

Notes

① Review def $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

② Split students into groups + get work for $f_1 \rightarrow f_6$ on the board. // Also cos/sin
(Assure students that work is posted.)

③ Return to seats.

Discuss remaining rules as a group

Big Goal : Rules are deduced, NOT just made up.

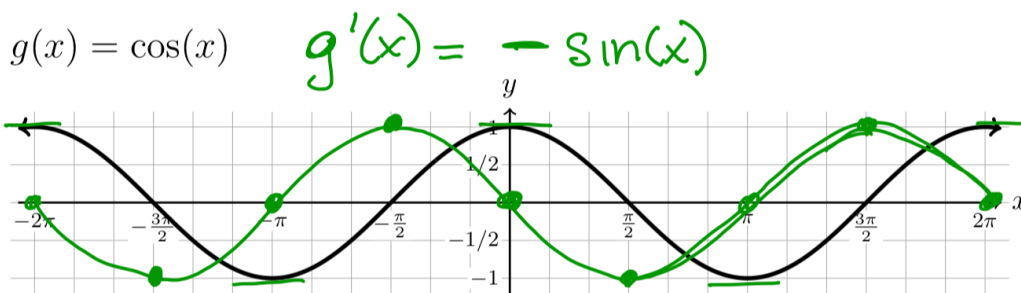
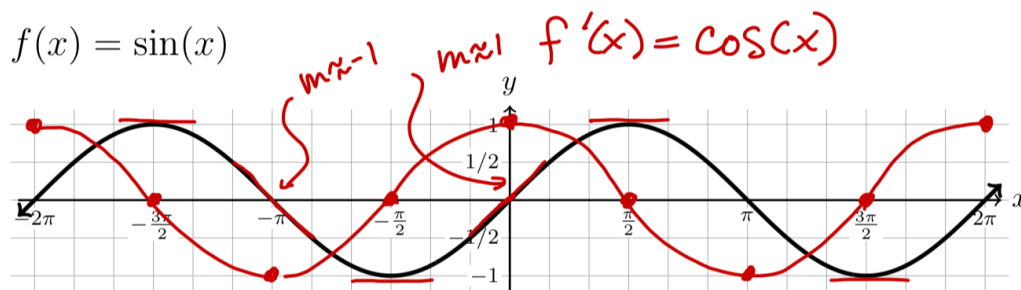
SECTION 3-3: DERIVATIVE RULES

1. Review from Section 3.2.

- (a) State the definition of $f'(x)$ using the "h"-notation and use it to find $f'(x)$ for at least one of the functions in the list: $f_1(x) = C$, $f_2(x) = x$, $f_3(x) = x^2$, $f_4(x) = x^3$, $f_5(x) = x^{-1}$, $f_6(x) = x^{1/2}$.

See all answers on
next sheet →

- (b) Use the graphs of $f(x) = \sin(x)$ and $g(x) = \cos(x)$ (below) to sketch the graph of their derivatives $f'(x)$ and $g'(x)$.



- (c) Use the work from the previous problems to fill in the blanks below:

i. $\frac{d}{dx}[C] = 0$
 ii. $\frac{d}{dx}[x^n] = nx^{n-1}$

iii. $\frac{d}{dx}[\sin(x)] = \cos(x)$
 iv. $\frac{d}{dx}[\cos(x)] = -\sin(x)$

$$f_1(x) = C$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{C - C}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = \lim_{h \rightarrow 0} 0 = 0; \text{ So } f'(x) = 0$$

$$f_2(x) = x$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h) - x}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = \lim_{h \rightarrow 0} 1 = 1; \text{ So } f'(x) = 1$$

$$f_3(x) = x^2$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} 2x + h = 2x; f'(x) = 2x$$

$$f_4(x) = x^3$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 = 3x^2, \quad f'(x) = 3x^2$$

$$f_5(x) = x^{-1} = \frac{1}{x}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right) = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{x - (x+h)}{(x+h)(x)} \right) = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{-h}{(x+h)(x)} \right)$$

$$= \lim_{h \rightarrow 0} \frac{-1}{(x+h)(x)} = \frac{-1}{(x+0)(x)} = -\frac{1}{x^2} = -x^{-2}; \quad f'(x) = -x^{-2}$$

$$f_6(x) = x^{\frac{1}{2}} = \sqrt{x}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f'_6(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

2. Use your intuition to evaluate the derivatives of the functions below and ask yourself what assumptions you are making.

(a) $S(x) = x^5 + \sin(x)$

$$S'(x) = 5x^4 + \cos(x)$$

Take derivative of each term separately.

(b) $M(x) = 20 \cos(x)$

$$M'(x) = 20(-\sin(x)) = -20 \sin(x)$$

Take constant outside of derivative.

3. Summary Rules

(a) Sum and Difference

$$\frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} [f(x)] \pm \frac{d}{dx} [g(x)]$$

(b) Constant Multiple

$$\frac{d}{dx} [cf(x)] = c \cdot \frac{d}{dx} [f(x)]$$

(c) Product

$$\begin{aligned} \frac{d}{dx} [f(x) \cdot g(x)] \\ = \frac{d}{dx} [f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx} [g(x)] \end{aligned}$$

(d) Quotient

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot \frac{d}{dx} [f(x)] - f(x) \cdot \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

4. Find the derivatives of the functions below.

(a) $K(\theta) = \theta^{1/3} \sin(\theta)$

(b) $j(x) = \frac{\cos(x) + \sqrt{2}}{3x+1}$

$$K'(\theta) = \frac{1}{3} \cdot \theta^{-\frac{2}{3}} \cdot \sin(\theta) + \theta^{\frac{1}{3}} \cdot \cos(\theta)$$

rule $\rightarrow f' \cdot g + f \cdot g'$

rule $\rightarrow g \cdot f' - f \cdot g'$

$$j'(x) = \frac{(x+1)(-\sin(x)) - (\cos(x) + \sqrt{2})(3)}{(3x+1)^2}$$

rule $\rightarrow g^2$

parentheses matter here!