

or commonly know as:

"a bit more transparent way to state
your research assumptions and questions'

Jose Rivera

April 29, 2022

## What are we going to talk about? I

- 1 About research
  - A typical scientific lab
  - Research hypothesis production
- 2 DAGs and PP
- 3 Anecdotal cases
  - Experimental design: the panacea
  - Simulation conventions
  - Fork bias: spurious relationships
  - Fork bias: masked relationships (a)
  - Fork bias: masked relationships (b)
  - Fork bias: multicollinearity



## What are we going to talk about? II

- lacktriangle Fork bias: bias amplification
- No more fork bias: neutral control
- Pipe bias: precision parasite
- Pipe bias: post-treatment
- Pipe bias: masked relationships

4 Concluding remarks

5 Do you wanna know more???



1. About research

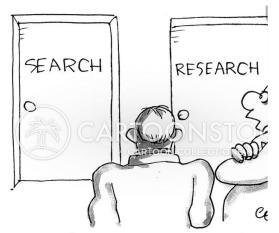
A typical scientific lab



# A typical scientific lab<sup>1</sup>

#### What is needed?

- 1. Quality of theory
- 2. Quality of data
- 3. Reliable procedures and code
- 4. Quality of data analysis
- 5. Documentation
- 6. Reporting



"DO WE NEED BOTH ?"

<sup>&</sup>lt;sup>1</sup>McElreath [12], lecture 20 and McElreath [13], chapter 17



# A typical scientific lab

What we "normally" focus on?

- 1. Quality of theory
- 2. Quality of data
- 3. Reliable procedures and code
- 4. Quality of data analysis
- 5. Documentation
- 6. Reporting





# A typical scientific lab

What can be improved? (with DAGs and PP)

- 1. Quality of theory
- 2. Quality of data
- 3. Reliable procedures and code
- 4. Quality of data analysis
- 5. Documentation
- 6. Reporting





#### 1. About research

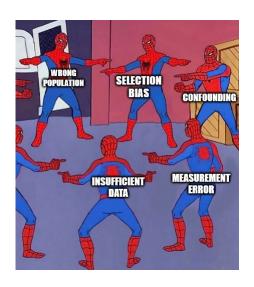
Research hypothesis production



# Research hypothesis production

#### Well known challenges<sup>a</sup>

- Insufficient data
- Wrong population
- Measurement error
- Selection bias
- Confounding



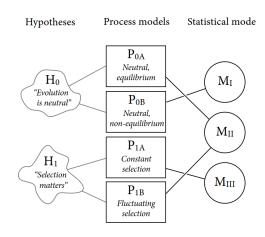


<sup>&</sup>lt;sup>a</sup>Hernán [8], lesson 4

# Research hypothesis production

#### but we should not forget<sup>a</sup>

- No one-to-one relationship exists between our process models and statistical models,
- Nor between our hypothesis and a process models





<sup>&</sup>lt;sup>a</sup>Figure 1.2 reproduced from chapter 1 McElreath [13]

# Research hypothesis production

#### and also

statistical models are just
 "machines to find association", not
 a reliable reflection of the theory
 (I can prove it!!).



# Research hypothesis schematics<sup>2</sup>

- a. Estimand and process model
- b. Synthetic data generation
- c. Statistical model design and testing
- d. Apply statistical model to data



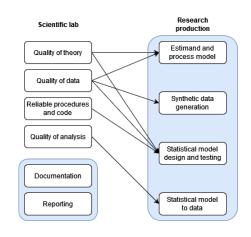
<sup>&</sup>lt;sup>2</sup>McElreath [13], lecture 20, Pearl [16]. Follow Fogarty et al. [6] on item (c).



# Research hypothesis schematic

#### Where does it match with the previous?

- a. Estimand and process model maps 1 (theory) and 2 (data) to a heuristic model.
- b. Synthetic data generation maps 2 (data) to an idealized data.
- c. Statistical model design and testing maps 1 (theory), 2 (data), and 3 (reliable code) to an statistical model.
- d. Apply statistical model to data maps 4 (analysis) onto a result.

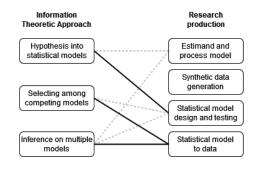




### Where does the ITA fit?

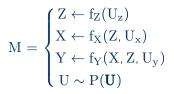
Information Theoretic Approach (ITA) is framework to select among competing models [1, 3]:

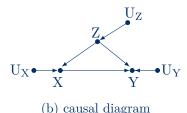
- 1. Hypothesis into statistical models, (how about a process model?)
- 2. Select among competing models, (do the code works as intended?)
- Make inferences based on one or multiple models. (do the code works as intended?, are there variables that can bias our results?)





- Directed acyclic graphs (DAGs), are a type of structural causal model (SCM) [15, 4]
- DAGs can be represented by a structural model, and its associated causal diagram<sup>a</sup>.
- we put distributional assumptions to the structural model through probabilistic programming (PP) [10]. (more in part 3)

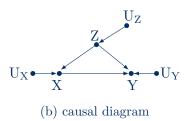




<sup>&</sup>lt;sup>a</sup>reproduced from Cinelli et al. [4].

- $V = \{Z, X, Y\}$  are called endogenous variables.
- **U** = {U<sub>Z</sub>, U<sub>X</sub>, U<sub>Y</sub>} are called exogenous variables.
  (drawn when strictly required)
- $\mathbf{F} = \{f_Z, f_X, f_Y\}$  are called structural equations.

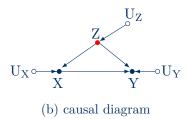
$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\textbf{U}) \end{cases}$$



#### Causal diagram conventions [4],

- black nodes are observed variables.
- white nodes are unobserved variables.
- red nodes are variables for which we will decide its inclusion or not.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\textbf{U}) \end{cases}$$

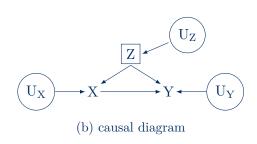




#### Other causal diagram conventions,

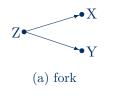
- no circle nodes are observed variables.
- circled nodes are unobserved variables.
- squared nodes are variables for which we will decide its inclusion or not.

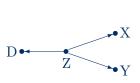
$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\textbf{U}) \end{cases}$$



## The benign case of DAG elementals

For everything can be depicted with them

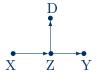




(d) descendant on fork



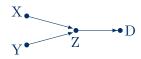
(b) pipe



(e) descendant on pipe



(c) collider



(f) descendant on collider

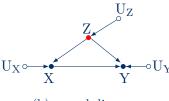
## About D-separation

Causal graph theory [14, 15, 17, 18, 19],

- descendant (child, grandchild), parent (grandparent).
   (path specific)
- 2. paths (directional, non-directional).
- 3. paths are blocked or open according to the D-separation rules.
  (also path specific)
- 4. there are only four (4) D-separation rules.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

## About D-separation

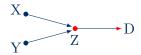
The D-separation (Directional) rules [8],

- If no variables being conditioned on, a path is blocked if and only if, two arrowheads on the path collide at some variable on the path.
- 2. Any path that contains a noncollider that has been conditioned on, is blocked (backdoor path)<sup>a</sup>.
- 3. A collider that has been conditioned on does not block a path.
- 4. A collider that has a descendant that has been conditioned on does not block a path.











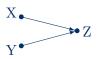
<sup>&</sup>lt;sup>a</sup>there is also a front-door path (if you wonder).

## About D-separation

The D-separation rules implications, (independent of distributional assumptions)

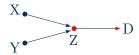
1. 
$$X \perp \!\!\!\perp Y \Longrightarrow P(X, Y) = P(X) \cdot P(Y)$$

- 2.  $X \perp \!\!\!\perp Y \mid Z \Longrightarrow$   $P(X, Y \mid Z) = P(X \mid Z) \cdot P(Y \mid Z)$ (same for fork or pipe)
- 3.  $X \not\perp \!\!\! \perp Y | Z \Longrightarrow P(X, Y|Z) \neq P(X|Z) \cdot P(Y|Z)$
- 4.  $X \not\perp \!\!\! \perp Y \mid D \Longrightarrow P(X, Y \mid D) \neq P(X \mid D) \cdot P(Y \mid D)$





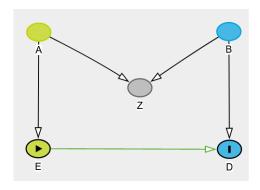






# Oh DAGitty!! mijn vriendin

- browser (R package) environment for creating, editing, and analyzing causal diagrams [20].
- available online: http://dagitty.net
- But there are more fish in the sea: http://www.causalfusion.net [2] (b\*\*\*\* better have my \$\$\$)





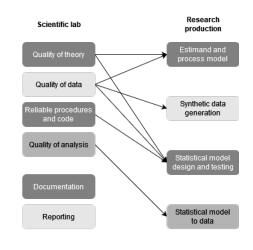
# Where do DAGs and PP fit?

#### starts with:

- A clear definition of the estimand and process model (assumptions).
- An improved the reliability of your procedures.
- As a documentation procedure.

#### and leads to:

- A sound analysis, and result (even when we cannot have an answer to our question)
- An improved planning to get data.





#### 3. Anecdotal cases

Experimental design: the panaces

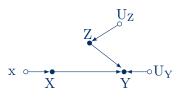


# Experimental design<sup>3</sup>

- Purpose: to control all factors responsible for the outcome's variation.
   (understand the system)
- It is modeled by modifying the structural model (and causal diagram).

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

<sup>&</sup>lt;sup>3</sup>Cinelli et al. [4], appendix A (p. 15)

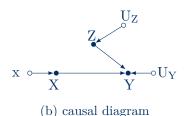


## Experimental design

- intervention on X can be written in do-calculus<sup>a</sup> as:  $P(\mathbf{V} \mid do(X = x))$ .
- remember:

$$\begin{aligned} & \boldsymbol{V} = \{Z, X, Y\}, \\ & \boldsymbol{U} = \{U_Z, U_X, U_Y\}, \text{ and } \\ & \boldsymbol{F} = \{f_Z, f_X, f_Y\}. \end{aligned}$$

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\textbf{U}) \end{cases}$$



<sup>&</sup>lt;sup>a</sup>an appropriate treatment can be found with the usual suspects [14, 15, 17, 18])

### Effects of interest

two types of effects,

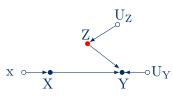
- 1. Average causal effect: ACE(x) = E[Y|do(x+1)] E[Y|do(x)]
- 2. Controlled direct effect: CDE(x, z) = E[Y|do(x + 1), do(z)] - E[Y|do(x), do(z)]

#### points to consider:

- CDE takes a particular relevance with observational data.
- There is also a distinction between total effect and direct effect.

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_x(x) \\ Y \leftarrow f_Y(X, Z, U_Y) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



(b) causal diagram

3. Anecdotal cases

Simulation conventions



### Simulation conventions

one way to defined it,

$$Z = U_Z$$
 ;  $U_Z \sim N(0, \sigma_Z)$ 

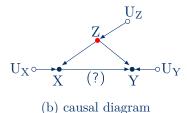
$$X = \beta_Z Z + U_X$$
 ;  $U_X \sim N(0, \sigma_X)$ 

$$Y = \beta_Z Z + \beta_X X + U_Y$$
;  $U_Y \sim N(0, \sigma_Y)$ 

a more succinct way,

$$Z \sim N(0, \sigma_Z)$$
  
 $X \sim N(\beta_Z Z, \sigma_X)$   
 $Y \sim N(\beta_Z Z + \beta_X X, \sigma_Y)$ 

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(Z, X, U_Y) \\ U \sim P(U) \end{cases}$$



3. Anecdotal cases



## Spurious relationships<sup>4</sup>

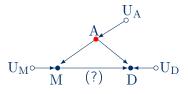
also known as,

- $\blacksquare$  spurious association
- confounder
- $\blacksquare$  an instance of fork bias

research question,

- Does M has a (direct) effect on D? variables,
  - A, median age at marriage
  - M, marriage rate
  - D, divorce rate

(a) structural model



(b) causal diagram

<sup>&</sup>lt;sup>4</sup>McElreath [12], chapter 05 (p. 125)



 $M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, M, U_D) \\ U \sim P(\boldsymbol{U}) \end{cases}$ 

## Simulation setting

```
# sim
A = rnorm( 100 )
M = rnorm( 100 , mean=-1*A )
D = rnorm( 100 , mean=-1*A + 0*M )
d = data.frame(A=A, M=M, D=D)
```

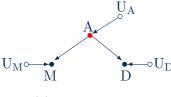
(c) R code

#### Implications,

- M #D
- M ⊥LD | A

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, U_D) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

(a) structural model

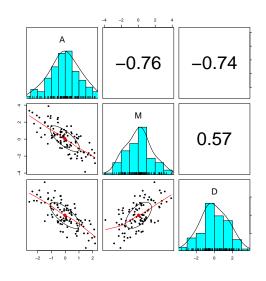


(b) causal diagram

# "Eyeballing" analysis

#### based on correlation analysis,

- cor(A, D) < 0 and cor(M, D) > 0 goes in line of our "rudimentary" understanding of the data.
- why there is cor(M, D) > 0? (hint: univariate correlation)
- we include M as a covariate in our statistical model
   (is our research hypothesis)





## Regression, regression!!

#### based on statistical analysis,

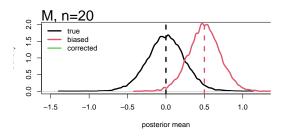
■ two regressions with two different results, which model is the "true"?

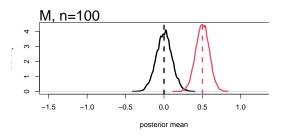
```
summary(lm(D ~ M, data=d)) # spurious relation
lm(formula = D \sim M, data = d)
Residuals:
               10 Median
-2.80012 -0.90447 -0.03866 0.80220 2.82970
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
                        0.12412 -1.877
                        0.08986 4.477 2.04e-05 ***
             0.40233
> summary(lm(D \sim A + M, data=d)) # controlled relation
lm(formula = D \sim A + M, data = d)
Residuals:
              10 Median
-2.27295 -0.68174 0.03781 0.78885 2.95320
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.18854
            -1.03121
            -0.06134
                        0.09362 -0.655
```

## I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples n = 20
- bottom: 10,000 samples n = 100 the larger the sample size,
  - the more certain you are about your estimates
  - the more mistaken you are about your research question (under the "incorrect" model)
     (the winner's curse)







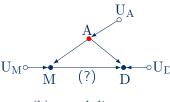
## The dream team!!

### based on DAG and statistical model,

- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path,
  i.e. M ⊥LD | A
- conditioning on A we can find, E[D|do(m)] = E[E[D|M = m, A]] (law of total expectation)
- then we can find the ACE(m) = E[D|do(m+1)] - E[D|do(m)](Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} A \leftarrow f_A(U_A) \\ M \leftarrow f_M(A, U_M) \\ D \leftarrow f_D(A, M, U_D) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

(a) structural model



## the dream team!!

based on DAG and statistical analysis,

■ the less biased model is the second, (assuming our DAG is true)



### 3. Anecdotal cases

Fork bias: masked relationships (a



# Masked relationships $(a)^5$

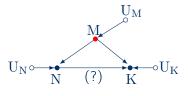
also known as,

- omitted variable bias
- an instance of fork bias

research question,

- Does N has a (direct) effect on K? variables.
  - M, mammal mass in kg.
  - N, ratio neocortex over total brain mass
  - K, Kcal. per gram of milk

(a) structural model



<sup>&</sup>lt;sup>5</sup>McElreath [12], chapter 05 (p. 144)



 $M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$ 

## Simulation setting

```
# sim
M = rnorm( 100 )
N = rnorm( 100 , 1*M )
K = rnorm( 100 , 1*N + -1*M )
d = data.frame(N=N, M=M, K=K)
```

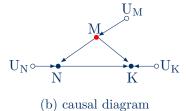
(c) R code

### Implications,

- N #K
- N #K | M

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$$

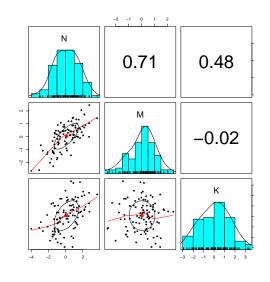
(a) structural model



# "Eyeballing" analysis

### based on correlation analysis,

- cor(N, K) > 0 goes in line of our "rudimentary" understanding of the data.
- but why there is  $cor(M, k) \approx 0$ ? (hint: univariate correlation)
- we might not include M as a covariate in our statistical model





## Regression, regression!!

#### based on statistical analysis,

■ two regressions with two different results, which model is the "true"?

```
summarv(lm(K \sim N. data=d)) # biased estimate
Call:
lm(formula = K \sim N, data = d)
Residuals:
    Min
             10 Median
-2.8355 -0.8110 0.0188 0.7897 3.4276
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.01401
                        0.09332 5.680 1.38e-07 ***
             0.53002
 summary(lm(K \sim N + M, data=d)) # less biased estima
Call:
lm(formula = K \sim N + M, data = d)
Residuals:
              10 Median
-2.50873 -0.72626 -0.01968 0.69016 2.93000
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.22096
                        0.09845
             0.95510
                        0.10089
                                  9.466 1.91e-15
            -1.06246
                        0.15462 -6.871 6.14e-10 ***
```

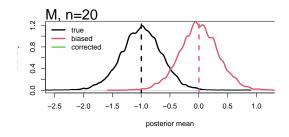
## I'll get more data!!

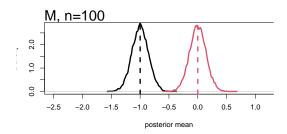
imagine we can continue sampling,

- $\bullet$  top: 10,000 samples n = 20
- $\blacksquare$  bottom: 10,000 samples n = 100

the larger the sample size,

- the more certain you are about your estimates
- the more mistaken you are about your research question (under the "incorrect" model)







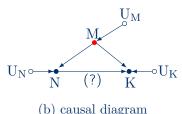
## The dream team!!

### based on DAG and statistical model,

- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path, i.e. N 业K | M
- conditioning on M we can find, E[K|do(n)] = E[E[K|N = n, M]] (law of total expectation)
- then we can find the ACE(n) = E[D|do(n+1)] E[D|do(n)] (Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} M \leftarrow f_M(U_M) \\ N \leftarrow f_N(M, U_N) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model





## the dream team!!

based on DAG and statistical analysis,

■ the less biased model is the second, (assuming our DAG is true)

## 3. Anecdotal cases



# Masked relationships $(b)^6$

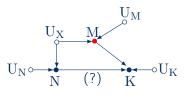
also known as,

- (unobserved) omitted variable bias
- an instance of fork bias

research question,

- Does N has a (direct) effect on K? variables,
  - $\blacksquare$  U<sub>X</sub>, unobservable (e.g. genetics)
  - M, mammal mass in kg.
  - N, neocortex over total brain mass
  - K, Kcal. per gram of milk

(a) structural model



<sup>&</sup>lt;sup>6</sup>McElreath [12], chapter 05 (p. 144)



 $M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$ 

## Simulation setting

```
# sim
U = rnorm( 100 )
N = rnorm( 100 , 1*U )
M = rnorm( 100 , 1*U )
K = rnorm( 100 , 1*N + -1*M )
d = data.frame(U=U,N=N,M=M,K=K)
```

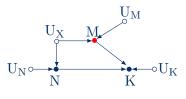
(c) R code

### Implications,

- N **¼**K
- N #K | M

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$$

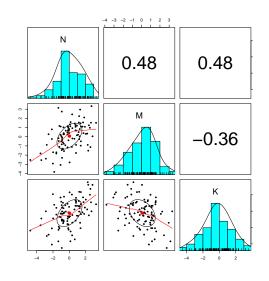
(a) structural model



# "Eyeballing" analysis

### based on correlation analysis,

- cor(N, K) > 0 goes in line of our "rudimentary" understanding of the data.
- cor(M, K) < 0 does NOT goes in line of our "rudimentary" understanding of the data. (hint: univariate correlation)
- we include M as a covariate in our statistical model (by chance?)



## Regression, regression!!

#### based on statistical analysis,

• two regressions with two different results, which model is the "true"?

```
summarv(lm(K \sim N. data=d)) # unobserved path still
Call:
lm(formula = K \sim N. data = d)
Residuals:
             10 Median
 -3.7763 -0.8480 0.1497 0.9874 3.3530
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.24867
                        0.14573 -1.706 0.0911
                        0.09502 5.410 4.46e-07 ***
             0.51406
 summary(lm(K \sim N + M, data=d)) # unobserved path c
Call:
lm(formula = K \sim N + M, data = d)
Residuals:
     Min
               1Q Median
-2.58218 -0.58434 -0.00579 0.72016 1.78724
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.19978
             0.90893
                        0.06958 13.064
                                           <2e-16 ***
            -0.89676
                        0.07572 -11.843
                                           <2e-16 ***
```

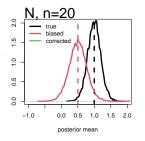
## I'll get more data!!

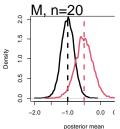
imagine we can continue sampling,

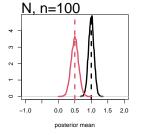
- $\blacksquare$  top: 10,000 samples n = 20
- $\blacksquare$  bottom: 10,000 samples n = 100

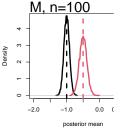
the larger the sample size,

- the more certain you are about your estimates
- the more mistaken you are about your research question (under the "incorrect" model)









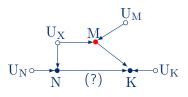
## The dream team!!

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,
   i.e. N ⊥K | U<sub>X</sub>
   (but it is unobservable)
- still we use the 2nd D-separation rule by controlling for M,
   i.e. N ⊥K | M
- conditioning on M we can still find, E[K|do(n)] = E[E[K|N = n, M]](law of total expectation)
- then we can find the ACE(n) = E[D|do(n+1)] E[D|do(n)] (Frisch-Waugh-Lovell theorem??)

$$M = \begin{cases} N \leftarrow f_N(U_N, U_X) \\ M \leftarrow f_M(U_M, U_X) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



## the dream team!!

based on DAG and statistical analysis,

■ the less biased model is the second, (assuming our DAG is true)

## Similar scenario<sup>7</sup>

### research question,

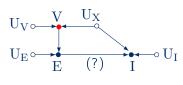
■ Does E has a (direct) effect on I?

## variables,

- U<sub>X</sub>, unobservable (e.g. family context)
- V, personal values
- E, education
- I, income

$$M = \begin{cases} V \leftarrow f_M(U_V, U_X) \\ E \leftarrow f_E(V, U_E) \\ I \leftarrow f_I(E, U_X, U_I) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



<sup>&</sup>lt;sup>7</sup>Cinelli et al. [4] (p. 3), McElreath [13], lecture 6



3. Anecdotal cases



# Multicollinearity<sup>8</sup>

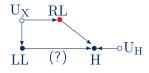
also known as,

- special case of masked relationships
- an instance of fork bias

research question,

- Should we include RL in our model? variables,
  - U<sub>X</sub>, unobservable (e.g. genetics)
  - LL, individual's left leg
  - RL, individual's right leg
  - H, individual's height

(a) structural model



<sup>&</sup>lt;sup>8</sup>McElreath [12], chapter 06 (p. 163)



 $M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(LL, U_H) \\ U \sim P(\textbf{U}) \end{cases}$ 

## Simulation setting

```
# backward simulation
H = round( rnorm( 100 , 170, 2), 1)
Lp = runif( 100 , 0.5-0.05, 0.5+0.05)
LL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
RL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
d = data.frame(LL,RL,H)
```

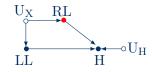
(c) R code

### Implications,

- N #K
- N #K | M

$$M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(LL, U_H) \\ U \sim P(U) \end{cases}$$

(a) structural model



# "Eyeballing" analysis

### based on correlation analysis,

- cor(N, K) > 0 goes in line of our "rudimentary" understanding of the data.
- cor(M, K) < 0 does NOT goes in line of our "rudimentary" understanding of the data. (hint: univariate correlation)
- we include M as a covariate in our statistical model (by chance?)

fork5\_panel.pdf

# Regression, regression!!

based on statistical analysis,

■ two regressions with two different results, which model is the "true"?

 $fork5\_reg.png$ 

## I'll get more data!!

imagine we can continue sampling,

- $\bullet$  top: 10,000 samples n = 20
- $\blacksquare$  bottom: 10,000 samples n = 100

the larger the sample size,

- the more certain you are about your estimates
- the more mistaken you are about your research question (under the "incorrect" model)

fork5\_samplesize.pd



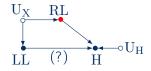
## The dream team!!

based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,
   i.e. N ⊥K | U<sub>X</sub>
   (but it is unobservable)
- still we use the 2nd D-separation rule by controlling for M,
  i.e. N ⊥K | M
- conditioning on M we can still find, E[K|do(n)] = E[E[K|N = n, M]](law of total expectation)
- then we can find the ACE(n) = E[D|do(n+1)] E[D|do(n)] (Frisch-Waugh-Lovell theorem??)

$$M = \begin{cases} LL \leftarrow f_L(U_X) \\ RL \leftarrow f_L(U_X) \\ H \leftarrow f_K(LL, U_H) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



## the dream team!!

based on DAG and statistical analysis,

■ the less biased model is the second, (assuming our DAG is true)

fork5\_reg2.png

3. Anecdotal cases



# Bias amplification<sup>9</sup>

also known as,

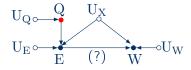
- (unobserved) omitted variable bias
- related to instrumental variables
- an instance of fork bias

research question,

- Does E has a (direct) effect on W? variables,
  - Q, instrumental variable (e.g. quarter of the year)
  - E, educational level
  - $\blacksquare$  U<sub>X</sub>, unobservables (e.g. ability)
  - W, future wages

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



<sup>&</sup>lt;sup>9</sup>McElreath [12], chapter 14 (p. 455), Cinelli et al. [4] (p. 5)

## Simulation setting

```
# sim
U = rnorm( 100 )
Q = sample( 1:4, 100, replace=T )
E = rnorm( 100 , 1*Q + 1*U )
W = rnorm( 100 , 0*E + 1*U )
d = data.frame(U=U,Q=Q,E=E,W=W)
```

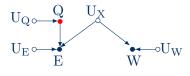
(c) R code

### Implications,

- E #W
- $\blacksquare$  E  $\bot\!\!\!\bot W \mid U_X \text{ (impossible)}$
- $\blacksquare$  Q  $\perp \!\!\! \perp U_X$  (cannot be tested)
- Q ⊥/E
- Q ⊥LW | E (cannot be tested) (exclusion restriction)

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(U_X, U_W) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model

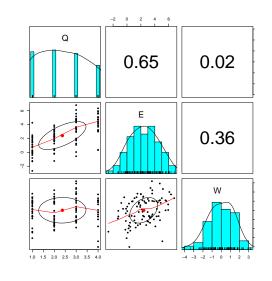


# "Eyeballing" analysis

#### based on correlation analysis,

- cor(Q, E) > 0 and cor(E, W) > 0 goes in line of our "rudimentary" understanding of the data.
- cor(Q, W) > 0 tells you about the exclusion restriction?

  (hint: No)
- we might NOT include Q as a covariate in our statistical model (but is the instrumental variable!!!)





## Regression, regression!!

### based on statistical analysis,

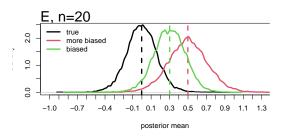
- two regressions with two different results, which model is the "true"?
- one is "worse"/"better" than the other

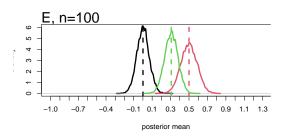
```
summary(lm(W ~ E, data=d)) # biased
lm(formula = W \sim E, data = d)
Residuals:
    Min
             10 Median
-4.0726 -0.9674 0.1771 0.9234 2.8787
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.60816
                       0.20506 -2.966 0.003793 **
             0.25408
                       0.06559 3.873 0.000194 ***
 summary(lm(W \sim E + Q, data=d)) # more biased
call.
lm(formula = W \sim E + 0. data = d)
Residuals:
             10 Median
-3.7405 -0.9774 0.0879 0.9162 2.9825
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.12229
                        0.30650
                                 0.399 0.69078
             0.42054
                        0.08262
                                 5.090 1.75e-06 ***
            -0.47716
                        0.15361 -3.106 0.00249 **
```

## I'll get more data!!

imagine we can continue sampling,

- top: 10,000 samples n = 20
- bottom: 10,000 samples n = 100 the larger the sample size,
  - the more certain you are about your estimates
  - the more mistaken you are about your research question (under the any model!!)(the winner's curse)







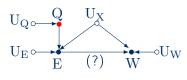
## Yo, what is going on??

### based on DAG and statistical model,

- the 2nd D-separation rule requires control on any noncollider to block the backdoor path,
   i.e. E ⊥⊥W | U<sub>X</sub>
   (but it is impossible)
- if we use Q in the model, the 3rd D-separation rule kicks in:
  "A collider that has been conditioned on does not block a path."
  i.e. Q ⊥ U<sub>X</sub> | E
  (e.g. switch, electricity, and light bulb)

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

(a) structural model



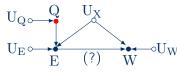
## Yo, what is going on??

### open paths?:

- $\blacksquare$  E  $\rightarrow$  W
- $\blacksquare$  E  $\rightarrow$  U<sub>x</sub>  $\rightarrow$  W
- $\blacksquare E \to U_x \to Q \to E \to W$
- $\blacksquare$  E  $\to$  U<sub>x</sub>  $\to$  Q  $\to$  E  $\to$  U<sub>X</sub>  $\to$  W

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



### What should I do then??

$$\begin{pmatrix} \mathbf{W} \\ \mathbf{E} \end{pmatrix} \sim \mathbf{MVN} \begin{bmatrix} \begin{pmatrix} \mu_{\mathbf{W}} \\ \mu_{\mathbf{E}} \end{pmatrix}, \mathbf{\Sigma} \end{bmatrix}$$

$$\mu_{\mathbf{W}} = \alpha_{\mathbf{W}} + \beta_{\mathbf{EW}} \mathbf{E}$$

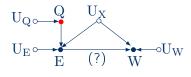
$$\mu_{\mathbf{E}} = \alpha_{\mathbf{E}} + \beta_{\mathbf{QE}} \mathbf{E}$$
(c) probabilistic model

based on DAG and statistical model, use the knowledge of the system

- $\blacksquare$  one model for  $Q \to E$
- $\blacksquare$  one model for  $E \to W$
- use the knowledge that cov(E, W) > 0 due to unobserved confounder U<sub>X</sub>, (i.e. cov(E, W) = Σ = SRS)

$$M = \begin{cases} Q \leftarrow f_Q(U_Q) \\ E \leftarrow f_E(Q, U_X, U_E) \\ W \leftarrow f_W(E, U_X, U_W) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

(a) structural model



### did it worked???

based on DAG and bayesian statistical analysis,

- appropriate value estimated, (assuming our DAG is true)
- it picks up the unobserved correlation R[1, 2]

```
mean
        0.02 \ 0.18 \ -0.26
       -0.14 0.16 -0.40
        1.00 0.07
boe
bEW
        0.05 0.07 -0.06
R[1,1]
                   1.00
R[1,2]
                    0.15
                          0.50
                    0.15
R[2,1]
                          0.50
        1.25 0.10
                    1.11
        1.39 0.10
                   1.24
```



### did it worked???

#### frequentists guys apply Two Stage Least Squares (2SLS)<sup>a</sup>:

- $\blacksquare$  regress  $Q \to E$ , predict  $\hat{E}$
- $\blacksquare$  regress  $\hat{E} \to W$

```
s1 = lm( E ~ Q, data=d)
Ehat = s1$fitted.values
s2 = lm( W ~ Ehat, data=d)
# se not corrected

require(AER)
tsls = ivreg( W ~ E | Q, data=d)
summary(tsls)
# se corrected
```

<sup>a</sup>Hanck et al. [7], section 12.1, See McElreath [12] chapter 14 (p. 460) for a discussion on the method. fork6\_reg4.png

### 3. Anecdotal cases

No more fork bias: neutral contro



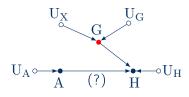
### Neutral control<sup>10</sup>

also known as,

- precision "booster"
- similar to experimental design research question,
- Does N has a (direct) effect on K? variables,
  - A, "hearing" age
  - G, gender
  - $\blacksquare$  U<sub>X</sub>, unobservable (e.g. no idea yet)
  - H, inverse logit of entropy (approximate of speech intelligibility)

$$M = \begin{cases} G \leftarrow f_G(U_G, U_X) \\ A \leftarrow f_A(U_A) \\ H \leftarrow f_H(A, G, U_H) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



<sup>&</sup>lt;sup>10</sup>Cinelli et al. [4] (p. 4)

## Simulation setting

```
# sim
G = sample( 0:1, 100 , replace=T )
A = rnorm( 100 )
H = rnorm( 100 , -1*A + -1*G )
d = data.frame(G=G,A=A,SI=SI)
```

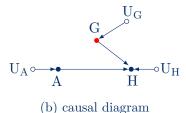
(c) R code

#### Implications,

- A ⊥LG
- A #H
- G #H

$$M = \begin{cases} G \leftarrow f_G(U_G, U_X) \\ A \leftarrow f_A(U_A) \\ H \leftarrow f_H(A, G, U_H) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

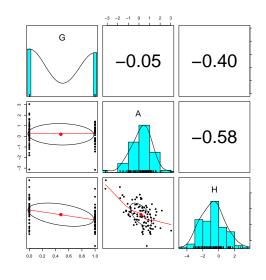
(a) structural model



## "Eyeballing" analysis

#### based on correlation analysis,

- $cor(G, H) \approx 0$ ,  $cor(G, A) \approx 0$  and cor(A, H) < 0 goes in line of our "rudimentary" understanding of the data.
- we include both as a covariate in our statistical model





## Regression, regression!!

#### based on statistical analysis,

- now there is no severe biasing
- notice the standard errors, lower for A when G is included

```
summary(lm(H \sim A, data=d)) # correct estimate
Call:
lm(formula = H \sim A, data = d)
Residuals:
             10 Median
-3.4714 -0.8797 -0.0633 0.8963 2.4346
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.5770
                         0.1216 -4.746 7.07e-06 ***
             -0 8410
                         0.1183 -7.108 1.92e-10 ***
 summary(lm(H ~ A + G, data=d)) # correct estimate,
Call:
lm(formula = H \sim A + G, data = d)
Residuals:
    Min
             10 Median
-2.7994 -0.6914 0.0579 0.7796 1.8274
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
            -0.87360
                        0.10090 -8.658 1.05e-13
            -1.25786
                        0.20371 -6.175 1.55e-08
```

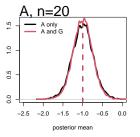
### I'll get more data!!

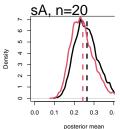
imagine we can continue sampling,

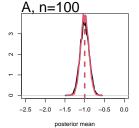
- $\blacksquare$  top: 10,000 samples n = 20
- $\blacksquare$  bottom: 10,000 samples n = 100

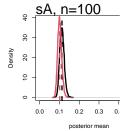
the larger the sample size,

- the (more) certain you are about your estimates
- the more correct you are about your research question (under the any model)









### 3. Anecdotal cases



## Precision parasite<sup>11</sup>

research question,

- Should we include Z in the model?
- variables,
  - **■** Z,
  - X,
  - Y,

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

(a) structural model

<sup>&</sup>lt;sup>11</sup>McElreath [13], lecture 6, Cinelli et al. [4] (p.5)



## Simulation setting

```
# Sim
Z = rnorm( 100 )
X = rnorm( 100 , 1*Z )
Y = rnorm( 100 , 1*X )
d = data.frame(Z,X,Y)
```

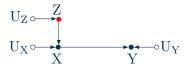
(c) R code

#### Implications,

- N #K
- N #K | M

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

(a) structural model



# "Eyeballing" analysis

#### based on correlation analysis,

- cor(M, K) < 0 does NOT goes in line of our "rudimentary" understanding of the data.
- and why there is  $cor(N, K) \approx 0$ ? (hint: univariate correlation)
- we include N as a covariate in our statistical model
   (is our research hypothesis)

pipe1\_panel.pdf



# Regression, regression!!

based on statistical analysis,

■ two regressions with two different results, which model is the "true"?

pipe1\_reg.png

## I'll get more data!!

imagine we can continue sampling,

- $\bullet$  top: 10,000 samples n = 20
- $\blacksquare$  bottom: 10,000 samples n = 100

the larger the sample size,

- the more certain you are about your estimates
- the more mistaken you are about your research question (under the "incorrect" model)

pipe1\_samplesize.pd

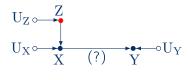
### The dream team!!

#### based on DAG and statistical model,

- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path,
   i.e. N ⊥K | M
- conditioning on M we can find, E[K|do(n)] = E[E[K|N = n, M]](law of total expectation)
- then we can find the ACE(n) = E[D|do(n+1)] E[D|do(n)] (Frisch-Waugh-Lovell theorem)

$$M = \begin{cases} Z \leftarrow f_Z(U_Z) \\ X \leftarrow f_X(Z, U_X) \\ Y \leftarrow f_Y(X, U_Y) \\ U \sim P(\boldsymbol{U}) \end{cases}$$

(a) structural model



### the dream team!!

based on DAG and statistical analysis,

■ the less biased model is the second, (assuming our DAG is true)

pipe1\_reg2.png

3. Anecdotal cases

Pipe bias: post-treatment



### Post-treatment bias<sup>12</sup>

also known as,

included variable bias (bigger grouping)

research question,

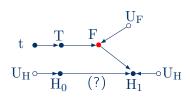
■ Does the treatment T works?

#### variables,

- $H_0$ , height of plant at t=0
- T, antifungal treatment
- F, presence of fungus
- $H_1$ , height of plant at t = 1

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_H(F, H_0, U_H) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



## Simulation setting

```
# backward simulation
H = round( rnorm( 100 , 170, 2), 1)
Lp = runif( 100 , 0.5-0.05, 0.5+0.05)
LL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
RL = round( Lp*H + rnorm( 100 , 0, 1 ), 1)
d = data.frame(LL,RL,H)
```

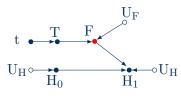
(c) R code

#### Implications,

- N #K
- N #K | M

$$M = \begin{cases} H_0 \leftarrow f_H(U_H) \\ T \leftarrow f_T(t) \\ F \leftarrow f_F(T, U_F) \\ H_1 \leftarrow f_H(F, H_0, U_H) \\ U \sim P(\textbf{U}) \end{cases}$$

(a) structural model



# "Eyeballing" analysis

#### based on correlation analysis,

- cor(M, K) < 0 does NOT goes in line of our "rudimentary" understanding of the data.
- and why there is  $cor(N, K) \approx 0$ ? (hint: univariate correlation)
- we include N as a covariate in our statistical model
   (is our research hypothesis)

pipe2\_panel.pdf

# Regression, regression!!

based on statistical analysis,

■ two regressions with two different results, which model is the "true"?

pipe2\_reg.png

## I'll get more data!!

imagine we can continue sampling,

- $\bullet$  top: 10,000 samples n = 20
- $\blacksquare$  bottom: 10,000 samples n = 100

the larger the sample size,

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pipe2\_samplesize.pd

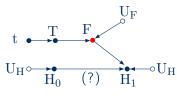
### The dream team!!

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- the 2nd D-separation rule requires you to control any noncollider to block the backdoor path, i.e. N 业K | M
- conditioning on M we can find, E[K|do(n)] = E[E[K|N = n, M]] (law of total expectation)
- then we can find the ACE(n) = E[D|do(n+1)] E[D|do(n)] (Frisch-Waugh-Lovell theorem)

$$\mathbf{M} = \begin{cases} \mathbf{H}_0 \leftarrow \mathbf{f}_{\mathrm{H}}(\mathbf{U}_{\mathrm{H}}) \\ \mathbf{T} \leftarrow \mathbf{f}_{\mathrm{T}}(\mathbf{t}) \\ \mathbf{F} \leftarrow \mathbf{f}_{\mathrm{F}}(\mathbf{T}, \mathbf{U}_{\mathrm{F}}) \\ \mathbf{H}_1 \leftarrow \mathbf{f}_{\mathrm{H}}(\mathbf{F}, \mathbf{H}_0, \mathbf{U}_{\mathrm{H}}) \\ \mathbf{U} \sim \mathbf{P}(\mathbf{U}) \end{cases}$$

(a) structural model



### the dream team!!

based on DAG and statistical analysis,

■ the less biased model is the second, (assuming our DAG is true)

pipe2\_reg2.png

### 3. Anecdotal cases

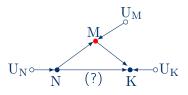


## Masked relationships<sup>13</sup>

also known as,

- mediation
- Simpson's paradox
- an instance of pipe bias research question,
- Does N has a (direct) effect on K? variables,
  - M, mammal mass in kg.
  - N, neocortex over total brain mass
  - K, Kcal. per gram of milk

(a) structural model



<sup>&</sup>lt;sup>13</sup>McElreath [12], chapter 05 (p. 144)



 $M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(N, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$ 

## Simulation setting

```
# sim
N = rnorm( 100 )
M = rnorm( 100 , 1*N )
K = rnorm( 100 , 1*N + -1*M )
d = data.frame(N=N, M=M, K=K)
```

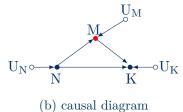
(c) R code

#### Implications,

- N #K
- N #K | M

$$M = \begin{cases} N \leftarrow f_N(U_N) \\ M \leftarrow f_M(M, U_M) \\ K \leftarrow f_K(M, N, U_K) \\ U \sim P(\textbf{U}) \end{cases}$$

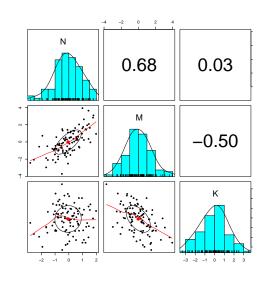
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## Regression, regression!!

#### based on statistical analysis,

• two regressions with two different results, which model is the "true"?

```
summary(lm(K \sim N, data=d)) # biased estimate
Call:
lm(formula = K \sim N, data = d)
Residuals:
             10 Median
                                     Max
-3.1751 -0.9009 0.1519 0.8574 3.6041
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.10412
                         0.13808
             0.05005
                        0.14487 0.345
                                            0.730
> summary(lm(K ~ N + M, data=d)) # less biased estimate
Call:
lm(formula = K \sim N + M, data = d)
Residuals:
              1Q Median
-2.58484 -0.59175 0.04378 0.61175 2.43360
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.06181
                        0.09825
            0.98297
                        0.13994
                                7.024 2.98e-10 ***
            -0.93107
                        0.09457 -9.846 2.89e-16 ***
```

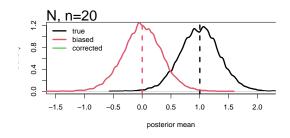
## I'll get more data!!

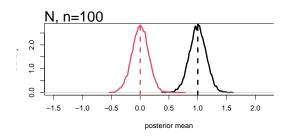
imagine we can continue sampling,

- top: 10,000 samples n = 20
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the larger the sample size,

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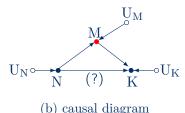
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(a) structural model





#### the dream team!!

based on DAG and statistical analysis,

■ the less biased model is the second, (assuming our DAG is true)

4. Concluding remarks



## Concluding remarks

- Research is filled with challenges, some obvious, some not (you: Duh!!)
- Statistical models are not theory (you: so obvious again!!)
- Don't trust your statistical model when no DAG is involved (me: how about that?!)
- For explanation, no sample size can save you when no DAG is involved (me: booya?!)
- For prediction, sometimes a DAG can help



5. Do you wanna know more???



5. Do you wanna know more????



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