Causes and effects in Dichotomous Comparative Judgments: an information-theoretical system with plausible mechanism

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Abstract

Dichotomous Comparative Judgment Pollitt (2012b) requires judges to evaluate the relative manifestation of traits between pairs of stimuli, resulting in a dichotomous outcome indicating which stimulus exhibits the trait more strongly. Research has demonstrated DCJ’s effectiveness and reliability in various domains (Pollitt 2012b; Bartholomew et al. 2018; van Daal et al. 2019; Lesterhuis 2018; Bartholomew and Williams 2020; Boonen, Kloots, and Gillis 2020). Nevertheless, despite the method’s widespread use, the literature lacks a transparent depiction of the DCJ system and the plausible mechanisms that generate the DCJ data. Particularly, there is no detailed explanation of how different assessment factors can potentially influence the observed DCJ data. This study aims to fill this gap by applying the framework of causal analysis and Directed Acyclic Graphs [DAG; Pearl (2009)]. Using this framework, the study will construct a scientific model to elucidate the causal assumptions and mechanisms inherent the system. This model will enable researchers to draw inferences about causal relationships from DCJ data. Subsequently, the study will translate this model into a probabilistic statistical model, aiming to derive statistical estimands for different targets of inference. The outcomes of this study will inform the planning of DCJ experiments and hold significance for researchers or analysts involved in education and assessment procedures who implement the DCJ methodology.

# Introduction

In contemporary contexts, Thurstone’s law of comparative judgment (1927) primarily refers to the method of *Dichotomous* Comparative Judgment (DCJ, Pollitt 2012a, 2012b). In DCJ, a judge assesses the relative manifestation of a *trait* within a pair of stimuli. This assessment results in a dichotomous value indicating which stimulus possesses a higher degree of the trait. After different judges perform multiple rounds of pairwise comparisons, an outcome vector is produced. This vector is modeled using the Bradley-Terry-Luce model (BTL, Bradley and Terry 1952; Luce 1959), which creates a score that corresponds with the trait of interest. This score is then used to rank the stimuli from lowest to highest or to evaluate the influence of certain variables on the stimuli’s positions in the ranking.

DCJ has proven effective in assessing competencies and traits predominantly within the educational realm, as demonstrated by Pollitt (2012b), Jones (2015), van Daal et al. (2019), Bartholomew et al. (2018), Lesterhuis (2018), Bartholomew and Williams (2020), and Marshall et al. (2020). However, its application transcends education, as exemplified by Boonen, Kloots, and Gillis (2020). The methodology has also evolved to include multiple, as opposed to pairwise comparisons (Luce 1959; Plackett 1975), and to accommodate comparisons with ordinal outcomes (Tutz 1986; Agresti 1992). Overall, research suggests that DCJ offers an alternative and efficient approach to measurement and evaluation, characterized by its reliability and validity (Lesterhuis 2018; van Daal 2020; Marshall et al. 2020). Nevertheless, despite the method’s widespread use, the literature does not offer a clear representation of the plausible mechanisms that generate the DCJ data. Particularly, there is no depiction of the complexity and the underlying assumptions of the DCJ system, nor how different assessment factors can potentially influence the observed DCJ outcome.

According to Verhavert et al. (2019) and van Daal (2020), several assessment factors interact and influence the DCJ outcome. These factors include the number and characteristics of the stimuli, their *proximity* in terms of the assessed trait, the number of comparison per stimulus, and the pairing algorithm used. Furthermore, since the method relies on judges’ assessments, the number and characteristics of judges, their *discrimination* abilities, and the number of comparisons per judge also play pivotal roles. Moreover, when the stimuli represent sub-units of higher-levels units, factors such as the number and characteristics of these units, along with their *proximity* in terms of the assessed trait, can significantly influence the outcome. For instance, van Daal et al. (2019) assessed university students’ skills in academic writing, utilizing multiple argumentative essays (stimuli, sub-units) originating from various students (units).

Although several studies have examined the individual impact of these factors on the method’s reliability (Bramley 2015; Pollitt 2012b; Bramley and Vitello 2019; Verhavert et al. 2019; Crompvoets, Béguin, and Sijtsma 2022; van Daal et al. 2017; Gijsen et al. 2021), none, to the best of the authors’ knowledge, have provided a transparent depiction of the DCJ system and the plausible mechanisms generating the DCJ outcome. This study aims to fill this gap by representing DCJ within the causal analysis framework (Pearl 2009; Pearl, Glymour, and Jewell 2016). Using Directed Acyclic Graphs (DAG, Gross, Yellen, and Anderson 2018; Neal 2020), the study will construct a scientific model to elucidate the underlying assumptions of the DCJ system, providing plausible mechanisms for how the DCJ outcome could be generated. Next, using a minimal set of assumptions embedded in the framework, the study will translate the scientific model into a probabilistic statistical model. This model derives statistical estimands to draw inferences about plausible causal relationships within the DCJ system. Ultimately, this research aims to extend the law of comparative judgment initially proposed by Thurstone (1927) and provide a sound probabilistic base for the statistical analysis of DCJ data. Consequently, this research holds significance for researchers and analysts involved in education and assessment procedures who implement or design DCJ experiments.

# Preliminaries

## Causal analysis and DAGs

## The flow of association and causation

# Theoretical framework

## A scientific model for the DCJ

## From the scientific to statistical model

## Where does it all fit?

# Discussion

## Findings

## Limitations and further research

# Conclusion

# Declarations

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# Appendix

## Why estimate judges appart?

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