Causes and effects in Dichotomous Comparative Judgments: an information-theoretical system of plausible mechanism

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Abstract

Dichotomous Comparative Judgment (DCJ) requires judges to compare pairs of stimuli to determine which one exhibits a higher degree of a specific trait. DCJ has proven effective and reliable across various fields (Pollitt, 2012b; Jones, 2015; van Daal et al., 2019; Bartholomew et al., 2018; Lesterhuis, 2018; Bartholomew and Williams, 2020; Marshall et al., 2020; Boonen et al., 2020). However, despite the method's widespread use, existing literature lacks a clear explanation of the complexities and assumptions underpinning the DCJ system, as well as the plausible mechanisms through which DCJ data could be generated. This study addresses these issues by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach, the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates this model into a probabilistic statistical model to estimate statistical relationships and infer causal effects within the system. This research provides a robust probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone's law of comparative judgment (1927). Its findings offer valuable insights for researchers and analysts designing and implementing DCJ experiments.

Keywords: causal inference, probability, Thurstone, comparative judgement, directed acyclic graph, structural causal models, statistical modeling

1. Introduction

In contemporary contexts, Thurstone's law of comparative judgment (1927) primarily refers to the method of *dichotomous* comparative judgment (DCJ, Pollitt, 2012a,b). In DCJ, a judge assesses the relative manifestation of a *trait* within a pair of stimuli. This assessment results in a dichotomous value indicating which stimulus possesses a

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Email addresses: JoseManuel.RiveraEspejo@uantwerpen.be (Jose Manuel Rivera Espejo), tine.vandaal@uantwerpen.be (Tine van Daal), sven.demaeyer@uantwerpen.be (Sven De Maeyer), steven.gillis@uantwerpen.be (Steven Gillis) higher degree of the trait. After different judges perform multiple rounds of pairwise comparisons, an outcome vector is produced. This vector is modeled using the Bradley-Terry-Luce model (BTL, Bradley and Terry, 1952; Luce, 1959), which creates a score that corresponds with the trait of interest. This score is then used to rank the stimuli from lowest to highest or to evaluate the influence of certain variables on the stimuli's positions in the ranking.

DCJ has proven effective in assessing competencies and traits predominantly within the educational realm, as demonstrated by Pollitt (2012b), Jones (2015), van Daal et al. (2019), Bartholomew et al. (2018), Lesterhuis (2018), Bartholomew and Williams (2020), and Marshall et al. (2020). However, its application transcends education, as exemplified by Boonen et al. (2020). The methodology has also evolved to include multiple, as opposed to pairwise comparisons (Luce, 1959; Plackett, 1975), and to accommodate comparisons with ordinal outcomes (Tutz, 1986; Agresti, 1992). Overall, research suggests that DCJ offers an alternative and efficient approach to measurement and evaluation, characterized by its reliability and validity (Lesterhuis, 2018; van Daal, 2020; Marshall et al., 2020; Bouwer et al., 2023). Nevertheless, despite the method's widespread use, existing literature lacks a clear representation of the plausible mechanisms through which DCJ data could be generated. Particularly, there is no depiction of the complexity and the assumptions underpinning the DCJ system, nor how different assessment factors can potentially influence the observed DCJ outcome.

According to Verhavert et al. (2019) and van Daal (2020), several assessment factors interact and influence the method's outcome. These factors include the number and characteristics of the stimuli, their *proximity* in terms of the assessed trait, the number of comparison per stimulus, and the pairing algorithm used. Furthermore, since the method relies on judges' assessments, the number and characteristics of judges, their *discrimination* abilities, and the number of comparisons per judge also play pivotal roles. Moreover, when the stimuli represent sub-units of higher-levels units, factors such as the number and characteristics of these units, along with their *proximity* in terms of the assessed trait, can significantly influence the outcome. For example, in van Daal et al. (2019), the authors assessed the academic writing skills of university students (units) using multiple argumentative essays (sub-units).

Although several studies have examined the individual impact of these factors on the method's reliability (Bramley, 2015; Pollitt, 2012b; Bramley and Vitello, 2019; Verhavert et al., 2019; Crompvoets et al., 2022; van Daal et al., 2017; Gijsen et al., 2021; Bouwer et al., 2023), none, to the best of the authors' knowledge, have provided a transparent depiction of the DCJ system and the mechanisms generating the DCJ outcome. This study aims to fill this gap by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach to causal inference, the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates the scientific model into a probabilistic statistical model. This model aims to produce statistical estimates to draw inferences about plausible causal relationships within the DCJ system.

Ultimately, this study provides a robust causal and probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone's law of comparative judgment (1927). Consequently, its findings offer valuable insights for researchers and analysts designing and implementing DCJ experiments.

2. Theoretical framework

This section introduces fundamental concepts in causal inference but does not offer a comprehensive description of causal inference methods. Readers interested in deeper exploration should consult introductory papers like Pearl (2010), Rohrer (2018), Pearl (2019), and Cinelli et al. (2020). They may also find introductory books such as Pearl and Mackenzie (2018), Neal (2020) and McElreath (2020) useful. For more advanced study, seminal intermediate papers like Neyman (1923), Rubin (1974), Spirtes et al. (1991), and Sekhon (2009), as well as books like Pearl (2009), Morgan and Winship (2014) and Hernán and Robins (2020) are recommended.

2.1. The structural approach to causal inference

Empirical research addresses real-world challenges by relying on evidence gathered through observation and experimentation. In this context, researchers typically frame their research questions as *estimands* or *targets of inference*. These estimands represent the specific quantities the study aims to determine (Everitt and Skrondal, 2010). For instance, a study might examine the question, "To what extent do different teaching methods (T) influence students' conceptual understanding of a topic (Y)?" To investigate this, the study could randomly assign students to two groups, each using a different teaching method ($T = \{1, 2\}$). Students' conceptual understanding of the topic could be evaluated through pairwise comparisons, resulting in a dichotomous outcome ($Y = \{0, 1\}$), indicating which student among those compared has a higher level of understanding. The research question could be then framed as the estimand, "*On average*, is there a difference in conceptual understanding of the topic between the two groups of students?" This estimand could be mathematically expressed by the random quantity $E[Y|T = 1] - E[Y|T = 2]$, where $E[\cdot]$ denotes the expected value. An example of this approach is seen in Jones et al. (2019).

Researchers then proceed to identify the estimands. *Identification* refers to the process of accurately computing an estimand using an estimator. An *estimator* is a method or function that transforms data into an estimate (Neal, 2020). *Estimates* are numerical values that approximate the estimand and are derived through *estimation*, which refers to the process of integrating data with an estimator (Everitt and Skrondal, 2010). Although various methods can approximate an estimand, researchers prioritize estimators with desirable properties that ensure the accuracy of estimates. For instance, the Z-test is an estimator known for its effectiveness in comparing groups' proportions, yielding accurate estimates when the underlying assumptions of the statistic are met (Kanji, 2006). The Z-test is expressed as a signal-to-noise ratio: $Z = (\hat{p}_1 - \hat{p}_2)/\hat{s}_p$. The signal is the difference between the group sample proportions, $\hat{p}_1 = \sum_{i=1}^{n_1} Y_i/n_1$ and $\hat{p}_2 = \sum_{i=1}^{n_2} Y_i/n_2$, analogous to $E[Y|T = 1]$ and $E[Y|T = 2]$, respectively. The noise $\hat{s}_p$ is the unpooled sample variability observed between the two groups.

However, many studies aim to understand the mechanisms underlying specific data and also seek to establish causal relationships rather than merely infer associations. In the earlier example, the differences between groups obtained using the Z-test, referred to as the associational estimate, can be interpreted as causal because the data were collected through a randomized experiment. Randomized experiments enable the causal interpretation of associational estimates by ensuring several key properties, including common support, no interference, and consistency (Morgan and Winship, 2014; Neal, 2020). The most crucial property, however, is that randomization effectively eliminates confounding. *Confounding* occurs when an external variable influences both the outcome and the variable of interest, leading to spurious associations (Everitt and Skrondal, 2010). Randomization mitigates this issue by decoupling the intervention assignment mechanism, such as assigning students to different groups, from other variables and outcomes (Morgan and Winship, 2014; Neal, 2020).

Experiments are widely recognized as the gold standard in evidence-based science (Hariton and Locascio, 2018; Hansson, 2014). However, researchers often face constraints that limit their ability to conduct experimental studies. These constraints include ethical concerns, such as the assignment of individuals to potentially harmful interventions, and practical limitations, such as the infeasibility of, for example, assigning individuals to genetic modifications or physical impairments (Neal, 2020). In these situations, causal inference provides a valuable alternative for generating causal estimates, particularly when the goal is to understand the mechanisms underlying specific data. Moreover, the framework provides significant theoretical insights that enhance the design of observational and experimental studies (McElreath, 2020).

Unlike classical statistical modeling, which focuses primarily on summarizing data and inferring associations, *causal inference* is a framework designed to identify causes and estimate their effects using data (Shaughnessy et al., 2010; Neal, 2020). The framework employs rigorous mathematical techniques to address the *fundamental problem of causality* (Pearl, 2009). This problem revolves around the question, “What would have happened ‘in the world’ under different circumstances?” a concept known as counterfactuals, essential for understanding and defining causal effects. *Counterfactuals* represent hypothetical scenarios that are *contrary to fact*, where alternative outcomes resulting from a specific cause are neither observed nor observable (Neal, 2020; Counterfactual, 2024).

Although a comprehensive discussion of causes and counterfactuals exceeds the scope of this document, a brief overview of how the framework addresses the fundamental problem of causality can be provided. The framework begins by defining *individual causal effects* (ICE) as the difference between students’ observed and unobserved potential outcomes: $\tau_i = (Y_i|T_i = 1) - (Y_i|T_i = 2)$. Notice when a student is assigned to $T_i = 1$, the potential outcome under $T_i = 2$ is no longer observed nor observable, and thus termed a *counterfactual*. To overcome the challenge posed by counterfactuals, the framework extends the ICE to *average causal effects* (ACE). The ACE is defined as $\tau = E[\tau_i] = E[Y_i|T_i = 1] - E[Y_i|T_i = 2]$, representing the difference between the average of observed potential outcomes and counterfactuals across the sample. Finally, similar to experimental studies, the counterfactuals are identified from associational estimates by ensuring the absence of confounding. This is accomplished by statistically conditioning on a *sufficient adjustment set* of variables (X). As a result, the ACE is expressed as $\tau = E_X\left[E[Y_i|T_i = 1, X] - E[Y_i|T_i = 2, X]\right]$, where $E_X[\cdot]$ denotes the marginal expected value over X (Morgan and Winship, 2014).

Several approaches to causal inference and counterfactuals exist, but two are particularly prominent: the potential outcomes approach, also known as the Neyman-Rubin causal model (Neyman, 1923; Rubin, 1974), and the structural approach (Pearl, 2009; Pearl et al., 2016). Both approaches employ rigorous mathematical notation to characterize the ACE, but they do so in different ways (Neal, 2020). The potential outcomes approach relies on counterfactual notation, whereas the structural approach employs do-calculus (Pearl, 2009). Despite these differences, both notations can be expressed in terms of the other, and both approaches provide methods for using experimental and observational data to estimate causal effects (Pearl, 2010).

The structural approach, however, offers a notable advantage over the potential outcomes approach by allowing the graphical and formal representation of a system through directed acyclic graphs (DAG, Pearl, 2009; Pearl et al., 2016; Gross et al., 2018; Neal, 2020). DAGs function as heuristics, effectively conveying the presumed causal structure of the system underlying the ACE, referred to as a *scientific model*. They do not represent detailed statistical models but allow researchers to deduce which statistical models can provide valid causal inferences, assuming the causal structure depicted in the DAGs are accurate (McElreath, 2020). The Identification-Estimation flowchart in Figure 1 visually represents the process of transitioning from estimands to estimates, as well as the application of the scientific model and data to identify and estimate causal effects.

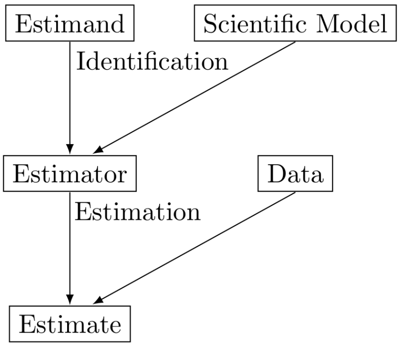


Figure 1: Identification-Estimation flowchart. Extracted and slightly modified from Neal (2020, 32)

2.2. DAGs and SCMs

Graph theory is a branch of mathematics focused on the study of graphs. Graphs are mathematical structures modeling pairwise relations between objects. They can represent physical relations, such as electrical circuits and roadways, and less tangible structures, such as ecosystems and sociological relations. Graphs have proven useful in various fields, including computer science, operations research, and the natural and social sciences (Gross et al., 2018).

In statistics, one application incorporating concepts from graph theory is causal inference. Specifically, the structural approach to causal inference uses directed acyclic graphs (DAG) to provide a graphical and formal representation of the causal structure of a system (Neal, 2020). In this context, a *graph* denotes a collection of nodes connected by edges, where nodes represent random variables. The term *directed* indicates the edges of the graph extend from one node to another, with arrows showing the direction of causal influence. Moreover, the term *acyclic* indicates the causal influences do not form a loop, meaning the influences do not cycle back on themselves (McElreath, 2020).

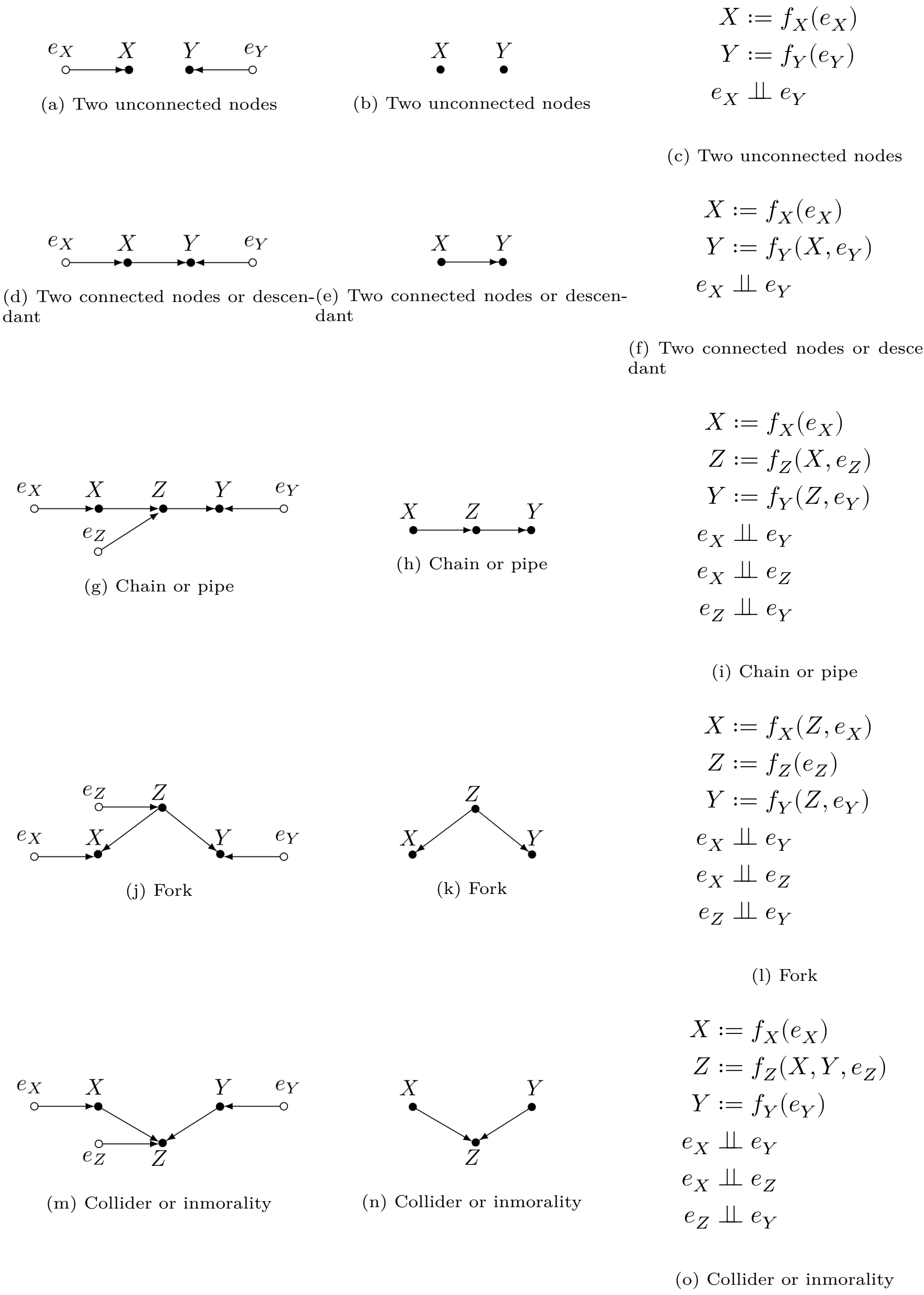
DAGs offer two key advantages for modeling causal structures. Firstly, they represent causal relations in a nonparametric and fully interactive manner. This feature allows for feasible causal analysis strategies without needing the specification of the type of data or the nature of the functional dependence among variables (Morgan and Winship, 2014). Secondly, regardless of complexity, DAGs can represent various causal structures using only five fundamental building blocks (Neal, 2020; McElreath, 2020). This feature enables the decomposition of complex structures into basic building blocks, facilitating the analysis of these structures by focusing on the causal assumptions associated with individual building blocks (McElreath, 2020). These building blocks can be represented in three ways: the magnified representation, the standard representation, and the structural causal model form (SCM, Morgan and Winship, 2014).

The left panels of Figure 2 illustrate the *magnified* representation. These graphs depict the *endogenous* variables $V = \{X, Z, Y\}$ alongside the *exogenous* variables $E = \{e_X, e_Z, e_Y\}$. Endogenous variables are those whose causal mechanisms the investigator chooses to model (Neal, 2020). In contrast, exogenous variables represent *errors* or *disturbances* arising from omitted factors that the investigator chooses not to model explicitly (Pearl, 2009, 27,68). The graphs show endogenous variables as solid black circles to signify that they are observed random variables, while endogenous variables are depicted as open circles to signify their unobserved (latent) nature. Lastly, the arrows in the graphs reflect the expected direction of causal influences among these variables.

Often, DAGs omit the exogenous variables for simplicity, resulting in the *standard* representation. However, including exogenous variables in a graph can be beneficial in some scenarios, as their presence can reveal potential issues related to conditioning and confounding (Cinelli et al., 2020), concepts explored in Section 2.3. The standard representation is illustrated in the middle panels of Figure 2.

Lastly, the right panels of Figure 2 depict the SCM form of the fundamental building blocks. SCMs are formal mathematical models defined by a set of endogenous variables V, a set of exogenous variables E, and a set of functions $F = \{f_X, f_Z, f_Y\}$ (Pearl, 2009; Neal, 2020). These functions, referred to as structural equations, specify each endogenous variable as nonparametric functions of other variables. Moreover, SCMs use the symbol '$:=$' to indicate the variables' asymmetrical causal dependence and the symbol '$\perp\!\!\!\perp$' to represent *d-separation*, which roughly equates to the concept of variable independence. The concepts of d-separation and causal (in)dependence are explored in Section 2.3.

A careful examination of Figure 2 highlights the assumptions underlying these building blocks. Figures 2a, 2b, and SCM 2c depict two unconnected nodes, representing a scenario where variables X and Y are not causally related. Figures 2d, 2e, and SCM 2f illustrate two connected nodes, showing a scenario where a *parent* node X exerts a causal influence on a *child* node Y. Consequently, Y is considered a *descendant* of X. Figures 2g, 2h, and SCM 2i depict a *chain* or *pipe*, where X influences Z, and Z influences Y. In this configuration, X is a parent node of Z, and Z is a parent node of Y. This creates


e_X X Y e_Y
X Y
e_X X Y e_Y
X Y
e_X X Z Y e_Y e_Z
X Z Y
e_Z Z e_X X Y e_Y
Z X Y
e_X X Y e_Y e_Z Z
X Y Z

(a) Two unconnected nodes

(b) Two unconnected nodes

$$X := f_X(e_X)$$
$$Y := f_Y(e_Y)$$
$$e_X \perp\!\!\!\perp e_Y$$

(c) Two unconnected nodes

(d) Two connected nodes or descendant

(e) Two connected nodes or descendant

$$X := f_X(e_X)$$
$$Y := f_Y(X, e_Y)$$
$$e_X \perp\!\!\!\perp e_Y$$

(f) Two connected nodes or descendant

(g) Chain or pipe

(h) Chain or pipe

$$X := f_X(e_X)$$
$$Z := f_Z(X, e_Z)$$
$$Y := f_Y(Z, e_Y)$$
$$e_X \perp\!\!\!\perp e_Y$$
$$e_X \perp\!\!\!\perp e_Z$$
$$e_Z \perp\!\!\!\perp e_Y$$

(i) Chain or pipe

(j) Fork

(k) Fork

$$X := f_X(Z, e_X)$$
$$Z := f_Z(e_Z)$$
$$Y := f_Y(Z, e_Y)$$
$$e_X \perp\!\!\!\perp e_Y$$
$$e_X \perp\!\!\!\perp e_Z$$
$$e_Z \perp\!\!\!\perp e_Y$$

(l) Fork

(m) Collider or inmorality

(n) Collider or inmorality

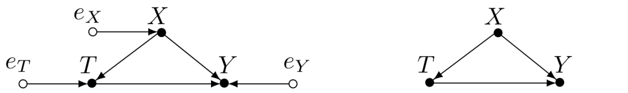
$$X := f_X(e_X)$$
$$Z := f_Z(X, Y, e_Z)$$
$$Y := f_Y(e_Y)$$
$$e_X \perp\!\!\!\perp e_Y$$
$$e_X \perp\!\!\!\perp e_Z$$
$$e_Z \perp\!\!\!\perp e_Y$$

(o) Collider or inmorality

Figure 2: The five fundamental building blocks of DAGs. **Note:** left panels show the magnified representation, middle panels show the standard representation, and the right panels show their corresponding SCM form.

a *directed path* between X and Y. Consequently, X is an *ancestor* of Y, and Z fully *mediates* the relationship between the two. Figures 2j, 2k, and SCM 2l illustrate a *fork*, where variables X and Y are both influenced by Z. Here, Z is a parent node of X and Y. Finally, Figures 2m, 2n, SCM 2o depict a *collider*, also known as *inmorality*, where variables X and Y are concurrent causes of Z. In this configuration, X and Y are not causally related to each other but both influence Z. Additionally, in all SCMs, the errors are assumed to be mutually independent of each other and of all other variables in the graph, as evidenced by the pairwise relations $e_X \perp\!\!\!\perp e_Y$, $e_X \perp\!\!\!\perp e_Z$, and $e_Z \perp\!\!\!\perp e_Y$.

The motivating example in Section 2.1 can be used to further illustrate how the five fundamental building blocks help construct a system's causal structure. In this scenario, an experiment cannot be conducted but the investigator still aims to determine whether, *on average*, there is a difference in conceptual understanding of a topic ($Y = \{0, 1\}$) between two groups of students ($T = \{1, 2\}$), described by the estimand $E[Y|T = 1] - E[Y|T = 2]$. The problem further suggests that the country to which a student belongs (X) may influence both T and Y. Such scenarios are plausible, especially when the teaching methods depend on software or access to technology, which may be limited in certain countries (maybe an example with more impact?). Figure 3 illustrates the plausible causal structure of this motivating example. A detailed examination of Figures 3a, 3b, and SCM 3c reveals the presence of at least four of the five fundamental building blocks. The figures display multiple descendants, as indicated by pairwise relations such as $X \rightarrow T$, $X \rightarrow Y$, and $T \rightarrow Y$. Additionally, the figures feature multiple pairs of unconnected nodes, evident from the relations $e_T \perp\!\!\!\perp e_X$, $e_T \perp\!\!\!\perp e_Y$, and $e_X \perp\!\!\!\perp e_Y$. Finally, the figures illustrate the fork $X \rightarrow \{T, Y\}$, and two colliders with $\{X, e_T\} \rightarrow T$ and $\{X, T, e_Y\} \rightarrow Y$.



(a) Magnified representation

(b) Standard representation

$$X := f_X(e_X)$$
$$T := f_Z(X, e_T)$$
$$Y := f_Y(T, X, e_Y)$$
$$e_T \perp\!\!\!\perp e_X$$
$$e_T \perp\!\!\!\perp e_Y$$
$$e_X \perp\!\!\!\perp e_Y$$

(c) Structural causal model

Figure 3: DAGs for a plausible causal structure in a system.

2.3. The flow of association and causation in graphs

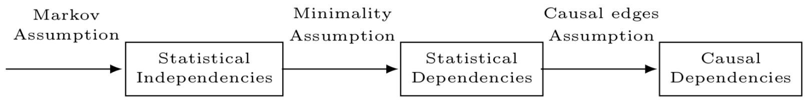


Figure 4: The flow of association and causation in graphs. Extracted and slightly modified from Neal (2020, 31)

3. Theory

3.1. A scientific model for the DCJ

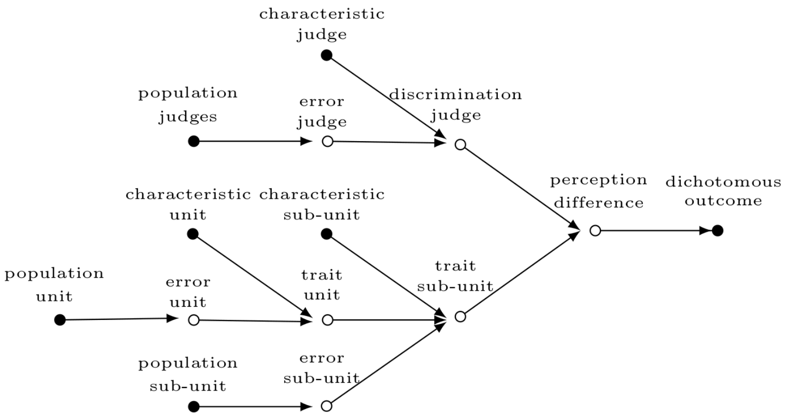


Figure 5: DCJ causal diagram, simplified description

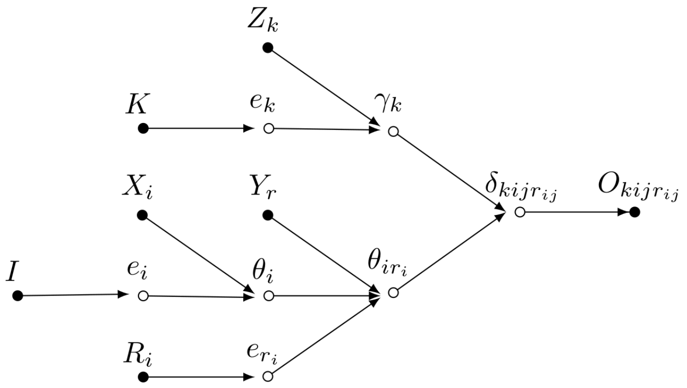


Figure 6: DCJ causal diagram, simplified mathematical description

3.2. Probabilitics assumptions of the scientific model

3.3. From the scientific to statistical model

3.4. Let's talk about Thurstone

4. Discussion

4.1. Findings

4.2. Limitations and further research

5. Conclusion

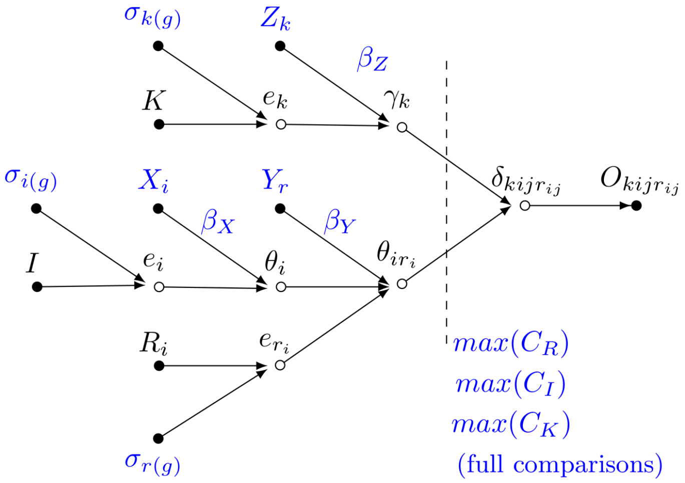


Figure 7: DCJ causal diagram, population mathematical description

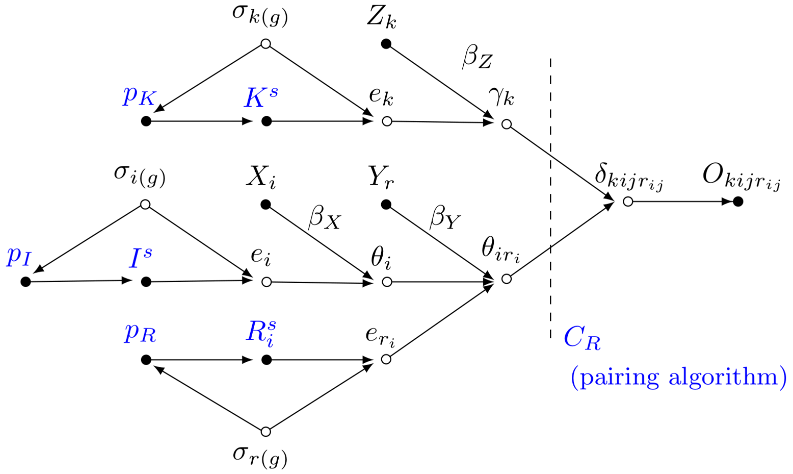


Figure 8: DCJ causal diagram, sample with comparisons mathematical description

Declarations

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Consent to participate: Not applicable

Consent for publication: All authors have read and agreed to the published version of the manuscript.

Availability of data and materials: No data was utilized in this study.

Code availability: All the code utilized in this research is available in the digital document located at: https://jriveraespejo.github.io/paper2_manuscript/.

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6. Appendix

6.1. Why do we need to estimate judges' abilities?

6.2. Latent variables as a mean of imputation

6.3. Other comparative scenarios

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