Causes and effects in Dichotomous Comparative Judgments: an information-theoretical system with plausible mechanism

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Abstract

Dichotomous Comparative Judgment (DCJ, Pollitt (2012a), Pollitt (2012b)) requires judges to evaluate the relative manifestation of traits between pairs of stimuli, resulting in a dichotomous outcome indicating which stimulus exhibits the trait more strongly. Research has demonstrated DCJ’s effectiveness and reliability in various domains (Pollitt 2012b; Bartholomew et al. 2018; van Daal et al. 2019; Lesterhuis 2018; Bartholomew and Williams 2020; Boonen, Kloots, and Gillis 2020). However, the literature lacks a clear and transparent depiction of the plausible mechanisms underlying DCJ data. Specifically, there is no detail explanation of how the different assessment factors can potentially influence the observed DCJ data. This study aims to fill this gap by applying the framework of causal analysis and Directed Acyclic Graphs (DAG; Pearl (2009)). Using this framework, the study will construct a scientific model to elucidate the causal assumptions and mechanisms inherent the system. This model will enable researchers to draw inferences about causal relationships from DCJ data. Subsequently, the study will translate this model into a probabilistic statistical model, aiming to derive statistical estimands for different targets of inference. The outcomes of this study will inform the planning of DCJ experiments and hold significance for researchers or analysts involved in education and assessment procedures who implement the DCJ methodology.

## Introduction

In contemporary contexts, Thurstone’s law of comparative judgment (1927) primarily refers to the method of *Dichotomous* Comparative Judgment (DCJ, Pollitt 2012a, 2012b). In DCJ, a judge assesses the relative manifestation of a *trait* within a pair of stimuli. This assessment results in a dichotomous value indicating which stimulus possesses a higher degree of the trait. After different judges perform multiple rounds of pairwise comparisons, an outcome vector is produced. This vector is modeled using the Bradley-Terry-Luce model (BTL, Bradley and Terry 1952; Luce 1959; Pollitt 2012a), which produces a latent variable corresponding to the trait of interest. This latent variable is then used to rank the stimuli from lowest to highest or to evaluate the influence of certain variables on the stimuli’s positions in the ranking.

DCJ has proven effective in assessing competencies and traits predominantly within the educational realm, as demonstrated by the works of Pollitt (2012b), Bartholomew et al. (2018), van Daal et al. (2019), Lesterhuis (2018), and Bartholomew and Williams (2020). However, its application transcends education, as exemplified by the work of Boonen, Kloots, and Gillis (2020). The methodology has also evolved to include multiple, as opposed to pairwise comparisons (Luce 1959; Plackett 1975), and to accommodate comparisons with ordinal outcomes (Tutz 1986; Agresti 1992). Overall, research suggests that DCJ offers an alternative and efficient approach to measurement and evaluation, characterized by its reliability and validity (Lesterhuis 2018; van Daal 2020; Marshall et al. 2020). Nevertheless, despite the method’s widespread use, the literature lacks a transparent depiction of the DCJ system and the plausible mechanisms that give rise to DCJ data. Particularly, there is no detailed explanation of how different assessment factors can potentially influence the observed DCJ data.

According to Verhavert et al. (2019) and van Daal (2020), several assessment factors interact and contribute to the reliability of the DCJ method. These factors include the number and characteristics of the stimuli, their *proximity* in terms of the assessed trait, the number of comparison per stimulus, and the pairing algorithm used. Furthermore, since the method relies on judges’ assessments, the number and characteristics of judges, their *discrimination* abilities, and the number of comparisons per judge also play pivotal roles. Moreover, when the stimuli represent sub-units of higher-levels units, factors such as the number and characteristics of these units, along with their *proximity* in terms of the assessed trait, can significantly influence the outcome. An example of this can be found in van Daal et al. (2019), where the authors assessed the *skills in academic writing* (trait) of Flemish university students, utilizing multiple argumentative essays (stimuli, sub-units) originating from various students (units).

## Theoretical framework

### Research questions and their estimands

### A scientific model for the DCJ procedure

### From the scientific to the Bradley-Terry-Luce model

## Discussion

### Limitations and further research

## Conclusion

## Declarations

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