Chapter 2

Consumer Theory

Consumer theory models individual decision making and consumer behavior. The goal of consumer theory is to derive the demand function and evaluate consumer welfare effects based on changes in prices and income. There are three components that form the basis of consumer theory:

1. The **budget constraint** is the income constraint faced by the consumer given prices and income.
2. **Consumer preferences** are expressed using utility functions and indifference curves. An indifference curve represent all the combinations of goods (including services) that provide a consumer with a given level of utility.
3. **Optimal consumer choice** results from the combination of the budget constraint and consumer preferences.

It is important to understand that consumer preferences are independent of the prices and income. A consumer can prefer a Bentley over a Toyota despite the fact that the Bentley is not affordable. It is consumer choice that determines what is purchased based on the budget constraint and consumer preferences.

2.1 Budget Constraint

A budget constraint represents all combinations of goods that can be purchased given income and prices. All examples in this section will be based on two goods, i.e., x and y. Of course, there are many more goods in reality but two goods are sufficient to explain the main concepts. If two goods seem insufficient, then good x can, for example, be milk and good y can represent all other goods.

With the two goods, the budget constraint is written as:

$$B = P_x \cdot Q_x + P_y \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{B}{P_y} - \frac{P_x}{P_y} \cdot Q_x$$

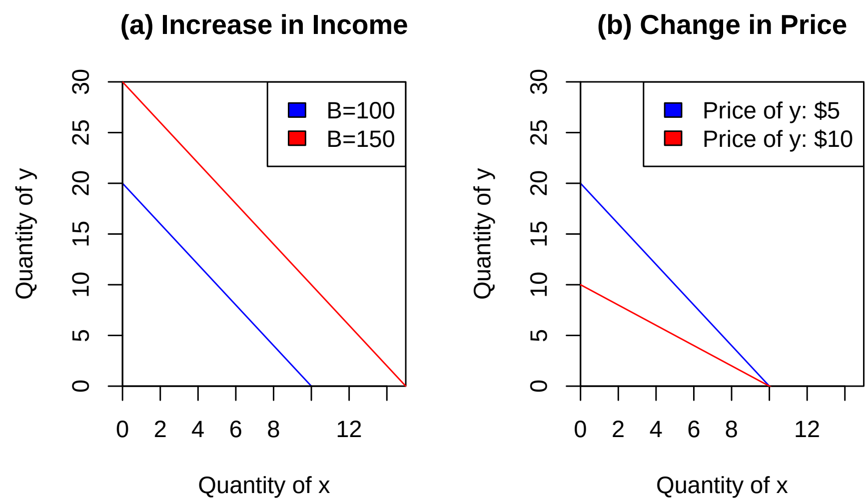

Figure 2.1: Changes in the budget constraint due to changes in income and price. Panel (a): If the income changes, the budget constraint shifts parallel. Panel (b): A change in price results in a different slope.

The income is represented by B—sometimes economists use M or I as well—and P and Q are used to denote prices and quantities. Expressing Q_y as a function of B, P_y, P_x, and Q_x allows for the representation in a two-dimensional graph (Figure 2.1).

A change in income results in a shift of the budget line but the slope remains the same. If prices change, the budget line rotates and the slope changes. To illustrate those concepts, consider the following situations (Figure 2.1):

- Situation 1: $B = 100$, $P_x = 10$, and $P_y = 5$

$$100 = 10 \cdot Q_x + 5 \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{100 - 10 \cdot Q_x}{5} = 20 - 2 \cdot Q_x$$

- Situation 2: $B = 150$, $P_x = 10$, and $P_y = 5$

$$150 = 10 \cdot Q_x + 5 \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{150 - 10 \cdot Q_x}{5} = 30 - 2 \cdot Q_x$$

- Situation 3: $B = 100$, $P_x = 10$, and $P_y = 10$

$$100 = 10 \cdot Q_x + 10 \cdot Q_y \quad \Rightarrow \quad Q_y = \frac{100 - 10 \cdot Q_x}{10} = 10 - Q_x$$

A budget constraint is linear only if prices are constant and independent of the quantity purchased. If there are restrictions on what can be bought, e.g., food stamps or quantity discounts, the budget constraint is not linear anymore.

2.2 Consumer Preferences

Before moving to consumer preference, consider a three-dimensional representation of Mount Saint Helens (Panel (a), Figure 2.1). In consumer theory, the height of the mountain and the contour lines represent utility and indifference curves, respectively.

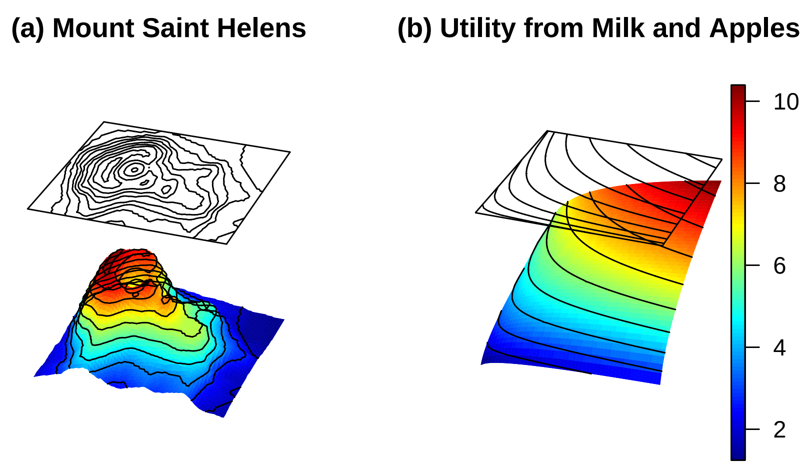

Figure 2.2: (a) Three-dimensional representation of Mount Saint Helens and contour lines. (b) Graphical representation of utility derived from various bundles of apples and milk.

Consumer theory makes the following assumptions about preferences:

- **Completeness**: Given two consumption bundles A and B, the consumer can make one of the following comparisons: (1) A is preferred to B, (2) B is preferred to A, or (3) A is indifferent to B.
- **Transitivity**: Assuming three consumption bundles A, B, and C and a consumer preferring A to B and B to C, then the consumer also prefers A to C. For example, if a consumer prefers a BMW to a Toyota and a Toyota to a Chevrolet, then the consumer also prefers a BMW to Chevrolet.
- **Non-satiation**: More is better than less, i.e., utility does not decrease if more goods are consumed by the consumer.
- **Diminishing marginal utility**: The more of a good is already consumed, the smaller the additional utility gained. This assumption has very important implications for the shapes of the utility function and the indifference curves.

2.2.1 Utility Functions and Indifference Curves

The assumptions about preferences lead us to the concept of utility, which is the satisfaction a consumer gets from consuming a good or undertaking an activity. Utility can be either ordinal or cardinal. Ordinal utility is only concerned about the rank-ordering of preferences, e.g., A is better than B. Cardinal utility measures the intensity of preferences, e.g., A is twice as good as B. In general, economists only use ordinal utility and utility cannot be compared between two consumers. Also, recall that utility is independent of income and prices.

To illustrate the concept of diminishing marginal utility, consider a utility function $U(x) = x^{0.5}$ (Figure 2.3). For this function, the utility is increasing in x but at a diminishing rate. That is, the change in utility from one more unit consumed, i.e., marginal utility, diminishes as more of the good is consumed, e.g., the fifth ice cream cone is not as desirable as the first one. This represents the law of diminishing marginal utility. As long as the marginal utility is positive, total utility increases.

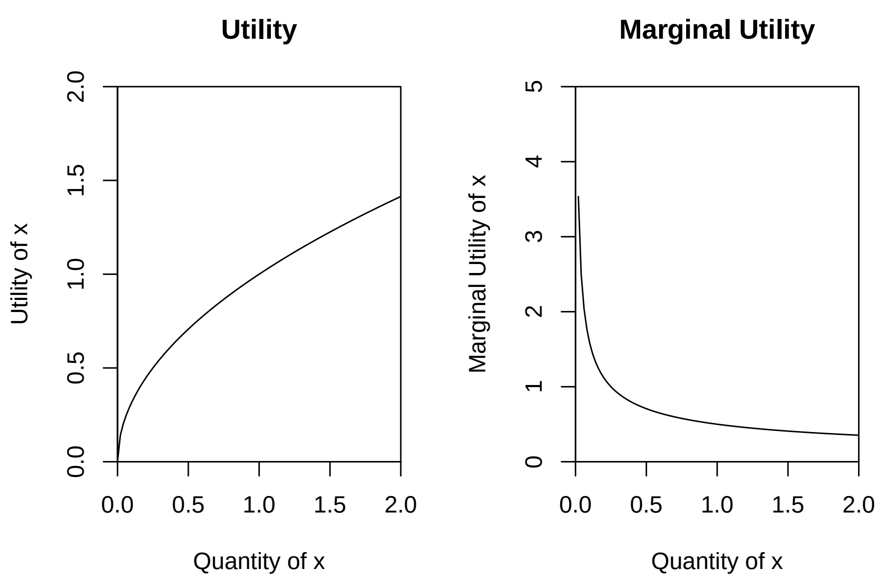

Figure 2.3: Univariate utility function and corresponding marginal utility.

Economics is about allocating scarce resources, i.e., asking what choices people make when faced with limited resources. Hence, analyzing utility for a single good is not enough and one or more goods need to be added. To do so, the above-used utility function of the form $U(x) = Q_x^{\alpha}$ can be expanded to what is called a Cobb-Douglas utility function written as follows:

$$U(Q_x, Q_y) = Q_x^{\alpha} \cdot Q_y^{\beta}$$

The shape of a Cobb-Douglas utility function is also depicted in Panel (b), Figure 2.2. Another commonly used utility function is called Constant Elasticity of Substitution or CES utility function that is written as follows:

$$U(Q_x, Q_y) = (\alpha_x \cdot Q_x^{\rho} + \alpha_y \cdot Q_y^{\rho})^{\frac{\gamma}{\rho}}$$

The CES utility function can accommodate a wide variety of preferences depending on the parameters of α, ρ, and γ.

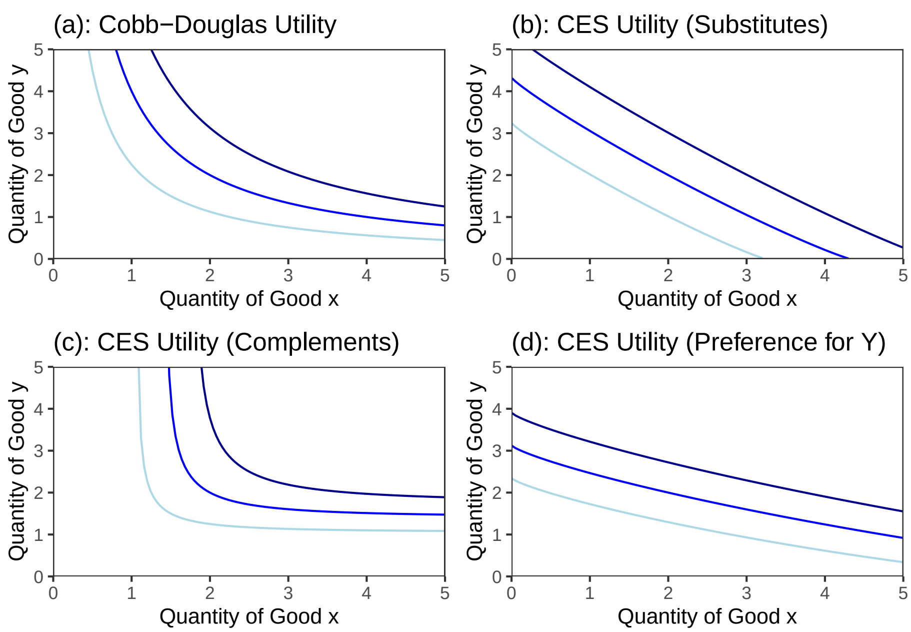

Figure 2.4: Indifference curves associated with various Cobb-Douglas and CES utility functions. (a) Cobb-Douglas, (b) CES with rho larger than zero leading to the two goods being substitutes, (c) CES with rho smaller than zero leading to the two goods being complements, and (d) CES with a preference for good y.

Because utility functions with two goodd can only be displayed in three dimensions, economics relies on so-called indifference curves that display all the combinations of goods Q_x and Q_y that result in the same level of utility. Indifference curves can be thought of as the contour lines of the utility function similar to the contour lines of Mount Saint Helens (Panel (a), Figure 2.2). Indifference curves (1) do not intersect, (2) slope downward, and (3) bend inward (are convex to the origin) (Figure 2.4). A point on a higher indifference curve is preferred to any point on a lower curve. The slope of the indifference curve at a given consumption bundle is called the Marginal Rate of Substitution (MRS). The indifference curve for the Cobb-Douglas utility function for a given level of utility U can be written as follows:

$$Q_y = \left(\frac{U}{Q_x^{\alpha}}\right)^{\frac{1}{\beta}}$$

And the equivalent for the CES utility function is as follows:

$$Q_y = \left(\frac{U^{\rho/\gamma} - \alpha_x}{\alpha_y \cdot Q_x^{\rho}}\right)^{\frac{1}{\rho}}$$

Let us consider an example in the 2-good space. We will be able to identify 3 regions: (1) Not preferred, (2) preferred, and (3) potentially indifferent.

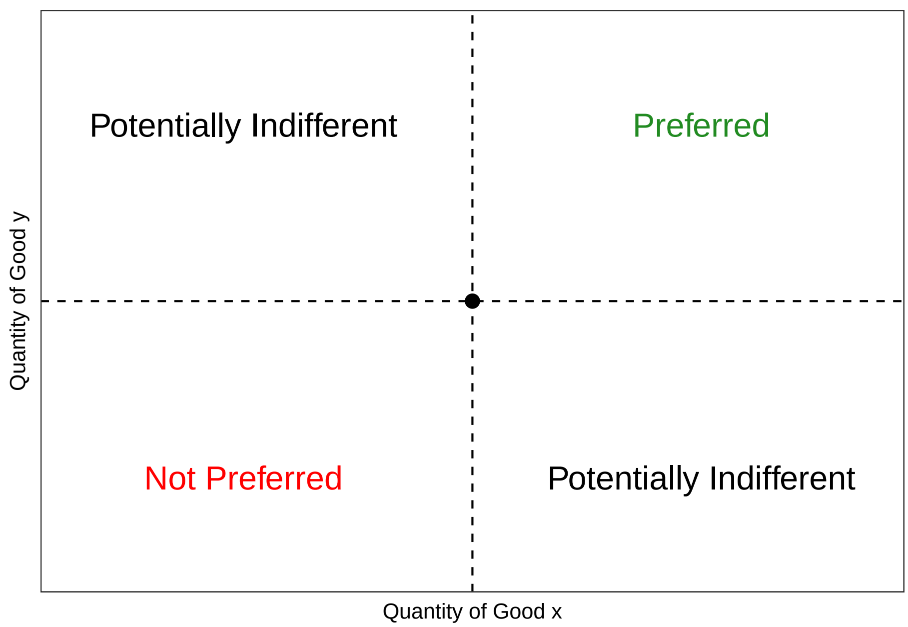

Figure 2.4 represents—for a given bundle of goods x and y—the areas of preference. The "Preferred" area gives more of both goods and hence, results in a higher level of utility due to the non-satiation assumption. The opposite is true for "Not Preferred" area. The "Potentially Indifferent" bundles may or may not lie on the same indifference curve than the initial bundle.

2.3 Consumer Choice

Consumers maximize their utility subject to their budget constraint. Deriving the optimal consumption bundle involves the use of calculus and for the purpose of this text, a graphical derivation is used.

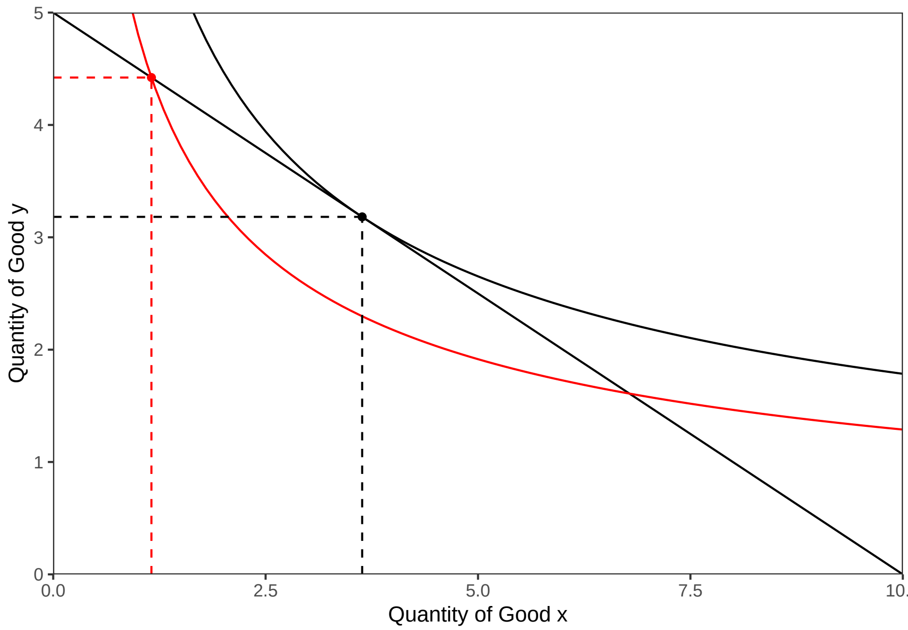

Figure 2.5: Optimal choice by the consumer.

Note that the slope of the budget constraint is

$$\frac{\Delta q_y}{\Delta q_x} = -\frac{p_x}{p_y}$$

The marginal rate of substitution (MRS) is the slope of the indifference curve:

$$MRS_{x,y} = -\frac{\Delta q_y}{\Delta q_x}\bigg|_{\Delta U=0}$$

So the optimality condition, i.e., optimal choice is

$$MRS_{x,y} = \frac{p_x}{p_y}$$

2.4 Derivation of the Demand Curve

Remember that the goal of utility theory is to derive the demand function of a product. The figure below illustrates the derivation of the demand function

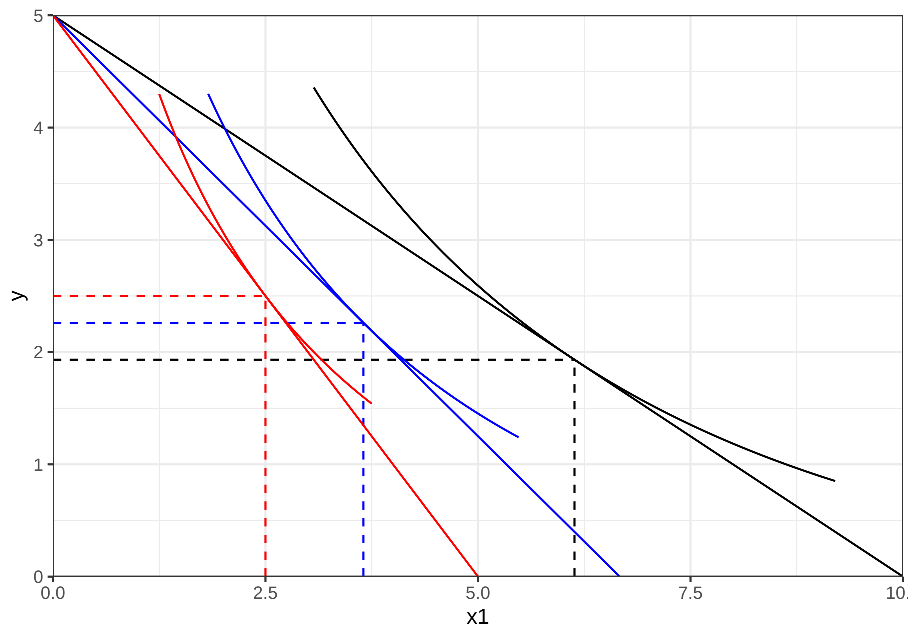

Figure 2.6: Effects of changing price for good x on optimal choice and quantity demanded for CES utility function.

for good x for two examples of utility functions that are used frequently in economics: The Constant Elasticity of Substitution (CES) and the Cobb-Douglas utility functions. If we start out with the optimal choice given initial prices for both goods and the optimal choice (the point where the indifference curve is tangent to the budget constraint), we have the consumption of x for a particular price p_x. If we start to increase the price, the original consumption bundle will not be achievable anymore and the consumer has to choose a new consumption point given the new price for p_x. This gives us a second point of the demand curve. If we continue this process, we can trace out the entire demand function for x for all prices.

A previous section introduced the Cobb-Douglas and Constant Elasticity of Substitution (CES) utility functions and their indifference curves. It can be shown that the demand functions associated with the Cobb-Douglas utility function are written as follows:

$$Q_x = \frac{\alpha}{\alpha + \beta} \cdot \frac{M}{P_x}$$
$$Q_y = \frac{\beta}{\alpha + \beta} \cdot \frac{M}{P_y}$$

And for the CES utility, the demand functions are written as follows with $\sigma =$

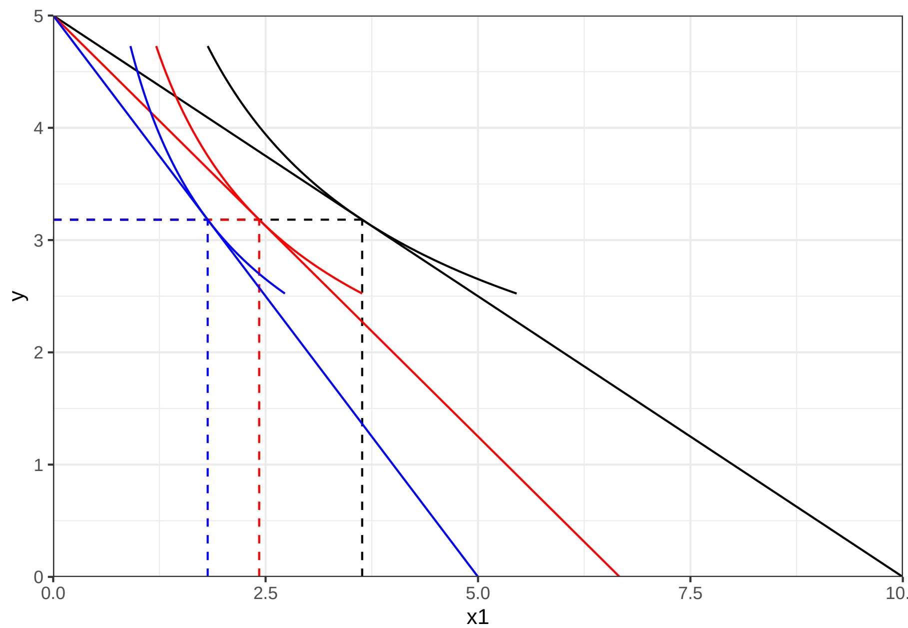

Figure 2.7: Effects of changing price for good x on optimal choce and quantity demanded for Cobb-Douglas utility function.

$1/(1-\rho)$:

$$Q_x = \frac{\alpha}{P_x} \cdot \frac{M}{\alpha^\sigma \cdot P_x^{(1-\sigma)} + (1-\alpha)^\sigma \cdot P_y^{(1-\sigma)}}$$
$$Q_y = \frac{1-\alpha}{P_x} \cdot \frac{M}{\alpha^\sigma \cdot P_x^{(1-\sigma)} + (1-\alpha)^\sigma \cdot P_y^{(1-\sigma)}}$$