Modelling Convection, Phase Change and Salt Fluxes from Mushy Sea Ice

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Introduction

- Sea ice is a mushy layer of ice crystals and brine.
- Dense brine drains during ice formation (Fig. 1), affecting ocean circulation.

Our goal: constrain brine fluxes using numerical simulations.

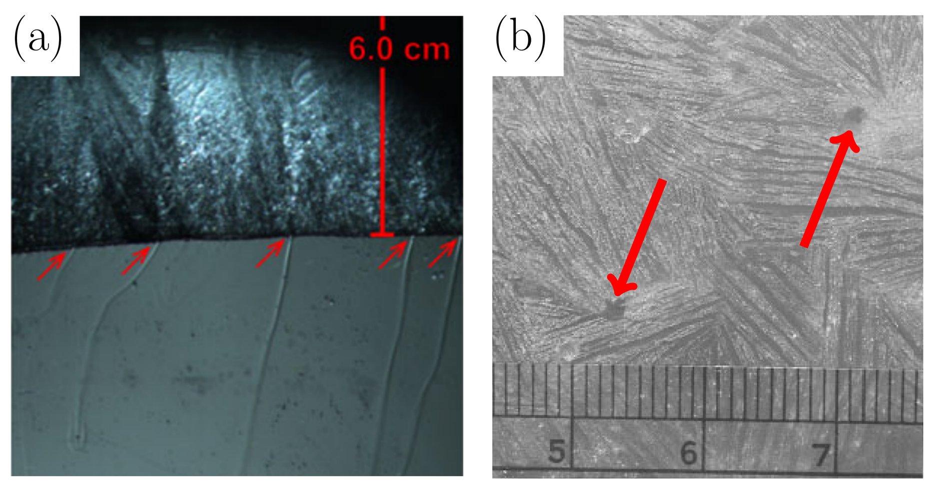

Fig. 1: Solidification of salt water viewed from (a) side; Middleton et. al. (2016) and (b) below; Wettlaufer et. al. (1997). Arrows indicate where salty plumes leave the ice.

What is a mushy layer?

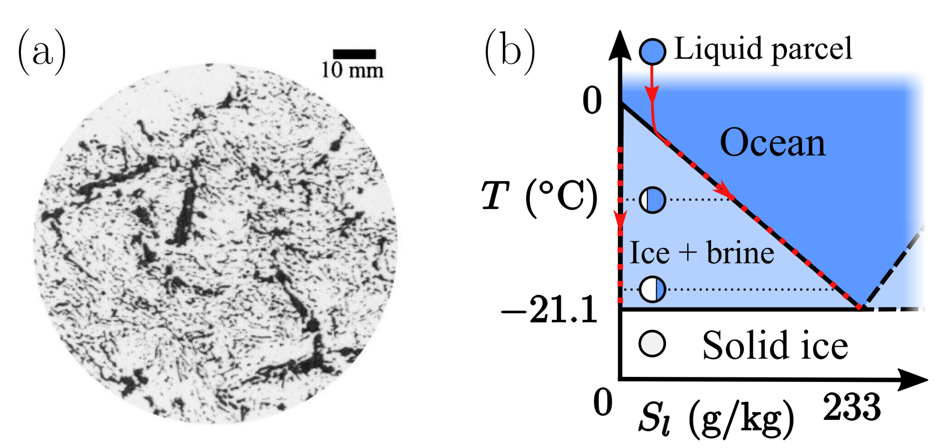

Fig. 2: (a) Sea ice is a porous mixture of solid ice crystals (white) and liquid brine (dark) [1]. (b) Trajectory (→) of a solidifying salt water parcel through the phase diagram. As the temperature T_l decreases, more ice forms and the residual brine salinity S_l increases making the fluid denser, which can drive convection. Below the eutectic (T_e, S_e) = (-21.1°, 233) the system is entirely solid.

Numerical Method

Solve (1)-(4) using Chombo finite volume toolkit:

- Momentum and mass: projection method [2].
- Energy and solute:
  - Advective terms: explicit, 2nd order unsplit Godunov method.
  - Nonlinear diffusive terms: semi implicit, geometric multigrid.
  - Timestepping: Backward Euler.

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[1] Hajo Eicken et al. *Cold Regions Science and Technology* 31.3 (2000), pp. 207–225. [2] D. F. Martin et al. *Journal of Computational Physics* 227.3 (2008), pp. 1863–1886. [3] J. S. Wettlaufer et al. *J. Fluid Mech.* 344 (1997), pp. 291–316. [4] M. G. Worster. *Journal of Fluid Mechanics* 224.-1 (1991), p. 335.

Summary:

- During transient growth, channel spacing increases over time.
- Scaling predictions for steady state growth are consistent with experimental observations

Simulated transient growth

Water of initial salinity $S_0 = 35\,\mathrm{g\,kg^{-1}}$ and temperature $-2°$C is frozen from above in a Hele-Shaw cell (plate separation d = 1mm). We assume $K_0 = 10^{-9}\,\mathrm{m}^2$, and vary the atmospheric temperature T_a.

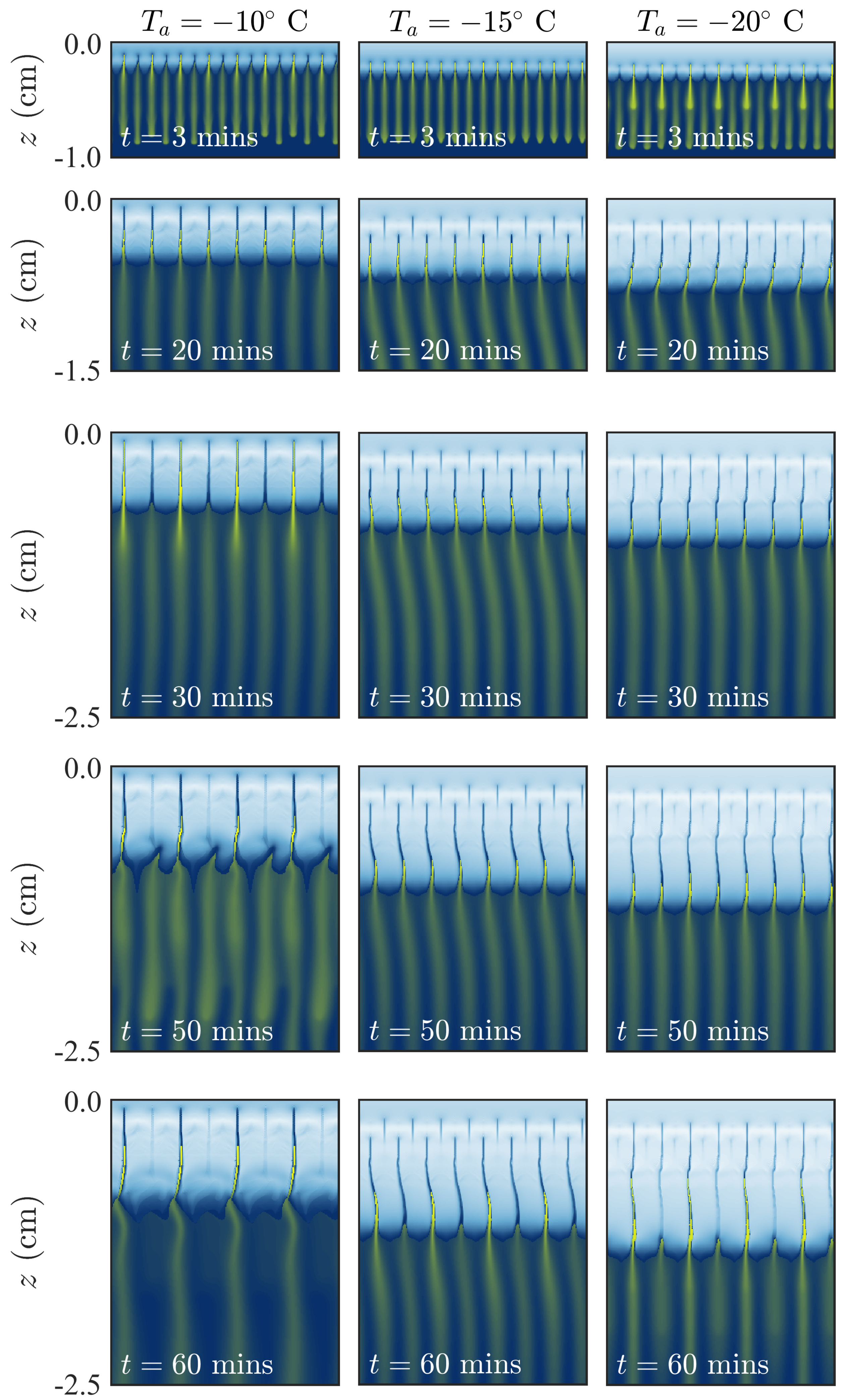


Fig. 4: Ice grown from a fixed boundary. https://goo.gl/4n9STV

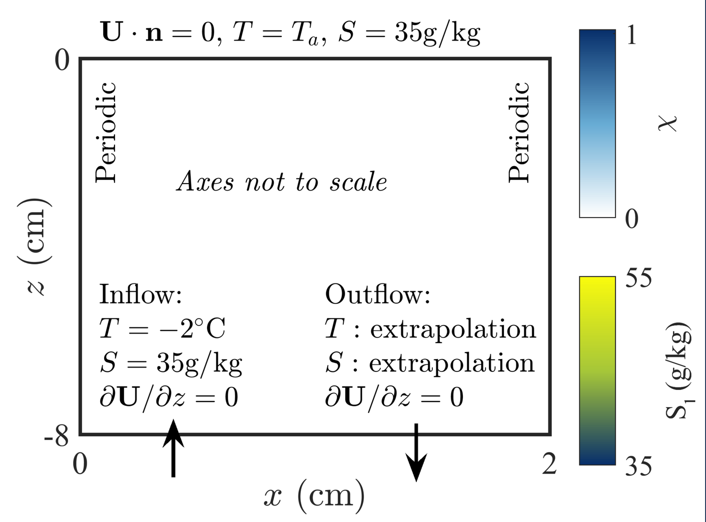


Fig. 3 (↑). Domain and colorbars for transient simulations (←).

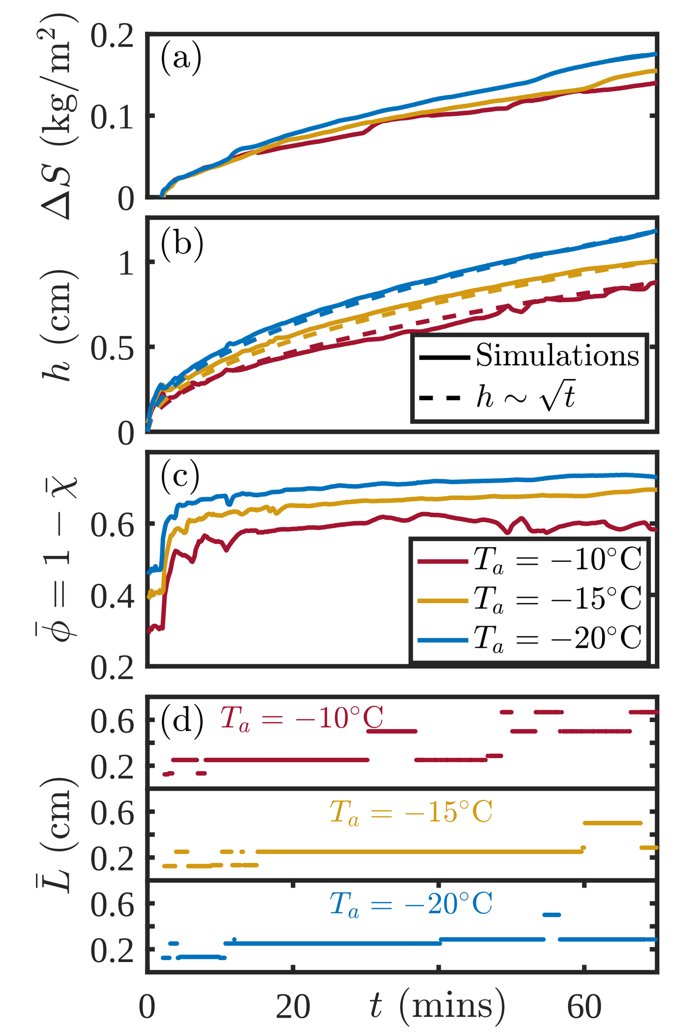


Fig. 5 (↑). (a) Total salt rejected from ice. (b) Ice depth increases roughly $\propto \sqrt{t}$. (c) Average solid fraction $\bar\phi$ increases following the onset of convection. (d) Average channel spacing $\bar L$ increases over time.

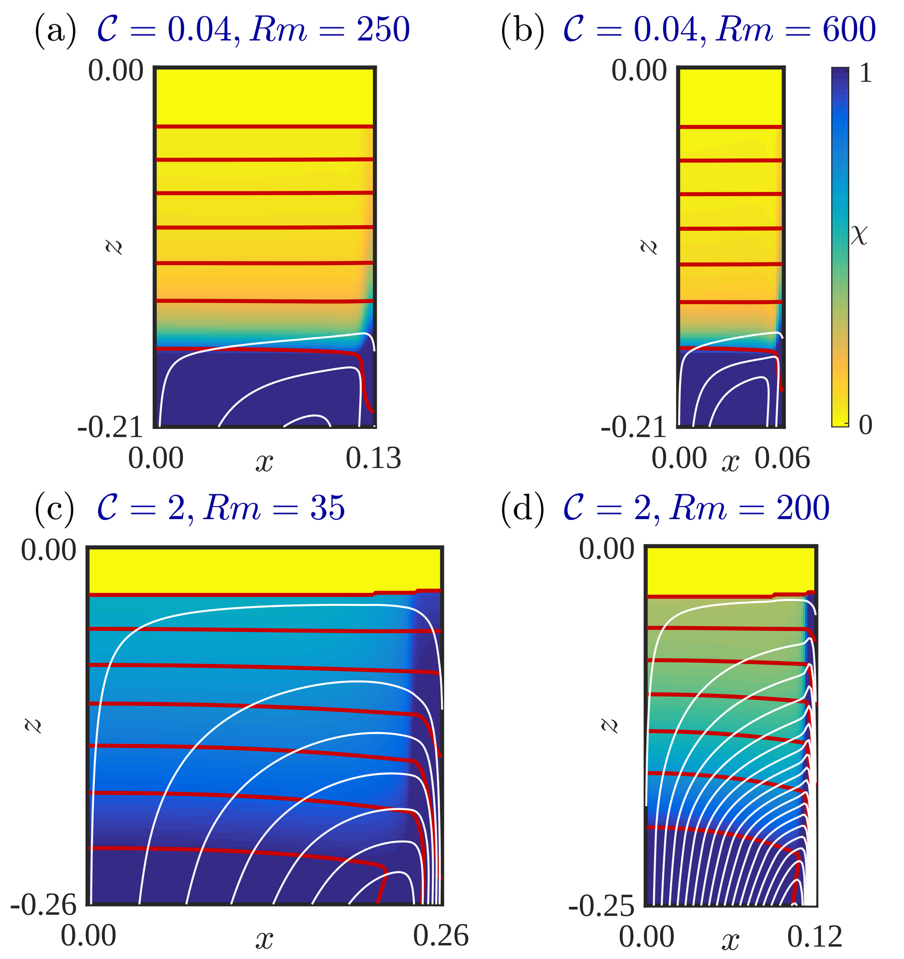
Steady state solutions

For ice grown at a constant rate V from water of initial salinity S_0 we investigate the sensitivity to the Rayleigh number Rm and concentration ratio $\mathcal{C}$,

$$\mathcal{C} = \frac{S_0}{S_e - S_0}.$$

In fig. 6 (right →), we plot the porosity χ, streamlines (white) and salinity contours (red) for steady state solutions, where the domain width is chosen to maximise the solute flux. **Decreasing $\mathcal{C}$ reduces the porosity, confining convection to a narrow porous layer at the mush-liquid interface.**

Below, we construct a scaling argument for $\mathcal{C} \ll 1$, then compare to numerical simulations (fig. 7) and experimental results (fig. 8).



Advection of heat and salt balanced by diffusion and solidification.

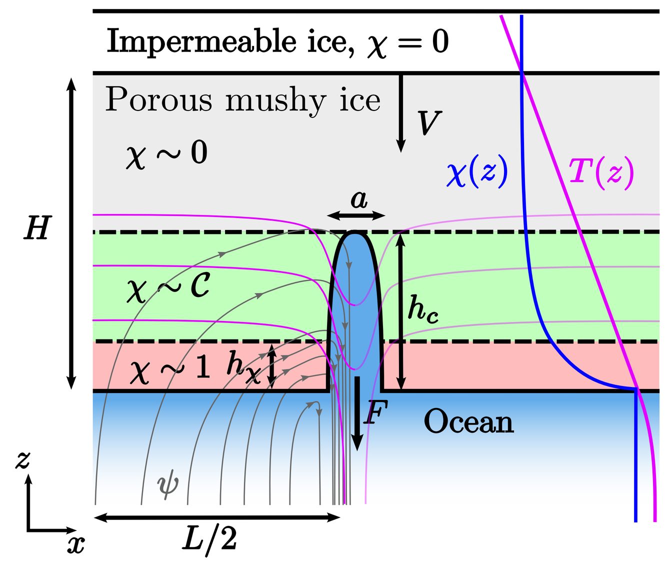
$$\mathbf{U}\cdot\nabla T \sim \frac{\partial^2 T}{\partial x^2} \qquad \mathbf{U}\cdot\nabla S_l \sim V\frac{\partial S}{\partial z}$$

Baroclinic torque drives flow in the porous layer.

$$Rm\frac{\partial S_l}{\partial x} \sim \frac{\partial^2 \psi}{\partial x^2}$$

Empirical scaling to close argument.

$$\frac{h_c}{h_\chi} \sim \sqrt{Rm}$$



Mushy layer height and vertical temperature profile well approximated by diffusive solution (as flow is weak).

$$S_l = -T \approx z/H \qquad \mathcal{S}\frac{\partial \chi}{\partial z} \sim \frac{\partial^2 T}{\partial z^2}$$

Flow in the narrow channels is driven by buoyancy. Salt advection is balanced by horizontal diffusion.

$$Rm\frac{\partial S_l}{\partial x} \sim \frac{\partial^2 \psi_c}{\partial x^2} \qquad \mathbf{U}\cdot\nabla S_l \sim \frac{1}{Le}\frac{\partial^2 S_l}{\partial x^2}$$

$$L \sim Rm_s^{-1/4},\ a \sim Rm^{-3/4},\ \psi \sim \mathcal{C}\,Rm^{1/2},\ \psi_c \sim \mathcal{C}\,Rm^{1/2},\ h_c \sim \mathcal{C}\,Rm^{1/4},\ h_\chi \sim \mathcal{C}\,Rm^{-1/4} \quad F \sim \frac{\psi_c}{L} \sim \mathcal{C}Rm^{3/4}$$

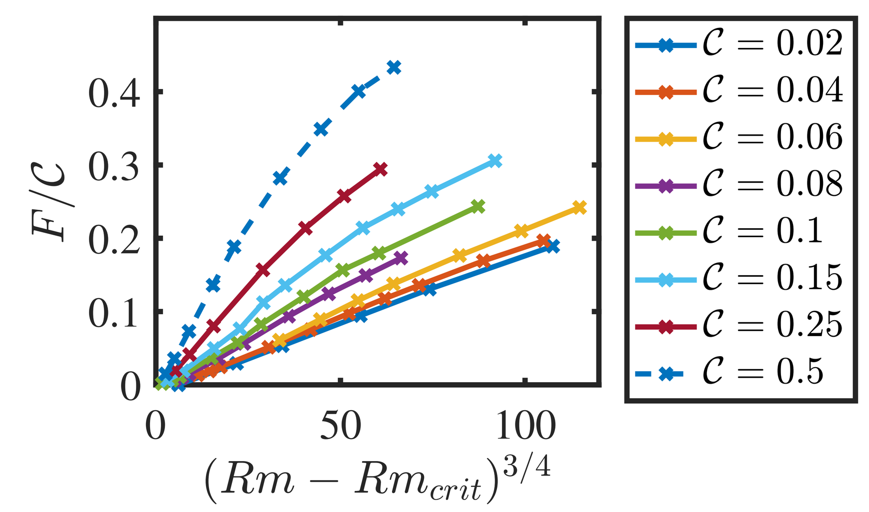


Fig. 7: Numerically calculated **salt flux increases nearly linearly with $(Rm - Rm_{crit})^{3/4}$** when $\mathcal{C} \ll 1$, where Rm_{crit} is the critical Rayleigh number for convection.

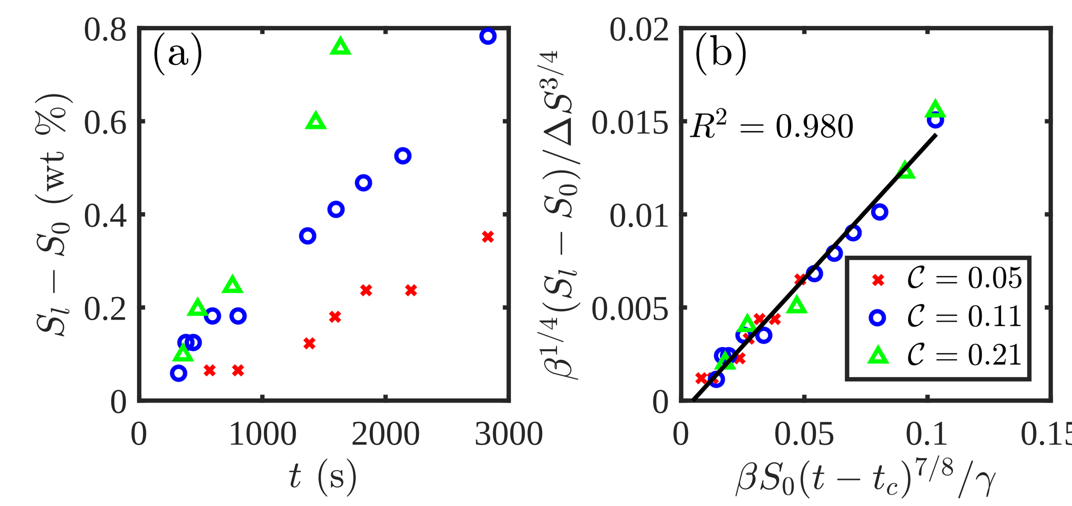


Fig. 8: (a) Change in salinity of the underlying liquid during transient growth measured by [3]. (b) Collapse of the data predicted by $F \sim \mathcal{C}Rm^{3/4}$, where t_c is the time when convection starts and $\gamma = l\,\kappa_l^{-3/8}(gK_0/\nu)^{-3/4}$.

Governing Equations for Flow in Porous Mushy Sea Ice

Solve for flow and solidification in reactive porous media [4] with Darcy's law applied in a narrow Hele-Shaw cell with variable ice porosity.

Scales: length $\sim l$, time $\sim l^2/\kappa_l$, velocity $\sim \kappa_l/l$, temperature $\Delta T \sim T_e - T_0$, salinity $\Delta S \sim S_e - S_0$, permeability $\sim K_0$, pressure $\sim \eta\kappa_l/K_0$.

Momentum $$\mathbf{U} = Rm\Pi\left(-\nabla p - S_l\right),$$

Energy $$\frac{DH}{Dt} + \mathbf{U}\cdot\nabla T = \nabla\cdot\left[\chi + (1-\chi)k\right]\nabla T,$$

Mass, salt $$\nabla\cdot\mathbf{U} = 0, \qquad \frac{DS}{Dt} + \mathbf{U}\cdot\nabla S = Le^{-1}\nabla\cdot\chi\nabla S.$$

Parameters $$Rm = \frac{\rho_0\, g\, \beta\, \Delta S\, K_0\, l}{\kappa_l\, \eta}, \quad Le = \frac{\kappa_l}{D_l}, \quad St = \frac{L}{c_{p,l}\Delta T}, \quad c_p = \frac{c_{p,s}}{c_{p,l}}, \quad k = \frac{k_s}{k_l}.$$

$\mathbf{U}$ (Darcy velocity), χ (porosity), p (pressure), T (temperature),
S_l (liquid salinity), $S = \chi S_l$ (bulk salinity),
$H = St\chi + [\chi + (1-\chi)c_p]\,T$ (enthalpy),
$\Pi(\chi)^{-1} = \Pi^{-1}_{\text{Hele-Shaw}} + \chi^{-3}$ (permeability),
η (viscosity); D_l (salt diffusivity); β (haline expansion);
$c_{p,l}$, $c_{p,s}$ (liquid/solid specific heat); k_l, k_s (liquid/solid heat conductivity);
d (Hele-Shaw cell thickness); K_0 (reference permeability);
l (box height); κ_l (liquid heat diffusivity); ρ_0 (reference density).

Future Work

- Consider growth over longer time periods (days to weeks), with time varying atmospheric forcing.
- Simulations with the liquid region governed by the Navier-Stokes equation, rather than flow in a Hele-Shaw cell.
- Three dimensional simulations, utilising the Adaptive Mesh Refinement capabilities of our code.