Name: Solutions ______ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (4 points)

   (a) Are any two planes through the origin in $\mathbb{R}^3$ isomorphic? Justify your answer.

   +3 Yes. Both have dimension 2, so they must be isomorphic

   (b) Are any two planes **not** necessarily through the origin in $\mathbb{R}^3$ isomorphic? Justify your answer.

   +1 The word "isomorphic" only applies to $\boxed{\text{vector spaces}}$. If the plane doesn't go through the origin, the objects do not form a vector space.

2. (6 points) Determine whether the map $f : \mathscr{P}_2 \to \mathbb{R}^2$ given by $ax^2 + bx + c \mapsto \begin{pmatrix} a+b \\ a-c \end{pmatrix}$ is a homomorphism (or linear map).

+1 check that f respects vector operations.

+1 Let $r_1, r_2 \in \mathbb{R}$ and $ax^2+bx+c, a'x^2+b'x+c' \in \mathcal{P}_2$.

$$f(r_1(ax^2+bx+c)+r_2(a'x^2+b'x+c'))$$

+2

$$= f((r_1a+r_2a')x^2+(r_1b+r_2b')x+r_1c+r_2c')$$

$$= \begin{pmatrix} r_1a+r_2a'+r_1b+r_2b' \\ r_1a+r_2a'-r_1c-r_1c' \end{pmatrix} = r_1\begin{pmatrix} a+b \\ a-c \end{pmatrix} + r_2\begin{pmatrix} a'+b' \\ a'-c' \end{pmatrix}$$

+2

$$= r_1 f(ax^2+bx+c) + r_2 f(a'x^2+b'x+c').$$