Let's talk about Thurstone & Co.
An information-theoretical model for comparative judgments, and its statistical translation

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Abstract

(to do)

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1. Introduction

Over the past decade, numerous studies have documented the effectiveness of the *comparative judgment* (CJ) method (Thurstone, 1927) for assessing competencies and traits. These studies have evaluated CJ from three main perspectives: its ability to produce reliable and valid trait scores, its practical applicability, and its time efficiency. Research on reliability and validity shows that CJ can generate precise and consistent scores (Pollitt, 2012a,b; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Verhavert et al., 2019; Crompvoets et al., 2022; Bouwer et al., 2023) that accurately represent the traits being measured (Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018; Bouwer et al., 2023). Research on practical applicability highlights CJ's versatility across both educational and non-educational contexts, presenting it as an efficient and effective alternative for measurement and evaluation (Pollitt, 2004; Jones, 2015; Bartholomew et al., 2018; Jones et al., 2019; Marshall et al., 2020; Bartholomew and Williams, 2020; Boonen et al., 2020). Lastly, research on time efficiency suggests that CJ can offer at least equal, if not significant, time savings when evaluating stimuli compared to traditional marking methods (Pollitt, 2012a,b; Coertjens et al., 2017; Goossens and De Maeyer, 2018).

Nevertheless, despite the growing number of studies on CJ, unsystematic and fragmented research approaches in the literature have left several critical issues unaddressed.

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These issues can be grouped into three main categories: the disconnect between CJ's structural and measurement model, the over-reliance on the assumptions of Thurstone's Case 5 (1927) in its measurement model, and the unclear role and impact of comparison algorithms and the number of comparisons on the method's reliability and validity. The following sections will discuss each of these issues in detail,, followed by the introduction of a theoretical model and its statistical translation, which aim to address all three concerns simultaneously.

2. The three critical issues in CJ literature

2.1. The disconnect between structural and measurement models

In a typical CJ study, the Bradley-Terry-Luce (BTL) model (Bradley and Terry, 1952; Luce, 1959) serves as the measurement model. This model specifies how latent variables are estimated from observed ones (Hoyle, 2023; Kline, 2023). In the context of CJ, multiple judges engage in several rounds of pairwise comparisons to assess the relative manifestation of a trait between two stimuli. Each comparison generates a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher degree of the trait. The BTL model then uses these observed outcomes to estimate scores that represent the latent trait of interest (Pollitt, 2012a,b; Whitehouse, 2012; Jones, 2015; van Daal et al., 2016; Lesterhuis, 2018; Boonen et al., 2020; Bouwer et al., 2023).

Moreover, researchers often use these BTL-generated scores or their transformations separately, applying additional analyses or hypothesis testing. The CJ literature shows that these scores have been employed to identify 'misfit' judges and stimuli (Pollitt, 2012b; van Daal et al., 2017; Goossens and De Maeyer, 2018), detect biases in judges' ratings (Pollitt and Elliott, 2003; Pollitt, 2012b), calculate correlations with other scoring methods (Goossens and De Maeyer, 2018; Bouwer et al., 2023), or test hypotheses related to the trait of interest (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijsen et al., 2021).

However, the statistical literature warns against using estimated scores for separate analyses and hypothesis testing. BTL-generated scores are parameter estimates that inherently carry uncertainty. Neglecting this uncertainty can inflate the precision and statistical power in independent analyses and tests. This oversight, in turn, may increase the risk of committing a type I error, where the null hypothesis is incorrectly rejected (McElreath, 2020). To mitigate this risk, principles from Structural Equation Modeling (SEM) (Hoyle, 2023; Kline, 2023) recommend conducting such analyses within the structural model of CJ rather than separately. Therefore, an integrated approach combining both structural and measurement models is advisable.

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6. Appendix

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