

# Speech intelligibility measurement

## A latent variable approach

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# Contents

<b>1. Introduction</b>	<b>4</b>
<b>2. Materials and Methods</b>	<b>5</b>
2.1. Experimental setup . . . . .	5
2.2. Children . . . . .	5
2.3. Stimuli . . . . .	6
2.4. Causal framework . . . . .	6
2.5. Analysis . . . . .	6
<b>3. Results</b>	<b>6</b>
<b>4. Discussion</b>	<b>7</b>
<b>5. Author contributions</b>	<b>7</b>
<b>6. Financial support</b>	<b>7</b>
<b>7. Conflicts of interest</b>	<b>8</b>
<b>8. Research transparency and reproducibility</b>	<b>8</b>
<b>A. Supplementary</b>	<b>9</b>
A.1. About Speech Intelligibility . . . . .	9
A.2. Sampling bias . . . . .	11
A.3. Children characteristics . . . . .	11
A.4. DAG: factors influencing Intelligibility . . . . .	11
A.5. Model details . . . . .	13
A.6. Simulation . . . . .	17
A.7. Model selection . . . . .	18
<b>Bibliography</b>	<b>19</b>

## List of Figures

1.	Structural diagram describing the relationships among the variables	6
2.	Variability in a Beta-Proportional distribution . . . . .	15
3.	Variability in a Beta-Proportional distribution . . . . .	16

## List of Tables

1.	Example of five aligned transcriptions and its corresponding entropy calculations. Extracted from Boonen et al. [5], and slightly modified with illustrative purposes. . . . .	10
2.	Characteristics of selected children. . . . .	12

## Abstract

### 1. Introduction

Intelligible speech can be defined as the extent to which the elements in an speaker’s acoustic signal, e.g. phonemes or words, can be correctly recovered by a listener [19, 39, 36, 14]. Because intelligible spoken language requires all core components of speech perception, cognitive processing, linguistic knowledge, and articulation to be mastered [14], its attainment carries an important societal value, as it is a milestone in children’s language development, the ultimate checkpoint for the success of speech therapy, and has been qualified as the “gold standard” for assessing the benefit of cochlear implantation [8].

The literature suggest two perspectives from which speech intelligibility (SI) can be assessed: the message and listener’s perspective [4, 5]. The first, also known as acoustic studies, is focused on separately assessing particular characteristics of speech samples, e.g. their pitch, duration or stress (supra segmental characteristics), or the articulation of vowels and consonants (segmental characteristics) [30]. Whereas the second, also known as perceptual studies, is centered on making holistic assessments of the speech stimuli, e.g. measure their perceived quality [4, 5]. On both instances, the children’s utterances can be generated from reading at loud, contextualized utterances, or spontaneous speech tasks<sup>1</sup>.

Moreover, perceptual studies can use multiple approaches to measure intelligibility, but they can be largely grouped into two: objective and subjective ratings (OR and SR, respectively) [17]. In OR, listeners transcribe children’s utterances orthographically or phonetically, and use such information to construct a score. In that sense, in the transcription task, intelligibility refers to the extent a transcriber can identify the words contained in an utterance [5]. In contrast, under SR, listeners directly produce the score by assessing the speech sample’s quality through specific procedures, e.g. absolute holistic (HJ), analytic (AJ), or comparative (CJ) judgments, among others. It is easy to infer that OR methods might produce more valid<sup>2</sup> and reliable<sup>3</sup> scores than the SR counterpart, and therefore as their name imply, are usually used as an objective measure of intelligibility [5, 11].

Considering the previous, this paper investigates the speech intelligibility levels of normal hearing versus hearing impaired children with cochlear implants (HI/CI). For this purpose, we measured the entropy of speech representations coming from spontaneous speech task, resulting from a transcription task. Moreover, the paper make three specific contributions to the measurement and analysis of speech intelligibility.

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<sup>1</sup>ordered on increasing level of ecological validity [13, 10]

<sup>2</sup>the extent to which scores are appropriate for their intended interpretation and use [22, 34].

<sup>3</sup>the extend to which a measure would give us the same result over and over again [34], i.e. measure something, free from error, in a consistent way.

## 2. Materials and Methods

### 2.1. Experimental setup

For the transcription task, 100 transcribers participated in the experiment. The participants and stimuli were divided into five groups, where each group of 20 students (100/5) transcribed 64 stimuli on their series (320/5), resulting in 20 transcriptions per utterance ( $64 \times 100/320$ ). In total we registered 6400 transcriptions. The transcription task followed the steps detailed in section A.1.1.

On the other hand, the calculation of the entropy measures followed the procedure detailed in section A.1.2.

### 2.2. Children

Thirty two (32) children were selected using a large corpus of *spontaneously spoken speech*, collected by the Computational Linguistics, Psycholinguistics and Sociolinguistics research center (CLiPS). The selection followed a two step procedure<sup>4</sup>. First, a [convenient sample](#) of hearing impaired children was selected based on the quality of their registered stimuli (utterances). And second, a [matched sample](#) of normal hearing children was also selected.

For the first step, a sample of 16 hearing impaired children with cochlear implant (HI/CI) was selected. All children in the sample were native speakers of Belgian Dutch, living in Flanders, the Dutch speaking area of Belgium. They were all raised in monolingual Dutch with a limited support of signs, and all were screened as hearing impaired by the Universal Neonatal Hearing Screening (UNHS) using automated Auditory Brainstem Response hearing tests for newborns.

For the second step, 12 normal hearing children (NH) were matched on gender, age, and regional background, to the groups selected in the previous step. The matching procedure was [manual](#), [explain the appropriate procedure](#).

Finally, the researcher considers important to highlight two relevant points from the children’s selection process. First, while the matching procedure for the NH group uses the child’s *age* (at recording), the method cannot use the same variable for the other two groups. This is due to the fact that *age* is merely used as a proxy, for the amount of time a child has been developing his(her) language. In that sense, more appropriate variables to use under the HI/CI group would be e.g. the *device length of use*, which approximates the “hearing age” of such children, or their *vocabulary size*, which resembles their “lexical age” [11]. For this research, we consider the *device length of use* as the simplest one to implement.

The characteristics of the selected children is detailed in Table 2 at the supplementary section.

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<sup>4</sup>similar to one outlined in Faes et al. [11]

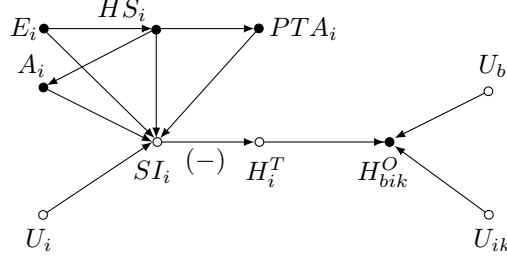


Figure 1: Structural diagram describing the relationships among the variables

### 2.3. Stimuli

The stimuli consisted of the children’s utterances, i.e. sentences of similar length, recovered from previously mentioned CLiPS corpus. More specifically, we use a portion of the corpus that consisted of 10 utterances recordings, for each of the 32 selected children. The stimuli were documented when the child was telling a story cued by the picture book “Frog, where are you” [23] to a caregiver “who does not know the story”. The quality of the stimuli was ensured by selecting utterances with no syntactically ill-formed or incomplete sentences, any background noise, cross-talk, long hesitations, revisions or non-words [5].

As a result, the data set consisted in a total of 320 utterances presented to the listeners in a random order, based on the adaptive pairing algorithm [27] implemented in Comproved<sup>5</sup>.

### 2.4. Causal framework

Where  $H_{ik}$  denoted the (observed) entropy replicates,  $H_i$  the (latent) “true” entropy,  $SI_i$  the (latent) speech intelligibility score (inversely related to  $H_i^T$ ). Moreover,  $A_i$  denoted the “hearing” age (subtracted the minimum age),  $E_i$  the etiology of disease that led to the hearing impairment,  $HS_i$  the hearing status and focus of our research,  $PTA_i$  the pure tone average (standardized). And Finally,  $B_b$  the block (will reduce  $\sigma_{U_{ik}}$ ). The variables are assumed independent, beyond the described relationships, i.e.

$$\begin{aligned} P(\mathbf{U}) &= P(U_{ik}, U_i, U_A, U_E, U_{HS}, U_P, B_{bk}) \\ &= P(U_{ik})P(U_i)P(U_A)P(U_E)P(U_{HS})P(U_P)P(B_{bk}) \end{aligned}$$

### 2.5. Analysis

## 3. Results

Research has shown that children with CI can attain spoken language skills similar to those of their normal hearing peers after three to four years of device use (i.e.,

<sup>5</sup>similar designs were used by Boonen et al. [4] and Faes et al. [11].

Bruijnzeel, Ziylan, Stegeman, Topsakal, & Grolman, 2016; Dettman, Dowell, Choo, Arnott, Abrahams, Davis, Dornan, Leigh, Constantinescu, Cowan, & Briggs, 2016; Geers, & Nicholas, 2013; Wie et al., 2020). However, the population of children with CI is characterized by remarkable variation. On the one hand, variation relates to differences between individual children: while a considerable number of children with CI appear to catch up with their NH peers, some do not catch up at all (Duchesne, & Marschark, 2019; Geers, Nicholas, Tobey, & Davidson, 2016; Nicholas, & Geers, 2007). On the other hand, variation also relates to differences between domains: some areas of speech and language appear to be more difficult to master than others (Duchesne, & Marschark, 2019). For instance, Faes, Gillis, & Gillis (2015) showed that in a group of children with CI acquiring Dutch, inflectional morphology and sentence length (as a proxy of syntagmatic development) were age-appropriate when the children were 7;0, but the former (and not the latter) was already age-appropriate at age 5;0. Moreover, the phonetics of the same childrens production of vowels was still significantly different from the vowels of their NH peers at the age of 7;0 (Verhoeven, Hide, De Maeyer, Gillis, & Gillis, 2016). Thus, although children with CI start with an initial delay in spoken language, a quite significant group eventually reaches age appropriate levels of linguistic functioning. But the individual variation is also quite large: while some children do catch up with their normally hearing peers, others do not achieve much language comprehension and production even after five years of device use (Barnard, Fisher, Johnson, Eisenberg, Wang, Quittner, Carson, & Niparko, 2015).

As to intelligibility, most studies found that CI childrens speech intelligibility is less well developed than that of their NH peers (i.a., Castellanos, Kronenberger, Beer Henning, Colson, & Pisoni, 2014; Chin, & Kuhns, 2014; Freeman et al., 2017; Grandon et al., 2020).

## 4. Discussion

talk about decision statements or thinking-at-loud tasks.. the listener provide a decision statement on why the selected stimulus sounded more intelligible

## 5. Author contributions

Jose Rivera performed the statistical analysis, Sven de Maeyer supervised the production of the documents and statistical results, and Steven Gillis collected the data.

## 6. Financial support

What is the financial support of the project

## **7. Conflicts of interest**

The authors declare they have no conflict of interest.

## **8. Research transparency and reproducibility**

The model's simulation procedures and testing that support the findings of this study are openly available at [https://github.com/jriveraespejo/PhD\\_UA\\_paper1](https://github.com/jriveraespejo/PhD_UA_paper1).

Due to the privacy and confidentiality of subjects, the data set in which the model was implemented cannot be put online.



## A. Supplementary

### A.1. About Speech Intelligibility

As it was specified in the document Intelligible speech can be defined as the extent to which the elements in an speaker’s acoustic signal, e.g. phonemes or words, can be correctly recovered by a listener [19, 39, 36, 14]. More specifically, in the context of the transcription task, speech intelligibility refers to the extent a transcriber can identify the words contained in an utterance [5].

#### A.1.1. Transcription task

The experimental settings for the transcription task followed the next steps [4, 5]:

1. the listener took a seat in front of a computer screen, located at his(her) home, workplace, or the experimental laboratory.
2. the listener opened Comproved<sup>6</sup> and select the transcription task.
3. the listener read two set of instructions presented on the computer screen about:
  - a) how to perform the task, and
  - b) the aspects not to consider for the task.
4. the listener hear the stimuli through high quality headphones, set at a comfortable volume.
5. the listener wrote the orthographic transcriptions of the utterances, in a free text field in the Comproved environment.

#### A.1.2. Entropy calculation

The outcome from the transcription task was obtained following a two step procedure [5]. First, we aligned the participant’s orthographic transcriptions, at the utterance level, in a column-like grid structure similar to the one presented in Table 1. This step was repeated for every one of the 6400 transcriptions. Lastly, we computed the entropy measure of the aligned transcriptions as in Shannon [31]:

$$H = H(\mathbf{p}) = \frac{-\sum_{i=1}^n p_i \cdot \log_2(p_i)}{\log_2(N)} \quad (1)$$

where  $H$  is bounded in the continuum  $[0, 1]$ ,  $n$  denotes the number of word occurrences within each utterance,  $p_i$  the probability of such word occurrence, and  $N$  the total number of aligned transcriptions per utterance.

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<sup>6</sup>software developed by the University of Antwerp designed to perform comparative judgments:  
<https://comproved.com/en/a>.

Transcription number	Utterance				
	1	2	3	4	5
1	de the	jongen boy	ziet see	een a	kikker frog
2	de the	jongen boy	ziet sees	de the	[X] [X]
3	de the	jongen boy	zag saw	[B] [B]	kokkin cook
4	de the	jongen boy	zag saw	geen no	kikkers frogs
5	de the	hond dog	zoekt searches	een a	[X] [X]
Entropy	0	0.3109	0.6555	0.8277	1

[B] = blank space, [X] = unidentifiable word

Table 1: Example of five aligned transcriptions and its corresponding entropy calculations. Extracted from Boonen et al. [5], and slightly modified with illustrative purposes.

Entropy was used as an objective measure of SI, i.e. a quantification of (dis)agreement between listeners' transcriptions. Utterances yielding a high degree of agreement between transcribers were considered highly intelligible, and therefore registered a lower entropy ( $H \rightarrow 0$ ). In contrast, utterances yielding a low degree of agreement were considered as exhibiting low intelligibility, and therefore registered a higher entropy ( $H \rightarrow 1$ ) [5, 11].

Using Table 1, we exemplify the entropy calculation for utterances 2, 4 and 5, which represent relevant scenarios for the procedure. Notice that every calculation considers five transcriptions in total ( $N = 5$ ).

For the second utterance, we observe that four transcriptions identify it with the word *jongen*, while the last with the word *hond*. Therefore, we registered two word occurrences ( $n = 2$ ), with probabilities  $\mathbf{p} = (p_1, p_2) = (4/5, 1/5)$ , and entropy measure equal to:

$$\begin{aligned}
H &= \frac{-\sum_{i=1}^2 p_i \cdot \log_2(p_i)}{\log_2(5)} \\
&= \frac{-[0.8 \log_2(0.8) + 0.2 \log_2(0.2)]}{\log_2(5)} \\
&\approx 0.3109
\end{aligned}$$

For the fourth utterance, we observe that two transcriptions identify it with the word *een*, one with *de*, one with *geen*, and one with a blank space [B]. Notice the blank space was not expected in such position, therefore, it was considered as a different word occurrence. As a result, the scenario had four word occurrences ( $n = 4$ ), with

probabilities  $\mathbf{p} = (p_1, p_2, p_3, p_4) = (2/5, 1/5, 1/5, 1/5)$ , and entropy measure equal to:

$$\begin{aligned} H &= \frac{-\sum_{i=1}^4 p_i \cdot \log_2(p_i)}{\log_2(5)} \\ &= \frac{-[0.4 \log_2(0.4) + 3 \cdot 0.2 \log_2(0.2)]}{\log_2(5)} \\ &\approx 0.8277 \end{aligned}$$

Finally, for the fifth utterance, we observe that all of the transcriptions identify it with different words. Notice we consider the unidentifiable word [X] in the second transcription, as being different from the one in the last. This is done to avoid the artificial reduction of the entropy measure, as [X] values already indicate the lack of intelligibility of the word. Therefore, we registered five word occurrences ( $n = 5$ ), with probabilities  $\mathbf{p} = (p_1, \dots, p_5) = (1/5, \dots, 1/5)$ , and entropy measure equal to:

$$\begin{aligned} H &= \frac{-\sum_{i=1}^5 p_i \cdot \log_2(p_i)}{\log_2(5)} \\ &= \frac{-5 \cdot 0.2 \log_2(0.2)}{\log_2(5)} \\ &= 1 \end{aligned}$$

## A.2. Sampling bias

Second, due to the nature of the sample selection procedure, we cannot ensure the HI/CI and HI/HA, nor the NH group, are representative of their respective populations. Therefore, inferences beyond this particular set of children must be taken with care.

## A.3. Children characteristics

The table includes all the variables used for the matching procedure in Section ??, and additionally, shows the child's etiology, i.e. the cause of their hearing impairment, and their post-implant pure tone average (PTA), i.e. the child's subjective hearing sensitivity, aided and unaided by their hearing apparatus. No other variables are included, as no known additional comorbidities, beside their hearing impairment, is suspected.

From the table, [describe summaries from the table](#).

## A.4. DAG: factors influencing Intelligibility

Many factors have been shown to contribute to the success of spoken language development of children with CI, including: (1) audiology related factors, such as the age at implantation, the duration of device use, bilateral (or contralateral) cochlear implantation and the childrens preoperative and postoperative hearing levels; (2) child

Child	Hearing	Gender	Regional background	Age (y;m)	Device use (y;m)	Etiology	PTA (dB.)	
	Status						unaided	aided
1	NH	male				genetic		
2	HI/CI	female				CMV infection		
3	HI/HA					unknown		
4								
5								
6								
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33								

(y;m) = (years;months)

NH = normal hearing,

HI/CI = hearing impaired / cochlear implant,

HI/HA = hearing impaired / hearing aid

Table 2: Characteristics of selected children.

related factors, such as the cause of the hearing impairment (genetic, infections),

gender, additional disabilities (mental retardation, speech motor problems); and (3) environmental factors, such as communication modality. An overview is provided in Boons, Brokx, Dhooge, Frijns, Peeraer, Vermeulen, Wouters, and van Wieringen, 2012, Fagan, Eisenberg, and Johnson, 2020, Gillis, 2018 and Niparko, Tobey, Thal, Eisenberg, Wang, Quittner, and Fink, 2010. A factor of particular importance here is age. Studies have shown that chronological age is an important factor for intelligibility: as they grow older, childrens intelligibility increases irrespective of their hearing status (Grandon et al., 2020). But in the case of children with CI, age is a complicated factor, since it can not only refer to childrens chronological age (as is the case for children with NH), but also to the childrens so-called hearing age, which is the amount of time between the activation of their device and their chronological age. For instance, a child implanted at the age of 1;0 has a hearing age of two years at the age of 3;0. In addition, the age at implantation has been shown to play a critical role in childrens spoken language achievements. In general, earlier implantation appears to lead to better results than later implantation in several domains (Boons et al., 2012; Niparko et al., 2010). But the research findings with respect to the effect of the variable age on children with CIs intelligibility are not unequivocal. In some studies, a significant effect of chronological age on childrens intelligibility was found (i.a., Flipsen, & Colvard, 2006; Grandon et al., 2020; Habib, Waltzman, Tajudeen, & Svirsky, 2010) but not in others (e.g., Khwaileh, & Flipsen, 2010). Hearing age was found to be a significant predictor of intelligibility by i.a., Flipsen and Colvard (2006), but hearing age was not always considered as a predictor. Age at implantation predicted childrens intelligibility in a considerable number of studies (i.a., Grandon et al., 2020; Habib et al., 2010; Montag, AuBuchon, Pisoni, & Kronenberger, 2014; Svirsky, Chin, & Jester, 2007) but this was not the case in other studies (i.a., Flipsen, & Colvard, 2006; Khwaileh, & Flipsen, 2010). Nevertheless, a general finding appears to be that earlier implantation leads to better results in speech and language development and in intelligibility. At present there is consistent evidence that implantation in the first two years of life leads to consistently better results in spoken language development in comparison to later implantation, and even (inconclusive) evidence for even better outcomes of implantation in the first year of life (Bruijnzeel et al., 2016; Dettman et al., 2016).

## **A.5. Model details**

### **A.5.1. Definition**

Previous research already used hierarchical models with the replicated entropy measures as outcomes [5, 11]. Hierarchical models are powerful to control for heterogeneity in the data, and also to avoid pre-aggregating procedures that could be pernicious for a proper statistical inference [24].

These claims are easier to understand using a thought experiment within our research. Consider we have two children with the same mean entropy, but the second child shows more variability across the 10 utterances than the first. It is clear that the average entropy measure informs about the child’s average SI, indicating that

both children have similar level. However, the entropy’s heterogeneity across the 10 utterances also informs about the child’s SI, as a higher variability imply transcribers agreed less about the second child’s intelligibility.

The intuition derived from the previous though experiment is similar to the one presented in Boonen et al. [5], and it is what justify our use of a hierarchical model. More specifically, we will use a Hierarchical (Mixed) Beta Regression model [12], for which we argue, its implementation is rather trivial under the bayesian framework, and we present it in the following lines.

First, figure ?? depicts the DAG representation of the model. For the measurement error part, section ?? reveals the (observed) entropy replicates  $H_{ik}^O$  can represent multiple realizations of a child’s *true* entropy  $H_i^T$ , measured with error  $e_i$ . As a result, we can say the  $k$ ’th entropy measure is nested within the  $i$ ’th child, where  $k = 1, \dots, K$ ,  $i = 1, \dots, I$ ,  $K = 10$  utterances, and  $I = 32$  children.

Second, for the hypothesis part, we can say the child’s *true* entropy  $H_i^T$  is inversely explained by the child’s speech intelligibility index  $SI_i$ , and in turn, the latter by a set of covariates. Notice from Figure ??, we propose two sets of models. The model in panel (a) use hearing status ( $HS_i$ ) and hearing age ( $A_i$ ) as covariates. The use of hearing status is justified as we are interested in comparing SI among groups, defined by the children’s hearing characteristics (NH, HI/CI, and HI/HA). On the other hand, we expect hearing age<sup>7</sup> and its interaction with hearing status, to also have an effect on the SI index, as previous evidence have shown the speech of HI children gradually approximate that of NH children [6].

Notice the model depicted in panel (a) is interested on (what we can call) *total effects*, i.e. the effects of the hearing characteristics, not independent from the effects of the hearing apparatus (cochlear implant or hearing aid). This is important to understand for two reasons. Since a hearing apparatus is fitted onto a child depending on aspects such as the locus and severity of his(her) hearing impairment [20]: (1) such specific children’s characteristics could confound the (beneficial) effects of using specific hearing apparatuses, while (2) because children are selected from a convenient sample, not representative of their respective populations (see section ??), the need to control for such characteristics is paramount, if we seek to obtain effects that can generalize better and beyond our sample<sup>8</sup>.

Considering the previous, we propose the model depicted in panel (b), where we control for the possible confounding variables etiology ( $E_i$ ), [as a proxy of locus](#), and unaided PTA ( $PTA_i$ ), as a proxy for hearing impairment severity. In that sense, the model would estimate (what we can call) the *direct effects* of the hearing apparatus, independent of the children’s characteristics.

Lastly, we proceed to use probabilistic programming to declare the algebraic structure of our models. Given the panel (a) model is nested within the panel (b) model,

<sup>7</sup>see section ?? to know how the variable is defined.

<sup>8</sup>follow the *notes* folder, to see a graphical though experiment.

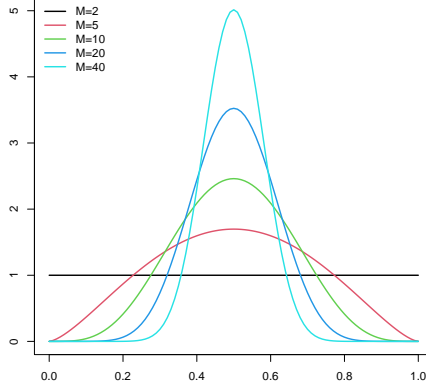


Figure 2: Variability in a Beta-Proportional distribution

we declare only the model structure for the latter:

$$H_{bik}^O \sim \text{BetaProp}(P_{bi}, M_{ik}) \quad (2)$$

$$P_{bi} = \alpha_b + H_i^T \quad (3)$$

$$H_i^T = \text{logit}^{-1}(-SI_i) \quad (4)$$

$$SI_i = a_i + \alpha + \alpha_{E[i], HS[i]} + \beta_{A, HS[i]}(A_i - \bar{A}) + \beta_P PTA_i \quad (5)$$

$$(6)$$

where  $\text{logit}(x) = \log[x/(1-x)]$ , and  $\text{logit}^{-1}(x) = \exp(x)/(1+\exp(x))$ . Additionally, a  $\text{BetaProp}(\mu, \theta)$  distribution is equal to a  $\text{Beta}(\alpha, \beta)$  distribution, with  $\alpha = \mu\theta$ ,  $\beta = (1-\mu)\theta$ . For our purposes,  $\mu = H_i^T$  and  $\theta = M_i$ , the latter denoting the “sample size” of the distribution. Moreover,  $a_i$  denote the children’s random effects,  $\alpha$  the fixed effects’ intercept,  $\alpha_{HS[i]}$  and  $\beta_{A, HS[i]}$  the intercept and slope of “hearing age” per hearing status group,  $\alpha_{E[i]}$  the intercept per etiology group, and  $\beta_P$  the slope for the standardized PTA levels.

Three important things need to be noticed from the previous algebraic structure. First, all the parameters are estimated in the logit scale and centered at  $PTA_i = 0$  and  $\bar{A}$ , which denotes the minimum hearing age in the sample. Second, instead of a latent measurement error  $U_{ik}$ , we use the latent “sample size” parameter  $M_{ik}$  to model the heterogeneity/variability of the duplicate entropies. This effectively works as a measurement error model for the replicates, as the parameter defines the shape of the distribution. Third, if we do not consider etiology and PTA values in equation (4), we obtain the panel (a) model.

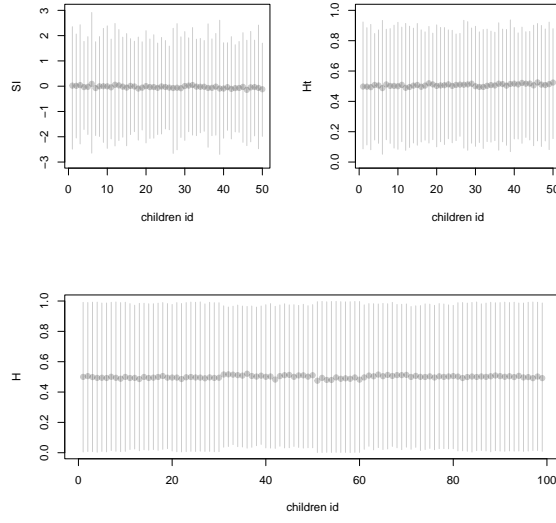


Figure 3: Variability in a Beta-Proportional distribution

#### A.5.2. Priors

$$M_i \sim \text{LN}(\mu_M, \sigma_M) \quad (7)$$

$$a_i \sim \text{N}(\mu_a, \sigma_a) \quad (8)$$

$$\alpha \sim \text{N}(0, 0.3) \quad (9)$$

$$\alpha_{HS[i]} \sim \text{N}(0, 0.3) \quad (10)$$

$$\beta_{A,HS[i]} \sim \text{N}(0, 0.3) \quad (11)$$

$$\alpha_{E[i]} \sim \text{N}(0, 0.5) \quad (12)$$

$$\beta_P \sim \text{N}(0, 0.3) \quad (13)$$

$$\mu_M \sim \text{N}(0, 5) \quad (14)$$

$$\sigma_M \sim \text{Exp}(1) \quad (15)$$

$$\mu_a \sim \text{N}(0, 0.5) \quad (16)$$

$$\sigma_a \sim \text{Exp}(1) \quad (17)$$

$$(18)$$

Third, we use mildly informative priors to state our uncertainty regarding the direction and magnitude of the effects<sup>9</sup>.

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<sup>9</sup>see Rivera [29] (p. 18-19) for an intuition on prior elicitation.



### A.5.3. Estimation

The models proposed in sections ?? and ?? will be estimated under the Bayesian framework<sup>10</sup>. More specifically, we will use the No-U-Turn Hamiltonian Monte Carlo algorithm (No-U-Turn HMC) [3, 9, 16, 25]. `Stan` [33] will be the software package that will provide us with the No-U-Turn HMC machinery, while `R` [28] and its integration packages [32], the software that will allow us to analyze its outputs.

### A.5.4. Pre-processing

Besides the exclusion of corrupted observations, e.g. no available rating, no other experimental run nor duplicate was eliminated before the modeling process. This decision departs from what it is observed in previous research, e.g. Boonen et al. [4] decided to eliminate "outlying" observations based on misfit analysis [22], while van Daal [35] and Boonen et al. [5] did the same based on univariate outlier analysis.

For the case of misfit analysis, we argue that such procedures cannot be used without caution. The literature points out that in the context of CJ, these statistics are always relative, i.e. they depend on other stimulus and judges included in the assessment [26, 27]. Moreover, they have been proven to be less sensitive, as they are calculated with a low number of judgments per representation [26].

On the other hand, for the case of univariate outlier analysis, we argue that outlying observations are interesting cases to analyze [24], and usually they cannot be identified properly outside the context of a full model [24], i.e. what can behave as an outlier based on a univariate analysis, can behave as expected under the appropriate model.

Considering the previous, if we still manage to identify outlying observations within the context of the proposed models (see Section ??), the researcher would rather make the model robust against their influence, playing on the strengths of the bayesian framework, than to eliminate the observations.

## A.6. Simulation

Preliminary to the data collection, we simulated data in silico to test the models and inform data collection procedure. The simulation code is available in the GitHub repository. Several functional correlation between age and knowledge have been simulated, and the model used in the analysis - which includes age as a ordinal categorical predictor of knowledge with monotonically increasing effect - has been able to recover the different shapes. Causal effect of activities, family composition and schooling have been simulated and tested.

The simulated data have been used -albeit in a previous version- to estimate the minimum number of interviewees necessary to recover the parameter values. If individuals were to name a maximum of 300 items in the freelist, 50 interviewees would have been sufficient to obtain reliable estimates of the parameters. Given that

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<sup>10</sup>see Rivera [29] (p. 11-13, 15-27) for a detailed description of its benefits and shortcomings.

data collection in vivo is much less regular and less controllable than in silico, we roughly doubled the number of interviewees and that of questions.

## A.7. Model selection

Following the successful and comprehensive analysis in van Daal [35] and Lesterhuis [22], the current research will also use the Information-Theoretic Approach (ITA) [2, 7] for the selection of competing models. The approach considers three steps: (1) state our hypothesis into statistical models, (2) select among competing models, and (3) make inferences based on one or multiple models.

First, for the translation of our working hypotheses into statistical models, we will use Directed Acyclic Graphs (DAG) and probabilistic programming [18]. A DAG is the simplest representation of a Graphical Causal Model (GCM), a heuristic model that contains information not purely statistical, but unlike a detailed statistical model, it allow us to deduce which variable relationships can provide valid causal inferences [15, 24]. In summary, a DAG is a reasonable way to state our hypothesis, and make our assumption more transparent. However, abide by the *no-free lunch* rule, the causal inferences produced under the DAG will only be valid if the assumed DAG is correct. In contrast, the probabilistic programming will serve as the algebraic formalist to define our statistical models.

Second, to select among competing models, we will use the Widely Applicable Information Criterion (WAIC) [38], and the Pareto-smoothed importance sampling cross-validation (PSIS) [37]<sup>11</sup>. Two reasons justify our decision. First, both criteria allow us to embrace the full flexibility and information of our bayesian implementation (outlined in Section ??). Last, and more important, both criteria provide us with the best approximations for the out-of-sample (cross-validated) deviance [24]. The deviance is the best approximation for the Kullback-Liebler (KL) divergence [21], i.e. a measure of how far a model is from describing the *true* distribution of our data. McElreath [24] points out that is a rather benign characteristic of the model's selection procedure that we do not need the KL divergence's absolute value, as the *true* distribution of our data is not available (otherwise, we would not need a statistical model). But rather, using the difference in deviance between competing models, we can measure which model is the farthest from *perfect (predictive) accuracy* for our data<sup>12</sup>.

Finally, considering the evidence provided by the previous step, we proceed to make inferences based on one or multiple models.

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<sup>11</sup>van Daal [35] used the Akaiques Information Criterion (AIC) [1] with similar purposes.

<sup>12</sup>see McElreath [24] (p. 202-211) for the intuition and detailed derivation of the argument.

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