Notes

Start

We are un-doing differentiation. Called $\boxed{\text{antiderivative}}$

$\boxed{\text{Ex}}$ If $F(x) = x^{1/2}$, then $F'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$.

$\boxed{\text{Ex}}$ Find an antiderivative of $f(x) = \frac{1}{2\sqrt{x}}$.

ANS: $F(x) = x^{1/2}$.

We say $F(x)$ is $\boxed{\text{an}}$ antiderivative of $f(x)$ if

$$F'(x) = f(x)$$

$\boxed{\text{Why?}}$

- algebraic
- graphical

$\boxed{\text{Ex}}$ Find the family of antiderivatives of $f(x) = \frac{1}{2\sqrt{x}}$.

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Use notation: to articulate the same idea:

"The family of antiderivatives of $f(x)$ is $F(x)$" $\equiv$ $\int \underbrace{f(x)}_{\text{integrand}} \overset{\text{huh?}}{dx} = F(x) + C$

integral elongated S

If time permits: Subtlety of

$$F(x) = \begin{cases} x^{-2} + 1 & \text{for } x > 0 \\ x^{-2} - 4 & \text{for } x < 0 \end{cases}$$

- (families of) antiderivatives
- definite integrals
- initial value problems

1. Find the (family of) antiderivatives for the following.

(a) $f(x) = x^3$

$F(x) = \frac{1}{4}x^4 + C$

Check:

$F'(x) = \frac{1}{4} \cdot 4x^3 + 0$

$= x^3$ ✓

(b) $f(x) = 5\sin(x)$

$F(x) = -5\cos(x) + C$

$F' = -5(-\sin(x))$

$= 5\sin(x)$ ✓

(c) $f(x) = \frac{e^x}{4}$

$F(x) = \frac{1}{4}e^x + C$

$F'(x) = \frac{1}{4}e^x$ ✓

(d) $f(x) = \sqrt{2}$

$F(x) = \sqrt{2}x$

(e) $f(x) = \frac{1}{x}$

$F(x) = \ln|x|$

* Where do absolute value bars come from?

(f) $f(x) = 1 - x + e^x$

$F(x) = x - \frac{1}{2}x^2 + e^x$

* What principle is being used here?

2. Confirm that $F(x) = \sin^2(2x)$ is an antiderivative of $f(x) = 4\sin(2x)\cos(2x)$.

$F'(x) = 2(\sin(2x))^1(\cos(2x)) \cdot 2 = 4\sin(2x)\cos(2x)$ ✓

antideriv of $f(x) = x^{-3}$?

why?

Function	Antiderivative
* x^k $(k \neq -1)$	$x^{k+1}/(k+1)$
x^{-1}	$\ln\|x\|$
1	x
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$

Function	Antiderivative
e^x	e^x
$1/(1+x^2)$	$\tan^{-1}(x)$
$\sec^2(x)$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$
$1/\sqrt{1-x^2}$	$\sin^{-1}(x)$

3. Evaluate the integrals.

(a) $\int (e^{-x} + \sec^2(x))\,dx$ $= -e^{-x} + \tan(x) + C$

(b) $\int \frac{x^2 + x^{1/2} + 1}{x^{1/2}}\,dx$ $= \int (x^{3/2} + 1 + x^{-1/2})\,dx = \frac{2}{5}x^{5/2} + x + 2x^{1/2} + C$

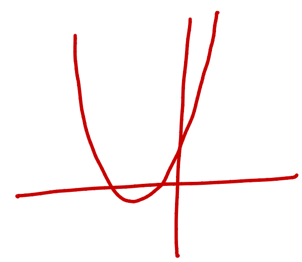
(c) $\int (\frac{1}{4}x^4 + \sec(x/\pi)\tan(x/\pi))\,dx$ $= \frac{1}{20}x^5 + \pi \sec(\frac{x}{\pi}) + C$

What is this?

4. Solve the initial value problem if $f'(x) = x + e^x$ and $f(0) = 4$.

$f(x) = \int (x + e^x)\,dx = \frac{1}{2}x^2 + e^x + C$

$f(0) = \frac{1}{2}(0^2) + e^0 + C = 1 + C = 4.$ So $C = 4.$



$\boxed{f(x) = \frac{1}{2}x^2 + e^x + 4}$

5. A particle moving along the x-axis has acceleration $a(t) = 10\sin(t)$ measured in cm/s^2. Assume the particle as initial velocity $v(0) = 0$ and initial position $s(0) = 0$, find a function that models its velocity, $v(t)$, and its position $s(t)$.

$a(t) = 10\sin(t)$

$v(t) = \int a(t)\,dt = \int 10\sin(t)\,dt = -10\cos(t) + C$

$v(0) = -10\cos(0) + C = -10 + C.$ So $C = 10.$

$\boxed{v(t) = -10\cos(t) + 10}$

$s(t) = \int v(t)\,dt = \int (-10\cos(t) + 10)\,dt = -10\sin(t) + 10t + C$

$s(0) = -10\sin(0) + 0 + C = 0.$ So $C = 0$

$\boxed{s(t) = -10\sin(t) + 10}$