

Name: \_\_\_\_\_

\_\_\_\_\_/ 25

Circle one: Faudree (F01) | Bueler (F02) | VanSpronsen (UX1)

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [9 points] Evaluate each limit below. Your answer for each should be either a real number,  $+\infty$ ,  $-\infty$ , or DNE. Show your work to receive full credit.

a.  $\lim_{x \rightarrow -3} \frac{x^2 + 4x + 3}{x^2 + x - 6}$ .  ~~$= \frac{9-12+3}{9-3-6} = \frac{0}{0}$~~  ← factor & cancel!

$$= \lim_{x \rightarrow -3} \frac{(x+3)(x+1)}{(x+3)(x-2)} = \lim_{x \rightarrow -3} \frac{x+1}{x-2} = \frac{-3+1}{-3-2} = \frac{-2}{-5} = \frac{2}{5}$$

b.  $\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9x - x^2}$  =  ~~$\frac{0}{0}$~~  Do algebra!

$$\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9x - x^2} \cdot \frac{3 + \sqrt{x}}{3 + \sqrt{x}} = \lim_{x \rightarrow 9} \frac{9 - x}{x(9-x)(3+\sqrt{x})} = \lim_{x \rightarrow 9} \frac{1}{x(3+\sqrt{x})} = \frac{1}{9(3+3)} = \frac{1}{54}$$

c.  $\lim_{h \rightarrow 0^-} \frac{2h^2 + 10h}{|h|}$  =  $\lim_{h \rightarrow 0^-} \frac{2h(h+5)}{|h|} = \lim_{h \rightarrow 0^-} \frac{2h(h+5)}{-h} = \lim_{h \rightarrow 0^-} -2(h+5)$

because if  $h < 0$ ,  
then  $|h| = -h$ .

$$= -2(0+5) = -10$$

2. [4 points] Use the Intermediate Value Theorem to **show** that the equation  $e^x = 4 - 5x$  has a root in the interval  $(0, 1)$ .

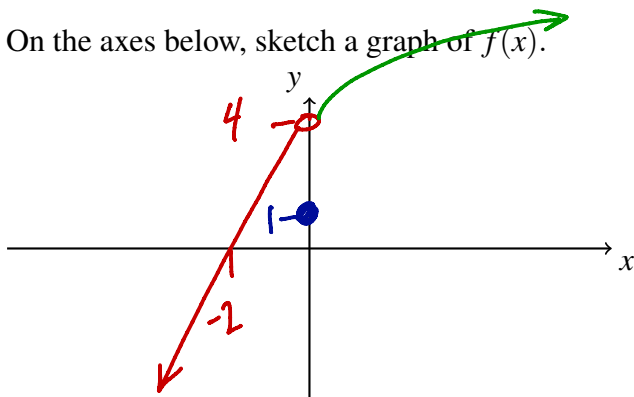
Let  $f(x) = e^x - 4 + 5x$ . Observe that  $f(x)$  is continuous.

Now  $f(0) = e^0 - 4 + 5 \cdot 0 = -3 < 0$  and  $f(1) = e^1 - 4 + 5 = e + 1 > 0$ .

Since  $f$  is negative at  $x=0$  and positive at  $x=1$ , it must be zero somewhere in between.

3. [8 points] Consider the function  $f(x) = \begin{cases} 2x+4 & x < 0 \\ 1 & x = 0 \\ \sqrt{x+16} & x > 0 \end{cases}$   $x=0: -2 \cdot 0 + 4 = 4$

a. On the axes below, sketch a graph of  $f(x)$ .



b. Evaluate the limit below or explain why the limit fails to exist.

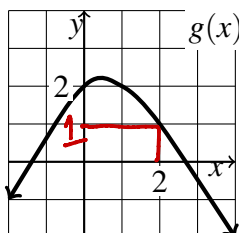
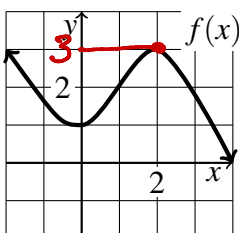
$$\lim_{x \rightarrow 0} f(x) = 4 \quad \text{because } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 2x+4 = 4 \text{ and}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \sqrt{x+16} = 4.$$

c. Is  $f$  continuous at  $x = 0$ ? Explain using the definition of continuity.

$$\text{No. } \lim_{x \rightarrow 0} f(x) = 4 \neq 1 = f(0)$$

4. [4 points] The graphs of  $f(x)$  and  $g(x)$  are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



$$\text{a. } \lim_{x \rightarrow 2} \left( \frac{5f(x)}{2+g(x)} \right) = \frac{5 \cdot 3}{2+1} = \frac{15}{2}$$

$$\text{b. } \lim_{x \rightarrow 2} (x^2 f(x)) = 2^2 \cdot 3 = 12$$