Causes and effects in Dichotomous Comparative Judgments: an information-theoretical system with plausible mechanism

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Abstract

Dichotomous Comparative Judgment (DCJ) requires judges to compare pairs of stimuli to determine which one exhibits a higher degree of a specific trait. DCJ has proven effective and reliable across various fields (Pollitt 2012b; Jones 2015; van Daal et al. 2019; Bartholomew et al. 2018; Lesterhuis 2018; Bartholomew and Williams 2020; Marshall et al. 2020; Boonen, Kloots, and Gillis 2020). However, despite the method’s widespread use, existing literature lacks a clear explanation of the complexities and assumptions underpinning the DCJ system, as well as the plausible mechanisms through which DCJ data could be generated. This study addresses these issues by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach, the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates this model into a probabilistic statistical framework to estimate statistical relationships and infer causal effects within the system. This research provides a robust probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone’s law of comparative judgment (1927). Its findings offer valuable insights for researchers and analysts designing and implementing DCJ experiments.

# Introduction

In contemporary contexts, Thurstone’s law of comparative judgment (1927) primarily refers to the method of *dichotomous* comparative judgment (DCJ, Pollitt 2012a, 2012b). In DCJ, a judge assesses the relative manifestation of a *trait* within a pair of stimuli. This assessment results in a dichotomous value indicating which stimulus possesses a higher degree of the trait. After different judges perform multiple rounds of pairwise comparisons, an outcome vector is produced. This vector is modeled using the Bradley-Terry-Luce model (BTL, Bradley and Terry 1952; Luce 1959), which creates a score that corresponds with the trait of interest. This score is then used to rank the stimuli from lowest to highest or to evaluate the influence of certain variables on the stimuli’s positions in the ranking.

DCJ has proven effective in assessing competencies and traits predominantly within the educational realm, as demonstrated by Pollitt (2012b), Jones (2015), van Daal et al. (2019), Bartholomew et al. (2018), Lesterhuis (2018), Bartholomew and Williams (2020), and Marshall et al. (2020). However, its application transcends education, as exemplified by Boonen, Kloots, and Gillis (2020). The methodology has also evolved to include multiple, as opposed to pairwise comparisons (Luce 1959; Plackett 1975), and to accommodate comparisons with ordinal outcomes (Tutz 1986; Agresti 1992). Overall, research suggests that DCJ offers an alternative and efficient approach to measurement and evaluation, characterized by its reliability and validity (Lesterhuis 2018; van Daal 2020; Marshall et al. 2020). Nevertheless, despite the method’s widespread use, existing literature lacks a clear representation of the plausible mechanisms through which DCJ data could be generated. Particularly, there is no depiction of the complexity and the assumptions underpinning the DCJ system, nor how different assessment factors can potentially influence the observed DCJ outcome.

According to Verhavert et al. (2019) and van Daal (2020), several assessment factors interact and influence the method’s outcome. These factors include the number and characteristics of the stimuli, their *proximity* in terms of the assessed trait, the number of comparison per stimulus, and the pairing algorithm used. Furthermore, since the method relies on judges’ assessments, the number and characteristics of judges, their *discrimination* abilities, and the number of comparisons per judge also play pivotal roles. Moreover, when the stimuli represent sub-units of higher-levels units, factors such as the number and characteristics of these units, along with their *proximity* in terms of the assessed trait, can significantly influence the outcome. For instance, van Daal et al. (2019) assessed academic writing skills of university students (units) using multiple argumentative essays (sub-units).

Although several studies have examined the individual impact of these factors on the method’s reliability (Bramley 2015; Pollitt 2012b; Bramley and Vitello 2019; Verhavert et al. 2019; Crompvoets, Béguin, and Sijtsma 2022; van Daal et al. 2017; Gijsen et al. 2021), none, to the best of the authors’ knowledge, have provided a transparent depiction of the DCJ system and the mechanisms generating the DCJ outcome. This study aims to fill this gap by representing DCJ within the framework of causal inference. Specifically, utilizing the structural approach (Wright 1927; Pearl 2009; Pearl, Glymour, and Jewell 2016), the study develops a scientific model to clarify plausible causal assumptions and mechanisms inherent in the DCJ system. The study then translates the scientific model into a probabilistic statistical model. This model aims to produce statistical estimates to draw inferences about plausible causal relationships within the DCJ system.

Ultimately, this research provides a robust probabilistic foundation for the statistical analysis of DCJ data, building upon Thurstone’s law of comparative judgment (1927). Consequently, its findings offer valuable insights for researchers and analysts designing and implementing DCJ experiments.

# Theoretical framework

## The structural approach to causal inference

In statistics, *causal inference* refers to the process of identifying the causes of a phenomenon and estimating their effects using data (Shaughnessy, Zechmeister, and Zechmeister 2010; Neal 2020). Unlike classical statistical modeling, which focuses solely on summarizing data and inferring associations, causal inference provides a coherent mathematical notation for analyzing causes and counterfactuals (Pearl 2009).

Counterfactuals represent scenarios *contrary to fact*, where alternative *potential* outcomes resulting from a cause are neither observed nor observable (Neal 2020; Counterfactual 2024). According to Pearl and Mackenzie (2018), counterfactuals form the foundation of causal inference and occupy the highest level of cognitive abstraction in the ladder of causation, followed by intervention and association. Nevertheless, despite their abstract nature, counterfactuals enable the development of a *theory of the world* that explains why specific causes have specific effects and what occurs in their absence (Pearl and Mackenzie 2018). They achieve this by translating causal statements into counterfactual statements, that is, statements about “what would have happened in the world under different circumstances.”

Several approaches to causal inference and counterfactuals exist, but two are particularly prominent: the potential outcomes approach, also known as the Neyman-Rubin causal model (Neyman 1923; Rubin 1974), and the structural approach (Wright 1927; Pearl 2009; Pearl, Glymour, and Jewell 2016). Both approaches employ rigorous mathematical notation to characterize causal inference, but they do so in different ways (Neal 2020). The potential outcomes approach relies on counterfactual notation, whereas the structural approach employs the do-operator and structural causal models (SCM, Pearl 2009; Pearl, Glymour, and Jewell 2016). Despite these differences, both notations can be expressed in terms of the other, and both approaches provide methods for using experimental and observational data to estimate causal effects (Pearl 2010).

Nevertheless, the structural approach offers an additional key advantage over the potential outcomes approach: it enables the graphical representation of any system through directed acyclic graphs (DAG, Gross, Yellen, and Anderson 2018; Neal 2020). DAGs are heuristics that can effectively convey the assumed causal structure of a system. They do not represent detailed statistical models but allow researchers to deduce which statistical models can provide valid causal inferences, assuming the causal structure depicted in the DAG is accurate (McElreath 2020).

## Directed acyclic graphs (DAG)

Graph theory is a branch of mathematics focused on the study of graphs. Graphs are mathematical structures that model pairwise relations between objects. They can represent physical relations, such as electrical circuits and roadways, and less tangible structures, such as ecosystems and sociological relations. Graphs have proven useful in various fields, including computer science, operations research, and the natural and social sciences (Gross, Yellen, and Anderson 2018).

In statistics, one application incorporating concepts from graph theory is causal inference. Specifically, the structural approach to causal inference uses directed acyclic graphs (DAG) to provide a formal and graphical representation of the causal structure of a system (Neal 2020). In this context, a *graph* is a collection of nodes connected by edges, where nodes represent random variables. The term *directed* indicates that the edges of the graph extend from one node to another, with arrows showing the direction of causal influence. Moreover, the term *acyclic* indicates the causal influences do not form a loop, meaning the influences do not cycle back on themselves (McElreath 2020).

Regardless of complexity, DAGs can represent various causal structures using only five fundamental building blocks (Neal 2020; McElreath 2020). Each panel of [Figure 1](#fig-dags) illustrates these building blocks. [Figure 1 (a)](#fig-dag_bb1) depicts two unconnected nodes, representing an scenario where variables and are not causally related. [Figure 1 (b)](#fig-dag_bb2) shows two connected nodes, illustrating a scenario where a parent node exerts a causal influence on a child node . Consequently, is considered a *descendant* of . [Figure 1 (c)](#fig-dag_bb3) depicts a *chain* (or *pipe*), where influences , and influences . In this configuration, is a parent node of , and a parent node of . Furthermore, the DAG shows that is an *ancestor* of , and that the relationship between these variables is entirely mediated by . [Figure 1 (d)](#fig-dag_bb4) illustrates a *fork*, where variables and are both influenced by . In this scenario is a parent node of both and . Finally, [Figure 1 (e)](#fig-dag_bb5) depicts a *collider*, also known as *inmorality*, where variables and are concurrent causes of . In this configuration, and are not causally related to each other but both influence .

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| |  | | --- | | (a) Two unconnected nodes | |  |

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| --- | --- | --- | --- | --- |
| |  | | --- | | (b) Two connected nodes or descendant | |  | |  | | --- | | (c) Chain or pipe | |

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| --- | --- | --- | --- |
|  | |  | | --- | | (d) Fork | |  |

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| --- | --- |
| |  | | --- | | (e) Collider or inmorality | |

Figure 1: DAG’s fundamental building blocks.

Given the heuristic nature of DAGs, the use of fundamental building block to construct a causal structure of a system is easier to understand using a motivating example. The motivating example can also serve to signal about other conventions when using DAGs. Consider a system where variables and influence a third variable . In this system, it is assumed that and are dependent on their own processes and are, therefore, independent from each other. [Figure 2 (a)](#fig-dag_example1) presents the plausible causal structure of this system. The DAG shows the endogenous variables as circled black nodes, indicating these variables are observed. The arrows connecting the variables indicate the direction of causal influence, while a lack of influence is indicated by the absence of arrows. Moreover, the exogenous variables represent everything else that is chosen not to be modeled explicitly. These exogenous variables, or *disturbances*, are usually represented by open circles to indicate their unobserved nature. Although this DAG explicitly shows the exogenous variables, conventionally these are omitted for brevity, resulting in an equivalent graph as shown in [Figure 2 (b)](#fig-dag_example2).

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| |  | | --- | | (a) Full DAG | |  |

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| --- | --- |
| |  | | --- | | (b) Simplified DAG | |

Figure 2: DAGs for a plausible causal structure in a system.

## The flow of association and causation in graphs

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| Figure 3: The flow of association and causation in graphs. Extracted from Neal (2020, 31) |

## But where does it all fit?

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| Figure 4: Identification-Estimation flowchart. Extracted from Neal (2020, 32) |

# Theory

## A scientific model for the DCJ

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| Figure 5: DCJ causal diagram, simplified description |

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| Figure 6: DCJ causal diagram, simplified mathematical description |

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| Figure 7: DCJ causal diagram, population mathematical description |

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| Figure 8: DCJ causal diagram, sample with comparisons mathematical description |

## Probabilitics assumptions of the scientific model

## From the scientific to statistical model

## Let’s talk about Thurstone

# Discussion

## Findings

## Limitations and further research

# Conclusion

# Declarations

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# Appendix

## Additional definitions

## Why do we need to estimate judges’ abilities?

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