



University of Antwerp
| Faculty of Social Sciences

DAGs and PP

or commonly known as:

“a bit more transparent way to state
your research assumptions and questions”

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(Josema, for the friends)

June 22, 2022

What are we going to talk about? I

1 About research

- A typical scientific lab
- Research hypothesis production

2 DAGs and PP

3 Example cases

- Experimental design: the panacea
- Simulation conventions
- Fork: spurious relationships
- Fork: masked relationships (a)
- Fork: masked relationships (b)
- Fork: multicollinearity

What are we going to talk about? II

- Fork: neutral control
- Pipe: precision parasite
- Pipe: post-treatment
- Pipe: Simpson's paradox
- Pipe/Fork: good controls
- Pipe/Fork: bias amplification
- Collider: Berkson's paradox
- Collider: M-bias
- Descendant: proxies (a)
- Descendant: case control

What are we going to talk about? III

- Descendant: measurement error, outcome only
- Descendant: measurement error, predictor only
- Descendant: measurement error, both

4 Concluding remarks

5 Do you wanna know more???

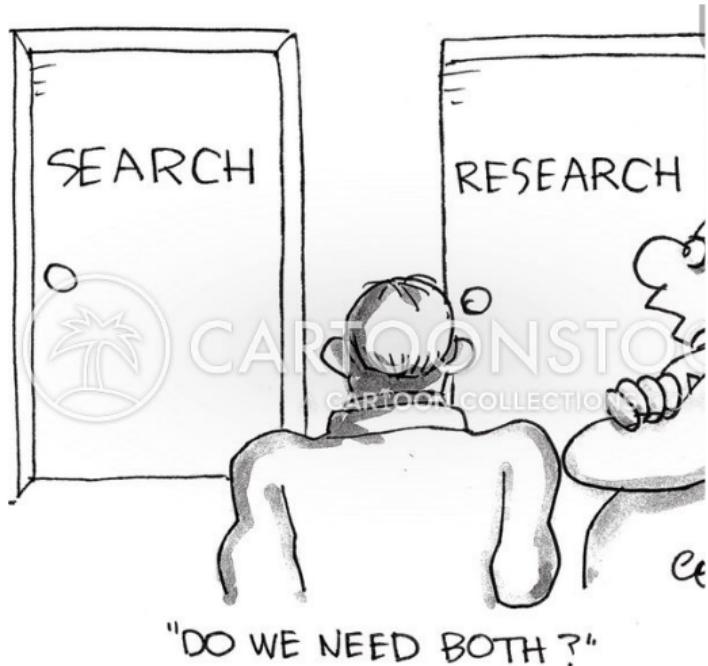
1. About research

A typical scientific lab

A typical scientific lab¹

what is needed?

1. Quality of theory
2. Quality of data
3. Reliable procedures and code
4. Quality of data analysis
5. Documentation
6. Reporting



¹McElreath [12], lecture 20 and McElreath [13], chapter 17

A typical scientific lab

what we “normally” focus on?

1. Quality of theory
2. Quality of data
3. Reliable procedures and code
4. Quality of data analysis
5. Documentation
6. Reporting

you said I only
need more data

more "quality"
data



A typical scientific lab²

what can be improved?
(with DAGs and PP)

1. Quality of theory
2. Quality of data
3. Reliable procedures and code
4. Quality of data analysis
5. Documentation
6. Reporting



²see Yarkoni [22], Rohrer et al. [19] for a discussion about the alignment between verbal and statistical expressions, and its problematics.

1. About research

Research hypothesis production

Research hypothesis production

well known challenges^a

- Insufficient data
- Wrong population
- Measurement error
- Selection bias
- Confounding

^aHernán [8], lesson 4



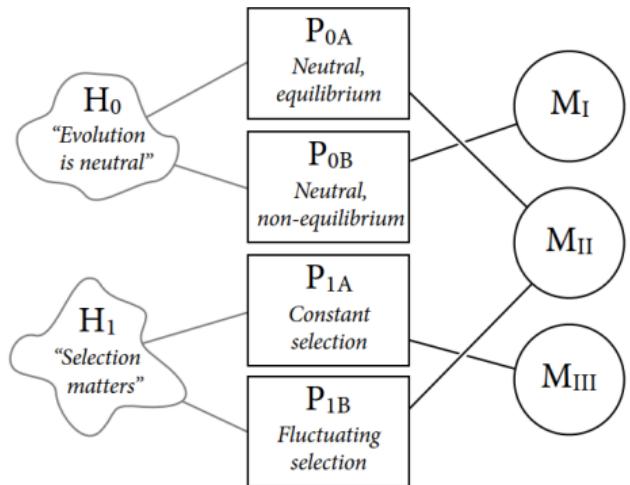
Research hypothesis production

but we should not forget^a

- No one-to-one relationship exists between our process models and statistical models,
- Nor between our hypothesis and a process models

^aFigure 1.2 reproduced from chapter 1
McElreath [13]

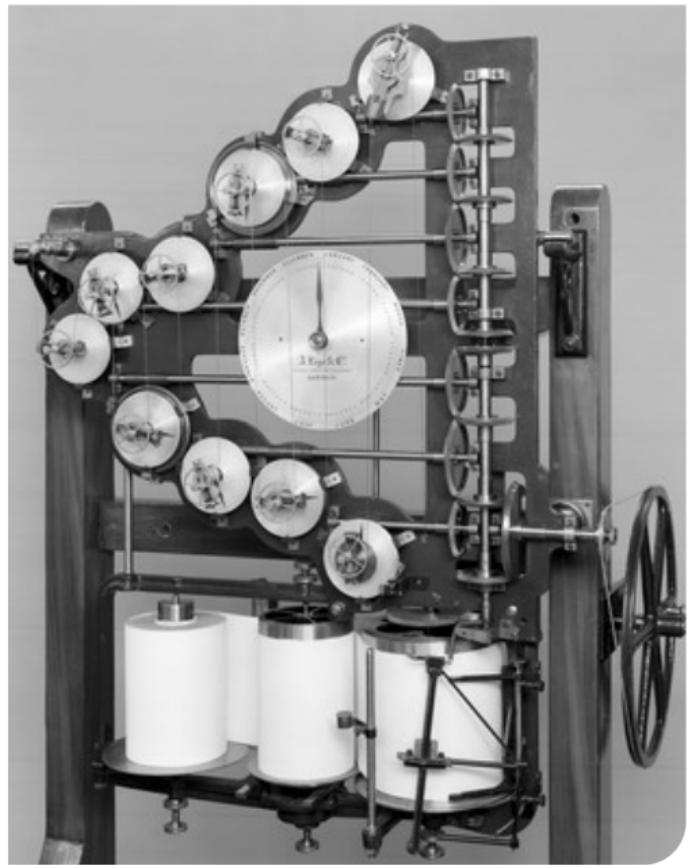
Hypotheses Process models Statistical mode



Research hypothesis production

and also

- statistical models are just “machines to find association”, not a reliable reflection of the theory (I can prove it!!).



Research hypothesis schematics³

- a. Estimand and process model
- b. Synthetic data generation
- c. Statistical model design and testing
- d. Apply statistical model to data

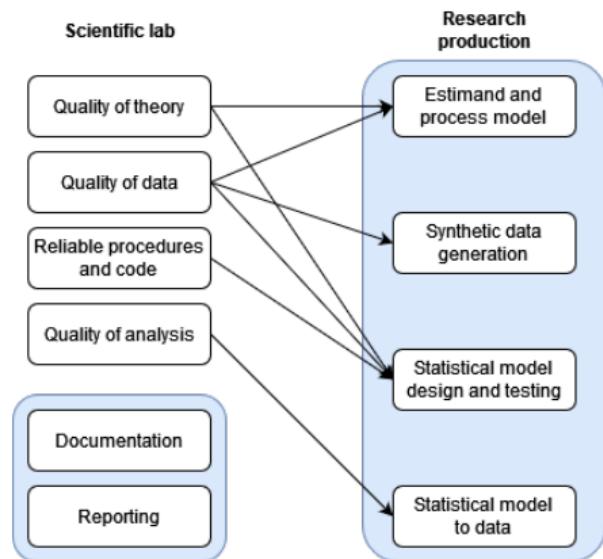


³McElreath [13], lecture 20, Pearl [16]. Follow Fogarty et al. [6] on item (c).

Research hypothesis schematic

where does it match with the previous?

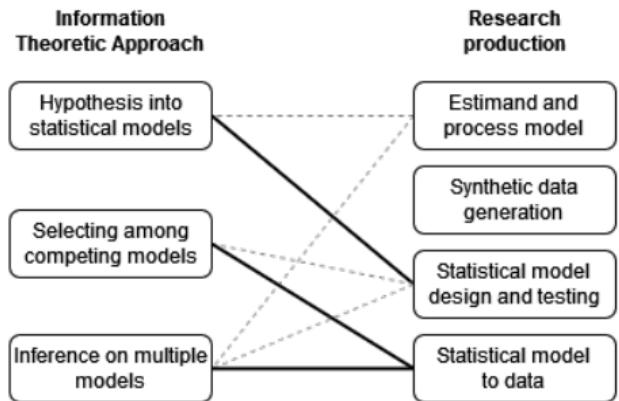
- a. Estimand and process model
maps 1 (theory) and 2 (data) to a heuristic model.
- b. Synthetic data generation
maps 2 (data) to an idealized data.
- c. Statistical model design and testing
maps 1 (theory), 2 (data), and 3 (reliable code) to an statistical model.
- d. Apply statistical model to data
maps 4 (analysis) onto a result.



Where does the ITA fit?

Information Theoretic Approach (ITA) is framework to select among competing models [1, 3]:

1. Hypothesis into statistical models,
(how about a process model?)
2. Select among competing models,
(do the code works as intended?)
3. Make inferences based on one or
multiple models.
(do the code works as intended?,
are there variables that can bias our
results?)



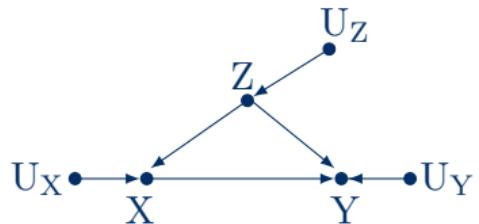
2. DAGs and PP

DAGs and PP

- Directed acyclic graphs (DAGs), are a type of structural causal model (SCM) [15, 4]
- DAGs can be represented by a structural model, and its associated causal diagram.
- they DO NOT represent specific functional forms, but rather general relationships (causal diagram).

$$M = \begin{cases} Z \leftarrow f_Z(U_z) \\ X \leftarrow f_X(Z, U_x) \\ Y \leftarrow f_Y(X, Z, U_y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

DAGs and PP

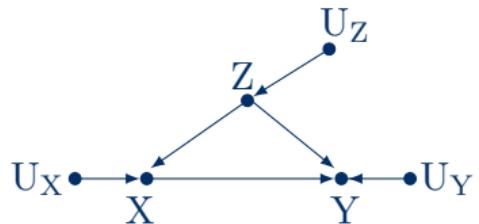
for the structural model (causal diagram),
we establish:

- the functional form f_Z , f_X , and f_Y , and
- the distributional assumptions $P(Z)$,
 $P(X)$, $P(Y)$, and $P(\mathbf{U})$

through probabilistic programming (PP)
[10] (more in part 3).

$$M = \begin{cases} Z \leftarrow f_Z(U_z) \\ X \leftarrow f_X(Z, U_x) \\ Y \leftarrow f_Y(X, Z, U_y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



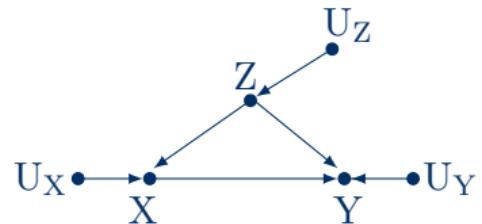
(b) causal diagram

DAGs and PP

- $\mathbf{V} = \{Z, X, Y\}$ are called endogenous variables.
- $\mathbf{U} = \{U_Z, U_X, U_Y\}$ are called exogenous variables.
(drawn when strictly required)
- $\mathbf{F} = \{f_Z, f_X, f_Y\}$ are called structural equations.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

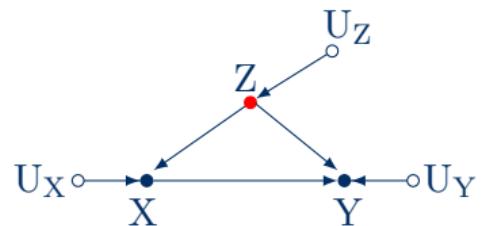
DAG conventions

causal diagram conventions [4],

- black nodes are observed variables.
- white nodes are unobserved variables.
- red nodes are variables for which we will decide its inclusion or not.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

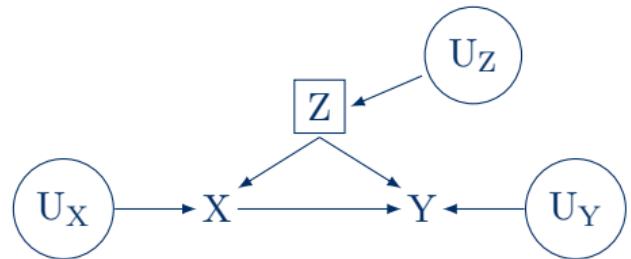
DAG conventions

other causal diagram conventions,

- no circle nodes are observed variables.
- circled nodes are unobserved variables.
- squared nodes are variables for which we will decide its inclusion or not.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

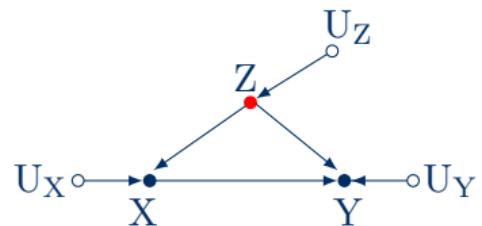
DAG conventions

based on causal graph theory
[14, 15, 17, 18, 20],

1. descendant (child, grandchild), parent (grandparent).
(path specific)
2. paths (directional, non-directional).
3. paths are **blocked** or **open** according to the **D-separation** (Directional) rules.
(also path specific)

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

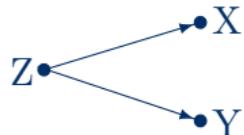
(a) structural model



(b) causal diagram

The benign case of DAG elementals

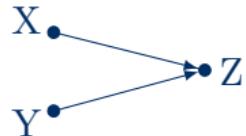
For everything can be depicted with them



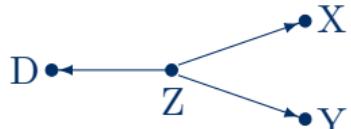
(a) fork



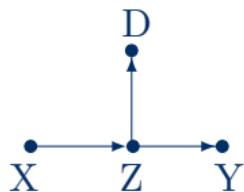
(b) pipe



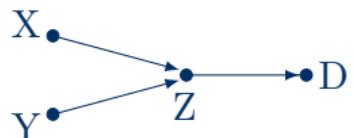
(c) collider



(d) descendant on fork



(e) descendant on pipe

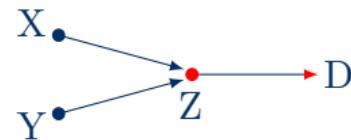
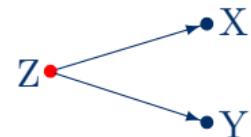
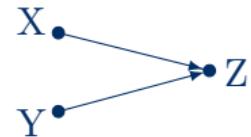


(f) descendant on collider

About D-separation

the four D-separation rules [8],

1. If no variables being conditioned on, a path is blocked if and only if, two arrowheads on the path collide at some variable on the path.
2. Any path that contains a noncollider that has been conditioned on, is blocked (**backdoor path**)^a.
3. A collider that has been conditioned on does not block a path.
4. A collider that has a descendant that has been conditioned on does not block a path.



^athere is also a **front-door** path (if you wonder).

About D-separation

the D-separation rules implications,
(independent of distributional assumptions)

1. $X \perp\!\!\!\perp Y \implies$

$$P(X, Y) = P(X) \cdot P(Y)$$

2. $X \perp\!\!\!\perp Y | Z \implies$

$$P(X, Y | Z) = P(X | Z) \cdot P(Y | Z)$$

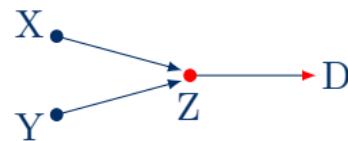
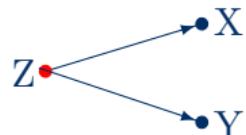
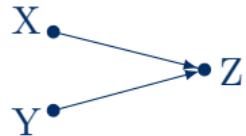
(same for fork or pipe)

3. $X \not\perp\!\!\!\perp Y | Z \implies$

$$P(X, Y | Z) \neq P(X | Z) \cdot P(Y | Z)$$

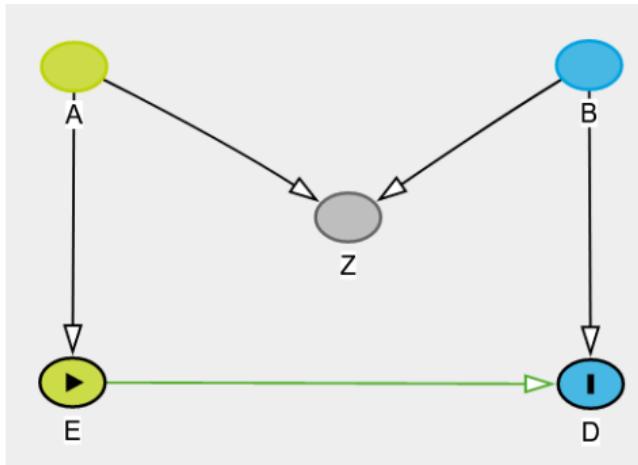
4. $X \not\perp\!\!\!\perp Y | D \implies$

$$P(X, Y | D) \neq P(X | D) \cdot P(Y | D)$$



Oh DAGitty!! mijn vriendin

- browser (R package) environment for creating, editing, and analyzing causal diagrams [21].
- available online: <http://dagitty.net>
- But there are more fish in the sea:
<http://www.causalfusion.net> [2]
(b**** better have my \$\$\$)



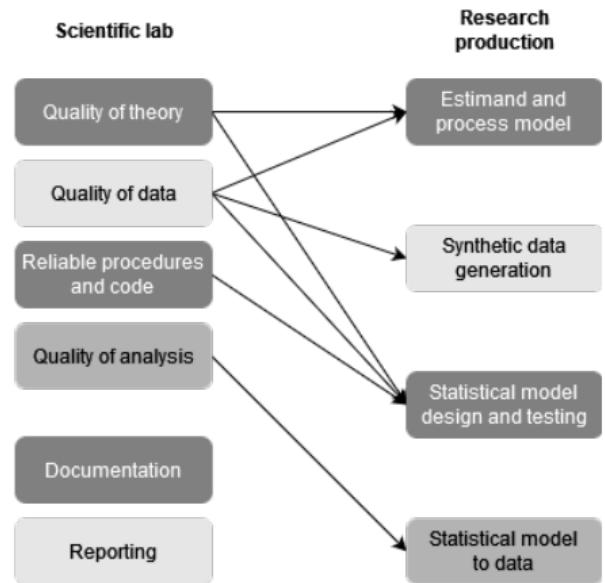
Where do they fit?

starts with:

- A clear definition of the estimand and process model (assumptions).
- An improved the reliability of your procedures.
- As a documentation procedure.

and leads to:

- A sound analysis, and result
(even when we cannot have an answer to our question)
- An improved planning to get data.



3. Example cases

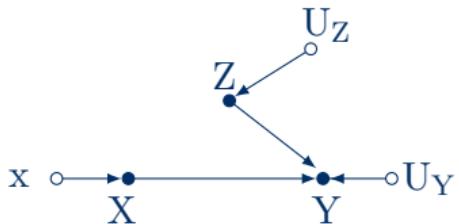
Experimental design: the panacea

Experimental design⁴

- Purpose: to control all factors responsible for the outcome's variation.
(understand the system)
- It is modeled by modifying the structural model (and causal diagram).

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

⁴Cinelli et al. [4], appendix A (p. 15)

Experimental design

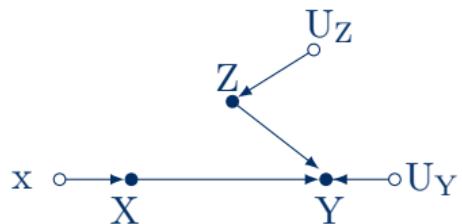
- intervention on X can be written in do-calculus^a as: $P(\mathbf{V} \mid \text{do}(X = x))$.
- remember:

$\mathbf{V} = \{Z, X, Y\}$,
 $\mathbf{U} = \{U_Z, U_X, U_Y\}$, and
 $\mathbf{F} = \{f_Z, f_X, f_Y\}$.

^aan appropriate treatment can be found with the usual suspects [14, 15, 17, 18])

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

Effects of interest

two types of effects,

1. Average causal effect:

$$\text{ACE}(x) = E[Y|do(x+1)] - E[Y|do(x)]$$

2. Controlled direct effect:

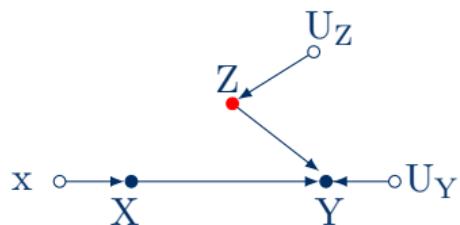
$$\text{CDE}(x, z) = E[Y|do(x+1), do(z)] - E[Y|do(x), do(z)]$$

points to consider:

- CDE takes a particular relevance with observational data.
- There is also a distinction between total effect and direct effect.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

3. Example cases

Simulation conventions

Simulation conventions

one way to define it,

$$Z = U_Z \quad ; \quad U_Z \sim N(0, \sigma_Z)$$

$$X = \beta_Z Z + U_X \quad ; \quad U_X \sim N(0, \sigma_X)$$

$$Y = \beta_Z Z + \beta_X X + U_Y \quad ; \quad U_Y \sim N(0, \sigma_Y)$$

a more succinct way,

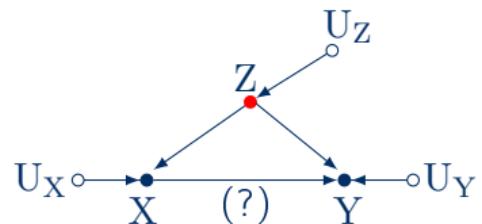
$$Z \sim N(0, \sigma_Z)$$

$$X \sim N(\beta_Z Z, \sigma_X)$$

$$Y \sim N(\beta_Z Z + \beta_X X, \sigma_Y)$$

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(Z, X, U_Y) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

3. Example cases

Fork: spurious relationships

Spurious relationships⁵

also known as,

- spurious association
- confounder
- an instance of Fork

research question,

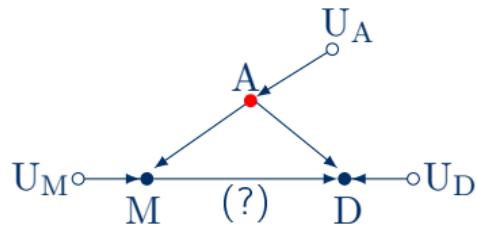
- Does M has a (direct) effect on D?

variables,

- A, median age at marriage
- M, marriage rate
- D, divorce rate

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, M, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

⁵McElreath [12], chapter 5 (p. 125)

Simulation setting

```
# sim
A = rnorm( 100 )
M = rnorm( 100 , mean=-1*A )
D = rnorm( 100 , mean=-1*A + 0*M )
d = data.frame(A=A, M=M, D=D)
```

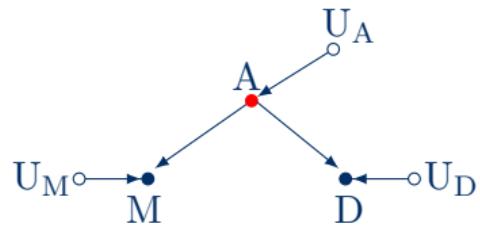
(c) R code

Implications,

- $M \not\perp\!\!\!\perp D$
- $M \perp\!\!\!\perp D \mid A$

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

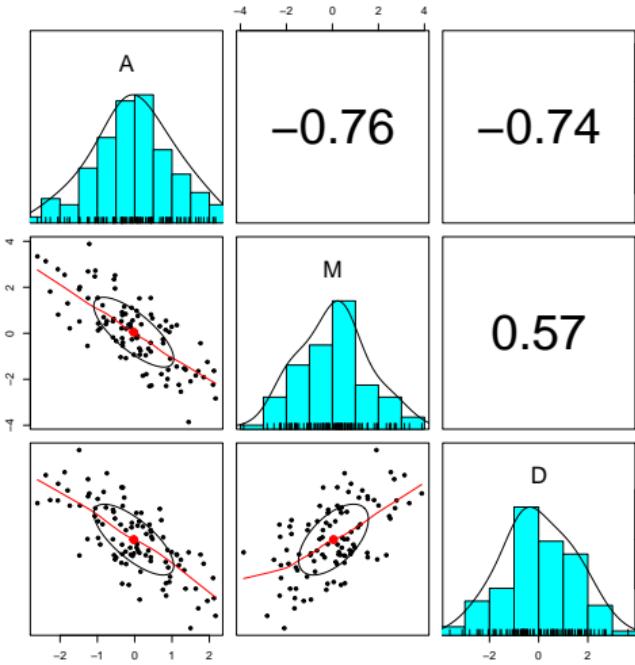


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(A, D) < 0$ and $\text{cor}(M, D) > 0$
goes in line of our “rudimentary”
understanding of the data.
- why there is $\text{cor}(M, D) > 0$?
(hint: univariate correlation)
- we include M as a covariate in our
statistical model
(is our research hypothesis)



Regression, regression!!

based on **statistical analysis**,

- we have two different stories,
(which one is the “truth”?)

```
> summary(lm(D ~ M, data=d)) # spurious relation

Call:
lm(formula = D ~ M, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.80012 -0.90447 -0.03866  0.80220  2.82970 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.23298   0.12412 -1.877  0.0635 .  
M            0.40233   0.08986  4.477 2.04e-05 *** 
> summary(lm(D ~ A + M, data=d)) # controlled relation

Call:
lm(formula = D ~ A + M, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.27295 -0.68174  0.03781  0.78885  2.95320 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.18854   0.09871 -1.910  0.0591 .  
A            -1.03121   0.13483 -7.648 1.49e-11 *** 
M            -0.06134   0.09362 -0.655  0.5139
```

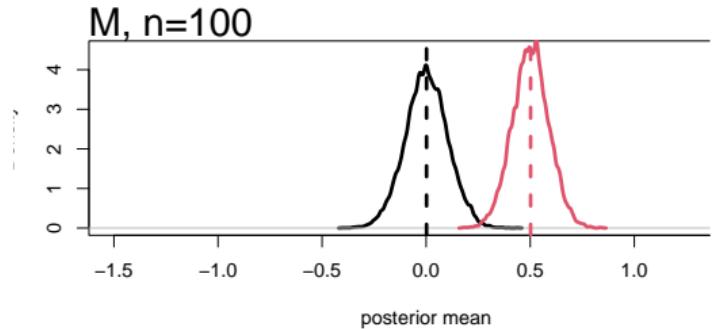
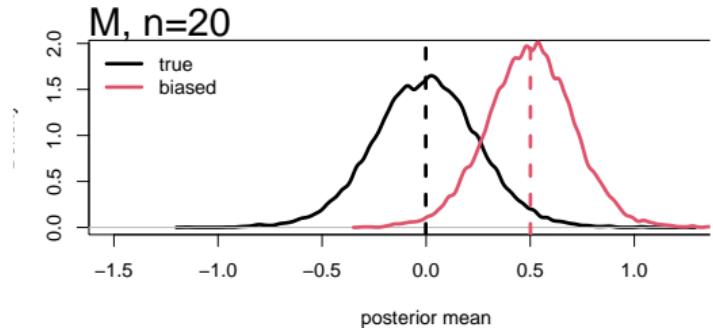
I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,
the larger the sample size,

- the more **certain** you are about
your **biased** estimates
(the winner's curse)



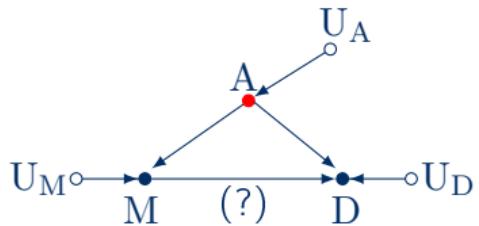
The dream team!!

based on DAG and statistical model,

- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path,
i.e. $M \perp\!\!\!\perp D \mid A$
- conditioning on A we can find,
 $E[D|do(m)] = E[E[D|M = m, A]]$
(law of total expectation)
- then we can find the
 $ACE(m) = E[D|do(m + 1)] - E[D|do(m)]$
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, M, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,
(assuming our DAG is true)

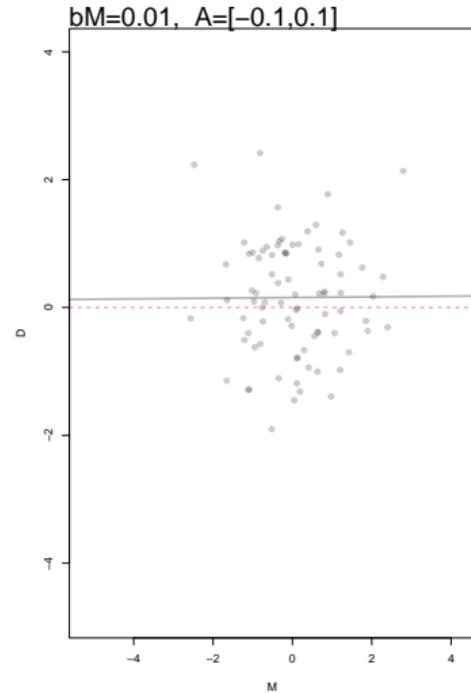
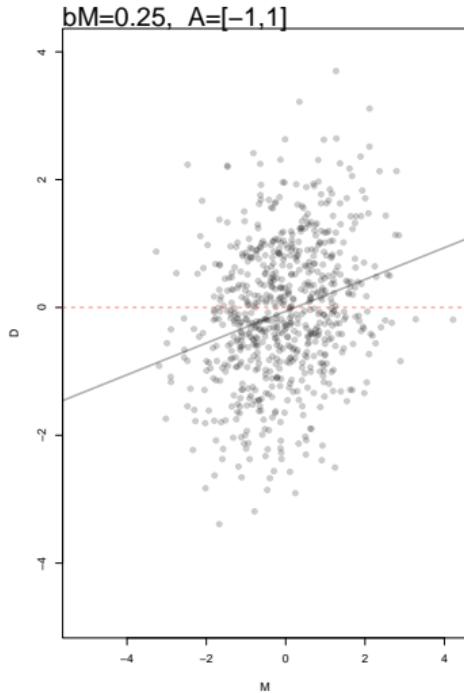
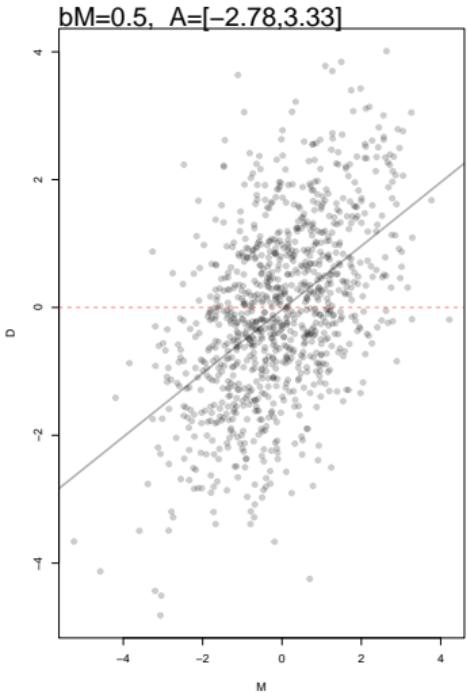
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Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.18854   0.09871 -1.910   0.0591 .  
A             -1.03121   0.13483 -7.648 1.49e-11 *** 
M             -0.06134   0.09362 -0.655   0.5139
```

So, what is going on?



3. Example cases

Fork: masked relationships (a)

Masked relationships (a)⁶

also known as,

- omitted variable bias

research question,

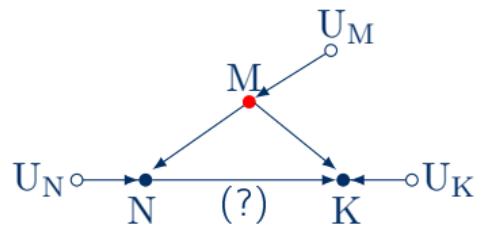
- Does N has a (direct) effect on K?

variables,

- M, mammal mass in kg.
- N, ratio neocortex over total brain mass
- K, Kcal. per gram of milk

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

⁶McElreath [12], chapter 5 (p. 144)

Simulation setting

```
# sim  
M = rnorm( 100 )  
N = rnorm( 100 , 1*M )  
K = rnorm( 100 , 1*N + -1*M )  
d = data.frame(N=N,M=M,K=K)
```

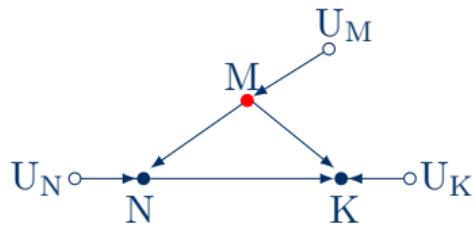
(c) R code

Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(U) \end{cases}$$

(a) structural model

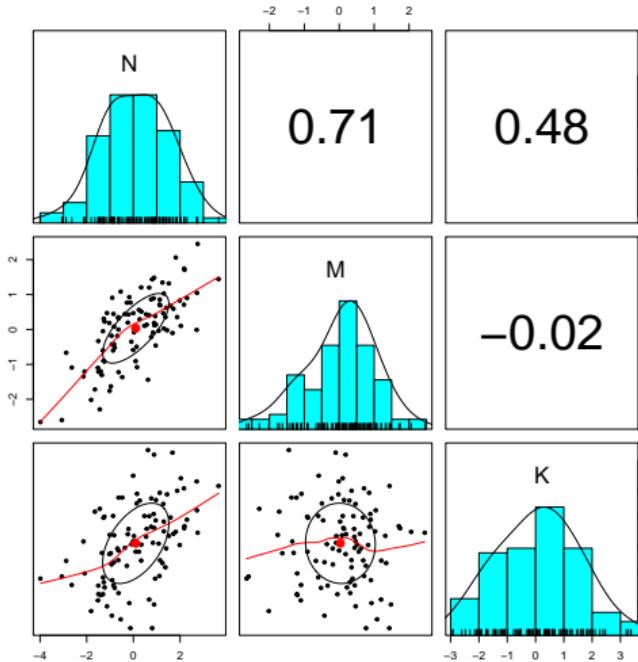


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(N, K) > 0$ goes in line of our “rudimentary” understanding of the data.
- but why there is $\text{cor}(M, k) \approx 0$?
(hint: univariate correlation)
- we **might not include M as a covariate in our statistical model**



Regression, regression!!

based on **statistical analysis**,

- we have two different stories,
(which one is the “truth”?)

```
> summary(lm(K ~ N, data=d)) # biased estimate

Call:
lm(formula = K ~ N, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.8355 -0.8110  0.0188  0.7897  3.4276 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.01401   0.12057   0.116   0.908    
N            0.53002   0.09332   5.680 1.38e-07 *** 
> summary(lm(K ~ N + M, data=d)) # less biased estimate

Call:
lm(formula = K ~ N + M, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.50873 -0.72626 -0.01968  0.69016  2.93000 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.22096   0.09845   2.244   0.0271 *  
N            0.95510   0.10089   9.466 1.91e-15 *** 
M           -1.06246   0.15462  -6.871 6.14e-10 ***
```

I'll get more data!!

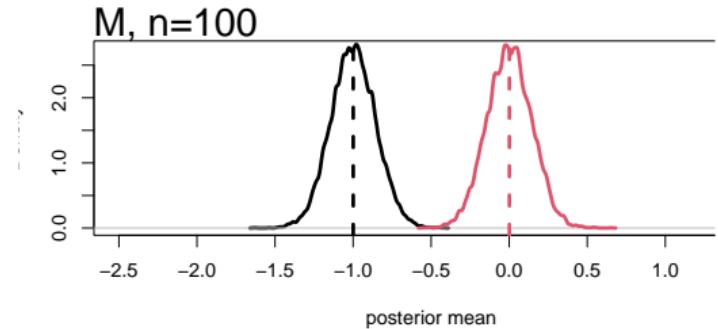
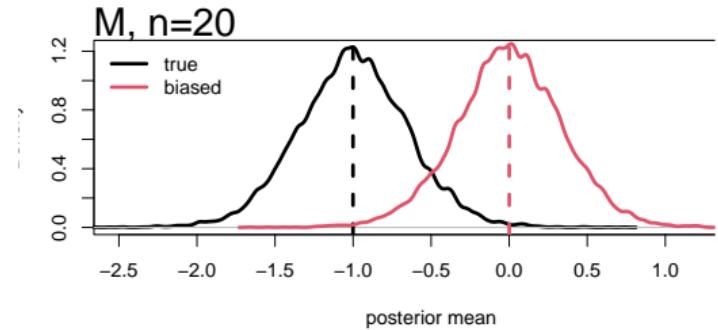
imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,

the larger the sample size,

- the more certain you are about your **biased** estimates



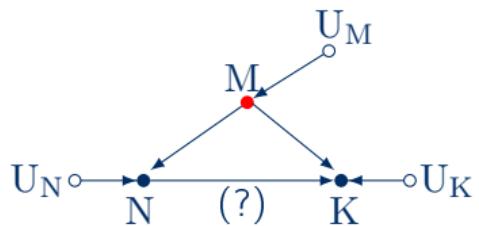
The dream team!!

based on DAG and statistical model,

- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path,
i.e. $N \not\perp\!\!\!\perp K \mid M$
- conditioning on M we can find,
 $E[K|do(n)] = E[E[K|N = n, M]]$
(law of total expectation)
- then we can find the
 $ACE(n) = E[D|do(n + 1)] - E[D|do(n)]$
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

the dream team!!

based on DAG and statistical analysis,

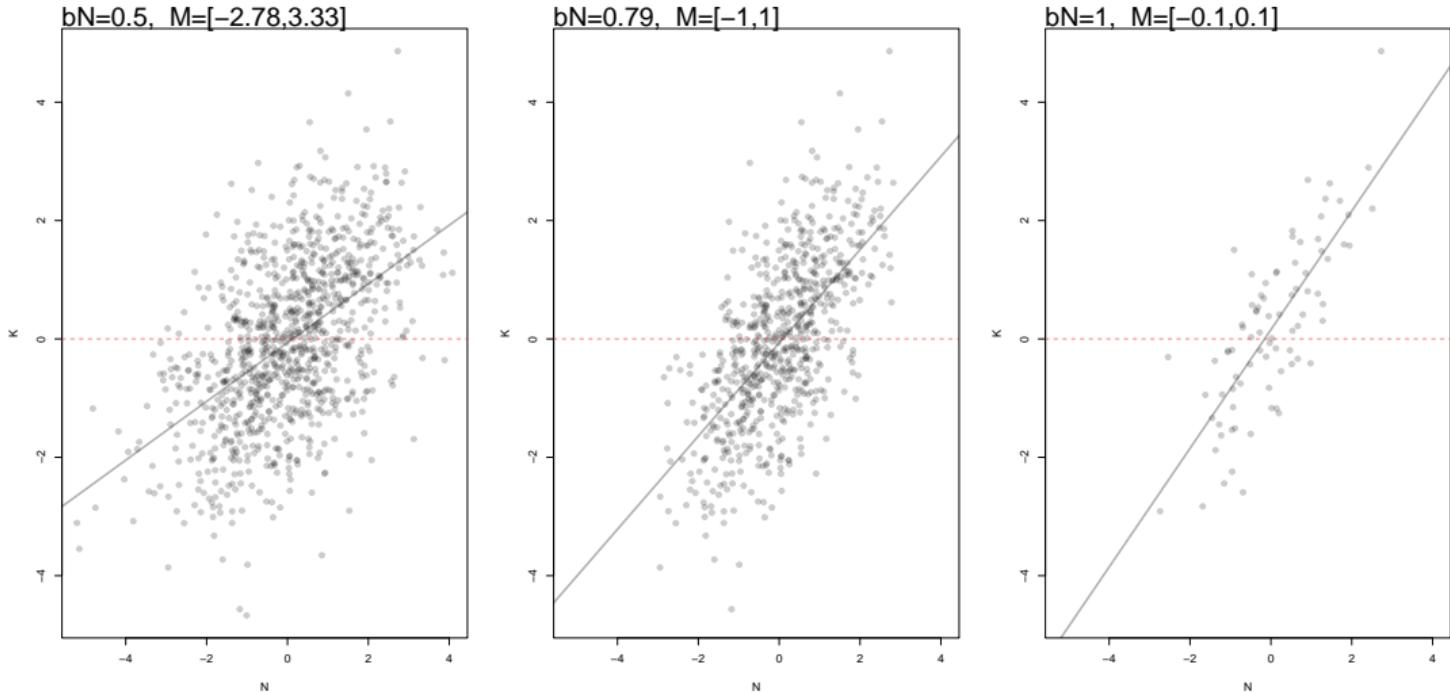
- the less biased model is the second,
(assuming our DAG is true)

```
> summary(lm(K ~ N + M, data=d)) # less biased estima
Call:
lm(formula = K ~ N + M, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.50873 -0.72626 -0.01968  0.69016  2.93000 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.22096   0.09845   2.244   0.0271 *  
N           0.95510   0.10089   9.466 1.91e-15 *** 
M          -1.06246   0.15462  -6.871 6.14e-10 ***
```

So, what is going on?



3. Example cases

Fork: masked relationships (b)

Masked relationships (b)⁷

also known as,

- (unobserved) omitted variable bias

research question,

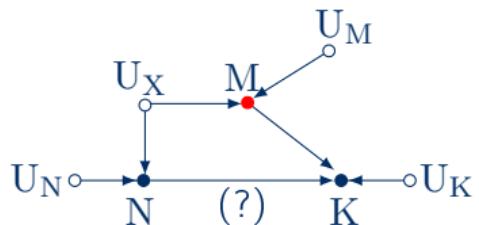
- Does N has a (direct) effect on K?

variables,

- U_X , unobservable (e.g. genetics)
- M, mammal mass in kg.
- N, neocortex over total brain mass
- K, Kcal. per gram of milk

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

⁷McElreath [12], chapter 5 (p. 144)

Simulation setting

```
# sim  
U = rnorm( 100 )  
N = rnorm( 100 , 1*U )  
M = rnorm( 100 , 1*U )  
K = rnorm( 100 , 1*N + -1*M )  
d = data.frame(U=U,N=N,M=M,K=K)
```

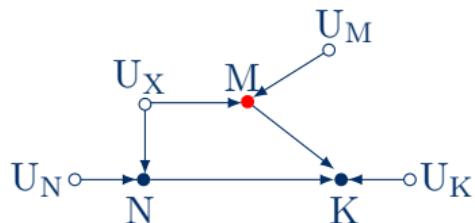
(c) R code

Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

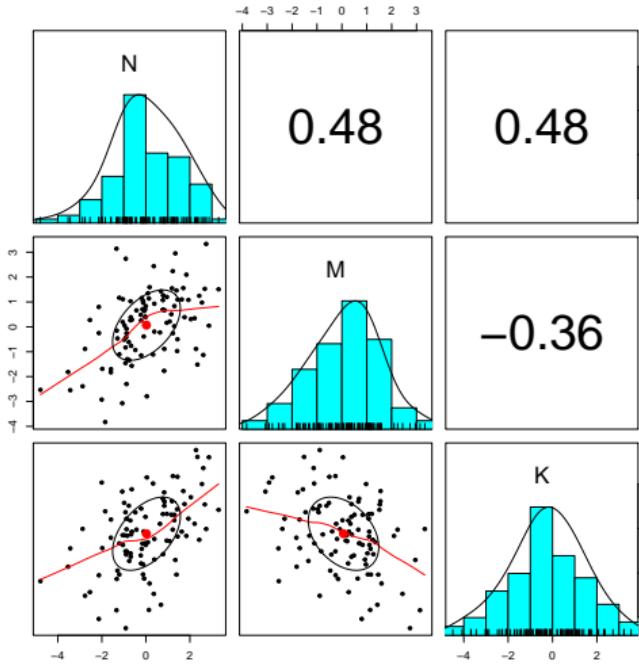


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(N, K) > 0$ goes in line of our “rudimentary” understanding of the data.
- $\text{cor}(M, K) < 0$ does NOT goes in line of our “rudimentary” understanding of the data.
(hint: univariate correlation)
- we include M as a covariate in our statistical model
(by chance?)



Regression, regression!!

based on **statistical analysis**,

- we have two different stories,
(which one is the “truth”?)

```
> summary(lm(K ~ N, data=d)) # unobserved path still  
Call:  
lm(formula = K ~ N, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-3.7763 -0.8480  0.1497  0.9874  3.3530  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) -0.24867   0.14573 -1.706  0.0911 .  
N            0.51406   0.09502  5.410 4.46e-07 ***  
> summary(lm(K ~ N + M, data=d)) # unobserved path cl  
Call:  
lm(formula = K ~ N + M, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-2.58218 -0.58434 -0.00579  0.72016  1.78724  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) -0.19978   0.09375 -2.131  0.0356 *  
N            0.90893   0.06958 13.064 <2e-16 ***  
M            -0.89676   0.07572 -11.843 <2e-16 ***
```

I'll get more data!!

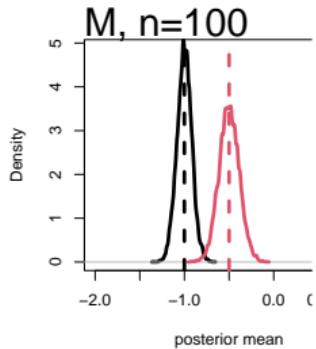
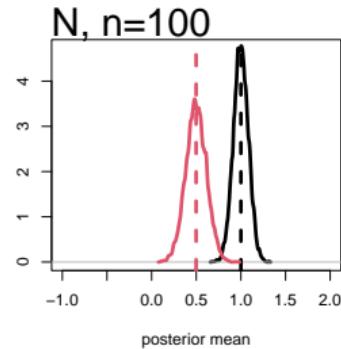
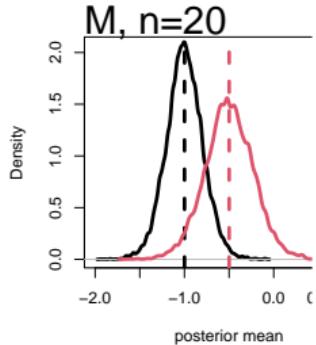
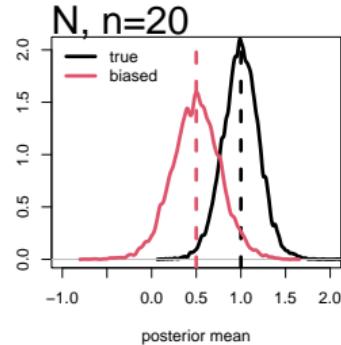
imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,

the larger the sample size,

- the more **certain** you are about your **biased** estimates



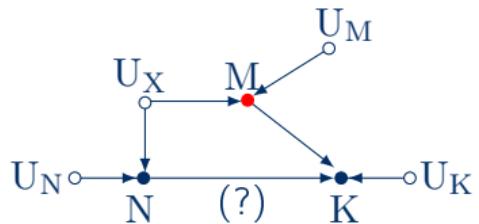
The dream team!!

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,
i.e. $N \not\perp\!\!\!\perp K \mid U_X$
(but it is unobservable)
- still we use the 2nd D-separation rule by controlling for M,
i.e. $N \not\perp\!\!\!\perp K \mid M$
- conditioning on M we can still find,
 $E[K|do(n)] = E[E[K|N = n, M]]$
(law of total expectation)
- then we can find the
 $ACE(n) = E[D|do(n + 1)] - E[D|do(n)]$
(Frisch-Waugh-Lovell theorem??)

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

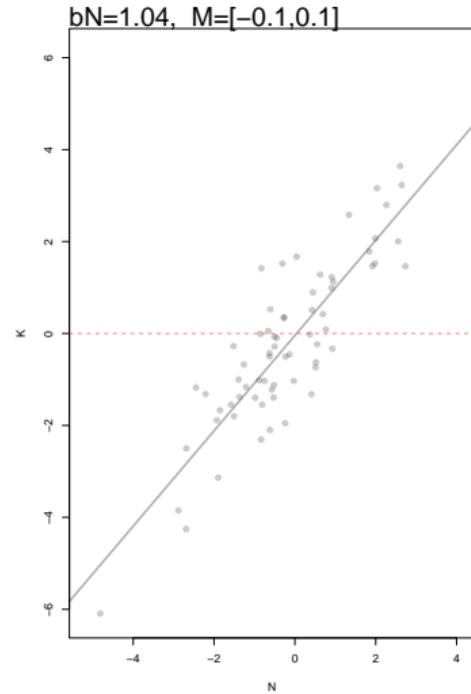
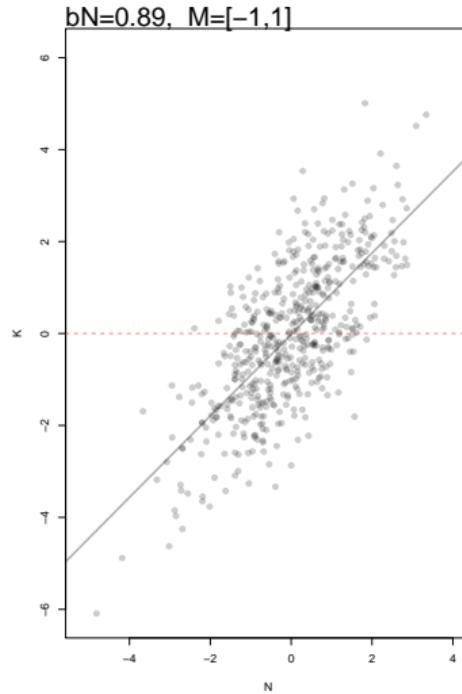
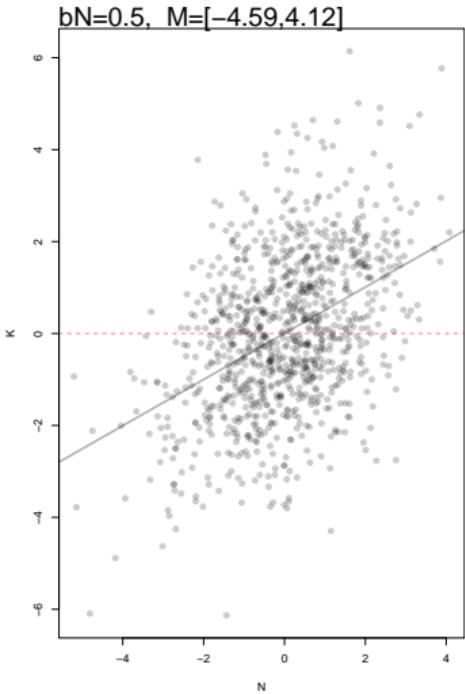
the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,
(assuming our DAG is true)

```
> summary(lm(K ~ N + M, data=d)) # unobserved path c1  
Call:  
lm(formula = K ~ N + M, data = d)  
Residuals:  
    Min      1Q  Median      3Q     Max  
-2.58218 -0.58434 -0.00579  0.72016  1.78724  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) -0.19978   0.09375 -2.131  0.0356 *  
N            0.90893   0.06958 13.064 <2e-16 ***  
M           -0.89676   0.07572 -11.843 <2e-16 ***
```

So, what is going on?



Similar scenario, unobserved masked⁸

research question,

- Does E has a (direct) effect on I?

variables,

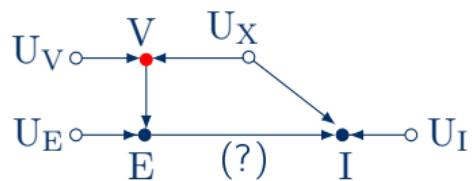
- U_X , unobservable
(e.g. family context)
- V, personal values
- E, education
- I, income

then,

- we need to control by V to get an unbiased estimate of $E \rightarrow I$

$$M = \begin{cases} V \leftarrow f_M(U_V, U_X) \\ E \leftarrow f_E(V, U_E) \\ I \leftarrow f_I(E, U_X, U_I) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

⁸Cinelli et al. [4] (p. 3), McElreath [13], lecture 6

3. Example cases

Fork: multicollinearity

Multicollinearity⁹

also known as,

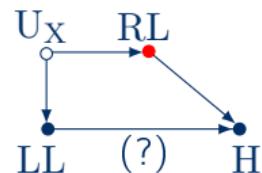
- extreme case of masked relationships research question,
 - Should we include RL in our model?

variables,

- U_X , unobservable
(e.g. genes and context)
- LL, individual's left leg
- RL, individual's right leg
- H, individual's height

$$M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(LL, RL) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

⁹McElreath [12], chapter 6 (p. 163)

Simulation setting

```
# backward simulation  
H = round( rnorm( 100 , 170, 2), 1)  
Lp = runif( 100 , 0.5-0.05, 0.5+0.05)  
LL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)  
RL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)  
d = data.frame(LL,RL,H)
```

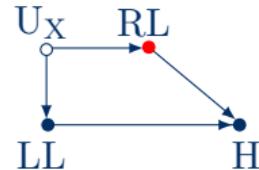
(c) R code

Implications,

- $LL \not\perp\!\!\!\perp RL$

$$M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(RL, LL) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

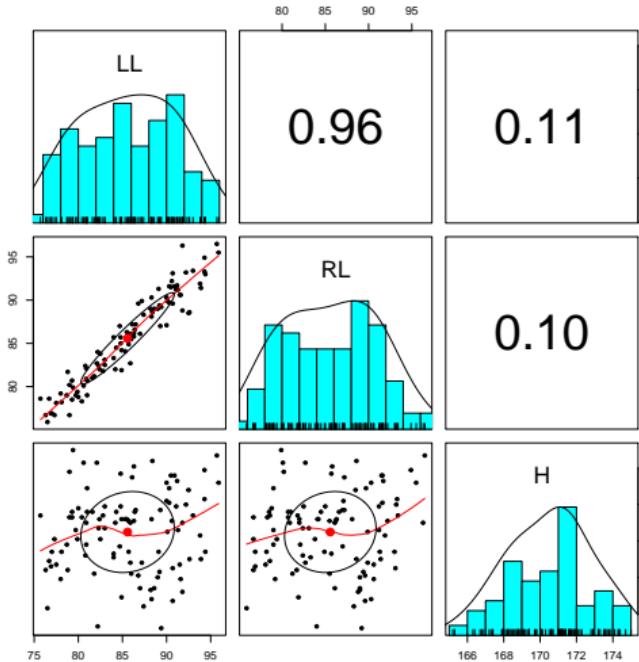


(b) causal diagram

“Eyeballing” analysis

based on **correlation analysis**,

- $\text{cor(LL, H)} > 0$, $\text{cor(RL, H)} > 0$
and $\text{cor(LL, RL)} > 0$ goes in line of
our “understanding” of the data.
- we might not include RL as a
covariate in our statistical model
(based on **univariate** correlation)



Regression, regression!!

based on statistical analysis,

- the second regression show a smaller effect of LL,
- the second regression show way larger SE values
(not rejecting the null)

```
> summary(lm(H ~ -1 + LL, data=d)) # unbiased,  
Call:  
lm(formula = H ~ -1 + LL, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-17.9704 -8.3662  0.7494 10.5256 21.1464  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
LL  1.98486   0.01229 161.6 <2e-16 ***  
> summary(lm(H ~ -1 + LL + RL, data=d)) # ine  
Call:  
lm(formula = H ~ -1 + LL + RL, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-16.780 -8.592  0.532 10.253 18.299  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
LL  1.0094    0.6969   1.448    0.151  
RL  0.9757    0.6970   1.400    0.165
```

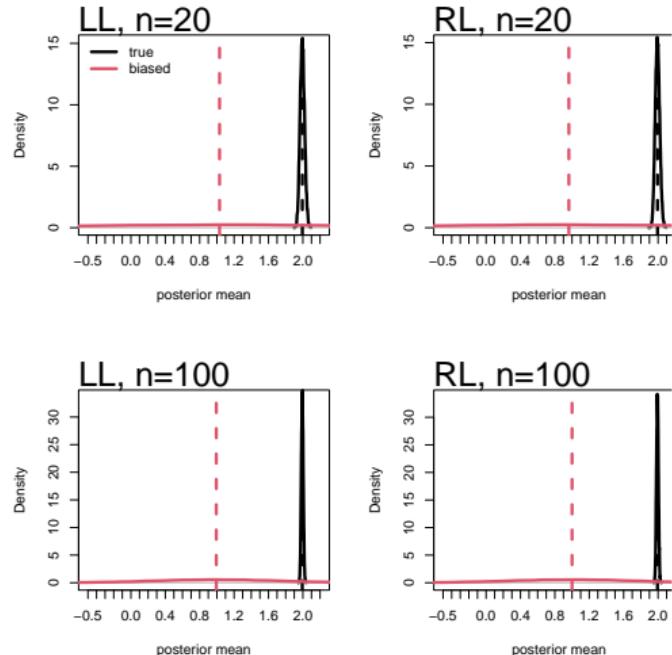
I'm sure data won't help

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,
the larger the sample size,

- the less **certain** are your **biased** estimates



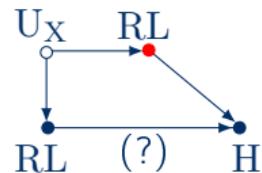
Not so great now??

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,
i.e. $LL \not\perp\!\!\!\perp H \mid U_X$
(but it is unobservable)
- we still use the 2nd D-separation rule by controlling for RL, but still we have $LL \not\perp\!\!\!\perp H \mid RL$
- issue goes beyond the backdoor path
the problem is that RL and LL give the same information to the model,
i.e. they form a singular matrix,
(is like having a causal model like b)

$$M = \begin{cases} LL = RL \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(RL) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

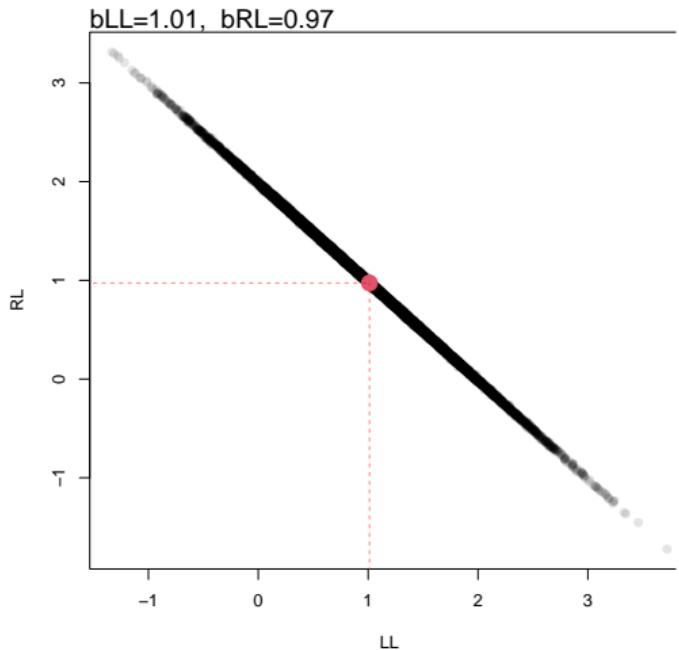


(b) causal diagram

So, what is going on then??

the estimated parameters under the second regression,

- are in a thin ridge
 $\text{Cor}(bLL, bRL) \approx -1$
- statistical model finds not only one solution (**red dot**), the thin ridge implies there are **infinite solutions** for the parameters,
(related to the singular matrix thing)



Not so great now??

based on DAG and statistical analysis,

- the less biased and more precise model is the first,
(assuming our DAG is true)

```
> summary(lm(H ~ -1 + LL, data=d)) # unbiased

Call:
lm(formula = H ~ -1 + LL, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-17.9704 -8.3662  0.7494 10.5256 21.1464 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
LL       1.98486   0.01229 161.6   <2e-16 ***
```

3. Example cases

Fork: neutral control

Neutral control¹⁰

also known as,

- precision “booster”
- similar to experimental design
- how block effects work (in a way)

research question,

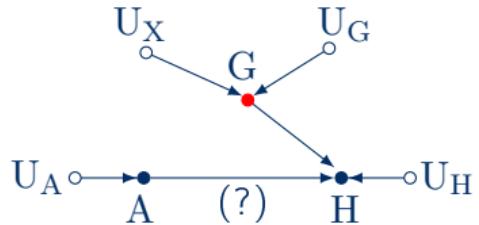
- Should we include G on our model?

variables,

- A, “hearing” age
- G, gender
- U_X , unobservable (e.g. no idea yet)
- H, entropy (logit)
(approximate of speech intelligibility)

$$M = \begin{cases} G \leftarrow f_G(U_G, U_X) \\ A \leftarrow f_A(U_A) \\ H \leftarrow f_H(A, G, U_H) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

¹⁰Cinelli et al. [4] (p. 4)

Simulation setting

```
# sim
G = sample( 0:1, 100 , replace=T )
A = rnorm( 100 )
H = rnorm( 100 , -1*A + -1*G )
d = data.frame(G=G,A=A,SI=SI)
```

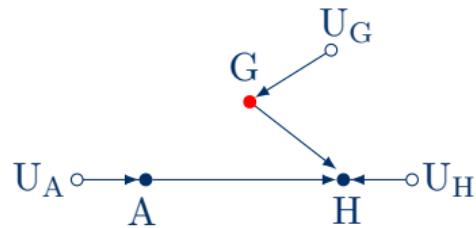
(c) R code

Implications,

- $A \perp\!\!\!\perp G$
- $A \not\perp\!\!\!\perp H$
- $G \not\perp\!\!\!\perp H$

$$M = \begin{cases} G \leftarrow f_G(U_G) \\ A \leftarrow f_A(U_A) \\ H \leftarrow f_H(A, G, U_H) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

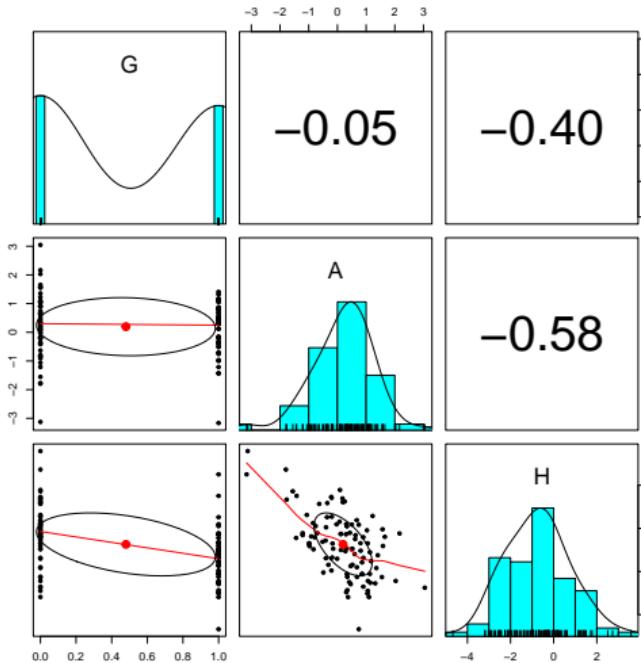


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(G, H) < 0$, $\text{cor}(G, A) \approx 0$ and $\text{cor}(A, H) < 0$ goes in line of our “rudimentary” understanding of the data.
- we include both as a covariate in our statistical model



Regression, regression!!

based on statistical analysis,

- almost no change on our estimates,
- lower SE for A when G is included
(because we have explained some variability in H, not related to A)

```
> summary(lm(H ~ A, data=d)) # correct estimate

Call:
lm(formula = H ~ A, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.4714 -0.8797 -0.0633  0.8963  2.4346 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.5770    0.1216 -4.746 7.07e-06 ***
A            -0.8410    0.1183 -7.108 1.92e-10 ***

> summary(lm(H ~ A + G, data=d)) # correct estimate, 

Call:
lm(formula = H ~ A + G, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.7994 -0.6914  0.0579  0.7796  1.8274 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.03317   0.14312   0.232    0.817  
A           -0.87360   0.10090  -8.658 1.05e-13 ***
G           -1.25786   0.20371  -6.175 1.55e-08 ***
```

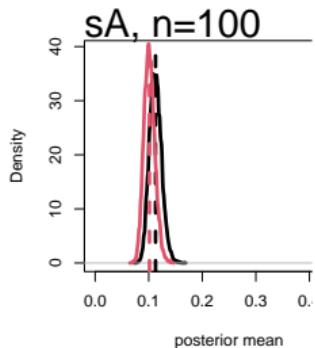
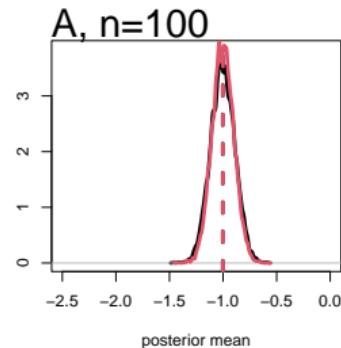
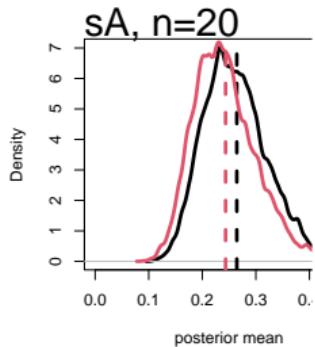
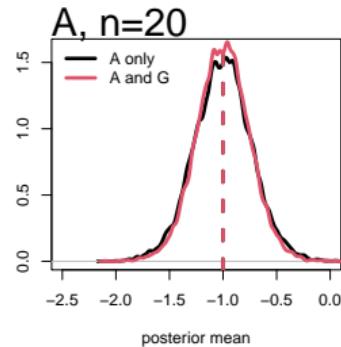
Does more data works in this case?

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **second model**,
the larger the sample size,

- the more **certain** you are about
your **non-biased** estimates
(under the **any** model)



3. Example cases

Pipe: precision parasite

Precision parasite¹¹

research question,

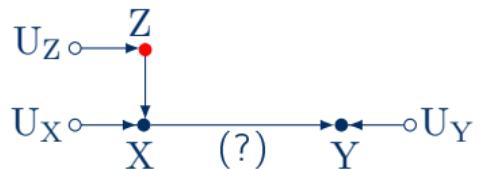
- What is the effect of X on Y?
- Should we include Z in the model?

variables,

- Z, “parent” of X
- X, exposure
- Y, outcome

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

¹¹McElreath [13], lecture 6; Cinelli et al. [4] (p. 5)

Simulation setting

```
# sim  
Z = rnorm( 100 )  
X = rnorm( 100 , 1*Z )  
Y = rnorm( 100 , 1*X )  
d = data.frame(Z,X,Y)
```

(c) R code

Implications,

- $X \not\perp\!\!\!\perp Y$
- $Z \not\perp\!\!\!\perp Y \mid X$

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

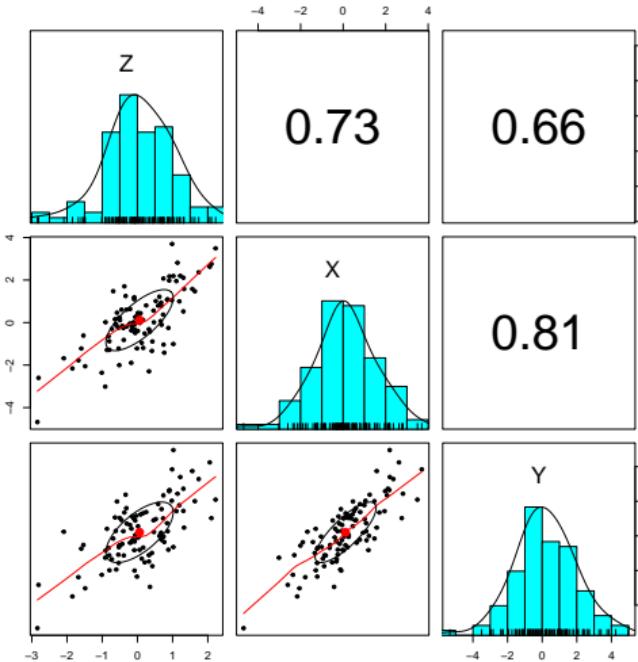


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(Z, X) > 0$ is not large enough to discard it as multicollinearity.
- $\text{cor}(Z, Y) > 0$ and $\text{cor}(X, Y) > 0$ indicate both should be in our model
(it might be our research hypothesis)



Regression, regression!!

based on **statistical analysis**,

- no bias in parameter if Z is in,
- but we loose precision on X

```
> summary(lm(Y ~ X, data=d)) # unbiased effect, more  
Call:  
lm(formula = Y ~ X, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-2.41746 -0.73659 -0.09384  0.63812  2.10338  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 0.03433   0.10274   0.334   0.739  
X           1.16908   0.06717  17.405 <2e-16 ***  
> summary(lm(Y ~ X + Z, data=d)) # unbiased effects,  
Call:  
lm(formula = Y ~ X + Z, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-2.27019 -0.74072 -0.06355  0.66643  2.20770  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 0.04722   0.10350   0.456   0.649  
X           1.08881   0.10358  10.512 <2e-16 ***  
Z           0.15431   0.15159   1.018   0.311
```

With more data??

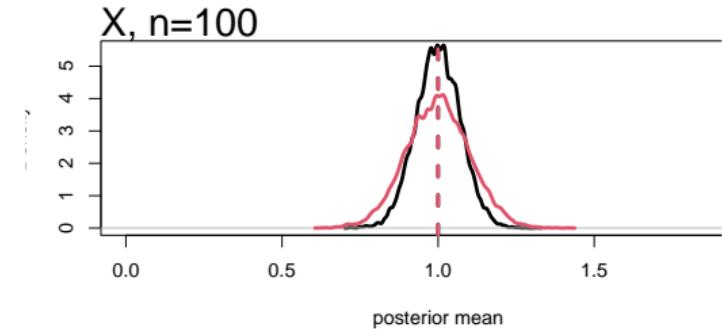
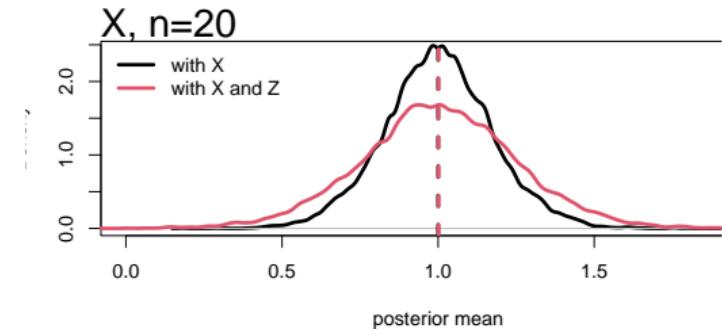
imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the second model,

the larger the sample size,

- still less precise estimates
- more difficult to test hypothesis



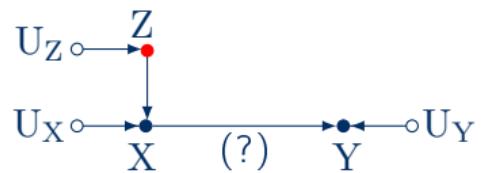
Now, what is going on here??

based on DAG and statistical model,

- conditioning on Z reduces variation on X, leaving less variability that can explain outcome Y

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

Now, what is going on here??

based on DAG and statistical analysis,

- the more appropriate model (for inference) is the first,
(assuming our DAG is true)

```
> summary(lm(Y ~ X, data=d)) # unbiased effect, more  
Call:  
lm(formula = Y ~ X, data = d)  
Residuals:  
    Min      1Q  Median      3Q     Max  
-2.41746 -0.73659 -0.09384  0.63812  2.10338  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 0.03433   0.10274   0.334   0.739  
X           1.16908   0.06717  17.405 <2e-16 ***
```

3. Example cases

Pipe: post-treatment

Post-treatment bias¹²

case of,

- full mediation

research question,

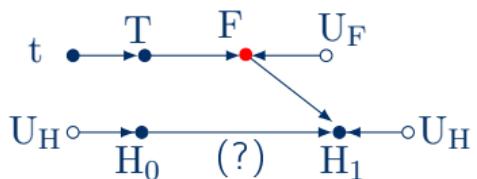
- Does the treatment T works?

variables,

- H_0 , height of plant at $t = 0$
- T, antifungal treatment
- F, presence of fungus
- H_1 , height of plant at $t = 1$

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_H(F, H_0, U_H) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

¹²McElreath [12], chapter 6 (p. 170)

Simulation setting

```
# sim
h0 = rnorm( 100 , 10, 2)
Tr = rep( 0:1 , each=100/2 )
Fu = rbinom( n , size=1 , prob=0.5 + -0.4*Tr )
h1 = h0 + rnorm( n , 5 + -3*Fu)
d = data.frame( h0=h0, h1=h1, Tr=Tr, Fu=Fu )
```

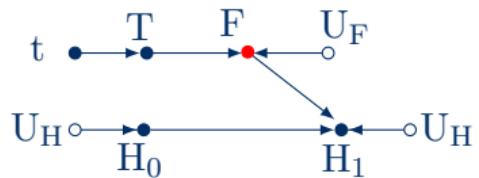
(c) R code

Implications,

- $T \perp\!\!\!\perp H_0$
- $T \not\perp\!\!\!\perp H_1$
- $T \not\perp\!\!\!\perp H_1 \mid F$

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_H(F, H_0, U_H) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

Descriptive analysis

based on descriptive analysis,

- positive change in height with treatment.
- negative change in height with fungus.
- diluted relationship for T when both are in the model
(hint: blocking path of information)

Tr	mean	sd	n	se	
<fct>	<dbl>	<dbl>	<int>	<dbl>	
0	2.96	1.75	50	0.248	
1	4.53	1.45	50	0.205	
Fu	mean	sd	n	se	
<fct>	<dbl>	<dbl>	<int>	<dbl>	
0	4.93	0.953	62	0.121	
1	1.81	0.902	38	0.146	
Tr	Fu	mean	sd	n	se
<fct>	<fct>	<dbl>	<dbl>	<int>	<dbl>
0	0	4.91	0.810	19	0.186
0	1	1.76	0.861	31	0.155
1	0	4.93	1.02	43	0.155
1	1	2.01	1.12	7	0.423

Again regression!!

based on statistical analysis we have two different stories (but not quite),

- treatment has a significant effect,
- but gets completely diluted when fungus is considered in the model

```
> summary(lm(h1-h0 ~ Tr, data=d)) # only treatment

Call:
lm(formula = h1 - h0 ~ Tr, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.1166 -1.0929  0.1755  1.2621  3.3990 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.9598    0.2273   13.02 <2e-16 ***
Tr1          1.5656    0.3215    4.87 4.29e-06 ***

> summary(lm(h1-h0 ~ Tr + Fu, data=d)) # only fungus

Call:
lm(formula = h1 - h0 ~ Tr + Fu, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.1691 -0.4823  0.0963  0.5315  2.0357 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.86397   0.19127  25.430 <2e-16 ***
Tr1          0.09138   0.21579   0.423   0.673  
Fu1         -3.07122   0.22229 -13.816 <2e-16 ***
```

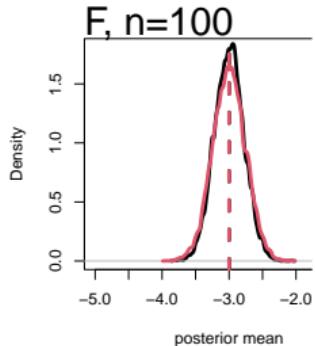
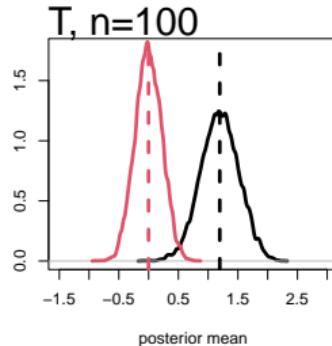
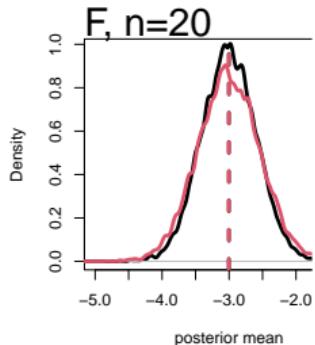
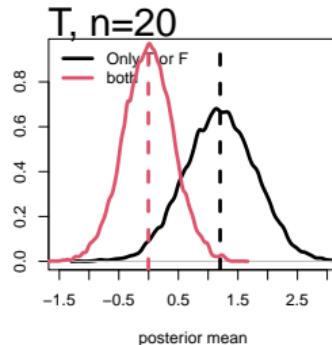
I can guess what happens with more data!!

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the “incorrect” model,
the larger the sample size,

- the more certain you are about your **biased** T estimates (not F)
(this result is not wrong!)



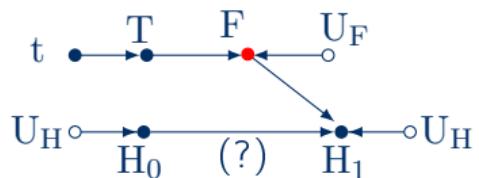
The dream team!!

based on DAG and statistical model,

- the 2nd D-separation states that if you control any noncollider you block the backdoor path,
i.e. $T \perp\!\!\!\perp H_1 \mid F$
- therefore if we want to find if $T = 1$ works, we **should not** stratify by F

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_{H_1}(F, H_0, U_H) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

the dream team!!

based on DAG and statistical analysis,

- the model that answers our research question is the first one, (assuming our DAG is true)

```
> summary(lm(h1-h0 ~ Tr, data=d)) # only treatment

call:
lm(formula = h1 - h0 ~ Tr, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.1166 -1.0929  0.1755  1.2621  3.3990 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.9598    0.2273   13.02 < 2e-16 ***
Tr1          1.5656    0.3215    4.87 4.29e-06 ***
```

3. Example cases

Pipe: Simpson's paradox

Masked relationships¹³

also known as,

- mediation
- masked relationships

research question,

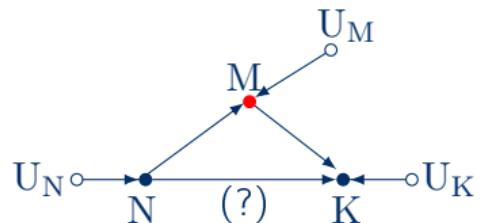
- Does N has a (direct) effect on K?

variables,

- M, mammal mass in kg.
- N, neocortex over total brain mass
- K, Kcal. per gram of milk

$$M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(N, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

¹³McElreath [12], chapter 6 (p. 170)

Simulation setting

```
# sim  
N = rnorm( 100 )  
M = rnorm( 100 , 1*N )  
K = rnorm( 100 , 1*N + -1*M )  
d = data.frame(N=N,M=M,K=K)
```

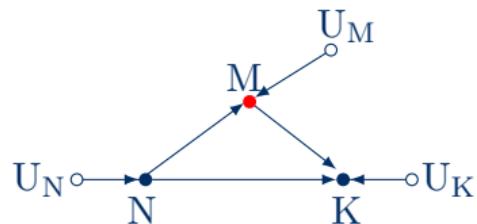
(c) R code

Implications,

- $N \not\perp\!\!\!\perp K$
- $N \not\perp\!\!\!\perp K \mid M$

$$M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(M, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(U) \end{cases}$$

(a) structural model

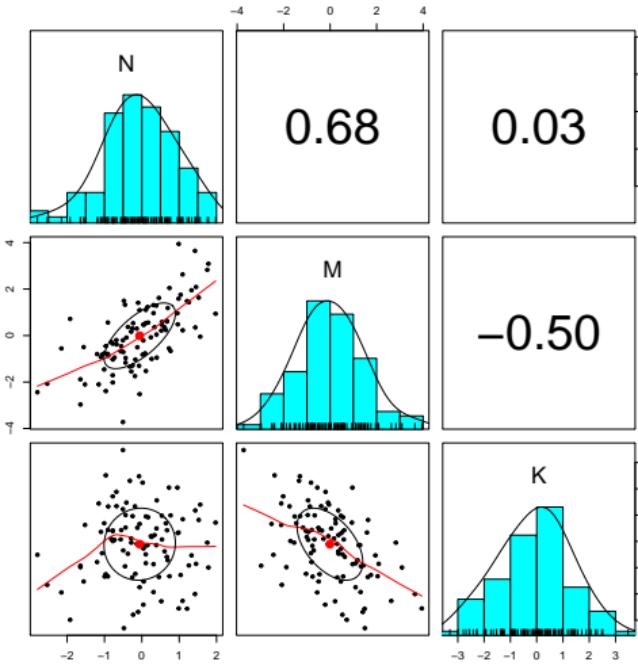


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(M, K) < 0$ does NOT goes in line of our “rudimentary” understanding of the data.
- and why there is $\text{cor}(N, K) \approx 0$?
(hint: univariate correlation)
- we include N as a covariate in our statistical model
(is our research hypothesis)



Regression, regression!!

based on **statistical analysis**,

- two regressions with two different results, which model is the “true”?

```
> summary(lm(K ~ N, data=d)) # biased estimate
Call:
lm(formula = K ~ N, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.1751 -0.9009  0.1519  0.8574  3.6041 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.10412   0.13808  -0.754   0.453    
N            0.05005   0.14487   0.345   0.730    
> summary(lm(K ~ N + M, data=d)) # less biased estimate
Call:
lm(formula = K ~ N + M, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.58484 -0.59175  0.04378  0.61175  2.43360 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.06181   0.09825  -0.629   0.531    
N            0.98297   0.13994   7.024 2.98e-10 ***
M           -0.93107   0.09457  -9.846 2.89e-16 ***
```

I'll get more data!!

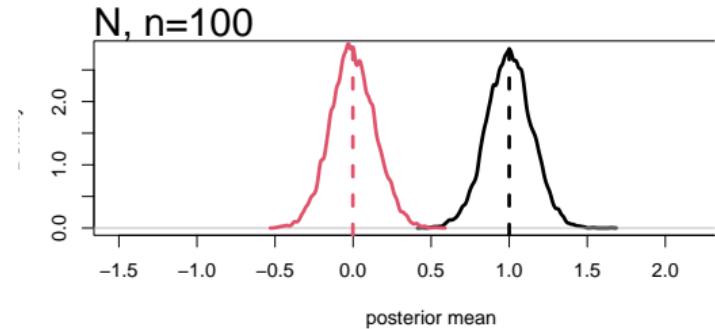
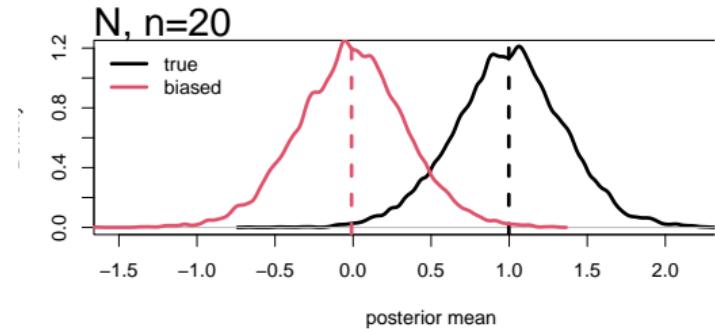
imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the “incorrect” model,

the larger the sample size,

- the more certain you are about your **biased** estimates



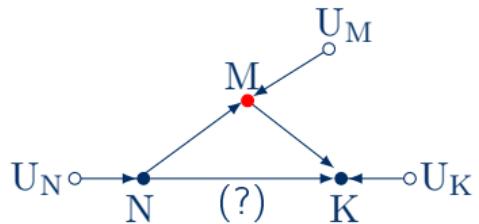
The dream team!!

based on DAG and statistical model,

- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path,
i.e. $N \not\perp\!\!\!\perp K \mid M$
- conditioning on M we can find,
 $E[K|do(n)] = E[E[K|N = n, M]]$
(law of total expectation)
- then we can find the
 $ACE(n) = E[D|do(n + 1)] - E[D|do(n)]$
(Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(M, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,
(assuming our DAG is true)

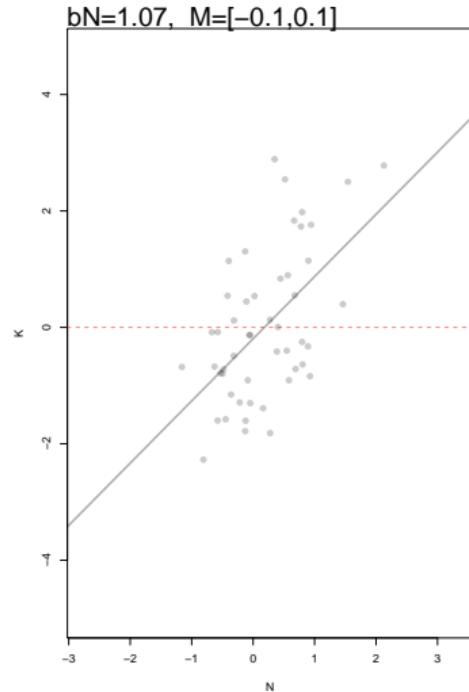
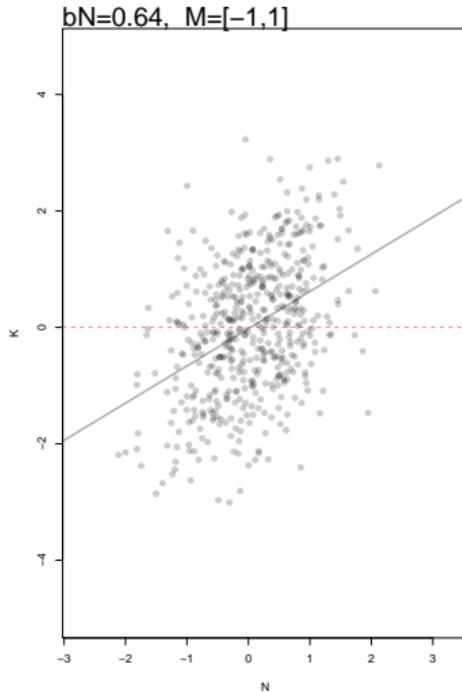
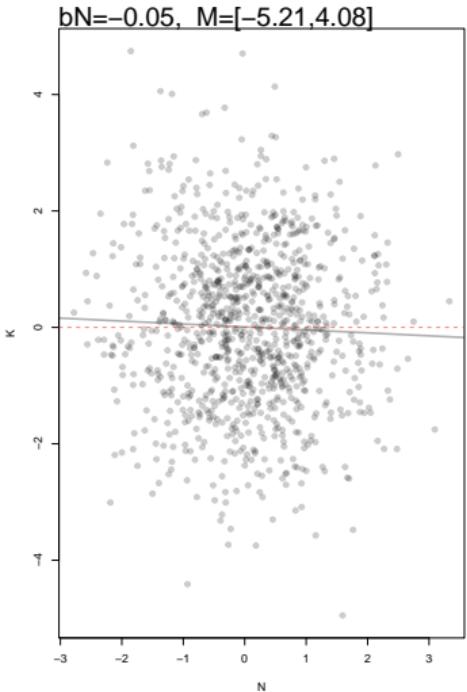
```
> summary(lm(K ~ N + M, data=d)) # Less biased estimate

Call:
lm(formula = K ~ N + M, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.58484 -0.59175  0.04378  0.61175  2.43360 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.06181   0.09825  -0.629   0.531    
N            0.98297   0.13994   7.024 2.98e-10 ***  
M           -0.93107   0.09457  -9.846 2.89e-16 ***
```

So, what is going on?



Similar case, gender discrimination¹⁴

research question,

- Do females are discriminated in school admissions, i.e. does $G \rightarrow A$?

variables,

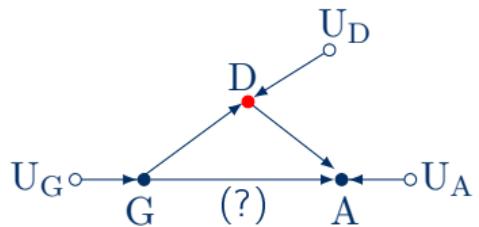
- G, gender
- D, department of application
- A, admission

then,

- stratification by D close mediation path (backdoor path on A)
- not stratifying by D finds the total effect of G on A (all paths)
(e.g. “structural” discrimination)

$$M = \begin{cases} G \leftarrow f_G(U_G) \\ D \leftarrow f_D(G, U_D) \\ A \leftarrow f_A(D, G, U_A) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

¹⁴McElreath [12], chapter 11 (p. 340)

Another similar case, children achievement¹⁵

research question,

- Does M has a (direct) effect on D?

variables,

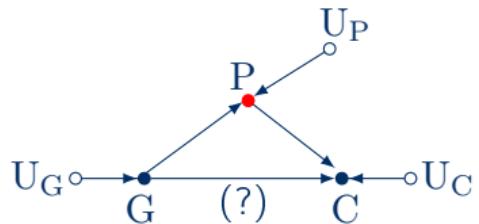
- G, grandparents' level of education
- P, parents' level of education
- C, children's educational achievement

then,

- we should stratify by P

$$M = \begin{cases} G \leftarrow f_G(U_G) \\ P \leftarrow f_P(G, U_P) \\ C \leftarrow f_C(G, P, U_C) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

¹⁵McElreath [12], chapter 6 (p. 180)

3. Example cases

Pipe/Fork: good controls

Pipe/fork good controls¹⁶

research question,

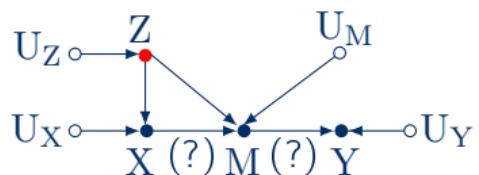
- What is the (total) effect of X on Y?
(all directional paths from X to Y)
(i.e. no mediators)

variables,

- Z, confounder
- X, exposure
- M, mediator
- Y, outcome

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ M \leftarrow f_M(X, Z, U_M) \\ Y \leftarrow f_Y(M, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

¹⁶Cinelli et al. [4] (p. 3)

Simulation setting

```
# sim  
Z = rnorm( 100 )  
X = rnorm( 100 , 1*Z )  
M = rnorm( 100 , 0*X + 1*Z )  
Y = rnorm( 100 , 0.5*M )  
d = data.frame(Z=Z,X=X,M=M,Y=Y)
```

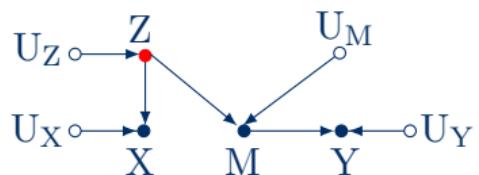
(c) R code

Implications,

- $X \not\perp\!\!\!\perp M \mid Z$
- $Z \perp\!\!\!\perp Y$

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ M \leftarrow f_M(X, Z, U_M) \\ Y \leftarrow f_Y(M, U_Y) \\ U \sim P(U) \end{cases}$$

(a) structural model

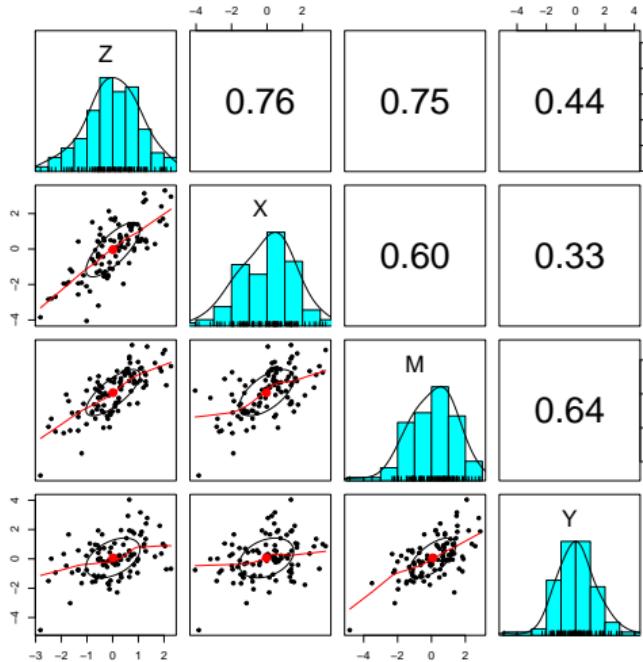


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- X and M should be in your model,
(but we know a full mediator closes
the backdoor path, and **we do not**
want that: not our research question)
- while $\text{cor}(Z, M) \approx 0.8$ prevents
from using Z as a covariate
(possibly a multicollinearity problem)
- since we will not include M, now Z
can be included



Regression, regression!!

based on statistical analysis,

- two different stories
- in one X has small but significant effect
- which model is the “truth”?

```
> summary(lm(Y ~ X, data=d)) # biased

Call:
lm(formula = Y ~ X, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8115 -0.7762 -0.1034  0.7210  3.5707 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.07480   0.12867   0.581  0.562354    
X           0.29099   0.08428   3.453  0.000821 *** 
> summary(lm(Y ~ X + Z, data=d)) # unbiased

Call:
lm(formula = Y ~ X + Z, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.3281 -0.7697 -0.0931  0.8735  3.6160 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.057974   0.123169   0.471  0.63892    
X           -0.005456   0.123103  -0.044  0.96474    
Z           0.571826   0.179473   3.186  0.00194 **
```

the telled tell of more data!!

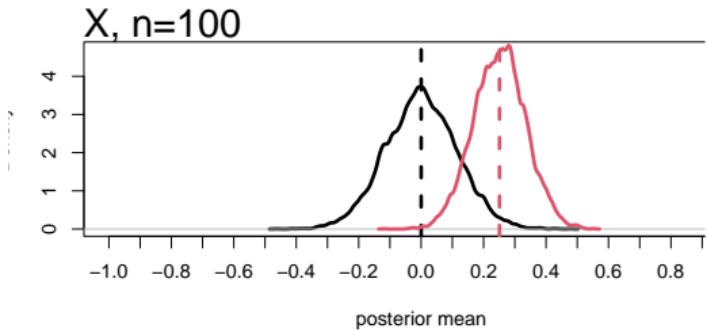
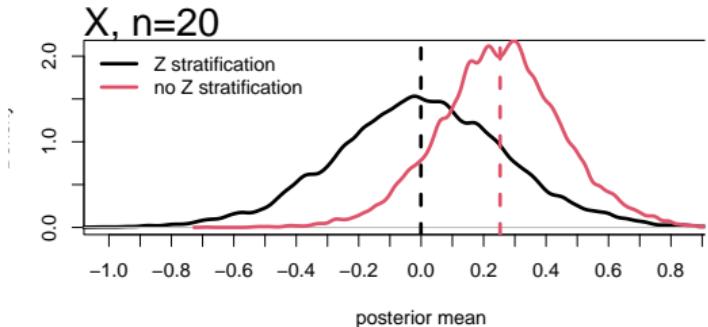
imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,

the larger the sample size,

- the more **certain** you are about your **biased** estimates



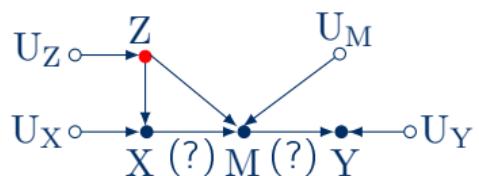
Yo, what is going on??

based on DAG and statistical model,

- there are two paths from X to Y
 $X \rightarrow M \rightarrow Y$
 $X \rightarrow Z \rightarrow M \rightarrow Y$
- stratifying by Z closes the second,
(a confounder path)
- estimate bZ corresponds to the total
effect of Z on Y,
i.e. $Z \rightarrow M \rightarrow Y$
(but this is not our main research
interest)

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ M \leftarrow f_M(X, Z, U_M) \\ Y \leftarrow f_Y(M, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

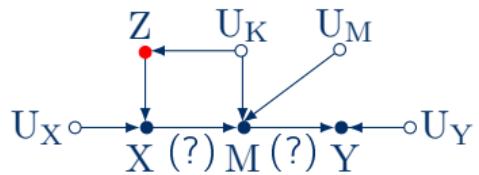
Similar cases¹⁷

research question,

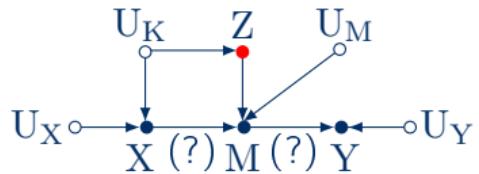
- What is the (total) effect of X on Y?
(all directional paths from X to Y)
(i.e. no mediators)

then,

- stratifying by Z is still a good idea
(U_Z is not drawn out of convenience)



(b) causal diagram



(b) causal diagram

¹⁷Cinelli et al. [4] (p. 3)

Careful what you control for (though)

```
# sim  
Z = rnorm( 100 )  
X = rnorm( 100 , 1*Z )  
M = rnorm( 100 , 0.5*X + 1*Z )  
Y = rnorm( 100 , 0*M )  
d = data.frame(Z=Z,X=X,M=M,Y=Y)
```

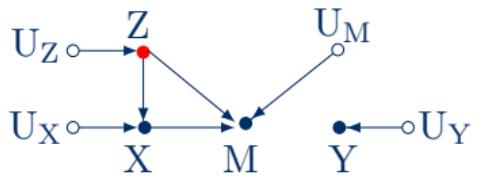
(c) R code

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ M \leftarrow f_M(X, Z, U_M) \\ Y \leftarrow f_Y(U_Y) \\ U \sim P(U) \end{cases}$$

(a) structural model

Implications,

- $X \perp\!\!\!\perp Y$
- $Z \perp\!\!\!\perp Y$
- $X \not\perp\!\!\!\perp M \mid Z$

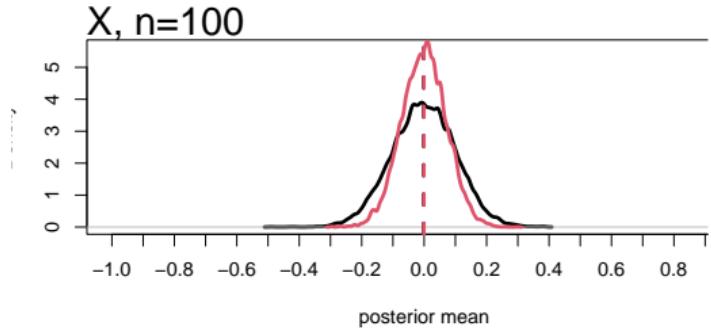
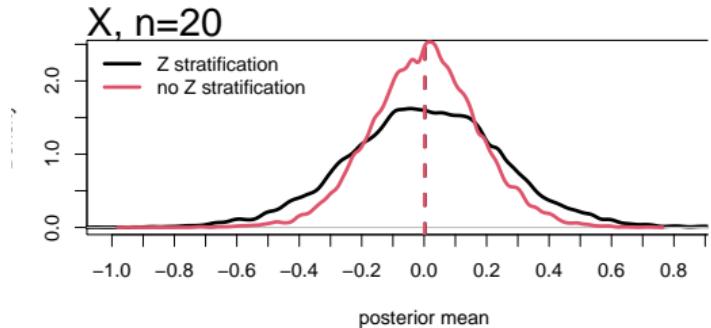


(b) causal diagram

Careful what you control for (though)

under the NO stratification model,
the larger the sample size,

- the more certain you are about your **correct** estimates
- Z now works as a **precision parasite**
i.e. conditioning on Z reduces variation on X, leaving less variability that can explain outcome Y



Not all is bad (though)

```
# sim
Z = rnorm( 100 )
X = rnorm( 100 , 0*Z )
M = rnorm( 100 , 0.5*X + 1*Z )
Y = rnorm( 100 , 0.5*M )
d = data.frame(Z=Z,X=X,M=M,Y=Y)
```

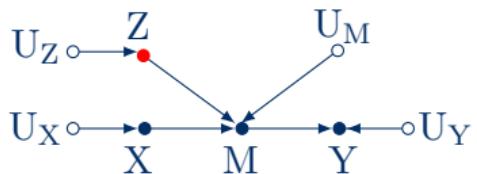
(c) R code

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(U_X) \\ M \leftarrow f_M(X, Z, U_M) \\ Y \leftarrow f_Y(M, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

Implications,

- $X \not\perp\!\!\!\perp Y$
- $X \perp\!\!\!\perp Y$
- $Z \perp\!\!\!\perp Y$

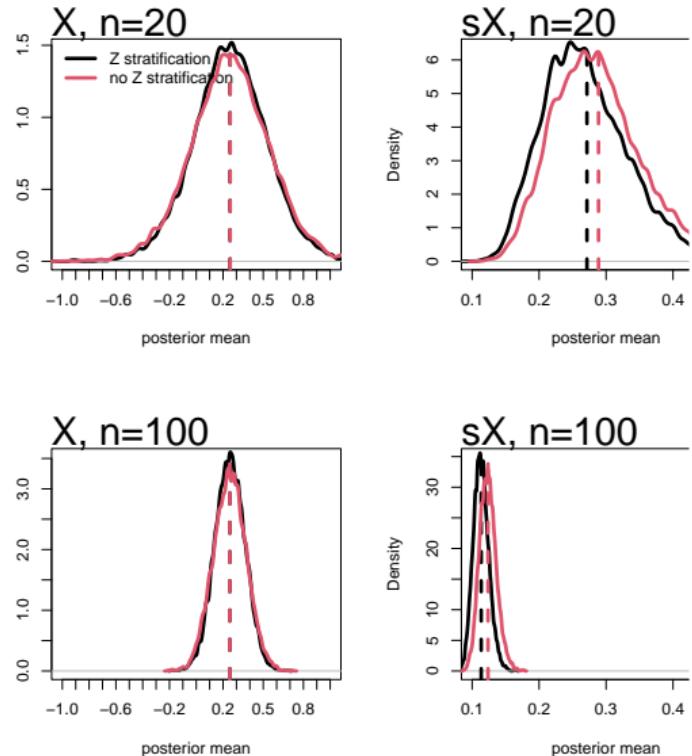


(b) causal diagram

Not all is bad (though)

under the **stratification model**,
the larger the sample size,

- the more **certain** you are about your **correct** estimates
- Z now works as a **precision booster**
i.e. conditioning on Z reduces variation on Y (as Z is a cause of M, and M of Y), leaving less variability to be explained by X



3. Example cases

Pipe/Fork: bias amplification

Bias amplification¹⁸

also known as,

- (unobserved) omitted variable bias
- related to **instrumental variables**
- an instance of **fork bias**

research question,

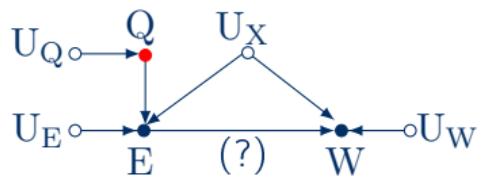
- Do E has a (direct) effect on W?

variables,

- Q, instrumental variable
(e.g. quarter of the year)
- E, educational level
- U_X, unobservables (e.g. ability)
- W, future wages

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

¹⁸McElreath [12], chapter 14 (p. 455), Cinelli et al. [4] (p. 5)

Simulation setting

```
# sim  
U = rnorm( 100 )  
Q = sample( 1:4, 100, replace=T )  
E = rnorm( 100 , 1*Q + 1*U )  
W = rnorm( 100 , 0*E + 1*U )  
d = data.frame(U=U,Q=Q,E=E,W=W)
```

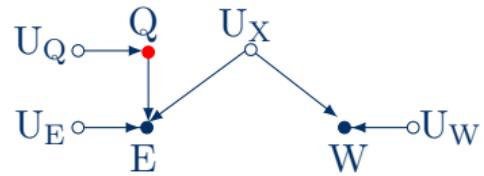
(c) R code

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

Implications,

- $E \not\perp\!\!\!\perp W$
- $E \perp\!\!\!\perp W \mid U_X$ (impossible)
- $Q \perp\!\!\!\perp U_X$ (cannot be tested)
- $Q \not\perp\!\!\!\perp E$
- $Q \perp\!\!\!\perp W \mid E$ (cannot be tested)
(exclusion restriction)

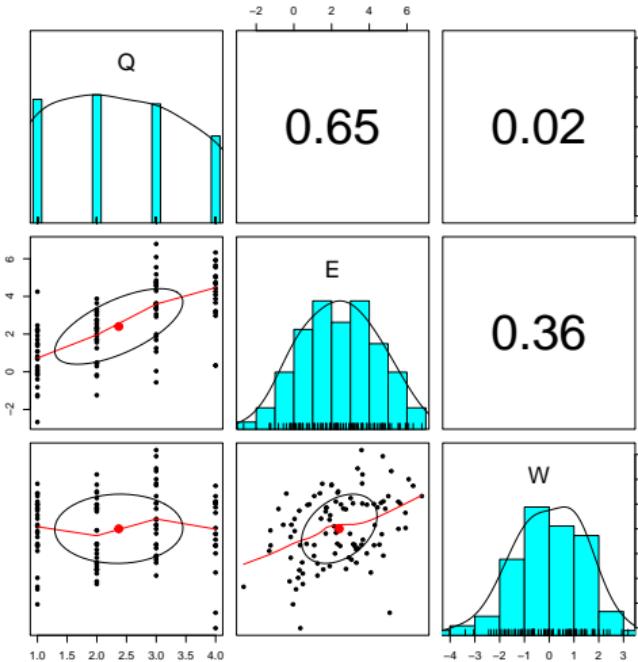


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(Q, E) > 0$ and $\text{cor}(E, W) > 0$
goes in line of our “rudimentary”
understanding of the data.
- $\text{cor}(Q, W) > 0$ tells you about the
exclusion restriction?
(hint: No)
- we might NOT include Q as a
covariate in our statistical model
(but is the instrumental variable!!!)



Regression, regression!!

based on **statistical analysis**,

- two different stories
(which model is the “truth”?)
- one is “worse”/“better” than the other?
- are both wrong?

```
> summary(lm(W ~ E, data=d)) # biased

Call:
lm(formula = W ~ E, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.0726 -0.9674  0.1771  0.9234  2.8787 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.60816   0.20506 -2.966 0.003793 ** 
E             0.25408   0.06559  3.873 0.000194 *** 
> summary(lm(W ~ E + Q, data=d)) # more biased

Call:
lm(formula = W ~ E + Q, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.7405 -0.9774  0.0879  0.9162  2.9825 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.12229   0.30650  0.399  0.69078  
E            0.42054   0.08262  5.090 1.75e-06 *** 
Q            -0.47716   0.15361 -3.106  0.00249 **
```

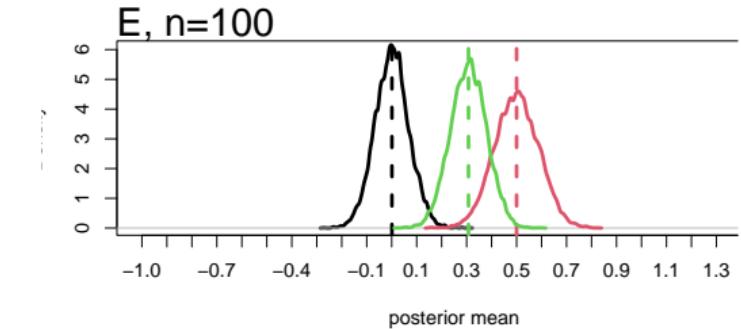
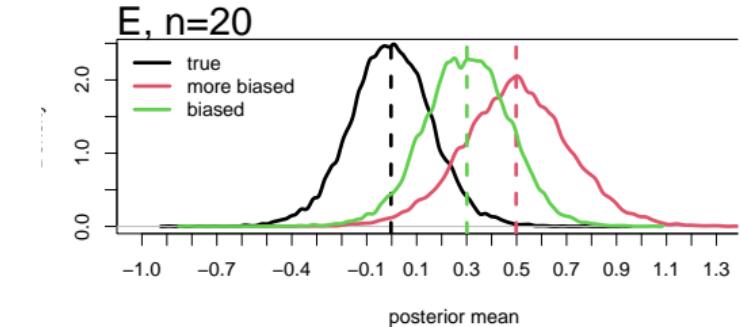
I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,
the larger the sample size,

- the more **certain** you are about
your **biased** estimates
(under the **any** model!!)



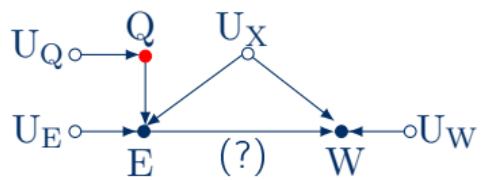
Yo, what is going on??

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,
i.e. $E \perp\!\!\!\perp W | U_X$
(but U_X is unobservable)
- if we use Q in the model, the 3rd D-separation rule kicks in:
“A collider that has been conditioned on does not block a path.”
i.e. $Q \not\perp\!\!\!\perp U_X | E$
(e.g. switch, electricity, and light bulb)

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

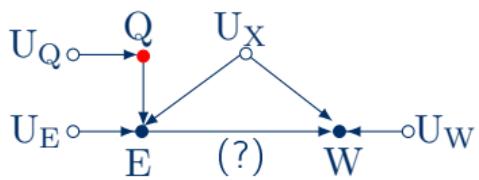
Yo, what is going on??

open paths?:

- $E \rightarrow W$
- $E \rightarrow U_x \rightarrow W$
- $E \rightarrow U_x \rightarrow Q \rightarrow E \rightarrow W$
- $E \rightarrow U_x \rightarrow Q \rightarrow E \rightarrow U_X \rightarrow W$

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

What should I do then??

$$\begin{pmatrix} W \\ E \end{pmatrix} \sim MVN \left[\begin{pmatrix} \mu_W \\ \mu_E \end{pmatrix}, \Sigma \right]$$

$$\mu_W = \alpha_W + \beta_{EW} E$$

$$\mu_E = \alpha_E + \beta_{QE} Q$$

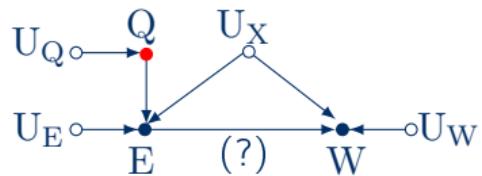
(c) probabilistic model

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

based on DAG and statistical model,
use the knowledge of the system

- one model for $Q \rightarrow E$
- one model for $E \rightarrow W$
- use the knowledge that $\text{cov}(E, W) > 0$
due to unobserved confounder U_X ,
(i.e. $\text{cov}(E, W) = \Sigma = \text{SRS}$)



(b) causal diagram

did it worked???

based on DAG and bayesian statistical analysis,

- appropriate value estimated,
(assuming our DAG is true)
- it picks up some of the unobserved correlation R[1, 2]

	mean	sd	5.5%	94.5%
aE	0.02	0.18	-0.26	0.30
aW	-0.14	0.16	-0.40	0.13
bQE	1.00	0.07	0.88	1.12
bEW	0.05	0.07	-0.06	0.16
R[1,1]	1.00	0.00	1.00	1.00
R[1,2]	0.33	0.11	0.15	0.50
R[2,1]	0.33	0.11	0.15	0.50
R[2,2]	1.00	0.00	1.00	1.00
S[1]	1.25	0.10	1.11	1.42
S[2]	1.39	0.10	1.24	1.56

did it worked???

frequentists guys apply

Two Stage Least Squares (2SLS)^a:

- regress $E \leftarrow Q$,
- predict \hat{E} ,
- regress $W \leftarrow \hat{E}$

```
s1 = lm( E ~ Q, data=d)
Ehat = s1$fitted.values
s2 = lm( W ~ Ehat, data=d)
# se not corrected

require(AER)
tsls = ivreg( W ~ E | Q, data=d)
# se corrected
```

```
> summary(s2)

Call:
lm(formula = W ~ Ehat, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.1911 -1.0670 -0.0643  1.3802  4.6813 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.5992     0.3667  -1.634   0.106    
Ehat         0.2093     0.1263   1.658   0.101    

> summary(tsls)

Call:
ivreg(formula = W ~ E | Q, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6571 -1.1852 -0.1819  1.0945  4.5328 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.5992     0.3325  -1.802   0.0746 .  
E             0.2093     0.1145   1.829   0.0705 .
```

^aHanck et al. [7], section 12.1,
See McElreath [12] chapter 14 (p. 460) for a
discussion on the method.

Similar case, contextual confounds¹⁹ research question,

- Does W has an effect on L?
- should we include H in our model?

variables,

- W, win the lottery
- H, happiness
- U_X , contextual confound
- L, lifespan

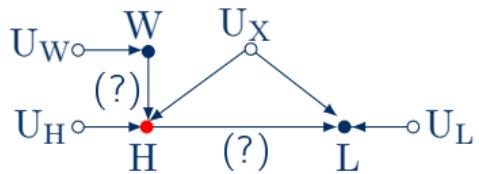
Short answer;

- for **total effects**: No
(two question marks together)
- for **direct effect** of $H \rightarrow L$:
will be always confounded

¹⁹McElreath [13], lecture 6

$$M = \begin{cases} W \leftarrow f_W(U_W) \\ H \leftarrow f_H(W, U_X, U_H) \\ L \leftarrow f_L(H, U_X, U_L) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

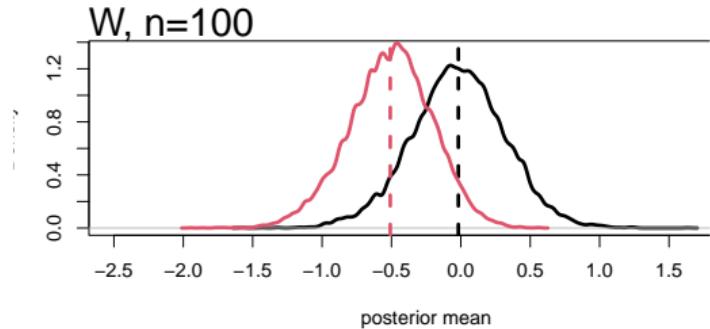
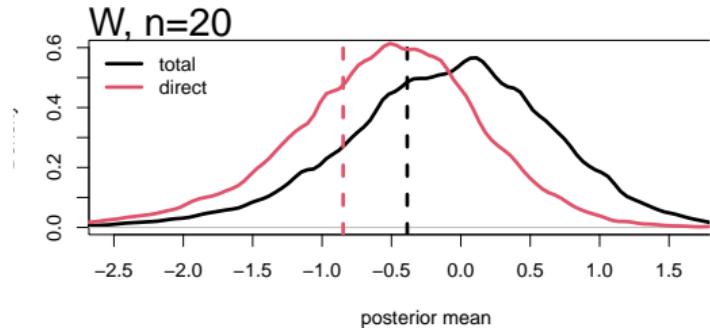
Similar case, contextual confounds

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

the larger the sample size,

- the more certain you are about your unbiased total effects
- the more certain you are about your biased direct effects



3. Example cases

Collider: Berkson's paradox

Berkson's paradox²⁰

also known as,

- selection bias
- selection-distortion effect
- “convenience” sample bias

research question,

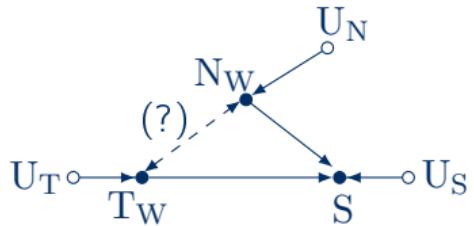
- Is there a true relationship between
N_W and T_W in studies?

variables,

- N_W, “news-worthiness” of a study
- T_W, “trust-worthiness” of a study
- S, selected studies

$$M = \begin{cases} T_W \leftarrow f_T(U_T) \\ N_W \leftarrow f_N(U_N) \\ S \leftarrow f_S(T_W, N_W, U_S) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

²⁰McElreath [12], chapter 6 (p. 161); Cinelli et al. [4] (p. 8)

Simulation setting

```
# sim  
NW = rnorm( 100 ) # uncorrelated  
TW = rnorm( 100 )  
Sc = NW + TW # total score  
q = quantile( Sc , 1-0.1 ) # top 10% threshold  
S = ifelse( Sc >= q , 1 , 0 ) # select top 10%  
d = data.frame( NW=NW, TW=TW, S=S )
```

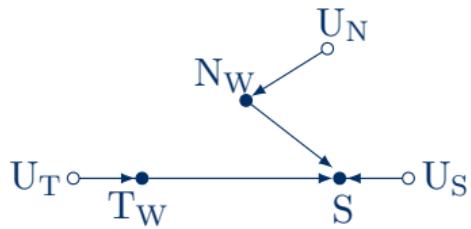
(c) R code

Implications,

- $T_W \perp\!\!\!\perp N_W$
- $T_W \not\perp\!\!\!\perp N_W \mid S$

$$M = \begin{cases} T_W \leftarrow f_T(U_T) \\ N_W \leftarrow f_N(U_N) \\ S \leftarrow f_S(T_W, N_W, U_S) \\ U \sim P(U) \end{cases}$$

(a) structural model

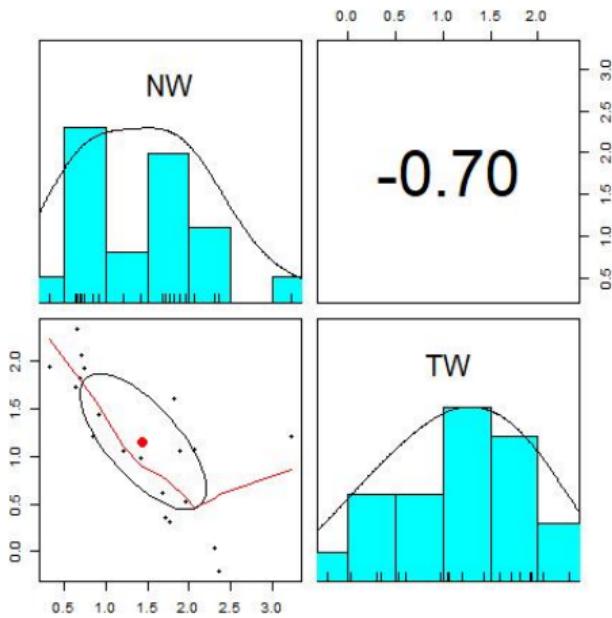


(b) causal diagram

Descriptive analysis

based on descriptive analysis,

- what we observe is $\text{Cor}(N_w, T_w | S)$,
(data has been stratified on a score)
- $\text{Cor}(N_w, T_w | S)$ does NOT go in line with our “rudimentary” understanding of the data
(N_w and T_w are related?)
- the issue is that we want to observe $\text{Cor}(N_w, T_w)$
(unconditional correlation)



Regression does not solve anything!!

based on **statistical analysis**,

- TW continues to “explain” Nw ,
(is the only model accessible)
- But is it correct though?

```
> summary(lm(NW ~ TW, data=d[d$S==1,])) # biased effe
Call:
lm(formula = NW ~ TW, data = d[d$S == 1, ])
Residuals:
    Min      1Q  Median      3Q     Max 
-0.5955 -0.2437 -0.1069  0.1720  0.8788 
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.3076     0.2176 10.605 3.59e-09 ***
TW          -0.7861     0.1402 -5.608 2.54e-05 ***
```

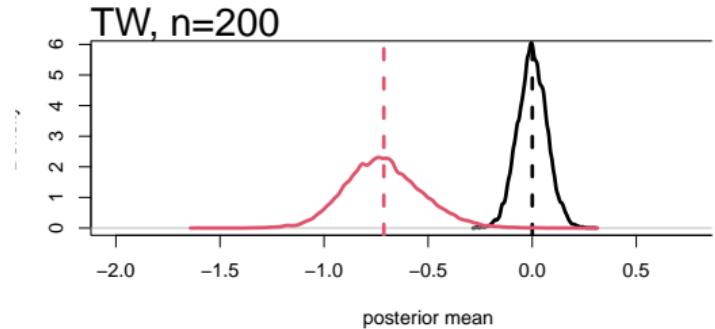
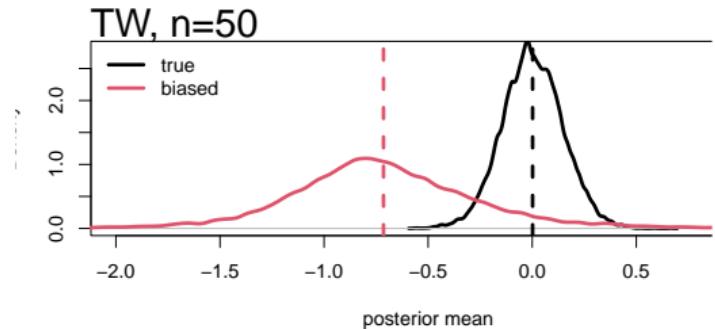
Let me guess?, more data,
more problems

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **only available model**,
the larger the sample size,

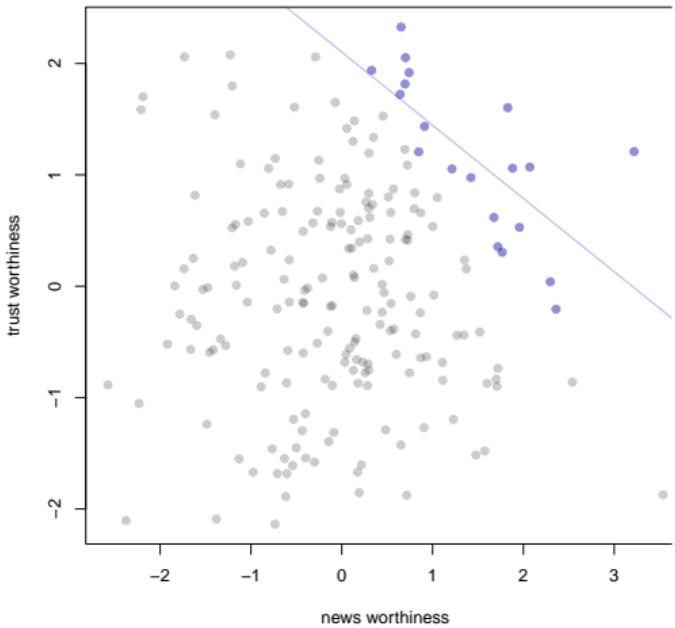
- the more **certain** you are about
your **biased** estimates



Ok, what is going on?

If we only observe data where $S = 1$,

- it means that even if your study had a low T_w , you know it necessarily has a high N_w to compensate and pass the threshold (e.g. switch, electricity, and light)



Similar case²¹

research question,

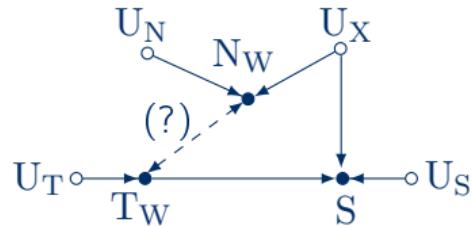
- Is there a true relationship between NW and TW in studies?

variables,

- NW, “news-worthiness” of a study
- TW, “trust-worthiness” of a study
- U_x , unobservable (e.g. no idea yet)
- S, selected studies

$$M = \begin{cases} T_W \leftarrow f_T(U_T) \\ N_W \leftarrow f_N(U_N) \\ S \leftarrow f_S(T_W, N_W, U_S) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

²¹Cinelli et al. [4] (p. 8)

Is there a way to fix it?

a simple ways,

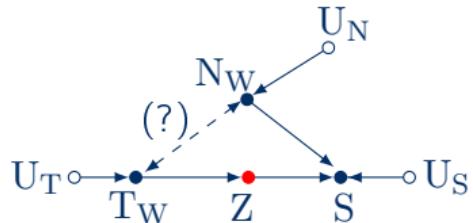
- use a variable that closes the selection path (2nd D-separation rule)^a,
i.e. stratify by a pipe to close the path
 $T_W \rightarrow Z \rightarrow S$ (see right)

or

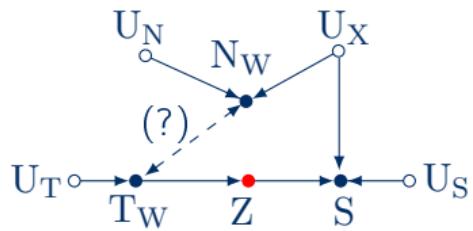
$N_W \rightarrow Z \rightarrow S$ (not shown)

$N_W \leftarrow U_X \rightarrow Z \rightarrow S$ (not shown)

^afront door criterion?



(b1) causal diagram

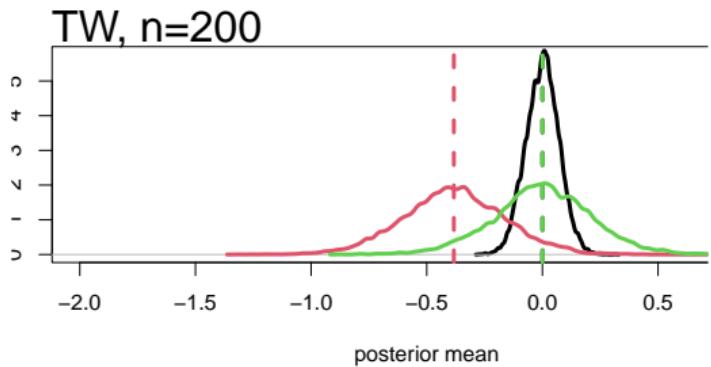
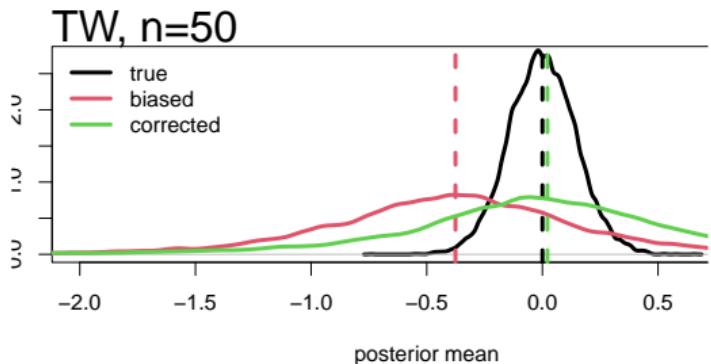


(b2) causal diagram

Does it work though?

based on the previous DAGs,
the statistical analysis reveals,

- stratifying by Z “corrects” the estimates,
(but we still lose some precision)



3. Example cases

Collider: M-bias

M-bias²²

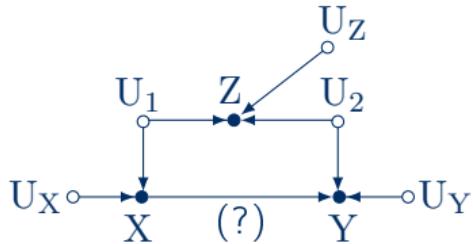
case of,

- bias on pre-treatment variable research question,
 - Should we include Z in our model? variables,

- Z, “health” quality of friends (defined as a continuum)
- X, health of individual 1
- U₁, hobbies of individual 1
- Y, health of individual 2
- U₂, hobbies of individual 2

$$M = \begin{cases} X \leftarrow f_X(U_1, U_X) \\ Z \leftarrow f_Z(U_1, U_2, U_Z) \\ Y \leftarrow f_Y(X, U_2, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

²²McElreath [13], lecture 6; Cinelli et al. [4] (p. 4)

Simulation setting

```
# sim
U1 = sample(1:5, 100 , replace=T)
U2 = sample(1:5, 100 , replace=T)
Z = rnorm( 100 , 0.5*U1 + 0.5*U2)
X = rnorm( 100 , 1*U1 + 0*Z)
Y = rnorm( 100 , 1*U2 + 0*X + 0*Z)
d = data.frame(U1,U2,Z,X,Y)
```

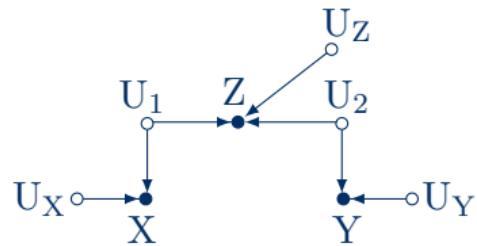
(c) R code

Implications,

- $X \perp\!\!\!\perp Y$
- $X \not\perp\!\!\!\perp Y | Z$

$$M = \begin{cases} X \leftarrow f_X(U_1, U_X) \\ Z \leftarrow f_Z(U_1, U_2, U_Z) \\ Y \leftarrow f_Y(X, U_2, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

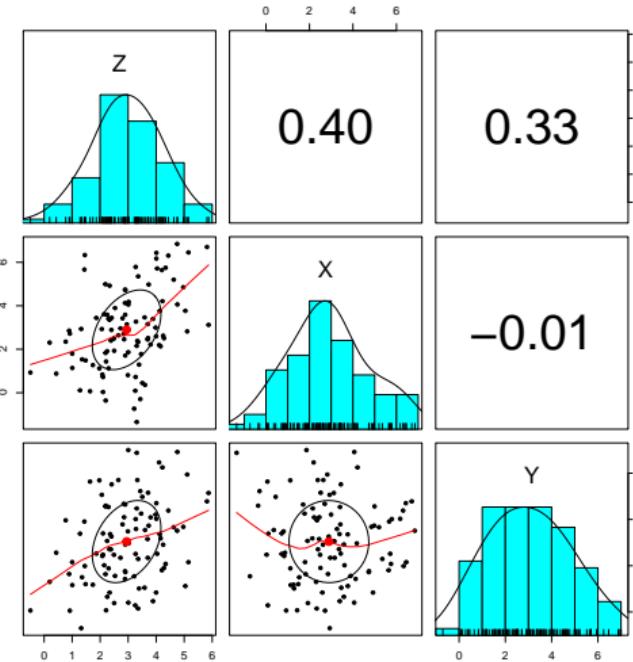


(b) causal diagram

Descriptive analysis

based on descriptive analysis,

- $\text{Cor}(X, Y) \approx 0$, quite low
- larger $\text{Cor}(Z, Y)$, while $\text{Cor}(Z, X)$ is not high enough to discard it as a cause of multicollinearity,
- we might include Z rather than X (but the effect of X is our interest!!)
- then we include Z and X



Again regression!!

based on statistical analysis,

- X does not have an effect of Y, (in the first nor the second model)
- but X has an (non-negligible) effect when Z is in the model (but we do not reject the null)
- The increase of the X effect might lead you to think that with more data, we can reject the null (and you would be right!!)
- But is it correct to include Z?

```
> summary(lm(Y ~ X, data=d)) # unbiased effects (efficacy)

Call:
lm(formula = Y ~ X, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.7452 -1.3300 -0.0253  1.2414  3.9479 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 3.067474   0.334586  9.168 7.74e-15 ***
X           -0.008255   0.097774  -0.084   0.933  
> summary(lm(Y ~ X + Z, data=d)) # biased effects (efficacy)

Call:
lm(formula = Y ~ X + Z, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.3756 -1.0898  0.0351  1.0540  3.7595 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.83324   0.45304   4.047 0.000105 ***
X           -0.15765   0.09991  -1.578 0.117835  
Z            0.56590   0.14974   3.779 0.000272 ***
```

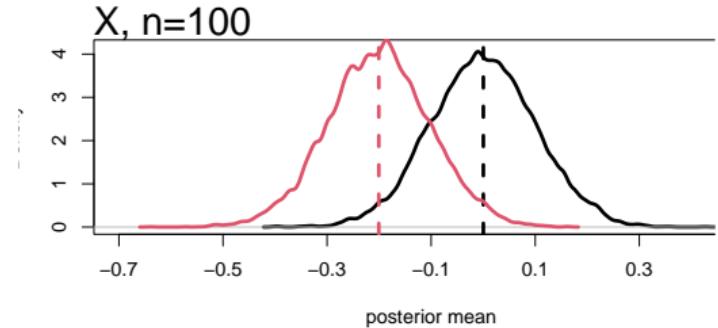
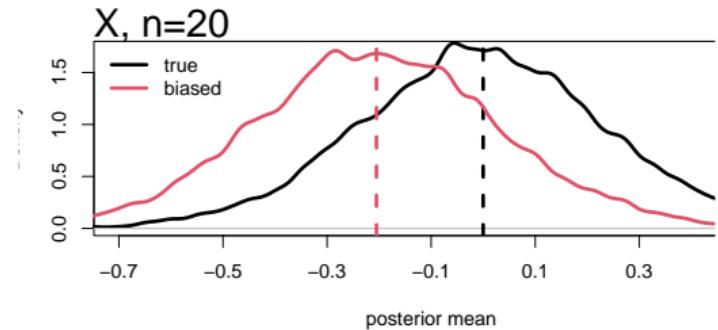
Ok, I get it!!, more data,
more bias!!

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the “incorrect” model,
the larger the sample size,

- the more certain you are about
your **biased** estimates
(with enough you could reject the
null)



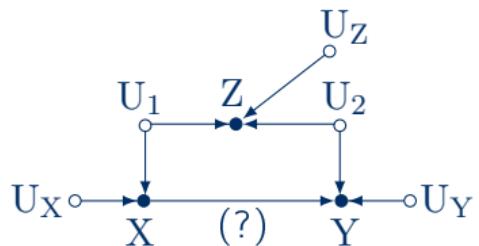
The dream team strikes back!!

based on DAG and statistical model,

- the 3rd D-separation states that a collider that has been conditioned on does not block a path,
in this case: $X \rightarrow U_1 \rightarrow Z \rightarrow U_2 \rightarrow Y$
i.e. $X \not\perp\!\!\!\perp Y \mid Z$
- therefore if we want to find the direct effect of $X \rightarrow Y$, we should not stratify by Z

$$M = \begin{cases} X \leftarrow f_X(U_1, U_X) \\ Z \leftarrow f_Z(U_1, U_2, U_Z) \\ Y \leftarrow f_Y(X, U_2, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

The dream team strikes back!!

based on DAG and statistical analysis,

- the model that answers our research question is the first one, (assuming our DAG is true)

```
> summary(lm(Y ~ X, data=d)) # unbiased effects (efficiency)

Call:
lm(formula = Y ~ X, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.7452 -1.3300 -0.0253  1.2414  3.9479 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 3.067474   0.334586  9.168 7.74e-15 ***
X           -0.008255   0.097774 -0.084    0.933  

```

3. Example cases

Descendant: proxies (a)

Proxies (a)²³

research question,

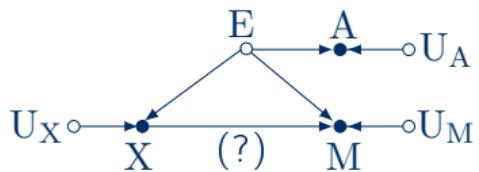
- Does X has a (direct) effect on M?

variables,

- X, teacher experience
- E, instruction quality (old, new)
(unobserved)
- A, proxy variable (e.g. age)
- M, teachers' score in mathematics
(from a standardized evaluation)

$$M = \begin{cases} A \leftarrow f_A(E, U_A) \\ X \leftarrow f_X(E, U_X) \\ M \leftarrow f_M(E, X, U_M) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

²³McElreath [12], chapter 11 (p. 340); McElreath [13], lecture 10

Simulation setting

```
# sim  
E = sample( 1:2, 100, replace=T)  
A = round( rnorm( 100 , 35 + bEA*E))  
X = rbinom( 100 , size=A , prob=0.5 + 0.2*E )  
M = rnorm( n , 0*X + 2*E )  
d = data.frame(X=X,A=A,E=E,M=M)
```

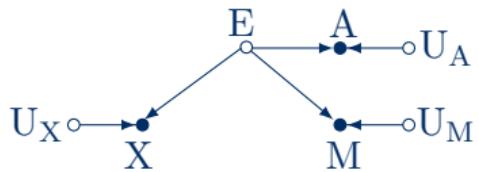
(c) R code

Implications,

- $A \not\perp\!\!\!\perp M$
- $A \not\perp\!\!\!\perp X$
- $X \perp\!\!\!\perp M \mid E$ (impossible)
- $X \perp\!\!\!\perp M \mid A$ (partially)

$$M = \begin{cases} A \leftarrow f_A(E, U_A) \\ X \leftarrow f_X(E, U_X) \\ M \leftarrow f_M(E, X, U_M) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

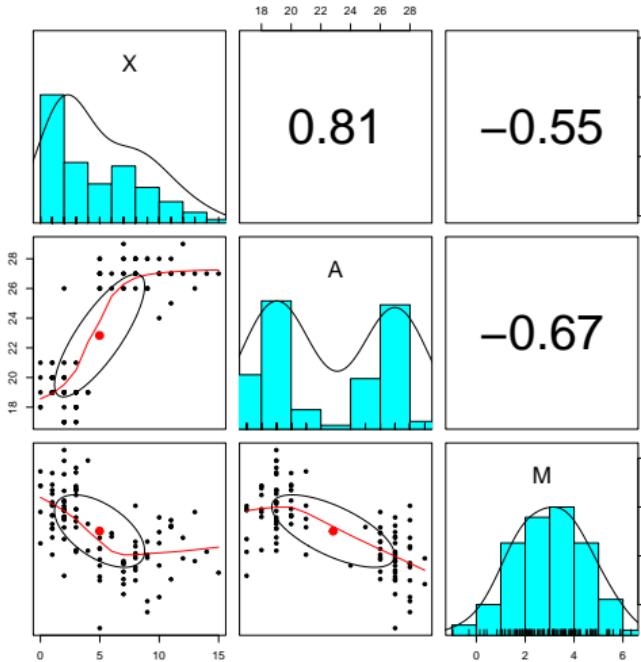


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{Cor}(A, M) < 0$ goes in line with our “rudimentary” understanding of the data,
(old teachers has old type of instruction, and lower score)
- but $\text{cor}(X, M) < 0$ does not go in line with our “understanding”
(the more experienced, the less the score?)
- but we might include Z in our statistical model
(based on univariate (linear) correlation)



Regression, regression!!

based on statistical analysis,

- we now have two models with two different effects
- which one is the “truth”?

```
> summary(lm(M ~ X, data=d)) # biased effect (X)
Call:
lm(formula = M ~ X, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.7322 -0.8688  0.1697  0.6983  3.0007 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 3.98199   0.21481 18.537 < 2e-16 ***
X           -0.23454   0.03389 -6.921 4.68e-10 ***
> summary(lm(M ~ X + A, data=d)) # unbiased effect I

Call:
lm(formula = M ~ X + A, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.80201 -0.73449  0.03422  0.86735  2.53160 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 7.24998   0.94373  7.682 1.26e-11 ***
X           -0.07091   0.05618 -1.262 0.209884  
A           -0.17473   0.04928 -3.546 0.000604 ***
```

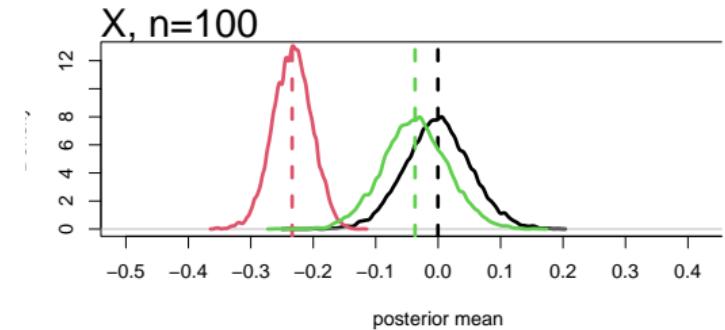
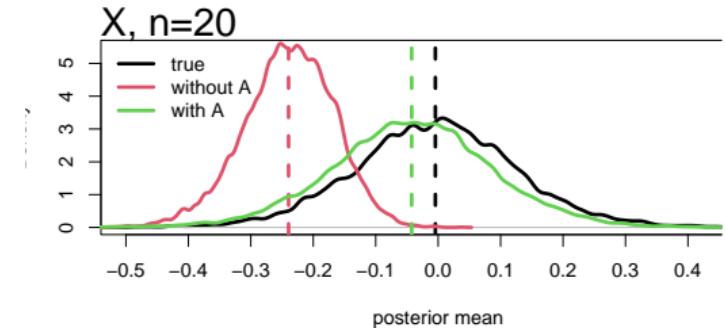
The data tell-tell story!!

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

the larger the sample size,

- under the model without A,
the more certain you are about
your biased estimates
- under the model with A,
the more certain you are about
your less biased estimates



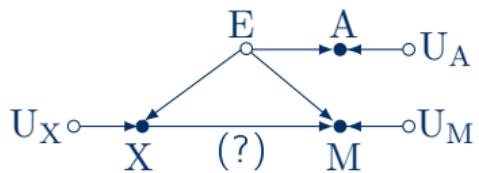
So, what is going on?

based on DAG and statistical model,

- stratifying on A partially blocks the backdoor path produced by the confounder E
i.e. when stratifying by A you are partially stratifying by E

$$M = \begin{cases} A \leftarrow f_A(E, U_A) \\ X \leftarrow f_X(E, U_X) \\ M \leftarrow f_M(E, X, U_M) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

the dream team!!

based on DAG and statistical analysis,

- the less biased model is the second,
(assuming our DAG is true)

```
> summary(lm(M ~ X + A, data=d)) # unbiased effect

Call:
lm(formula = M ~ X + A, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.80201 -0.73449  0.03422  0.86735  2.53160 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 7.24998   0.94373   7.682 1.26e-11 ***
X           -0.07091   0.05618  -1.262 0.209884    
A           -0.17473   0.04928  -3.546 0.000604 ***
```

But age cause experience, right?

research question,

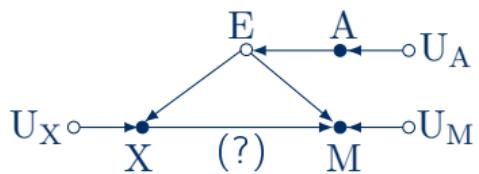
- Does X has a (direct) effect on M?

variables,

- X, teacher experience
- E, instruction quality (old, new)
(unobserved)
- A, cause variable (e.g. age)
- M, teachers' score in mathematics
(from a standardized evaluation)

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ X \leftarrow f_X(E, U_X) \\ M \leftarrow f_M(E, X, U_M) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

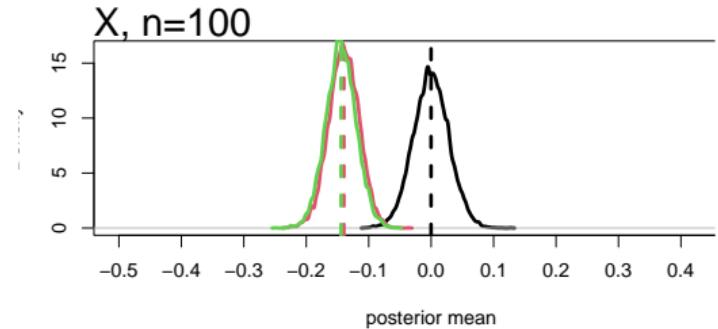
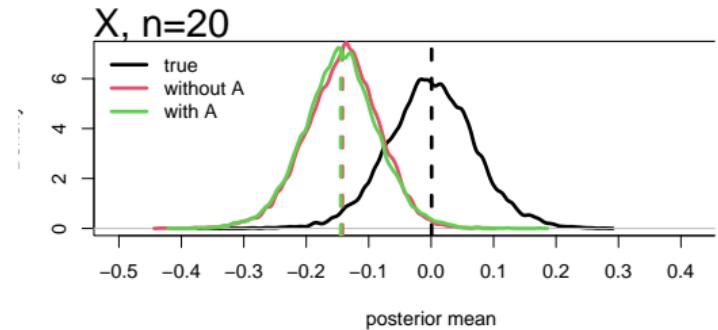
But age cause experience, right?

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

the larger the sample size,

- under any model including A,
the more certain you are about
your **biased** estimates



3. Example cases

Descendant: case control

Case control²⁴

also,

- virtual collider

research question,

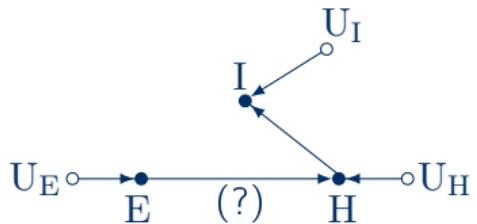
- Does E has a (direct) effect on H?
- Should we include I on our model?

variables,

- E, education
- H, hours in occupation
(standardized)
- I, income

$$M = \begin{cases} E \leftarrow f_E(U_E) \\ H \leftarrow f_H(E, U_H) \\ I \leftarrow f_I(H, U_I) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

²⁴McElreath [13], lecture 6; Cinelli et al. [4] (p. 8, 19), Hernán [8], lesson 3

Simulation setting

```
# sim  
E = rnorm( 100 )  
H = rnorm( 100 , -1*E )  
I = rnorm( 100 , -1*H )  
d = data.frame(E,H,I)
```

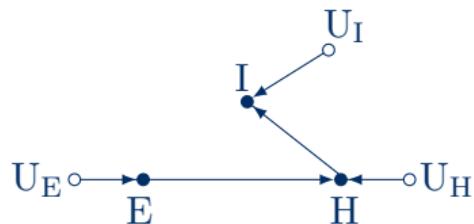
(c) R code

$$M = \begin{cases} E \leftarrow f_E(U_E) \\ H \leftarrow f_H(E, U_H) \\ I \leftarrow f_I(H, U_I) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

Implications,

- $E \not\perp\!\!\!\perp H$
- $E \perp\!\!\!\perp I \mid H$
- $E \not\perp\!\!\!\perp U_H \mid I$
(virtual collider)

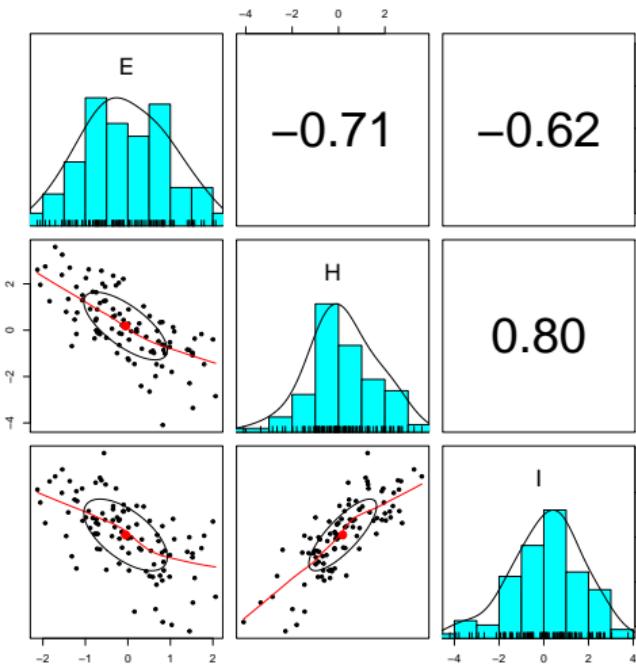


(b) causal diagram

“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(E, I) < 0$ does NOT goes in line of our “rudimentary” understanding of the data.
- while $\text{cor}(H, I) > 0$ indicate the more you work the more you gain (but is it the only way?)
- since $\text{cor}(H, I)$ is high we might include it as a covariate in our statistical model (to improve the precision?)



Regression, regression!!

based on statistical analysis,

- we now have two models with two different “levels” of effects
- which one is the “truth”?

```
> summary(lm(H ~ E, data=d)) # unbiased effects

Call:
lm(formula = H ~ E, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.19319 -0.60621 -0.06694  0.55674  2.77776 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.04689   0.09534   0.492   0.624    
E          -0.90223   0.08930 -10.104  <2e-16 *** 
> summary(lm(H ~ E + I, data=d)) # biased effects

Call:
lm(formula = H ~ E + I, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.22991 -0.54882  0.01153  0.46886  1.79879 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.05036   0.06861   0.734   0.465    
E          -0.45847   0.07915  -5.793 8.57e-08 *** 
I          0.55017   0.05728   9.604 9.61e-16 ***
```

The data tell-tell story!!

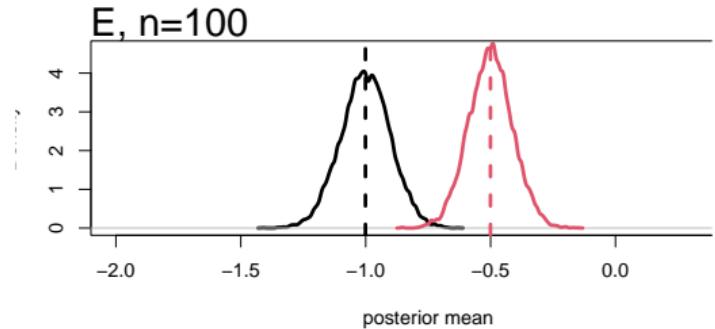
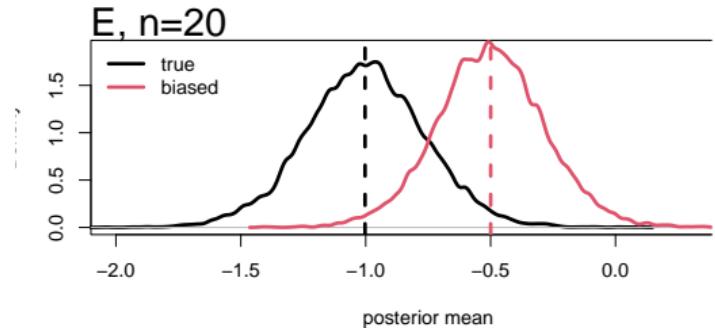
imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,

the larger the sample size,

- the more **certain** you are about your **biased** estimates



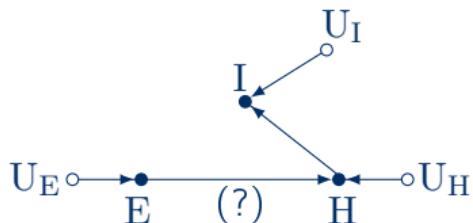
So, what is going on?

based on DAG and statistical model,

- stratifying on I explains variability in H (is his descendant)
remaining variance is explained by E
(big chunk is already explained)
- but there is more!!:
H is now a collider in the path
 $E \rightarrow H \leftarrow U_H$ (virtual collider),
then, stratifying by I opens that path
(biasing the estimates).

$$M = \begin{cases} E \leftarrow f_E(U_E) \\ H \leftarrow f_H(E, U_H) \\ I \leftarrow f_I(H, U_I) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

the dream team!!

based on DAG and statistical analysis,

- the less biased model is the first,
(assuming our DAG is true)

```
> summary(lm(H ~ E, data=d)) # unbiased effects

Call:
lm(formula = H ~ E, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.19319 -0.60621 -0.06694  0.55674  2.77776 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.04689   0.09534   0.492   0.624    
E           -0.90223   0.08930 -10.104 <2e-16 ***
```

Similar, but different²⁵

research question,

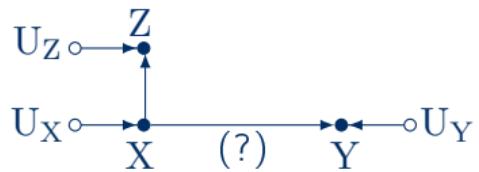
- What is the effect of X on Y?
- Should we include Z in the model?

variables,

- Z, “child” of X
- X, exposure
- Y, outcome

$$M = \begin{cases} X \leftarrow f_X(U_X) \\ Z \leftarrow f_Z(X, U_Z) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

²⁵Cinelli et al. [4] (p. 7)

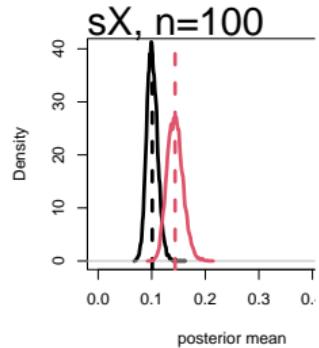
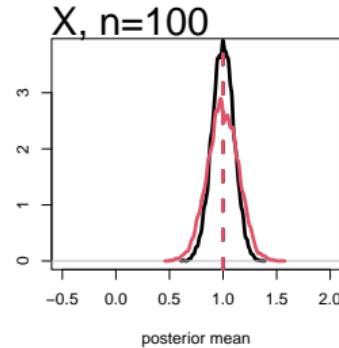
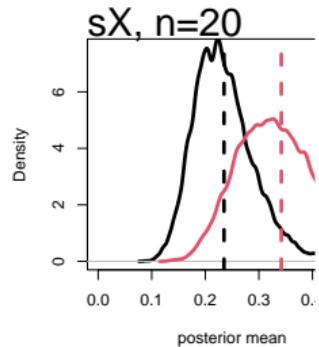
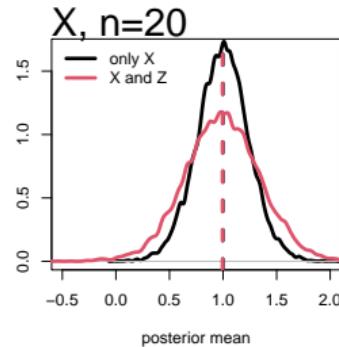
Similar, but different

imagine we can continue sampling,

- top: 10,000 samples $n = 20$
- bottom: 10,000 samples $n = 100$

under the **incorrect model**,
the larger the sample size,

- you loose **certainty** in your
non-biased estimates



3. Example cases

Descendant: measurement error, outcome only

Measurement error, outcome only²⁶

also,

- residual confounding
- think meta analysis

research question,

- Does M has a (direct) effect on D?

variables,

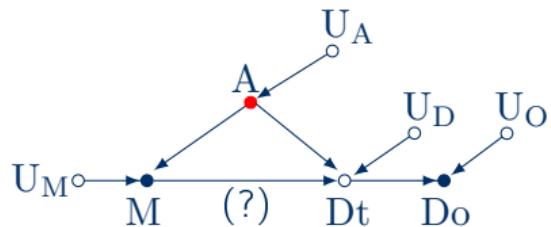
- A, median age at marriage
- M, marriage rate
- Dt, divorce rate (true rate)
- Do, divorce rate (observed rate)

extras,

- U_D , between variability (true rate)
- U_O , measurement error

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ Dt \leftarrow f_D(A, M, U_D) \\ Do \leftarrow f_D(Dt, U_O) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

²⁶McElreath [12], chapter 15 (p. 491)

Simulation setting

```
# sim
A = rnorm( 100 )
M = rnorm( 100 , -1*A )
D_true = rnorm( 100 , -1*A + 0*M )
D_obs = rnorm( 100 , mean=D_true , sd=1 )
d = data.frame( A , M , D_true , D_obs )
```

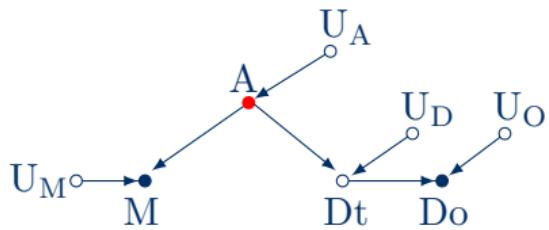
(c) R code

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ Dt \leftarrow f_D(A, U_D) \\ Do \leftarrow f_D(Dt, U_O) \\ U \sim P(U) \end{cases}$$

(a) structural model

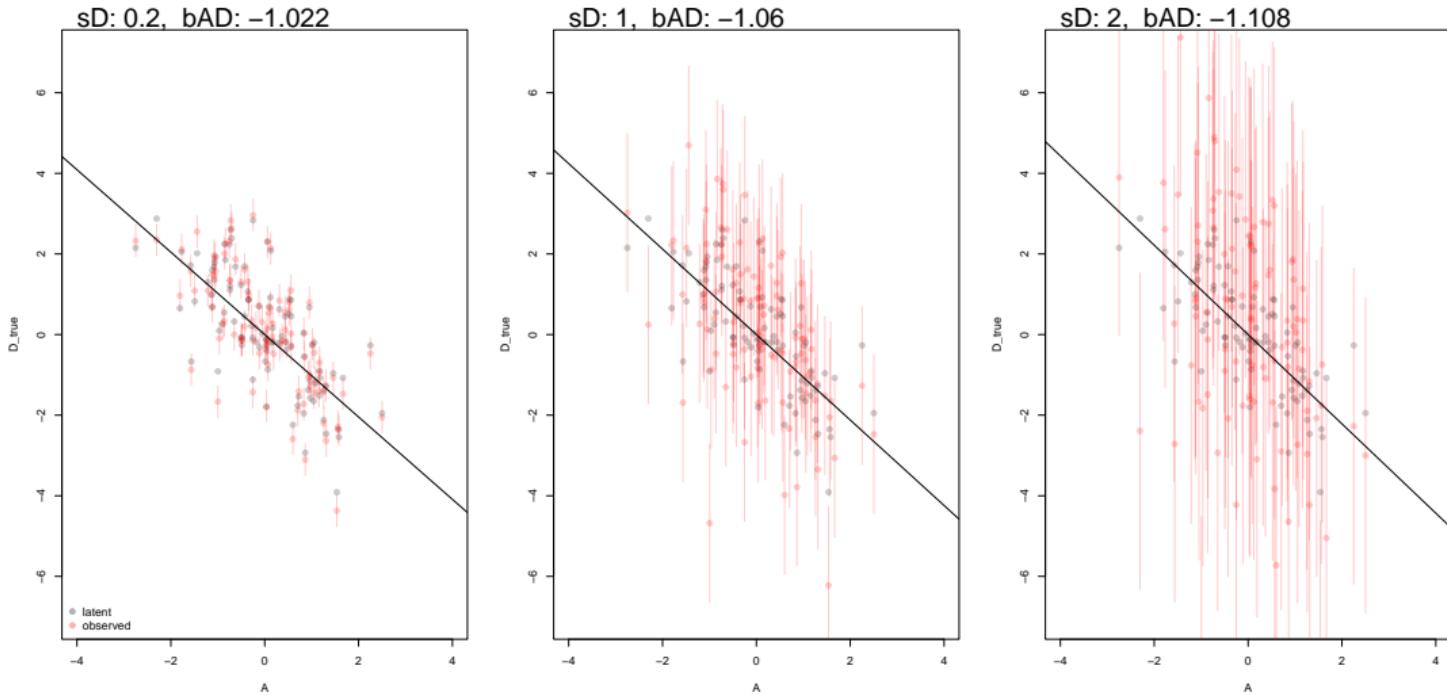
Implications,

- $M \not\perp\!\!\!\perp Dt$
- $M \not\perp\!\!\!\perp Do$
- $M \perp\!\!\!\perp Dt \mid A$
- $M \perp\!\!\!\perp Do \mid A$

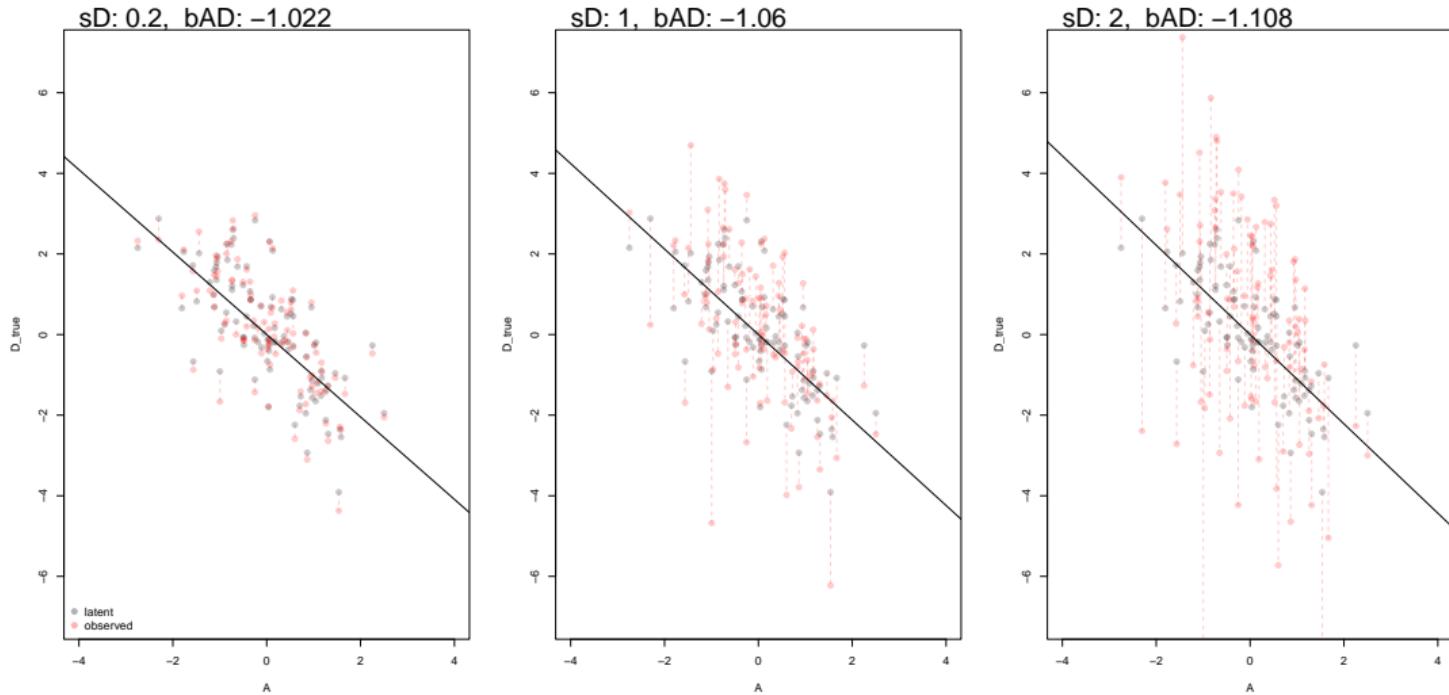


(b) causal diagram

So, what does the simulation means?



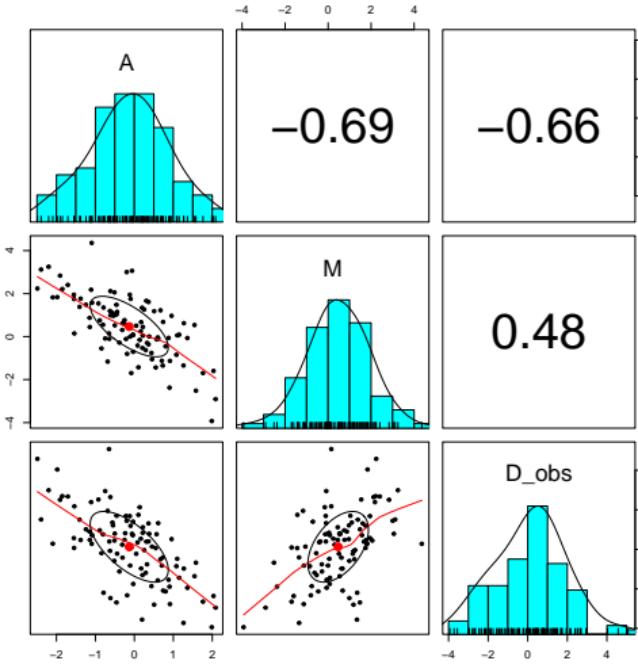
So, what does the simulation means?



“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(A, D_{\text{obs}}) < 0$ and $\text{cor}(M, D_{\text{obs}}) > 0$ goes in line of our “rudimentary” understanding of the data.
- why there is $\text{cor}(M, D_{\text{obs}}) > 0$?
(hint: univariate correlation)
- we include M as a covariate in our statistical model
(besides, it is our research hypothesis)



Regression, regression!!

based on statistical analysis,

- first model is impossible (unobservable)
- no effect bias in observable data
- we have larger standard error (in this case does not change the hypothesis, but with smaller effects it could happen)

```
> summary(lm(D_true ~ A + M, data=d))  
  
Call:  
lm(formula = D_true ~ A + M, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-2.47208 -0.62476  0.01248  0.59219  2.55400  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 0.11134   0.09911  1.123  0.264  
A           -0.94270   0.16062 -5.869 6.11e-08 ***  
M           0.05942   0.10356  0.574  0.567  
  
> summary(lm(D_obs ~ A + M, data=d))  
  
Call:  
lm(formula = D_obs ~ A + M, data = d)  
  
Residuals:  
    Min      1Q  Median      3Q     Max  
-9.9413 -1.5996  0.2249  1.8544  5.6810  
  
Coefficients:  
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 0.2263    0.2629   0.861  0.39143  
A           -1.2958    0.4261  -3.041  0.00303 **  
M           -0.1516    0.2747  -0.552  0.58234
```

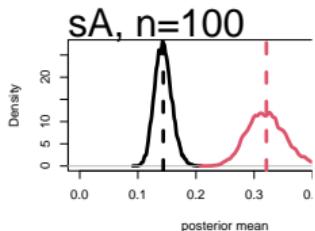
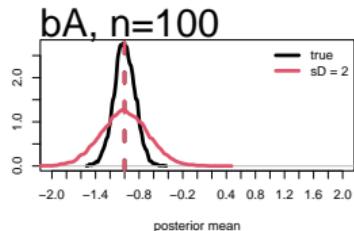
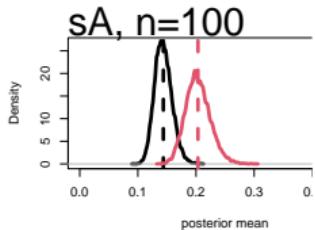
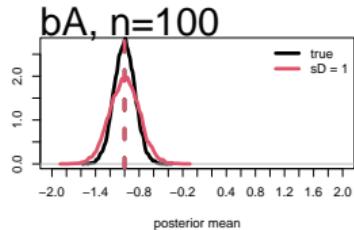
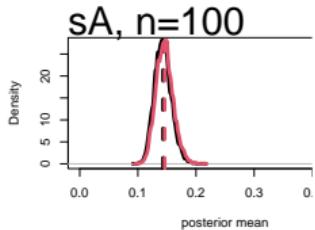
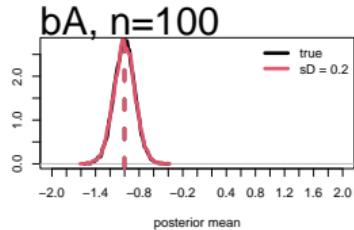
So, what is going on?

imagine we can continue sampling,

- 10,000 samples $n = 100$

notice,

- the larger the measurement error, the less confident you are of your estimates,
- larger measurement error (in outcomes) does not bias the effects,



How can I fix it???

$$D_{\text{obs}} \sim N(D_{\text{true}}, \sigma_O)$$

$$D_{\text{true}} \sim N(\mu, \sigma_D)$$

$$\mu = \alpha + \beta_{AD}A + \beta_{MD}M$$

(c) probabilistic model

based on DAG and statistical model,
use the knowledge of the system

■ measurement error part:

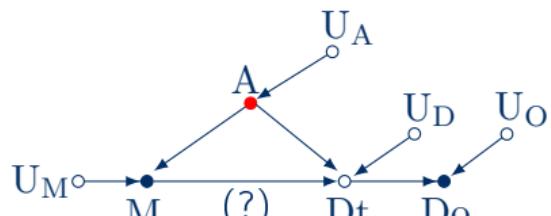
$$D_{\text{obs}} \sim N(D_{\text{true}}, \sigma_O),$$

■ latent structure: $D_{\text{true}} \sim N(\mu, \sigma_D)$,

■ latent regression: $\mu = f(A, M)$

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D_t \leftarrow f_D(A, M, U_D) \\ D_o \leftarrow f_D(D_t, U_O) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

How can I fix it???

```
me_model = ulam(  
  alist(  
    D_obs ~ dnorm( D_true , D_sd ),  
    vector[N]:D_true ~ dnorm( mu , sigma ),  
    mu <- a + bA*A + bM*M,  
    a ~ dnorm(0,0.2),  
    bA ~ dnorm(0,0.5),  
    bM ~ dnorm(0,0.5),  
    sigma ~ dexp(1)),  
  data=dlist , chains=4 , cores=4 )
```

	mean	sd	5.5%	94.5%
a	0.09	0.16	-0.16	0.36
bA	-0.78	0.32	-1.28	-0.25
bM	0.09	0.22	-0.26	0.44
sigma	2.43	0.21	2.14	2.78

Using DAG and Bayesian analysis:

- measurement error part:

$$D_{\text{obs}} \sim N(D_{\text{true}}, S_{\text{obs}}),$$

- latent structure: $D_{\text{true}} \sim N(\mu, \sigma)$,

- latent regression: $\mu = f(A, M)$

How can I fix it???

```
require(metafor)
me_model1 = rma(yi = D_obs,
                 sei = rep(1, 100),
                 data = d,
                 method = "REML",
                 mods = ~ A + M,
                 test = "t")
```

Using DAG and frequentist analysis:

- measurement error part:

$$D_{\text{obs}} = \mu + \delta + \epsilon,$$

- 2-level variability: $\delta \sim N(0, \tau)$,
- residual variability: $\epsilon \sim N(0, \sigma_\epsilon)$,
- 2-level regression: $\mu = f(A, M)$

```
Mixed-Effects Model (k = 100; tau^2 estimator: REML)

logLik deviance      AIC      BIC      AICC
-231.2985  462.5970  470.5970  480.8958  471.0317

tau^2 (estimated amount of residual heterogeneity): 5.8975
tau (square root of estimated tau^2 value):        2.4285
I^2 (residual heterogeneity / unaccounted variability): 85.50%
H^2 (unaccounted variability / sampling variability): 6.90
R^2 (amount of heterogeneity accounted for):       15.75%

Test for Residual Heterogeneity:
QE(df = 97) = 669.0606, p-val < .0001

Test of Moderators (coefficients 2:3):
F(df1 = 2, df2 = 97) = 8.9110, p-val = 0.0003

Model Results:

estimate      se      tval    df   pval    ci.lb    ci.ub
intcpt  0.2263  0.2629  0.8609  97  0.3914  -0.2955  0.7481
A       -1.2958  0.4261 -3.0414  97  0.0030  -2.1414 -0.4502
M       -0.1516  0.2747 -0.5518  97  0.5823  -0.6967  0.3936
```

3. Example cases

Descendant: measurement error, predictor only

Measurement error, predictor only²⁷

also,

- residual confounding

research question,

- Does M has a (direct) effect on D?

variables,

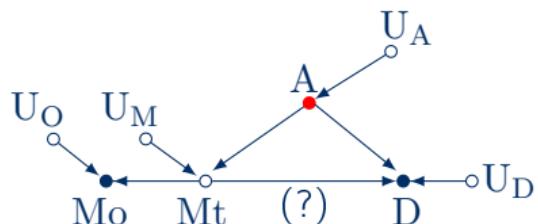
- A, median age at marriage
- Mt, (true) marriage rate
- Mo, (observed) marriage rate
- D, divorce rate

extras,

- U_M, between variability (true rate)
- U_O, measurement error

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ Mt \leftarrow f_M(A, U_M) \\ Mo \leftarrow f_M(Mt, U_O) \\ D \leftarrow f_D(A, Mt, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram



Simulation setting

```
# sim
A = rnorm( 100 )
M_true = rnorm( 100 , -1*A )
M_obs = rnorm( 100 , mean=M_true, sd=1 )
D = rnorm( 100 , -1*A + 1*M_true)
d = data.frame(A, M_true, M_obs, D)
```

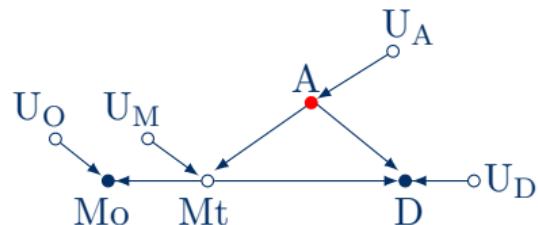
(c) R code

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ Mt \leftarrow f_M(A, U_M) \\ Mo \leftarrow f_M(Mt, U_O) \\ D \leftarrow f_D(A, Mt, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

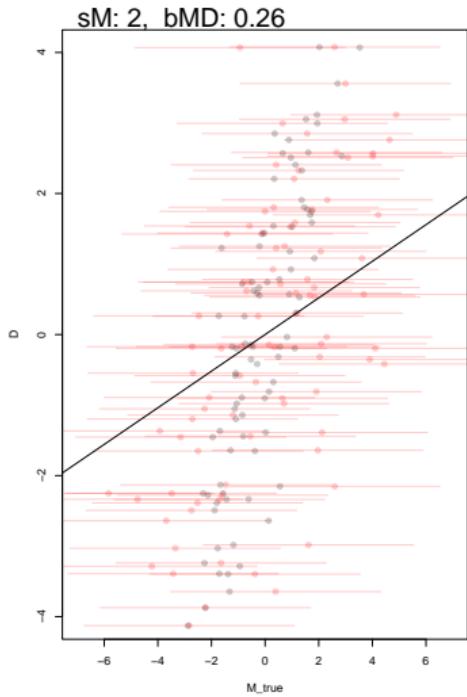
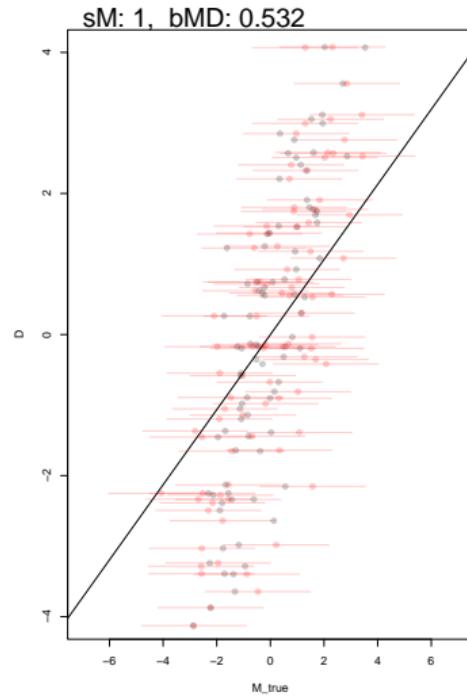
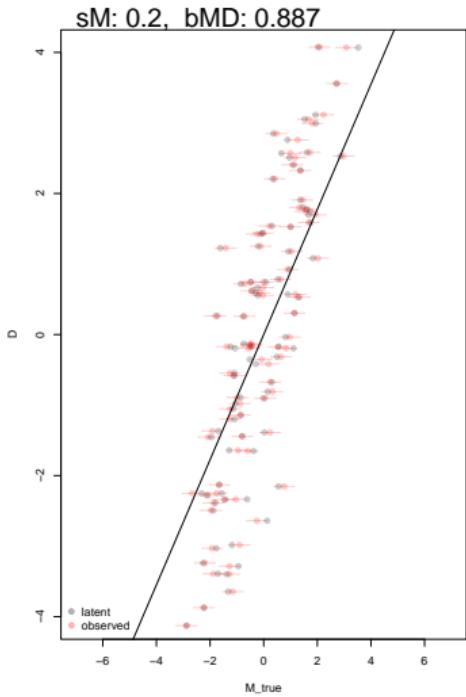
Implications,

- $Mt \not\perp\!\!\!\perp D$
- $Mt \not\perp\!\!\!\perp D \mid A$
(not possible)
- $Mo \not\perp\!\!\!\perp D \mid A$
(partially)

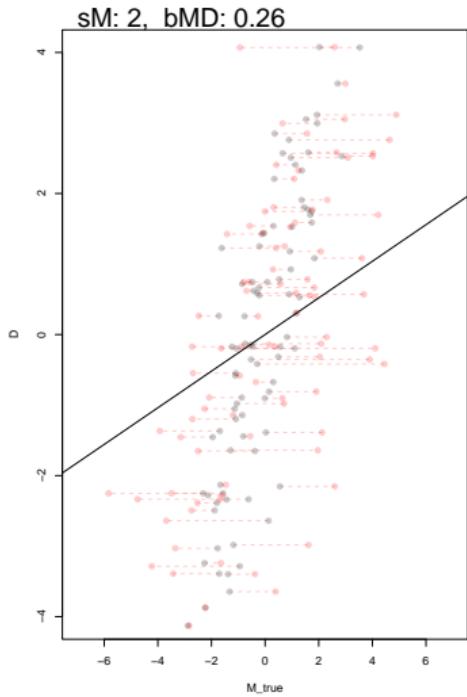
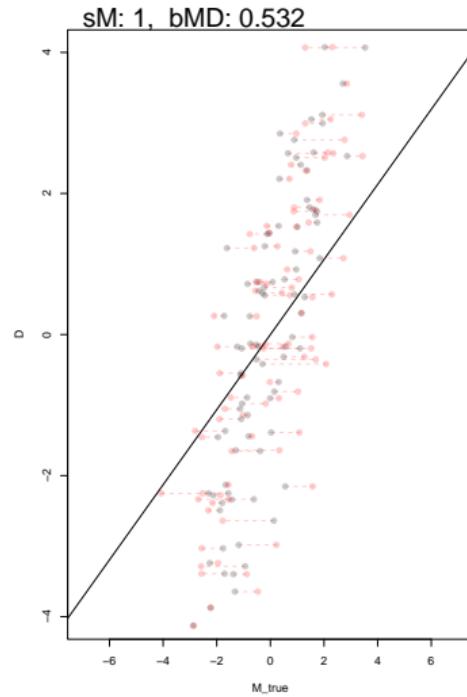
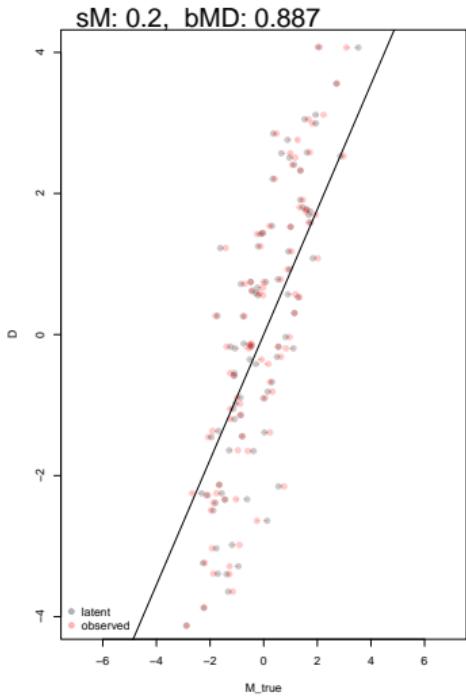


(b) causal diagram

So, what does the simulation means?



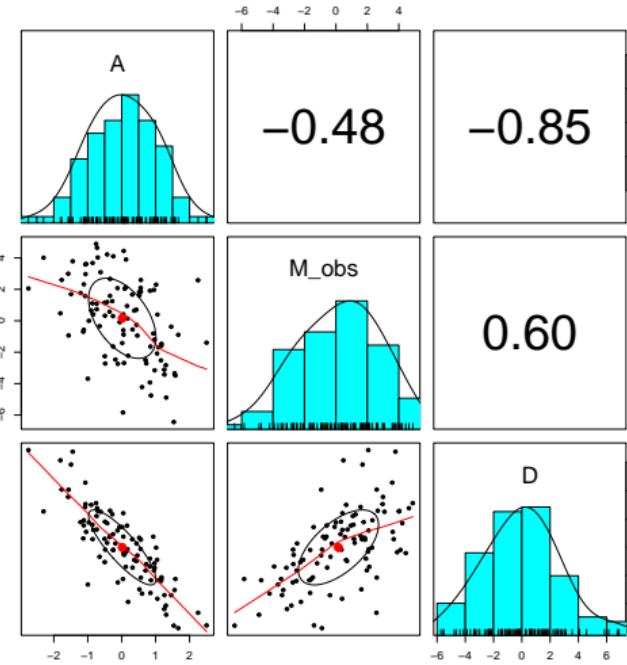
So, what does the simulation means?



“Eyeballing” analysis

based on correlation analysis,

- $\text{cor}(A, D) < 0$ and $\text{cor}(M_{\text{obs}}, D) > 0$ goes in line of our “rudimentary” understanding of the data.
- we include M as a covariate in our statistical model
(besides, it is our research hypothesis)



Regression, regression!!

based on statistical analysis,

- first model is impossible (unobservable)
- huge effect bias in observable data (M and A)
(effect of measurement error, A has less error in its measures)
- we have smaller standard error
(not only changes the hypothesis, but also makes you believe you have more precision)

```
> summary(lm(D ~ A + M_true, data=d))

Call:
lm(formula = D ~ A + M_true, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.9855 -0.7037  0.1587  0.8698  2.6449 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.05749   0.11354   0.506   0.614    
A           -1.17654   0.18400  -6.394  5.66e-09 ***  
M_true      0.89451   0.11862   7.541  2.51e-11 ***  
> summary(lm(D ~ A + M_obs, data=d))

Call:
lm(formula = D ~ A + M_obs, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.5852 -0.8332  0.0857  0.9001  3.0001 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.02454   0.13060  -0.188   0.851    
A           -1.93761   0.15041 -12.883 < 2e-16 ***  
M_obs       0.26051   0.05852   4.452  2.28e-05 ***
```

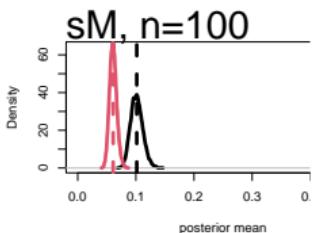
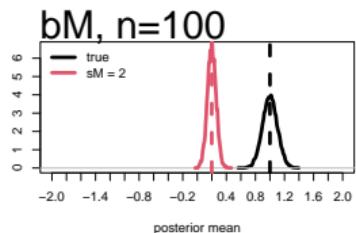
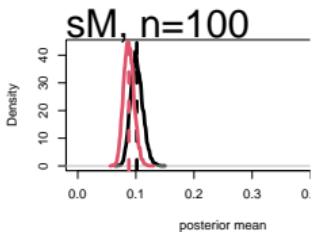
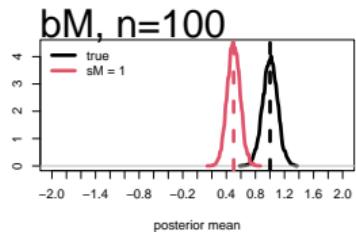
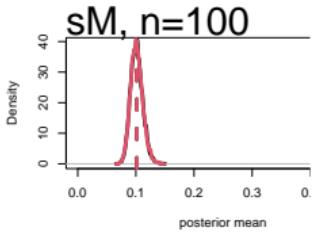
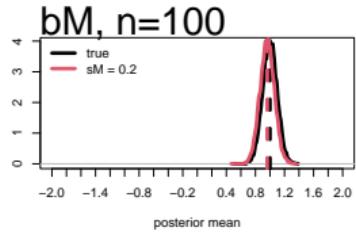
So, what is going on?

imagine we can continue sampling,

- 10,000 samples $n = 100$

notice,

- the larger the measurement error, the more confident you are of your estimates,
- larger measurement error (in predictor) hugely bias the effects, (also for A, not shown)



How can I fix it???

$$D \sim N(\mu, \sigma_D)$$

$$\mu = \alpha + \beta_{AD}A + \beta_{MD}M_{\text{true}}$$

$$M_{\text{obs}} \sim N(M_{\text{true}}, \sigma_O)$$

$$M_{\text{true}} \sim N(0, \sigma_M)$$

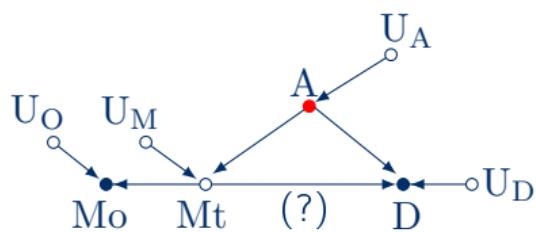
(c) probabilistic model

based on DAG and statistical model,
use the knowledge of the system

- measurement error part:
 $M_{\text{obs}} \sim N(M_{\text{true}}, \sigma_O)$,
- latent structure: $M_{\text{true}} \sim N(0, \sigma_M)$,
- latent regression: $\mu = f(A, M_{\text{true}})$

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ Mt \leftarrow f_M(A, U_M) \\ Mo \leftarrow f_M(Mt, U_O) \\ D \leftarrow f_D(A, Mt, U_D) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

How can I fix it???

```
me_model = ulam(  
  alist(  
    D ~ dnorm( mu , sigma ),  
    mu <- a + bA*A + bM*M_true[i],  
    M_obs ~ dnorm( M_true , M_sd ),  
    vector[N]:M_true ~ dnorm( 0 , sigma ),  
    a ~ dnorm(0,0.2),  
    bA ~ dnorm(0,0.5),  
    bM ~ dnorm(0,0.5),  
    sigma ~ dexp(1)),  
  data=dlist , chains=4 , cores=4 )
```

	mean	sd	5.5%	94.5%
a	-0.02	0.13	-0.22	0.19
bA	-1.71	0.19	-2.02	-1.41
bM	0.36	0.10	0.20	0.51
sigma	1.76	0.11	1.59	1.95

Using DAG and Bayesian analysis:
(frequentist analysis falls short here)

- measurement error part:
 $M_{obs} \sim N(M_{true}, \sigma_O)$,
- latent structure: $M_{true} \sim N(0, \sigma_M)$,
- latent regression: $\mu = f(A, M_{true})$

3. Example cases

Descendant: measurement error, both

Measurement error, both

variables^a,

- A, median age at marriage
- Mt, (true) marriage rate
- Mo, (observed) marriage rate
- Dt, (true) divorce rate
- Do, (observed) divorce rate

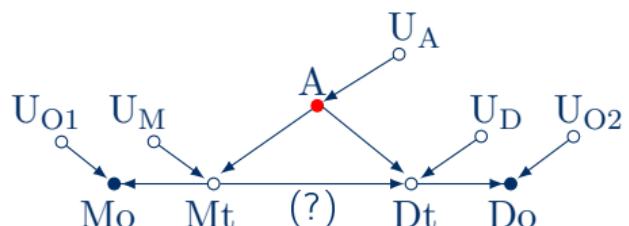
extras,

- U_M , between variability (true rate)
- U_D , between variability (true rate)
- U_{O1} , measurement error
- U_{O2} , measurement error

^aMcElreath [12], chapter 15 (p. 495)

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ Mt \leftarrow f_M(A, U_M) \\ Mo \leftarrow f_M(Mt, U_O) \\ Dt \leftarrow f_D(A, M, U_D) \\ Do \leftarrow f_D(Dt, U_O) \\ U \sim P(U) \end{cases}$$

(a) structural model



(b) causal diagram

Simulation setting

```
# sim  
A = rnorm( 100 )  
M_true = rnorm( 100 , -1*A )  
M_obs = rnorm( 100 , mean=M_true, sd=1 )  
D_true = rnorm( 100 , -1*A + 1*M_true )  
D_obs = rnorm( 100 , mean=D_true, sd=1 )  
d = data.frame(A, M_true, M_obs, D_true, D_obs)
```

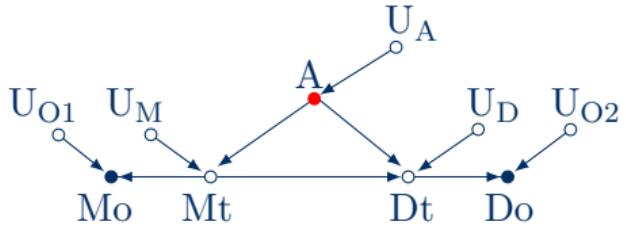
(c) R code

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ Mt \leftarrow f_M(A, U_M) \\ Mo \leftarrow f_M(Mt, U_O) \\ Dt \leftarrow f_D(A, M, U_D) \\ Do \leftarrow f_D(Dt, U_O) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model

Implications,

- $Mt \not\perp\!\!\!\perp D$
- $Mt \not\perp\!\!\!\perp D \mid A$
(not possible)
- $Mo \not\perp\!\!\!\perp D \mid A$
(partially)



(b) causal diagram

Regression, regression!!

based on statistical analysis,

- first model is impossible (unobservable)
- huge effect bias in observable data (M and A)
(effect of measurement error, A has less error in its measures)
- we have smaller standard error
(not only changes the hypothesis, but also makes you believe you have more precision)

```
> summary(lm(D_true ~ A + M_true, data=d))

Call:
lm(formula = D_true ~ A + M_true, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.11359 -0.45535  0.05458  0.56348  2.06299 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.13861   0.10146   1.366   0.175    
A           -1.01718   0.12996  -7.827 6.26e-12 ***  
M_true      0.97700   0.09338  10.462 < 2e-16 ***  
> summary(lm(D_obs ~ A + M_obs, data=d))

Call:
lm(formula = D_obs ~ A + M_obs, data = d)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.0935 -1.0855 -0.2568  1.1535  4.5252 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.1779    0.1737   1.024  0.308355  
A           -1.4942    0.1942  -7.695 1.18e-11 ***  
M_obs       0.4387    0.1132   3.875  0.000194 ***
```

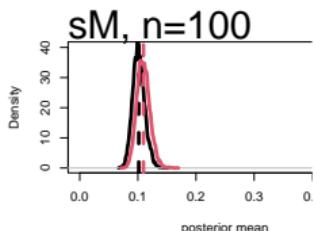
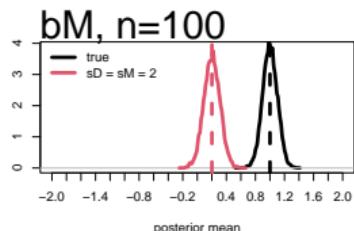
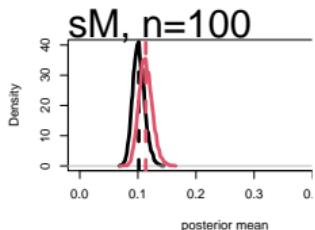
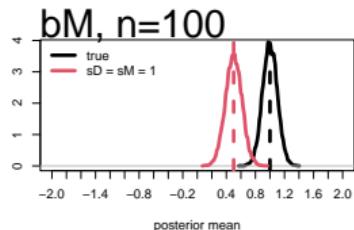
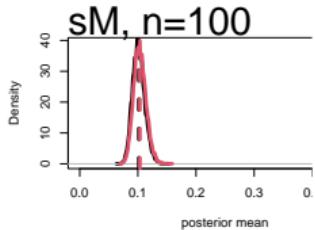
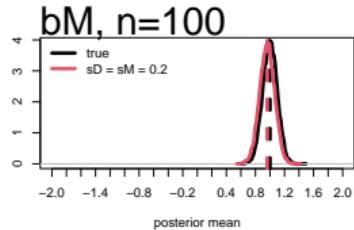
So, what is going on?

imagine we can continue sampling,

- 10,000 samples $n = 100$

notice,

- now it is unclear the direction our precision (standard error) would go with larger measurement error,
- however, the larger measurement error (in predictor) hugely bias the effects,
(also for A, not shown)



How can I fix it???

$$D_{\text{obs}} \sim N(D_{\text{true}}, \sigma_{O1})$$

$$D_{\text{true}} \sim N(\mu, \sigma_D)$$

$$\mu = \alpha + \beta_{AD} A + \beta_{MD} M_{\text{true}}$$

$$M_{\text{obs}} \sim N(M_{\text{true}}, \sigma_{O2})$$

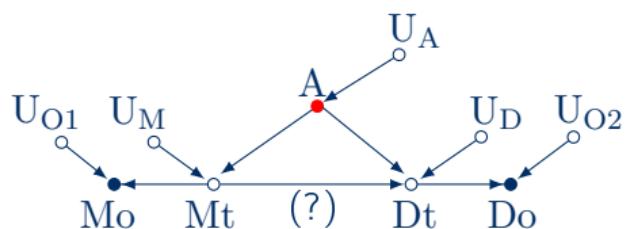
$$M_{\text{true}} \sim N(0, \sigma_M)$$

(c) probabilistic model

based on DAG and statistical model,
(use the knowledge of the system).

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ Mt \leftarrow f_M(A, U_M) \\ Mo \leftarrow f_M(Mt, U_O) \\ Dt \leftarrow f_D(A, M, U_D) \\ Do \leftarrow f_D(Dt, U_O) \\ U \sim P(\mathbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

How can I fix it???

```
me_model = ulam(  
alist(  
  D_obs ~ dnorm( D_true , D_sd ),  
  vector[N]:D_true ~ dnorm( mu , sigma ),  
  mu <- a + bA*A + bM*M_true[i],  
  M_obs ~ dnorm( M_true , M_sd ),  
  vector[N]:M_true ~ dnorm( 0 , 1 ),  
  a ~ dnorm(0,0.2),  
  bA ~ dnorm(0,0.5),  
  bM ~ dnorm(0,0.5),  
  sigma ~ dexp( 1 ) ),  
  data=d , chains=4 , cores=4 )
```

	mean	sd	5.5%	94.5%
a	0.12	0.12	-0.07	0.31
bA	-1.29	0.15	-1.52	-1.04
bM	1.31	0.15	1.06	1.55
sigma	0.39	0.30	0.00	0.88

Using DAG and Bayesian analysis:
(frequentist analysis falls short here)

4. Concluding remarks

Are there other ways to solve previous problems?²⁸

many other (much more complex) ways,

- sensitivity analysis
- post-stratification?
- matching?
- inverse-probability weighting?
- g-formula?
- g-estimations?

²⁸most also apply for previous examples, but are beyond the scope of this document (for now)

Concluding remarks

- research is filled with challenges
(you: duh!!)
- statistical models are not theory
(you: so obvious again!!)
- don't trust your statistical model
when no DAG is involved
(me: how about that?!)
- for explanation, without a DAG
the (sample) size does not matter
(me: sorry?!)
- even a good experimental design
can lead you to wrong conclusions
if no DAG is involved
(me: did you expect this one?!)



Concluding remarks

Additionally, simulation can serve as,

- a reflection of a hypothesis, and its research complexities
(me: with DAGs baby!!)
- a place where you can reflect the status of a population
(test “what happens if”)
- a data where you can test your statistical model
(parameter recovery, power)
- a tool for planning (all kinds of) experiments.



5. Do you wanna know more???

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