Dumortier

Pooled Data

Wage

Incinerat

Panel Data Theoretical

Fixed Effect

Random Effects Mode

Panel Data

Jerome Dumortier

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Introduction

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Required R packages

- plm
- Imtest

Documentation

Panel data econometrics in R

Note regarding presentation of results

• Use of the package stargazer due to the large number of variables

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Pooled vs. Panel Data

Pooled data: Combination of multiple cross-sectional data over time

- Two or more different observational units over time
- Examples: Grades in an economics class from multiple semesters, American Community Survey (ACS), or General Social Survey (GSS)

Panel data: Repeated measurement of the same individual i over time t

- Individual units can be people, states, firms, counties, countries, etc.
- Necessary adjustments of standard error due to correlation across time

Assumptions about panel data and models

- Regular time intervals
- Errors are correlated
- Parameters may vary across individuals or time
- Intercept: Individual specific effects (fixed or random)

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National Longitudinal Survey

• For NLSY79: Accessing data \Rightarrow investigator \Rightarrow Begin searching as guest \Rightarrow Pick income as an example

Panel Study of Income Dynamics (PSID)

 Data on approximately 5,000 families on various socioeconomic and demographic variables

Survey of Income and Program Participation (SIPP)

Interviews about economic condition of respondents

GSS is not a panel data set because different respondents are questioned every year

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Advantages of Panel Data

Controlling for unobserved heterogeneity among observational units (e.g., firms)

• Time-invariant individual-specific effects (e.g., institutional, personality traits) that would otherwise bias estimates if omitted

Dynamic behavior

- Better understanding of the dynamic changes for observational units over time
- Combination of cross-sectional with time series data leading to more complete behavioral model

Efficiency of parameter estimates

Increases in data variability improving estimates and avoiding multicollinearity

Less omitted variable bias

• Implicit inclusion of time-invariant, unobserved variables leading to less bias

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Disadvantages of Panel Data

Data availability and collection complexity

- Very difficult and expensive to collect with potentially high attrition rates
- Accumulation of measurement error over time leading to estimation biases
- Example: NLSY79 requires tracking of individuals since 1979

Advanced estimation and interpretation

- High likelihood of autocorrelated errors and heteroskedastic
- Time-invariant variables are excluded in fixed effects models

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Terminology and Types

Balanced versus unbalance panel

 A balanced panel has the same number of time-series observations for each subject or observational unit, whereas an unbalanced panel does not

Short versus long panel

- Short panel: Larger number of subjects or observational units than time periods.
- Long panel: Greater number of time periods than observational units

Types of regression models

- Pooled Ordinary Least Square model
- Fixed effects model
- Random effects model

Data fertil1 from the GSS (1974-1984)

- year: 72 to 84, even
- educ: years of schooling
- meduc and feduc: mother's and father's education
- kids: number children ever born
- east, northcentral, and west: 1 if lived in at 16
- farm: 1 if on farm at 16
- otherrural: 1 if other rural at 16
- town: 1 if lived in town at 16
- smallcity: 1 if in small city at 16

Source: Jeffrey Wooldridge, Introductory Econometrics: A Modern Approach

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Estimation Results

```
Dependent variable:
                                  kids
## educ
                            -0.130***(0.019)
                            0.499*** (0.141)
## age
## I(age2)
                            -0.005*** (0.002)
## east
                              0.061 (0.133)
## northcentral
                             0.220* (0.121)
## west
                              0.051 (0.168)
## v82
                            -0.414**(0.174)
## v84
                            -0.565***(0.177)
## Constant
                            -6.785** (3.099)
## Observations
                                  1,129
## R2
                                  0.099
## Adjusted R2
                                  0.086
## Residual Std. Error
                            1.581 (df = 1112)
## F Statistic
                        7.671*** (df = 16: 1112)
## Note:
                       *p<0.1: **p<0.05: ***p<0.01
```

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Interpretation

Evolution of fertility rates over time after controlling of other observable factors:

- Base year: 1972
- Negative coefficients indicate a drop in fertility in the early 1980's
- Coefficient of y82 (-0.41) indicates that women had on average 0.41 less children, i.e., 100 women had 41 kids less than 1972
- This drop is independent from education since we are controlling for education.
- More educated women have fewer children
- Assumes that the effect of each explanatory variable remains constant.

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Random Effects Mod Data cps7885 from the Current Population Survey:

• educ, exper, union, female, and year dummy variable y85

Interact year dummy with key explanatory variables to see if the effect of that variable has changed over time:

$$\ln(wage) = \beta_0 + \gamma_0 \cdot y85 + \beta_1 \cdot educ + \gamma_1 \cdot y85 \cdot educ + \beta_2 \cdot exper + \beta_3 \cdot exper^2 + \beta_4 \cdot union + \beta_5 \cdot female + \gamma_5 \cdot y85 \cdot female$$

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Estimation Results

```
Dependent variable:
                                 log(wage)
## y85
                               0.118 (0.124)
## educ
                            0.075*** (0.007)
## exper
                            0.030*** (0.004)
## I(exper2)
                            -0.0004***(0.0001)
## union
                            0.202*** (0.030)
                             -0.317*** (0.037)
## female
## v85:educ
                             0.018** (0.009)
## v85:female
                             0.085* (0.051)
## Constant
                             0.459*** (0.093)
## Observations
                                   1,084
## R2
                                   0.426
## Adjusted R2
                                   0.422
## Residual Std. Error
                            0.413 \text{ (df = 1075)}
## F Statistic
                        99.804*** (df = 8: 1075)
## Note:
                       *p<0.1: **p<0.05: ***p<0.01
```

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Return to education

- β_1 is the return in 1978: 7.5%
- $\beta_1 + \gamma_1$ is the return in 1985: 7.5%+1.8% = 9.3%

Change in the return of education

• γ_1 as the change in return over the 7 year period

Gender gap

- 1978 gender gap: 31.67%
- 1985 gender gap: 31.67%-8.51% = 23.16%

Intercept

ullet eta_0 and $eta_0+\gamma_0$ as the 1978 and 1985 intercepts, respectively

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Data and Variables

Data set kiel about home values near the location of an garbage incinerator

- Run 1981 data
- Run 1978 data

Naive Implementation in R

```
kiel81 = subset(kiel, year==1981)
```

bhat81 = lm(rprice~nearinc,data=kiel81)

kiel78 = subset(kiel, year==1978)

bhat78 = lm(rprice~nearinc,data=kiel78)

Naive Results

```
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```

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```
##
##
                                 Dependent variable:
##
##
                                       rprice
##
                               (1)
                                                    (2)
## nearing
                      -30,688***(5,828) -18,824***(4,745)
## Constant
                       101.308*** (3.093) 82.517*** (2.654)
## Observations
                               142
                                                    179
## R2
## Adjusted R2
## Residual Std. Error 31,238 (df = 140) 29,432 (df = 177)
## F Statistic
                       28*** (df = 1: 140) 16*** (df = 1: 177)
                                   *p<0.1; **p<0.05; ***p<0.01
## Note:
```

Estimation Equation

To determine statistical significance

$$price = \beta_0 + \gamma_0 \cdot y81 + \beta_1 \cdot nearinc + \gamma_1 \cdot y81 \cdot nearinc$$

Interpretation

- β_0 : Average home value which is not near the garbage incinerator
- $\gamma_0 \cdot y81$: Average change in housing values for all homes
- β_1 · nearinc: Location effect that is not due to the incinerator
- γ_1 : Decline in housing values due to incinerator

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Implementation in R

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bhat1

```
= lm(rprice~y81+nearinc+y81:nearinc,data=kiel)
            lm(rprice~v81+nearinc+v81:nearinc+age+I(age^2)
bhat2
               .data=kiel)
bhat.3
          = lm(rprice~y81+nearinc+y81:nearinc+age+I(age^2)+
```

cbd+rooms+area+land+baths,data=kiel)

```
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```

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##				
## ##		Dependent variable:		
## ##		(1)	rprice (2)	(3)
## ##	y81	18,790.290***	21,321.040***	14,115.710***
##	nearinc	(4,050.065) -18,824.370***	(3,443.631) 9,397.936*	(2,802.303) 3,618.020
##		(4,875.322)	(4,812.222)	(4,644.530) 3,310.163**
##	200111			(1,665.357) 17.920***
##	land			(2.310) 0.136***
##		44 000 000	04 000 070	(0.031)
##	y81:nearinc	(7,456.646)	-21,920.270*** (6,359.745)	(4,999.499)
##	Constant	(2,726.910)	89,116.540*** (2,406.051)	13,055.760 (11,272.270)
## ##	Observations	321	321	321
	R2 Adjusted R2	0.174 0.166	0.414 0.405	0.658 0.647
## ##	Residual Std. Error F Statistic		44.591*** (df = 5; 315)	59.742*** (df = 10; 310)
***	Note:			<pre></pre>

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Interpretation

 $Difference-in-difference\ estimator:\ -\$30,688-(-\$18,824) = -\$11,864$

$$\hat{\delta}_1 = (\textit{price}_{81,\textit{near}} - \textit{price}_{81,\textit{far}}) - (\textit{price}_{78,\textit{near}} - \textit{price}_{78,\textit{far}})$$

where $\hat{\delta}_1$ represents the difference over time in average differences in housing prices in the two locations.

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Grunfeld Data

The data set is used in many textbooks and comes also with the package plm. Data on 10 companies over the period 1935 to 1954:

• *inv*: Investment

• value: Value of the firm

• capital: Capital stock

Companies of interest for this class: GM (firm 1), U.S. Steel (firm 2), GE (firm 3), Westinghouse (firm 8)

Panel Data: Theoretical Aspects

Fixed Effect Model

Random Effects Mode Pooling all cross-sectional and time series observations into a single data set and running an OLS regression.

$$inv_i = \beta_0 + \beta_1 \cdot value_i + \beta_2 \cdot capital_i$$

General formulation of the pooled model

$$y_{it} = \beta_0 + \beta_1 \cdot x_i + \epsilon_i$$

Issues with pooled OLS model:

Ignores heterogeneity among the observations and time.

With heterogeneity: Biased and inconsistent estimates due to correlation between independent variables and error term.

```
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```

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Data Preparation and Pooled OLS Model

Use of firms 1, 2, 3, and 8:

```
grunfeld = subset(grunfeld,grunfeld$firm %in% c(1,2,3,8))
```

To use the functions from plm, define data as a panel data set:

```
grunfeld = pdata.frame(grunfeld,index=c("firm","year"))
```

Running a simple OLS model on the data:

- Using the regular lm() function
- Using the plm() function and specifying the model as pooling
- Name the outputs grunfeld.ols and grunfeld.pooling

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Panel Data: Theoretical **Aspects**

Pooled OLS Model

```
grunfeld.ols
grunfeld.pooling
```

- = lm(inv~value+capital,data=grunfeld)
- = plm(inv~value+capital,data=grunfeld, model="pooling")

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Pooled OLS Model

```
##
                                     Dependent variable:
                                             inv
##
                                  OLS
                                                     panel
                                                     linear
                                   (1)
                                                      (2)
## value
                            0.111*** (0.014) 0.111*** (0.014)
## capital
                           0.300*** (0.049) 0.300*** (0.049)
                            -62.832** (29.725) -62.832** (29.725)
## Constant
## Observations
                                   80
## R2
                                 0.755
                                                    0.755
## Adjusted R2
                                 0.748
                                                    0.748
## Residual Std. Error
                           142.916 (df = 77)
## F Statistic (df = 2: 77)
                                118.424***
                                                   118.424***
## Note:
                                     *p<0.1; **p<0.05; ***p<0.01
```

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Theoretical Concepts I

Fixed effects model or Least-Squares Dummy Variable (LSDV) regression

Constant slope coefficients but varying intercept over i

Regression equation:

$$inv_{it} = \beta_{0i} + \beta_1 \cdot value_{it} + \beta_2 \cdot capital_{it}$$

with i = 1, 2, 3, 4 and $t = 1, 2, \dots, 20$. This model can also be written as

$$\mathit{inv}_{it} = \alpha_0 + \alpha_1 \cdot D_{1i} + \alpha_2 \cdot D_{2i} + \alpha_3 \cdot D_{3i} + \beta_1 \cdot \mathit{value}_{it} + \beta_2 \cdot \mathit{capital}_{it}$$

Individual specific effects:

$$y_{it} = \alpha_i + \beta_i \cdot x_{it} + \epsilon_{it}$$

 α_i can be fixed or random

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Theoretical Concepts II

Fixed effects model

- Intercept β_{0i} is firm specific.
- For an individual, this could be education and/or ability, possibly correlated with independent variables
- Intercept is time-invariant.
- Slope coefficients do not vary across individuals (firms) or time

Implementation in R

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Results

```
##
                  Dependent variable:
##
              _____
                         inv
## value
                  0.108*** (0.018)
                  0.345*** (0.027)
## capital
## Observations
## R2
                        0.806
## Adjusted R2
                        0.792
## F Statistic 153.291*** (df = 2: 74)
## Note:
              *p<0.1; **p<0.05; ***p<0.01
```

```
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```

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Firm-Specific Intercepts and Hypothesis Test

In order to get the firm specific intercepts:

fixef(grunfeld.fixed)

```
## 1 2 3 8
## -85.515 94.988 -246.228 -59.386
```

Testing whether a fixed effects or OLS is appropriate (H_0 : OLS better):

pFtest(grunfeld.fixed,grunfeld.ols)

```
##
## F test for individual effects
##
## data: inv ~ value + capital
## F = 67.215, df1 = 3, df2 = 74, p-value < 2.2e-16
## alternative hypothesis: significant effects</pre>
```

If the p-value is below 0.05 then the fixed effects model is a better choice.

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Implementation in R using lm()

Implementation of a fixed effects model with the command lm()

bhat = lm(inv~value+capital+factor(firm),data=grunfeld)

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Theoretical Concepts

The general fixed effects model can be expressed as

$$y_{it} = \beta_{0i} + \beta_1 \cdot x_{1,it} + \beta_2 \cdot x_{2,it} + \epsilon_{it}$$

Instead of treating β_{0i} as fixed, the random model assumes

$$\beta_{0i} = \beta_0 + v_i$$

where v_i is random error term with a mean of zero and variance σ_v^2 . According to Gujarati: What we are essentially saying is that the four firms included in our sample are a drawing from a much larger universe of such companies and that they have a common mean value for the intercept β_0 and the individual differences in the intercept values of each company are reflected in the error term v_i .

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Implementation in R

```
grunfeld.random = plm(inv-value+capital,data=grunfeld,model="random")
stargazer(grunfeld.random,no.space=TRUE,single.row=TRUE,type="text")
```

```
##
               Dependent variable:
## value
               0.108*** (0.017)
## capital
              0.345*** (0.027)
## Constant
           -73.085 (81.172)
  _____
## Observations
## R2
                    0.803
## Adjusted R2
                    0.798
## F Statistic
                  314.851***
  _____
## Note:
            *p<0.1: **p<0.05: ***p<0.01
```

```
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```

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Breusch-Pagan Lagrange Multiplier (LM) Test

For random effects models: Null hypothesis of no panel effect, i.e., OLS is better. If p-value is below 0.05, we reject the null hypothesis and thus, a random effects model is more appropriate than the OLS.

```
plmtest(grunfeld.pooling,type=c("bp"))
```

```
##
## Lagrange Multiplier Test - (Breusch-Pagan)
##
## data: inv ~ value + capital
## chisq = 378.44, df = 1, p-value < 2.2e-16
## alternative hypothesis: significant effects</pre>
```

```
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Hausman Test: Fixed or Random Model

The Hausman Test tests the null hypothesis that the preferred model is a random effects model. It basically tests whether the unique errors are correlated with the regressors.

phtest(grunfeld.random,grunfeld.fixed)

```
##
## Hausman Test
##
## data: inv ~ value + capital
## chisq = 0.14882, df = 2, p-value = 0.9283
## alternative hypothesis: one model is inconsistent
```

```
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```

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Testing for Heteroscedasticity

bptest(inv~value+capital+factor(firm),data=grunfeld)

```
##
## studentized Breusch-Pagan test
##
## data: inv ~ value + capital + factor(firm)
## BP = 25.375, df = 5, p-value = 0.0001179
```

If the p-value is below 0.05, then we have heteroscedasticity.

```
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```

Random Effects Model

Heteroscedasticity Consistent Coefficients and Standard Frrors

coeftest(grunfeld.fixed,vcovHC)

```
##
## t test of coefficients:
##
        Estimate Std. Error t value Pr(>|t|)
##
## value
        0.108400
                 0.014293 7.5839 7.902e-11 ***
 ##
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```