Game Theory

Jerome Dumortier

Overview

Game theory models players–such as individuals, firms, or political parties–making decisions strategically in response to the actions of others

- Pricing strategies among firms competing in an oligopolistic market
- Positions of political parties on issues before elections
- Nuclear deterrence (1983 Movie WarGames)

Goal of game theory: Predict and design behavior in strategic settings

Examples in Economic Policy

Game theory applications

- Tax competition: Jurisdictions choose rates strategically
- Climate agreements: Countries decide emission levels anticipating others' choices
- Oligopoly pricing: Firms set prices or quantities strategically
- Public goods: Individuals decide contribution levels under interdependence

Everyday example (self-fulfilling prophecy)

- Early in a crisis (e.g., pandemics), consumers decide whether to hoard or buy normally
- If everyone buys normally: No shortage
- If some hoard: Others face empty shelves unless they hoard too
- Rational response: Hoard early to avoid being left out

Game Components

Players: Number of players (at least 2) who are decision makers

- Examples: Firms, countries, people

Actions: Set of finite actions for each player

- Example of pricing actions of firms in a competitive environment: (1) Keep prices at current levels or (2) Cut prices likely triggering a price war

Payoffs for each player

- Utility or benefits depend on one's own action as well as the actions of others
- Payoffs can at least be ranked ordinally
- Payoffs may be in the form of a change in (marginal) utility, revenue, profit, or some non-monetary change in satisfaction

Cooperative versus Non-Cooperative Games

Strategic interdependence

- Player's welfare as a function of own action and actions of other players
- Optimal action of a player depends on expectations of what other players do

Non-cooperative game theory

- No binding agreements among players
- Cooperation may or may not emerge as a result of rational, self-interested behavior (e.g., trigger strategies)
- Example: Two sports teams competing in a match

Complete versus Incomplete Information I

Complete information leads to all players knowing the following

- Structure of the game (players, strategies, payoffs)
- Payoff functions of all players
- Order of play and information sets

Examples

- Prisoner's Dilemma
- Cournot or Bertrand competition
- Entry deterrence with known costs

Common knowledge: Each player knows that everyone else knows all of the above

- Uncertainty only from strategic interaction and not from private data and information

Complete versus Incomplete Information II

Incomplete information arises from at least one player having private information unknown to others

- Payoff function or type (e.g., high-cost vs. low-cost firm)
- Available actions or constraints
- Beliefs about external conditions (e.g., policy environment)

Other players must form beliefs about unknown information

Nash Equilibrium

Central idea in non-cooperative game theory

- Stable outcome where no player benefits from changing their own action
- Each player chooses the best response to others' choices
- No player has an incentive to deviate unilaterally

Characteristics

- Mutual best responses among all players
- Players' strategies are consistent and expectations are fulfilled
- Each player acts rationally given others' behavior
- Describes strategic stability and not cooperation

Identification of a Nash Equilibrium

- List each player's possible strategies
- Determine best responses for each combination of others' choices
- Intersection of best responses with resulting combination as Nash Equilibrium

2001 Movie *A Beautiful Mind*

If we all go for the blonde and block each other, not a single one of us is going to get her. So then we go for her friends, but they will all give us the cold shoulder because no one likes to be second choice. But what if none of us goes for the blonde? We won't get in each other's way and we won't insult the other girls. It's the only way to win.

Economic Models and Game Theory

Economic models based on game theory as abstractions from strategic interaction of agents

- Ability to model tractable interactions
- Ability to produce implications and draw conclusions that can be used to understand real-world strategic interactions

Role of game theory in economic modeling

- Extends traditional models by adding strategic interdependence
- Each agent's payoff depends on the actions of others
- Captures competition, cooperation, and negotiation within formal structure
- Links individual rationality with collective outcomes
- Core framework for understanding incentives and interaction

Strategy

A player's strategy is their complete contingent plan

- If it could be written down, any other agent could follow the plan and duplicate the player's actions
- Thus, a strategy is a player's course of action involving a set of actions (moves) dependent on actions of other players

A unique equilibrium or a set of equilibria may occur within a set of strategies; this is called a Nash equilibrium (after mathematician John Nash)

- Not all games have a Nash equilibrium, and some games may have multiple Nash equilibria

Prisoner's Dilemma: Setup

Situation: Arrest of two suspects for a joint crime

- Separate interrogation of both suspects
- Prosecutors lack enough evidence for a major conviction without a confession but have evidence for a smaller crime leading to prison time

Identical deal for each prisoner

- Confession by one with the other remaining silent: Confessor goes free and silent partner gets heavy sentence
- Confession by both: Moderate sentence for both due to cooperation
- Both remaining silent: Light sentence on minor charges for both

Prisoner's Dilemma: Payoff Matrix

		Player B	
		Deny	Confess
Player A	Deny	1,1	20,0
	Confess	0,20	5,5

Prisoner must decide without knowing what the other does

- Rational reasoning: Confessing (dominant strategy) is better regardless of what the other does

Outcome: Both confess and get moderate sentences despite worse outcome compared to remaining silent

Cartel Game

Cartels as repeated games

- Repeated games in which firms cooperate to keep prices high but face constant incentives to cheat for short-term gain
- Collusion is fragile and depends on trust, monitoring, and the expectation of future interactions
- Antitrust policies, especially leniency programs, exploit these strategic tensions by encouraging defection and transforming stable cooperation into a one-shot prisoner's dilemma that breaks the cartel

		Firm B	
		Don't Cheat	Cheat
Firm A	Don't Cheat	50,50	45,54
	Cheat	54,45	48,48

Rock, Paper, Scissors

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	Draw, Draw	Loss, Win	Win, Loss
	Paper	Win, Loss	Draw, Draw	Loss, Win
	Scissors	Loss, Win	Win, Loss	Draw, Draw

Stag Hunt

		Hunter B	
		Stag	Hare
Hunter A	Stag	2,2	0,1
	Hare	1,0	1,1

Dominant and Mixed Strategies

Dominant strategy

- One strategy is preferred to another no matter what other players do
- When all players have a dominant strategy, an equilibrium of dominant strategies exists that is determined without a player having to consider behavior of other players

		Player B	
		Action 1	Action 2
Player A	Action 1	10,10	7,7
	Action 2	7,7	5,5

Mixed Strategy

		Firm B	
		High Price (λ)	Low Price ($1-\lambda$)
Firm A	High Price (ρ)	60,40	10,70
	Low Price ($1-\rho$)	30,30	50,20

Mixed strategy equilibrium for Firm A

$$40 \cdot \rho + 30 \cdot (1-\rho) = 70 \cdot \rho + 20 \cdot (1-\rho) \Rightarrow \rho = 1/4$$

Mixed strategy equilibrium for Firm B

$$60 \cdot \lambda + 10 \cdot (1-\lambda) = 30 \cdot \lambda + 50 \cdot (1-\lambda) \Rightarrow \lambda = 4/7$$

Trigger Strategy I

Identification of optimal strategy for an infinitely repeated game where players interact over time rather than just once

- Each player cooperates initially and continues cooperating as long as everyone else does
- If any player deviates from the cooperative action, the others “trigger” a punishment phase
- The punishment is often permanent (grim trigger) or temporary (tit-for-tat or finite punishment)
- The threat of future punishment makes cooperation rational if players value future payoffs highly enough

Trigger Strategy II

Key idea

- Fear of losing long-term gains outweighs the short-term benefit of cheating

Trigger strategies can sustain outcomes that would not be equilibria in one-shot games, such as mutual cooperation in a prisoner's dilemma. Nash equilibrium in a static game is always a Nash Equilibrium in a repeated game.

Trigger Strategy: Cartel

		Firm 2	
		High Price	Low Price
Firm 1	High Price	12,12	6,17
	Low Price	17,6	8,8

Trigger strategy

- One-time win from low price: 5
- Loss in repeated games: 4 (discount rate becomes important)

Sequential Games: Overview and Examples

In a sequential (dynamic) game, one player knows the other player's choice before taking their action (perfect information)

- Entry deterrence: An incumbent firm decides whether to fight or accommodate after a potential competitor chooses to enter the market
- Investment or trust game: One party invests or sends money first, and the second party decides how much to return
- Ultimatum game: A proposer offers how to split a sum of money, and a responder either accepts (both get the proposed shares) or rejects (both get nothing)

Additional Examples

- Bargaining between unions and firms: One side proposes a wage or contract, and the other accepts or rejects
- Political negotiation: A government proposes a policy, and an opposition party chooses to support or block it
- Stackelberg competition: One firm sets output first, and a follower firm observes this and chooses its own output afterward
- Environmental regulation: A regulator sets a standard, and firms decide how to comply or whether to lobby for changes
- Foreign policy crisis: One country makes an initial threat or action, and others respond after observing that move

Backward Induction

Use a technique called backward induction- to determine a subgame perfect Nash equilibrium, by working backward toward the root in a game tree

- Once the game is understood through backward induction, players play it forward
- To apply backward induction, first determine optimal actions at last decision nodes that result in terminal nodes
- Then determine optimal actions at next-to-last decision nodes, assuming that optimal actions will follow at next decision nodes
- Backward induction implicitly assumes that a player's strategy will consist of optimal actions at every node in the game tree

First-Mover Advantage

Firms who act first have an advantage lead to preemption games

- Strategic pre-commitments can affect future payoffs
- For example, a firm adopting a relatively large production capacity in a new market can saturate the market and make it difficult for ensuing firms to enter
- Ability to seize a market first depends on the market's contestability
- Governments, concerned with the ability of firms to saturate a market and forestall entry of other firms, have attempted to place restrictions on such behavior