

Math 265

Homework #500

due 1/11/2026

§4.1, #3: Evaluate $\int_1^2 \frac{1}{x^2} dx$.*Answer:*

$$\int_1^2 \frac{1}{x^2} dx = \int_1^2 x^{-2} dx = -x^{-1} \Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

§4.2, #17 Prove $\sqrt{2}$ is irrational.*Proof.* Suppose, to the contrary, that $\sqrt{2}$ is rational. Then

$$\sqrt{2} = \frac{a}{b}$$

where $a, b \in \mathbb{Z}$, $b \neq 0$ with a, b having no common factors. Squaring yields

$$2 = \frac{a^2}{b^2},$$

so

$$2b^2 = a^2.$$

This shows 2 divides a^2 , and so since 2 is prime by a lemma proved in class, we see 2 divides a . Letting $a = 2c$ for some $c \in \mathbb{Z}$, this implies

$$2b^2 = 4c^2,$$

so

$$b^2 = 2c^2.$$

Now the same argument as above, but with b, a replaced by c, b , shows 2 divides b . Therefore 2 divides both a and b . But this contradicts that a, b had no common factors. \square

§4.2, #18 Find the product of x and y supposing that both are odd.

This shows you an aligned string of equations.

$$\begin{aligned} xy &= (2a + 1)(2b + 1) \\ &= (2a)(2b) + (2a)(1) + 1(2b) + 1(1) \\ &= 4ab + 2a + 2b + 1 \\ &= 2(2ab + a + b) + 1 \\ &= 2k + 1, \end{aligned}$$

This shows you an aligned string of equations with justifications.

$xy = (2a + 1)(2b + 1)$	a, b integers, (by definition of odd)
$= (2a)(2b) + (2a)(1) + 1(2b) + 1(1)$	(expanding binomial multiplication)
$= 4ab + 2a + 2b + 1$	(simplifying)
$= 2(2ab + a + b) + 1$	(factoring out a 2)
$= 2k + 1,$	$(k = 2ab + a + b)$

§4.2, #19 Make a table of useful L^AT_EX symbols.

words	what you type into L ^A T _E X	what appears in the PDF	example
is an element of	<code>\in</code>	\in	$x \in \mathbb{R}$
is not an element of	<code>\not \in</code>	\notin	$x \notin \mathbb{R}$
is a subset of	<code>\subteq</code>	\subseteq	$\mathbb{Q} \subseteq \mathbb{R}$
for all	<code>\forall</code>	\forall	$\forall x \in \mathbb{R}$
there exists	<code>\exists</code>	\exists	
logical and	<code>\land</code>	\wedge	
logical or	<code>\lor</code>	\vee	
logical negation	<code>\sim</code>	\sim	
subscript	<code>A_2</code>	A_2	
superscript	<code>x^2</code>	x^2	
number sign	<code>\#</code>	$\#$	
ampersand	<code>\&</code>	$\&$	
new line	<code>\cr</code> or <code>\\</code>		
backslash	<code>\textbackslash</code>	\backslash	