Your Name	e (prir	nt clea	rly)			

Wednesday, December 15, 2015

Problem	Total Points	Score		
1	20			
2	10			
3	10			
4	15			
5	10			
6	15			
7	10			
8	10			
Total	100			

Instructions and information:

- Please turn off cell phones or any other thing that will go BEEP.
- $\bullet\,$ You are allowed to use your textbook
- Read the directions for each problem.

- 1. (5 points each) Find the number of ways to distribute all 30 books to 10 libraries in each situation.
 - (a) Assume the books are distinct and the libraries are distinct and there are no restrictions on where the books go.

(b) Assume 5 of the 30 books are identical and that these 5 books should go to different libraries, but there are no other restrictions.

BDistributi remains books: 10

AM. (10) 1025

(c) Assume all 30 books are identical and every library should get at least one book.

Distrubite 10 bodes to 10 libraries: Distribute remains 36 books to 10 library: 5

NSwer: (29) or (10)

= (29) 20)

ANSWer: (29) or (20)

(d) Assume all 30 books are distinct and each library will receive exactly three books.

 $= {\binom{30}{3}} {\binom{27}{3}} {\binom{24}{3}} ... {\binom{6}{3}} {\binom{3}{3}}$

ANSWERZ Dermut books: 30!, divide by overcout (3!) answer 30!

(3!)10

2. (10 points) How many numbers must be chosen from the set {1, 2, 3, 4, 5, 6, 7, 8} to guarantee that at least one pair of these numbers adds up to 9? You must prove your answer is correct. (Hint: Use the Pigeon Hole Principle)

ANS: 5 numbers.

Just FYI clear 4 is too small summer the 8t {1,2,3,43 doesn't have this property

Proof: Poortition the set into following blocks: 2183, 22, \$3,63 and 24,53.

Observe that the pair in each block sums to 9. If 5 numbers are chosen, by PHP, two must come from same block.

3. (10 points) Give a combinatorial proof that $\sum_{i=1}^{n} i \binom{n}{i} = n2^{n-1}$ for all positive integers, n.

Let S be the of all ordered pairs (A, X) where A = [n] and XEA. (In language of committees, let S be the set of all nonempty committees w/ a clesignated chair. I from a set of n people)

Count 1: Pick Committee clair: n ways. Then fill commuttee in 2n-1 ways.

Count 2: Partition committees according to the number of members. For $i \in [n]$, $\binom{n}{i}$ counts the number of committees $w \mid i$ members and then there are i ways to choose the chair Thus, $n \cdot 2^{n-1} = \sum_{i=1}^{\infty} i \binom{n}{i}$

4. (10 points) Let \mathcal{C} be a q-ary code with codewords of length n. (So \mathcal{C} is a subset of all n-length words using the alphabet $X = \{0, 1, 2, \dots, q-1\}$.) As with the Hamming distance on binary words, assume that the distance between two q-ary words is the number of positions in which the two words differ. Prove that if code \mathcal{C} can correct up to e errors, than

$$|\mathcal{C}| \leq \frac{q^n}{\sum_{i=0}^{e} \binom{n}{i} (q-1)^i}$$

- 5. (15 points) Theorem 7.2.4 (page 288) says that if \mathcal{D} is a symmetric BIBD with parameters (v, k, λ) , then \mathcal{D}' , the derived design obtained from \mathcal{D} , has parameters $(v-1, k, k-1, \lambda, \lambda-1)$.
 - (a) Explain directly (not using the other parameters of \mathcal{D}') why the derived design has v-1 blocks.

To make D', one block is deleted; 80 one is lost. More over, the other blocks must have A elements in common withe the deleted block. Thus, the other V-I blocks remain.

(b) Explain directly (not using the other parameters of \mathcal{D}') why the derived design is regular.

Because de in Symmetrie, it is 2-linked.

Thus the intersect V

Every variety in D appears K times. Since Every variety in D appears K times. Since exactly one block is deleted, the remaining exactly one block is deleted, the remaining varieties appear in one fewer block, namely K-1.

- 6. (15 points) Define a partially ordered set P(X,R) where $X = \{1,2,3,4,5,6,7,8\}$ and for all $a,b \in X, (a,b) \in R$ if and only if $a \mid b$.
 - (a) Draw the Hasse Diagram for P.

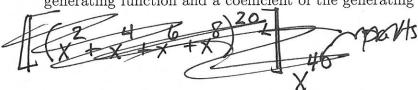
- (b) Find all maximal elements of P.
- (c) Find the meet of 4 and 6 (i.e. $4 \wedge 6$) and show that 4 and 6 have a lower bound distinct from $4 \wedge 6$.
- (d) Find a maximal chain in P that is not a maximum chain.

- 7. (15 points)
 - (a) Give ONE example of a partition of 40 into 3 parts.

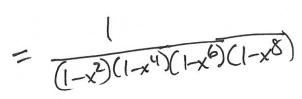
(b) Give ONE example of a partition of 40 into 3 parts each of which is an even number (i.e. a partition with even parts)

(c) Give TWO examples of a partition of 40 into even parts each of which is at most 8.

(d) Express the number of partitions of 40 into even parts each of which is at most 8 as a suitable coefficient of a certain generating function. (That is, you must specify a generating function and a coefficient of the generating function.)



 $(1+x^2+x^4+...)(1+x^4+x^8+...)(1+x^6+x^{12}+...)(1+x^8+x^4-.)$



ANS. (-x)(1-x8)(1-x8)

8. (10 points) Show that if $\delta(G) \geq k$ then G must contain a cycle on at least k+1 vertices.

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