

# Causes and effects in Dichotomous Comparative Judgments: an information-theoretical system with plausible mechanism

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## Abstract

Dichotomous Comparative Judgment (DCJ) requires judges to compare pairs of stimuli to determine which one exhibits a higher degree of a specific trait. DCJ has proven effective and reliable across various fields (Pollitt, 2012b; Jones, 2015; van Daal et al., 2019; Bartholomew et al., 2018; Lesterhuis, 2018; Bartholomew and Williams, 2020; Marshall et al., 2020; Boonen et al., 2020). However, despite the method's widespread use, existing literature lacks a clear explanation of the complexities and assumptions underpinning the DCJ system, as well as the plausible mechanisms through which DCJ data could be generated. This study addresses these issues by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach, the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates this model into a probabilistic statistical model to estimate statistical relationships and infer causal effects within the system. This research provides a robust probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone's law of comparative judgment (1927). Its findings offer valuable insights for researchers and analysts designing and implementing DCJ experiments.

*Keywords:* causal inference, probability, Thurstone, comparative judgement, directed acyclic graph, structural causal models, statistical modeling

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## 1. Introduction

In contemporary contexts, Thurstone's law of comparative judgment (1927) primarily refers to the method of *dichotomous* comparative judgment (DCJ, Pollitt, 2012a,b). In DCJ, a judge assesses the relative manifestation of a *trait* within a pair of stimuli. This assessment results in a dichotomous value indicating which stimulus possesses a

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higher degree of the trait. After different judges perform multiple rounds of pairwise comparisons, an outcome vector is produced. This vector is modeled using the Bradley-Terry-Luce model (BTL, [Bradley and Terry, 1952](#); [Luce, 1959](#)), which creates a score that corresponds with the trait of interest. This score is then used to rank the stimuli from lowest to highest or to evaluate the influence of certain variables on the stimuli's positions in the ranking.

DCJ has proven effective in assessing competencies and traits predominantly within the educational realm, as demonstrated by [Pollitt \(2012b\)](#), [Jones \(2015\)](#), [van Daal et al. \(2019\)](#), [Bartholomew et al. \(2018\)](#), [Lesterhuis \(2018\)](#), [Bartholomew and Williams \(2020\)](#), and [Marshall et al. \(2020\)](#). However, its application transcends education, as exemplified by [Boonen et al. \(2020\)](#). The methodology has also evolved to include multiple, as opposed to pairwise comparisons ([Luce, 1959](#); [Plackett, 1975](#)), and to accommodate comparisons with ordinal outcomes ([Tutz, 1986](#); [Agresti, 1992](#)). Overall, research suggests that DCJ offers an alternative and efficient approach to measurement and evaluation, characterized by its reliability and validity ([Lesterhuis, 2018](#); [van Daal, 2020](#); [Marshall et al., 2020](#)). Nevertheless, despite the method's widespread use, existing literature lacks a clear representation of the plausible mechanisms through which DCJ data could be generated. Particularly, there is no depiction of the complexity and the assumptions underpinning the DCJ system, nor how different assessment factors can potentially influence the observed DCJ outcome.

According to [Verhavert et al. \(2019\)](#) and [van Daal \(2020\)](#), several assessment factors interact and influence the method's outcome. These factors include the number and characteristics of the stimuli, their *proximity* in terms of the assessed trait, the number of comparison per stimulus, and the pairing algorithm used. Furthermore, since the method relies on judges' assessments, the number and characteristics of judges, their *discrimination* abilities, and the number of comparisons per judge also play pivotal roles. Moreover, when the stimuli represent sub-units of higher-levels units, factors such as the number and characteristics of these units, along with their *proximity* in terms of the assessed trait, can significantly influence the outcome. For instance, [van Daal et al. \(2019\)](#) assessed academic writing skills of university students (units) using multiple argumentative essays (sub-units).

Although several studies have examined the individual impact of these factors on the method's reliability ([Bramley, 2015](#); [Pollitt, 2012b](#); [Bramley and Vitello, 2019](#); [Verhavert et al., 2019](#); [Crompvoets et al., 2022](#); [van Daal et al., 2017](#); [Gijzen et al., 2021](#)), none, to the best of the authors' knowledge, have provided a transparent depiction of the DCJ system and the mechanisms generating the DCJ outcome. This study aims to fill this gap by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach ([Wright, 1927](#); [Pearl, 2009](#); [Pearl et al., 2016](#)), the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates the scientific model into a probabilistic statistical model. This model aims to produce statistical estimates to draw inferences about plausible causal relationships within the DCJ system.

Ultimately, this research provides a robust probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone's law of comparative judgment ([1927](#)). Consequently, its findings offer valuable insights for researchers and analysts designing

and implementing DCJ experiments.

## 2. Theoretical framework

This section introduces fundamental concepts in causal inference, such as directed acyclic graphs (DAGs), structural causal models (SCMs), and the flow of association and causation in graphs. It is not a comprehensive description of causal inference methods. Readers interested in deeper exploration should consult introductory papers like [Pearl \(2010\)](#), [Rohrer \(2018\)](#), [Pearl \(2019\)](#), and [Cinelli et al. \(2020\)](#). They may also find introductory books such as [Pearl and Mackenzie \(2018\)](#), [Neal \(2020\)](#) and [McElreath \(2020\)](#) useful. For more advanced study, seminal intermediate papers like [Neyman \(1923\)](#), [Rubin \(1974\)](#), [Spirtes et al. \(1991\)](#), and [Sekhon \(2009\)](#), as well as books such as [Pearl \(2009\)](#), [Morgan and Winship \(2014\)](#) and [Hern'án and Robins \(2020\)](#) are recommended.

### 2.1. The structural approach to causal inference

In statistics, *causal inference* refers to the process of identifying the causes of a phenomenon and estimating their effects using data ([Shaughnessy et al., 2010](#); [Neal, 2020](#)). Unlike classical statistical modeling, which focuses solely on summarizing data and inferring associations, causal inference provides a coherent mathematical notation for analyzing causes and counterfactuals ([Pearl, 2009](#)).

Counterfactuals represent scenarios *contrary to fact*, where alternative *potential* outcomes resulting from a cause are neither observed nor observable ([Neal, 2020](#); [Counterfactual, 2024](#)). According to [Pearl and Mackenzie \(2018\)](#), counterfactuals form the foundation of causal inference and occupy the highest level of cognitive abstraction in the ladder of causation, followed by intervention and association. Nevertheless, despite their abstract nature, counterfactuals enable the development of a *theory of the world* that explains why specific causes have specific effects and what occurs in their absence ([Pearl and Mackenzie, 2018](#)). They achieve this by translating causal statements into counterfactual statements, that is, statements about “what would have happened in the world under different circumstances.”

Several approaches to causal inference and counterfactuals exist, but two are particularly prominent: the potential outcomes approach, also known as the Neyman-Rubin causal model ([Neyman, 1923](#); [Rubin, 1974](#)), and the structural approach ([Wright, 1927](#); [Pearl, 2009](#); [Pearl et al., 2016](#)). Both approaches employ rigorous mathematical notation to characterize causal inference, but they do so in different ways ([Neal, 2020](#)). The potential outcomes approach relies on counterfactual notation, whereas the structural approach employs the do-operator and structural causal models (SCM, [Pearl, 2009](#); [Pearl et al., 2016](#)). Despite these differences, both notations can be expressed in terms of the other, and both approaches provide methods for using experimental and observational data to estimate causal effects ([Pearl, 2010](#)).

Nevertheless, the structural approach offers an additional key advantage over the potential outcomes approach: it enables the graphical representation of any system through directed acyclic graphs (DAG, [Gross et al., 2018](#); [Neal, 2020](#)). DAGs are heuristics that can effectively convey the assumed causal structure of a system. They do not represent detailed statistical models but allow researchers to deduce which statistical models can

provide valid causal inferences, assuming the causal structure depicted in the DAG is accurate (McElreath, 2020).

## 2.2. Directed acyclic graphs (DAG)

Graph theory is a branch of mathematics focused on the study of graphs. Graphs are mathematical structures that model pairwise relations between objects. They can represent physical relations, such as electrical circuits and roadways, and less tangible structures, such as ecosystems and sociological relations. Graphs have proven useful in various fields, including computer science, operations research, and the natural and social sciences (Gross et al., 2018).

In statistics, one application incorporating concepts from graph theory is causal inference. Specifically, the structural approach to causal inference uses directed acyclic graphs (DAG) to provide a formal and graphical representation of the causal structure of a system (Neal, 2020). In this context, a *graph* is a collection of nodes connected by edges, where nodes represent random variables. The term *directed* indicates that the edges of the graph extend from one node to another, with arrows showing the direction of causal influence. Moreover, the term *acyclic* indicates the causal influences do not form a loop, meaning the influences do not cycle back on themselves (McElreath, 2020).

One key advantage of DAGs is that, regardless of complexity, they can represent various causal structures using only five fundamental building blocks (Neal, 2020; McElreath, 2020). Figure 1 illustrates these building blocks. Figure 1a depicts two unconnected nodes, representing an scenario where variables  $X_1$  and  $X_2$  are not causally related. Figure 1b shows two connected nodes, illustrating a scenario where a parent node  $X_1$  exerts a causal influence on a child node  $X_2$ . Consequently,  $X_2$  is considered a *descendant* of  $X_1$ . Figure 1c depicts a *chain* (or *pipe*), where  $X_1$  influences  $X_2$ , and  $X_2$  influences  $X_3$ . In this configuration,  $X_1$  is a parent node of  $X_2$ , and  $X_2$  a parent node of  $X_3$ . Furthermore, the DAG shows that  $X_1$  is an *ancestor* of  $X_3$  and that the relationship between these variables is entirely mediated by  $X_2$ . Figure 1d illustrates a *fork*, where variables  $X_1$  and  $X_3$  are both influenced by  $X_2$ . In this scenario  $X_2$  is a parent node of both  $X_1$  and  $X_3$ . Finally, Figure 1e depicts a *collider*, also known as *immorality*, where variables  $X_1$  and  $X_3$  are concurrent causes of  $X_2$ . In this configuration,  $X_1$  and  $X_3$  are not causally related to each other but both influence  $X_2$ .

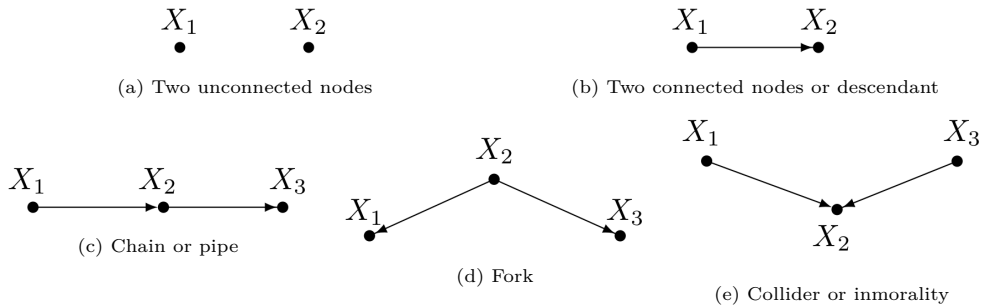


Figure 1: DAG's fundamental building blocks.

Given the heuristic nature of DAGs, a motivating example can illustrate how these five fundamental building blocks can be used to construct the causal structure of a system. This example can also help clarify additional rules and conventions for drawing DAGs.

Consider a research problem where the causal effect of a variable  $T$  on an outcome  $Y$  needs to be investigated. Additionally, the problem indicates that a variable  $X$  potentially influences both  $Y$  and  $T$ . Such scenarios are not hard to imagine. For instance,  $T$  could describe different treatments potentially influencing the binary onset of cancer ( $Y$ ) while  $X$  could denote the age of a patient. Using an example from DCJ, we need an example. Figure 2 presents two graphs describing the plausible causal structure of this system, the *simplified* and the *magnified* DAG.

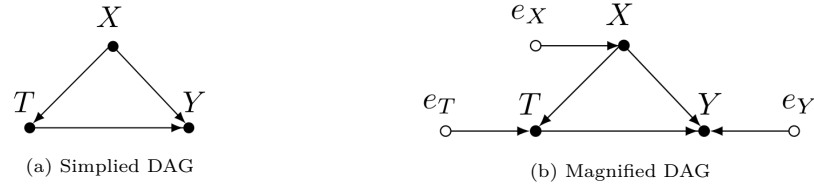


Figure 2: DAGs for a plausible causal structure in a system.

### 2.3. The flow of association and causation in graphs

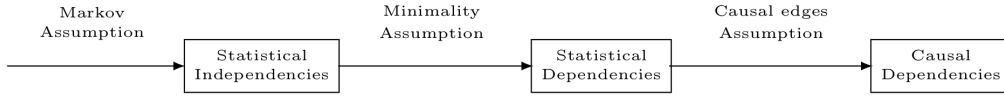


Figure 3: The flow of association and causation in graphs. Extracted from Neal (2020, 31)

### 2.4. But where does it all fit?

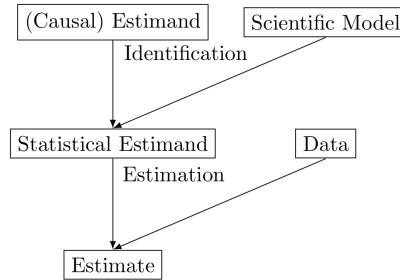


Figure 4: Identification-Estimation flowchart. Extracted from Neal (2020, 32)

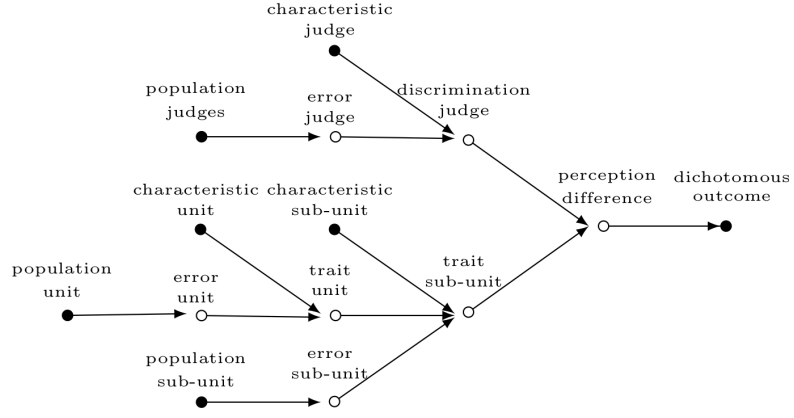


Figure 5: DCJ causal diagram, simplified description

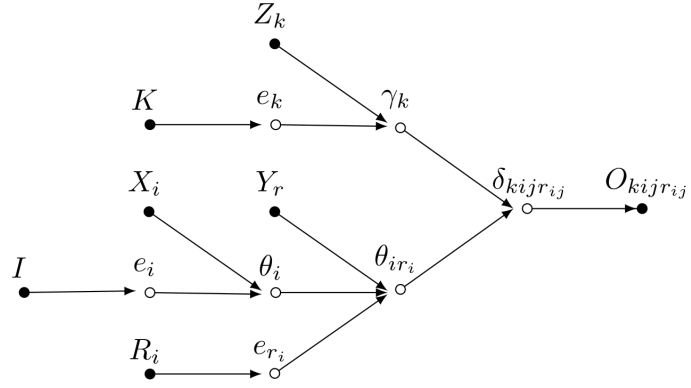


Figure 6: DCJ causal diagram, simplified mathematical description

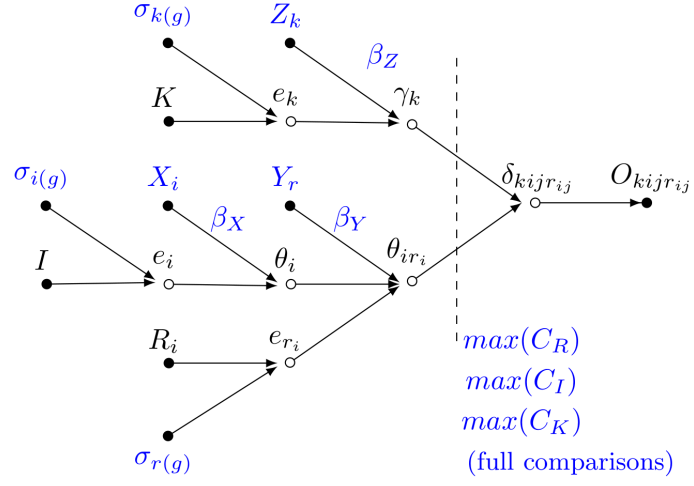


Figure 7: DCJ causal diagram, population mathematical description

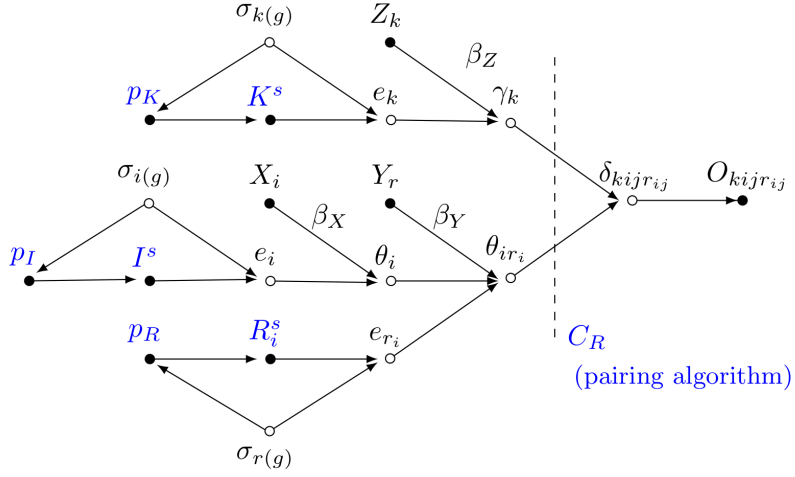


Figure 8: DCJ causal diagram, sample with comparisons mathematical description

### **3. Theory**

*3.1. A scientific model for the DCJ*

*3.2. Probabilistic assumptions of the scientific model*

*3.3. From the scientific to statistical model*

*3.4. Let's talk about Thurstone*

### **4. Discussion**

*4.1. Findings*

*4.2. Limitations and further research*

### **5. Conclusion**



## Declarations

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**Consent to participate:** Not applicable

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**Code availability:** All the code utilized in this research is available in the digital document located at: [https://jriverspejo.github.io/paper2\\_manuscript/](https://jriverspejo.github.io/paper2_manuscript/).

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## **6. Appendix**

- 6.1. Why do we need to estimate judges' abilities?*
- 6.2. Latent variables as a mean of imputation*
- 6.3. Other comparative scenarios*

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