Let's talk about Thurstone & Co.: An information-theoretical model for comparative judgments, and its statistical translation

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Abstract

(to do)

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1. Introduction

Over the past decade, numerous studies have documented the effectiveness of the *comparative judgment* (CJ) method (Thurstone, 1927) for assessing competencies and traits. These studies have evaluated CJ from three main perspectives: its ability to produce reliable and valid trait scores, its practical applicability, and its time efficiency. Research on reliability and validity shows that CJ can generate precise and consistent scores (Pollitt, 2012a,b; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Verhavert et al., 2019; Crompvoets et al., 2022; Bouwer et al., 2023) that accurately represent the traits being measured (Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018; Bouwer et al., 2023). Research on practical applicability highlights CJ's versatility across both educational and non-educational contexts, presenting it as an efficient and effective alternative for measurement and evaluation (Pollitt, 2004; Jones, 2015; Bartholomew et al., 2018; Jones et al., 2019; Marshall et al., 2020; Bartholomew and Williams, 2020; Boonen et al., 2020). Lastly, research on time efficiency suggests that CJ can offer at least equal, if not significant, time savings when evaluating stimuli compared to traditional marking methods (Pollitt, 2012a,b; Coertjens et al., 2017; Goossens and De Maeyer, 2018).

Nevertheless, despite the growing number of studies on CJ, unsystematic and fragmented research approaches in the literature have left several critical issues unaddressed.

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This research primarily focuses on three: the apparent disconnect between CJ's structural and measurement model, the over-reliance on the assumptions of Thurstone's Case 5 (1927) in CJ's measurement model, and the unclear role and impact of comparison algorithms on the method's reliability and validity. The following sections will discuss each of these issues in detail, followed by the introduction of a theoretical model and its statistical translation, which aim to address all three concerns simultaneously.

2. Three critical issues in CJ literature

2.1. The disconnect between structural and measurement models

In a typical CJ study, the Bradley-Terry-Luce (BTL) model (Bradley and Terry, 1952; Luce, 1959) serves as the measurement model (Andrich, 1978; Bramley, 2008). This model specifies how latent variables are estimated from manifest variables (Everitt and Skrondal, 2010). In CJ, multiple judges engage in several rounds of pairwise comparisons to assess the relative manifestation of a trait between two stimuli. Each comparison generates a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher degree of the trait. The BTL model then uses these observed outcomes to estimate scores that represent the latent trait of interest (Pollitt, 2012a,b; Whitehouse, 2012; Jones, 2015; van Daal et al., 2016; Lesterhuis, 2018; Boonen et al., 2020; Bouwer et al., 2023).

Moreover, researchers often use these BTL-generated scores or their transformations separately, conducting further analyses or hypothesis testing. The CJ literature shows how these scores have been employed to identify 'misfit' judges and stimuli (Pollitt, 2012b; van Daal et al., 2017; Goossens and De Maeyer, 2018), detect biases in judges' ratings (Pollitt and Elliott, 2003; Pollitt, 2012b), calculate correlations with other scoring methods (Goossens and De Maeyer, 2018; Bouwer et al., 2023), or test hypotheses related to the trait of interest (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijsen et al., 2021).

However, the statistical literature cautions against using estimated scores for separate analyses and tests. A key consideration is that BTL-generated scores are parameter estimates that inherently carry uncertainty. Ignoring this uncertainty when conducting separate analyses and tests can inflate their precision and statistical power. This, in turn, increases the risk of committing a type I error (McElreath, 2020), which is when a null hypothesis is incorrectly rejected (Everitt and Skrondal, 2010). To mitigate this risk, principles from Structural Equation Modeling (SEM) (Hoyle, 2023; Kline, 2023) and Item Response Theory (IRT) (de Ayala, 2009; Fox, 2010; van der Linden, 2017) recommend conducting these analyses and tests within a structural model that accounts for both the scores and their uncertainties, rather than treating them separately. Thus, an integrated approach combining CJ's structural and measurement models can offer significant advantages.

2.2. The assumptions of Case 5 and the measurement model
2.3. The role and impact of comparison algorithms
3. Theory
3.1. A theoretical model for CJ
3.2. From theory to statistics
4. Discussion
4.1. Findings
4.2. Limitations and further research
5. Conclusion

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6. Appendix

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