This is a 1-hour sheet.

Recall that students will effectively be repeating all this algebra again in the definition of the derivative section... They will get more practice.

Goals:
- To understand that limits can pass through a lot of algebraic expressions (including composition) if nothing bad happens. This is used to understand derivative rules
- To recognize the bad stuff (ie $\frac{0}{0}$) and have tools for dealing with it.

In-class plan:
- Get students to give reasons for the example limit values. Get them to generalize. (Page 1)
- (Page 2) Write the algebra in words. Get them to start. Then do on board.

address pink questions as you go...

- Tell them they will need these techniques in Hmwk.

If time permits:
- $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$

0 in denom. but algebra won't help.

Use $-1 \le \sin\left(\frac{1}{x}\right) \le 1$. So $\underset{\downarrow\, 0}{-x^2} \le \underbrace{x^2\sin\left(\tfrac{1}{x}\right)}_{\text{so } \downarrow\, 0} \le \underset{\downarrow\, 0}{x^2}$ as $x \to 0$

(ie Squeeze)

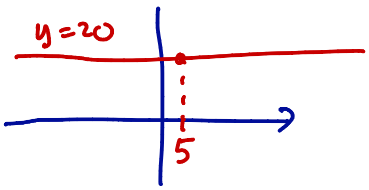
- If $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{C}{0}$ where $C \neq 0$, then you DO know roughly what is happening.

- Don't be afraid of

$$\lim_{x\to 10} \sqrt{x+1}\,\ln\left(x^2+7\right) = \sqrt{11}\,\ln(107)$$

1. Let's use some concrete examples to figure out some rules.

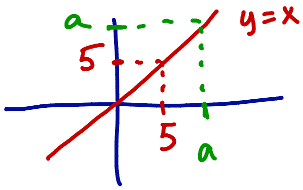
(a) $\lim_{x\to5} 20 =$ $20$




$\lim_{x\to a} c = c$ (c is a fixed constant)

The limit of a constant is constant.

(b) $\lim_{x\to5} x =$ $5$




$\lim_{x\to a} x = a$

The limit of $f(x)=x$ is evaluated by plugging in

(c) $\lim_{x\to5}(x+20) =$ $5+20=25$

$\lim_{x\to5} x \to 5$, $\lim_{x\to5} 20 \to 20$

$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

The limit of a sum is the sum of limits.

or The limit will pass through a sum.

(d) $\lim_{x\to\pi/2} x\sin(x) =$ $\frac{\pi}{2}\cdot\sin(\pi/2) = \frac{\pi}{2}\cdot 1 = \pi/2$

$x \to \frac{\pi}{2}$, $\sin(x) \to 1$

$\lim_{x\to a}[f(x)\cdot g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$

The limit will pass through a product.

(e) $\lim_{x\to\pi/2} 100(x\sin(x)) =$ $100\cdot\frac{\pi}{2} = 50\pi$

$100 \to 100$, $x\sin(x) \to \pi$

$\lim_{x\to a}[c\,f(x)] = c\cdot\left(\lim_{x\to a} f(x)\right)$

A (multiplied) constant can be moved outside the limit.

2. *ALL* rules are formally listed in Theorem 2.5 in your textbook. The nutshell version of these rules is

we can evaluate the limit of a function piece-by-piece

provided the resulting number makes sense.

what bad thing could happen?

Ex) $\lim_{x\to2}\frac{x^2-4}{x-2} = \frac{\lim_{x\to2} x^2-4}{\lim_{x\to2} x-2} = \frac{0}{0}$ ← uh oh...

Not a number.

If we get a zero in the denominator, try some algebra.

**What happens when the rules don't apply?**

3. lesson: factor and cancel

$$\lim_{t\to 2}\frac{t^2-4}{\underbrace{t-2}_{0/0 \text{ is bad.}}} = \lim_{t\to 2}\frac{(t+2)(t-2)}{t-2} = \lim_{t\to 2} t+2 = 2+2 = 4$$

Are you sure this is fair?

4. lesson: Get a common denominator.

$$\lim_{x\to 2}\frac{\frac{1}{4}-\frac{1}{2+x}}{\underbrace{x-2}_{0/0 \text{ is bad.}}} = \lim_{x\to 2}\left(\frac{1}{x-2}\right)\left(\frac{1}{4}-\frac{1}{2+x}\right) = \lim_{x\to 2}\left(\frac{1}{x-2}\right)\left(\frac{2+x-4}{4(2+x)}\right) = *$$

rearrange

get common denominator

$$* = \lim_{x\to 2}\left(\frac{1}{x-2}\right)\left(\frac{x-2}{4(2+x)}\right) = \lim_{x\to 2}\frac{1}{4(2+x)} = \frac{1}{4(2+2)} = \frac{1}{16}$$

How are the letters "a" and "h" different?

5. lesson: Rationalize.

$$\lim_{h\to 0}\frac{\sqrt{a}-\sqrt{a+h}}{\underbrace{h}_{0/0 \text{ is bad}}}\cdot\underbrace{\frac{\sqrt{a}+\sqrt{a+h}}{\sqrt{a}+\sqrt{a+h}}}_{\text{multiply by conjugate}} = \lim_{h\to 0}\frac{a-(a+h)}{h(\sqrt{a}+\sqrt{a+h})} = \lim_{h\to 0}\frac{-h}{h(\sqrt{a}+\sqrt{a+h})} = *$$

use $(c-d)(c+d) = c^2-d^2$

$$* = \lim_{h\to 0}\frac{-1}{\sqrt{a}+\sqrt{a+h}} = \frac{-1}{\sqrt{a}+\sqrt{a}} = \frac{-1}{2\sqrt{a}}$$