Name: _____ _____ / 10

There are 10 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (6 points) Put the system of equations into row echelon form. You may convert to matrix form if you like, but your answer must be in the form of a system of equations. You do not need to find a solution to the system. You must show your elementary row operations.

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 1 \\ 2x_1 + 6x_2 - 2x_3 - 2x_4 = 1 \\ 3x_3 + x_4 = 3 \\ -x_1 + 2x_2 + x_3 + 2x_4 = 2 \end{cases}$$

$r_2 - 2r_1 \mapsto r_2$, $r_3 + r_1 \mapsto r_3$

$$\begin{pmatrix} 1 & 2 & -1 & 1 & 1 \\ 0 & 2 & 0 & -4 & -1 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 4 & 0 & 3 & 3 \end{pmatrix}$$

$0 \quad -4 \quad 0 \quad +8 \quad +2 \quad 4 \quad \cdot -2$

$r_3 - 2r_2 \mapsto r_3$

$$\begin{pmatrix} 1 & 2 & -1 & 1 & 1 \\ 0 & 2 & 0 & -4 & -1 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 11 & 5 \end{pmatrix}$$

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 1 \\ 2x_2 - 4x_4 &= -1 \\ 3x_3 + x_4 &= 3 \\ 11x_4 &= 5 \end{aligned}$$

2. (3 points) The system of linear equations below is in echelon form. Write the solution set of the system in vector form.

$$\begin{cases} x-2y+z=8 \\ y-4z=0 \end{cases}$$

Answer

$$\left\{ \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix} z \; : \; z \in \mathbb{R} \right\}$$

$$\begin{cases} x = 8+2y-z \\ y = 4z \end{cases}$$

back substitute:

$x = 8+2(4z)-z$

$x = 8+7z$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8+7z \\ 4z \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix} z$$

3. (1 points) Give an example of a system of equations which has no solution.

$x+y=1$
$x+y=2$] These are parallel lines that are distinct.

$x+y+z=1$
$2x+3y+4z=8$
$3x+4y+5z=10$] Since the 3rd row is r_1+r_2 on the left but NOT r_1+r_2 on right, it must be inconsistent.