Let’s talk about Thurstone & Co.: An information-theoretical model for comparative judgments, and its statistical translation

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Abstract

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# Introduction

In *comparative judgment* (CJ) studies, judges assess a specific trait or attribute across various stimuli by performing pairwise comparisons (Thurstone 1927b, 1927a). Each comparison produces a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher trait level. For example, when assessing text quality, judges compare pairs of written texts (the stimuli) to determine the relative quality each text exhibit (the trait) (Laming 2004; Pollitt 2012; Whitehouse 2012; van Daal et al. 2016; Lesterhuis 2018a; Coertjens et al. 2017; Goossens and De Maeyer 2018; Bouwer et al. 2023).

Numerous studies have documented the effectiveness of CJ in assessing traits and competencies over the past decade. These studies have emphasized three aspects of the method’s effectiveness: its reliability, validity, and practical applicability. Research on reliability indicates that CJ requires a relatively small number of pairwise comparisons (Verhavert et al. 2019; Crompvoets, Béguin, and Sijtsma 2022) to produce trait scores that are as precise and consistent as those generated by other assessment methods (Coertjens et al. 2017; Goossens and De Maeyer 2018; Bouwer et al. 2023). Furthermore, evidence suggests that the reliability and time efficiency of CJ are comparable, if not superior, to those of other assessment methods when employing adaptive comparison algorithms (Pollitt 2012; Verhavert, Furlong, and Bouwer 2022; Mikhailiuk et al. 2021). Meanwhile, research on validity suggests that scores generated by CJ can accurately represent the traits under measurement (Whitehouse 2012; van Daal et al. 2016; Lesterhuis 2018a; Bartholomew et al. 2018; Bouwer et al. 2023), while research on practical applicability highlights the method’s versatility across both educational and non-educational contexts (Kimbell 2012; Jones and Inglis 2015; Bartholomew et al. 2018; Jones et al. 2019; Marshall et al. 2020; Bartholomew and Williams 2020; Boonen, Kloots, and Gillis 2020).

Nevertheless, despite the increasing number of CJ studies, unsystematic and fragmented research approaches have left several critical issues unaddressed. The present study primarily focuses on two: the over-reliance on the assumptions of Thurstone’s Case V in the statistical analysis of CJ data, and the apparent disconnect between CJ’s trait measurement and hypothesis testing. The following sections begin with a brief overview of Thurstone’s theory and a detailed discussion of these issues. Subsequently, the study introduces a theoretical model for CJ that builds upon Thurstone’s theory, alongside its statistical translation, designed to address the two concerns simultaneously.

# Thurstone’s theory

In its most general form, Thurstone’s theory deals with pairwise comparisons of stimuli made by a single judge (Thurstone 1927a, 267). The theory proposes that two key factors determine the dichotomous outcome of these comparisons: the discriminal process of each stimulus and their discriminal difference. The *discriminal process* represents the psychological impact each stimulus has on judges or, more simply, their underlying perception of the stimulus’ trait level. According to the theory, the discriminal process for each stimulus follows a Normal distribution, where its mode (mean), referred to as the *modal discriminal process*, indicates the stimulus’ position on the trait continuum, and its dispersion, known as the *discriminal dispersion*, reflects the variability in the perceived trait level of the stimulus.

For instance, [Figure 1](#fig-discriminal_process) illustrates the discriminal process distributions along a quality trait continuum for two written texts. The figure shows that these processes follow a Normal distribution. Moreover, it depicts differences in the texts’ positions along the quality trait continuum, where text B is positioned further along the continuum than text A, as indicated by their modal discriminal processes ( and ). Finally, it highlights differences in the texts’ discriminal dispersions ( and ), showing that text B exhibits a greater variability in its perceived quality than text A, as reflected by its wider distribution.

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| Figure 1: Example distributions of the discriminal processes for two written texts |

However, because the discriminal process of a single stimulus is not directly observable, the theory introduces the *law of comparative judgment*. This law posits that in pairwise comparisons, a judge perceives the stimulus positioned further along the trait continuum as having a higher level of that trait. This principle highlights that the outcome of a pairwise comparison likely depends on the relative distance between stimuli rather than their absolute positions on the trait continuum.

Indeed, the theory assumes that the observed dichotomous outcome arises from the distribution of the difference between the underlying discriminal processes of the stimuli, known as the *discriminal difference*. Since the individual discriminal processes follow a Normal distribution, their difference also follows a Normal distribution (Andrich 1978). The mode (mean) of this distribution, representing the (average) relative separation, is given by the difference between the modal discriminal processes of the stimuli . Meanwhile, the dispersion of the distribution, reflecting the variability in the relative separation, is calculated as . Here, and are the previously defined discriminal dispersions, while measures the correlation between the discriminal processes of the stimuli. This correlation quantifies the dependence of the judge’s perception of the trait in one stimulus on his perception of the same trait in another.

[Figure 2](#fig-discriminal_difference) shows the distribution of the discriminal difference for the texts depicted in [Figure 1](#fig-discriminal_process), assuming a correlation of . The figure reveals that, under these conditions, the judge perceives text B as having significantly higher quality than text A, as indicated by the shaded gray area under the curve . As a result, the dichotomous outcome of this comparison would mostly favor text B over text A.

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| Figure 2: Example distribution of the discriminal difference for the two texts shown in [Figure 1](#fig-discriminal_process), assuming a correlation of 0.6 |

Notably, the correlation between the discriminal processes, , plays a pivotal role in determining comparison outcomes by shaping the distribution of the discriminal difference between the stimuli. Specifically, as the correlation increases, reflecting a stronger dependence of the judge’s perception of quality in one stimulus on his perception of the other, the distribution of the discriminal difference narrows. This narrowing ultimately impacts the area under the curve that determines the comparison outcome and, consequently, the conclusions drawn from this outcome.

[Figure 3](#fig-correlation) illustrates how varying correlations influence the distribution of the discriminal difference for the two texts depicted in [Figure 1](#fig-discriminal_process). Since the texts differ in quality, higher correlations increase the likelihood that the discriminal difference distinctly favors text B over text A. This is evident from the larger proportion of area under the curve, , that lies above zero (shaded gray area in [Figure 3](#fig-correlation)). Conversely, it is not hard to infer that if the texts had similar or identical quality levels, higher correlations would likely reduce the chance that the discriminal difference distinctly favors one text over the other. This probability reduction occurs because the distribution of the discriminal difference would become more narrowly centered around zero (not illustrated).

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| Figure 3: The effect of correlation on the distribution of the discriminal difference of the same two written text |

# Three critical issues in CJ literature

## The Case V and the statistical analysis of CJ data

The previous section outlines the general form of Thurstone’s theory, which applies to a CJ design where a single judge evaluates multiple stimuli. For the practical application of the theory, Thurstone developed four additional cases derived from this general form, where each successive case incorporates additional simplifying assumptions. Case I represents the general form of the theory. Case II extends this by allowing multiple judges to make comparisons rather than restricting the comparisons to a single judge. Case III introduces the assumption of zero correlation between stimuli. Case IV builds on this by assuming that the stimuli have similar dispersions. Finally, Case V replaces this assumption with the condition that the stimuli have equal discriminal dispersions. [Table 1](#tbl-thurstone_cases) summarizes these cases and their assumptions. For a detailed discussion of this progression, refer to Thurstone (1927a) and Bramley (2008, 248–53).

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| Table 1: Thurstones cases and asumptions |

Despite its reliance on the largest number of simplifying assumptions (Bramley 2008, 253; Kelly, Richardson, and Isaacs 2022, 677), Case V remains the most widely used case in the CJ literature. This popularity stems mainly from its simplified statistical representation in the Bradley-Terry-Luce (BTL) model (Bradley and Terry 1952; Luce 1959). The BTL model mirrors the assumptions of Case V, with one key difference: while Case V assumes a Normal distribution for the discriminal processes of the stimuli, the BTL model uses the more mathematically tractable Logistic distribution (Andrich 1978; Bramley 2008, 254) (see [Table 1](#tbl-thurstone_cases)). This substitution has little impact on the model’s estimation or interpretation, as the Normal and Logistic distributions share similar statistical properties, differing only by a scaling factor of approximately (van der Linden 2017a, 1:16) (see [Figure 4](#fig-logistic_vs_normal)).

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| |  | | --- | | (a) Probability density | |  |

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| |  | | --- | | (b) Cummulative probability | |

Figure 4: Probability density and cumulative probability of the logistic and Normal distributions. Extracted from Bramley (2008, 254–55).

However, Thurstone originally developed Case V to provide a “rather coarse scaling” of traits (Thurstone 1927a, 269), prioritizing statistical simplicity over precision in trait measurement (Kelly, Richardson, and Isaacs 2022, 677). As a result, its assumptions may not be suitable for applications beyond the psycho-physical contexts for which it was created. Thurstone himself cautioned that its use “should not be made without (an) experimental test” (Thurstone 1927a, 270), acknowledging that some assumptions could prove problematic in the presence of complex traits or heterogeneous stimuli (Thurstone 1927b, 376). Consequently, given that modern CJ applications frequently involve these traits and stimuli, two main assumptions of Case V and, by extension, of the BTL model may not consistently hold in theory or practice: the equal dispersion and zero correlation between stimuli.

On the one hand, the assumption of *equal dispersion between stimuli* asserts that the variability in perceived trait levels remains consistent across all stimuli. However, Thurstone noted that this assumption may fail when researchers assess complex traits or heterogeneous stimuli (Thurstone 1927b, 376), as these can introduce greater discrepancies into judgments due to their unique characteristics. One example of this is the evaluation of creativity. In this context, for instance, it is plausible that a student completes an assignment in a highly creative yet unexpected way. Judges, unprepared for such originality, may experience greater discrepancies in their judgments of the assignment. As a result, the discriminal difference associated with this assignment is expected to show a broader dispersion, reflecting the larger variation in its discriminal process compared to others. Although direct evidence for this specific scenario is lacking, studies such as van Daal (2020, chap. 8) suggest that misfitting texts tend to exhibit larger dispersion, likely due to increased confusion among judges regarding the texts.

what are the challenges when the equal dispersion assumption is violated?

how do we mitigate these risks?

On the other hand hand, the assumption of *zero correlation between stimuli* implies that, during a pairwise comparison, a judge’s perception of quality in one text does not influence his perception of the same trait in another text (see [Section 2](#sec-thurstone_theory)). Thurstone attributed this independence to the cancellation of potential judges’ biases, driven by two opposing and equally weighted effects occurring during the pairwise comparisons (Thurstone 1927a, 268). Andrich (1978) mathematically demonstrated this cancellation using the BTL model under the assumption of discriminal processes with additive biases. However, it is easy to imagine at least two scenarios where the zero correlation assumption almost certainly does not hold: when the pairwise comparison involves multidimensional, complex traits with heterogeneous stimuli and when an additional hierarchical structure is relevant to the stimuli.

In the first scenario, the intricate aspects of multidimensional, complex traits may introduce dependencies between the stimuli due to certain judges’ biases that resist cancellation. Research on text quality suggests that when judges evaluate these traits, they often rely on various intricate characteristics of the stimuli to form their judgments (van Daal et al. 2016; Lesterhuis 2018b; Chambers and Cunningham 2022). These additional relevant characteristics, which are unlikely to be equally weighted or opposing, can unevenly influence judges’ perceptions, creating biases in their judgments and, ultimately, introducing dependencies between stimuli (van der Linden 2017b, 2:346). For example, this could occur when a judge assessing the argumentative quality of a text places disproportionate emphasis on grammatical accuracy, ultimately favoring texts with fewer errors but weaker arguments. While direct evidence for this specific scenario is lacking, studies such as Pollitt and Elliott (2003) demonstrate the presence of such biases, supporting the idea that the factors influencing pairwise comparisons may not always cancel out.

In the second scenario, the shared context or inherent connections created by additional hierarchical structures may further introduce dependencies between stimuli, a statistical phenomenon commonly known as clustering (Everitt and Skrondal 2010). Although the CJ literature acknowledges the presence of such hierarchical structures, the statistical handling of this extra source of dependency between stimuli has been inadequate. For example, when CJ data includes multiple samples of stimuli from the same individuals, researchers often rely on (average) estimated BTL scores to conduct subsequent analyses and tests at the individual hierarchical level (Bramley and Vitello 2019; Boonen, Kloots, and Gillis 2020; Bouwer et al. 2023; van Daal et al. 2017; Jones et al. 2019; Gijsen et al. 2021). This approach, however, has the significant limitation of ignoring the uncertainty associated with the BTL scores, which generates additional statistical and measurement issues, as discussed in section [Section 3.2](#sec-theory-issue2).

In any case, the psychometric and statistical literature emphasizes the importance of addressing the factors that create dependencies between stimuli, as failing to do so can lead to inaccurate conclusions about a trait’s reliability and, by extension, its validity. For instance, neglecting additional traits relevant to the stimuli, such as judges’ biases, often leads to a dimensional mismatch in the statistical model used for analysis. This mismatch can potentially inflate the reliability of the trait (Hoyle 2023, 341) or, worse, introduce bias into the trait’s estimates (Ackerman 1989). Similarly, failing to account for hierarchical (grouping) structures reduces the precision of model parameter estimates, further amplifying the overestimation of reliability (Hoyle 2023, 482). These issues collectively undermine the trait’s reliability and ultimately compromise the validity of the trait’s estimates (Perron and Gillespie 2015, 2).

Fortunately, the same literature offers solutions for addressing these issues. Andrich (1978) and Wainer, TimbersFairbank, and Hough (1978) recommend integrating judges’ biases into the BTL model. Moreover, the literature advocates for the incorporation of relevant hierarchical structures into the statistical model to account for these dependencies. Together, these additions can result in a model resembling a Multilevel Structural Equation Model (MSEM) (Hoyle 2023, chap. 26) combined with a multidimensional or two-parameter logistic IRT model (Hoyle 2023, chap. 15), depending on the theoretical and statistical treatment of judges’ biases.

## The disconnect between trait measurement and hypothesis testing

Building on the previous section, it is evident that the BTL model commonly functions as the trait’s measurement model in CJ experiments (Andrich 1978; Bramley 2008). A measurement model specifies how manifest variables contribute to the estimation of latent variables (Everitt and Skrondal 2010). For example, when evaluating text quality, researchers use the BTL model to process the dichotomous outcomes resulting from the pairwise comparisons (the manifest variables) to estimate scores that reflect the underlying quality level of texts (the latent variable) (Laming 2004; Pollitt 2012; Whitehouse 2012; van Daal et al. 2016; Lesterhuis 2018a; Coertjens et al. 2017; Goossens and De Maeyer 2018; Bouwer et al. 2023).

Researchers then typically use the estimated BTL scores, or their transformations, to conduct additional analyses or hypothesis tests. For example, these scores have been used to identify ‘misfit’ judges and stimuli (Pollitt 2012; van Daal et al. 2017; Goossens and De Maeyer 2018), detect biases in judges’ ratings (Pollitt and Elliott 2003; Pollitt 2012), calculate correlations with other assessment methods (Goossens and De Maeyer 2018; Bouwer et al. 2023), or test hypotheses related to the underlying trait of interest (Bramley and Vitello 2019; Boonen, Kloots, and Gillis 2020; Bouwer et al. 2023; van Daal et al. 2017; Jones et al. 2019; Gijsen et al. 2021).

However, the statistical literature advises caution when using estimated scores for additional analyses and tests. A key consideration is that BTL scores are parameter estimates that inherently carry uncertainty. Ignoring this uncertainty can bias the analysis and reduce the precision of hypothesis tests. Notably, the direction and magnitude of such biases are often unpredictable. Results may be attenuated, exaggerated, or remain unaffected depending on the degree of uncertainty in the scores and the actual effects being tested (Kline 2023, 25; Hoyle 2023, 137). Finally, the reduced precision in hypothesis tests diminishes their statistical power, increasing the likelihood of committing type-I or type-II errors (McElreath 2020).

To mitigate these risks, principles from Structural Equation Modeling (SEM) (Hoyle 2023, 138) and Item Response Theory (IRT) (Fox 2010, chap. 6; van der Linden 2017a, vol. 1, chap. 24) recommend conducting these analyses and tests within a structural model. A structural model specifies how different manifest or latent variables influence the latent variable of interest (Everitt and Skrondal 2010). This approach allows analyses that can account for both the BTL scores and their uncertainties simultaneously, rather than treating them as separate elements. Therefore, an integrated approach that combines CJ’s measurement and structural models can offer significant advantages.

# An updated theoretical and statistical model for CJ

## The theoretical model

## From theory to statistics

# Discussion

## Findings

## Limitations and further research

# Conclusion

# Declarations

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