

2.5 Income and Substitution Effect

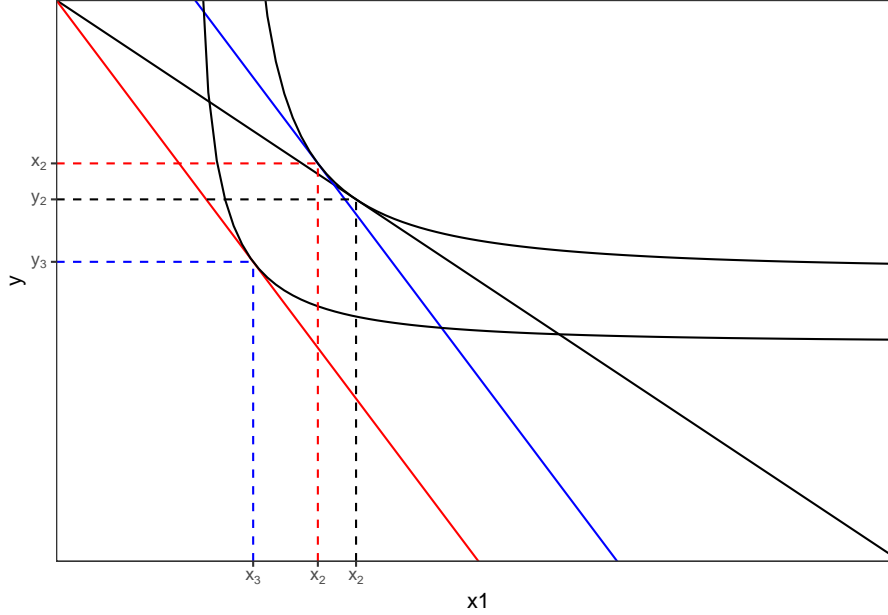


Figure 2.10: Income and (Hicks) substitution effect of an increase in price of good x . The original quantities consumed are x_1 and y_1 . The price increase makes good x relatively more expensive. This substitution effect is displayed by going from x_1 to x_2 . The income effect is x_2 to x_3 .

A change in the price of a good can be decomposed into an income and a substitution effect. That is, a price change makes a good relatively more (or less) expensive relative to other goods resulting in the income effect. The consumer also changes the allocation to the good in question and hence, the allocation to other goods, which is called the substitution effect.

Consider the example of a consumer who has an income of 100 faces prices $P_x = 10$ and $P_y = 20$. The utility function of the consumer is written as follows:

$$U(Q_x, Q_y) = (0.4 \cdot Q_x^{-2} + 0.6 \cdot Q_y^{-2})^{(-1/2)}$$

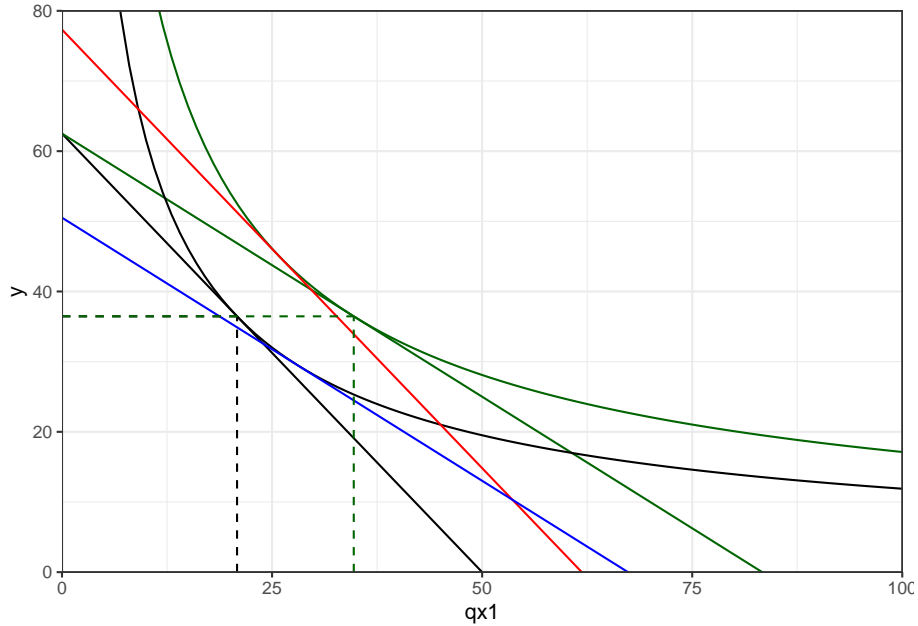
Given those preferences, income, and prices, the optimal allocation Q_x and Q_y is 3.55 and 3.23, respectively, which gives the consumer a utility of 3.34. If the price of good x increases by \$15, then the optimal allocation Q_x and Q_y is 2.33 and 2.67, respectively, which gives the consumer a utility of 2.52. The change in consumption from x_1 to x_2 is considered the total effect. Next, we are going to divide the total effect into the income and substitution effect. To do so, we have to increase the income of the consumer under the new prices such that

their income is getting them back to the old utility function.

2.6 Compensating and Equivalent Variation

Compensating variation (CV) and equivalent variation (EV) are two measures in welfare economics used to evaluate changes in consumer well-being when prices change. CV is the amount of money that would need to be given to or taken away from a consumer after a price change to make them as well off as they were before the change. In contrast, EV is the amount of money that would need to be given to or taken away from a consumer before the price change to make them as well off as they would be after the change. While both capture the monetary value of a change in utility, CV uses the new utility level as a reference point whereas EV uses the original utility level. Thus, CV and EV can differ depending on whether the price change is beneficial or harmful.

To illustrate the difference between CV and EV, consider a consumer with income $M = 300$. The original prices are $P_x = P_y = 4$ and the new price levels are $P_x = 2$ and $P_y = 4$. That is, the price of good x decreases.



2.7 Market Demand

To obtain the market demand for a good, the individual demand functions have to be added horizontally. For example, consider three consumers with the following demand functions: $Q_1 = 10 - 2 \cdot P$, $Q_2 = 24 - 3 \cdot P$, and $Q_3 = 15 - P$.

The inverse demand functions can be written as:

$$P = 5 - \frac{Q_1}{2}$$

$$P = 8 - \frac{Q_2}{3}$$

$$P = 15 - Q_3$$

The inverse demand functions are valid $Q_1 \in [0, 10]$, $Q_2 \in [0, 24]$, and $Q_3 \in [0, 15]$.

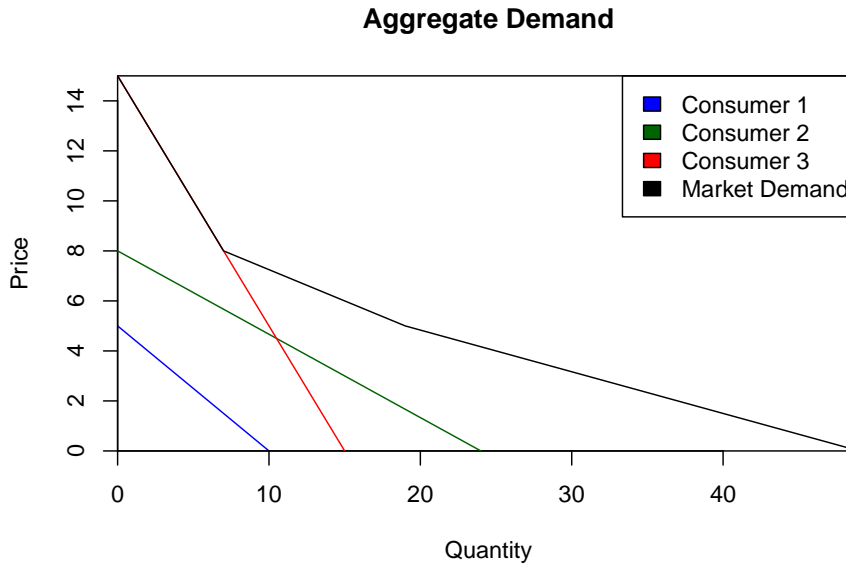


Figure 2.11: Derivation of the market demand curve given three individual demand functions.

2.8 Policy Analysis with Consumer Theory

This section covers some theoretical examples that can be translated to policy decisions as well as some applied examples from the economics literature.

2.8.1 Theoretical Examples

Switching food delivery services: Pamela uses a food delivery service which sends you the ingredients for meals and she just needs to combine and cook them at home for a nice meal. Currently, she purchases this service 20 times a month. Her income is \$1,000 per month and the meals cost \$20 per meal. All

other goods cost \$10 per unit. A new online service offers similar meals but charges a flat fee of \$200 per month plus \$10 per meal. This package lets her consume the same amount of food. However, because the slope of the budget constraint is different, the initial choice is not optimal anymore. By reallocating her consumption, she can get on a higher indifference curve.

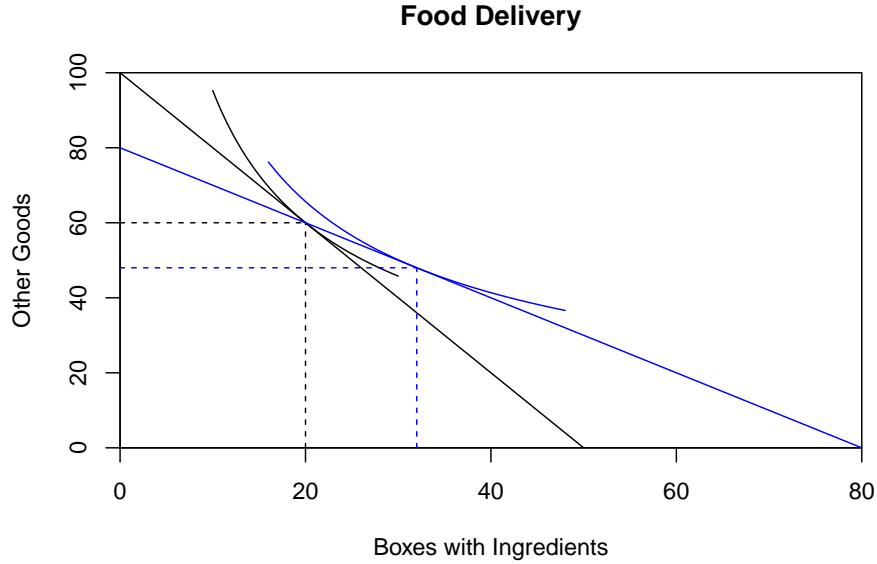


Figure 2.12: Theoretical examples about individual choice. Panel (a) demonstrates the switch from a per-unit purchasing system to a scheme which has a fixed cost and a lower per-unit price for subsequent purchases. Panel (b) contrasts the difference between a lump-sum tax and a per-unit tax.

Policy Example of a Lump-Sum Tax versus a Per-Unit Tax: A per-unit tax is the institution of an excise tax on some (but not all) goods a consumer purchases. A lump-sum tax is the collection of a single sum, independent of the consumer's choices. Assume the following notation: p_x and p_y as the price of goods x and y , M as the income, t as the per-unit tax, T as the lump-sum tax. Table 2.1 and panel "Lump-Sum vs. Per-Unit Tax" in Figure 2.13 represent this problem. The graph in figure 2.13 is based on the following parameters: $M = 100$ (income), $P_x = 3$, $P_y = 2$, $t = 3$ (per unit tax), and the utility function is written as $U(x, y) = Q_x^a \cdot Q_y^{1-a}$ with $a = 0.4$.

Subsidy for Low-Income Housing: Suppose you have to decide how to subsidize low-income housing. Assume that the family has an income of \$1000 and that the price of other goods is \$1. There are two subsidy plans: (1) a dollar-for-dollar subsidy or (2) a lump-sum payment. The initial consumption on

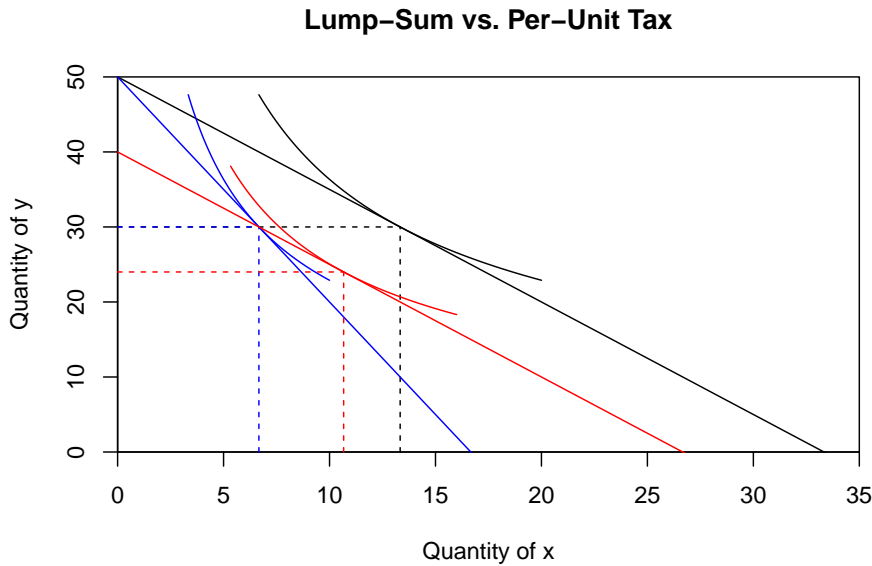


Figure 2.13: Taxation

	Budget line	Choice
Original	$P_x \cdot Q_x + P_y \cdot Q_y = B$	(x_1^*, y_1^*)
Per-unit	$(P_x + t) \cdot Q_x + P_y \cdot Q_y = B$	(x_2^*, y_2^*)
Lump-sum	$P_x \cdot Q_x + P_y \cdot Q_y = B - T$	(x_3^*, y_3^*)

Table 2.1: Comparison in budget constraint without a tax (Original), a per-unit tax, and a lump-sum tax.

Option	Condition 1		Condition 2	
	Drop. Calls per 100	\$/Year	Drop. Calls per 1000	\$/Month
A	4.2	384	42	32
B	6.5	324	65	27

Table 2.2: Two cell phone plans presented to consumers with different rates of dropped calls and cost. Note that the plans A and B are identical and that only the scaling of the interruptions and cost is changed.

housing is \$500 (\$250 from personal income and \$250 subsidy from government). How does this differ from food stamps?

2.8.2 Policy Examples

Economists assume that consumers are rational agents who—given all information—make informed decisions. This may not be true even in everyday situations as illustrated in the first three examples below. On the other hand, the law of diminishing marginal utility may have some practical implications as demonstrated in the last example.

Consumer Rationality: [Burson et al. \(2009\)](#) demonstrate how people reverse their preferences if faced with numbers that are scaled differently. In their experiment, they have respondents choose a cell phone plan (Table 2.2). Under condition 1, 31% favor A and 53% favor B. Under condition 2, 69% favor A and 23% favor B. Note that both plans are identical (e.g., $\$32 \cdot 12 = \384) and people are simply attracted to a smaller number. If you listen to or watch advertisement, you always hear “it’s only \$1 a day” and never “it’s only \$365 per year.”

MPG Illusion: The second example illustrates the concept of [MPG Illusion](#). Suppose that three people drive 10,000 miles per year. The fuel economy of the current cars are 10 MPG for person A, 16.5 MPG for person B, and 33 MPG for person C. They all trade-in their current cars for new cars that get 11, 20, and 50 MPG for A, B, and C, respectively (Figure 2.14). It turns out that all three consumers save the same amount of gasoline. Most people assume a linear relationship between MPG and gasoline consumption, which is not correct. The reverse measure gallons per mile does not suffer from this flaw.

Marginal Utility and Undernourishment: A paper by [Jensen and Miller \(2010\)](#), which was also covered in an article by The Economist titled [People’s spending choices are a good way to assess levels of hunger](#), questions the usefulness of a fixed calorie threshold to quantify undernourishment. Their argument is based on the observation that in some countries, caloric intake decreases while real income increases. Instead of measuring the number of calories consumed, researchers and policymakers should measure the amount of staples consumed

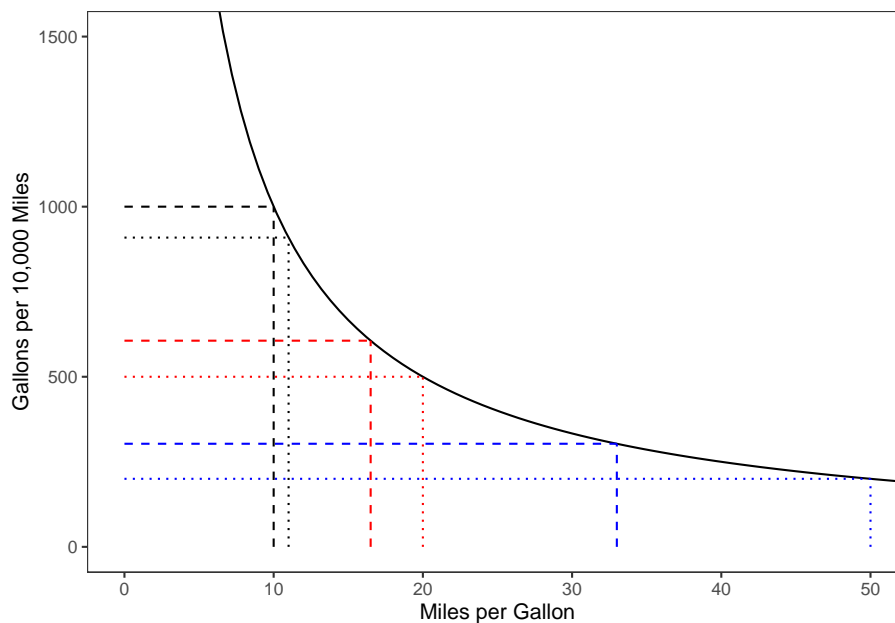


Figure 2.14: Gallons of gasoline consumed based on driving 10,000 miles per year. The different colored lines represent consumers switching from their current cars (dashed lines) to new cars (dotted lines).

such as rice and millet. Both goods are low cost sources of calories and thus, people consuming a large share of their total calories from those foods are likely to be malnourished. Consuming those foods results in a significant increase in utility very quickly and very cheaply. The authors in the article use the term staple calorie share as a measure of undernourishment.

Automatic Bill Payment and Electricity Consumption: [Sexton \(2015\)](#) assesses the effect of enrolling in automatic bill payment (ABP) on electricity consumption. ABP is a service that allows automatic deduction of the electricity bill from the consumer's bank account. The results show that ABP enrollment increases residential and commercial electricity consumption by up to 4.0% and 8.1%, respectively. The result is attributed to consumers not following closely their electricity consumption and associated cost anymore.

2.9 Exercises

For all questions, assume that quantities are perfectly divisible, i.e., purchasing 2.5 textbooks is possible.

1. **Quantity Discounts (***)**: The budget constraints are straight lines

if prices do not change over the entire range of quantities purchased. If there are quantity discounts, the budget constraints may be kinked and/or discontinuous. Draw the budget constraints for the following situations:

- a. $M = 100$, $P_y = 2$, and $P_x = 4$
 - b. $M = 100$, $P_y = 2$, and $P_x = 4$ if less than 10 units are bought and $P_x = 3$ for units after 10.
 - c. $M = 100$, $P_y = 2$, and $P_x = 4$ if less than 10 units are bought and $P_x = 3$ on all units if more than 10 are bought.
2. **Veal for a Vegetarian** (*): Draw the indifference curves of a vegetarian for veal and spinach.
 3. **Muesli** (**): Sunny is a spoiled kid and forces her daddy to add a tablespoon of nuts for each tablespoon of fruits to her Muesli. The price of nuts and fruit per tablespoon is \$0.10 and \$0.20, respectively. He allocates a total of \$3 for her entire breakfast. Draw her indifference curves for nuts and fruits. How many tablespoons are used for her breakfast each morning?
 4. **Revealed Preferences** (***) : To better understand preferences, researchers often present respondents with different price and income situations and ask them to choose between various consumption bundles (see Grebitus and Dumortier (2016) for an example). Consider the following two situations:
 - Situation 1: Income is \$100 and prices are $p_x = \$10$ and $p_y = \$20$. In this situation, the respondent has the choice between two bundles: Either bundle A with $Q_x^A = 2$ and $Q_y^A = 4$ or bundle B with $Q_x^B = 4$ and $Q_y^B = 3$. The consumer picks bundle A .
 - Situation 2: Income is \$80 and prices are $p_x = \$20$ and $p_y = \$10$. In this situation, the respondent has the choice between two bundles: Either bundle C with $Q_x^C = 1$ and $Q_y^C = 6$ or bundle A with $Q_x^A = 2$ and $Q_y^A = 4$. The consumer picks bundle C .

Rank the consumer's preferences for bundles A , B , and C from highest to lowest preference. Next, assume there is a fourth bundle D with $Q_x^D = 5$ and $Q_y^D = 5$. How does bundle D compare in terms of preference ordering compared to the other three bundles. Do you have all the information you need to determine bundle's D place in the ranking?

5. **Textbook Gift Voucher** (***) : Tom is in college and receives \$500 from his parents each semester to spend on textbooks and other goods. The price of books is \$20 and the price of other goods is \$10. Assume that the number of textbooks is a choice. In the initial situation, Tom purchases six textbooks and the rest is spend on other goods. Draw Tom's budget constraint and the indifference curve indicating the optimal choice. To ensure that Tom spends sufficient amounts of money on textbooks, his parents replace the cash payment of \$500 with a bookstore gift certificate

of \$300 and \$300 in cash. Draw the budget constraint under the new regime. Is Tom better off? On a new graph, draw a situation under which he is better off with a cash payment from his parents of \$600 than the \$300 cash/\$300 gift certificate split.

6. **Obesity in Pawnee** (**): Because of rampant obesity in Pawnee, the city government decides to tax candy. In the figure below, you have the original budget constraint with the optimal choice in the absence of any tax. The income of the consumer is \$100, the price of candy is \$2 per unit, and the price of other goods is \$1 per unit. Without the tax, consumers buy 20 units of candy and 60 units of other goods. With the tax of \$2 per unit, the consumer buys 10 units of candy and 60 units of other goods. How much does the city government collect in taxes? Assume that political pressure forces the city government to return the amount of the tax collected to the consumer in form of a lump-sum payment at the end of the year. Draw the new budget constraint given the lump-sum payment using the figure provided. Is the consumer as happy as in the case with neither tax nor subsidy. Interpret. See Figure for a template.
7. **Per-Diem Meal and Hotel Rates** (**): Suppose you work for a company which subsidizes your traveling with \$1,000 per month. However, you cannot exceed \$600 for hotel and \$400 for meals. Assume that everywhere you travel, the price for one night in a hotel is \$100 and the price of one meal is \$20. Assume that you always exhaust your budget for traveling. Draw the budget constraint for this problem with hotel on the horizontal axis and meals on the vertical axis. Explain to your boss why you would (most likely) be better off with \$1,000 but no restrictions on spending. Draw the relevant indifference curves.
8. **Church Roof** (**): Philip is a rich atheist but has a soft spot for the leaking roof of his hometown church. The church currently spends \$2,000 on the roof and \$8,000 on other activities.
 - a. Draw the budget constraint for his hometown church with roof repairs on the horizontal axis and other activities on the vertical axis. Assuming utility maximization of the church, draw the indifference curve corresponding to the current situation.
 - b. Philip offers the church a \$1,000 grant with the constraint that the money can only go towards roof repairs. Put differently, it cannot be used for other activities. Will the amount of money spent on roof repairs increase by exactly \$1,000 or will it change by a different amount. Justify your answers.
9. **Gasoline in Iran** (**): The Iranian government has [provided substantial subsidies on gasoline](#) to its citizens in past decades. After international sanctions were imposed on the country due to its suspect nuclear program, those subsidies were not sustainable in the long-run from a financial perspective. Thus Iran eliminated the subsidies but replaced them with a

cash transfer to ease the financial burden for its citizens. Assume that gasoline is on the horizontal axis and all other goods are on the vertical axis. The price of other goods does not change throughout the question.

- a. Draw the initial budget constraint and the optimal consumption bundle. Assume that both goods are initially consumed.
- b. How does the budget constraint change if the government eliminates subsidies on gasoline and the price of gasoline increase subsequently? Show this effect in the graph.
- c. Given the new prices, the government provides a subsidy that raises income such that the consumers are as happy as before. Illustrate in your graph.
- d. Why is this increase in income cheaper than the subsidy on gasoline? Illustrate in your graph.