



University of Antwerp  
Faculty of Social Sciences

# DAGs and PP

or commonly know as:

“a bit more transparent way to state  
your research assumptions and questions”

Jose Rivera

(Josema, for the friends)

April 30, 2022

# What are we going to talk about? I

## 1 About research

- A typical scientific lab
- Research hypothesis production

## 2 DAGs and PP

## 3 Anecdotal cases

- Experimental design: the panacea
- Simulation conventions
- Fork bias: spurious relationships
- Fork bias: masked relationships (a)
- Fork bias: masked relationships (b)
- Fork bias: multicollinearity

# What are we going to talk about? II

- Fork bias: bias amplification
- No more fork bias: neutral control
- Pipe bias: precision parasite
- Pipe bias: post-treatment
- Pipe bias: masked relationships

## 4 Concluding remarks

## 5 Do you wanna know more???

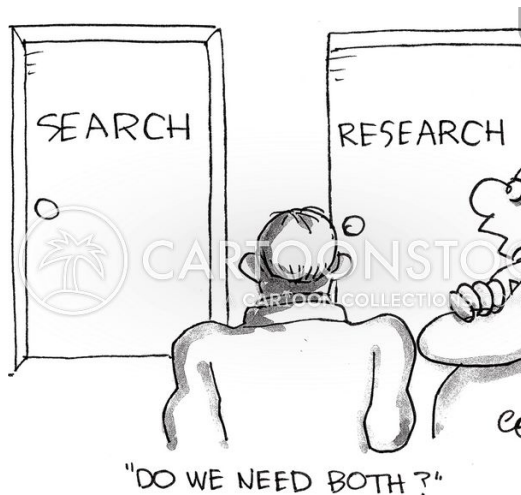
# 1. About research

A typical scientific lab

# A typical scientific lab<sup>1</sup>

What is needed?

1. Quality of theory
2. Quality of data
3. Reliable procedures and code
4. Quality of data analysis
5. Documentation
6. Reporting



<sup>1</sup>McElreath [12], lecture 20 and McElreath [13], chapter 17

# A typical scientific lab

What we “normally” focus on?

1. Quality of theory
2. Quality of data
3. Reliable procedures and code
4. Quality of data analysis
5. Documentation
6. Reporting

you said I only  
need more data

more "quality"  
data



# A typical scientific lab

What can be improved?  
(with DAGs and PP)

1. Quality of theory
2. Quality of data
3. Reliable procedures and code
4. Quality of data analysis
5. Documentation
6. Reporting

**THIS BAD BOYS CAN HELP  
YOU STATE YOUR ASSUMPTIONS**



# 1. About research

Research hypothesis production



# Research hypothesis production

Well known challenges<sup>a</sup>

- Insufficient data
- Wrong population
- Measurement error
- Selection bias
- Confounding

---

<sup>a</sup>Hernán [8], lesson 4

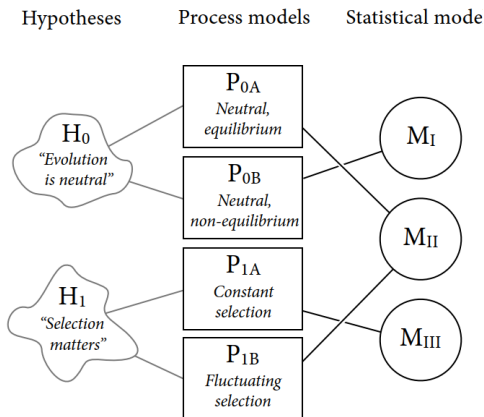


# Research hypothesis production

but we should not forget<sup>a</sup>

- No one-to-one relationship exists between our **process models** and **statistical models**,
- Nor between our hypothesis and a process models

<sup>a</sup>Figure 1.2 reproduced from chapter 1 McElreath [13]



# Research hypothesis production

and also

- statistical models are just  
“machines to find association”, not  
a reliable reflection of the theory  
(I can prove it!!).



# Research hypothesis schematics<sup>2</sup>

- a. Estimand and **process model**
- b. Synthetic data generation
- c. Statistical model design and testing
- d. Apply **statistical model** to data



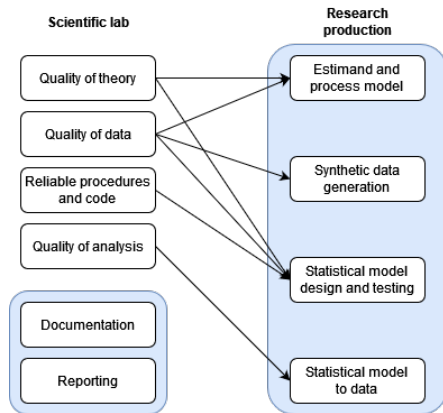
---

<sup>2</sup>McElreath [13], lecture 20, Pearl [16]. Follow Fogarty et al. [6] on item (c).

# Research hypothesis schematic

Where does it match with the previous?

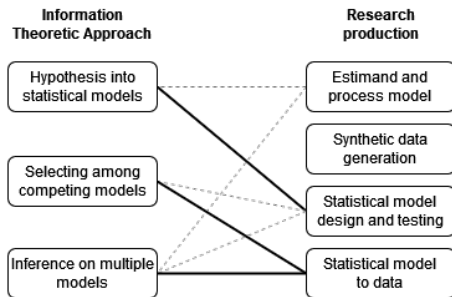
- Estimand and process model maps 1 (theory) and 2 (data) to a heuristic model.
- Synthetic data generation maps 2 (data) to an idealized data.
- Statistical model design and testing maps 1 (theory), 2 (data), and 3 (reliable code) to an statistical model.
- Apply statistical model to data maps 4 (analysis) onto a result.



# Where does the ITA fit?

Information Theoretic Approach (ITA) is framework to select among competing models [1, 3]:

1. Hypothesis into statistical models,  
(how about a process model?)
2. Select among competing models,  
(do the code works as intended?)
3. Make inferences based on one or multiple models.  
(do the code works as intended?,  
are there variables that can bias our  
results?)



## 2. DAGs and PP

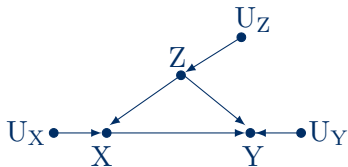
# DAGs and PP

- Directed acyclic graphs (DAGs), are a type of structural causal model (SCM) [15, 4]
- DAGs can be represented by a structural model, and its associated causal diagram<sup>a</sup>.
- we put distributional assumptions to the structural model through probabilistic programming (PP) [10].  
(more in part 3)

<sup>a</sup>reproduced from Cinelli et al. [4].

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

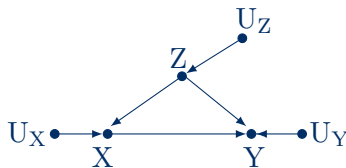


# DAGs and PP

- $\mathbf{V} = \{Z, X, Y\}$  are called **endogenous variables**.
- $\mathbf{U} = \{U_Z, U_X, U_Y\}$  are called **exogenous variables**.  
(drawn when strictly required)
- $\mathbf{F} = \{f_Z, f_X, f_Y\}$  are called **structural equations**.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

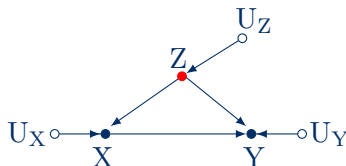
# DAGs and PP

Causal diagram conventions [4],

- **black nodes** are **observed variables**.
- **white nodes** are **unobserved variables**.
- **red nodes** are variables for which we will decide its inclusion or not.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

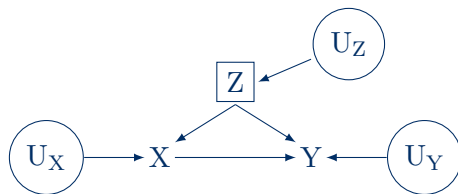
# DAGs and PP

Other causal diagram conventions,

- no circle nodes are **observed variables**.
- circled nodes are **unobserved variables**.
- squared nodes are variables for which we will decide its inclusion or not.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

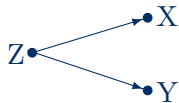
(a) structural model



(b) causal diagram

# The benign case of DAG elementals

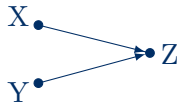
For everything can be depicted with them



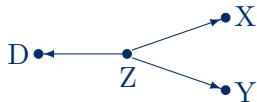
(a) fork



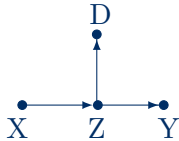
(b) pipe



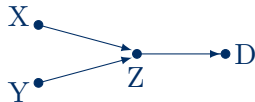
(c) collider



(d) descendant on fork



(e) descendant on pipe



(f) descendant on collider

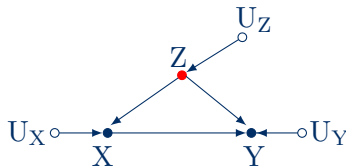
# About D-separation

Causal graph theory [14, 15, 17, 18, 19],

1. descendant (child, grandchild), parent (grandparent).  
(path specific)
2. paths (directional, non-directional).
3. paths are blocked or open according to the D-separation rules.  
(also path specific)
4. there are only four (4) D-separation rules.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

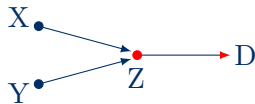
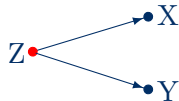
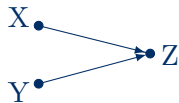
# About D-separation

The D-separation (Directional) rules [8],

1. If no variables being conditioned on, a path is blocked if and only if, two arrowheads on the path collide at some variable on the path.
2. Any path that contains a noncollider that has been conditioned on, is blocked (**backdoor path**)<sup>a</sup>.
3. A collider that has been conditioned on does not block a path.
4. A collider that has a descendant that has been conditioned on does not block a path.

---

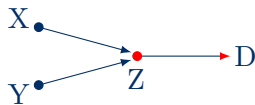
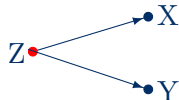
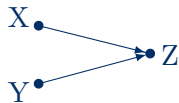
<sup>a</sup>there is also a **front-door path** (if you wonder).



# About D-separation

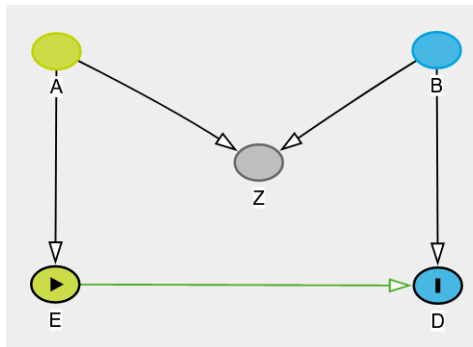
The D-separation rules **implications**,  
(independent of distributional assumptions)

1.  $X \perp\!\!\!\perp Y \implies$   
 $P(X, Y) = P(X) \cdot P(Y)$
2.  $X \perp\!\!\!\perp Y \mid Z \implies$   
 $P(X, Y \mid Z) = P(X \mid Z) \cdot P(Y \mid Z)$   
(same for fork or pipe)
3.  $X \not\perp\!\!\!\perp Y \mid Z \implies$   
 $P(X, Y \mid Z) \neq P(X \mid Z) \cdot P(Y \mid Z)$
4.  $X \not\perp\!\!\!\perp Y \mid D \implies$   
 $P(X, Y \mid D) \neq P(X \mid D) \cdot P(Y \mid D)$



# Oh DAGitty!! mijn vriendin

- browser (R package) environment for creating, editing, and analyzing causal diagrams [20].
- available online: <http://dagitty.net>
- But there are more fish in the sea: <http://www.causalfusion.net> [2]  
(b\*\*\*\* better have my \$\$\$)





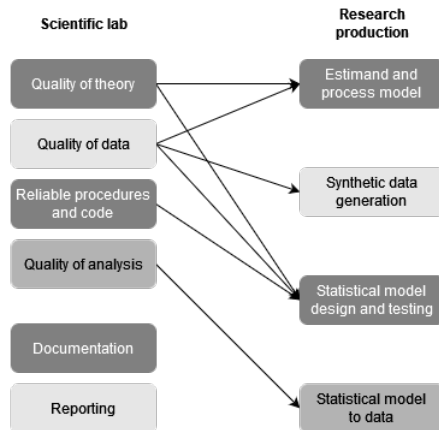
# Where do DAGs and PP fit?

starts with:

- A clear definition of the estimand and process model (assumptions).
- An improved the reliability of your procedures.
- As a documentation procedure.

and leads to:

- A sound analysis, and result  
(even when we cannot have an answer to our question)
- An improved planning to get data.



### 3. Anecdotal cases

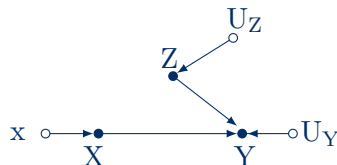
Experimental design: the panacea

# Experimental design<sup>3</sup>

- Purpose: to control all factors responsible for the outcome's variation.  
(understand the system)
- It is modeled by modifying the structural model (and causal diagram).

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

---

<sup>3</sup>Cinelli et al. [4], appendix A (p. 15)

# Experimental design

- **intervention** on  $X$  can be written in do-calculus<sup>a</sup> as:  $P(\mathbf{V} \mid \text{do}(X = x))$ .

- remember:

$$\mathbf{V} = \{Z, X, Y\},$$

$$\mathbf{U} = \{U_Z, U_X, U_Y\}, \text{ and}$$

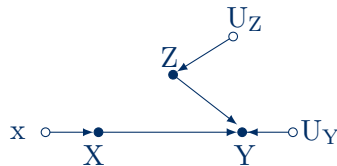
$$\mathbf{F} = \{f_Z, f_X, f_Y\}.$$

---

<sup>a</sup>an appropriate treatment can be found with the usual suspects [14, 15, 17, 18])

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

# Effects of interest

two types of effects,

1. Average causal effect:

$$ACE(x) = E[Y|do(x+1)] - E[Y|do(x)]$$

2. Controlled direct effect:

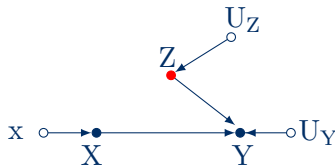
$$CDE(x, z) = E[Y|do(x+1), do(z)] - E[Y|do(x), do(z)]$$

points to consider:

- CDE takes a particular relevance with observational data.
- There is also a distinction between total effect and direct effect.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

### 3. Anecdotal cases

Simulation conventions

# Simulation conventions

one way to defined it,

$$Z = U_Z \quad ; U_Z \sim N(0, \sigma_Z)$$

$$X = \beta_Z Z + U_X \quad ; U_X \sim N(0, \sigma_X)$$

$$Y = \beta_Z Z + \beta_X X + U_Y \quad ; U_Y \sim N(0, \sigma_Y)$$

a more succinct way,

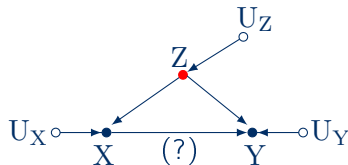
$$Z \sim N(0, \sigma_Z)$$

$$X \sim N(\beta_Z Z, \sigma_X)$$

$$Y \sim N(\beta_Z Z + \beta_X X, \sigma_Y)$$

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(Z, X, U_Y) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

### 3. Anecdotal cases

Fork bias: spurious relationships



# Spurious relationships<sup>4</sup>

also known as,

- spurious association
- confounder
- an instance of **fork bias**

research question,

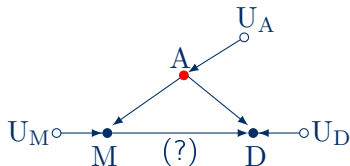
- Does M has a (direct) effect on D?

variables,

- A, median age at marriage
- M, marriage rate
- D, divorce rate

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, M, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

---

<sup>4</sup>McElreath [12], chapter 05 (p. 125)

# Simulation setting

```
# sim
A = rnorm( 100 )
M = rnorm( 100 , mean=-1*A )
D = rnorm( 100 , mean=-1*A + 0*M )
d = data.frame(A=A,M=M,D=D)
```

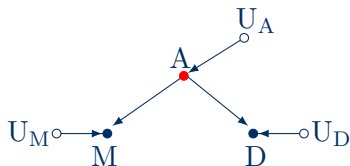
(c) R code

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

## Implications,

- $M \not\perp\!\!\!\perp D$
- $M \perp\!\!\!\perp D \mid A$

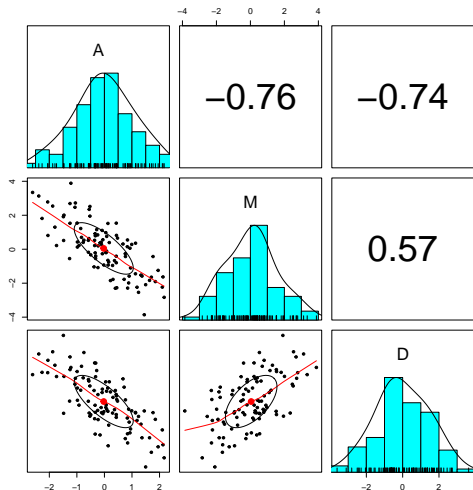


(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(A, D) < 0$  and  $\text{cor}(M, D) > 0$  goes in line of our “rudimentary” understanding of the data.
- why there is  $\text{cor}(M, D) > 0$ ?  
(hint: univariate correlation)
- we include M as a covariate in our statistical model  
(is our research hypothesis)



# Regression, regression!!

based on statistical analysis,

- two regressions with two different results, which model is the “true”?

```
> summary(lm(D ~ M, data=d)) # spurious relation
Call:
lm(formula = D ~ M, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-2.80012 -0.90447 -0.03866  0.80220  2.82970

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.23298    0.12412  -1.877   0.0635 .
M             0.40233    0.08986   4.477 2.04e-05 ***
---
> summary(lm(D ~ A + M, data=d)) # controlled relation
Call:
lm(formula = D ~ A + M, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-2.27295 -0.68174  0.03781  0.78885  2.95320

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.18854    0.09871  -1.910   0.0591 .
A            -1.03121    0.13483  -7.648 1.49e-11 ***
M            -0.06134    0.09362  -0.655   0.5139
---

```

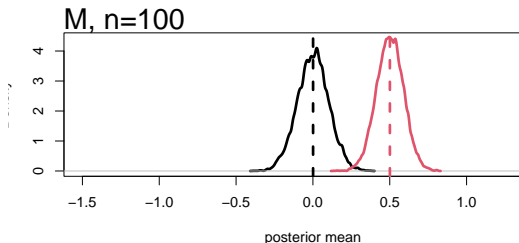
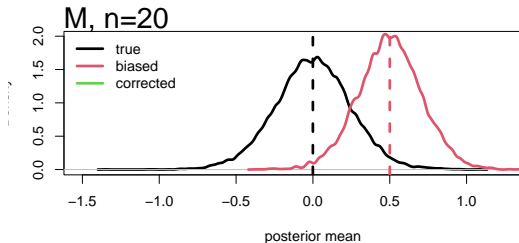
# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

under the **incorrect model**,  
the larger the sample size,

- the more **certain** you are about your estimates
- the more **mistaken** you are about your research question  
(the winner's curse)



# The dream team!!

based on **DAG** and **statistical model**,

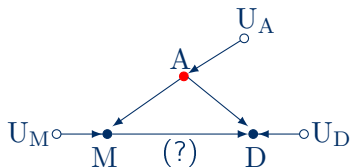
- the 2nd D-separation rule requires you to control any noncollider to block the **backdoor path**,  
i.e.  $M \perp\!\!\!\perp D \mid A$

- conditioning on A we can find,  
 $E[D|\text{do}(m)] = E[ E[D|M = m, A] ]$   
(law of total expectation)

- then we can find the  
 $ACE(m) = E[D|\text{do}(m+1)] - E[D|\text{do}(m)]$   
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, M, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

# the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,  
(assuming our DAG is true)

```
> summary(lm(D ~ A + M, data=d)) # controlled relation
```

Call:  
lm(formula = D ~ A + M, data = d)

Residuals:

Min	1Q	Median	3Q	Max
-2.27295	-0.68174	0.03781	0.78885	2.95320

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.18854	0.09871	-1.910	0.0591 .
A	-1.03121	0.13483	-7.648	1.49e-11 ***
M	-0.06134	0.09362	-0.655	0.5139

### 3. Anecdotal cases

Fork bias: masked relationships (a)



## Masked relationships (a)<sup>5</sup>

also known as,

- omitted variable bias
- an instance of **fork bias**

research question,

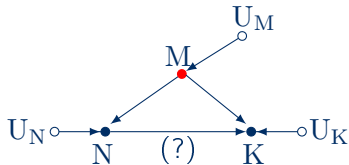
- Does N has a (direct) effect on K?

variables,

- M, mammal mass in kg.
- N, ratio neocortex over total brain mass
- K, Kcal. per gram of milk

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

---

<sup>5</sup>McElreath [12], chapter 05 (p. 144)

# Simulation setting

```
# sim
M = rnorm( 100 )
N = rnorm( 100 , 1*M )
K = rnorm( 100 , 1*N + -1*M )
d = data.frame(N=N,M=M,K=K)
```

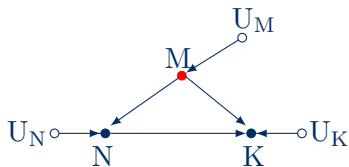
(c) R code

## Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(U) \end{cases}$$

(a) structural model

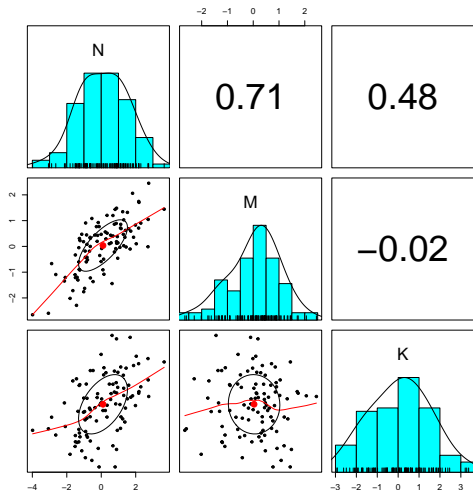


(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(N, K) > 0$  goes in line of our “rudimentary” understanding of the data.
- but why there is  $\text{cor}(M, k) \approx 0$ ?  
(hint: univariate correlation)
- we might not include M as a covariate in our statistical model



# Regression, regression!!

based on [statistical analysis](#),

- two regressions with two different results, which model is the “true”?

```
> summary(lm(K ~ N, data=d)) # biased estimate
Call:
lm(formula = K ~ N, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-2.8355 -0.8110  0.0188  0.7897  3.4276

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.01401    0.12057   0.116   0.908
N            0.53002    0.09332   5.680 1.38e-07 ***
> summary(lm(K ~ N + M, data=d)) # less biased estimate
Call:
lm(formula = K ~ N + M, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-2.50873 -0.72626 -0.01968  0.69016  2.93000

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.22096    0.09845   2.244  0.0271 *
N            0.95510    0.10089   9.466 1.91e-15 ***
M           -1.06246    0.15462  -6.871 6.14e-10 ***
```

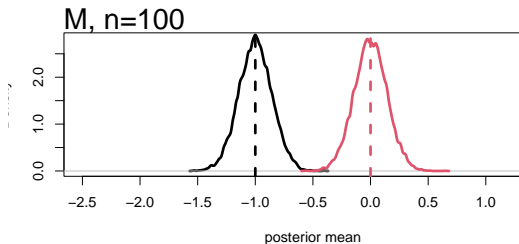
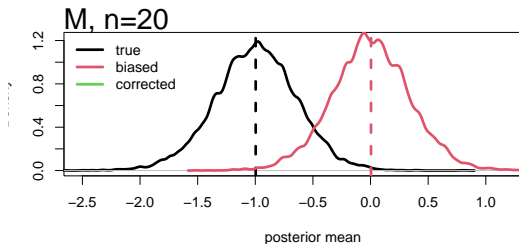
# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

under the **incorrect model**,  
the larger the sample size,

- the more **certain** you are about your estimates
- the more **mistaken** you are about your research question



# The dream team!!

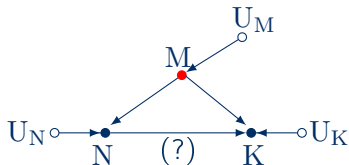
based on **DAG** and **statistical model**,

- the 2nd D-separation rule requires you to control any noncollider to block the **backdoor path**,  
i.e.  $N \not\perp\!\!\!\perp K \mid M$

- conditioning on M we can find,  
 $E[K|\text{do}(n)] = E[ E[K|N = n, M] ]$   
(law of total expectation)
- then we can find the  
 $ACE(n) = E[D|\text{do}(n + 1)] - E[D|\text{do}(n)]$   
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

# the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,  
(assuming our DAG is true)

```
> summary(lm(K ~ N + M, data=d)) # less biased estima  
Call:  
lm(formula = K ~ N + M, data = d)  
Residuals:  
    Min       1Q   Median       3Q      Max   
-2.50873 -0.72626 -0.01968  0.69016  2.93000  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)      
(Intercept)  0.22096    0.09845   2.244  0.0271 *      
N             0.95510    0.10089   9.466 1.91e-15 ***    
M            -1.06246    0.15462  -6.871 6.14e-10 ***  
```

### 3. Anecdotal cases

Fork bias: masked relationships (b)



## Masked relationships (b)<sup>6</sup>

also known as,

- (unobserved) omitted variable bias
- an instance of **fork bias**

research question,

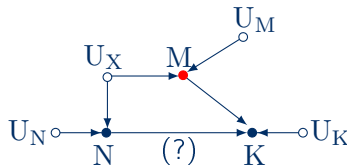
- Does N has a (direct) effect on K?

variables,

- $U_X$ , unobservable (e.g. genetics)
- M, mammal mass in kg.
- N, neocortex over total brain mass
- K, Kcal. per gram of milk

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

---

<sup>6</sup>McElreath [12], chapter 05 (p. 144)

# Simulation setting

```
# sim
U = rnorm( 100 )
N = rnorm( 100 , 1*U )
M = rnorm( 100 , 1*U )
K = rnorm( 100 , 1*N + -1*M )
d = data.frame(U=U,N=N,M=M,K=K)
```

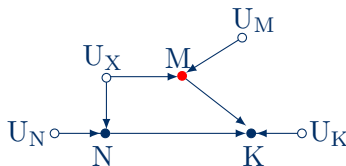
(c) R code

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

## Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$

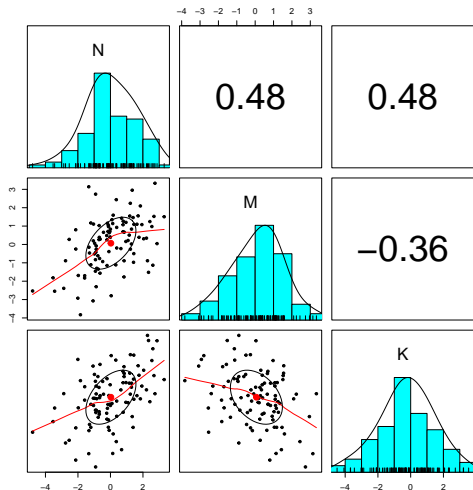


(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(N, K) > 0$  goes in line of our “rudimentary” understanding of the data.
- $\text{cor}(M, K) < 0$  does NOT goes in line of our “rudimentary” understanding of the data.  
(hint: univariate correlation)
- we **include** M as a covariate in our statistical model  
(by chance?)



# Regression, regression!!

based on [statistical analysis](#),

- two regressions with two different results, which model is the “true”?

```
> summary(lm(K ~ N, data=d)) # unobserved path still  
Call:  
lm(formula = K ~ N, data = d)  
  
Residuals:  
    Min       1Q   Median       3Q      Max   
-3.7763 -0.8480  0.1497  0.9874  3.3530  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)      
(Intercept) -0.24867    0.14573  -1.706   0.0911 .      
N             0.51406    0.09502   5.410 4.46e-07 ***  
> summary(lm(K ~ N + M, data=d)) # unobserved path c1  
Call:  
lm(formula = K ~ N + M, data = d)  
  
Residuals:  
    Min       1Q   Median       3Q      Max   
-2.58218 -0.58434 -0.00579  0.72016  1.78724  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)      
(Intercept) -0.19978    0.09375  -2.131   0.0356 *      
N             0.90893    0.06958  13.064 <2e-16 ***    
M            -0.89676    0.07572 -11.843 <2e-16 ***
```

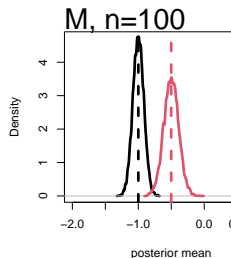
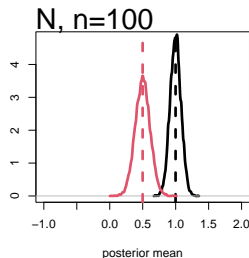
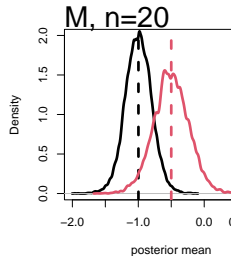
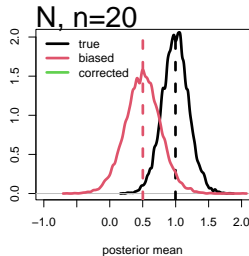
# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

under the **incorrect model**,  
the larger the sample size,

- the more **certain** you are about your estimates
- the more **mistaken** you are about your research question



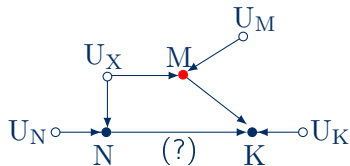
# The dream team!!

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the **backdoor path**,  
i.e.  $N \not\perp\!\!\!\perp K \mid U_X$   
(but it is unobservable)
- still we use the 2nd D-separation rule by controlling for M,  
i.e.  $N \not\perp\!\!\!\perp K \mid M$
- conditioning on M we can still find,  
 $E[K|\text{do}(n)] = E[ E[K|N = n, M] ]$   
(law of total expectation)
- then we can find the  
 $ACE(n) = E[D|\text{do}(n+1)] - E[D|\text{do}(n)]$   
(Frisch-Waugh-Lovell theorem??)

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

# the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,  
(assuming our DAG is true)

```
> summary(lm(K ~ N + M, data=d)) # unobserved path c1
```

Call:  
lm(formula = K ~ N + M, data = d)

Residuals:

	Min	1Q	Median	3Q	Max
	-2.58218	-0.58434	-0.00579	0.72016	1.78724

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.19978	0.09375	-2.131	0.0356 *
N	0.90893	0.06958	13.064	<2e-16 ***
M	-0.89676	0.07572	-11.843	<2e-16 ***

## Similar scenario<sup>7</sup>

research question,

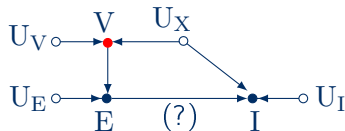
- Does E has a (direct) effect on I?

variables,

- $U_X$ , unobservable  
(e.g. family context)
- $V$ , personal values
- $E$ , education
- $I$ , income

$$M = \begin{cases} V \leftarrow f_M(U_V, U_X) \\ E \leftarrow f_E(V, U_E) \\ I \leftarrow f_I(E, U_X, U_I) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

<sup>7</sup>Cinelli et al. [4] (p. 3), McElreath [13], lecture 6



### 3. Anecdotal cases

Fork bias: multicollinearity

# Multicollinearity<sup>8</sup>

also known as,

- extreme case of masked relationships
- an instance of **fork bias**

research question,

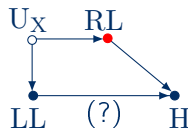
- **Should we include RL in our model?**

variables,

- $U_X$ , unobservable  
(e.g. genetics and context)
- LL, individual's left leg
- RL, individual's right leg
- H, individual's height

$$M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(RL, LL) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

---

<sup>8</sup>McElreath [12], chapter 06 (p. 163)

# Simulation setting

```
# backward simulation
H = round( rnorm( 100 , 170, 2), 1)
Lp = runif( 100 , 0.5-0.05, 0.5+0.05)
LL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
RL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
d = data.frame(LL,RL,H)
```

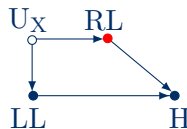
(c) R code

Implications,

■  $LL \not\perp RL$

$$M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(RL, LL) \\ U \sim P(U) \end{cases}$$

(a) structural model

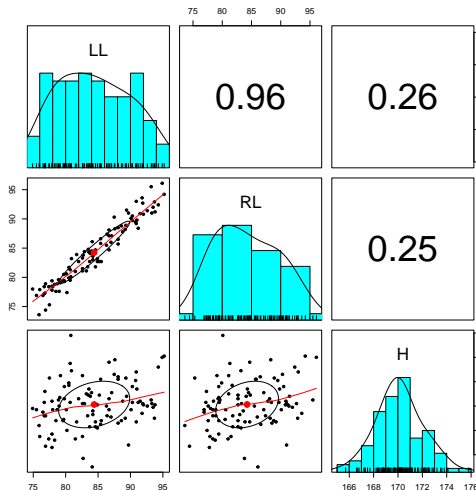


(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(\text{LL}, \text{H}) > 0$ ,  $\text{cor}(\text{RL}, \text{H}) > 0$   
and  $\text{cor}(\text{LL}, \text{RL}) > 0$  goes in line of  
our understanding of the data.  
(hint: univariate correlation)
- we might not include RL as a  
covariate in our statistical model  
(based on univariate correlation)



# Regression, regression!!

based on statistical analysis,

- the second regression show larger SE values

```
> summary(lm(H ~ -1 + RL, data=d)) # unbiased,
Call:
lm(formula = H ~ -1 + RL, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-18.1223  -7.6047   0.8014   9.3119  17.3240

Coefficients:
      Estimate Std. Error t value Pr(>|t|)
RL  2.00234    0.01198   167.2   <2e-16 ***
> summary(lm(H ~ -1 + LL + RL, data=d)) # ine
Call:
lm(formula = H ~ -1 + LL + RL, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-20.357  -8.608   1.473  10.207  19.770

Coefficients:
      Estimate Std. Error t value Pr(>|t|)
LL  0.5090    0.7271    0.700   0.4856
RL  1.5017    0.7284    2.062   0.0419 *
```

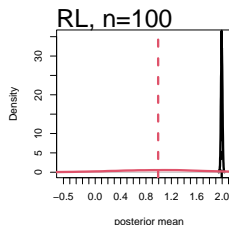
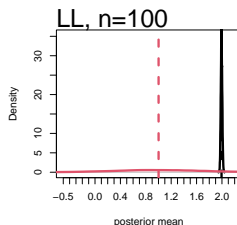
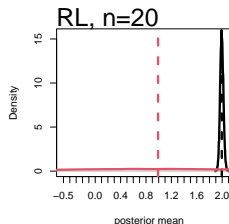
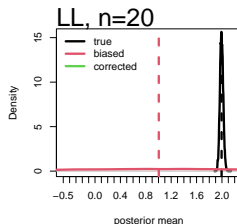
# I'm sure data won't help

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

under the **incorrect model**,  
the larger the sample size,

- the more **incorrect** you are about your estimates
- the less **certain** you are about your estimates
- the more **mistaken** you are about your research question



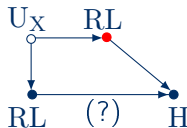
## Not so great now??

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,  
i.e.  $LL \not\perp H \mid U_X$   
(but it is unobservable)
- we still use the 2nd D-separation rule by controlling for RL,  
i.e.  $LL \not\perp H \mid RL$
- but RL and LL give the same information, i.e. singular matrix,  
(is like having a causal model like b)

$$M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(RL) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

# Not so great now??

based on DAG and statistical analysis,

- the less biased model is the second,  
(assuming our DAG is true)

```
> summary(lm(H ~ -1 + RL, data=d)) # unbiased
```

Call:  
lm(formula = H ~ -1 + RL, data = d)

Residuals:

Min	1Q	Median	3Q	Max
-18.1223	-7.6047	0.8014	9.3119	17.3240

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
RL	2.00234	0.01198	167.2	<2e-16 ***



### 3. Anecdotal cases

Fork bias: bias amplification

# Bias amplification<sup>9</sup>

also known as,

- (unobserved) omitted variable bias
- related to **instrumental variables**
- an instance of **fork bias**

research question,

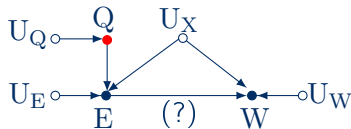
- **Does E has a (direct) effect on W?**

variables,

- Q, instrumental variable  
(e.g. quarter of the year)
- E, educational level
- $U_X$ , unobservables (e.g. ability)
- W, future wages

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

<sup>9</sup>McElreath [12], chapter 14 (p. 455), Cinelli et al. [4] (p. 5)

# Simulation setting

```
# sim
U = rnorm( 100 )
Q = sample( 1:4, 100, replace=T )
E = rnorm( 100 , 1*Q + 1*U )
W = rnorm( 100 , 0*E + 1*U )
d = data.frame(U=U,Q=Q,E=E,W=W)
```

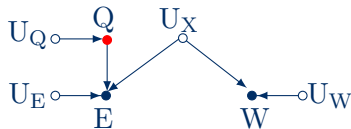
(c) R code

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

## Implications,

- $E \not\perp\!\!\!\perp W$
- $E \perp\!\!\!\perp W \mid U_X$  (impossible)
- $Q \perp\!\!\!\perp U_X$  (cannot be tested)
- $Q \not\perp\!\!\!\perp E$
- $Q \perp\!\!\!\perp W \mid E$  (cannot be tested)  
(exclusion restriction)

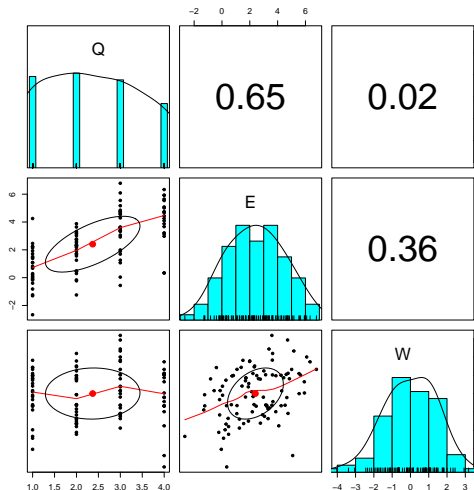


(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(Q, E) > 0$  and  $\text{cor}(E, W) > 0$  goes in line of our “rudimentary” understanding of the data.
- $\text{cor}(Q, W) > 0$  tells you about the exclusion restriction?  
(hint: No)
- we might NOT include  $Q$  as a covariate in our statistical model  
(but is the instrumental variable!!!)



# Regression, regression!!

based on statistical analysis,

- two regressions with two different results, which model is the “true”?
- one is “worse”/“better” than the other?
- are both wrong?

```
> summary(lm(W ~ E, data=d)) # biased

Call:
lm(formula = W ~ E, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-4.0726 -0.9674  0.1771  0.9234  2.8787

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.60816    0.20506  -2.966  0.003793 **
E             0.25408    0.06559   3.873  0.000194 ***
> summary(lm(W ~ E + Q, data=d)) # more biased

Call:
lm(formula = W ~ E + Q, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-3.7405 -0.9774  0.0879  0.9162  2.9825

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.12229    0.30650   0.399  0.69078
E             0.42054    0.08262   5.090 1.75e-06 ***
Q            -0.47716    0.15361  -3.106  0.00249 **
```

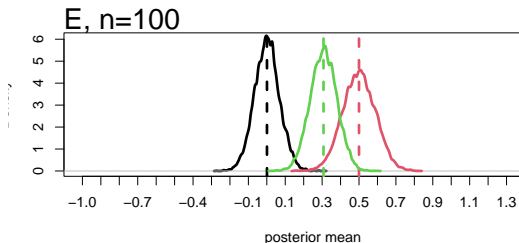
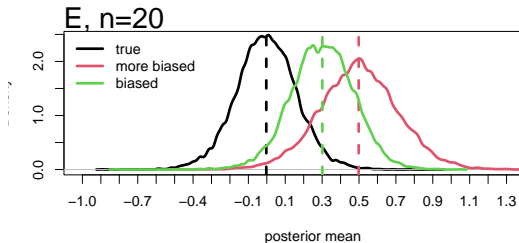
# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

under the **incorrect model**,  
the larger the sample size,

- the more **certain** you are about your estimates
- the more **mistaken** you are about your research question (under the any model!!)  
(the winner's curse)



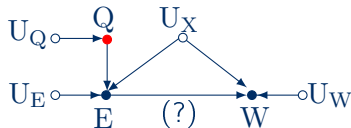
# Yo, what is going on??

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,  
i.e.  $E \perp\!\!\!\perp W \mid U_X$   
(but it is impossible)
- if we use  $Q$  in the model, the 3rd D-separation rule kicks in:  
“A collider that has been conditioned on does not block a path.”  
i.e.  $Q \not\perp\!\!\!\perp U_X \mid E$   
(e.g. switch, electricity, and light bulb)

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

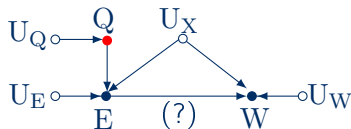
# Yo, what is going on??

open paths?:

- $E \rightarrow W$
- $E \rightarrow U_X \rightarrow W$
- $E \rightarrow U_X \rightarrow Q \rightarrow E \rightarrow W$
- $E \rightarrow U_X \rightarrow Q \rightarrow E \rightarrow U_X \rightarrow W$

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram



# What should I do then??

$$\begin{pmatrix} W \\ E \end{pmatrix} \sim \text{MVN} \left[ \begin{pmatrix} \mu_W \\ \mu_E \end{pmatrix}, \Sigma \right]$$

$$\mu_W = \alpha_W + \beta_{EW}E$$

$$\mu_E = \alpha_E + \beta_{QE}E$$

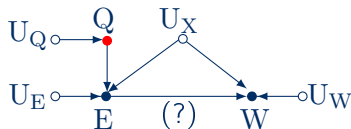
(c) probabilistic model

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

based on **DAG** and **statistical model**,  
use the knowledge of the system

- one model for  $Q \rightarrow E$
- one model for  $E \rightarrow W$
- use the knowledge that  $\text{cov}(E, W) > 0$   
due to unobserved confounder  $U_X$ ,  
(i.e.  $\text{cov}(E, W) = \Sigma = \mathbf{SRS}$ )



(b) causal diagram

## did it worked???

based on DAG and bayesian statistical analysis,

- appropriate value estimated, (assuming our DAG is true)
- it picks up the unobserved correlation  $R[1, 2]$

	mean	sd	5.5%	94.5%
aE	0.02	0.18	-0.26	0.30
aW	-0.14	0.16	-0.40	0.13
bQE	1.00	0.07	0.88	1.12
bEW	0.05	0.07	-0.06	0.16
R[1,1]	1.00	0.00	1.00	1.00
R[1,2]	0.33	0.11	0.15	0.50
R[2,1]	0.33	0.11	0.15	0.50
R[2,2]	1.00	0.00	1.00	1.00
S[1]	1.25	0.10	1.11	1.42
S[2]	1.39	0.10	1.24	1.56

# did it worked???

frequentists guys apply

Two Stage Least Squares (2SLS)<sup>a</sup>:

- regress  $Q \rightarrow E$ , predict  $\hat{E}$
- regress  $\hat{E} \rightarrow W$

```
s1 = lm( E ~ Q, data=d)
Ehat = s1$fitted.values
s2 = lm( W ~ Ehat, data=d)
# se not corrected

require(AER)
tsls = ivreg( W ~ E | Q, data=d)
# se corrected
```

<sup>a</sup>Hanck et al. [7], section 12.1,  
See McElreath [12] chapter 14 (p. 460) for a  
discussion on the method.

```
> summary(s2)

Call:
lm(formula = W ~ Ehat, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-4.1911 -1.0670 -0.0643  1.3802  4.6813

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.5992     0.3667  -1.634   0.106
Ehat           0.2093     0.1263   1.658   0.101
> summary(tsls)

Call:
ivreg(formula = W ~ E | Q, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-3.6571 -1.1852 -0.1819  1.0945  4.5328

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.5992     0.3325  -1.802   0.0746 .
E             0.2093     0.1145   1.829   0.0705 .
```

### 3. Anecdotal cases

No more fork bias: neutral control

## Neutral control<sup>10</sup>

also known as,

- precision “booster”
- similar to experimental design

research question,

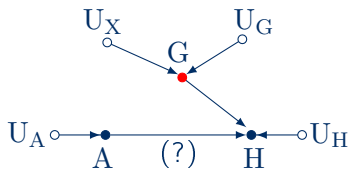
- Does N has a (direct) effect on K?

variables,

- A, “hearing” age
- G, gender
- $U_X$ , unobservable (e.g. no idea yet)
- H, inverse logit of entropy  
(approximate of speech intelligibility)

$$M = \begin{cases} G \leftarrow f_G(U_G, U_X) \\ A \leftarrow f_A(U_A) \\ H \leftarrow f_H(A, G, U_H) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

---

<sup>10</sup>Cinelli et al. [4] (p. 4)

# Simulation setting

```
# sim  
G = sample( 0:1, 100 , replace=T )  
A = rnorm( 100 )  
H = rnorm( 100 , -1*A + -1*G )  
d = data.frame(G=G,A=A,SI=SI)
```

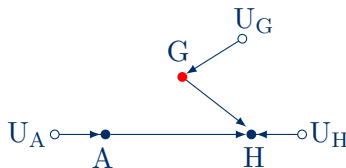
(c) R code

$$M = \begin{cases} G \leftarrow f_G(U_G, U_X) \\ A \leftarrow f_A(U_A) \\ H \leftarrow f_H(A, G, U_H) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

## Implications,

- $A \perp\!\!\!\perp G$
- $A \not\perp\!\!\!\perp H$
- $G \not\perp\!\!\!\perp H$

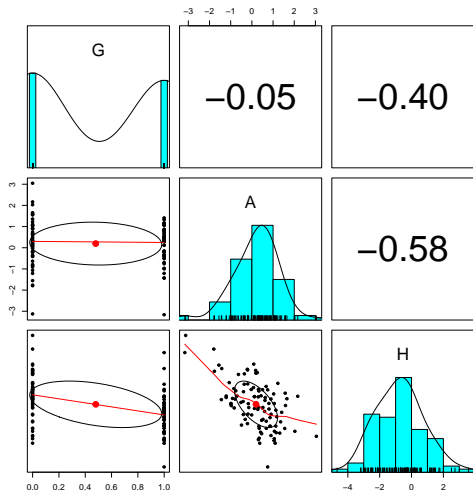


(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(G, H) \approx 0$ ,  $\text{cor}(G, A) \approx 0$  and  $\text{cor}(A, H) < 0$  goes in line of our “rudimentary” understanding of the data.
- we include both as a covariate in our statistical model



# Regression, regression!!

based on statistical analysis,

- now there is no severe biasing
- notice the standard errors, lower for A when G is included

```
> summary(lm(H ~ A, data=d)) # correct estimate
Call:
lm(formula = H ~ A, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-3.4714 -0.8797 -0.0633  0.8963  2.4346

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.5770     0.1216  -4.746 7.07e-06 ***
A           -0.8410     0.1183  -7.108 1.92e-10 ***
> summary(lm(H ~ A + G, data=d)) # correct estimate,
Call:
lm(formula = H ~ A + G, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-2.7994 -0.6914  0.0579  0.7796  1.8274

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.03317    0.14312   0.232   0.817
A           -0.87360    0.10090  -8.658 1.05e-13 ***
G           -1.25786    0.20371  -6.175 1.55e-08 ***
```



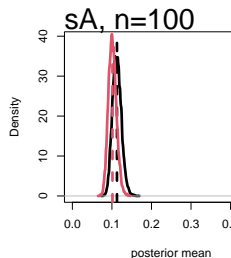
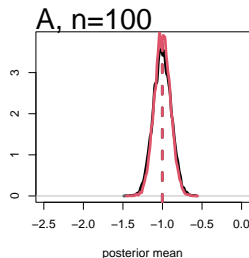
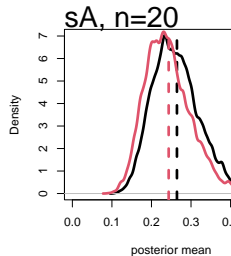
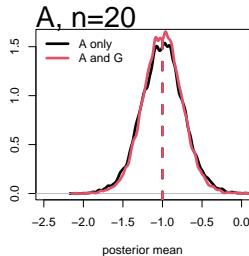
# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

the larger the sample size,

- the (more) certain you are about your estimates
- the more correct you are about your research question (under the any model)



### 3. Anecdotal cases

Pipe bias: precision parasite

# Precision parasite<sup>11</sup>

research question,

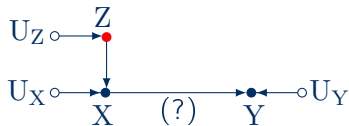
- Should we include Z in the model?

variables,

- Z,
- X,
- Y,

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

<sup>11</sup>McElreath [13], lecture 6, Cinelli et al. [4] (p.5)

# Simulation setting

```
# sim
Z = rnorm( 100 )
X = rnorm( 100 , 1*Z )
Y = rnorm( 100 , 1*X )
d = data.frame(Z,X,Y)
```

(c) R code

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(U) \end{cases}$$

(a) structural model

Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$



(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(M, K) < 0$  does NOT goes in line of our “rudimentary” understanding of the data.
- and why there is  $\text{cor}(N, K) \approx 0$ ?  
(hint: univariate correlation)
- we include  $N$  as a covariate in our statistical model  
(is our research hypothesis)



# Regression, regression!!

based on statistical analysis,

- two regressions with two different results, which model is the “true”?



# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

the larger the sample size,

- the more **certain** you are about your estimates
- the more **mistaken** you are about your research question  
(under the “incorrect” model)

pipe1\_samplesize.pd

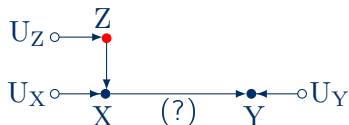
# The dream team!!

based on **DAG** and **statistical model**,

- the 2nd D-separation rule requires you to control any noncollider to block the **backdoor path**,  
i.e.  $N \not\perp\!\!\!\perp K \mid M$
- conditioning on  $M$  we can find,  
 $E[K|\text{do}(n)] = E[ E[K|N = n, M] ]$   
(law of total expectation)
- then we can find the  
 $ACE(n) = E[D|\text{do}(n + 1)] - E[D|\text{do}(n)]$   
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram



# the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,  
(assuming our DAG is true)



### 3. Anecdotal cases

Pipe bias: post-treatment

# Post-treatment bias<sup>12</sup>

also known as,

- included variable bias (bigger grouping)

research question,

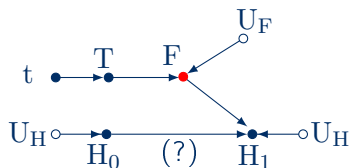
- Does the treatment  $T$  works?

variables,

- $H_0$ , height of plant at  $t = 0$
- $T$ , antifungal treatment
- $F$ , presence of fungus
- $H_1$ , height of plant at  $t = 1$

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_H(F, H_0, U_H) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

# Simulation setting

```
# backward simulation
H = round( rnorm( 100 , 170, 2), 1)
Lp = runif( 100 , 0.5-0.05, 0.5+0.05)
LL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
RL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
d = data.frame(LL,RL,H)
```

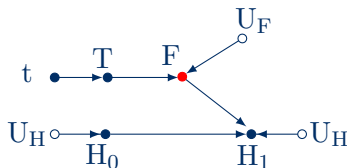
(c) R code

## Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_H(F, H_0, U_H) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(M, K) < 0$  does NOT goes in line of our “rudimentary” understanding of the data.
- and why there is  $\text{cor}(N, K) \approx 0$ ?  
(hint: univariate correlation)
- we include N as a covariate in our statistical model  
(is our research hypothesis)



# Regression, regression!!

based on statistical analysis,

- two regressions with two different results, which model is the “true”?

pipe2\_reg.png

# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

the larger the sample size,

- the more **certain** you are about your estimates
- the more **mistaken** you are about your research question  
(under the “incorrect” model)



pipe2\_samplesize.pd

# The dream team!!

based on DAG and statistical model,

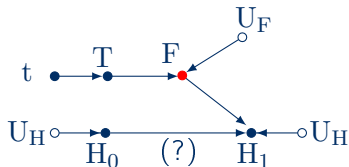
- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path, i.e.  $N \not\perp\!\!\!\perp K \mid M$

- conditioning on M we can find,  
 $E[K|\text{do}(n)] = E[ E[K|N = n, M] ]$   
(law of total expectation)

- then we can find the  
 $ACE(n) = E[D|\text{do}(n+1)] - E[D|\text{do}(n)]$   
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_H(F, H_0, U_H) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



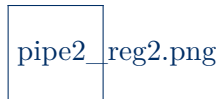
(b) causal diagram



# the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,  
(assuming our DAG is true)



### 3. Anecdotal cases

Pipe bias: masked relationships

## Masked relationships<sup>13</sup>

also known as,

- mediation
- Simpson's paradox
- an instance of **pipe bias**

research question,

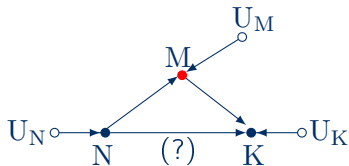
- Does N has a (direct) effect on K?

variables,

- M, mammal mass in kg.
- N, neocortex over total brain mass
- K, Kcal. per gram of milk

$$M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(N, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

---

<sup>13</sup>McElreath [12], chapter 05 (p. 144)

# Simulation setting

```
# sim
N = rnorm( 100 )
M = rnorm( 100 , 1*N )
K = rnorm( 100 , 1*N + -1*M )
d = data.frame(N=N,M=M,K=K)
```

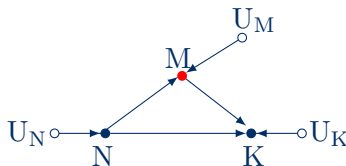
(c) R code

$$M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(M, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(U) \end{cases}$$

(a) structural model

## Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$

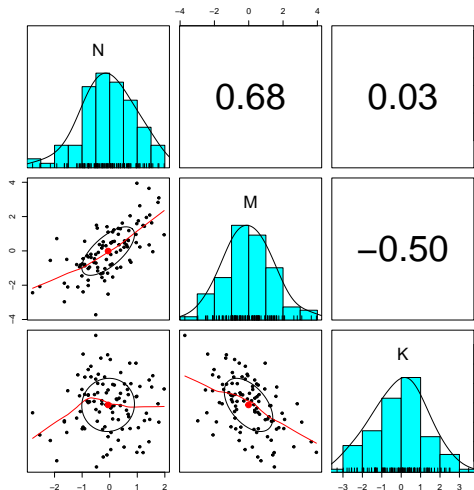


(b) causal diagram

# “Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(M, K) < 0$  does NOT goes in line of our “rudimentary” understanding of the data.
- and why there is  $\text{cor}(N, K) \approx 0$ ?  
(hint: univariate correlation)
- we include N as a covariate in our statistical model  
(is our research hypothesis)



# Regression, regression!!

based on statistical analysis,

- two regressions with two different results, which model is the “true”?

```
> summary(lm(K ~ N, data=d)) # biased estimate

Call:
lm(formula = K ~ N, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-3.1751 -0.9009  0.1519  0.8574  3.6041

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.10412    0.13808  -0.754   0.453
N             0.05005    0.14487   0.345   0.730
> summary(lm(K ~ N + M, data=d)) # less biased estimate

Call:
lm(formula = K ~ N + M, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-2.58484 -0.59175  0.04378  0.61175  2.43360

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.06181    0.09825  -0.629   0.531
N             0.98297    0.13994   7.024 2.98e-10 ***
M            -0.93107    0.09457  -9.846 2.89e-16 ***
```

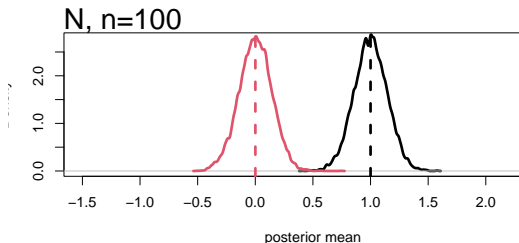
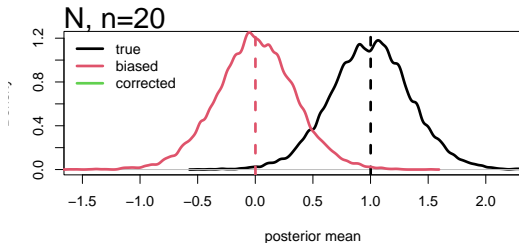
# I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples  $n = 20$
- bottom: 10,000 samples  $n = 100$

the larger the sample size,

- the more **certain** you are about your estimates
- the more **mistaken** you are about your research question (under the “incorrect” model)



# The dream team!!

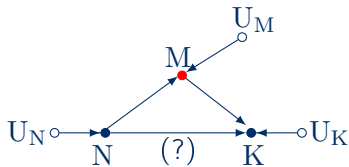
based on **DAG** and **statistical model**,

- the 2nd D-separation rule requires you to control any noncollider to block the **backdoor path**,  
i.e.  $N \not\perp\!\!\!\perp K \mid M$

- conditioning on M we can find,  
 $E[K|\text{do}(n)] = E[ E[K|N = n, M] ]$   
(law of total expectation)
- then we can find the  
 $ACE(n) = E[D|\text{do}(n + 1)] - E[D|\text{do}(n)]$   
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(M, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram



# the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,  
(assuming our DAG is true)

```
> summary(lm(K ~ N + M, data=d)) # less biased estimate

Call:
lm(formula = K ~ N + M, data = d)

Residuals:
    Min       1Q   Median       3Q      Max
-2.58484 -0.59175  0.04378  0.61175  2.43360

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.06181   0.09825   -0.629    0.531
N             0.98297   0.13994    7.024 2.98e-10 ***
M            -0.93107   0.09457   -9.846 2.89e-16 ***
```

## 4. Concluding remarks

# Concluding remarks

- Research is filled with challenges  
(you: D]duh!!)
- Statistical models are not theory  
(you: so obvious again!!)
- **Don't trust** your statistical model  
when no DAG is involved  
(me: how about that?!)
- For **explanation**, without a DAG  
the (sample) size does not matter  
(me: sorry?!)
- For **prediction**, sometimes a DAG  
can help  
(me: did you expect this one?!)



5. Do you wanna know more???

5. Do you wanna know more???

- [1] Anderson, D. [2008]. Model Based Inference in the Life Sciences: A Primer on Evidence, Springer.
- [2] Bareinboim, E. and Pearl, J. [2016]. Causal inference and the data-fusion problem, *Proceedings of the National Academy of Sciences* 113(27): 7345–7352. doi: <https://doi.org/10.1073/pnas.1510507113>.
- [3] Chamberlain, T. [1965]. The method of multiple working hypotheses, *Science* 148(3671): 754–759. url: <https://www.jstor.org/stable/1716334>.
- [4] Cinelli, C., Forney, A. and Pearl, J. [2021]. A crash course in good and bad controls, Technical report.
- [5] Cunningham, S. [2022]. Causal inference: The mixtape. url: <https://mixtape.scunning.com/index.html>.
- [6] Fogarty, L., Madeleine, A., Holding, T., Powell, A. and Kandler, A. [2022]. Ten simple rules for principled simulation modelling, *PLOS Computational Biology* 18(3): 1–8. doi: <https://doi.org/10.1371/journal.pcbi.1009917>.
- [7] Hanck, C., Arnold, M., Gerber, A. and Schmelzer, M. [2021]. Introduction to econometrics with r. url: <https://www.econometrics-with-r.org/index.html>.

- [8] Hernán, M. [2020]. Causal diagrams: Draw your assumptions before your conclusions.  
url: <https://www.edx.org/course/causal-diagrams-draw-your-assumptions-before-your>.
- [9] Hernán, M. and Robins, J. [2020]. Causal Inference: What If, 1 edn, Chapman and Hall/CRC.  
url: <https://www.hsph.harvard.edu/miguel-hernan/causal-inference-book>.
- [10] Jaynes, E. [2003]. Probability Theory: The Logic of Science, Cambridge University Press.
- [11] McElreath, R. [2019]. Statistical rethinking, 2019 course.  
url: [https://github.com/rmcelreath/statrethinking\\_winter2019](https://github.com/rmcelreath/statrethinking_winter2019).
- [12] McElreath, R. [2020]. Statistical Rethinking: A Bayesian Course with Examples in R and STAN, Chapman and Hall/CRC.
- [13] McElreath, R. [2022]. Statistical rethinking, 2022 course.  
url: [https://github.com/rmcelreath/stat\\_rethinking\\_2022](https://github.com/rmcelreath/stat_rethinking_2022).
- [14] Pearl, J. [1988]. Probabilistic reasoning in intelligent systems: Networks of plausible inference, The Journal of Philosophy 88(8): 434–437.  
doi: <https://doi.org/10.2307/2026705>.  
url: <https://www.jstor.org/stable/2026705>.

- [15] Pearl, J. [2009]. Causality: Models, Reasoning and Inference, Cambridge University Press.
- [16] Pearl, J. [2019]. The seven tools of causal inference, with reflections on machine learning, Communications of the ACM 62(3): 54–60.  
doi: <https://doi.org/10.1177/0962280215586010>.
- [17] Pearl, J., Glymour, M. and Jewell, N. [2016]. Causal Inference in Statistics: A Primer, John Wiley Sons, Inc.
- [18] Pearl, J. and Mackenzie, D. [2018]. The Book of Why: The New Science of Cause and Effect, 1st edn, Basic Books, Inc.
- [19] Spirtes, P., Glymour, C. and Scheines, R. [1991]. From probability to causality, Philosophical Studies 64(1): 1–36.  
url: <https://www.jstor.org/stable/4320244>.
- [20] Textor, J., van der Zander, B., Gilthorpe, M., Liskiewicz, M. and Ellison, G. [2016]. Robust causal inference using directed acyclic graphs: the r package 'dagitty', Int J Epidemiol 45(6): 1887–1894.  
doi: <https://doi.org/10.1093/ije/dyw341>.