Finding Thurstone: modeling comparative judgment data with R (and Stan)

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Abstract

A particular data analysis workflow has become the standard approach for analyzing comparative judgment (CJ) data, because it provides a simple method for measuring traits and conducting statistical inferences. The workflow's simplicity stems from two key features: (1) the use of the Bradley-Terry-Luce (BTL) model, which imposes an extensive set of simplifying assumptions about traits, judges, and stimuli in CJ assessments; and (2) the use of ad hoc procedures to handle inferences, including hypothesis testing. However, recent studies question whether the BTL assumptions hold in contemporary CJ applications and whether the ad hoc procedures effectively fulfill their intended analytical goals.

To address these concerns, Rivera et al. (2025) proposed an approach that extends the general form of Thurstone's law of comparative judgment. The approach enables the development of a model tailored to the assumed data-generating process of the CJ system under study, eliminating the need to rely on simplifying assumptions. Moreover, by integrating measurement and inference within a single analytical framework, the approach also removes the dependence on ad hoc hypothesis-testing procedures.

Keywords: tutorial, causal inference, bayesian inference, thurstonian model, comparative judgement, statistical modeling

1. Introduction

Comparative judgment (CJ) has emerged as a valuable methodology for measuring latent traits across diverse fields, including education (Kimbell, 2012; Jones and Inglis, 2015; van Daal et al., 2016; Bartholomew et al., 2018), political sciences (ZuccoJr. et al., 2019), linguistics (Boonen et al.,

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2020), and criminology (Seymour and Hernandez, 2025). In CJ studies, judges actively compare pairs of stimuli to determine which stimulus exhibits more of the latent trait of interest (Thurstone, 1927b,a).

A particular data analysis workflow has become the standard approach for analyzing CJ data (see e.g., Thwaites and Paquot, 2024). Researchers favor this approach because it provides a simple method for measuring traits and conducting statistical inferences (Andrich, 1978; Pollitt, 2012). This simplicity, in turn, arises from two key features. First, the workflow relies on the Bradley-Terry-Luce (BTL) model (Bradley and Terry, 1952; Luce, 1959), which imposes an extensive set of simplifying assumptions about traits, judges, and stimuli in CJ assessments (Thurstone, 1927a; Bramley, 2008). Second, the workflow uses ad hoc procedures to handle inferences, including data summaries and hypothesis testing (Pollitt, 2012).

Recent studies, however, question whether the assumptions of the BTL model hold in contemporary CJ applications and whether the ad hoc procedures achieve their intended analytical goals (Bramley, 2008; Kelly et al., 2022; Rivera et al., 2025). For instance, Rivera et al. (2025) argue that while the assumptions of equal dispersions and zero correlations between stimuli simplify trait measurement, they may fail to represent complex traits or heterogeneous stimuli adequately (Thurstone, 1927b; Andrich, 1978; van Daal et al., 2016; Lesterhuis et al., 2018; Chambers and Cunningham, 2022). As a result, such assumptions can compromise the reliability and accuracy of trait estimates (Ackerman, 1989; Zimmerman, 1994; McElreath, 2020; Wu et al., 2022; Miller, 2023; Hoyle, 2023). Furthermore, the same authors note that although ad hoc procedures simplify data analyses, the use of untested methods can also undermine the validity of statistical inferences derived from CJ data (McElreath, 2020; Kline, 2023; Hoyle, 2023).

To address these concerns, Rivera et al. (2025) proposed an approach that extends the general form of Thurstone's law of comparative judgment (Thurstone, 1927b,a). This approach leverages causal and Bayesian inference methods to combine Thurstone's core theoretical principles with key design features of CJ assessment. By doing so, it enables the development of a model tailored to the assumed data-generating process of the CJ system under study. This tailoring effectively removes the need to rely on the simplifying assumptions of the BTL model. Moreover, by integrating measurement and inference within a single analytical framework, the approach also eliminates the dependence on ad hoc hypothesis-testing procedures. Ultimately, this approach has the potential to produce reliable trait estimates and accurate statistical inferences. However, its effectiveness still requires empirical validation.

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7. Appendix

7.1. Appendix A: Stationarity, converge and mixing

7.2. Appendix B: Misfit observations

7.3. Appendix C: Sample size calculations

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