Let’s talk about Thurstone & Co.

An information-theoretical model for comparative judgments, and its statistical translation

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2024-10-23

Abstract

(to do)

# Introduction

Over the past decade, numerous studies have documented the effectiveness of the *comparative judgment* (CJ) method (Thurstone 1927) for assessing competencies and traits. These studies have evaluated CJ from three main perspectives: its ability to produce reliable and valid trait scores, its practical applicability, and its time efficiency. Research on reliability and validity shows that CJ can generate precise and consistent scores (Pollitt 2012a, 2012b; Coertjens et al. 2017; Goossens and De Maeyer 2018; Verhavert et al. 2019; Crompvoets, Béguin, and Sijtsma 2022; Bouwer et al. 2023) that accurately represent the traits being measured (Whitehouse 2012; van Daal et al. 2016; Lesterhuis 2018; Bouwer et al. 2023). Research on practical applicability highlights CJ’s versatility across both educational and non-educational contexts, presenting it as an efficient and effective alternative for measurement and evaluation (Pollitt 2004; Jones 2015; Bartholomew et al. 2018; Jones et al. 2019; Marshall et al. 2020; Bartholomew and Williams 2020; Boonen, Kloots, and Gillis 2020). Lastly, research on time efficiency suggests that CJ can offer at least equal, if not significant, time savings when evaluating stimuli compared to traditional marking methods (Pollitt 2012a, 2012b; Coertjens et al. 2017; Goossens and De Maeyer 2018).

Nevertheless, despite the growing number of studies on CJ, unsystematic and fragmented research approaches in the literature have left several critical issues unaddressed. These issues can be grouped into three main categories: the disconnect between CJ’s structural and measurement model, the over-reliance on the assumptions of Thurstone’s Case 5 (1927) in its measurement model, and the unclear role and impact of comparison algorithms and the number of comparisons on the method’s reliability and validity. The following sections will discuss each of these issues in detail,, followed by the introduction of a theoretical model and its statistical translation, which aim to address all three concerns simultaneously.

# The three critical issues in CJ literature

## The disconnect between structural and measurement models

In a typical CJ study, the Bradley-Terry-Luce (BTL) model (Bradley and Terry 1952; Luce 1959) serves as the measurement model. This model specifies how latent variables are estimated from observed ones (Hoyle 2023; Kline 2023). In the context of CJ, multiple judges engage in several rounds of pairwise comparisons to assess the relative manifestation of a trait between two stimuli. Each comparison generates a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher degree of the trait. The BTL model then uses these observed outcomes to estimate scores that represent the latent trait of interest (Pollitt 2012a, 2012b; Whitehouse 2012; Jones 2015; van Daal et al. 2016; Lesterhuis 2018; Boonen, Kloots, and Gillis 2020; Bouwer et al. 2023).

Moreover, researchers often use these BTL-generated scores or their transformations separately, applying additional analyses or hypothesis testing. The CJ literature shows that these scores have been employed to identify ‘misfit’ judges and stimuli (Pollitt 2012b; van Daal et al. 2017; Goossens and De Maeyer 2018), detect biases in judges’ ratings (Pollitt and Elliott 2003; Pollitt 2012b), calculate correlations with other scoring methods (Goossens and De Maeyer 2018; Bouwer et al. 2023), or test hypotheses related to the trait of interest (Bramley and Vitello 2019; Boonen, Kloots, and Gillis 2020; Bouwer et al. 2023; van Daal et al. 2017; Jones et al. 2019; Gijsen et al. 2021).

However, the statistical literature warns against using estimated scores for separate analyses and hypothesis testing. BTL-generated scores are parameter estimates that inherently carry uncertainty. Neglecting this uncertainty can inflate the precision and statistical power in independent analyses and tests. This oversight, in turn, may increase the risk of committing a type I error, where the null hypothesis is incorrectly rejected (McElreath 2020). To mitigate this risk, principles from Structural Equation Modeling (SEM) (Hoyle 2023; Kline 2023) recommend conducting such analyses within the structural model of CJ rather than separately. Therefore, an integrated approach combining both structural and measurement models is advisable.

## The assumptions of Case 5 and the measurement model

## The role and impact of comparison algorithms

# Theory

## A theoretical model for CJ

## From theory to statistics

# Discussion

## Findings

## Limitations and further research

# Conclusion

# Declarations

**Funding:** The project was founded through the Research Fund of the University of Antwerp (BOF).

**Financial interests:** The authors have no relevant financial interest to disclose.

**Non-financial interests:** Author XX serve on advisory broad of Company Y but receives no compensation this role.

**Ethics approval:** The University of Antwerp Research Ethics Committee has confirmed that no ethical approval is required.

**Consent to participate:** Not applicable

**Consent for publication:** All authors have read and agreed to the published version of the manuscript.

**Availability of data and materials:** No data was utilized in this study.

**Code availability:** All the code utilized in this research is available in the digital document located at: <https://jriveraespejo.github.io/paper2_manuscript/>.

**Authors’ contributions:** *Conceptualization:* S.G., S.DM., T.vD., and J.M.R.E; *Methodology:* S.DM., T.vD., and J.M.R.E; *Software:* J.M.R.E.; *Validation:* J.M.R.E.; *Formal Analysis:* J.M.R.E.; *Investigation:* J.M.R.E; *Resources:* S.G., S.DM., and T.vD.; *Data curation:* J.M.R.E.; *Writing - original draft:* J.M.R.E.; *Writing - review & editing:* S.G., S.DM., and T.vD.; *Visualization:* J.M.R.E.; *Supervision:* S.G. and S.DM.; *Project administration:* S.G. and S.DM.; *Funding acquisition:* S.G. and S.DM.

# Appendix

## References

Bartholomew, S., L. Nadelson, W. Goodridge, and E. Reeve. 2018. “Adaptive Comparative Judgment as a Tool for Assessing Open-Ended Design Problems and Model Eliciting Activities.” *Educational Assessment* 23 (2): 85–101. <https://doi.org/10.1080/10627197.2018.1444986>.

Bartholomew, S., and P. Williams. 2020. “STEM Skill Assessment: An Application of Adaptive Comparative Judgment.” In *Integrated Approaches to STEM Education. Advances in STEM Education*, edited by J. Anderson and Y. Li, 331–49. Springer. <https://doi.org/10.1007/978-3-030-52229-2_18>.

Boonen, N., H. Kloots, and S. Gillis. 2020. “Rating the Overall Speech Quality of Hearing-Impaired Children by Means of Comparative Judgements.” *Journal of Communication Disorders* 83: 1675–87. <https://doi.org/10.1016/j.jcomdis.2019.105969>.

Bouwer, R., M. Lesterhuis, F. De Smedt, H. Van Keer, and S. De Maeyer. 2023. “Comparative Approaches to the Assessment of Writing: Reliability and Validity of Benchmark Rating and Comparative Judgement.” *Journal of Writing Research* 15 (3): 497–518. <https://doi.org/10.17239/jowr-2024.15.03.03>.

Bradley, R., and M. Terry. 1952. “Rank Analysis of Incomplete Block Designs: I. The Method of Paired Comparisons.” *Biometrika* 39 (3-4): 324–45. <https://doi.org/10.2307/2334029>.

Bramley, T., and S. Vitello. 2019. “The Effect of Adaptivity on the Reliability Coefficient in Adaptive Comparative Judgement.” *Assessment in Education: Principles, Policy and Practice* 71 (9): 1–25. <https://doi.org/10.1080/0969594X.2017.1418734>.

Coertjens, L., M Lesterhuis, S. Verhavert, R. Van Gasse, and S. De Maeyer. 2017. “Teksten Beoordelen Met Criterialijsten of via Paarsgewijze Vergelijking: Een Afweging van Betrouwbaarheid En Tijdsinvestering.” *Pedagogische Studien* 94: 283–303. <https://repository.uantwerpen.be/docman/irua/e71ea9/147930.pdf>.

Crompvoets, Elise A. V., Anton A. Béguin, and Klaas Sijtsma. 2022. “On the Bias and Stability of the Results of Comparative Judgment.” *Frontiers in Education* 6. <https://doi.org/10.3389/feduc.2021.788202>.

Gijsen, M., T. van Daal, Marije Lesterhuis, David Gijbels, and Sven De Maeyer. 2021. “The Complexity of Comparative Judgments in Assessing Argumentative Writing: An Eye Tracking Study.” *Frontiers in Education* 5. <https://doi.org/10.3389/feduc.2020.582800>.

Goossens, M., and S. De Maeyer. 2018. “How to Obtain Efficient High Reliabilities in Assessing Texts: Rubrics Vs Comparative Judgement.” In *Technology Enhanced Assessment*, edited by E. Ras and A. Guerrero Roldán, 13–25. Springer International Publishing. <https://doi.org/10.1007/978-3-319-97807-9_2>.

Hoyle, R. (eds.). 2023. *Handbook of Structural Equation Modeling*. Guilford Press.

Jones, I. 2015. “The Problem of Assessing Problem Solving: Can Comparative Judgement Help?” *Educational Studies in Mathematics* 89 (3): 337–55. <https://doi.org/10.1007/s10649-015-9607-1>.

Jones, I., M. Bisson, C. Gilmore, and M. Inglis. 2019. “Measuring Conceptual Understanding in Randomised Controlled Trials: Can Comparative Judgement Help?” *British Educational Research Journal* 45 (3): 662–80. <https://doi.org/10.1002/berj.3519>.

Kline, R. 2023. *Principles and Practice of Structural Equation Modeling*. Methodology in the Social Sciences. Guilford Press.

Lesterhuis, M. 2018. “The Validity of Comparative Judgement for Assessing Text Quality: An Assessor’s Perspective.” PhD thesis, University of Antwerp.

Luce, R. 1959. “On the Possible Psychophysical Laws.” *The Psychologcal Review* 66 (2): 482–99. <https://doi.org/10.1037/h0043178>.

Marshall, N., K Shaw, J. Hunter, and I. Jones. 2020. “Assessment by Comparative Judgement: An Application to Secondary Statistics and English in New Zealand.” *New Zealand Journal of Educational Studies* 55: 49–71. <https://doi.org/10.1007/s40841-020-00163-3>.

McElreath, R. 2020. *Statistical Rethinking: A Bayesian Course with Examples in r and STAN*. Chapman; Hall/CRC.

Pollitt, A. 2004. “Let’s Stop Marking Exams.” In *Proceedings of the IAEA Conference*. Philadelphia: University of Cambridge Local Examinations Syndicate. <https://www.cambridgeassessment.org.uk/images/109719-let-s-stop-marking-exams.pdf>.

———. 2012a. “Comparative Judgement for Assessment.” *International Journal of Technology and Design Education* 22 (2): 157--170. <https://doi.org/10.1007/s10798-011-9189-x>.

———. 2012b. “The Method of Adaptive Comparative Judgement.” *Assessment in Education: Principles, Policy and Practice* 19 (3): 281--300. <https://doi.org/10.1080/0969594X.2012.665354>.

Pollitt, A., and G. Elliott. 2003. “Finding a Proper Role for Human Judgement in the Examination System.” University of Cambridge Local Examinations Syndicate. <https://www.cambridgeassessment.org.uk/Images/109707-monitoring-and-investigating-comparability-a-proper-role-for-human-judgement.pdf>.

Thurstone, L. 1927. “A Law of Comparative Judgment.” *Psychological Review* 34 (4): 482–99. <https://doi.org/10.1037/h0070288>.

van Daal, T., M. Lesterhuis, L. Coertjens, V. Donche, and S. De Maeyer. 2016. “Validity of Comparative Judgement to Assess Academic Writing: Examining Implications of Its Holistic Character and Building on a Shared Consensus.” *Assessment in Education: Principles, Policy & Practice* 26 (1): 59–74. <https://doi.org/10.1080/0969594X.2016.1253542>.

van Daal, T., M. Lesterhuis, L. Coertjens, MT. van de Kamp, V. Donche, and Sven De Maeyer. 2017. “The Complexity of Assessing Student Work Using Comparative Judgment: The Moderating Role of Decision Accuracy.” *Frontiers in Education* 2. <https://doi.org/10.3389/feduc.2017.00044>.

Verhavert, S., R. Bouwer, V Donche, and S. De Maeyer. 2019. “A Meta-Analysis on the Reliability of Comparative Judgement.” *Assessment in Education: Principles, Policy and Practice* 26 (5): 541–62. <https://doi.org/10.1080/0969594X.2019.1602027>.

Whitehouse, C. 2012. “Testing the Validity of Judgements about Geography Essays Using the Adaptive Comparative Judgement Method.” Centre for Education Research & Policy. <https://filestore.aqa.org.uk/content/research/CERP_RP_CW_24102012_0.pdf?download=1>.