# Causes and effects in Dichotomous Comparative Judgments: an information-theoretical system of plausible mechanism

Jose Manuel Rivera Espejo<sup>a,\*</sup>, Tine van Daal<sup>a</sup>, Sven De Maeyer<sup>a</sup>, Steven Gillis<sup>b</sup>

 $^a$  University of Antwerp, Training and education sciences,  $^b$  University of Antwerp, Linguistics,

### Abstract

Dichotomous Comparative Judgment (DCJ) requires judges to compare pairs of stimuli to determine which one exhibits a higher degree of a specific trait. DCJ has proven effective and reliable across various fields (Pollitt, 2012b; Jones, 2015; van Daal et al., 2019; Bartholomew et al., 2018; Lesterhuis, 2018; Bartholomew and Williams, 2020; Marshall et al., 2020; Boonen et al., 2020). However, despite the method's widespread use, existing literature lacks a clear explanation of the complexities and assumptions underpinning the DCJ system, as well as the plausible mechanisms through which DCJ data could be generated. This study addresses these issues by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach, the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates this model into a probabilistic statistical model to estimate statistical relationships and infer causal effects within the system. This research provides a robust probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone's law of comparative judgment (1927). Its findings offer valuable insights for researchers and analysts designing and implementing DCJ experiments.

Keywords: causal inference, probability, Thurstone, comparative judgement, directed acyclic graph, structural causal models, statistical modeling

## 1. Introduction

In contemporary contexts, Thurstone's law of comparative judgment (1927) primarily refers to the method of *dichotomous* comparative judgment (DCJ, Pollitt, 2012a,b). In DCJ, a judge assesses the relative manifestation of a *trait* within a pair of stimuli. This assessment results in a dichotomous value indicating which stimulus possesses a

<sup>\*</sup>Corresponding author

Email addresses: JoseManuel.RiveraEspejoQuantwerpen.be (Jose Manuel Rivera Espejo), tine.vandaalQuantwerpen.be (Tine van Daal), sven.demaeyerQuantwerpen.be (Sven De Maeyer), steven.gillisQuantwerpen.be (Steven Gillis)

higher degree of the trait. After different judges perform multiple rounds of pairwise comparisons, an outcome vector is produced. This vector is modeled using the Bradley-Terry-Luce model (BTL, Bradley and Terry, 1952; Luce, 1959), which creates a score that corresponds with the trait of interest. This score is then used to rank the stimuli from lowest to highest or to evaluate the influence of certain variables on the stimuli's positions in the ranking.

DCJ has proven effective in assessing competencies and traits predominantly within the educational realm, as demonstrated by Pollitt (2012b), Jones (2015), van Daal et al. (2019), Bartholomew et al. (2018), Lesterhuis (2018), Bartholomew and Williams (2020), and Marshall et al. (2020). However, its application transcends education, as exemplified by Boonen et al. (2020). The methodology has also evolved to include multiple, as opposed to pairwise comparisons (Luce, 1959; Plackett, 1975), and to accommodate comparisons with ordinal outcomes (Tutz, 1986; Agresti, 1992). Overall, research suggests that DCJ offers an alternative and efficient approach to measurement and evaluation, characterized by its reliability and validity (Lesterhuis, 2018; van Daal, 2020; Marshall et al., 2020; Bouwer et al., 2023). Nevertheless, despite the method's widespread use, existing literature lacks a clear representation of the plausible mechanisms through which DCJ data could be generated. Particularly, there is no depiction of the complexity and the assumptions underpinning the DCJ system, nor how different assessment factors can potentially influence the observed DCJ outcome.

According to Verhavert et al. (2019) and van Daal (2020), several assessment factors interact and influence the method's outcome. These factors include the number and characteristics of the stimuli, their proximity in terms of the assessed trait, the number of comparison per stimulus, and the pairing algorithm used. Furthermore, since the method relies on judges' assessments, the number and characteristics of judges, their discrimination abilities, and the number of comparisons per judge also play pivotal roles. Moreover, when the stimuli represent sub-units of higher-levels units, factors such as the number and characteristics of these units, along with their proximity in terms of the assessed trait, can significantly influence the outcome. For example, in van Daal et al. (2019), the authors assessed the academic writing skills of university students (units) using multiple argumentative essays (sub-units).

Although several studies have examined the individual impact of these factors on the method's reliability (Bramley, 2015; Pollitt, 2012b; Bramley and Vitello, 2019; Verhavert et al., 2019; Crompvoets et al., 2022; van Daal et al., 2017; Gijsen et al., 2021; Bouwer et al., 2023), none, to the best of the authors' knowledge, have provided a transparent depiction of the DCJ system and the mechanisms generating the DCJ outcome. This study aims to fill this gap by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach to causal inference, the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates the scientific model into a probabilistic statistical model. This model aims to produce statistical estimates to draw inferences about plausible causal relationships within the DCJ system.

Ultimately, this study provides a robust causal and probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone's law of comparative judgment (1927). Consequently, its findings offer valuable insights for researchers and analysts

designing and implementing DCJ experiments.

#### 2. Theoretical framework

This section introduces fundamental concepts in causal inference, such as directed acyclic graphs (DAGs), structural causal models (SCMs), and the flow of association and causation in graphs. The section is not a comprehensive description of causal inference methods. Readers interested in deeper exploration should consult introductory papers like Pearl (2010), Rohrer (2018), Pearl (2019), and Cinelli et al. (2020). They may also find introductory books such as Pearl and Mackenzie (2018), Neal (2020) and McElreath (2020) useful. For more advanced study, seminal intermediate papers like Neyman (1923), Rubin (1974), Spirtes et al. (1991), and Sekhon (2009), as well as books like Pearl (2009), Morgan and Winship (2014) and Hernán and Robins (2020) are recommended.

### 2.1. The structural approach to causal inference

Empirical research addresses real-world challenges by relying on evidence gathered through observation and experimentation. In this context, researchers typically frame their research questions as estimands or targets of inference. These estimands represent the specific quantities the study aims to determine (Everitt and Skrondal, 2010). For instance, a study might seek to answer the question, "To what extent do daily practice hours (X) influence students' ability to produce high-quality written texts (Y)?". To investigate this, the study randomly assigns students to two groups with different levels of daily practice, low (X=1) and medium (X=2). The quality of the written texts is evaluated using pairwise comparisons, resulting in a dichotomous outcome  $(Y=\{0,1\})$ . The research question is then framed as the estimand, "On average, which group of students produces higher-quality written texts?" This estimand is mathematically expressed as E[Y|X=1]-E[Y|X=2], where  $E[\cdot]$  represents the expected value.

Researchers then proceed to identify the estimands. *Identification* is the process of accurately computing an estimand using an estimator. An *estimator* is a method or function that transforms data into an estimate (Neal, 2020). *Estimates* are numerical values that approximate the estimand and are derived through *estimation*, which is the process that integrates data with an estimator (Everitt and Skrondal, 2010). While various methods can approximate an estimand, researchers prioritize estimators with desirable properties that ensure accurate estimates. For example, Fisher's exact test (Fisher, 1922) is an estimator with desirable properties for comparing  $2 \times 2$  tables. The test provides accurate estimates for comparing proportions when its assumptions are satisfied. Figure 1, known as the Identification-Estimation flowchart (McElreath, 2020; Neal, 2020), illustrates the aforementioned process of transitioning from estimands to estimates.

However, many studies aim to establish causal relationships rather than merely infer associations. In the earlier example, the estimates obtained using Fisher's exact test can be interpreted as causal because the data were collected through a randomized experiment. In randomized experiments, the researcher fully controls the treatment assignment mechanism, such as assigning students to different groups. This control allows causal estimates to be derived from associational estimates, such as those provided by the Fisher

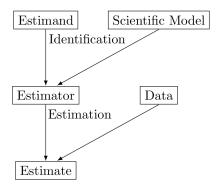


Figure 1: Identification-Estimation flowchart. Extracted and slightly modified from Neal (2020, 32)

statistic, since randomization effectively eliminates confounding (Neal, 2020). Confounding occurs when an external variable influences both the outcome and the variable of interest, leading to spurious association (Everitt and Skrondal, 2010).

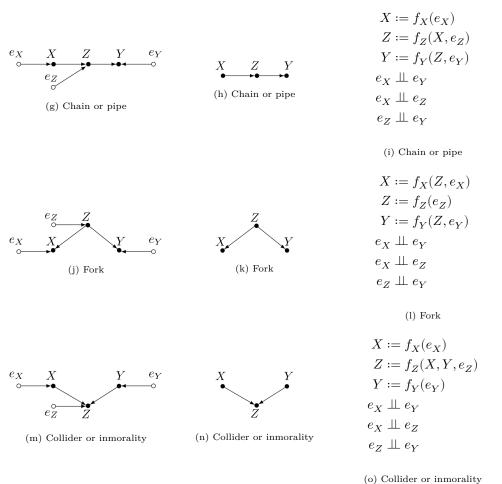
### 2.2. DAGs and SCMs

Graph theory is a branch of mathematics focused on the study of graphs. Graphs are mathematical structures modeling pairwise relations between objects. They can represent physical relations, such as electrical circuits and roadways, and less tangible structures, such as ecosystems and sociological relations. Graphs have proven useful in various fields, including computer science, operations research, and the natural and social sciences (Gross et al., 2018).

In statistics, one application incorporating concepts from graph theory is causal inference. Specifically, the structural approach to causal inference uses directed acyclic graphs (DAG) to provide a graphical and formal representation of the causal structure of a system (Neal, 2020). In this context, a graph denotes a collection of nodes connected by edges, where nodes represent random variables. The term directed indicates the edges of the graph extend from one node to another, with arrows showing the direction of causal influence. Moreover, the term acyclic indicates the causal influences do not form a loop, meaning the influences do not cycle back on themselves (McElreath, 2020).

DAGs offer two key advantages for modeling causal structures. Firstly, they represent causal relations in a nonparametric and fully interactive manner. This feature allows for feasible causal analysis strategies without needing the specification of the type of data or the nature of the functional dependence among variables (Morgan and Winship, 2014). Secondly, regardless of complexity, DAGs can represent various causal structures using only five fundamental building blocks (Neal, 2020; McElreath, 2020). This feature enables the decomposition of complex structures into basic building blocks, facilitating the analysis of these structures by focusing on the causal assumptions associated with each building block individually (McElreath, 2020). These building blocks can be represented in three ways: the magnified representation, the standard representation, and the structural causal model form (SCM, Morgan and Winship, 2014).

(f) Two connected nodes or descendant



(b) Conider or inmoranty

Figure 2: The five fundamental building blocks of DAGs. Note: left panels show the magnified representation, middle panels show the standard representation, and the right panels show their corresponding SCM form. 5

The left panels of Figure 2 illustrate the magnified representation. These graphs depict the endogenous variables  $V = \{X, Z, Y\}$  alongside the exogenous variables  $E = \{e_X, e_Z, e_Y\}$ . Endogenous variables are those whose causal mechanisms the investigator chooses to model (Neal, 2020). In contrast, exogenous variables represent errors or disturbances arising from omitted factors that the investigator chooses not to model explicitly (Pearl, 2009, 27,68). The graphs show endogenous variables as solid black circles to signify that they are observed random variables, while endogenous variables are depicted as open circles to signify their unobserved (latent) nature. Lastly, the arrows in the graphs reflect the expected direction of causal influences among these variables.

Often, DAGs omit the exogenous variables for simplicity, resulting in the *standard* representation. However, including exogenous variables in a graph can be beneficial in some scenarios, as their presence can reveal potential issues related to conditioning and confounding (Cinelli et al., 2020), concepts explored in Section 2.3. The standard representation is illustrated in the middle panels of Figure 2.

Lastly, the right panels of Figure 2 depict the SCM form of the fundamental building blocks. SCMs are formal mathematical models defined by a set of endogenous variables V, a set of exogenous variables E, and a set of functions  $F = \{f_X, f_Z, f_Y\}$  (Pearl, 2009; Neal, 2020). These functions, referred to as structural equations, specify each endogenous variable as nonparametric functions of other variables. Moreover, SCMs use the symbol ':=' to indicate the variables' asymmetrical causal dependence and the symbol ' $\perp$ L' to represent d-separation, which roughly equates to the concept of variable independence. The concepts of d-separation and causal (in)dependence are explored in Section 2.3.

A careful examination of Figure 2 highlights the assumptions underlying these building blocks. Figures 2a, 2b, and SCM 2c depict two unconnected nodes, representing a scenario where variables X and Y are not causally related. Figures 2d, 2e, and SCM 2f illustrate two connected nodes, showing a scenario where a parent node X exerts a causal influence on a child node Y. Consequently, Y is considered a descendant of X. Figures 2g, 2h, and SCM 2i depict a chain or pipe, where X influences Z, and Z influences Y. In this configuration, X is a parent node of Z, and Z is a parent node of Y. This creates a directed path between X and Y. Consequently, X is an ancestor of Y, and Z fully mediates the relationship between the two. Figures 2j, 2k, and SCM 2l illustrate a fork, where variables X and Y are both influenced by Z. Here, Z is a parent node of X and Y. Finally, Figures 2m, 2n, SCM 2o depict a collider, also known as inmorality, where variables X and Y are concurrent causes of Z. In this configuration, X and Y are not causally related to each other but both influence Z. Additionally, in all SCMs, the errors are assumed to be mutually independent of each other and of all other variables in the graph, as evidenced by the pairwise relations  $e_X \perp \!\!\!\!\perp e_Y$ ,  $e_X \perp \!\!\!\!\perp e_Z$ , and  $e_Z \perp \!\!\!\!\perp e_Y$ .



Figure 3: The flow of association and causation in graphs. Extracted and slightly modified from Neal (2020, 31)

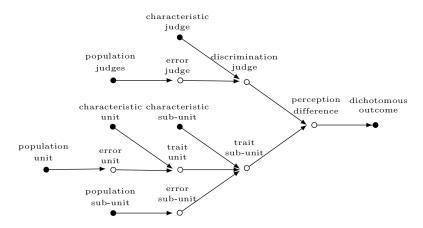


Figure 4: DCJ causal diagram, simplified description

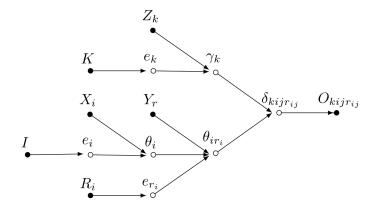


Figure 5: DCJ causal diagram, simplified mathematical description  $\,$ 

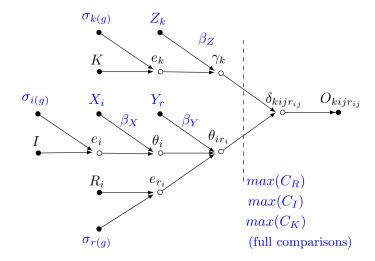


Figure 6: DCJ causal diagram, population mathematical description

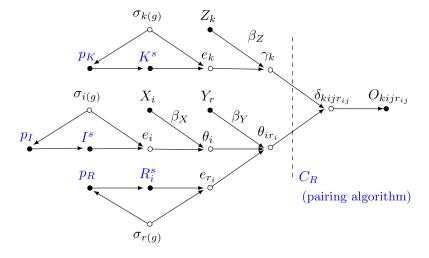


Figure 7: DCJ causal diagram, sample with comparisons mathematical description

- 2.3. The flow of association and causation in graphs
- 2.4. A motivating example

## 3. Theory

- 3.1. A scientific model for the DCJ
- 3.2. Probabilities assumptions of the scientific model
- 3.3. From the scientific to statistical model
- 3.4. Let's talk about Thurstone

## 4. Discussion

- 4.1. Findings
- 4.2. Limitations and further research

## 5. Conclusion

### **Declarations**

**Funding:** The project was founded through the Research Fund of the University of Antwerp (BOF).

Financial interests: The authors have no relevant financial interest to disclose.

**Non-financial interests:** Author XX serve on advisory broad of Company Y but receives no compensation this role.

**Ethics approval:** The University of Antwerp Research Ethics Committee has confirmed that no ethical approval is required.

Consent to participate: Not applicable

Consent for publication: All authors have read and agreed to the published version of the manuscript.

Availability of data and materials: No data was utilized in this study.

Code availability: All the code utilized in this research is available in the digital document located at: https://jriveraespejo.github.io/paper2\_manuscript/.

**Authors' contributions:** Conceptualization: S.G., S.DM., T.vD., and J.M.R.E; Methodology: S.DM., T.vD., and J.M.R.E; Software: J.M.R.E.; Validation: J.M.R.E.; Formal Analysis: J.M.R.E.; Investigation: J.M.R.E; Resources: S.G., S.DM., and T.vD.; Data curation: J.M.R.E.; Writing - original draft: J.M.R.E.; Writing - review & editing: S.G., S.DM., and T.vD.; Visualization: J.M.R.E.; Supervision: S.G. and S.DM.; Project administration: S.G. and S.DM.; Funding acquisition: S.G. and S.DM.

## 6. Appendix

- 6.1. Why do we need to estimate judges' abilities?
- 6.2. Latent variables as a mean of imputation
- $6.3.\ Other\ comparative\ scenarios$

#### References

- Agresti, A., 1992. Analysis of ordinal paired comparison data. Journal of the Royal Statistical Society 41, 287–297. URL: https://www.jstor.org/stable/2347562, doi:10.2307/2347562.
- Bartholomew, S., Nadelson, L., Goodridge, W., Reeve, E., 2018. Adaptive comparative judgment as a tool for assessing open-ended design problems and model eliciting activities. Educational Assessment 23, 85–101. doi:10.1080/10627197.2018.1444986.
- Bartholomew, S., Williams, P., 2020. Stem skill assessment: An application of adaptive comparative judgment, in: Anderson, J., Li, Y. (Eds.), Integrated Approaches to STEM Education. Advances in STEM Education. Springer, pp. 331–349. doi:10.1007/978-3-030-52229-2\_18.
- Boonen, N., Kloots, H., Gillis, S., 2020. Rating the overall speech quality of hearing-impaired children by means of comparative judgements. Journal of Communication Disorders 83, 1675–1687. doi:10.1016/j.jcomdis.2019.105969.
- Bouwer, R., Lesterhuis, M., De Smedt, F., Van Keer, H., De Maeyer, S., 2023. Comparative approaches to the assessment of writing: Reliability and validity of benchmark rating and comparative judgement. Journal of Writing Research 15, 497–518. URL: https://www.jowr.org/index.php/jowr/article/view/867, doi:10.17239/jowr-2024.15.03.03.
- Bradley, R., Terry, M., 1952. Rank analysis of incomplete block designs: I. the method of paired comparisons. Biometrika 39, 324–345. URL: http://www.jstor.com/stable/2334029, doi:10.2307/2334029.
- Bramley, T., 2015. Investigating the reliability of adaptive comparative judgment. URL <a href="http://www.cambridgeassessment.org.uk/Images/232694-investigating-the-reliability-of-adaptive-comparative-judgment.pdf">http://www.cambridgeassessment.org.uk/Images/232694-investigating-the-reliability-of-adaptive-comparative-judgment.pdf</a>. cambridge Assessment Research Report.
- Bramley, T., Vitello, S., 2019. The effect of adaptivity on the reliability coefficient in adaptive comparative judgement. Assessment in Education: Principles, Policy and Practice 71, 1–25. doi:10.1080/0969594X.2017.1418734.
- Cinelli, C., Forney, A., Pearl, J., 2020. A crash course in good and bad controls. SSRN URL: https://ssrn.com/abstract=3689437, doi:10.2139/ssrn.3689437.
- Crompvoets, E.A.V., Béguin, A.A., Sijtsma, K., 2022. On the bias and stability of the results of comparative judgment. Frontiers in Education 6. URL: url{https://www.frontiersin.org/articles/10.3389/feduc.2021.788202}, doi:10.3389/feduc.2021.788202.
- Everitt, B., Skrondal, A., 2010. The Cambridge Dictionary of Statistics. Cambridge University Press. Fisher, R., 1922. On the interpretation of 2 from contingency tables, and the calculation of p. Journal of the Royal Statistical Society 85, 87–94. URL: http://www.jstor.org/stable/2340521, doi:10.2307/2340521
- Gijsen, M., van Daal, T., Lesterhuis, M., Gijbels, D., De Maeyer, S., 2021. The complexity of comparative judgments in assessing argumentative writing: An eye tracking study. Frontiers in Education 5. URL: url{https://www.frontiersin.org/articles/10.3389/feduc.2020.582800}, doi:10.3389/feduc.2020.582800.
- Gross, J., Yellen, J., Anderson, M., 2018. Graph Theory and Its Applications. Textbooks in Mathematics, Chapman and Hall/CRC. doi:https://doi.org/10.1201/9780429425134. 3rd edition.
- Hernán, M., Robins, J., 2020. Causal Inference: What If. 1 ed., Chapman and Hall/CRC. URL: https://www.hsph.harvard.edu/miguel-hernan/causal-inference-book. last accessed 31 July 2024.
- Jones, I., 2015. The problem of assessing problem solving: can comparative judgement help? Educational Studies in Mathematics 89, 337–355. doi:10.1007/s10649-015-9607-1.
- Lesterhuis, M., 2018. The validity of comparative judgement for assessing text quality: An assessor's perspective. Ph.D. thesis. University of Antwerp.
- Luce, R., 1959. On the possible psychophysical laws. The Psychological Review 66, 482–499. doi:10.1037/h0043178.
- Marshall, N., Shaw, K., Hunter, J., Jones, I., 2020. Assessment by comparative judgement: An application to secondary statistics and english in new zealand. New Zealand Journal of Educational Studies 55, 49–71. doi:10.1007/s40841-020-00163-3.
- McElreath, R., 2020. Statistical Rethinking: A Bayesian Course with Examples in R and STAN. Chapman and Hall/CRC.
- Morgan, S., Winship, C., 2014. Counterfactuals and Causal Inference: Methods and Principles for Social Research. Analytical Methods for Social Research. 2 ed., Cambridge University Press.
- Neal, B., 2020. Introduction to causal inference from a machine learning perspective. URL: https://www.bradyneal.com/Introduction\_to\_Causal\_Inference-Dec17\_2020-Neal.pdf. last accessed 30 April 2024.

- Neyman, J., 1923. On the application of probability theory to agricultural experiments. essay on principles. section 9. Statistical Science 5, 465–472. URL: <a href="http://www.jstor.org/stable/2245382">http://www.jstor.org/stable/2245382</a>. translated by Dabrowska, D. and Speed, T. (1990).
- Pearl, J., 2009. Causality: Models, Reasoning and Inference. Cambridge University Press.
- Pearl, J., 2010. An introduction to causal inference. The international journal of biostatistics 6, 855–859. URL: https://www.degruyter.com/document/doi/10.2202/1557-4679.1203/html, doi:10.2202/1557-4679.1203.
- Pearl, J., 2019. The seven tools of causal inference, with reflections on machine learning. Communications of the ACM 62, 54–60. doi:10.1177/0962280215586010.
- Pearl, J., Mackenzie, D., 2018. The Book of Why: The New Science of Cause and Effect. 1st ed., Basic Books. Inc.
- Plackett, R., 1975. The analysis of permutations. Journal of the Royal Statistical Society 24, 193–202. URL: https://www.jstor.org/stable/2346567, doi:10.2307/2346567.
- Pollitt, A., 2012a. Comparative judgement for assessment. International Journal of Technology and Design Education 22, 157—170. doi:10.1007/s10798-011-9189-x.
- Pollitt, A., 2012b. The method of adaptive comparative judgement. Assessment in Education: Principles, Policy and Practice 19, 281—300. doi:10.1080/0969594X.2012.665354.
- Rohrer, J., 2018. Thinking clearly about correlations and causation: Graphical causal models for observational data. Advances in Methods and Practices in Psychological Science 1, 27–42. doi:10.1177/2515245917745629.
- Rubin, D., 1974. Estimating causal effects of treatments in randomized and nonrandomized studies. Journal of Educational Psychology 66, 688–701. doi:10.1037/h0037350.
- Sekhon, J., 2009. The neyman-rubin model of causal inference and estimation via matching methods, in: Box-Steffensmeier, J., Brady, H., Collier, D. (Eds.), The Oxford Handbook of Political Methodology. Oxford University Press, pp. 271–299. doi:10.1093/oxfordhb/9780199286546.003.0011.
- Spirtes, P., Glymour, C., Scheines, R., 1991. From probability to causality. Philosophical Studies 64, 1–36. URL: https://www.jstor.org/stable/4320244.
- Thurstone, L., 1927. A law of comparative judgment. Psychological Review 34, 482–499. doi:10.1037/h0070288.
- Tutz, G., 1986. Bradley-terry-luce model with an ordered response. Journal of Mathemathical Psychology 30, 306–316. doi:10.1016/0022-2496(86)90034-9.
- van Daal, T., 2020. Making a choice is not easy?!: Unravelling the task difficulty of comparative judgement to assess student work. Ph.D. thesis. University of Antwerp.
- van Daal, T., Lesterhuis, M., Coertjens, L., Donche, V., De Maeyer, S., 2019. Validity of comparative judgement to assess academic writing: examining implications of its holistic character and building on a shared consensus. Assessment in Education: Principles, Policy & Practice 26, 59–74. doi:10.1080/0969594X.2016.1253542.
- van Daal, T., Lesterhuis, M., Coertjens, L., van de Kamp, M., Donche, V., De Maeyer, S., 2017. The complexity of assessing student work using comparative judgment: The moderating role of decision accuracy. Frontiers in Education 2. URL: https://www.frontiersin.org/articles/10.3389/feduc.2017.00044, doi:10.3389/feduc.2017.00044.
- Verhavert, S., Bouwer, R., Donche, V., De Maeyer, S., 2019. A meta-analysis on the reliability of comparative judgement. Assessment in Education: Principles, Policy and Practice 26, 541–562. doi:10.1080/0969594X.2019.1602027.