\left[g(x)\right]$$

   (d) Quotient

   $$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}\left[f(x)\right] - f(x) \cdot \frac{d}{dx}\left[g(x)\right]}{\left[g(x)\right]^2}$$

4. Find the derivatives of the functions below.

   (a) $K(\theta) = \underset{f}{\theta^{1/3}} \cdot \underset{g}{\sin(\theta)}$

   $$K'(\theta) = \frac{1}{3} \cdot \theta^{-\frac{2}{3}} \cdot \sin(\theta) + \theta^{\frac{1}{3}} \cdot \cos(\theta)$$

   rule $\rightarrow$ $f' \cdot g + f \cdot g'$

   (b) $j(x) = \dfrac{\overset{f}{\cos(x)+\sqrt{2}}}{\underset{g}{3x+1}}$

   rule $\rightarrow$ $g \cdot f' - f \cdot g'$

   $$j'(x) = \frac{(x+1)(-\sin(x)) - (\cos(x)+\sqrt{2})(3)}{(3x+1)^2}$$

   parantheses matter here!

   rule $\rightarrow$ g^2