Let's talk about Thurstone & Co.: An information-theoretical model for comparative judgments, and its statistical translation

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Abstract

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1. Introduction

In *comparative judgment* (CJ) studies, judges assess a specific trait or attribute across various stimuli by performing pairwise comparisons (Thurstone, 1927b,a). Each comparison produces a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher trait level. For example, when assessing text quality, judges compare pairs of written texts (the stimuli) to determine the relative quality each text exhibit (the trait) (Laming, 2004; Pollitt, 2012; Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023).

Numerous studies have documented the effectiveness of CJ in assessing traits and competencies over the past decade. These studies have emphasized three aspects of the method's effectiveness: its reliability, validity, and practical applicability. Research on reliability indicates that CJ requires a relatively small number of pairwise comparisons (Verhavert et al., 2019; Crompvoets et al., 2022) to produce trait scores that are as precise and consistent as those generated by other assessment methods (Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023). Furthermore, evidence suggests that the reliability and time efficiency of CJ are comparable, if not superior, to those of other assessment methods when employing adaptive comparison algorithms (Pollitt,

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2012; Verhavert et al., 2022; Mikhailiuk et al., 2021). Meanwhile, research on validity suggests that scores generated by CJ can accurately represent the traits under measurement (Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Bartholomew et al., 2018; Bouwer et al., 2023), while research on practical applicability highlights the method's versatility across both educational and non-educational contexts (Kimbell, 2012; Jones and Inglis, 2015; Bartholomew et al., 2018; Jones et al., 2019; Marshall et al., 2020; Bartholomew and Williams, 2020; Boonen et al., 2020).

Nevertheless, despite the increasing number of CJ studies, unsystematic and fragmented research approaches have left several critical issues unaddressed. The present study primarily focuses on two: the over-reliance on the assumptions of Thurstone's Case V in the statistical analysis of CJ data, and the apparent disconnect between CJ's trait measurement and hypothesis testing. The following sections begin with a brief overview of Thurstone's theory and a detailed discussion of these issues. Subsequently, the study introduces a theoretical model for CJ that builds upon Thurstone's theory, alongside its statistical translation, designed to address the two concerns simultaneously.

2. Thurstone's theory

In its most general form, Thurstone's theory deals with pairwise comparisons of stimuli made by a single judge (Thurstone, 1927a, pp. 267). The theory proposes that two key factors determine the dichotomous outcome of these comparisons: the discriminal process of each stimulus and their discriminal difference. The *discriminal process* represents the psychological impact each stimulus has on judges or, more simply, their underlying perception of the stimulus' trait level. According to the theory, the discriminal process for each stimulus follows a Normal distribution, where its mode (or the mean), referred to as the *modal discriminal process*, indicates the stimulus' position on the trait continuum, and its dispersion, known as the *discriminal dispersion*, reflects the variability in the perceived trait level of the stimulus.

For instance, Figure 1 illustrates the discriminal process distributions along a quality trait continuum for two written texts. The figure shows that these processes follow a Normal distribution. Moreover, it depicts differences in the texts' positions along the quality trait continuum, where text B is positioned further along the continuum than text A, as indicated by their modal discriminal processes (S_B and S_A). Finally, it highlights differences in the texts' discriminal dispersions (σ_B and σ_A), showing that text B exhibits a greater variability in its perceived quality than text A, as reflected by its wider distribution.

However, because the discriminal process of a single stimulus is not directly observable, the theory introduces the *law of comparative judgment*. This law posits that in pairwise comparisons, a judge perceives the stimulus positioned further along the trait continuum as having a higher level of that trait. This principle highlights that the outcome of a pairwise comparison likely depends on the relative distance between stimuli rather than their absolute positions on the trait continuum.

Indeed, the theory assumes that the observed dichotomous outcome arises from the distribution of the difference between the underlying discriminal processes of the stimuli,

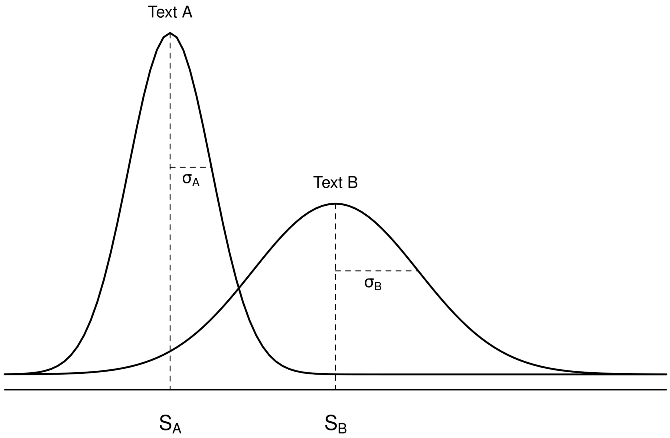


Figure 1: Example distributions of the discriminal processes for two written texts

known as the *discriminal difference*. Since the individual discriminal processes follow a Normal distribution, their difference also follows a Normal distribution. The mode (or the mean) of this distribution, representing the (average) relative separation, is given by the difference between the modal discriminal processes of the stimuli $S_{BA} = S_B - S_A$. Meanwhile, the dispersion of the distribution, reflecting the variability in the relative separation, is calculated as $\sigma_{BA} = \sqrt{\sigma_B^2 + \sigma_A^2 - \rho\sigma_B\sigma_A}$. Here, σ_B and σ_A are the previously defined discriminal dispersions, while ρ measures the correlation between the discriminal processes of the stimuli. This correlation quantifies the dependence of the judge's perception of the trait in one stimulus on his perception of the same trait in the other.

Figure 2 shows the distribution of the discriminal difference for the texts depicted in Figure 1, assuming a correlation of $\rho = 0.6$. The figure reveals that, under these conditions, the judge perceives text B as having significantly higher quality than text A, as indicated by the shaded gray area under the curve $P(B > A)$. As a result, the dichotomous outcome of this comparison almost certainly favors text B over text A.

Notably, the correlation between the discriminal processes, ρ, plays a pivotal role in determining comparison outcomes by shaping the distribution of the discriminal difference between the stimuli. Specifically, as the correlation increases, reflecting a stronger dependence of the judge's perception of quality in one stimulus on his perception of the other, the distribution of the discriminal difference narrows. This narrowing ultimately impacts the area under the curve that determines the comparison outcome and, consequently, the conclusions drawn from the comparison.

Figure 3 illustrates how varying correlations influence the distribution of the discriminal difference for the two texts depicted in Figure 1. Since the texts differ in quality, higher correlations increase the likelihood that the discriminal difference distinctly favors text B over text A. This is evident from the larger proportion of the area under the curve, $P(B > A)$, that lies above zero (shaded gray area in Figure 3). Conversely, it is not hard

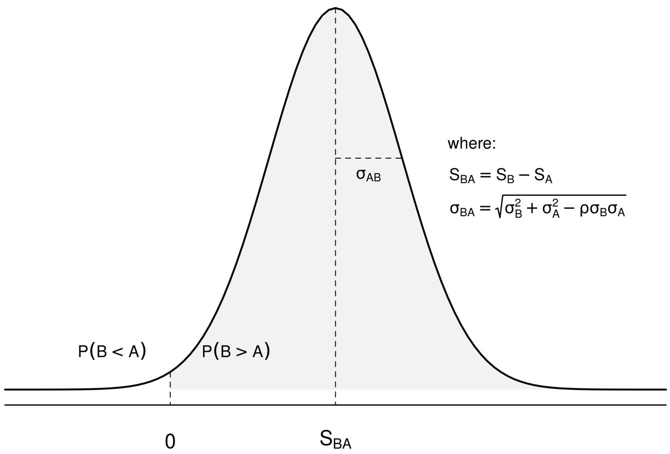


Figure 2: Example distribution of the discriminal difference for the two texts shown in Figure 1, assuming a correlation of 0.6

to infer that if the texts had similar or identical quality levels, higher correlations would likely reduce the chance that the discriminal difference distinctly favors one text over the other. This probability reduction occurs because the distribution of the discriminal difference would become more narrowly centered around zero (not illustrated).

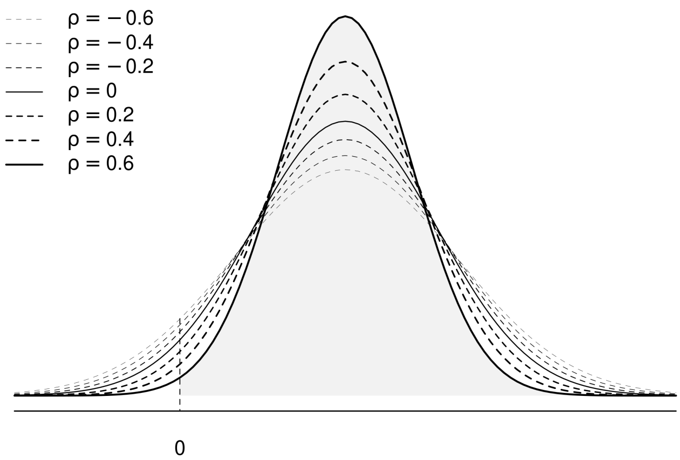


Figure 3: The effect of correlation on the distribution of the discriminal difference of the same two written text

Table 1: Thurstones cases and asumptions

Assumption	**Thurstone's** **Case I**	**Case II**	**Case III**	**Case IV**	**Case V**	**BTL model**
Discriminal process (distribution)	Normal	Normal	Normal	Normal	Normal	Logistic
Discriminal dispersion (between stimuli)	Different	Different	Different	Similar	Equal	Equal
Correlation (between stimuli)	Constant	Constant	Zero	Zero	Zero	Zero
How many judges compare?	Single	Multiple	Multiple	Multiple	Multiple	Multiple

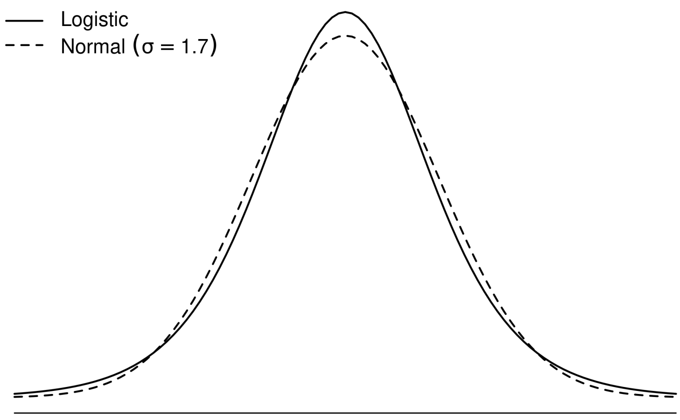
3. Three critical issues in CJ literature

3.1. The Case V and the statistical analysis of CJ data

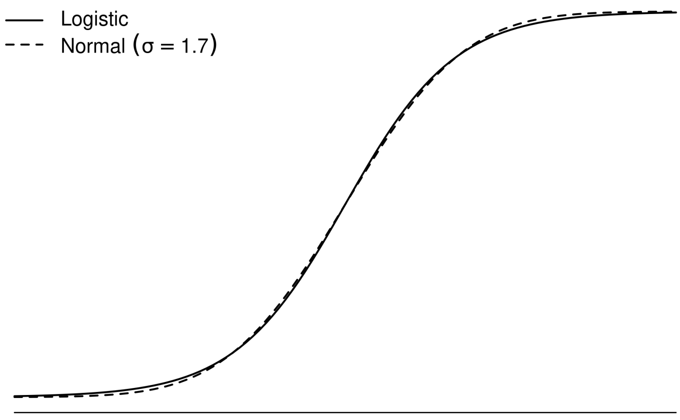
The previous section outlines the general form of Thurstone's theory, which applies to a CJ design where a single judge evaluates multiple stimuli. For the practical application of the theory, Thurstone developed four additional cases derived from this general form, where each successive case incorporates additional simplifying assumptions. Case I represents the general form of the theory. Case II extends this by allowing multiple judges to make comparisons rather than restricting the comparisons to a single judge. Case III introduces the assumption of zero correlation between stimuli. Case IV builds on this by assuming that the stimuli have similar dispersions. Finally, Case V replaces this assumption with the condition that the stimuli have equal discriminal dispersions. Table 1 summarizes these cases and their assumptions. For a detailed discussion of this progression, refer to Thurstone (1927a) and Bramley (2008, pp. 248-253).

Despite its reliance on the largest number of simplifying assumptions (Bramley, 2008, pp. 253; Kelly et al., 2022, pp. 677), Case V remains the most widely used case in the CJ literature. This popularity stems mainly from its simplified statistical representation in the Bradley-Terry-Luce (BTL) model (Bradley and Terry, 1952; Luce, 1959). The BTL model mirrors the assumptions of Case V, with one key difference: while Case V assumes a Normal distribution for the discriminal processes of the stimuli, the BTL model uses the more mathematically tractable Logistic distribution (Andrich, 1978; Bramley, 2008, pp. 254) (see Table 1). This substitution has little impact on the model's estimation or interpretation, as the Normal and Logistic distributions share similar statistical properties, differing only by a scaling factor of approximately 1.7 (van der Linden, 2017a, pp. 16) (see Figure 4).

However, Thurstone originally developed Case V to provide a "rather coarse scaling" of traits (Thurstone, 1927a, pp. 269), prioritizing statistical simplicity over precision in trait measurement (Kelly et al., 2022, pp. 677). As a result, its assumptions may not be suitable for applications beyond the psycho-physical contexts for which it was created. Thurstone himself cautioned that its use "should not be made without (an) experimental test" (Thurstone, 1927a, pp. 270), acknowledging that some assumptions could prove problematic in the presence of complex traits or heterogeneous stimuli, such as handwriting or English compositions (Thurstone, 1927b, pp. 374). Consequently, given that



(a) Probability density



(b) Cummulative probability

Figure 4: Probability density and cumulative probability of the logistic and Normal distributions. Extracted from Bramley (2008, pp. 254-255).

modern CJ applications frequently involve these traits and stimuli, two main assumptions of Case V may not consistently hold in theory or practice: the zero correlation and equal dispersion between stimuli.

As outlined in the previous section, the assumption of *zero correlation between stimuli* suggests that during a pairwise comparison, a judge's perception of quality in one text does not influence his perception of the same trait in another text (see Figure 3, $\rho = 0$). Thurstone attributed this independence to the cancellation of potential judges' biases, driven by two opposing and equally weighted effects occurring during the pairwise comparisons (Thurstone, 1927a, pp. 268). Andrich (1978) mathematically demonstrated this cancellation using the BTL model under the assumption of discriminal processes with additive biases. However, it is easy to imagine at least two scenarios where the zero correlation assumption almost certainly does not hold: when the pairwise comparison involves multidimensional, complex traits with heterogeneous stimuli and when an additional hierarchical structure is relevant to the stimuli.

In the first scenario, the intricate aspects of multidimensional, complex traits may introduce dependencies between heterogeneous stimuli due to certain judges' biases that resist cancellation. Research on text quality indicates that when judges evaluate these traits, they often rely on various intricate aspects of the stimuli to form their judgments (van Daal et al., 2016; Lesterhuis, 2018b; Chambers and Cunningham, 2022). These aspects, which are likely neither equally weighted nor opposing, may influence judges' perceptions unevenly. For example, this could occur when a judge assessing the argumentative quality of a text places disproportionate emphasis on grammatical accuracy, ultimately favoring texts with fewer errors but weaker arguments. Ignoring these relevant additional "traits" can result in biases that resist cancellation, introducing dependencies between stimuli (van der Linden, 2017b, pp. 346) and ultimately violating the assumption of zero correlation. While direct evidence for the specific mechanisms causing biases is lacking, studies such as Pollitt and Elliott (2003) demonstrate the presence of such biases, supporting the idea that the factors influencing pairwise comparisons may not always cancel out.

In the second scenario, the shared context or inherent connections created by additional hierarchical structures may introduce dependencies between stimuli, a statistical phenomenon commonly known as clustering (Everitt and Skrondal, 2010). Although the CJ literature acknowledges the presence of such hierarchical structures in CJ data, the statistical handling of this extra source of dependency has been inadequate. For example, when CJ data includes multiple samples of stimuli from the same individuals, researchers often rely on (average) estimated BTL scores to conduct subsequent analyses and tests at the individual hierarchical level (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijsen et al., 2021). This approach, however, has the significant limitation of ignoring the uncertainty associated with the BTL scores, which generates additional statistical and measurement issues, as discussed in section Section 3.2.

In any case, the psychometric and statistical literature strongly advises against ignoring relevant additional traits or overlooking clustering (grouping) structures that may introduce dependencies between stimuli. First, ignoring additional aspects pertinent to the trait of interest can cause a dimensional mismatch, which can lead to an overestimation of the precision (reliability) of the trait's measurement (Hoyle, 2023, pp. 340-341) or, worse, introduce bias into the measurement process (Ackerman, 1989). Additionally, overlooking relevant hierarchical (or grouping) structures can exacerbate the inflation of reliability by reducing the accuracy of the model's parameters (Hoyle, 2023, pp. 482). Taken together, these oversights undermine the reliability of the measurement process and, since validity cannot exist without reliability (Perron and Gillespie, 2015, pp. 2), ultimately compromise the validity of the results derived from it.

3.2. The disconnect between trait measurement and hypothesis testing

Building on the previous section, it is evident that the BTL model commonly functions as the trait's measurement model in CJ experiments (Andrich, 1978; Bramley, 2008). A measurement model specifies how manifest variables contribute to the estimation of latent variables (Everitt and Skrondal, 2010). For example, when evaluating text quality, researchers use the BTL model to process the dichotomous outcomes resulting from the pairwise comparisons (the manifest variables) to estimate scores that reflect the underlying quality level of texts (the latent variable) (Laming, 2004; Pollitt, 2012; Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023).

Researchers then typically use the estimated BTL scores, or their transformations, to conduct additional analyses or hypothesis tests. For example, these scores have been used to identify 'misfit' judges and stimuli (Pollitt, 2012; van Daal et al., 2017; Goossens and De Maeyer, 2018), detect biases in judges' ratings (Pollitt and Elliott, 2003; Pollitt, 2012), calculate correlations with other assessment methods (Goossens and De Maeyer, 2018; Bouwer et al., 2023), or test hypotheses related to the underlying trait of interest (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijsen et al., 2021).

However, the statistical literature advises caution when using estimated scores for additional analyses and tests. A key consideration is that BTL scores are parameter estimates that inherently carry uncertainty. Ignoring this uncertainty can bias the analysis and reduce the precision of hypothesis tests. Notably, the direction and magnitude of such biases are often unpredictable. Results may be attenuated, exaggerated, or remain unaffected depending on the degree of uncertainty in the scores and the actual effects being tested (Kline, 2023, pp. 25; Hoyle, 2023, pp. 137). Finally, the reduced precision in hypothesis tests diminishes their statistical power, increasing the likelihood of committing type-I or type-II errors (McElreath, 2020).

To mitigate these risks, principles from Structural Equation Modeling (SEM) (Hoyle, 2023, pp. 138) and Item Response Theory (IRT) (Fox, 2010, chap. 6; van der Linden, 2017a, chap. 24) recommend conducting these analyses and tests within a structural model. A structural model specifies how different manifest or latent variables influence the latent variable of interest (Everitt and Skrondal, 2010). This approach allows analyses that can account for both the BTL scores and their uncertainties simultaneously, rather than treating them as separate elements. Therefore, an integrated approach that combines CJ's measurement and structural models can offer significant advantages.

4. An updated theoretical and statistical model for CJ

4.1. The theoretical model

4.2. From theory to statistics

5. Discussion

5.1. Findings

5.2. Limitations and further research

6. Conclusion

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