Let’s talk about Thurstone & Co.: An information-theoretical model for comparative judgments, and its statistical translation

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Abstract

This study revisits Thurstone’s law of comparative judgments (CJ) by addressing two key limitations in traditional approaches. Firstly, it addresses the overreliance on the assumptions of Thurstone’s Case V in the statistical analysis of CJ data. Secondly, it addresses the apparent disconnect between CJ’s approach to trait measurement and hypothesis testing. We put forward a systematic approach based on causal analysis and Bayesian statistical methods, which results in a model that facilitates a more comprehensive understanding of the factors influencing CJ experiments while offering a robust statistical translation. The new model accommodates unequal dispersions and correlations between stimuli, enhancing the reliability and validity of CJ’s trait estimation, thereby ensuring the accurate measurement and interpretation of comparative data. The paper highlights the relevance of this updated framework for modern empirical research, particularly in education and social sciences. This contribution advances current research methodologies, providing a robust foundation for future applications in diverse fields.

# Introduction

In *comparative judgment* (CJ) studies, judges assess a specific trait or attribute across various stimuli by performing pairwise comparisons (Thurstone 1927b, 1927a). Each comparison produces a dichotomous outcome, indicating which stimulus is perceived to exhibit a higher trait level. For example, when assessing text quality, judges compare pairs of written texts (the stimuli) to determine the relative quality each text exhibit (the trait) (Laming 2004; Pollitt 2012b; Whitehouse 2012; van Daal et al. 2016; Lesterhuis 2018a; Coertjens et al. 2017; Goossens and De Maeyer 2018; Bouwer et al. 2023).

Numerous studies have documented the effectiveness of CJ in assessing traits and competencies over the past decade. These studies have emphasized three aspects of the method’s effectiveness: its reliability, validity, and practical applicability. Research on reliability indicates that CJ requires a relatively small number of pairwise comparisons (Verhavert et al. 2019; Crompvoets, Béguin, and Sijtsma 2022) to produce trait scores that are as precise and consistent as those generated by other assessment methods (Coertjens et al. 2017; Goossens and De Maeyer 2018; Bouwer et al. 2023). Furthermore, evidence suggests that the reliability and time efficiency of CJ are comparable, if not superior, to those of other assessment methods when employing adaptive comparison algorithms (Pollitt 2012b; Verhavert, Furlong, and Bouwer 2022; Mikhailiuk et al. 2021). Meanwhile, research on validity suggests that scores generated by CJ can accurately represent the traits under measurement (Whitehouse 2012; van Daal et al. 2016; Lesterhuis 2018a; Bartholomew et al. 2018; Bouwer et al. 2023), while research on practical applicability highlights the method’s versatility across both educational and non-educational contexts (Kimbell 2012; Jones and Inglis 2015; Bartholomew et al. 2018; Jones et al. 2019; Marshall et al. 2020; Bartholomew and Williams 2020; Boonen, Kloots, and Gillis 2020).

Nevertheless, despite the increasing number of CJ studies, unsystematic and fragmented research approaches have left several critical issues unaddressed. The present study primarily focuses on two: the overreliance on the assumptions of Thurstone’s Case V in the statistical analysis of CJ data, and the apparent disconnect between CJ’s approach to trait measurement and hypothesis testing. The following sections begin with a brief overview of Thurstone’s theory and a detailed examination of these issues. Subsequently, the study introduces a theoretical model for CJ that builds upon Thurstone’s theory, alongside its statistical translation, designed to address the two concerns simultaneously.

# Thurstone’s theory

In its most general form, Thurstone’s theory addresses pairwise comparisons where a single judge evaluates multiple stimuli (Thurstone 1927a, 267). The theory posits that two key factors determine the dichotomous outcome of these comparisons: the discriminal process of each stimulus and their discriminal difference. The *discriminal process* captures the psychological impact each stimulus exerts on the judge or, more simply, his perception of the stimulus trait. The theory assumes that the DP for any given stimulus forms a Normal distribution along the trait continuum (Thurstone 1927a, 266). The mode (mean) of this distribution, known as the *modal discriminal process*, indicates the stimulus position on this continuum, while its dispersion, referred to as the *discriminal dispersion*, reflects variability in the perceived trait of the stimulus.

[Figure 1 (a)](#fig-discriminal_process) illustrates hypothetical discriminal processes along a quality trait continuum for two written texts. The figure indicates that the mDP for Text B is positioned further along the continuum than that of Text A , suggesting that Text B exhibits higher quality. Additionally, the figure highlights that Text B has a broader distribution compared to Text A, which arises from its larger discriminal dispersion .

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| |  | | --- | | (b) Discriminal difference | |

Figure 1: Hypothetical discriminal processes and discriminant difference along a quality trait continuum for two written texts.

However, since the individual discriminal processes of the stimuli are not directly observable, the theory introduces *the law of comparative judgment*. This law posits that in pairwise comparisons, a judge perceives the stimulus with a discriminal process positioned further along the trait continuum as possessing more of the trait (Bramley 2008, 251). This suggests that the relative distance between stimuli, rather than their absolute positions on the continuum, likely defines the outcome of pairwise comparisons. Indeed, the theory assumes that the difference between the underlying discriminal processes of the stimuli, referred to as *the discriminal difference*, determines the observed dichotomous outcome. Moreover, the theory assumes that because the individual discriminal processes form a Normal distribution on the continuum, the discriminal difference will also conform to a Normal distribution (Andrich 1978). In this distribution, the mode (mean) represents the relative separation between the stimuli, and its dispersion indicates the variability of that separation.

[Figure 1 (b)](#fig-discriminal_difference) illustrates the distribution of the discriminal difference for the hypothetical texts depicted in [Figure 1 (a)](#fig-discriminal_process). The figure indicates that the judge perceives Text B as having significantly higher quality than Text A. This conclusion rests on two key observations: the positive difference between their modal discriminal processes and the probability area where the discriminal difference distinctly favors Text B over Text A, represented by the shaded gray area denoted as . As a result, the dichotomous outcome of this comparison is more likely to favor Text B over Text A.

# The two critical issues in CJ literature

This section examines the two critical issues in the CJ literature that serve as the primary focus of this study. The first is the overreliance on Thurstone’s Case V assumptions in the statistical analysis of CJ data. The second is the apparent disconnect between CJ’s approach to trait measurement and hypothesis testing.

## The Case V and the statistical analysis of CJ data

Thurstone noted that the general form of the theory, as delineated in [Section 2](#sec-thurstone_theory), gave rise to a trait scaling problem. The model required estimating more “unknown” parameters than the available pairwise comparisons (Thurstone 1927a, 267). To address this issue and facilitate the practical implementation of the theory, he developed five cases derived from this general form, each case progressively incorporated additional simplifying assumptions into the model.

In Case I, Thurstone postulated that pairs of stimuli would maintain a constant correlation across all comparisons. In Case II, he allowed multiple judges to undertake comparisons instead of confining evaluations to a single judge. In Case III, he posited that there was no correlation between stimuli. In Case IV, he assumed that the stimuli exhibited similar dispersions. Finally, in Case V, he replaced this assumption with the condition that stimuli had equal discriminal dispersions. [Table 1](#tbl-thurstone_cases) summarizes the assumptions of the general form and the five cases. For a detailed discussion of these cases and their progression, refer to Thurstone (1927a) and Bramley (2008, 248–53).

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| Table 1: Thurstones cases and their asumptions |

Despite relying on the most extensive set of simplifying assumptions (Bramley 2008, 253; Kelly, Richardson, and Isaacs 2022, 677), Case V remains the most widely used case in the CJ literature. This popularity stems mainly from its simplified statistical representation in the Bradley-Terry-Luce (BTL) model (Bradley and Terry 1952; Luce 1959). The BTL model mirrors the assumptions of Case V, with one notable distinction: whereas Case V assumes a Normal distribution for the stimuli’s discriminal processes, the BTL model uses the more mathematically tractable Logistic distribution (Andrich 1978; Bramley 2008, 254) (see [Table 1](#tbl-thurstone_cases)). This substitution has little impact on the model’s estimation or interpretation, as the Normal and Logistic distributions exhibit analogous statistical properties, differing only by a scaling factor of approximately (van der Linden 2017a, 1:16).

However, Thurstone originally developed Case V to provide a “rather coarse scaling” of traits (Thurstone 1927a, 269), prioritizing statistical simplicity over precision in trait measurement (Kelly, Richardson, and Isaacs 2022, 677). He explicitly warned against its untested application, stating that its use “should not be made without (an) experimental test” (Thurstone 1927a, 270). Furthermore, he acknowledged that some assumptions could prove problematic when researchers assess complex traits or heterogeneous stimuli (Thurstone 1927b, 376). Consequently, given that modern CJ applications frequently involve such traits and stimuli, two main assumptions of Case V and, by extension, of the BTL model may not consistently hold in theory or practice, namely the assumption of equal dispersion and zero correlation between stimuli.

### The assumption of equal dispersions between stimuli

According to the theory, discrepancies in the discriminal dispersions of stimuli shape the distribution of the discriminal difference, exerting a direct influence on the outcome of pairwise comparisons. [Figure 2 (a)](#fig-dispersion) presents a thought experiment to illustrate this idea. In this experiment, a researcher can observe the discriminal processes for the texts depicted in [Figure 1 (a)](#fig-discriminal_process). Furthermore, the figure assumes that the discriminal dispersion for Text A remains constant and that the texts are uncorrelated . The figure reveals that an increase in the uncertainty associated with the perception of Text B in comparison to Text A, , broadens the distribution of their discriminal difference. This broadening affects the probability area where the discriminal difference distinctly favors Text B over Text A, expressed as , ultimately influencing the comparison outcome. Additionally, the figure reveals that when the discriminal dispersions of the texts are equal , the discriminal difference is more likely to favor Text B over Text A (shaded gray area), compared to situations where their dispersions differ.

In experimental practice, however, this process occurs in reverse. Researchers first observe the comparison outcome and then use the BTL model to infer the discriminal difference between the stimuli and their respective discriminal processes (Thurstone 1927b, 373). Therefore, it is reasonable to assume that the outcome’s ability to reflect the “true” differences between stimuli largely depends on the validity of the model’s assumptions (Kohler, Kreuter, and Stuart 2019, 150), particularly the assumption of equal dispersions. For instance, when researchers observe a sample of outcomes favoring Text B over Text A and correctly assume equal dispersions between the texts, the BTL model estimates a discriminal difference distribution that accurately represents the “true” discriminal difference of the texts. This scenario is illustrated in [Figure 2 (a)](#fig-dispersion), where the model’s discriminal difference distribution aligns with the “true” distribution, represented by the thick continuous line corresponding to . The accuracy of these discriminal difference ensures reliable estimates for the texts’ discriminal processes (citation needed?).

However, Thurstone argued that the assumption of equal dispersions may not be applicable when researchers assess complex traits or heterogeneous stimuli (Thurstone 1927b, 376), as these traits and stimuli can introduce judgment discrepancies due to their unique features (van Daal et al. 2016; Lesterhuis 2018b; Chambers and Cunningham 2022). Indeed, evidence of this violation may already be present in the CJ literature in the form of misfit statistics, which measure judgment discrepancies associated with specific stimuli (Pollitt 2004, 12; Goossens and De Maeyer 2018, 20). For example, labeling texts as “misfits” indicates that comparisons involving these texts result in more judgment discrepancies than those involving other texts (Pollitt 2012a, 2012b; van Daal et al. 2016; Goossens and De Maeyer 2018). These discrepancies, in turn, suggest that the discriminal differences for “misfit” texts have broader distributions, indicating that their discriminal processes may also exhibit more variation than that of other texts. A similar line of reasoning applies to the concept of “misfit” judges, whose evaluations deviate substantially from the shared consensus due to the unique characteristics of the stimuli or the judges themselves. These “misfit” judges and their associated deviations can give rise to additional statistical and measurement issues, which we discuss in more detail in [Section 3.1.2](#sec-theory-issue1b).

Nevertheless, model misspecification, in the form of an erroneous assumption of equal dispersions between stimuli, can give rise to significant statistical and measurement issues. For instance, the model may overestimate the degree to which the outcome accurately reflects the “true” discriminal differences between stimuli. This overestimation can result in researchers drawing spurious conclusions about these differences (McElreath 2020, 370) and, by extension, about the underlying discriminal processes of stimuli. [Figure 2 (a)](#fig-dispersion) illustrates this issue when the model’s discriminal difference distribution aligns with the thick continuous line for , while the “true” discriminal difference follows any discontinuous line where .

Additionally, if researchers recognize that misfit statistics highlight these critical differences in dispersions, the conventional CJ practice of excluding stimuli based on these statistics (Pollitt 2012a, 2012b; van Daal et al. 2016; Goossens and De Maeyer 2018) can unintentionally discard valuable information. Such exclusions can introduce bias into trait estimates (Zimmerman 1994; McElreath 2020, chap. 12). The direction and magnitude of these biases are often unpredictable, as they depend on which stimuli are excluded from the analysis.

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| |  | | --- | | (b) Correlation between stimuli | |

Figure 2: The effect of dispersion discrepancies and stimulus correlation on the distribution of the discriminal difference.

### The assumption of zero correlation between stimuli

The correlation, represented by the symbol , measures how much a judge’s perception of a specific trait in one stimulus depends on their perception of the same trait in another. As with the discriminal dispersions, this correlation shapes the distribution of the discriminal difference, directly impacting the outcomes of pairwise comparisons. [Figure 2 (b)](#fig-correlation) presents a similar thought experiment as in [Section 3.1.1](#sec-theory-issue1a) to illustrate this idea. The illustration now assumes that the discriminal dispersions for both texts remain constant. The figure shows that as the correlation between the texts increases, the distribution of their discriminal difference becomes narrower. This narrowing affects the area under the curve where the discriminal difference distinctly favors Text B over Text A, denoted as , thus influencing the comparison outcome. Furthermore, the figure shows that when two texts are independent or uncorrelated , their discriminal difference is less likely to favor Text B over Text A (shaded gray area) compared to scenarios where the texts are highly correlated.

Off course, in experimental practice, researchers approach this process in reverse. They begin by observing the sample of outcomes favoring Text B over Text A and then use the BTL model to estimate the discriminal difference and the discriminal processes of the stimuli. Given that the BTL model assumes independent discriminal processes across comparisons, if this assumption holds, then the model estimates a discriminal difference distribution that accurately reflects the “true” discriminal difference of the texts. This scenario is illustrated with [Figure 2 (b)](#fig-correlation) when the discriminal difference distribution of the model aligns with the “true” distribution, represented by the thick continuous line corresponding to . Once more, the estimation accuracy of the discriminal difference ensures reliable estimates for the discriminal processes of the texts (citation needed?).

Notably, Thurstone attributed the independence of stimuli to the cancellation of potential judges’ biases. He argued that this cancellation resulted from two opposing and equally weighted effects occurring during pairwise comparisons (Thurstone 1927a, 268). Andrich (1978) provided a mathematical demonstration of this cancellation using the BTL model under the assumption of discriminal processes with additive biases. However, it is easy to imagine at least two scenarios in which the zero correlation assumption is almost certainly invalid: when the pairwise comparison involves multidimensional, complex traits with heterogeneous stimuli and when an additional hierarchical structure is relevant to the stimuli.

In the first scenario, the intricate aspects of multidimensional, complex traits may introduce dependencies between the stimuli due to certain judges’ biases that resist cancellation. Research on text quality suggests that when judges evaluate these traits, they often rely on various intricate characteristics of the stimuli to form their judgments (van Daal et al. 2016; Lesterhuis 2018b; Chambers and Cunningham 2022). These additional relevant characteristics, which are unlikely to be equally weighted or opposing, can exert an uneven influence on judges’ perceptions, creating biases in their judgments and, ultimately, introducing dependencies between stimuli (van der Linden 2017b, 2:346). For example, this could occur when a judge assessing the argumentative quality of a text places more weight on its grammatical accuracy than other judges, thereby favoring texts with fewer errors but weaker arguments. While direct evidence for this particular scenario is lacking, studies such as Pollitt and Elliott (2003) demonstrate the presence of such biases, supporting the notion that the factors influencing pairwise comparisons may not always cancel out.

In the second scenario, the shared context or inherent connections created by additional hierarchical structures may further introduce dependencies between stimuli, a statistical phenomenon commonly known as clustering (Everitt and Skrondal 2010). Despite the CJ literature acknowledging the existence of such hierarchical structures, the statistical handling of this additional source of dependence between stimuli has been inadequate. For instance, when CJ data incorporates multiple samples of stimuli from the same individuals, researchers frequently rely on (average) estimated BTL scores to conduct subsequent analyses and tests at the individual hierarchical level (Bramley and Vitello 2019; Boonen, Kloots, and Gillis 2020; Bouwer et al. 2023; van Daal et al. 2017; Jones et al. 2019; Gijsen et al. 2021). However, this approach can introduce additional statistical and measurement issues, which we discuss in greater detail in [Section 3.2](#sec-theory-issue2).

In any case, similar to [Section 3.1.1](#sec-theory-issue1a), model misspecification due to an erroneous assumption of zero correlation between stimuli can lead to significant statistical and measurement issues. For instance, the model may over- or underestimate how accurately the outcome reflects the “true” discriminal differences between stimuli. Such inaccuracies can result in spurious inferences about these differences and, by extension, about the stimuli’s discriminal processes. This scenario is illustrated in [Figure 2 (b)](#fig-correlation), where the model’s discriminal difference distribution aligns with the thick continuous line for , while the “true” discriminal difference follows any discontinuous line where .

This misspecification may arise from neglecting additional relevant traits, excluding judges based on misfit statistics, or ignoring hierarchical (grouping) structures. Neglecting relevant traits, such as judges’ biases, can cause dimensional mismatches in the BTL model, artificially inflating the trait’s reliability (Hoyle 2023, 341) or, worse, introducing bias into the trait’s estimates (Ackerman 1989). Excluding judges based on misfit statistics risks discarding valuable information, which may further bias the trait’s estimates (Zimmerman 1994; McElreath 2020, chap. 12). Finally, ignoring hierarchical structures may reduce the precision of model parameter estimates, potentially amplifying the overestimation of the trait’s reliability (Hoyle 2023, 482).

## The disconnect between trait measurement and hypothesis testing

Building on the previous section, it is clear that, despite its limitations, the BTL model is commonly used as the measurement model in CJ assessments. A measurement model specifies how manifest variables contribute to the estimation of latent variables (Everitt and Skrondal 2010). For example, when evaluating text quality, researchers use the BTL model to process the dichotomous outcomes resulting from the pairwise comparisons (the manifest variables) to estimate scores that reflect the underlying quality level of the texts (the latent variable) (Laming 2004; Pollitt 2012b; Whitehouse 2012; van Daal et al. 2016; Lesterhuis 2018a; Coertjens et al. 2017; Goossens and De Maeyer 2018; Bouwer et al. 2023).

Researchers then typically use these estimated BTL scores, or their transformations, to conduct additional analyses or hypothesis tests. For example, these scores have been used to identify ‘misfit’ judges and stimuli (Pollitt 2012b; van Daal et al. 2016; Goossens and De Maeyer 2018), detect biases in judges’ ratings (Pollitt and Elliott 2003; Pollitt 2012b), calculate correlations with other assessment methods (Goossens and De Maeyer 2018; Bouwer et al. 2023), or test hypotheses related to the underlying trait of interest (Bramley and Vitello 2019; Boonen, Kloots, and Gillis 2020; Bouwer et al. 2023; van Daal et al. 2017; Jones et al. 2019; Gijsen et al. 2021).

However, the statistical literature advises caution when using estimated scores for additional analyses and tests. A key consideration is that BTL scores are parameter estimates that inherently carry uncertainty. Ignoring this uncertainty can bias the analysis and reduce the precision of hypothesis tests. Notably, the direction and magnitude of such biases are often unpredictable. Results may be attenuated, exaggerated, or remain unaffected depending on the degree of uncertainty in the scores and the actual effects being tested (Kline 2023, 25; Hoyle 2023, 137). Finally, the reduced precision in hypothesis tests diminishes their statistical power, increasing the likelihood of committing type-I or type-II errors (McElreath 2020).

In aggregate, researchers’ inadequate handling of violations to the assumptions of equal dispersion and zero correlation between stimuli, coupled with the apparent disconnect between CJ’s approach to trait measurement and hypothesis testing, can potentially compromise the reliability of the trait estimates and, by extension, their validity (Perron and Gillespie 2015, 2). Consequently, adopting a more systematic and integrated approach to examine the factors influencing CJ experiments could offer several statistical and measurement benefits, including the ability to address these issues.

# An updated theoretical and statistical model for CJ

This section presents a theoretical model for CJ that extends Thurstone’s theory. It uses causal analysis (Pearl 2009; Pearl, Glymour, and Jewell 2016; Morgan and Winship 2014; Hernán and Robins 2020) and, in particular, directed acyclic graphs (DAGs) (Gross, Yellen, and Anderson 2018; Neal 2020), to articulate the core theoretical principles of CJ theory. The model also incorporates several practical factors that influence judges in CJ experiments, such as the selection of judges, stimuli, and the number of comparisons. In addition, the study uses Bayesian statistical methods (McElreath 2020) to translate these theoretical and practical elements into a statistical model that facilitates the analysis of pairwise comparison data.

## The theoretical model

The (latent) discriminal difference of the stimuli directly determines the (manifest) outcome of the pairwise comparisons

The (latent) “perceived” discriminal processes for the stimuli directly determines their discriminal difference

The (latent) “true” discriminal processes for the stimuli and the judges’ biases directly determines their (latent) “perceived” discriminal processes

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without loosing generality, the (latent) “perceived” and “true” discriminal processes for the stimuli can be depicted in a vector for each judge, as in

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Considering the sampling and comparison mechanisms

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| Figure 15 |

## From theory to statistics

# Discussion

## Findings

## Limitations and further research

# Conclusion

# Declarations

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