§ 3.8 Exponential Growth & Decay

Ex y = # people world wide who have been infected w/ novel coronavirus (COVID-19)

listen to news: yesterday 1000 new infections

$$\left.\frac{dy}{dt}\right|_{t=60} = 1000$$ for t = days since 1 december

What is $\frac{dy}{dt}$?

t = days

$$\frac{dy}{dt} = 10 \qquad y = 10t$$

$$\frac{dy}{dt} = 8t \qquad y = 4t^2$$

$$\frac{dy}{dt} = 8y \qquad y = e^{8t}$$

$$y' = e^{8t} \cdot 8 = 8y$$

<u>In general</u>: if $\frac{dy}{dt} = ky$

then $y = Ce^{kt} = y(0) \cdot e^{kt}$

initial population

Ex Bacteria
- growth is proportional to size of pop.
- start w/ 20 cells
- 2 hours later 100 cells.

→ write expression for population as a fcn. of time

$$P = P_0 e^{kt}$$

$$P = 20e^{kt}$$

$$100 = 20e^{k \cdot 2}$$

$$5 = e^{2k}$$

$$\ln 5 = 2k$$

$$k = \frac{1}{2}\ln 5$$

$$\frac{1000}{20} = e^{\frac{\ln 5}{2}t}$$

$$\frac{2 \cdot \ln 50}{\ln 5} = t \approx 4.8614$$

<u>ANS</u>: $P(t) = 20e^{\frac{\ln 5}{2}t}$

- Find & interpret $P'(3)$ & $P(3)$
- When does pop reach 1000?

$P(3) = 223.6$ units bacteria

$$P'(t) = \frac{20 \cdot \ln 5}{2} e^{\frac{\ln 5}{2}t}$$

$$P'(3) = 10\ln 5 \, e^{\frac{3}{2}\ln 5} = 179.9 \text{ bacteria/hour}$$