

# Let's talk about Thurstone & Co.: An information-theoretical model for comparative judgments, and its statistical translation

Jose Manuel Rivera Espejo<sup>a,\*</sup>, Tine van Daal<sup>a</sup>, Sven De Maeyer<sup>a</sup>, Steven Gillis<sup>b</sup>

<sup>a</sup>*University of Antwerp, Training and education sciences,*

<sup>b</sup>*University of Antwerp, Linguistics,*

---

## Abstract

(to do)

*Keywords:* Probability, Directed Acyclic Graphs, Bayesian methods, Thurstonian model, Comparative judgement, Structural Causal Models, Statistical modeling

---

## 1. Introduction

In *comparative judgment* (CJ) studies, judges assess a specific trait or attribute across various stimuli by performing pairwise comparisons (Thurstone, 1927b,a). Each comparison produces a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher trait level. For example, when assessing text quality, judges compare pairs of written texts (the stimuli) to determine the relative quality each text exhibit (the trait) (Laming, 2004; Pollitt, 2012; Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023).

Numerous studies have documented the effectiveness of CJ in assessing traits and competencies over the past decade. These studies have emphasized three aspects of the method's effectiveness: its reliability, validity, and practical applicability. Research on reliability indicates that CJ requires a relatively small number of pairwise comparisons (Verhavert et al., 2019; Crompvoets et al., 2022) to produce trait scores that are as precise and consistent as those generated by other assessment methods (Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023). Furthermore, evidence suggests that the reliability and time efficiency of CJ are comparable, if not superior, to those of other assessment methods when employing adaptive comparison algorithms (Pollitt,

---

\*Corresponding author

Email addresses: JoseManuel.RiveraEspejo@uantwerpen.be (Jose Manuel Rivera Espejo), tine.vandaal@uantwerpen.be (Tine van Daal), sven.demaeyer@uantwerpen.be (Sven De Maeyer), steven.gillis@uantwerpen.be (Steven Gillis)

2012; Verhavert et al., 2022; Mikhailiuk et al., 2021). Meanwhile, research on validity suggests that scores generated by CJ can accurately represent the traits under measurement (Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Bartholomew et al., 2018; Bouwer et al., 2023), while research on practical applicability highlights the method’s versatility across both educational and non-educational contexts (Kimbell, 2012; Jones and Inglis, 2015; Bartholomew et al., 2018; Jones et al., 2019; Marshall et al., 2020; Bartholomew and Williams, 2020; Boonen et al., 2020).

Nevertheless, despite the increasing number of CJ studies, unsystematic and fragmented research approaches have left several critical issues unaddressed. The present study primarily focuses on two: the over-reliance on the assumptions of Thurstone’s Case V in the statistical analysis of CJ data, and the apparent disconnect between CJ’s trait measurement and hypothesis testing. The following sections begin with a brief overview of Thurstone’s theory and a detailed discussion of these issues. Subsequently, the study introduces a theoretical model for CJ that builds upon Thurstone’s theory, alongside its statistical translation, designed to address the two concerns simultaneously.

## 2. Thurstone’s theory

In its most general form, Thurstone’s theory deals with pairwise comparisons of stimuli made by a single judge (Thurstone, 1927a, pp. 267). The theory proposes that two key factors determine the dichotomous outcome of these comparisons: the discriminial process of each stimulus and their discriminial difference. The *discriminal process* represents the psychological impact each stimulus has on judges or, more simply, their underlying perception of the stimulus’ trait level. According to the theory, the discriminial process for each stimulus follows a Normal distribution, where its mode (mean), referred to as the *modal discriminial process*, indicates the stimulus’ position on the trait continuum, and its dispersion, known as the *discriminal dispersion*, reflects the variability in the perceived trait level of the stimulus.

For instance, Figure 1 illustrates the discriminial process distributions along a quality trait continuum for two written texts. The figure shows that these processes follow a Normal distribution. Moreover, it depicts differences in the texts’ positions along the quality trait continuum, where text B is positioned further along the continuum than text A, as indicated by their modal discriminial processes ( $S_B$  and  $S_A$ ). Finally, it highlights differences in the texts’ discriminial dispersions ( $\sigma_B$  and  $\sigma_A$ ), showing that text B exhibits a greater variability in its perceived quality than text A, as reflected by its wider distribution.

However, because the discriminial process of a single stimulus is not directly observable, the theory introduces the *law of comparative judgment*. This law posits that in pairwise comparisons, a judge perceives the stimulus positioned further along the trait continuum as having a higher level of that trait. This principle highlights that the outcome of a pairwise comparison likely depends on the relative distance between stimuli rather than their absolute positions on the trait continuum.

Indeed, the theory assumes that the observed dichotomous outcome arises from the distribution of the difference between the underlying discriminial processes of the stimuli,

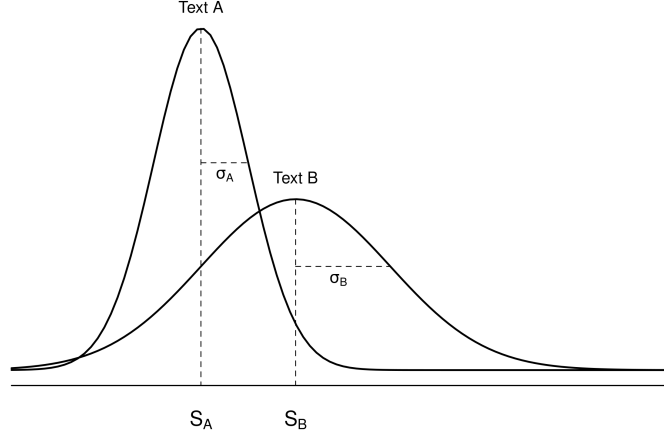


Figure 1: Example distributions of the discriminational processes for two written texts

known as the *discriminal difference*. Since the individual discriminational processes follow a Normal distribution, their difference also follows a Normal distribution (Andrich, 1978). The mode (mean) of this distribution, representing the (average) relative separation, is given by the difference between the modal discriminational processes of the stimuli  $S_{BA} = S_B - S_A$ . Meanwhile, the dispersion of the distribution, reflecting the variability in the relative separation, is calculated as  $\sigma_{BA} = \sqrt{\sigma_B^2 + \sigma_A^2 - \rho\sigma_B\sigma_A}$ . Here,  $\sigma_B$  and  $\sigma_A$  are the previously defined discriminational dispersions, while  $\rho$  measures the correlation between the discriminational processes of the stimuli. This correlation quantifies the dependence of the judge's perception of the trait in one stimulus on his perception of the same trait in another.

Figure 2 shows the distribution of the discriminational difference for the texts depicted in Figure 1, assuming a correlation of  $\rho = 0.6$ . The figure reveals that, under these conditions, the judge perceives text B as having significantly higher quality than text A, as indicated by the shaded gray area under the curve  $P(B > A)$ . As a result, the dichotomous outcome of this comparison would mostly favor text B over text A.

Notably, the correlation between the discriminational processes,  $\rho$ , plays a pivotal role in determining comparison outcomes by shaping the distribution of the discriminational difference between the stimuli. Specifically, as the correlation increases, reflecting a stronger dependence of the judge's perception of quality in one stimulus on his perception of the other, the distribution of the discriminational difference narrows. This narrowing ultimately impacts the area under the curve that determines the comparison outcome and, consequently, the conclusions drawn from this outcome.

Figure 3 illustrates how varying correlations influence the distribution of the discriminational difference for the two texts depicted in Figure 1. Since the texts differ in quality, higher correlations increase the likelihood that the discriminational difference distinctly favors text B over text A. This is evident from the larger proportion of area under the curve,  $P(B > A)$ ,

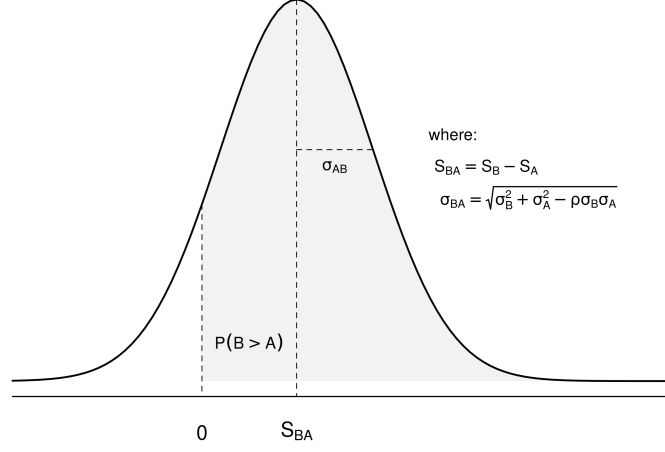


Figure 2: Example distribution of the discrimininal difference for the two texts shown in Figure 1, assuming a correlation of 0.6

that lies above zero (shaded gray area in Figure 3). Conversely, it is not hard to infer that if the texts had similar or identical quality levels, higher correlations would likely reduce the chance that the discrimininal difference distinctly favors one text over the other. This probability reduction occurs because the distribution of the discrimininal difference would become more narrowly centered around zero (not illustrated).

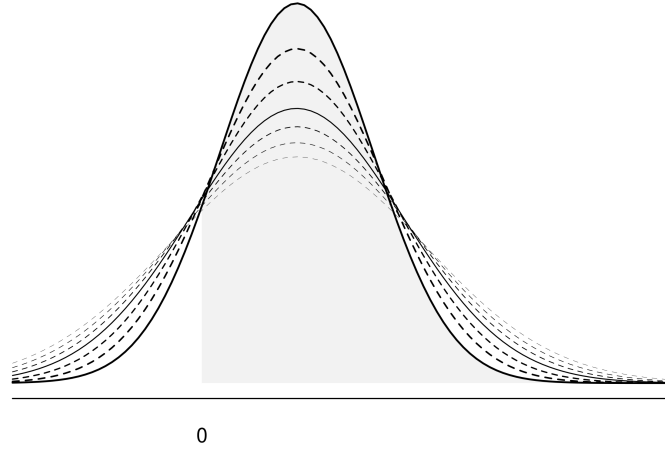


Figure 3: The effect of correlation on the distribution of the discrimininal difference of the same two written text

Table 1: Thurstone’s cases and assumptions

Assumption	Thurstone’s					BTL model
	Case I	Case II	Case III	Case IV	Case V	
Discriminal process (distribution)	Normal	Normal	Normal	Normal	Normal	Logistic
Discriminal dispersion (between stimuli)	Different	Different	Different	Similar	Equal	Equal
Correlation (between stimuli)	Constant	Constant	Zero	Zero	Zero	Zero
How many judges compare?	Single	Multiple	Multiple	Multiple	Multiple	Multiple

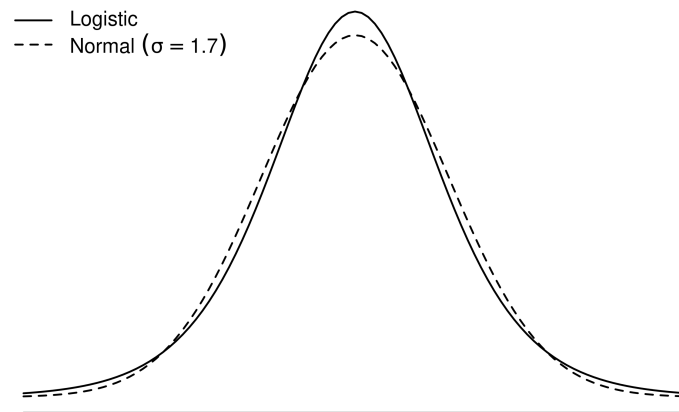
### 3. Three critical issues in CJ literature

#### 3.1. The Case V and the statistical analysis of CJ data

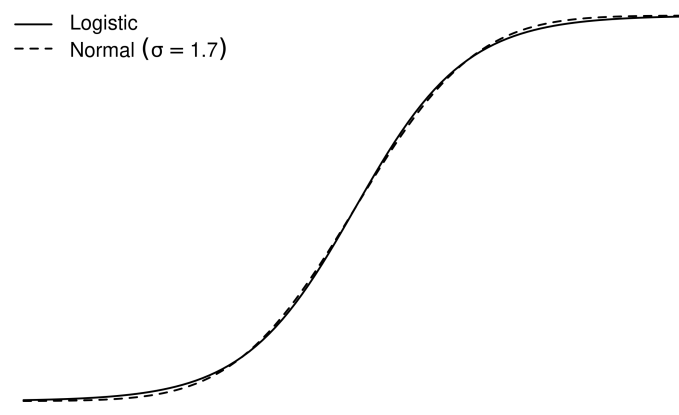
The previous section outlines the general form of Thurstone’s theory, which applies to a CJ design where a single judge evaluates multiple stimuli. For the practical application of the theory, Thurstone developed four additional cases derived from this general form, where each successive case incorporates additional simplifying assumptions. Case I represents the general form of the theory. Case II extends this by allowing multiple judges to make comparisons rather than restricting the comparisons to a single judge. Case III introduces the assumption of zero correlation between stimuli. Case IV builds on this by assuming that the stimuli have similar dispersions. Finally, Case V replaces this assumption with the condition that the stimuli have equal discriminative dispersions. Table 1 summarizes these cases and their assumptions. For a detailed discussion of this progression, refer to [Thurstone \(1927a\)](#) and [Bramley \(2008, pp. 248-253\)](#).

Despite its reliance on the largest number of simplifying assumptions ([Bramley, 2008, pp. 253](#); [Kelly et al., 2022, pp. 677](#)), Case V remains the most widely used case in the CJ literature. This popularity stems mainly from its simplified statistical representation in the Bradley-Terry-Luce (BTL) model ([Bradley and Terry, 1952](#); [Luce, 1959](#)). The BTL model mirrors the assumptions of Case V, with one key difference: while Case V assumes a Normal distribution for the discriminative processes of the stimuli, the BTL model uses the more mathematically tractable Logistic distribution ([Andrich, 1978](#); [Bramley, 2008, pp. 254](#)) (see Table 1). This substitution has little impact on the model’s estimation or interpretation, as the Normal and Logistic distributions share similar statistical properties, differing only by a scaling factor of approximately 1.7 ([van der Linden, 2017a, pp. 16](#)) (see Figure 4).

However, Thurstone originally developed Case V to provide a “rather coarse scaling” of traits ([Thurstone, 1927a, pp. 269](#)), prioritizing statistical simplicity over precision in trait measurement ([Kelly et al., 2022, pp. 677](#)). As a result, its assumptions may not be suitable for applications beyond the psycho-physical contexts for which it was created. Thurstone himself cautioned that its use “should not be made without (an) experimental test” ([Thurstone, 1927a, pp. 270](#)), acknowledging that some assumptions could prove problematic in the presence of complex traits or heterogeneous stimuli ([Thurstone, 1927b, pp. 376](#)). Consequently, given that modern CJ applications frequently involve these traits



(a) Probability density



(b) Cumulative probability

Figure 4: Probability density and cumulative probability of the logistic and Normal distributions. Extracted from [Bramley \(2008, pp. 254-255\)](#).

and stimuli, two main assumptions of Case V and, by extension, of the BTL model may not consistently hold in theory or practice: the equal dispersion and zero correlation between stimuli.

On the one hand, the assumption of *equal dispersion between stimuli* asserts that the variability in perceived trait levels remains consistent across all stimuli. However, Thurstone noted that this assumption may fail when researchers assess complex traits or heterogeneous stimuli (Thurstone, 1927b, pp. 376), as these can introduce greater discrepancies into judgments due to their unique characteristics. One example of this is the evaluation of creativity. In this context, for instance, it is plausible that a student completes an assignment in a highly creative yet unexpected way. Judges, unprepared for such originality, may experience greater discrepancies in their judgments of the assignment. As a result, the discriminial difference associated with this assignment is expected to show a broader dispersion, reflecting the larger variation in its discriminial process compared to others. Although direct evidence for this specific scenario is lacking, studies such as van Daal (2020, chap. 8) suggest that misfitting texts tend to exhibit larger dispersion, likely due to increased confusion among judges regarding the texts.

what are the challenges when the equal dispersion assumption is violated?

how do we mitigate these risks?

On the other hand, the assumption of *zero correlation between stimuli* implies that, during a pairwise comparison, a judge’s perception of quality in one text does not influence his perception of the same trait in another text (see Section 2). Thurstone attributed this independence to the cancellation of potential judges’ biases, driven by two opposing and equally weighted effects occurring during the pairwise comparisons (Thurstone, 1927a, pp. 268). Andrich (1978) mathematically demonstrated this cancellation using the BTL model under the assumption of discriminial processes with additive biases. However, it is easy to imagine at least two scenarios where the zero correlation assumption almost certainly does not hold: when the pairwise comparison involves multidimensional, complex traits with heterogeneous stimuli and when an additional hierarchical structure is relevant to the stimuli.

In the first scenario, the intricate aspects of multidimensional, complex traits may introduce dependencies between the stimuli due to certain judges’ biases that resist cancellation. Research on text quality suggests that when judges evaluate these traits, they often rely on various intricate characteristics of the stimuli to form their judgments (van Daal et al., 2016; Lesterhuis, 2018b; Chambers and Cunningham, 2022). These additional relevant characteristics, which are unlikely to be equally weighted or opposing, can unevenly influence judges’ perceptions, creating biases in their judgments and, ultimately, introducing dependencies between stimuli (van der Linden, 2017b, pp. 346). For example, this could occur when a judge assessing the argumentative quality of a text places disproportionate emphasis on grammatical accuracy, ultimately favoring texts with fewer errors but weaker arguments. While direct evidence for this specific scenario is lacking, studies such as Pollitt and Elliott (2003) demonstrate the presence of such biases, supporting the idea that the factors influencing pairwise comparisons may not always cancel out.

In the second scenario, the shared context or inherent connections created by additional

hierarchical structures may further introduce dependencies between stimuli, a statistical phenomenon commonly known as clustering (Everitt and Skrondal, 2010). Although the CJ literature acknowledges the presence of such hierarchical structures, the statistical handling of this extra source of dependency between stimuli has been inadequate. For example, when CJ data includes multiple samples of stimuli from the same individuals, researchers often rely on (average) estimated BTL scores to conduct subsequent analyses and tests at the individual hierarchical level (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijzen et al., 2021). This approach, however, has the significant limitation of ignoring the uncertainty associated with the BTL scores, which generates additional statistical and measurement issues, as discussed in section Section 3.2.

In any case, the psychometric and statistical literature emphasizes the importance of addressing the factors that create dependencies between stimuli, as failing to do so can lead to inaccurate conclusions about a trait’s reliability and, by extension, its validity. For instance, neglecting additional traits relevant to the stimuli, such as judges’ biases, often leads to a dimensional mismatch in the statistical model used for analysis. This mismatch can potentially inflate the reliability of the trait (Hoyle, 2023, pp. 341) or, worse, introduce bias into the trait’s estimates (Ackerman, 1989). Similarly, failing to account for hierarchical (grouping) structures reduces the precision of model parameter estimates, further amplifying the overestimation of reliability (Hoyle, 2023, pp. 482). These issues collectively undermine the trait’s reliability and ultimately compromise the validity of the trait’s estimates (Perron and Gillespie, 2015, pp. 2).

Fortunately, the same literature offers solutions for addressing these issues. Andrich (1978) and Wainer et al. (1978) recommend integrating judges’ biases into the BTL model. Moreover, the literature advocates for the incorporation of relevant hierarchical structures into the statistical model to account for these dependencies. Together, these additions can result in a model resembling a Multilevel Structural Equation Model (MSEM) (Hoyle, 2023, chap. 26) combined with a multidimensional or two-parameter logistic IRT model (Hoyle, 2023, chap. 15), depending on the theoretical and statistical treatment of judges’ biases.

### 3.2. *The disconnect between trait measurement and hypothesis testing*

Building on the previous section, it is evident that the BTL model commonly functions as the trait’s measurement model in CJ experiments (Andrich, 1978; Bramley, 2008). A measurement model specifies how manifest variables contribute to the estimation of latent variables (Everitt and Skrondal, 2010). For example, when evaluating text quality, researchers use the BTL model to process the dichotomous outcomes resulting from the pairwise comparisons (the manifest variables) to estimate scores that reflect the underlying quality level of texts (the latent variable) (Laming, 2004; Pollitt, 2012; Whitehouse, 2012; van Daal et al., 2016; Lesterhuis, 2018a; Coertjens et al., 2017; Goossens and De Maeyer, 2018; Bouwer et al., 2023).

Researchers then typically use the estimated BTL scores, or their transformations, to conduct additional analyses or hypothesis tests. For example, these scores have been used to identify ‘misfit’ judges and stimuli (Pollitt, 2012; van Daal et al., 2017; Goossens and De Maeyer, 2018), detect biases in judges’ ratings (Pollitt and Elliott, 2003; Pollitt,



2012), calculate correlations with other assessment methods (Goossens and De Maeyer, 2018; Bouwer et al., 2023), or test hypotheses related to the underlying trait of interest (Bramley and Vitello, 2019; Boonen et al., 2020; Bouwer et al., 2023; van Daal et al., 2017; Jones et al., 2019; Gijzen et al., 2021).

However, the statistical literature advises caution when using estimated scores for additional analyses and tests. A key consideration is that BTL scores are parameter estimates that inherently carry uncertainty. Ignoring this uncertainty can bias the analysis and reduce the precision of hypothesis tests. Notably, the direction and magnitude of such biases are often unpredictable. Results may be attenuated, exaggerated, or remain unaffected depending on the degree of uncertainty in the scores and the actual effects being tested (Kline, 2023, pp. 25; Hoyle, 2023, pp. 137). Finally, the reduced precision in hypothesis tests diminishes their statistical power, increasing the likelihood of committing type-I or type-II errors (McElreath, 2020).

To mitigate these risks, principles from Structural Equation Modeling (SEM) (Hoyle, 2023, pp. 138) and Item Response Theory (IRT) (Fox, 2010, chap. 6; van der Linden, 2017a, chap. 24) recommend conducting these analyses and tests within a structural model. A structural model specifies how different manifest or latent variables influence the latent variable of interest (Everitt and Skrondal, 2010). This approach allows analyses that can account for both the BTL scores and their uncertainties simultaneously, rather than treating them as separate elements. Therefore, an integrated approach that combines CJ's measurement and structural models can offer significant advantages.

## **4. An updated theoretical and statistical model for CJ**

### *4.1. The theoretical model*

### *4.2. From theory to statistics*

## **5. Discussion**

### *5.1. Findings*

### *5.2. Limitations and further research*

## **6. Conclusion**

## Declarations

**Funding:** The project was founded through the Research Fund of the University of Antwerp (BOF).

**Financial interests:** The authors have no relevant financial interest to disclose.

**Non-financial interests:** The authors have no relevant non-financial interest to disclose.

**Ethics approval:** The University of Antwerp Research Ethics Committee has confirmed that no ethical approval is required.

**Consent to participate:** Not applicable

**Consent for publication:** All authors have read and agreed to the published version of the manuscript.

**Availability of data and materials:** No data was utilized in this study.

**Code availability:** All the code utilized in this research is available in the digital document located at: [https://jriverspejo.github.io/paper2\\_manuscript/](https://jriverspejo.github.io/paper2_manuscript/).

**AI-assisted technologies in the writing process:** The authors used ChatGPT, an AI language model, during the preparation of this work. They occasionally employed the tool to refine phrasing and optimize wording, ensuring appropriate language use and enhancing the manuscript’s clarity and coherence. The authors take full responsibility for the final content of the publication.

**CRedit authorship contribution statement:** *Conceptualization:* S.G., S.D.M., T.vD., and J.M.R.E.; *Methodology:* S.D.M., T.vD., and J.M.R.E.; *Software:* J.M.R.E.; *Validation:* J.M.R.E.; *Formal Analysis:* J.M.R.E.; *Investigation:* J.M.R.E.; *Resources:* S.G., S.D.M., and T.vD.; *Data curation:* J.M.R.E.; *Writing - original draft:* J.M.R.E.; *Writing - review and editing:* S.G., S.D.M., and T.vD.; *Visualization:* J.M.R.E.; *Supervision:* S.G. and S.D.M.; *Project administration:* S.G. and S.D.M.; *Funding acquisition:* S.G. and S.D.M.

## References

- Ackerman, T., 1989. Unidimensional irt calibration of compensatory and noncompensatory multidimensional items. *Applied Psychological Measurement* 13, 113–127. doi:[10.1177/014662168901300201](https://doi.org/10.1177/014662168901300201).
- Andrich, D., 1978. Relationships between the thurstone and rasch approaches to item scaling. *Applied Psychological Measurement* 2, 451–462. doi:[10.1177/014662167800200319](https://doi.org/10.1177/014662167800200319).
- Bartholomew, S., Nadelson, L., Goodridge, W., Reeve, E., 2018. Adaptive comparative judgment as a tool for assessing open-ended design problems and model eliciting activities. *Educational Assessment* 23, 85–101. doi:[10.1080/10627197.2018.1444986](https://doi.org/10.1080/10627197.2018.1444986).
- Bartholomew, S., Williams, P., 2020. Stem skill assessment: An application of adaptive comparative judgment, in: Anderson, J., Li, Y. (Eds.), *Integrated Approaches to STEM Education*. *Advances in STEM Education*. Springer, pp. 331–349. doi:[10.1007/978-3-030-52229-2\\_18](https://doi.org/10.1007/978-3-030-52229-2_18).
- Boonen, N., Kloots, H., Gillis, S., 2020. Rating the overall speech quality of hearing-impaired children by means of comparative judgements. *Journal of Communication Disorders* 83, 1675–1687. doi:[10.1016/j.jcomdis.2019.105969](https://doi.org/10.1016/j.jcomdis.2019.105969).
- Bouwer, R., Lesterhuis, M., De Smedt, F., Van Keer, H., De Maeyer, S., 2023. Comparative approaches to the assessment of writing: Reliability and validity of benchmark rating and comparative judgement. *Journal of Writing Research* 15, 497–518. doi:[10.17239/jowr-2024.15.03.03](https://doi.org/10.17239/jowr-2024.15.03.03).
- Bradley, R., Terry, M., 1952. Rank analysis of incomplete block designs: I. the method of paired comparisons. *Biometrika* 39, 324–345. doi:[10.2307/2334029](https://doi.org/10.2307/2334029).
- Bramley, T., 2008. Paired comparison methods, in: Newton, P., Baird, J., Goldsteing, H., Patrick, H., Tymms, P. (Eds.), *Techniques for monitoring the comparability of examination standards*. GOV.UK., pp. 246–300. URL: <https://assets.publishing.service.gov.uk/media/5a80d75940f0b62305b8d734/2007-comparability-exam-standards-i-chapter7.pdf>.
- Bramley, T., Vitello, S., 2019. The effect of adaptivity on the reliability coefficient in adaptive comparative judgement. *Assessment in Education: Principles, Policy and Practice* 71, 1–25. doi:[10.1080/0969594X.2017.1418734](https://doi.org/10.1080/0969594X.2017.1418734).
- Chambers, L., Cunningham, E., 2022. Exploring the validity of comparative judgement: Do judges attend to construct-irrelevant features? *Frontiers in Education* doi:[10.3389/feduc.2022.802392](https://doi.org/10.3389/feduc.2022.802392).
- Coertjens, L., Lesterhuis, M., Verhavert, S., Van Gasse, R., De Maeyer, S., 2017. Teksten beoordelen met criterialijsten of via paarsgewijze vergelijking: een afweging van betrouwbaarheid en tijdsinvestering. *Pedagogische Studien* 94, 283–303. URL: <https://repository.uantwerpen.be/docman/irua/e71ea9/147930.pdf>.
- Cromptvoets, E., Béguin, A., Sijtsma, K., 2022. On the bias and stability of the results of comparative judgment. *Frontiers in Education* 6. doi:[10.3389/feduc.2021.788202](https://doi.org/10.3389/feduc.2021.788202).
- Everitt, B., Skrondal, A., 2010. *The Cambridge Dictionary of Statistics*. Cambridge University Press.
- Fox, J., 2010. *Bayesian Item Response Modeling, Theory and Applications*. Statistics for Social and Behavioral Sciences, Springer.
- Gijzen, M., van Daal, T., Lesterhuis, M., Gijbels, D., De Maeyer, S., 2021. The complexity of comparative judgments in assessing argumentative writing: An eye tracking study. *Frontiers in Education* 5. doi:[10.3389/feduc.2020.582800](https://doi.org/10.3389/feduc.2020.582800).
- Goossens, M., De Maeyer, S., 2018. How to obtain efficient high reliabilities in assessing texts: Rubrics vs comparative judgement, in: Ras, E., Guerrero Roldán, A. (Eds.), *Technology Enhanced Assessment*, Springer International Publishing. pp. 13–25. doi:[10.1007/978-3-319-97807-9\\_2](https://doi.org/10.1007/978-3-319-97807-9_2).
- Hoyle, R.e., 2023. *Handbook of Structural Equation Modeling*. Guilford Press.
- Jones, I., Bisson, M., Gilmore, C., Inglis, M., 2019. Measuring conceptual understanding in randomised controlled trials: Can comparative judgement help? *British Educational Research Journal* 45, 662–680. doi:[10.1002/berj.3519](https://doi.org/10.1002/berj.3519).
- Jones, I., Inglis, M., 2015. The problem of assessing problem solving: can comparative judgement help? *Educational Studies in Mathematics* 89, 337–355. doi:[10.1007/s10649-015-9607-1](https://doi.org/10.1007/s10649-015-9607-1).
- Kelly, K., Richardson, M., Isaacs, T., 2022. Critiquing the rationales for using comparative judgement: a call for clarity. *Assessment in Education: Principles, Policy & Practice* 29, 674–688. doi:[10.1080/0969594X.2022.2147901](https://doi.org/10.1080/0969594X.2022.2147901).
- Kimbell, R., 2012. Evolving project e-scape for national assessment. *International Journal of Technology and Design Education* 22, 135–155. doi:[10.1007/s10798-011-9190-4](https://doi.org/10.1007/s10798-011-9190-4).
- Kline, R., 2023. *Principles and Practice of Structural Equation Modeling*. Methodology in the Social Sciences, Guilford Press.
- Laming, D., 2004. Marking university examinations: Some lessons from psychophysics. *Psychology Learning & Teaching* 3, 89–96. doi:[10.2304/plat.2003.3.2.89](https://doi.org/10.2304/plat.2003.3.2.89).

- Lesterhuis, M., 2018a. The validity of comparative judgement for assessing text quality: An assessor's perspective. Ph.D. thesis. University of Antwerp. URL: <https://hdl.handle.net/10067/1548280151162165141>.
- Lesterhuis, M., 2018b. When teachers compare argumentative texts: Decisions informed by multiple complex aspects of text quality. *L1-Educational Studies in Language and Literature* 18, 1–22. doi:[10.17239/L1ESLL-2018.18.01.02](https://doi.org/10.17239/L1ESLL-2018.18.01.02).
- Luce, R., 1959. On the possible psychophysical laws. *The Psychological Review* 66, 482–499. doi:[10.1037/h0043178](https://doi.org/10.1037/h0043178).
- Marshall, N., Shaw, K., Hunter, J., Jones, I., 2020. Assessment by comparative judgement: An application to secondary statistics and english in new zealand. *New Zealand Journal of Educational Studies* 55, 49–71. doi:[10.1007/s40841-020-00163-3](https://doi.org/10.1007/s40841-020-00163-3).
- McElreath, R., 2020. *Statistical Rethinking: A Bayesian Course with Examples in R and STAN*. Chapman and Hall/CRC.
- Mikhailiuk, A., Wilmot, C., Perez-Ortiz, M., Yue, D., Mantiuk, R., 2021. Active sampling for pairwise comparisons via approximate message passing and information gain maximization, in: 2020 25th International Conference on Pattern Recognition (ICPR), pp. 2559–2566. doi:[10.1109/ICPR48806.2021.9412676](https://doi.org/10.1109/ICPR48806.2021.9412676).
- Perron, B., Gillespie, D., 2015. Reliability and Measurement Error, in: *Key Concepts in Measurement*. Oxford University Press. Pocket guides to social work research methods. chapter 4. doi:[10.1093/acprof:oso/9780199855483.003.0004](https://doi.org/10.1093/acprof:oso/9780199855483.003.0004).
- Pollitt, A., 2012. The method of adaptive comparative judgement. *Assessment in Education: Principles, Policy and Practice* 19, 281–300. doi:[10.1080/0969594X.2012.665354](https://doi.org/10.1080/0969594X.2012.665354).
- Pollitt, A., Elliott, G., 2003. Finding a proper role for human judgement in the examination system. URL: <https://www.cambridgeassessment.org.uk/Images/109707-monitoring-and-investigating-comparability-a-proper-role-for-human-judgement.pdf>. research & Evaluation Division.
- Thurstone, L., 1927a. A law of comparative judgment. *Psychological Review* 34, 482–499. doi:[10.1037/h0070288](https://doi.org/10.1037/h0070288).
- Thurstone, L., 1927b. Psychophysical analysis. *American Journal of Psychology* , 368–89URL: [https://brocku.ca/MeadProject/Thurstone/Thurstone\\_1927g.html](https://brocku.ca/MeadProject/Thurstone/Thurstone_1927g.html). last accessed 20 december 2024.
- van Daal, T., 2020. Making a choice is not easy?!: Unravelling the task difficulty of comparative judgement to assess student work. Ph.D. thesis. University of Antwerp.
- van Daal, T., Lesterhuis, M., Coertjens, L., Donche, V., De Maeyer, S., 2016. Validity of comparative judgement to assess academic writing: examining implications of its holistic character and building on a shared consensus. *Assessment in Education: Principles, Policy & Practice* 26, 59–74. doi:[10.1080/0969594X.2016.1253542](https://doi.org/10.1080/0969594X.2016.1253542).
- van Daal, T., Lesterhuis, M., Coertjens, L., van de Kamp, M., Donche, V., De Maeyer, S., 2017. The complexity of assessing student work using comparative judgment: The moderating role of decision accuracy. *Frontiers in Education* 2. doi:[10.3389/feduc.2017.00044](https://doi.org/10.3389/feduc.2017.00044).
- van der Linden, W. (Ed.), 2017a. *Handbook of Item Response Theory: Models*. volume 1 of *Statistics in the Social and Behavioral Sciences Series*. CRC Press.
- van der Linden, W. (Ed.), 2017b. *Handbook of Item Response Theory: Statistical Tools*. volume 2 of *Statistics in the Social and Behavioral Sciences Series*. CRC Press.
- Verhavert, S., Bouwer, R., Donche, V., De Maeyer, S., 2019. A meta-analysis on the reliability of comparative judgement. *Assessment in Education: Principles, Policy and Practice* 26, 541–562. doi:[10.1080/0969594X.2019.1602027](https://doi.org/10.1080/0969594X.2019.1602027).
- Verhavert, S., Furlong, A., Bouwer, R., 2022. The accuracy and efficiency of a reference-based adaptive selection algorithm for comparative judgment. *Frontiers in Education* 6. doi:[10.3389/feduc.2021.785919](https://doi.org/10.3389/feduc.2021.785919).
- Wainer, H., TimbersFairbank, D., Hough, R., 1978. Predicting the impact of simple and compound life change events. *Applied Psychological Measurement* 2, 313–322. doi:[10.1177/014662167800200301](https://doi.org/10.1177/014662167800200301).
- Whitehouse, C., 2012. Testing the validity of judgements about geography essays using the adaptive comparative judgement method. URL: [https://filestore.aqa.org.uk/content/research/CERP\\_RP\\_CW\\_24102012\\_0.pdf?download=1](https://filestore.aqa.org.uk/content/research/CERP_RP_CW_24102012_0.pdf?download=1). aQA Education.