1. Find the derivative of each of the following. Use whatever rule you choose. Simplify if you have time.

(a) $G(x) = \frac{x^2}{8+x^2}$ quotient rule

$$G'(x) = \frac{(8+x^2)(2x) - x^2(2x)}{(8+x^2)^2} = \frac{2x(8+x^2-x^2)}{(8+x^2)^2} = \boxed{\frac{16x}{(8+x^2)^2}}$$

NOT quotient rule!

(b) $K(x) = \frac{8+x^2}{x^2} = \frac{8}{x^2} + \frac{x^2}{x^2} = 8x^{-2} + 1$

$$\boxed{K'(x) = -16x^{-3}}$$

(c) $v(\theta) = \sqrt{\theta}\cos(\theta) = \theta^{1/2}\cos\theta$ product rule

$$V'(\theta) = \frac{1}{2}\theta^{-1/2}\cos(\theta) + \theta^{1/2}(-\sin\theta) = \boxed{\frac{1}{2}\theta^{-1/2}\cos\theta - \theta^{1/2}\sin\theta}$$

NOT product rule

(d) $H(x) = \frac{1}{3x}(8+x^2) = \frac{1}{3}(8x^{-1} + x)$

$$H'(x) = \frac{1}{3}(-8x^{-2} + 1) = \boxed{\frac{1}{3}\left(1 - \frac{8}{x^2}\right)}$$

(e) $f(x) = 5e^2 + 4x^{3/4} + \underbrace{5x\sin(x)}_{\text{product rule here only}}$

$$f'(x) = 0 + 4\left(\frac{3}{4}\right)x^{-1/4} + 5\left(1 \cdot \sin(x) + x\cos(x)\right)$$

$$= \boxed{3x^{-1/4} + 5(\sin(x) + x\cos(x))}$$

2. Determine the point (or points) where the graph $f(x) = x^3$ has a slope of 2.

$f'(x) = 3x^2$

$m=2$

So $3x^2=2$ or

$x = \pm\sqrt{2/3}$

points : $\left(\sqrt{\frac{2}{3}}, \left(\sqrt{\frac{2}{3}}\right)^3\right) = \left(\left(\frac{2}{3}\right)^{1/2}, \left(\frac{2}{3}\right)^{3/2}\right)$

$\left(-\sqrt{\frac{2}{3}}, -\left(\sqrt{\frac{2}{3}}\right)^3\right) = \left(-\left(\frac{2}{3}\right)^{1/2}, -\left(\frac{2}{3}\right)^{3/2}\right)$

3. An ant walking along a sidewalk has traveled $s(t) = t^4 - 2t^2$ inches in t minutes. Find the acceleration of the ant (with units) when the velocity of the ant is 0.

$S'(t) = V(t) = 4t^3 - 4t$ ← units: inches/minute

$S''(t) = V'(t) = a(t) = 12t^2 - 4$ ← units inches/minute/minute = in/min²

$V = S' = 0$ when $4t^3 - 4t = 4t(t^2-1) = 4t(t-1)(t+1) = 0$ or $t = 0, 1, -1$.

$a(0) = -4$, $a(1) = 8$, $a(-1) = 8$ all in in/min²

4. The concentration of an antibiotic in the bloodstream t hours after being injected is given by $C(t) = \dfrac{2t^2+t}{t^3+50}$ where C is measured in milligrams per liter of blood.

(a) Find $C(0)$ and $C(10)$ and explain what these numbers mean in the context of the problem.

$C(0) = 0$

$C(10) = \frac{210}{1050} = 0.20$

Before the injection, the concentration in the blood is zero. Ten hours after the injection, the concentration in the blood is 0.2 mg/L.

(b) It is a fact that $C'(t) = \frac{-2(t^4+t^3-100t-25)}{(t^3+50)^2}$. What are the units of $C'(x)$?

(milligrams per liter) per hour or mg/L/hr

(c) It is a fact that $C'(10) = -0.018$. Interpret this fact in the context of the problem. Use language a Precalculus student could understand.

Ten hours after the injection, the concentration of antibiotic in the blood is decreasing at a rate of 0.018 mg/L each hour.

(d) Use the fact from parts (a) and (c) to make a guess about $C(11)$.

$C(11) \approx C(10) + C'(10) = 0.20 - 0.018 = 0.182$ mg/L