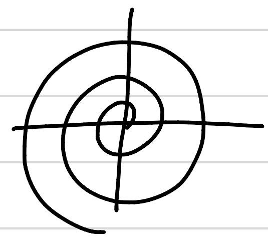
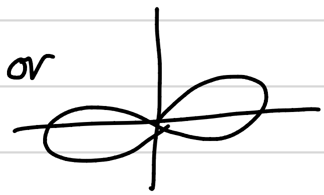
	Specific	general
Start:	$z = (x^2 + e^{x-1})^8$	$z = (g(x))^8$
	$z' = 8(x^2+e^{x-1})^7(2x+e^{x-1})$	$z' = 8(g(x))^8 \cdot g'(x)$

Observation: - $x^2+y^2=5$ is surprisingly challenging to get slope!

- Curves:



or



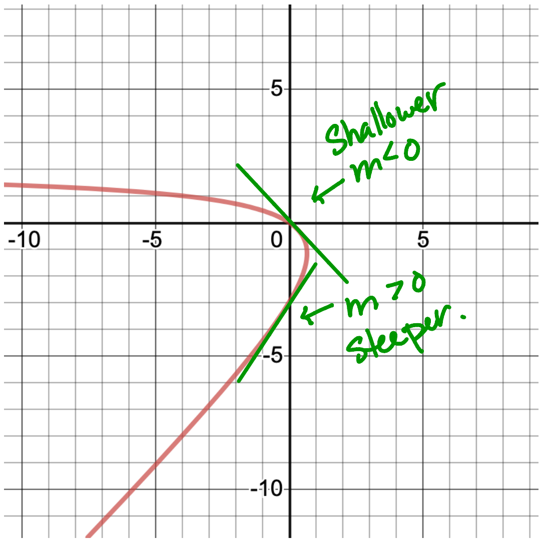
Solution: Implicit Differentiation $\rightarrow$ treat y like $g(x)$.

Ex1
$$x^2+y^2=5$$
$$2x+2y\left(\frac{dy}{dx}\right)=0$$
$$\frac{dy}{dx} = \frac{-x}{y} \leftarrow \text{plausible?}$$

Ex2
$$x^3 + \sin(y) = 5 + x + y$$
$$3x^2 + (\cos y)\left(\frac{dy}{dx}\right) = 0 + 1 + \frac{dy}{dx}$$
$$\frac{dy}{dx}(\cos y - 1) = 1 - 3x^2$$
$$\frac{dy}{dx} = \frac{1-3x^2}{\cos y - 1}$$

(Probs 1-3)

1. Find $\frac{dy}{dx}$ for $2x + 3y = xy - y^2$ and find the equations of tangents to the graph when $x = 0$. Use the graph below as an aid and to determine the plausibility of your answers.



$$2x + 3y = xy - y^2$$

$$2 + 3y' = 1 \cdot y + x \cdot y' - 2yy'$$

$$2 - y = (x - 2y - 3)y'$$

$$\boxed{y' = \frac{2-y}{x-2y-3}}$$

at $(0,0)$, $y' = -2/3$

at $(0,3)$, $y' = \frac{5}{3}$

lines:

$y = \frac{-2}{3}x$

$y = \frac{5}{3}x - 3$

2. Find $\frac{da}{db}$ for $a^3 \sin(3b) = a^2 - b^2$

$$\left(3a^2 \cdot \frac{da}{db}\right)\sin(3b) + a^3\cos(3b)(3) = 2a\frac{da}{db} - 2b$$

$$\left(3a^2\sin(3b) - 2a\right)\frac{da}{db} = -3a^3\cos(3b) - 2b$$

$$\boxed{\frac{da}{db} = \frac{-3a^3\cos(3b) - 2b}{3a^2\sin(3b) - 2a}}$$

3. Find $\frac{dy}{dx}$ for $e^{xy} = x + y + 1$

$$\left(e^{xy}\right)\frac{d}{dx}[xy] = 1 + \frac{dy}{dx} + 0$$

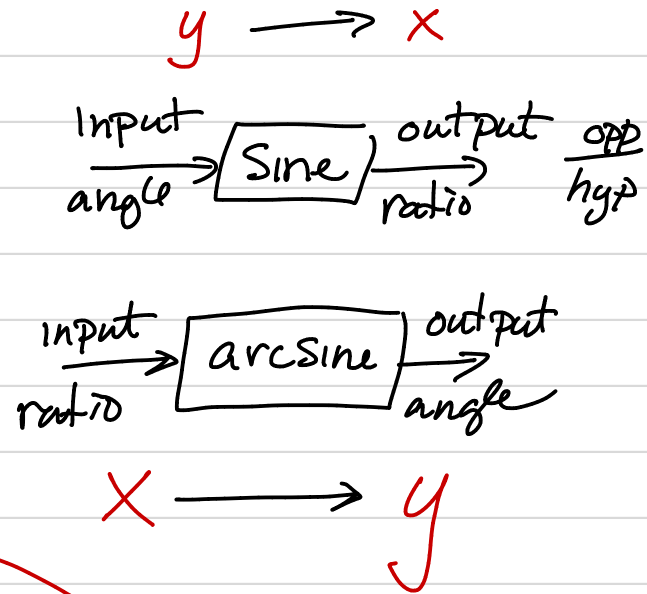
$$e^{xy}\left(1 \cdot y + x \cdot \frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

$$ye^{xy} + xe^{xy}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\left(xe^{xy} - 1\right)\frac{dy}{dx} = 1 - ye^{xy}$$

$$\boxed{\frac{dy}{dx} = \frac{1 - ye^{xy}}{xe^{xy} - 1}}$$

Implict Diff is a useful tool for DERIVING derivative formulas



Ex | $f(x) = \arcsin(x)$

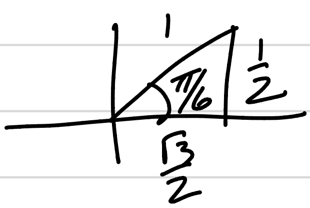
$y = \arcsin(x)$

the same as

$$\boxed{x = \sin(y)}$$

Plan a substitution.

x y



$\arcsin\left(\frac{1}{2}\right) = \pi/6$

Is $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$? yup!

Find $\frac{dy}{dx}$ implicitly.

$$1 = \cos(y) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\underbrace{\cos y}} = \frac{1}{\sqrt{1-x^2}}$$

We want our answer w/ x's, not y's!!

Use $\sin^2 y + \cos^2 y = 1$

$\cos y = +\sqrt{1-\sin^2 y} = \sqrt{1-x^2}$

4. You are going to derive the formula for the derivative of inverse tangent the way we found the derivative of inverse sine in class.

   (a) Find dy/dx for the expression $x = \tan(y)$.

   $$1 = \sec^2(y) \cdot \frac{dy}{dx}, \quad \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

   (b) Use the identity $1 + \tan^2(\theta) = \sec^2(\theta)$ to rewrite you answer in part (a) and *write your dy/dx in terms of x only*.

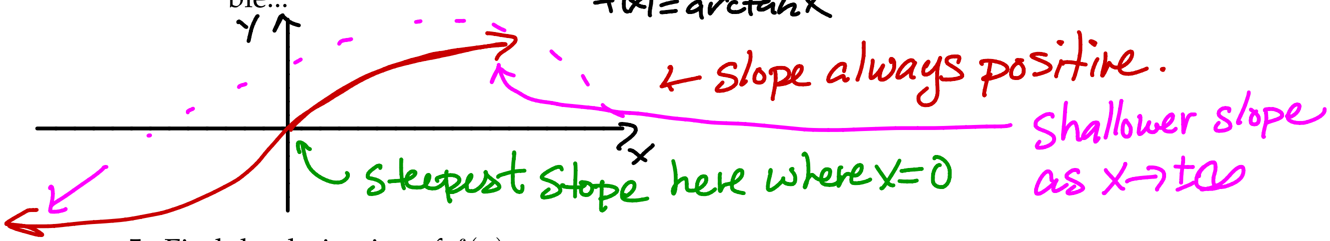
   $$\frac{dy}{dx} = \frac{1}{1+\tan^2(y)} = \frac{1}{1+x^2}$$

   because $x = \tan y$

   (c) Now fill in the blank $\dfrac{d}{dx}[\arctan(x)] = \dfrac{1}{1+x^2}$

   (d) Use your knowledge of the *graph* of $f(x) = \arctan(x)$ to decide if your answer seems plausible...

   $f(x) = \arctan x$



5. Find the derivative of $f(x) = x \arctan x$.

   $$f'(x) = \boxed{1 \cdot \arctan x + \frac{x}{1+x^2}}$$

6. Find the derivative of $f(x) = \arctan(4 - x^2)$.

   $$f'(x) = \frac{1}{1+(4-x^2)^2} \cdot \frac{d}{dx}(4-x^2) = \boxed{\frac{-2x}{1+(4-x^2)^2}}$$