Risk and Uncertainty

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Introduction

Often interchangeable use of the terms "risk" and "uncertainty"

- Risk: Outcomes are random but probabilities are known (e.g., insurance, lotteries)
- Uncertainty: Probabilities are unknown or ambiguous (e.g., pandemic, war, political change)

Public management relevance

- Decision-making in budgeting, project evaluation, and regulation under uncertain (economic) outcomes
- Risk and uncertainty about macroeconomic development

Most decisions made are forward-looking and subject to risk and uncertainty

- Example: Price and income changes, weather uncertainty

Risk and Uncertainty for Consumers and Firms

Examples of risks affecting consumers and firms

- Evolution of retirement funds
- Future gasoline, electricity, and natural gas prices
- Occurs of adverse events such as accidents or natural disasters

Economic questions associated with risk

- Quantification of risk
- Decision making under uncertainty
- Existence of a market and price for risk
- Risk reduction and management

Probability Distribution

Link between outcomes and associated probabilities

- Probability associated with each outcome
- Sum of probabilities across all different outcomes equals 1

Example: Investment opportunity in one of two mutually exclusive projects

		Profits	
State of the Economy	Probability	Project A	Project B
Recession	40%	4000	0
Normal	35%	5000	5000
Boom	25%	1000	8000

Expected Value

Random variable (X)

- Profit from investing in either project A or B

Expected value with i indicating outcomes

$$E[X] = \sum_i X_i \cdot Pr(X_i)$$

Expected value of projects A and B:

$$E(X_A) = 0.4 \cdot 4000 + 0.35 \cdot 5000 + 0.25 \cdot 1000 = 3600$$
$$E(X_B) = 0.4 \cdot 0 + 0.35 \cdot 5000 + 0.25 \cdot 8000 = 3750$$

Higher expected payoff for project B

- Possibility of no payoff at all in case of a recession for project B

Variance and Standard Deviation

Variance associated with random variable X

$$Var(X) = \sum_i [X_i - E(X)]^2 \cdot Pr(X_i)$$

Standard deviation (σ) associated with random variable X

- Square root of the variance

Variance and standard deviation of projects A and B

$$Var(X_A) = 2,440,000 \Rightarrow \sigma_A = 1560$$
$$Var(X_B) = 10,687,500 \Rightarrow \sigma_B = 3269$$

Higher standard deviation of project B compared to A

- In general: Trade-off between return and variance (e.g., stock market)

Risky Game

Consider a casino offering the following game

- 5 coin flips with the payout (P) to the player determined by the number of heads (h): $P = h^2 - 4 \cdot h$
- Profit ($P > 0$) or loss ($P < 0$) to the player

Payoff table

Heads	0	1	2	3	4	5
Payoff	0.00	-3.00	-4.00	-3.00	0.00	5.00
Probability	0.03	0.16	0.31	0.31	0.16	0.03

Expected value to the player: $-\$2.50$, i.e., loss from perpetual play

Motivation

Consider three games that each cost \$10 to play

1. You receive \$10
2. You flip a coin and receive \$0 for heads and \$20 for tails
3. You roll a die and get \$60 for a six and \$0 otherwise

Payoff for each game is \$10 (\$0 after deducting the cost to play) but people clearly have preferences which game to play

- Note: Standard deviation for games 1–3 are \$0.00, \$10.00, and \$22.36, respectively

Expected Utility

Expected utility

- Probability-weighted sum of utilities associated with all possible states (i.e., outcomes)
- Differentiation between risk neutral, risk averse, and risk seeking individuals

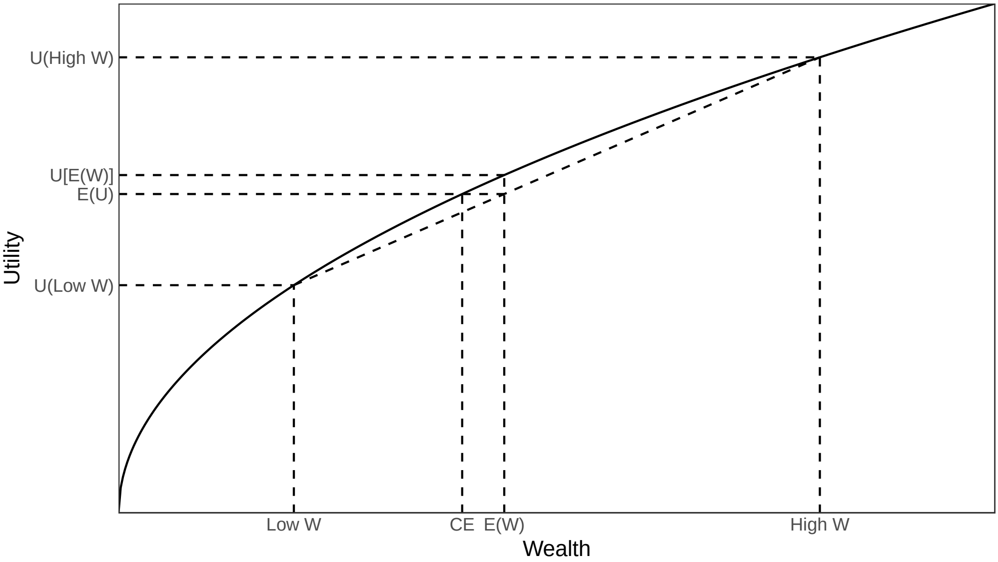
Jensen's inequality with w designating wealth

- $E[U(w)] = U(E(w)) \Rightarrow$ Risk neutral
- $E(U(w)) < U(E(w)) \Rightarrow$ Risk averse

Risk premium

- Maximum amount of money a risk-averse person will pay to avoid taking a risk.

Risk Preferences: Graphical Representation



Expected Utility: Example

Two possible outcomes

- Getting \$1 with a probability of 60%
- Getting \$8 with a probability of 40%

Expected value of wealth

$$E(w) = 0.6 \cdot \$1 + 0.4 \cdot \$8 = \$3.80$$

Expected utility

$$E(U) = 0.6 \cdot U(\$1) + 0.4 \cdot U(\$8)$$

We also have $U(\$3.8)$. Note that

$$u(\$3.8) > 0.6 \cdot U(\$1) + 0.4 \cdot U(\$8)$$

Risk Diversification

Two investment possibilities

1. Coat company: $p_0^C = \$10$ and $p_1^C = \$5$ if sunny or $p_1^C = \$20$ if rainy
2. Sunglasses company: $p_0^G = \$10$ and $p_1^G = \$20$ if sunny or $p_1^G = \$10$ if rainy

Diversification given cold weather and sunshine chances of 60% and 40%, respectively

- 50-50 diversification $\Rightarrow$ \$12.50

There are gains from diversification as long as assets are not perfectly correlated

- Stock market
- Mutual funds

Application: Insurance Market

Generally: Risk aversion for individuals and firms

- Why do insurance companies exist?

Coin flip with two possible outcomes: Heads or tails

- Key condition: independence
- Expected value of heads (or tails): $E(H) = E(T) = 0.5$

Variance of N coin flips:

$$Var(N) = \frac{P \cdot (1 - P)}{N}$$

Examples:

- $Var(1) = 0.5$, $Var(10) = 0.025$, or $Var(1000) = 0.00025$

Difficulty predicting share of heads from single coin flip but not from 1000 flips

Insurance Market: Setup

Example:

- $Pr(fire) = 1/250$
- House value: $250,000
- Value of the house after fire: $0
- Insurance premium: $1000 (expected loss).

Simulation

1. Simulate the damage of n homeowners.
2. Calculate the share.
3. Repeat 1,000 times.
4. Generate histogram.

Insurance Market: Simulation Results