SECTION 5.7 EXTRA: INTEGRATION PALOOZA

Evaluate the following integrals. A correct answer requires organized, correct supporting work. Once all are completed, describe in words the strategy used or the type of integration problem it represents.

1. $\displaystyle\int_1^8 \frac{x^{1/3} - (\pi x)^2}{x}\,dx = \int_1^8 \left(x^{-2/3} - \pi^2 x\right) dx = 3x^{\frac{1}{3}} - \frac{\pi^2}{2}x^2 \Big]_1^8$

$= \left(3\cdot 8^{\frac{1}{3}} - \frac{\pi^2}{2}8^2\right) - \left(3(1)^{1/3} - \frac{\pi^2}{2}1^2\right) = \left(6 - \pi^2\cdot\frac{64}{2}\right) - \left(3 - \frac{\pi^2}{2}\right)$

$= 3 - \frac{63}{2}\pi^2$

type/strategy: simplify before integrating!

2. $\displaystyle\int \frac{5x}{\sqrt{3-6x^2}}\,dx = -\frac{5}{12}\int u^{-1/2}\,du = -\frac{5}{12}\cdot\frac{2}{1}\cdot u^{\frac{1}{2}} + C = -\frac{5}{6}\sqrt{3-6x^2} + C$

let $u = 3 - 6x^2$

$du = -12x\,dx$

$-\frac{1}{12}du = x\,dx$

type/strategy: Pick u inside the $\sqrt{\ }$. Get exponent correct!

3. $\displaystyle\int \frac{5}{\sqrt{1-4x^2}}\,dx = 5\int \frac{dx}{\sqrt{1-(2x)^2}} = \frac{5}{2}\int\frac{du}{\sqrt{1-u^2}} = \frac{5}{2}\arcsin(u) + C$

$= \frac{5}{2}\arcsin(2x) + C$

Let $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

Strategy/type: arcsine

4. $\int \left(8x + \frac{1}{e^{2x-9}}\right) dx = \int \left(8x + e^{9-2x}\right) dx = \int 8x\,dx + \int e^{9-2x}\,dx$

$= 4x^2 - \frac{1}{2} e^{9-2x} + C$

Strategy/type: • Get e^u in numerator
• Treat different terms differently

5. $\int x^3(1+x^2)^{0.1}\,dx = \int x^2 (1+x^2)^{0.1}\, x\,dx = \frac{1}{2}\int (u-1)\,u^{0.1}\,du = \frac{1}{2}\int \left(u^{1.1} - u^{0.1}\right) du$

let $u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$u-1 = x^2$

$= \frac{1}{2}\cdot\frac{1}{2.1} u^{2.1} - \frac{1}{1.1} u^{1.1} + C$

$= \frac{1}{4.2}(1+x^2)^{2.1} - \frac{1}{1.1}(1+x^2)^{1.1} + C$

strategy/type: Sophisticated u-sub

6. $\int \left(\frac{1}{2x} + \sec(3x)\tan(3x)\right) dx = \int \frac{1}{2} x^{-1}\,dx + \int \sec(3x)\tan(3x)\,dx$

$= \frac{1}{2}\ln|x| + \frac{1}{3}\sec(3x) + C$

Strategy/type: - ln
- treat different parts differently
- guess-n-check.