# Causes and effects in Dichotomous Comparative Judgments: an information-theoretical system of plausible mechanism

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#### Abstract

(to do)

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#### 1. Introduction

Over the past decade, numerous studies have documented the effectiveness of the *comparative judgment* (CJ) method (Thurstone, 1927) for assessing competencies and traits. These studies have evaluated CJ from two main perspectives: its ability to produce reliable and valid trait scores, and its practical applicability. Research on reliability and validity has shown that CJ can generate precise and consistent scores (Pollitt, 2012a,b; Bramley and Vitello, 2019; Verhavert et al., 2019; Crompvoets et al., 2022; Bouwer et al., 2023) that accurately represent the traits being measured (Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018). Regarding practical applicability, several studies have highlighted CJ's versatility across both educational and non-educational contexts, presenting it as an efficient and effective alternative for measurement and evaluation (Jones, 2015; Bartholomew et al., 2018; Jones et al., 2019; Marshall et al., 2020; Bartholomew and Williams, 2020; Boonen et al., 2020).

Nevertheless, despite the growing number of CJ studies, the unsystematic and fragmented research approaches employed in the literature have overlooked several critical issues concerning the method. These issues fall into three main categories: concerns about the structural model, the measurement model, and the experimental design of CJ. In the following sections, each issue will be discussed in detail, followed by the introduction of an approach that addresses all three concerns simultaneously.

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## 2. Let's talk about Thurstone co.

## 2.1. The disconect between structural and measurement model

A key issue in the first category is the apparent disconnect between the method's structural and measurement models. In CJ literature, it is common to perform data analysis and hypothesis testing on scores estimated using the Bradley-Terry-Luce (BTL) model (Bradley and Terry, 1952; Luce, 1959). Several studies use the scores generated by the BTL model or their transformations to identify 'misfit' judges and stimuli (Pollitt, 2012b; van Daal et al., 2017), detect 'bias' in judges' ratings (Pollitt and Elliott, 2003; Pollitt, 2012b), or test various hypotheses about the underlying trait being measured (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijsen et al., 2021). In the motivating example, ...

However, since these scores are parameter estimates with inherent uncertainty, the statistical literature suggests that separating the analysis from this uncertainty may artificially inflate the precision and power of the results. This, in turn, could increase the risk of committing a type I error, where a null hypothesis is wrongly rejected (McElreath, 2020). In the motivating example

To address this issue properly, the approach should follow a strategy similar to that used in Structural Equation Modeling (SEM), where data analysis and hypothesis testing occur at the structural model level, while the BTL model functions as the measurement model.

### 3. Theory

- 3.1. A scientific model for the CJ
- 3.2. From theory to statistical model

# 4. Discussion

- 4.1. Findings
- 4.2. Limitations and further research
- 5. Conclusion

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6. Appendix

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