## Notes from Fri 9 Sept.

## I. Recap of One. I. 3 thus far

- The number of solutions in the set of all solutions of a system of linear equations will be
  - none (zero)
  - exactly 1
  - an Do number

No way to get exactly 5 solns.

· Every Bystem of linear equations will have a solution set with form

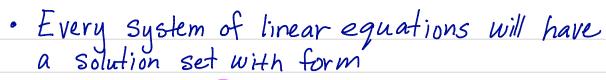
{ \vec{p} + \vec{h} \cdot \choose \square \squ

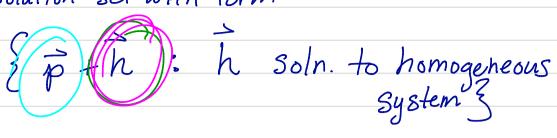
 $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0$   $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0$ 

ak, x1 + akz x2+ ... + akn xn = 0

b rector is all zeros. b column

$$\frac{\mathsf{b}}{\mathsf{b}} = \begin{pmatrix} \mathsf{o} \\ \mathsf{o} \\ \mathsf{o} \end{pmatrix}$$





Example of this Thm in action.

(i) Observe that 
$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -1/5 \\ 0 \\ 0 \end{pmatrix}$$
 is a Garticular Solution

(i) Solve homogeneous system 
$$x + 2y - z = 0$$
  
 $2x - y - 2z + w = 0$ 

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 0 \\ 2 & -1 & -2 & 1 & 0 \end{bmatrix} \xrightarrow{-2e_1 + e_2 + 9e_2} \begin{bmatrix} 1 & 2 & -1 & 0 & 0 \\ 0 & -5 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{rcl}
x + 2y - \overline{z} &= 0 \\
-5y & + w = 0
\end{array}$$

So 
$$y = \frac{w}{5}$$
  
 $x = -2y + 2 = -\frac{2}{5}w + 2$ 

$$= \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -2/5 \\ 1/5 \\ 0 \\ 1 \end{pmatrix}$$

Solution to the Original SOLE.

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## The last of One.I. 2 definition: A square matrix is nonsingular If it is the coefficient matrix of a homogeneous system with a unique solution. Otherwise, the matrix is <u>Singular</u>. Examples Solt matrix form coefficient matrix A x + y = 7 x + 2y = 4 I 2:4 I 1 I 2

$$\begin{bmatrix} A & x+y=7 \\ x+2y=4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 2 & 4 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -1 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -1 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -1 & -2 & 1 \end{bmatrix}$$

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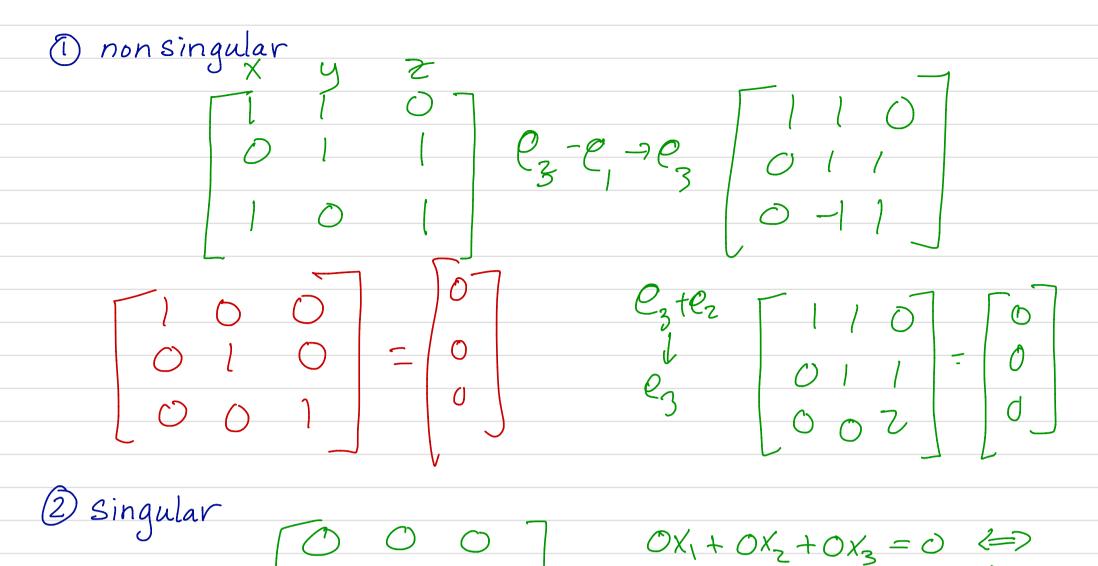
$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -1$$

Does X+9=0 hove I soln or AD # Solns? Yes

row ops 
$$x+y=0$$
  $\longrightarrow x=y=0$   $V=0$ 

geometrically, lins. So intersect in 1 point.

## Let's make 3×3 matrices that are



0=0