Homework 2 - Solutions

Section 1.3 Subsets: #2,3,4,7,9,11,13-16*

2. Here is the list of 8 subsets:
$$\emptyset, \{1\}, \{2\}, \{\emptyset\}, \{1,2\}, \{1,\emptyset\}, \{2,\emptyset\}, \{1,2,\emptyset\}$$

3. There are two subsets: $\emptyset, \{\ \{\mathbb{R}\}\ \}$

4. There is only one subset: $\emptyset$

7. There are four subsets: $\emptyset, \{\mathbb{R}\}, \{\ \{\mathbb{Q}, \mathbb{N}\}\ \}, \{\mathbb{R}, \{\mathbb{Q}, \mathbb{N}\}\}$

9. There are three subsets: $\{3,2\}, \{3,a\}, \{2,a\}$

11. There are no subsets with 4 elements, so there is nothing to write down.

13. True. Every set is a subset of itself.

14. False. The sets $\mathbb{R}^2$ and $\mathbb{R}^3$ have no elements in common since the first is a set of 2-tuples and the second is a set of 3-tuples.

15. True. If $x-1=0$, then we know $x^2-x=x(x-1)=0$. So any ordered pair in the set $\{(x,y) \in \mathbb{R}^2 : x-1=0\}$ must be in $\{(x,y) \in \mathbb{R}^2 : x^2-x=0\}$

16. False. The ordered pair $(0,0) \in \{(x,y) \in \mathbb{R}^2 : x^2-x=0\}$ but $(0,0) \notin \{(x,y) \in \mathbb{R}^2 : x-1=0\}$.

Section 1.4 Power Sets: # 2, 5, 6, 8, 14, 15, 16, 20

2. Since $2^4=16$, we know there will be 16 elements in $\mathscr{P}(\{1,2,3,4\})$. I will order them by the number of elements and lexicographically.

   ANS: $\mathscr{P} = \{\emptyset,$
   $\{1\}, \{2\}, \{3\}, \{4\},$
   $\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\},$
   $\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\},$
   $\{1,2,3,4\}\}$

5. First, I observe that $\mathscr{P}(\{2\}) = \{\emptyset, \{2\}\}$. Now, we see $\mathscr{P}(\mathscr{P}(\{2\})) = \{\emptyset, \{\emptyset\}, \{\{2\}\}, \{\emptyset, \{2\}\}$.

6. We know $\mathscr{P}(\{1,2\} = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ and $\mathscr{P}(\{3\}) = \{\emptyset, \{3\}\}$. Now,

   $\mathscr{P}(\{1,2\} \times \mathscr{P}(\{3\}) = \{$
   $(\emptyset,\emptyset), (\emptyset, \{3\})$
   $(\{1\},\emptyset), (\{1\}, \{3\})$
   $(\{2\},\emptyset), (\{2\}, \{3\})$
   $(\{1,2\},\emptyset), (\{1,2\}, \{3\})$
   $\}$

8. In this case, we first find $\{1,2\} \times \{3\} = \{(1,3),(2,3)\}$. Now,
   $\mathscr{P}(\{1,2\} \times \{3\}) = \{\emptyset, \{(1,3)\}, \{(2,3)\}, \{(1,3),(2,3)\}\}$

14. Since $|A| = m$, we know $|\mathscr{P}(A)| = 2^m$. Thus, $|\mathscr{P}(\mathscr{P}(A))| = 2^{(2^m)}$.

15. Since $|A \times B| = mn$, we know $|\mathscr{P}(A \times B)| = 2^{mn}$.

16. $|\mathscr{P}(A) \times \mathscr{P}(B)| = 2^m \cdot 2^n = 2^{(m+n)}$.

20. In words, we are being asked to count the number of subsets of A with at most 1 element. There are m subsets with exactly 1 element. Thus, including the empty set, we conclude: $|\{X \subseteq \mathscr{P}(A) : |X| \leq 1\}| = m+1$.

Section 1.5 Unions, Intersection, Difference: # 2, 4, 6, 9

2. For reference: $A = \{0,2,4,6,8\}$, $B = \{1,3,5,7\}$, $C = \{2,8,4\}$

   a. $A \cup B = \{0,1,2,3,4,5,6,7,8\}$
   b. $A \cap B = \emptyset$
   c. $A - B = A$
   d. $A - C = \{0,6\}$
   e. $B - A = B$
   f. $A \cap C = C$
   g. $B \cap C = \emptyset$
   h. $C - A = \emptyset$
   i. $C - B = C$

4. For reference: $A = \{b,c,d\}$ and $B = \{a,b\}$

   a. $(A \times B) \cap (B \times B) = \{b\} \times B$
   b. $(A \times B) \cup (B \times B) = \{a,b,c,d\} \times B$
   c. $(A \times B) - (B \times B) = \{c,d\} \times B$
   d. $(A \cap B) \times A = \{b\} \times A$
   e. $\emptyset$

f. $\mathscr{P}(A) \cap \mathscr{P}(B) = \{\emptyset, \{b\}\}$

g. $\mathscr{P}(A) - \mathscr{P}(B) = \{\{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\},\}$

h. $\mathscr{P}(A \cup B) = \{\emptyset, \{b\}\}$

i. The $\mathscr{P}(A) \times \mathscr{P}(B)$ will have $2^3 \cdot 2^2 = 32$ elements. So I will need to be efficient and systematic about how I write them down. I will list them according to their first element.

$\mathscr{P}(A) \times \mathscr{P}(B) = \{$
$(\emptyset,\emptyset), (\emptyset, \{a\}), (\emptyset, \{b\}), (\emptyset, \{a,b\}),$
$(\{b\},\emptyset), (\{b\}, \{a\}), (\{b\}, \{b\}), (\{b\}, \{a,b\}),$
$(\{c\},\emptyset), (\{c\}, \{a\}), (\{c\}, \{b\}), (\{c\}, \{a,b\}),$
$(\{d\},\emptyset), (\{d\}, \{a\}), (\{d\}, \{b\}), (\{d\}, \{a,b\}),$
$(\{b,c\},\emptyset), (\{b,c\}, \{a\}), (\{b,c\}, \{b\}), (\{b,c\}, \{a,b\}),$
$(\{b,d\},\emptyset), (\{b,d\}, \{a\}), (\{b,d\}, \{b\}), (\{b,d\}, \{a,b\}),$
$(\{c,d\},\emptyset), (\{c,d\}, \{a\}), (\{c,d\}, \{b\}), (\{c,d\}, \{a,b\}),$
$(\{b,c,d\},\emptyset), (\{b,c,d\}, \{a\}), (\{b,c,d\}, \{b\}), (\{b,c,d\}, \{a,b\})$
$\}$

6. FIX ME

9. **These are informal justifications, not a formal proofs.**

$(\mathbb{R} \times \mathbb{Z}) \cap (\mathbb{Z} \times \mathbb{R}) = \mathbb{Z} \times \mathbb{Z}$ is TRUE.

(Fact 1:) Any ordered pair in $\mathbb{R} \times \mathbb{Z}$ must have an integer second coordinate.
(Fact 2:) Similarly, any ordered pair in $\mathbb{Z} \times \mathbb{R}$ must have an integer first coordinate.

Thus, by the definition of intersection, any ordered pair in $(\mathbb{R} \times \mathbb{Z}) \cap (\mathbb{Z} \times \mathbb{R}) = \mathbb{Z} \times \mathbb{Z}$ must satisfy **both** facts.

$(\mathbb{R} \times \mathbb{Z}) \cup (\mathbb{Z} \times \mathbb{R}) = \mathbb{R} \times \mathbb{R}$ is FALSE.
The ordered pair (π, π) is in $\mathbb{R} \times \mathbb{R}$ but it is in neither $(\mathbb{R} \times \mathbb{Z})$ nor $(\mathbb{Z} \times \mathbb{R})$. Thus, $(\pi, \pi) \notin (\mathbb{R} \times \mathbb{Z}) \cup (\mathbb{Z} \times \mathbb{R})$.

Section 1.6 Complements # 1, 4, 6

2. For reference $A = \{1,3,4,6,7,9\}$, $B = \{4,5,6,8\}$, and $U = \{0,1,2,\ldots,10\}$.

a.

b.

c.

d.

e.

f.

g.

h.

i.

4.

6.