Let's talk about Thurstone & Co.: An information-theoretical model for comparative judgments, and its statistical translation

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Abstract

(to do)

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1. Introduction

In *comparative judgment* (CJ) studies, judges assess a specific trait or attribute across various stimuli by performing pairwise comparisons (Thurstone, 1927b,a). Each comparison produces a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher trait level. For example, when assessing text quality, judges compare pairs of written texts (the stimuli) to determine the relative quality each text exhibit (the trait) (Laming, 2004; Pollitt, 2012b; Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023).

Numerous studies have documented the effectiveness of CJ in assessing traits and competencies over the past decade. These studies have emphasized three aspects of the method's effectiveness: its reliability, validity, and practical applicability. Research on reliability indicates that CJ requires a relatively small number of pairwise comparisons (Verhavert et al., 2019; Crompvoets et al., 2022) to produce trait scores that are as precise and consistent as those generated by other assessment methods (Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023). Furthermore, evidence suggests that the reliability and time efficiency of CJ are comparable, if not superior, to those of other assessment methods when employing adaptive comparison algorithms (Pollitt, 2012b; Verhavert et al., 2022; Mikhailiuk et al., 2021). Meanwhile, research on

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validity suggests that scores generated by CJ can accurately represent the traits under measurement (Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Bartholomew et al., 2018; Bouwer et al., 2023), while research on practical applicability highlights the method's versatility across both educational and non-educational contexts (Kimbell, 2012; Jones and Inglis, 2015; Bartholomew et al., 2018; Jones et al., 2019; Marshall et al., 2020; Bartholomew and Williams, 2020; Boonen et al., 2020).

Nevertheless, despite the increasing number of CJ studies, unsystematic and fragmented research approaches have left several critical issues unaddressed. The present study primarily focuses on two: the over-reliance on the assumptions of Thurstone's Case V in the statistical analysis of CJ data, and the apparent disconnect between CJ's trait measurement and hypothesis testing. The following sections begin with a brief overview of Thurstone's theory and a detailed discussion of these issues. Subsequently, the study introduces a theoretical model for CJ that builds upon Thurstone's theory, alongside its statistical translation, designed to address the two concerns simultaneously.

2. Thurstone's theory

In its most general form, Thurstone's theory deals with pairwise comparisons of stimuli made by a single judge (Thurstone, 1927a, pp. 267). The theory proposes that two key factors determine the dichotomous outcome of these comparisons: the discriminal process of each stimulus and their discriminal difference. The *discriminal process* represents the psychological impact each stimulus has on judges or, more simply, their underlying perception of the stimulus' trait. According to the theory, the discriminal process for each stimulus follows a Normal distribution, where its mode (mean), referred to as the *modal discriminal process*, indicates the stimulus' position on the trait continuum, and its dispersion, known as the *discriminal dispersion*, reflects the variability of the stimulus' perceived trait.

For instance, Figure 1 illustrates the discriminal process distributions along a quality trait continuum for two written texts. The figure shows that these processes follow a Normal distribution. Moreover, it depicts differences in the texts' positions along the quality trait continuum, where text B is positioned further along the continuum than text A, as indicated by their modal discriminal processes (S_B and S_A). Finally, it highlights differences in the texts' discriminal dispersions (σ_B and σ_A), showing that text B exhibits a greater variability in its perceived quality than text A, as reflected by its wider distribution.

However, because the discriminal process of a single stimulus is not directly observable, the theory introduces the *law of comparative judgment*. This law posits that in pairwise comparisons, a judge perceives the stimulus positioned further along the trait continuum as having a higher level of that trait. This emphasizes that the outcome of a pairwise comparison likely depends on the relative distance between stimuli rather than their absolute positions on the trait continuum.

Indeed, the theory assumes that the observed dichotomous outcome arises from the distribution of the difference between the underlying discriminal processes of the stimuli, known as the *discriminal difference*. Since the individual discriminal processes follow a

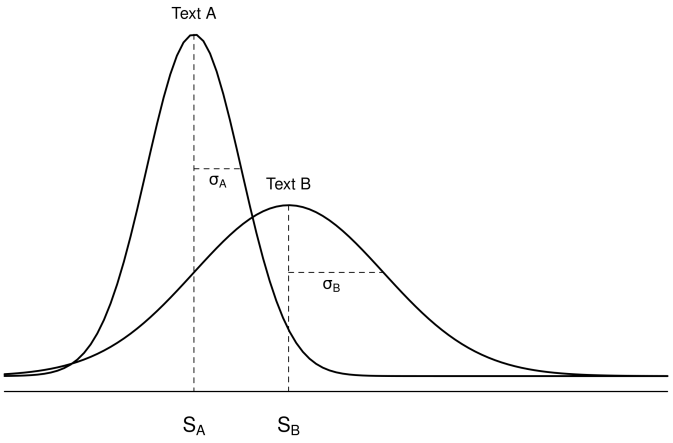


Figure 1: Example distributions of the discriminal processes for two written texts

Normal distribution, their difference also follows a Normal distribution (Andrich, 1978). The mode (mean) of this distribution, representing the (average) relative separation, is given by the difference between the modal discriminal processes of the stimuli $S_{BA} = S_B - S_A$. Meanwhile, the dispersion of the distribution, reflecting the variability in the relative separation, is calculated as $\sigma_{BA} = \sqrt{\sigma_B^2 + \sigma_A^2 - \rho\sigma_B\sigma_A}$. Here, σ_B and σ_A denote the discriminal dispersions of the stimuli, while ρ represents the correlation between their discriminal processes. This correlation quantifies the dependence of the judge's perception of the trait in one stimulus on his perception of the same trait in another.

Figure 2 shows the distribution of the discriminal difference for the texts depicted in Figure 1, assuming a correlation of $\rho = 0.6$. The figure reveals that, under these conditions, the judge perceives text B as having significantly higher quality than text A, as indicated by the shaded gray area under the curve $P(B > A)$. As a result, the dichotomous outcome of this comparison would likely favor text B over text A.

Notably, the correlation between the discriminal processes, ρ, plays a pivotal role in determining the comparison outcome by shaping the distribution of the discriminal difference between the stimuli. Specifically, as the correlation increases, reflecting a stronger dependence of the judge's perception of quality in one stimulus on his perception of the other, the distribution of the discriminal difference narrows. This narrowing ultimately impacts the area under the curve that determines the comparison outcome and, consequently, the conclusions drawn from this outcome.

Figure 3 illustrates how varying correlations influence the distribution of the discriminal difference for the texts depicted in Figure 1. Since the texts differ in quality, higher correlations increase the likelihood that the discriminal difference distinctly favors text B over text A. This is evident from the larger proportion of the area under the curve, $P(B > A)$, that lies above zero compared to curves with lower correlations. Moreover, although the figure does not illustrate this scenario, it is reasonable to infer that if the

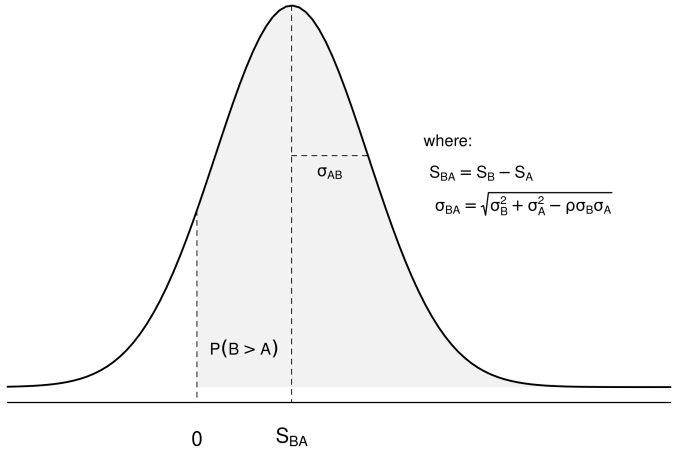


Figure 2: Example distribution of the discriminal difference for the two texts shown in Figure 1, assuming a correlation of 0.6

texts had similar or identical quality levels, higher correlations would likely reduce the chance of the discriminal difference distinctly favoring one text over the other. This probability reduction occurs because the distribution of the discriminal difference would become more narrowly centered around zero.

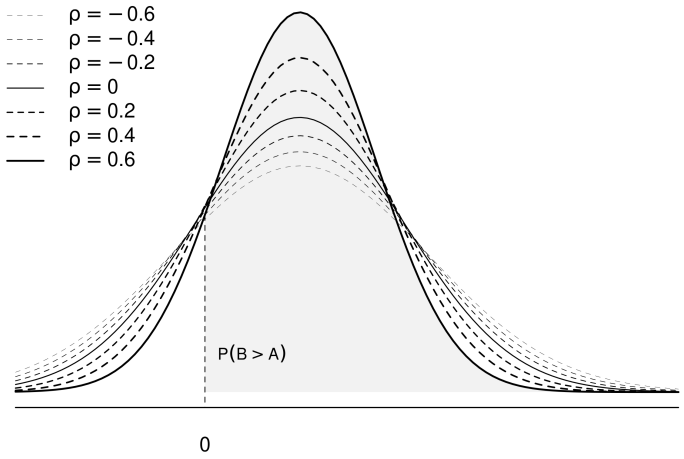


Figure 3: The effect of correlation on the distribution of the discriminal difference of the same two written text

Table 1: Thurstones cases and asumptions

Assumption	Thurstone's					BTL
	Case I	**Case II**	**Case III**	**Case IV**	**Case V**	**model**
Discriminal process (distribution)	Normal	Normal	Normal	Normal	Normal	Logistic
Discriminal dispersion (between stimuli)	Different	Different	Different	Similar	Equal	Equal
Correlation (between stimuli)	Constant	Constant	Zero	Zero	Zero	Zero
How many judges compare?	Single	Multiple	Multiple	Multiple	Multiple	Multiple

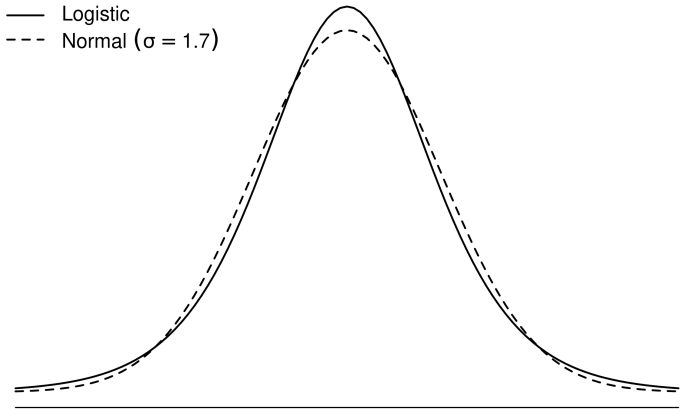
3. Three critical issues in CJ literature

3.1. The Case V and the statistical analysis of CJ data

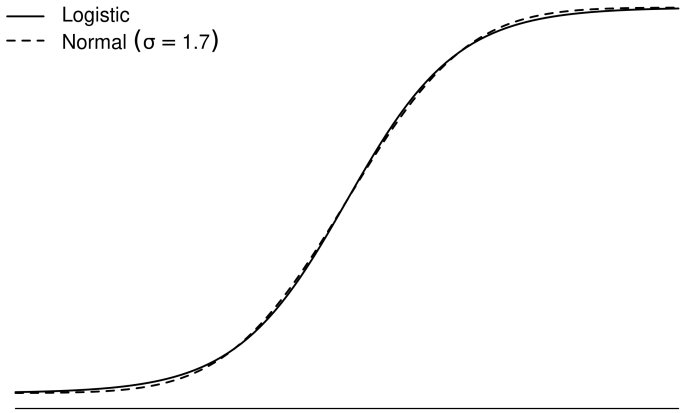
The previous section outlines the general form of Thurstone's theory, which applies to a CJ design where a single judge evaluates multiple stimuli. For the practical application of the theory, Thurstone developed four additional cases derived from this general form, where each successive case incorporates additional simplifying assumptions. Case I represents the general form of the theory. Case II extends this by allowing multiple judges to make comparisons rather than restricting the comparisons to a single judge. Case III introduces the assumption of zero correlation between stimuli. Case IV builds on this by assuming that the stimuli have similar dispersions. Finally, Case V replaces this assumption with the condition that the stimuli have equal discriminal dispersions. Table 1 summarizes these cases and their assumptions. For a detailed discussion of this progression, refer to Thurstone (1927a) and Bramley (2008, pp. 248-253).

Despite its reliance on the largest number of simplifying assumptions (Bramley, 2008, pp. 253; Kelly et al., 2022, pp. 677), Case V remains the most widely used case in the CJ literature. This popularity stems mainly from its simplified statistical representation in the Bradley-Terry-Luce (BTL) model (Bradley and Terry, 1952; Luce, 1959). The BTL model mirrors the assumptions of Case V, with one key difference: while Case V assumes a Normal distribution for the discriminal processes of the stimuli, the BTL model uses the more mathematically tractable Logistic distribution (Andrich, 1978; Bramley, 2008, pp. 254) (see Table 1). This substitution has little impact on the model's estimation or interpretation, as the Normal and Logistic distributions share similar statistical properties, differing only by a scaling factor of approximately 1.7 (van der Linden, 2017a, pp. 16) (see Figure 4).

Nevertheless, Thurstone originally developed Case V to provide a "rather coarse scaling" of traits (Thurstone, 1927a, pp. 269), prioritizing statistical simplicity over precision in trait measurement (Kelly et al., 2022, pp. 677). As a result, its assumptions may not be suitable for applications beyond the psycho-physical contexts for which it was created. Thurstone himself cautioned that its use "should not be made without (an) experimental test" (Thurstone, 1927a, pp. 270), acknowledging that some assumptions could prove problematic in the presence of complex traits or heterogeneous stimuli (Thurstone, 1927b, pp. 376). Consequently, given that modern CJ applications frequently involve these traits



(a) Probability density



(b) Cummulative probability

Figure 4: Probability density and cumulative probability of the logistic and Normal distributions. Extracted from Bramley (2008, pp. 254-255).

and stimuli, two main assumptions of Case V and, by extension, of the BTL model may not consistently hold in theory or practice: the equal dispersion and zero correlation between stimuli.

On the one hand, the assumption of *equal dispersion between stimuli* suggests that the variability of the perceived trait remains consistent across all stimuli. However, Thurstone contended that this assumption may not hold when researchers assess complex traits or heterogeneous stimuli (Thurstone, 1927b, pp. 376), as these traits and stimuli can introduce judgment discrepancies due to their unique features. Indeed, evidence of such violation may already exist in the CJ literature as misfit statistics. These statistics measure the judgment discrepancies associated with a given stimulus (Pollitt, 2004, pp. 12; Goossens and De Maeyer, 2018, pp. 20). For instance, labeling texts as "misfits" indicates that comparisons involving these texts result in more judgment discrepancies than comparisons involving other texts (Pollitt, 2012a,b; van Daal et al., 2016; Goossens and De Maeyer, 2018). This finding implies that the discriminal differences associated with "misfits" texts usually display a broader dispersion, suggesting that their discriminal processes also exhibit more variation than other texts. Notably, this reasoning also applies to "misfit" judges, whose evaluations reflect substantial deviations from the shared consensus due to the unique characteristics of the stimuli or the judges themselves.

Nonetheless, assuming equal dispersions between stimuli despite its violation can lead Case V (and the BTL model) to overlook critical differences in the reliability of the trait across stimuli, resulting in inaccurate conclusions about the trait's estimates (McElreath, 2020, pp. 370). Furthermore, if researchers acknowledge that misfit statistics help identify these critical differences, the common practice in the CJ literature of excluding stimuli based on these statistics (Pollitt, 2012b; van Daal et al., 2017; Goossens and De Maeyer, 2018) risks discarding valuable information and introducing bias into the trait's estimates (Zimmerman, 1994; McElreath, 2020, chap. 12). The direction and magnitude of these biases are unpredictable, as they depend on the specific stimuli researchers exclude from the analysis. Taken together, these oversights undermine the reliability of the trait and ultimately compromise its validity (Perron and Gillespie, 2015, pp. 2).

Fortunately, the statistical literature offers a solution to address the abovementioned issues. This solution involves using models that extend traditional approaches to account for the different variability in the stimuli, referred to as over-dispersed models (McElreath, 2020, chap. 12).

On the other hand, the assumption of *zero correlation between stimuli* implies that, during a pairwise comparison, a judge's perception of quality in one text does not influence his perception of the same trait in another text (see Section 2). Thurstone attributed this independence to the cancellation of potential judges' biases, driven by two opposing and equally weighted effects occurring during the pairwise comparisons (Thurstone, 1927a, pp. 268). Andrich (1978) mathematically demonstrated this cancellation using the BTL model under the assumption of discriminal processes with additive biases. However, it is easy to imagine at least two scenarios where the zero correlation assumption almost certainly does not hold: when the pairwise comparison involves multidimensional, complex traits with heterogeneous stimuli and when an additional hierarchical structure is relevant to the stimuli.

In the first scenario, the intricate aspects of multidimensional, complex traits may introduce dependencies between the stimuli due to certain judges' biases that resist cancellation. Research on text quality suggests that when judges evaluate these traits, they often rely on various intricate characteristics of the stimuli to form their judgments (van Daal et al., 2016; Lesterhuis, 2018b; Chambers and Cunningham, 2022). These additional relevant characteristics, which are unlikely to be equally weighted or opposing, can unevenly influence judges' perceptions, creating biases in their judgments and, ultimately, introducing dependencies between stimuli (van der Linden, 2017b, pp. 346). For example, this could occur when a judge assessing the argumentative quality of a text places more weight on grammatical accuracy than other judges, ultimately favoring texts with fewer errors but weaker arguments. While direct evidence for this specific scenario is lacking, studies such as Pollitt and Elliott (2003) demonstrate the presence of such biases, supporting the idea that the factors influencing pairwise comparisons may not always cancel out.

In the second scenario, the shared context or inherent connections created by additional hierarchical structures may further introduce dependencies between stimuli, a statistical phenomenon commonly known as clustering (Everitt and Skrondal, 2010). Although the CJ literature acknowledges the presence of such hierarchical structures, the statistical handling of this extra source of dependency between stimuli has been inadequate. For example, when CJ data includes multiple samples of stimuli from the same individuals, researchers often rely on (average) estimated BTL scores to conduct subsequent analyses and tests at the individual hierarchical level (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijsen et al., 2021). This approach, however, has the significant limitation of ignoring the uncertainty associated with the BTL scores, which generates additional statistical and measurement issues, as discussed in section Section 3.2.

In any case, the psychometric and statistical literature emphasizes the importance of addressing the factors that create dependencies between stimuli, as failing to do so can lead to inaccurate conclusions about a trait's reliability and, by extension, its validity. For instance, neglecting additional traits relevant to the stimuli, such as judges' biases, often leads to a dimensional mismatch in the statistical model used for analysis. This mismatch can potentially inflate the reliability of the trait (Hoyle, 2023, pp. 341) or, worse, introduce bias into the trait's estimates (Ackerman, 1989). Similarly, failing to account for hierarchical (grouping) structures reduces the precision of model parameter estimates, further amplifying the overestimation of reliability (Hoyle, 2023, pp. 482). These issues collectively also undermine the trait's reliability and ultimately compromise the validity of the trait's estimates (Perron and Gillespie, 2015, pp. 2).

Fortunately, the same literature offers solutions for addressing these issues. Andrich (1978) and Wainer et al. (1978) recommend integrating judges' biases into the BTL model. Moreover, the literature advocates for the incorporation of relevant hierarchical structures into the statistical model to account for these dependencies. Together, these additions can result in a model resembling a Multilevel Structural Equation Model (MSEM) (Hoyle, 2023, chap. 26) combined with a multidimensional or two-parameter logistic Item Response Theory (IRT) model (Hoyle, 2023, chap. 15), depending on the theoretical and statistical treatment of judges' biases.

3.2. The disconnect between trait measurement and hypothesis testing

Building on the previous section, it is evident that, besides its inconveniences, the BTL model commonly functions as the trait's measurement model in CJ assessments. A measurement model specifies how manifest variables contribute to the estimation of latent variables (Everitt and Skrondal, 2010). For example, when evaluating text quality, researchers use the BTL model to process the dichotomous outcomes resulting from the pairwise comparisons (the manifest variables) to estimate scores that reflect the underlying quality level of texts (the latent variable) (Laming, 2004; Pollitt, 2012b; Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023).

Researchers then typically use these estimated BTL scores, or their transformations, to conduct additional analyses or hypothesis tests. For example, these scores have been used to identify 'misfit' judges and stimuli (Pollitt, 2012b; van Daal et al., 2016; Goossens and De Maeyer, 2018), detect biases in judges' ratings (Pollitt and Elliott, 2003; Pollitt, 2012b), calculate correlations with other assessment methods (Goossens and De Maeyer, 2018; Bouwer et al., 2023), or test hypotheses related to the underlying trait of interest (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijsen et al., 2021).

However, the statistical literature advises caution when using estimated scores for additional analyses and tests. A key consideration is that BTL scores are parameter estimates that inherently carry uncertainty. Ignoring this uncertainty can bias the analysis and reduce the precision of hypothesis tests. Notably, the direction and magnitude of such biases are often unpredictable. Results may be attenuated, exaggerated, or remain unaffected depending on the degree of uncertainty in the scores and the actual effects being tested (Kline, 2023, pp. 25; Hoyle, 2023, pp. 137). Finally, the reduced precision in hypothesis tests diminishes their statistical power, increasing the likelihood of committing type-I or type-II errors (McElreath, 2020).

To mitigate these risks, principles from Structural Equation Modeling (SEM) (Hoyle, 2023, pp. 138) and IRT (Fox, 2010, chap. 6; van der Linden, 2017a, chap. 24) recommend conducting these analyses and tests within a structural model. A structural model specifies how different manifest or latent variables influence the latent variable of interest (Everitt and Skrondal, 2010). This approach allows analyses that can account for both the BTL scores and their uncertainties simultaneously, rather than treating them as separate elements.

4. An updated theoretical and statistical model for CJ

4.1. The theoretical model

4.2. From theory to statistics

5. Discussion

5.1. Findings

5.2. Limitations and further research

6. Conclusion

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