§2.3   A few last principles
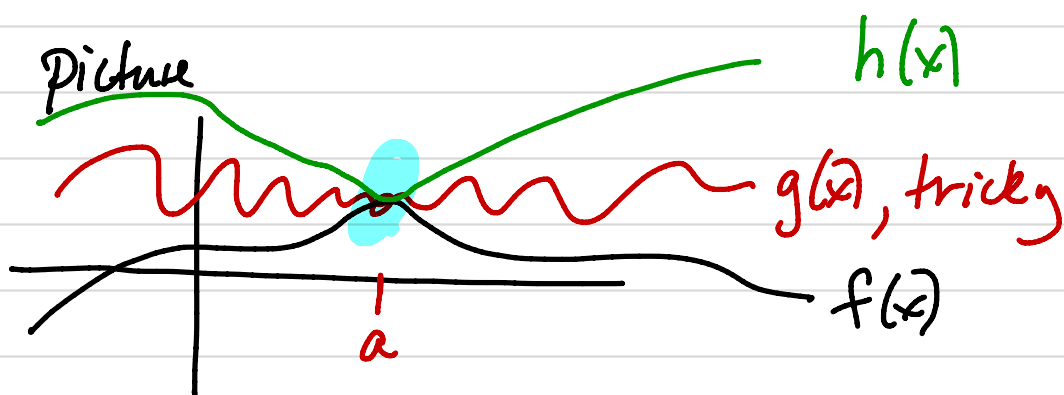
- If $f(x)$ is a poly or rational fcn, $\left(\text{aka: } \begin{array}{l} f(x) = 3x^5 - 12x + 20 \\ g(x) = \dfrac{x^3 - 3x^2}{\pi + \sqrt{2}x^{17}} \end{array}\right)$

  then $\lim\limits_{x \to a} f(x) = f(a)$ provided

  a in domain of $f(x)$

  (why?)

- What is the relationship between 2-sided & 1-sided limits?

  $\lim\limits_{x \to a} f(x) = L$ is equivalent $\lim\limits_{x \to a^+} f(x) = L = \lim\limits_{x \to a^-} f(x)$

   to

- Some $\overset{\text{challenging}}{\underset{\llcorner}{\text{limits}}}$ can be evaluated by Squeeze lemma

  Picture

  $h(x)$

  $g(x)$, tricky

  $f(x)$

  a

  If $f < g < h$ for all x-values close to a and $\lim\limits_{x \to a} f(x) = L = \lim\limits_{x \to a} h(x)$

  then $\lim\limits_{x \to a} g(x) = L$

- Illustration: $\lim\limits_{x \to 0} \left(\sqrt{x^2 + x}\right) \sin\left(\tfrac{1}{x}\right) = \boxed{}$

  $-\sqrt{x^2 + x} \leq \sqrt{x^2 + x} \sin\left(\tfrac{1}{x}\right) \leq \sqrt{x^2 + x}$