Producer Theory
Jerome Dumortier
Production Functions
Concepts
Production with Two Inputs
Production in the Short- and Long-Run
Short-Run Production
Long-Run Production
Profit Maximization
Production Function
Marginal Cost and Marginal Revenue

Producer Theory

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Overview

Goal of producer theory

- Derivation of the supply function (ultimately derived in the chapter “Cost Theory”)
- Explaining changes in output based on changes in input prices

Components

- Production function
- Production in the short- and long-run
- Profit maximization

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Producer Theory vs. Consumer Theory

Similar concepts between producer and consumer theory

- Preferences $\Leftrightarrow$ Production technology
- Budget constraint $\Leftrightarrow$ Total cost function (unconstrained)
- Consumption choices $\Leftrightarrow$ Production choices
- Indifference curves and budget lines become isoquants and isocost lines

Production and Production Functions

Production

- Process of combining inputs to produce goods and services

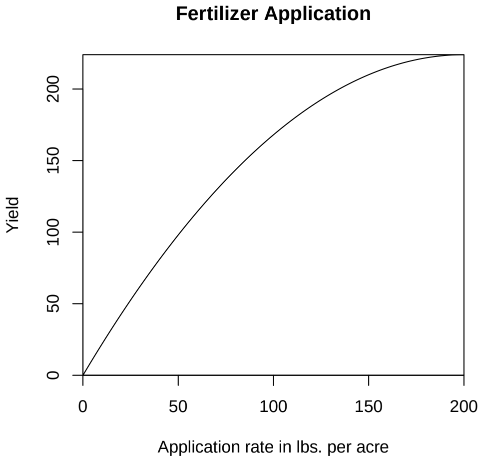
Production function

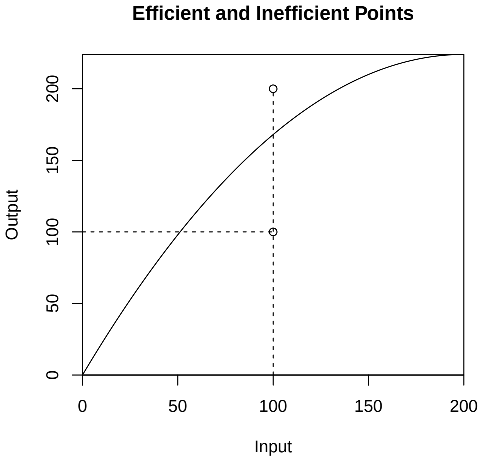
- Maximum amount of output a firm can produce over some period of time from each combination of inputs

Broad applications of production technology

- Production of chairs (output) as a function of inputs (e.g., wood, nails)
- Production of an exam score in a university class based on inputs such as ability and time studying (among others)
- Fertilizer input to produce crop yield

Crop Yield and Nitrogen Fertilization





Cobb-Douglas Production Function

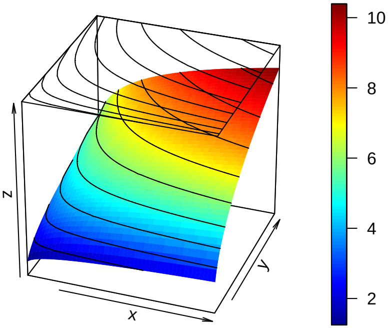
Cobb-Douglas production function with an added parameter A representing total factor productivity (TFP):

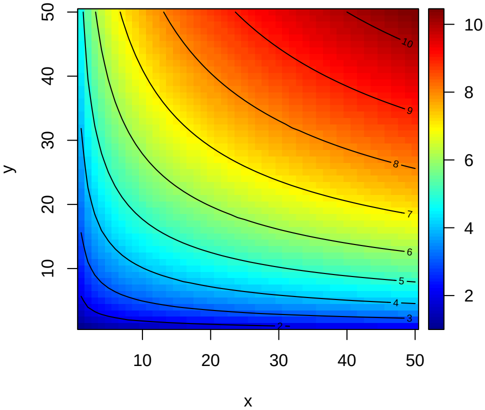
$$Q = f(K, L) = A \cdot L^{\alpha} \cdot K^{\beta}$$

Modeling of returns to scale with Cobb-Douglas

- Constant returns to scale: $\alpha + \beta = 1$
- Increasing returns to scale: $\alpha + \beta > 1$
- Decreasing returns to scale: $\alpha + \beta < 1$

Cobb-Douglas Function: Graphical Representation





Cobb-Douglas Function: Returns to Scale

α	β	L	K	Q	t	$f(t \cdot K, t \cdot L)$	$f(t \cdot K, t \cdot L)/Q$
0.4	0.6	10	20	151.57	2	303.14	2.00
0.8	0.4	10	20	209.13	2	480.45	2.30
0.2	0.3	10	20	38.93	2	55.06	1.41

Table 1: Illustration of economies of scale for a Cobb-Douglas production function.

Production in the Short- and Long-Run

Common inputs in the production process

- Workers, energy, machinery, factories, etc.

Short-run

- Fixed inputs cannot be adjusted as output changes in the short run
- Examples of fixed inputs: Machinery and factories
- Classification of fixed input as capital (K)

Long-run

- A time horizon long enough for a firm to vary all of its inputs
- Variable inputs can be adjusted up or down as the quantity of output changes and/or input prices change

Short-Run Production Function

Consider the following production function

$$Q = 10 \cdot K^{0.5} \cdot L^{0.5} = 10 \cdot \sqrt{K \cdot L}$$

Capital fixed at $\bar{K} = 9$, then the short-run production function is written as

$$Q = 30 \cdot \sqrt{L}$$

Or more generally

$$Q = f(\bar{K}, L)$$

Average and Marginal Product of Labor

Marginal product of labor: Additional output produced from one additional worker

$$MPL = \frac{\Delta Q}{\Delta L}$$

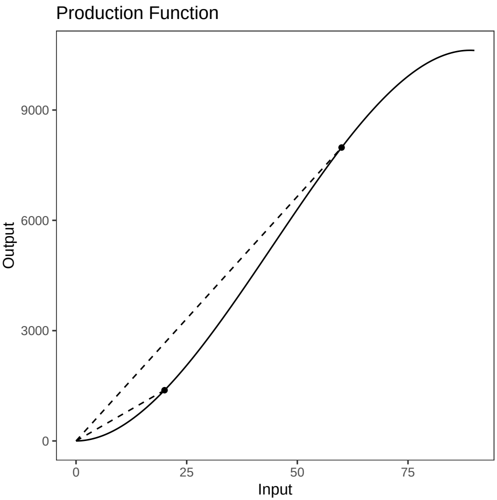
Average product of labor: Average quantity per worker

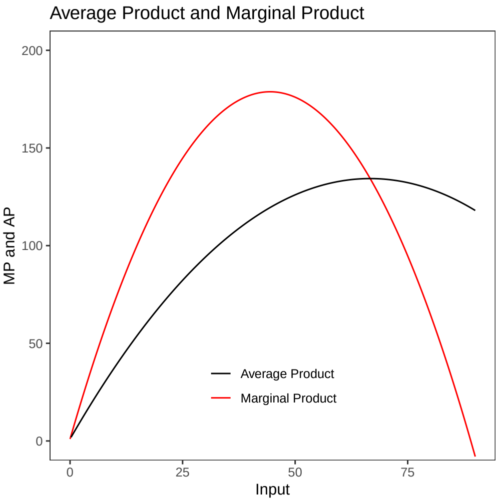
$$AP = \frac{Q}{L}$$

Law of diminishing marginal product

- As more of any input is added to a fixed amount of other inputs, its marginal product eventually declines.

Marginal and Average Product: Graphical Analysis





Isocost

Let w be the wage per worker (per unit of L) and let r be the rental rate of capital per unit of K. The total cost of production given a certain level of K and L can be written as follows:

$$TC = w \cdot L + r \cdot K$$

$$K = \frac{TC}{r} - \frac{w}{r} \cdot L$$

Marginal Rate of Technical Substitution

Marginal rate of technical substitution

- The rate at which a firm can substitute one input for another while keeping output constant.

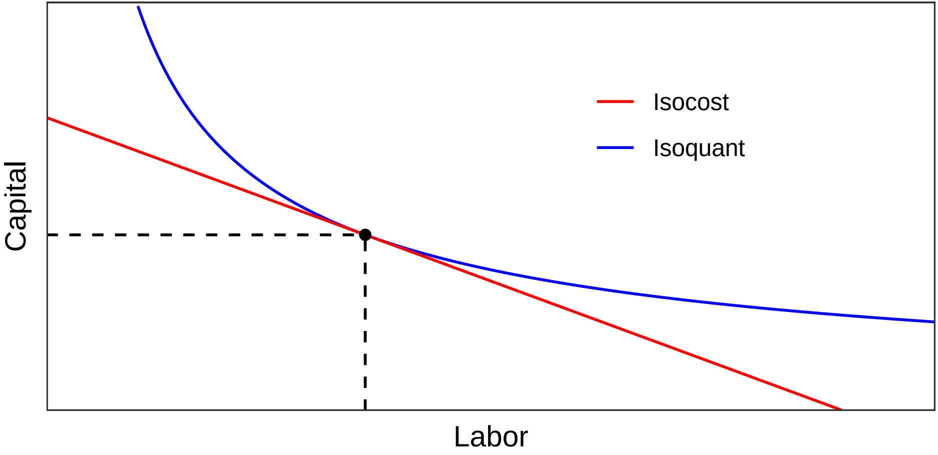
Marginal product of capital and labor

$$MP_K = \frac{\Delta Q}{\Delta K}$$
$$MP_L = \frac{\Delta Q}{\Delta L}$$

Marginal rate of technical substitution, i.e., slope of the isoquant

$$MRTS = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

Optimal Production



One Input and One Output

Two approaches

1. Profit maximizing input choice based on production function
2. Profit maximization using marginal revenue and marginal cost

Production function

$$Q = 30 \cdot \sqrt{L}$$

Parameters

- Output price: $p = 5$
- Wage per worker: $w = 10$

Production Function: Solution

Maximizing profit (π) with respect to L

$$\pi = p \cdot f(L) - w \cdot L$$

Solution

$$p \cdot f'(L) = w \quad \Leftrightarrow \quad f'(L) = \frac{w}{p}$$

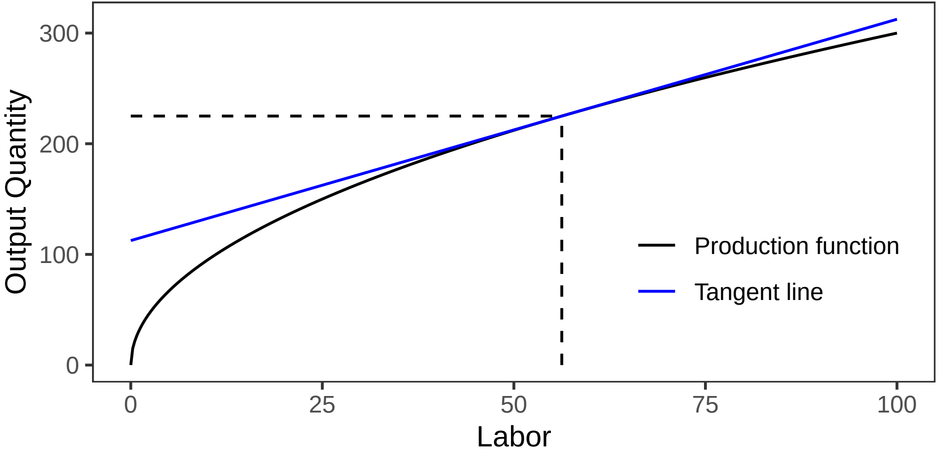
where $f'(L)$ represents the slope of the production function. Put differently

$$\pi = p \cdot Q - w \cdot L$$

Solving for Q

$$Q = \frac{\pi - w \cdot L}{p} = \frac{\pi}{p} + \frac{w}{p} \cdot L$$

Production Function: Graphical Interpretation



Setup

Profit maximization as a function of output

$$\pi(Q) = R(Q) - C(Q)$$

where $R = p \cdot q$. Cost as a function of output and marginal cost

$$C(Q) = w \cdot \frac{Q^2}{900}$$

$$MC(Q) = 2 \cdot w \cdot \frac{Q}{900}$$

Solution: MR(Q)=MC(Q)

Marginal Revenue

$$\frac{\Delta R}{\Delta Q} = p$$

Marginal Cost

$$\frac{\Delta C}{\Delta Q} = MC$$

Profit maximization condition is

$$MC(Q) = MR(Q)$$

True for any market structure with differences across market structures due to marginal revenue

MR(Q)=MC(Q): Graphical Interpretation