Binary Choice Models

Jerome Dumortier

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Overview

Binary choice models and possible research questions

- Did you vote during the last election?
- Does an individual get arrested after being released from prison?
- Does an individual participate in the labor market?

Dependent variable y takes on one of two values: 0 or 1

Example

Consider the following equations

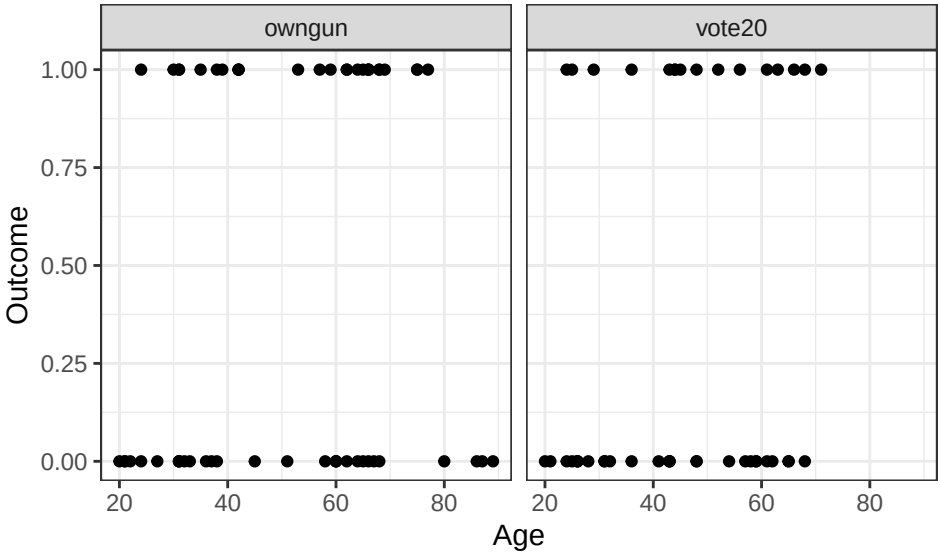
$$y = x^2$$
$$y = x^2 + \sqrt{x}$$

Questions

- What is the value of y if $x = 4$ or $x = 9$?
- What is the value of x if $y = 81$ or $x = 14$?

Need for numerical methods to determine the answer to the last question more generally for the second equation

Sample Binary Choice Data from the GSS



Linear Probability Model (LPM)

Most rudimentary model for binary choice

- Use of linear regression model, i.e., $y_i = \beta_0 + \beta_1 \cdot x_i + \epsilon$

Problems

- Possibility of $E(y_i|x_i) > 1$ or $E(y_i|x_i) < 0$
- Error terms are neither normally distributed nor homoscedastic

Alternative: Logit and Probit

- Calculation of the probability that the outcome equals 1 given exogenous variables, i.e, $Pr(y = 1|x)$

Common Setup for Logit and Probit Models

General assumption about some function $G(\cdot)$ for all values of z

$$0 \leq G(z) \leq 1$$

Let

$$z = \beta_0 + \beta_1 \cdot x_1 + \cdots + \beta_k \cdot x_k$$

Then, we have

$$P(y = 1|x) = G(\beta_0 + \beta_1 \cdot x_1 + \cdots + \beta_k \cdot x_k)$$

Logit Model

Remember the Bernoulli distribution from statistics:

$$Pr(y = 1) = p$$
$$Pr(y = 0) = 1 - p$$

with $E(y) = p$. We have the following for the logit model

$$Pr(y = 1) = G(z) = \frac{e^z}{1 + e^z} = \frac{1}{1 + e^{-z}}$$

where $z = \beta_0 + \beta_1 \cdot x_1 + \cdots + \beta_k \cdot x_k$

Probit Model

Instead of using the cumulative logistic distribution, the probit model uses the cumulative normal distribution:

$$G(z) = \Phi(z)$$

Both models lead to similar results (not similar coefficients!).

Example using `organic`

Data description

- *income* of the respondent in $ 1,000
- *buying* of organic food: yes (1) or no (0)

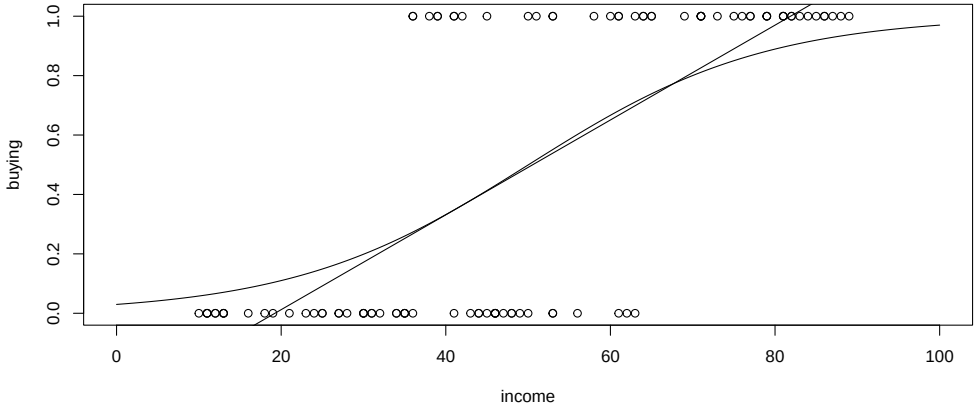
Results of interest for the binary choice model (for other models as well)

- Coefficient estimates
- Marginal effects
- Predicted probabilities

Notes

- Estimation through Maximum Likelihood
- Difficulty interpreting the values of coefficient

Comparison LPM vs. Logit



Estimation with R

Coefficient estimates using the built-in R command:

```
bhatglm = glm(buying~income,
              family=binomial(link="logit"),
              data=organic)
library(mfx)
bhatmfx = logitmfx(buying~income,data=organic)
```

Obtaining summary from `bhatmfx`

```
summary(bhatmfx$fit)
```

Base Results

```
##
## Call:
## glm(formula = buying ~ income, family = binomial(link = "logit"),
##     data = organic)
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -5.87557    1.13842  -5.161 2.45e-07 ***
## income       0.11709    0.02247   5.211 1.87e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 138.469  on 99  degrees of freedom
## Residual deviance:  70.931  on 98  degrees of freedom
## AIC: 74.931
##
## Number of Fisher Scoring iterations: 6
```

Results from mfx

```
##
## Call:
## glm(formula = formula, family = binomial(link = "logit"), data = data,
##     start = start, control = control, x = T)
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -5.87557    1.13842  -5.161 2.45e-07 ***
## income       0.11709    0.02247   5.211 1.87e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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## Number of Fisher Scoring iterations: 6
```

Marginal Effects with mfx package

Advantage of mfx package: Estimation of marginal effects

```
bhatmfx$mfxest
```

```
##              dF/dx   Std. Err.        z        P>|z|
## income 0.02919553 0.005634262 5.181785 2.197728e-07
```

Important note:

- Marginal effects are estimated at the mean of the independent variable(s)!

Fitted Values in a Binary Choice Model

Example:

- What are the predicted probabilities of a person purchasing organic given their annual income (in $ 1,000) of 25, 50, and 75?

Solution in R:

```
datablock = data.frame(income=c(25,50,75))
test = predict(bhatglm,newdata=datablock,type="response")
```

Probit Model

Very similar results compared to Logit:

```
bhatmfx = probitmfx(buying~income,data=organic)
bhatmfx$mfxest
```

```
##              dF/dx   Std. Err.        z        P>|z|
## income 0.02771441 0.004753676 5.830101 5.539374e-09
```

Food Purchases `fpdata`

Food purchases data:

- `strawberries_org`: Frequency of strawberry purchases per month
- `tomatoes_org`: Frequency of strawberry purchases per month
- `age`: Age of the respondent
- `kidsunder12`: Presence of kids under the age of 12
- `rootsurban`: Urban (as opposed to rural) upbringing of respondent
- `education`: Education level
- `income`: Income

Data Preparation and Estimation

```
fpdata$strawberriesorg  = ifelse(fpdata$strawberriesorg==0,0,1)
fpdata$tomatoesorg      = ifelse(fpdata$tomatoesorg==0,0,1)
```

```
bhats = glm(strawberriesorg~age+kidsunder12+rootsurban+
            education+income,
            family=binomial(link="logit"),
            data=fpdata)
bhatt = glm(tomatoesorg~age+kidsunder12+rootsurban+
            education+income,
            family=binomial(link="logit"),
            data=fpdata)
```

Results Strawberries

```
##
## Call:
## glm(formula = strawberriesorg ~ age + kidsunder12 + rootsurban +
##     education + income, family = binomial(link = "logit"), data = fpdata)
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)  6.961e-02  6.909e-01   0.101  0.91974
## age         -8.478e-03  1.121e-02  -0.756  0.44947
## kidsunder12  8.526e-02  3.709e-01   0.230  0.81820
## rootsurban   3.507e-01  3.312e-01   1.059  0.28972
## education   -1.203e-01  1.329e-01  -0.905  0.36528
## income       1.524e-05  5.597e-06   2.722  0.00649 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 232.45  on 171  degrees of freedom
## Residual deviance: 222.77  on 166  degrees of freedom
##   (4 observations deleted due to missingness)
## AIC: 234.77
##
## Number of Fisher Scoring iterations: 4
```

Results Tomatoes

```
##
## Call:
## glm(formula = tomatoesorg ~ age + kidsunder12 + rootsurban +
##     education + income, family = binomial(link = "logit"), data = fpdata)
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.329e-01  7.010e-01  -0.190  0.84967
## age         -5.728e-03  1.138e-02  -0.503  0.61466
## kidsunder12 -1.104e-01  3.770e-01  -0.293  0.76956
## rootsurban   3.603e-01  3.364e-01   1.071  0.28417
## education   -4.158e-02  1.338e-01  -0.311  0.75606
## income       1.708e-05  5.892e-06   2.899  0.00374 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 227.06  on 171  degrees of freedom
## Residual deviance: 216.18  on 166  degrees of freedom
##   (4 observations deleted due to missingness)
## AIC: 228.18
##
## Number of Fisher Scoring iterations: 4
```

Additional Questions

For the strawberries and tomatoes regression, do the following:

- Calculate the marginal effects of all independent variables
- Calculate the predicted probability for each observation