

Subsets, Power Set, Set Operations

1. Definitions

(a) The set B is a **subset** of the set A if every element in B is also an element in A . Written $B \subseteq A$

Ex: $\{1,3\} \subseteq \{1,2,3\}$, $\{1,4\} \not\subseteq \{1,2,3\}$; $\mathbb{N} \subseteq \mathbb{R}$

(b) The **power set** of a set A is the set of all subsets of A , Denoted $\mathcal{P}(A)$.

Ex] $A = \{1,2\}$, $\mathcal{P}(A) = \{\{1\}, \{2\}, \{1,2\}, \emptyset\}$

Not "1" (pointing to {1})
or {3} but NOT {∅} (pointing to ∅)

(c) The **union** of sets A and B is the set of elements in A or in B .

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Ex] $A = \{1,2\}$, $B = \{2,3\}$, $A \cup B = \{1,2,3\}$

(d) The **intersection** of sets A and B is the set of elements in both A and B .

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Ex] $A \cap B = \{2\}$, $A \cap A = \{1,2\} = A$, $A \cap \{5,6\} = \emptyset$

(e) The **difference** of sets A and B , say $A - B$, is the set of elements in A and not in B .

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

Ex] $A - B = \{1,2\} - \{2,3\} = \{1\}$

(f) The **complement** of the set A is denoted \bar{A} (elsewhere A^c) and requires a universal set U . $\bar{A} = \{x \in U : x \notin A\}$.

Ex] Say $U = \{1,2,3,4\}$, $\bar{A} = \{3,4\}$

2. Let $C = \{1, 2, 3\}$

(a) List all the elements of the set C .

1, 2, 3

(b) List all the subsets of the set C .

$$\mathcal{P}(C) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

(c) Find $\mathcal{P}(C)$. (You have permission to answer this by small changes to your answer in (b).)

(d) Find $\mathcal{P}(\emptyset)$. $= \{\emptyset\}$

3. Determine whether each statement below is true or false and be prepared to give a reason for your answer.

T (a) $a \in \{a, b, c, d, \{a\}, \{a, e\}\}$

F (b) $a \subseteq \{a, b, c, d, \{a\}, \{a, e\}\}$ *Not a set. Nonsense.*

T (c) $\{a\} \in \{a, b, c, d, \{a\}, \{a, e\}\}$

T (d) $\{a\} \subseteq \{a, b, c, d, \{a\}, \{a, e\}\}$

T (e) $\emptyset \subseteq \{a, b, c, d, \{a\}, \{a, e\}\}$ *always*

F (f) $\emptyset \in \{a, b, c, d, \{a\}, \{a, e\}\}$ *No " \emptyset " in here.*

F (g) $\{a, e\} \subseteq \{a, b, c, d, \{a\}, \{a, e\}\}$ *← it's an element*

T (h) $\{\{a, e\}\} \subseteq \{a, b, c, d, \{a\}, \{a, e\}\}$

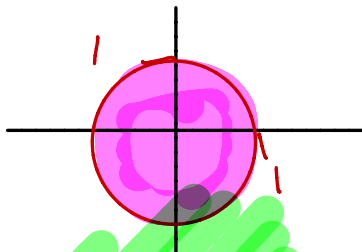
F (i) $e \in \{a, b, c, d, \{a\}, \{a, e\}\}$ *← No "e" by itself*

F (j) $\{e\} \subseteq \{a, b, c, d, \{a\}, \{a, e\}\}$ *←*

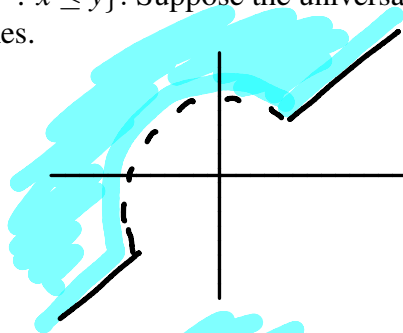
$$\{\{a\}\} \subseteq \{a, b, c, d, \{a\}, \{a, e\}\} \quad \mathbf{T}$$

4. Let $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and let $B = \{(x, y) \in \mathbb{R}^2 : x \leq y\}$. Suppose the universal set U is $\mathbb{R} \times \mathbb{R}$. Sketch each of the following on separate axes.

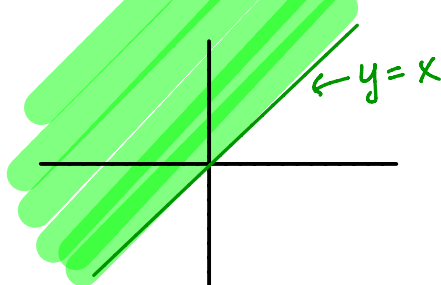
(a) A



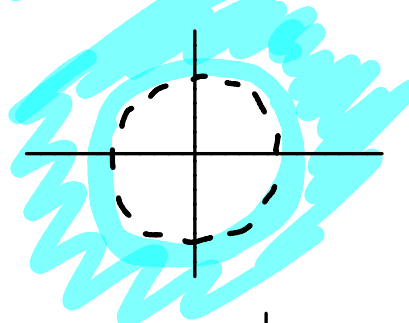
(f) $B - A$



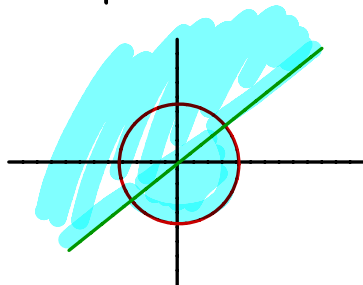
(b) B



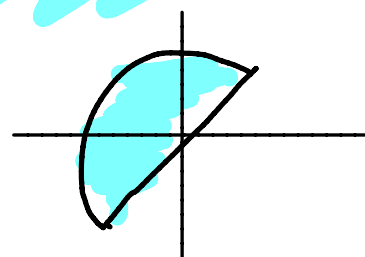
(g) \bar{A}



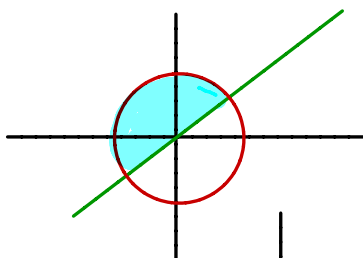
(c) $A \cup B$



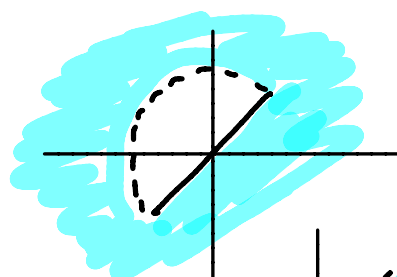
(h) $A - \bar{B}$



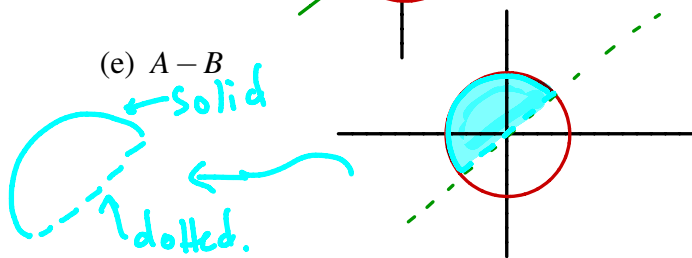
(d) $A \cap B$



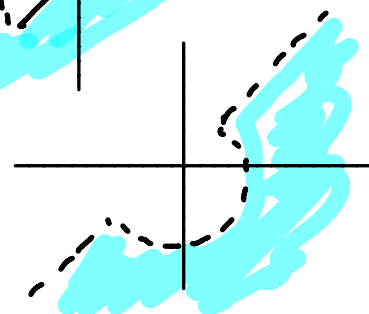
(i) $\overline{A - B}$



(e) $A - B$



(j) $\overline{A \cup B}$



5. How many subsets does $D = \{1, 2, 3, 4, 5\}$ have? Justify your answer.

$2^5 = 32$. For each element, there are 2 choices: in or out.

6. Determine $|\mathcal{P}(A)|$ where A is a set with n elements.

$$|\mathcal{P}(A)| = 2^n$$