

Chapter 2

Consumer Theory

Consumer theory models individual decision making and consumer behavior. The goal of consumer theory is to derive the demand function and evaluate consumer welfare effects based on changes in prices and income. There are three components that form the basis of consumer theory:

1. The **budget constraint** is the income constraint faced by the consumer given prices and income.
2. **Consumer preferences** are expressed using utility functions and indifference curves. An indifference curve represent all the combinations of goods (including services) that provide a consumer with a given level of utility.
3. **Optimal consumer choice** results from the combination of the budget constraint and consumer preferences.

It is important to understand that consumer preferences are independent of the prices and income. A consumer can prefer a Bentley over a Toyota despite the fact that the Bentley is not affordable. It is consumer choice that determines what is purchased based on the budget constraint and consumer preferences.

2.1 Budget Constraint

A budget constraint represents all combinations of goods that can be purchased given income and prices. All examples in this section will be based on two goods, i.e., x and y . Of course, there are many more goods in reality but two goods are sufficient to explain the main concepts. If two goods seem insufficient, then good x can, for example, be milk and good y can represent all other goods.

With the two goods, the budget constraint is written as:

$$B = P_x \cdot Q_x + P_y \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{B}{P_y} - \frac{P_x}{P_y} \cdot Q_x$$

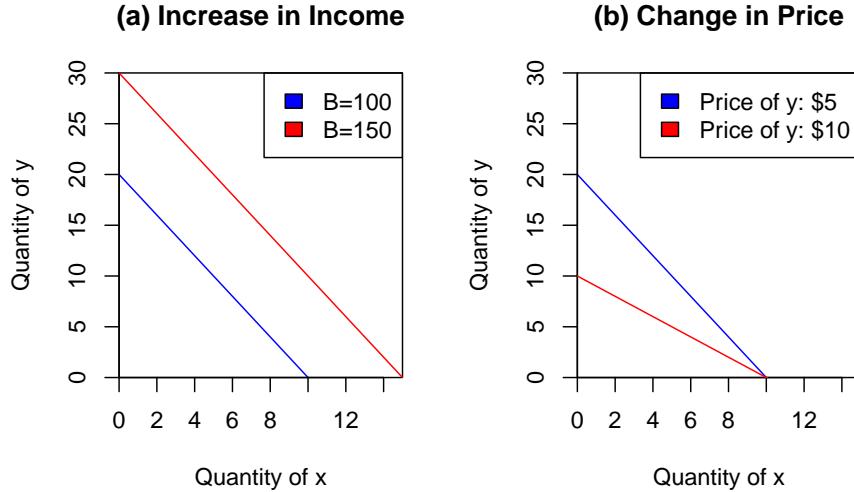


Figure 2.1: Changes in the budget constraint due to changes in income and price. Panel (a): If the income changes, the budget constraint shifts parallel. Panel (b): A change in price results in a different slope.

The income is represented by B —sometimes economists use M or I as well—and P and Q are used to denote prices and quantities. Expressing Q_y as a function of B , P_y , P_x , and Q_x allows for the representation in a two-dimensional graph (Figure 2.1).

A change in income results in a shift of the budget line but the slope remains the same. If prices change, the budget line rotates and the slope changes. To illustrate those concepts, consider the following situations (Figure 2.1):

- Situation 1: $B = 100$, $P_x = 10$, and $P_y = 5$

$$100 = 10 \cdot Q_x + 5 \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{100 - 10 \cdot Q_x}{5} = 20 - 2 \cdot Q_x$$

- Situation 2: $B = 150$, $P_x = 10$, and $P_y = 5$

$$150 = 10 \cdot Q_x + 5 \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{150 - 10 \cdot Q_x}{5} = 30 - 2 \cdot Q_x$$

- Situation 3: $B = 100$, $P_x = 10$, and $P_y = 10$

$$100 = 10 \cdot Q_x + 10 \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{100 - 10 \cdot Q_x}{10} = 10 - Q_x$$

A budget constraint is linear only if prices are constant and independent of the quantity purchased. If there are restrictions on what can be bought, e.g., food stamps or quantity discounts, the budget constraint is not linear anymore.

2.2 Consumer Preferences

Before moving to consumer preference, consider a three-dimensional representation of Mount Saint Helens (Panel (a), Figure 2.1). In consumer theory, the height of the mountain and the contour lines represent utility and indifference curves, respectively.

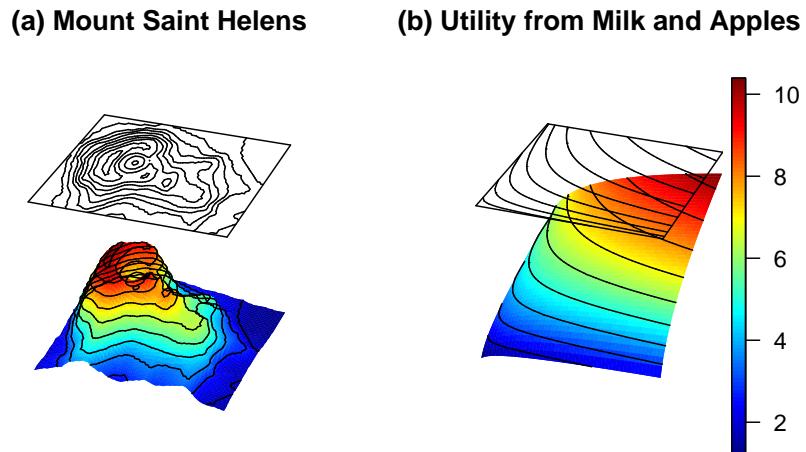


Figure 2.2: (a) Three-dimensional representation of Mount Saint Helens and contour lines. (b) Graphical representation of utility derived from various bundles of apples and milk.

Consumer theory makes the following assumptions about preferences:

- **Completeness:** Given two consumption bundles A and B , the consumer can make one of the following comparisons: (1) A is preferred to B , (2) B is preferred to A , or (3) A is indifferent to B .
- **Transitivity:** Assuming three consumption bundles A , B , and C and a consumer preferring A to B and B to C , then the consumer also prefers A to C . For example, if a consumer prefers a BMW to a Toyota and a Toyota to a Chevrolet, then the consumer also prefers a BMW to Chevrolet.
- **Non-satiation:** More is better than less, i.e., utility does not decrease if more goods are consumed by the consumer.
- **Diminishing marginal utility:** The more of a good is already consumed, the smaller the additional utility gained. This assumption has very important implications for the shapes of the utility function and the indifference curves.

2.2.1 Utility Functions and Indifference Curves

The assumptions about preferences lead us to the concept of utility, which is the satisfaction a consumer gets from consuming a good or undertaking an activity. Utility can be either ordinal or cardinal. Ordinal utility is only concerned about the rank-ordering of preferences, e.g., A is better than B . Cardinal utility measures the intensity of preferences, e.g., A is twice as good as B . In general, economists only use ordinal utility and utility cannot be compared between two consumers. Also, recall that utility is independent of income and prices.

To illustrate the concept of diminishing marginal utility, consider a utility function $U(x) = x^{0.5}$ (Figure 2.3). For this function, the utility is increasing in x but at a diminishing rate. That is, the change in utility from one more unit consumed, i.e., marginal utility, diminishes as more of the good is consumed, e.g., the fifth ice cream cone is not as desirable as the first one. This represents the law of diminishing marginal utility. As long as the marginal utility is positive, total utility increases.

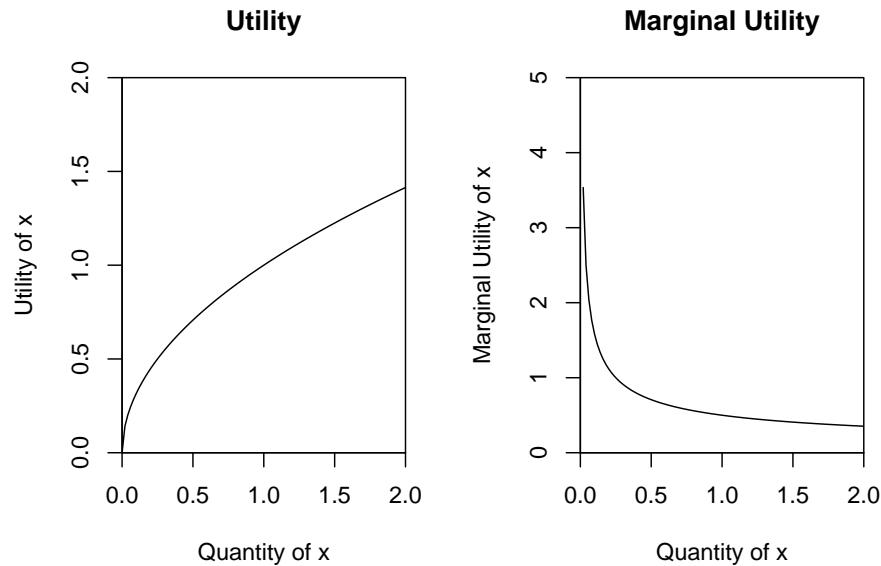


Figure 2.3: Univariate utility function and corresponding marginal utility.

Economics is about allocating scarce resources, i.e., asking what choices people make when faced with limited resources. Hence, analyzing utility for a single good is not enough and one or more goods need to be added. To do so, the above-used utility function of the form $U(x) = Q_x^\alpha$ can be expanded to what is

called a Cobb-Douglas utility function written as follows:

$$U(Q_x, Q_y) = Q_x^\alpha \cdot Q_y^\beta$$

The shape of a Cobb-Douglas utility function is also depicted in Panel (b), Figure 2.2. Another commonly used utility function is called Constant Elasticity of Substitution or CES utility function that is written as follows:

$$U(Q_x, Q_y) = (\alpha_x \cdot Q_x^\rho + \alpha_y \cdot Q_y^\rho)^{\frac{\gamma}{\rho}}$$

The CES utility function can accommodate a wide variety of preferences depending on the parameters of α , ρ , and γ .

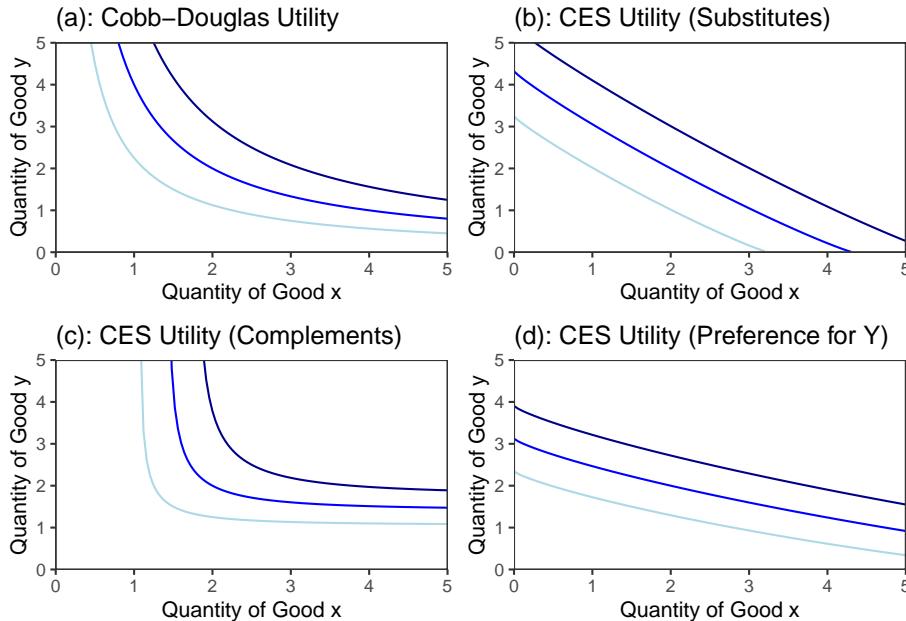


Figure 2.4: Indifference curves associated with various Cobb-Douglas and CES utility functions. (a) Cobb-Douglas, (b) CES with rho larger than zero leading to the two goods being substitutes, (c) CES with rho smaller than zero leading to the two goods being complements, and (d) CES with a preference for good y.

Because utility functions with two goods can only be displayed in three dimensions, economics relies on so-called indifference curves that display all the combinations of goods Q_x and Q_y that result in the same level of utility. Indifference curves can be thought of as the contour lines of the utility function similar to the contour lines of Mount Saint Helens (Panel (a), Figure 2.2). Indifference curves (1) do not intersect, (2) slope downward, and (3) bend inward

(are convex to the origin) (Figure 2.4). A point on a higher indifference curve is preferred to any point on a lower curve. The slope of the indifference curve at a given consumption bundle is called the Marginal Rate of Substitution (MRS). The indifference curve for the Cobb-Douglas utility function for a given level of utility U can be written as follows:

$$Q_y = \left(\frac{U}{Q_x^\alpha} \right)^{\frac{1}{\beta}}$$

And the equivalent for the CES utility function is as follows:

$$Q_y = \left(\frac{U^{\rho/\gamma} - \alpha_x}{\alpha_y \cdot Q_x^\rho} \right)^{\frac{1}{\rho}}$$

Let us consider an example in the 2-good space. We will be able to identify 3 regions: (1) Not preferred, (2) preferred, and (3) potentially indifferent.

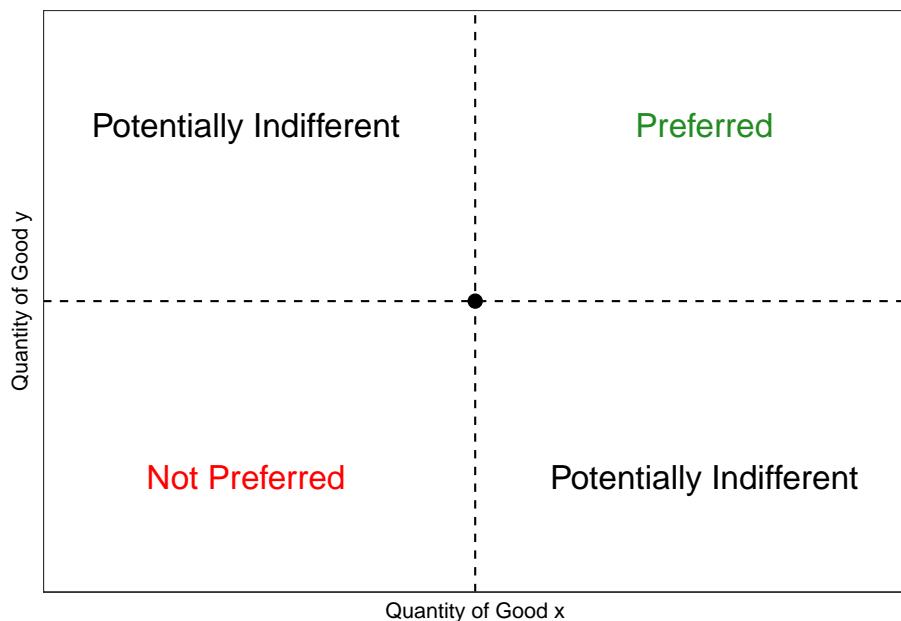


Figure 2.4 represents—for a given bundle of goods x and y —the areas of preference. The “Preferred” area gives more of both goods and hence, results in a higher level of utility due to the non-satiation assumption. The opposite is true for “Not Preferred” area. The “Potentially Indifferent” bundles may or may not lie on the same indifference curve than the initial bundle.

2.3 Consumer Choice

Consumers maximize their utility subject to their budget constraint. Deriving the optimal consumption bundle involves the use of calculus and for the purpose of this text, a graphical derivation is used.

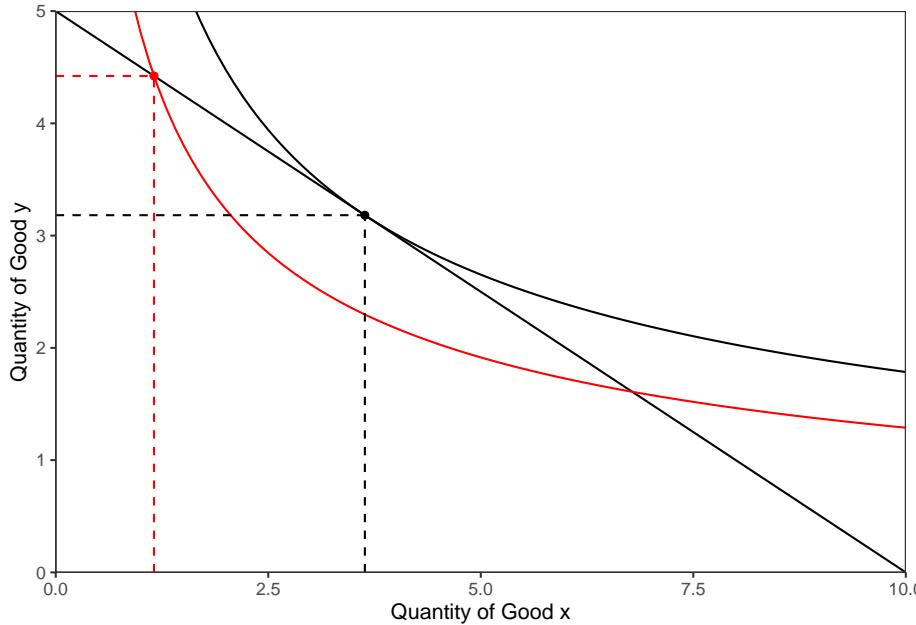


Figure 2.5: Optimal choice by the consumer.

Note that the slope of the budget constraint is

$$\frac{\Delta q_y}{\Delta q_x} = -\frac{p_x}{p_y}$$

The marginal rate of substitution (MRS) is the slope of the indifference curve:

$$MRS_{x,y} = -\left. \frac{\Delta q_y}{\Delta q_x} \right|_{\Delta U=0}$$

So the optimality condition, i.e., optimal choice is

$$MRS_{x,y} = \frac{p_x}{p_y}$$

2.4 Derivation of the Demand Curve

Remember that the goal of utility theory is to derive the demand function of a product. The figure below illustrates the derivation of the demand function

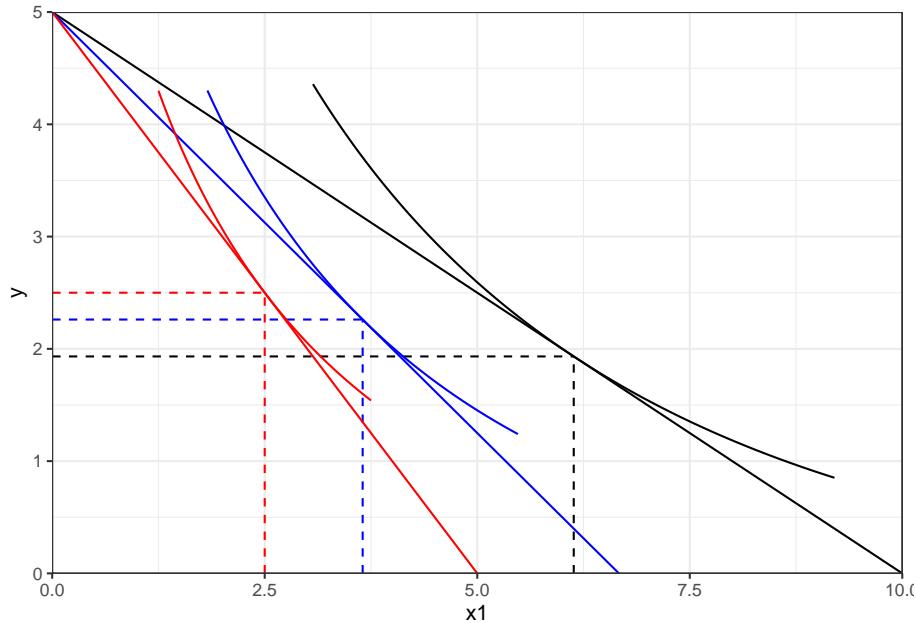


Figure 2.6: Effects of changing price for good x on optimal choice and quantity demanded for CES utility function.

for good x for two examples of utility functions that are used frequently in economics: The Constant Elasticity of Substitution (CES) and the Cobb-Douglas utility functions. If we start out with the optimal choice given initial prices for both goods and the optimal choice (the point where the indifference curve is tangent to the budget constraint), we have the consumption of x for a particular price p_x . If we start to increase the price, the original consumption bundle will not be achievable anymore and the consumer has to choose a new consumption point given the new price for p_x . This gives us a second point of the demand curve. If we continue this process, we can trace out the entire demand function for x for all prices.

A previous section introduced the Cobb-Douglas and Constant Elasticity of Substitution (CES) utility functions and their indifference curves. It can be shown that the demand functions associated with the Cobb-Douglas utility function are written as follows:

$$Q_x = \frac{\alpha}{\alpha + \beta} \cdot \frac{M}{P_x}$$

$$Q_y = \frac{\beta}{\alpha + \beta} \cdot \frac{M}{P_y}$$

And for the CES utility, the demand functions are written as follows with $\sigma =$

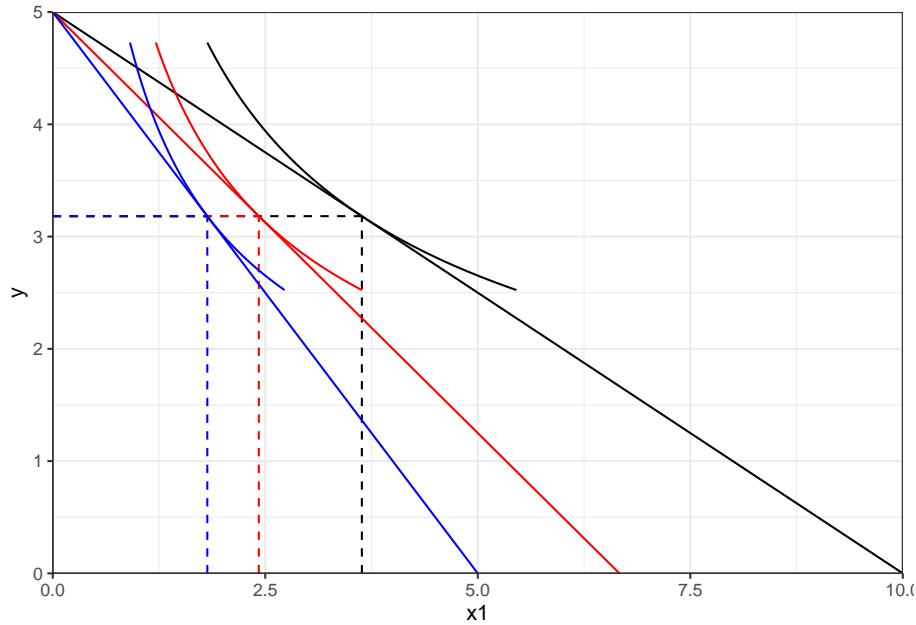


Figure 2.7: Effects of changing price for good x on optimal choice and quantity demanded for Cobb-Douglas utility function.

$1/(1 - \rho)$:

$$Q_x = \frac{\alpha}{P_x} \cdot \frac{M}{\alpha^\sigma \cdot P_x^{(1-\sigma)} + (1-\alpha)^\sigma \cdot P_y^{(1-\sigma)}}$$

$$Q_y = \frac{1-\alpha}{P_x} \cdot \frac{M}{\alpha^\sigma \cdot P_x^{(1-\sigma)} + (1-\alpha)^\sigma \cdot P_y^{(1-\sigma)}}$$

2.5 Income and Substitution Effect

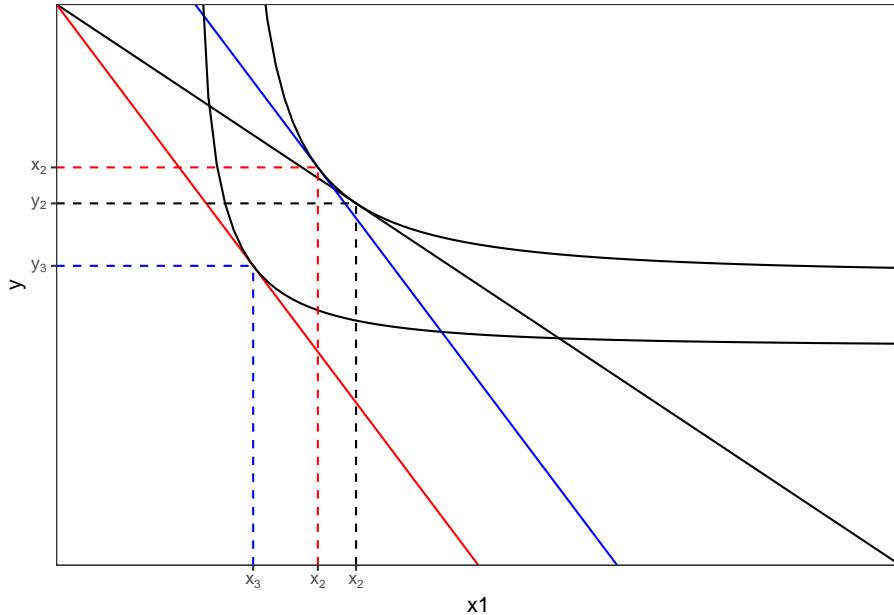


Figure 2.10: Income and (Hicks) substitution effect of an increase in price of good x . The original quantities consumed are x_1 and y_1 . The price increase makes good x relatively more expensive. This substitution effect is displayed by going from x_1 to x_2 . The income effect is x_2 to x_3 .

A change in the price of a good can be decomposed into an income and a substitution effect. That is, a price change makes a good relatively more (or less) expensive relative to other goods resulting in the income effect. The consumer also changes the allocation to the good in question and hence, the allocation to other goods, which is called the substitution effect.

Consider the example of a consumer who has an income of 100 faces prices $P_x = 10$ and $P_y = 20$. The utility function of the consumer is written as follows:

$$U(Q_x, Q_y) = (0.4 \cdot Q_x^{-2} + 0.6 \cdot Q_y^{-2})^{(-1/2)}$$

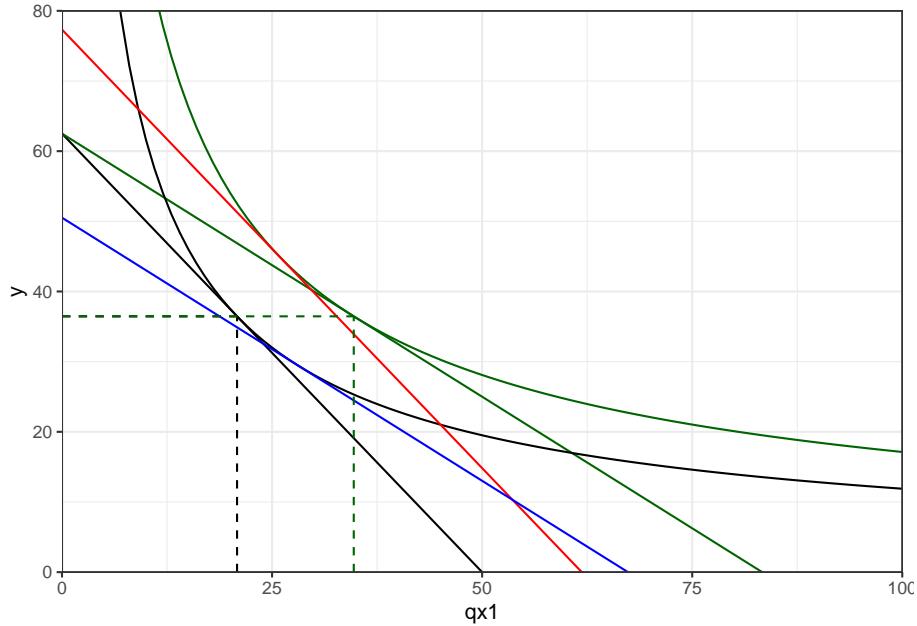
Given those preferences, income, and prices, the optimal allocation Q_x and Q_y is 3.55 and 3.23, respectively, which gives the consumer a utility of 3.34. If the price of good x increases by \$15, then the optimal allocation Q_x and Q_y is 2.33 and 2.67, respectively, which gives the consumer a utility of 2.52. The change in consumption from x_1 to x_2 is considered the total effect. Next, we are going to divide the total effect into the income and substitution effect. To do so, we have to increase the income of the consumer under the new prices such that

their income is getting them back to the old utility function.

2.6 Compensating and Equivalent Variation

Compensating variation (CV) and equivalent variation (EV) are two measures in welfare economics used to evaluate changes in consumer well-being when prices change. CV is the amount of money that would need to be given to or taken away from a consumer after a price change to make them as well off as they were before the change. In contrast, EV is the amount of money that would need to be given to or taken away from a consumer before the price change to make them as well off as they would be after the change. While both capture the monetary value of a change in utility, CV uses the new utility level as a reference point whereas EV uses the original utility level. Thus, CV and EV can differ depending on whether the price change is beneficial or harmful.

To illustrate the difference between CV and EV, consider a consumer with income $M = 300$. The original prices are $P_x = P_y = 4$ and the new price levels are $P_x = 2$ and $P_y = 4$. That is, the price of good x decreases.



2.7 Market Demand

To obtain the market demand for a good, the individual demand functions have to be added horizontally. For example, consider three consumers with the following demand functions: $Q_1 = 10 - 2 \cdot P$, $Q_2 = 24 - 3 \cdot P$, and $Q_3 = 15 - P$.

The inverse demand functions can be written as:

$$P = 5 - \frac{Q_1}{2}$$

$$P = 8 - \frac{Q_2}{3}$$

$$P = 15 - Q_3$$

The inverse demand functions are valid $Q_1 \in [0, 10]$, $Q_2 \in [0, 24]$, and $Q_3 \in [0, 15]$.

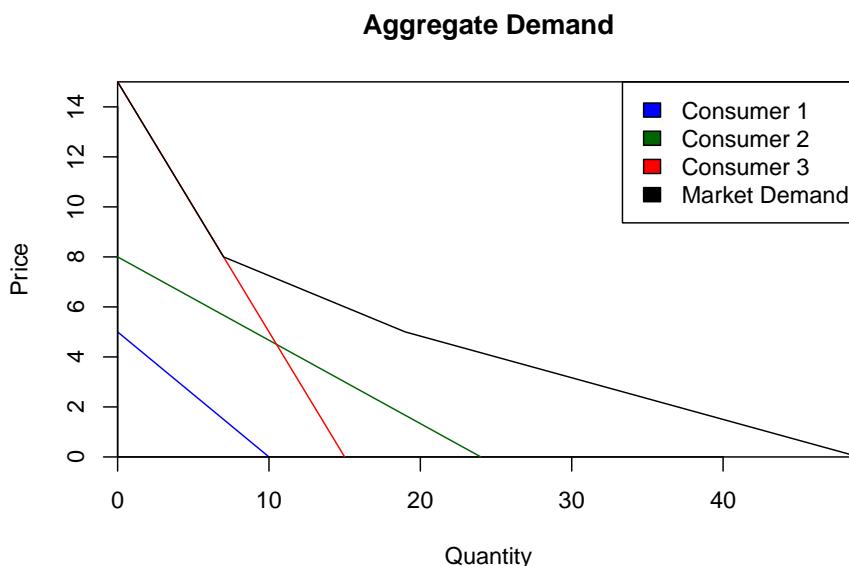


Figure 2.11: Derivation of the market demand curve given three individual demand functions.

2.8 Policy Analysis with Consumer Theory

This section covers some theoretical examples that can be translated to policy decisions as well as some applied examples from the economics literature.

2.8.1 Theoretical Examples

Switching food delivery services: Pamela uses a food delivery service which sends you the ingredients for meals and she just needs to combine and cook them at home for a nice meal. Currently, she purchases this service 20 times a month. Her income is \$1,000 per month and the meals cost \$20 per meal. All

other goods cost \$10 per unit. A new online service offers similar meals but charges a flat fee of \$200 per month plus \$10 per meal. This package lets her consume the same amount of food. However, because the slope of the budget constraint is different, the initial choice is not optimal anymore. By reallocating her consumption, she can get on a higher indifference curve.



Figure 2.12: Theoretical examples about individual choice. Panel (a) demonstrates the switch from a per-unit purchasing system to a scheme which has a fixed cost and a lower per-unit price for subsequent purchases. Panel (b) contrasts the difference between a lump-sum tax and a per-unit tax.

Policy Example of a Lump-Sum Tax versus a Per-Unit Tax: A per-unit tax is the institution of an excise tax on some (but not all) goods a consumer purchases. A lump-sum tax is the collection of a single sum, independent of the consumer's choices. Assume the following notation: p_x and p_y as the price of goods x and y , M as the income, t as the per-unit tax, T as the lump-sum tax. Table 2.1 and panel ‘‘Lump-Sum vs. Per-Unit Tax’’ in Figure 2.13 represent this problem. The graph in figure 2.13 is based on the following parameters: $M = 100$ (income), $P_x = 3$, $P_y = 2$, $t = 3$ (per unit tax), and the utility function is written as $U(x, y) = Q_x^a \cdot Q_y^{1-a}$ with $a = 0.4$.

Subsidy for Low-Income Housing: Suppose you have to decide how to subsidize low-income housing. Assume that the family has an income of \$1000 and that the price of other goods is \$1. There are two subsidy plans: (1) a dollar-for-dollar subsidy or (2) a lump-sum payment. The initial consumption on

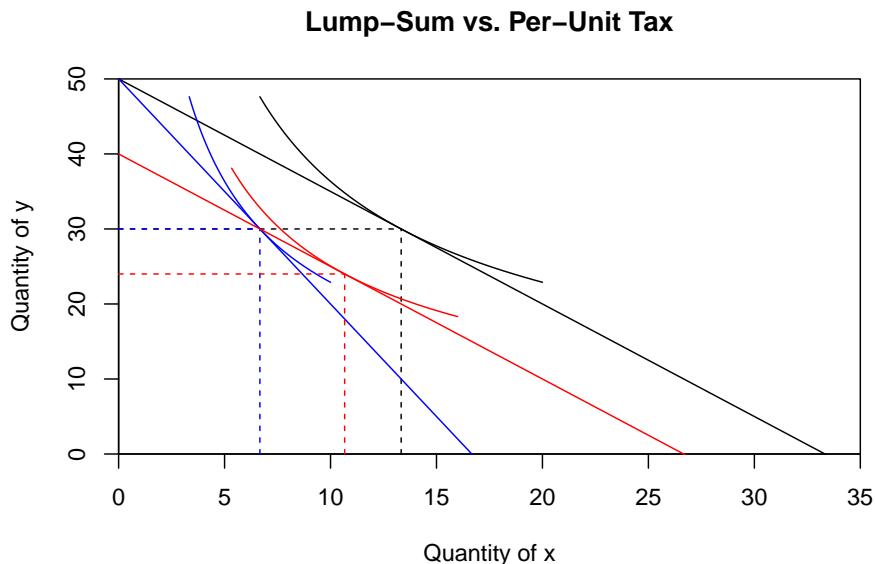


Figure 2.13: Taxation

	Budget line	Choice
Original	$P_x \cdot Q_x + P_y \cdot Q_y = B$	(x_1^*, y_1^*)
Per-unit	$(P_x + t) \cdot Q_x + P_y \cdot Q_y = B$	(x_2^*, y_2^*)
Lump-sum	$P_x \cdot Q_x + P_y \cdot Q_y = B - T$	(x_3^*, y_3^*)

Table 2.1: Comparison in budget constraint without a tax (Original), a per-unit tax, and a lump-sum tax.

Option	Condition 1		Condition 2	
	Drop. Calls per 100	\$/Year	Drop. Calls per 1000	\$/Month
A	4.2	384	42	32
B	6.5	324	65	27

Table 2.2: Two cell phone plans presented to consumers with different rates of dropped calls and cost. Note that the plans A and B are identical and that only the scaling of the interruptions and cost is changed.

housing is \$500 (\$250 from personal income and \$250 subsidy from government). How does this differ from food stamps?

2.8.2 Policy Examples

Economists assume that consumers are rational agents who—given all information—make informed decisions. This may not be true even in everyday situations as illustrated in the first three examples below. On the other hand, the law of diminishing marginal utility may have some practical implications as demonstrated in the last example.

Consumer Rationality: [Burson et al. \(2009\)](#) demonstrate how people reverse their preferences if faced with numbers that are scaled differently. In their experiment, they have respondents choose a cell phone plan (Table 2.2). Under condition 1, 31% favor A and 53% favor B. Under condition 2, 69% favor A and 23% favor B. Note that both plans are identical (e.g., $\$32 \cdot 12 = \384) and people are simply attracted to a smaller number. If you listen to or watch advertisement, you always hear “it’s only \$1 a day” and never “it’s only \$365 per year.”

MPG Illusion: The second example illustrates the concept of [MPG Illusion](#). Suppose that three people drive 10,000 miles per year. The fuel economy of the current cars are 10 MPG for person *A*, 16.5 MPG for person *B*, and 33 MPG for person *C*. They all trade-in their current cars for new cars that get 11, 20, and 50 MPG for *A*, *B*, and *C*, respectively (Figure 2.14). It turns out that all three consumers save the same amount of gasoline. Most people assume a linear relationship between MPG and gasoline consumption, which is not correct. The reverse measure gallons per mile does not suffer from this flaw.

Marginal Utility and Undernourishment: A paper by [Jensen and Miller \(2010\)](#), which was also covered in an article by The Economist titled [People’s spending choices are a good way to assess levels of hunger](#), questions the usefulness of a fixed calorie threshold to quantify undernourishment. Their argument is based on the observation that in some countries, caloric intake decreases while real income increases. Instead of measuring the number of calories consumed, researchers and policymakers should measure the amount of staples consumed

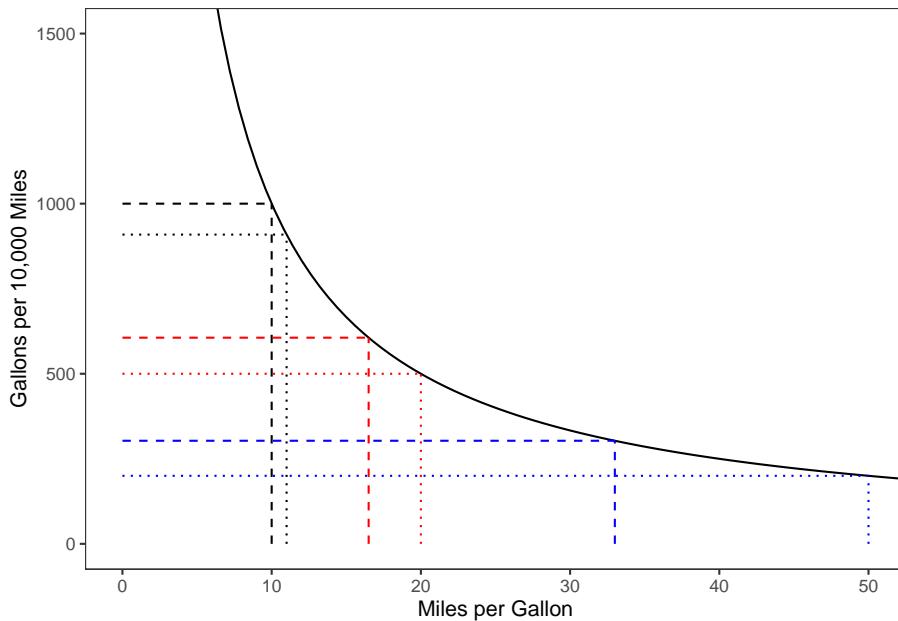


Figure 2.14: Gallons of gasoline consumed based on driving 10,000 miles per year. The different colored lines represent consumers switching from their current cars (dashed lines) to new cars (dotted lines).

such as rice and millet. Both goods are low cost sources of calories and thus, people consuming a large share of their total calories from those foods are likely to be malnourished. Consuming those foods results in a significant increase in utility very quickly and very cheaply. The authors in the article use the term staple calorie share as a measure of undernourishment.

Automatic Bill Payment and Electricity Consumption: [Sexton \(2015\)](#) assesses the effect of enrolling in automatic bill payment (ABP) on electricity consumption. ABP is a service that allows automatic deduction of the electricity bill from the consumer's bank account. The results show that ABP enrollment increases residential and commercial electricity consumption by up to 4.0% and 8.1%, respectively. The result is attributed to consumers not following closely their electricity consumption and associated cost anymore.

2.9 Exercises

For all questions, assume that quantities are perfectly divisible, i.e., purchasing 2.5 textbooks is possible.

1. **Quantity Discounts (***)**: The budget constraints are straight lines

if prices do not change over the entire range of quantities purchased. If there are quantity discounts, the budget constraints may be kinked and/or discontinuous Draw the budget constraints for the following situations:

- a. $M = 100$, $P_y = 2$, and $P_x = 4$
- b. $M = 100$, $P_y = 2$, and $P_x = 4$ if less than 10 units are bought and $P_x = 3$ for units after 10.
- c. $M = 100$, $P_y = 2$, and $P_x = 4$ if less than 10 units are bought and $P_x = 3$ on all units if more than 10 are bought.
2. ***Veal for a Vegetarian*** (*): Draw the indifference curves of a vegetarian for veal and spinach.
3. ***Muesli*** (**): Sunny is a spoiled kid and forces her daddy to add a tablespoon of nuts for each tablespoon of fruits to her **Muesli**. The price of nuts and fruit per tablespoon is \$0.10 and \$0.20, respectively. He allocates a total of \$3 for her entire breakfast. Draw her indifference curves for nuts and fruits. How many tablespoons are used for her breakfast each morning?
4. ***Revealed Preferences*** (***): To better understand preferences, researchers often present respondents with different price and income situations and ask them to choose between various consumption bundles (see [Grebitus and Dumortier \(2016\)](#) for an example). Consider the following two situations:
 - Situation 1: Income is \$100 and prices are $p_x = \$10$ and $p_y = \$20$. In this situation, the respondent has the choice between two bundles: Either bundle A with $Q_x^A = 2$ and $Q_y^A = 4$ or bundle B with $Q_x^B = 4$ and $Q_y^B = 3$. The consumer picks bundle A.
 - Situation 2: Income is \$80 and prices are $p_x = \$20$ and $p_y = \$10$. In this situation, the respondent has the choice between two bundles: Either bundle C with $Q_x^C = 1$ and $Q_y^C = 6$ or bundle A with $Q_x^A = 2$ and $Q_y^A = 4$. The consumer picks bundle C.
 Rank the consumer's preferences for bundles A, B, and C from highest to lowest preference. Next, assume there is a fourth bundle D with $Q_x^D = 5$ and $Q_y^D = 5$. How does bundle D compare in terms of preference ordering compared to the other three bundles. Do you have all the information you need to determine bundle's D place in the ranking?
5. ***Textbook Gift Voucher*** (***): Tom is in college and receives \$500 from his parents each semester to spend on textbooks and other goods. The price of books is \$20 and the price of other goods is \$10. Assume that the number of textbooks is a choice. In the initial situation, Tom purchases six textbooks and the rest is spent on other goods. Draw Tom's budget constraint and the indifference curve indicating the optimal choice. To ensure that Tom spends sufficient amounts of money on textbooks, his parents replace the cash payment of \$500 with a bookstore gift certificate

of \$300 and \$300 in cash. Draw the budget constraint under the new regime. Is Tom better off? On a new graph, draw a situation under which he is better off with a cash payment from his parents of \$600 than the \$300 cash/\$300 gift certificate split.

6. ***Obesity in Pawnee*** (**): Because of rampant obesity in Pawnee, the city government decides to tax candy. In the figure below, you have the original budget constraint with the optimal choice in the absence of any tax. The income of the consumer is \$100, the price of candy is \$2 per unit, and the price of other goods is \$1 per unit. Without the tax, consumers buy 20 units of candy and 60 units of other goods. With the tax of \$2 per unit, the consumer buys 10 units of candy and 60 units of other goods. How much does the city government collect in taxes? Assume that political pressure forces the city government to return the amount of the tax collected to the consumer in form of a lump-sum payment at the end of the year. Draw the new budget constraint given the lump-sum payment using the figure provided. Is the consumer as happy as in the case with neither tax nor subsidy. Interpret. See Figure for a template.
7. ***Per-Diem Meal and Hotel Rates*** (**): Suppose you work for a company which subsidizes your traveling with \$1,000 per month. However, you cannot exceed \$600 for hotel and \$400 for meals. Assume that everywhere you travel, the price for one night in a hotel is \$100 and the price of one meal is \$20. Assume that you always exhaust your budget for traveling. Draw the budget constraint for this problem with hotel on the horizontal axis and meals on the vertical axis. Explain to your boss why you would (most likely) be better off with \$1,000 but no restrictions on spending. Draw the relevant indifference curves.
8. ***Church Roof*** (**): Philip is a rich atheist but has a soft spot for the leaking roof of his hometown church. The church currently spends \$2,000 on the roof and \$8,000 on other activities.
 - a. Draw the budget constraint for his hometown church with roof repairs on the horizontal axis and other activities on the vertical axis. Assuming utility maximization of the church, draw the indifference curve corresponding to the current situation.
 - b. Philip offers the church a \$1,000 grant with the constraint that the money can only go towards roof repairs. Put differently, it cannot be used for other activities. Will the amount of money spent on roof repairs increase by exactly \$1,000 or will it change by a different amount. Justify your answers.
9. ***Gasoline in Iran*** (**): The Iranian government has [provided substantial subsidies on gasoline](#) to its citizens in past decades. After international sanctions were imposed on the country due to its suspect nuclear program, those subsidies were not sustainable in the long-run from a financial perspective. Thus Iran eliminated the subsidies but replaced them with a

cash transfer to ease the financial burden for its citizens. Assume that gasoline is on the horizontal axis and all other goods are on the vertical axis. The price of other goods does not change throughout the question.

- a. Draw the initial budget constraint and the optimal consumption bundle. Assume that both goods are initially consumed.
- b. How does the budget constraint change if the government eliminates subsidies on gasoline and the price of gasoline increase subsequently? Show this effect in the graph.
- c. Given the new prices, the government provides a subsidy that raises income such that the consumers are as happy as before. Illustrate in your graph.
- d. Why is this increase in income cheaper than the subsidy on gasoline? Illustrate in your graph.