

Generalized Linear Latent and Mixed Modeling:

method, estimation procedures, advantages, and applications to educational policy.

Jose Manuel Rivera Espejo

Supervisor: Prof. Geert Molenbegrhs
Affiliation (optional)

Co-supervisor: Prof. Wim Van den
Noortgate *(optional)*
Affiliation (optional)

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Dedication

To Manuel, for being my friend and father.
To Margarita, Susan and Ana, for their relentless encouragement.
To both of you, as you are always in my mind.
And to all that knowingly or not, help me to get here.
I am lucky due to all of you.
I hope I make you all proud.

A Manuel, por ser mi amigo y mi padre.
A Margarita, Susan y Ana, por su incansable aliento.
A ustedes dos, que siempre las tengo en mente.
Y a todos los que sabiendolo o no, me ayudaron a llegar aquí
Soy un suertudo gracias todos ustedes.
Espero llenarlos de orgullo.

Acknowledgment

(in the works)

Abstract

(in the works)

Keywords:

Contents

1	Introduction	1
1.1	Preliminar considerations	1
1.2	Objectives	3
1.3	Organization	4
2	The Generalized Linear Latent and Mixed Model	5
2.1	Definition	5
2.1.1	Response model	5
2.1.2	Structural model for the latent variables	11
2.1.3	Distribution of the latent variables	12
2.1.4	Model identification	12
2.2	Relationship with other modeling schemes	12
2.2.1	Factor Models	13
2.2.2	Item Response Theory and Generalized Latent Models	13
2.2.3	Multilevel Models	13
2.3	Advantages and Disadvantages	13
3	Estimation	14
3.1	Likelihood methods	14
3.1.1	Likelihood function	14
3.1.2	Adaptive Quadrature	14
3.2	Bayesian methods	14
3.2.1	Prior distributions	14
3.2.2	Initial start	14
3.2.3	Posterior distributions	14
4	Application	15
4.1	Instruments	15
4.2	Data	15
4.2.1	Collection	15
4.2.2	Sample scheme	15
4.3	Results	15
4.3.1	Hypothesis 1:	15
4.3.2	Hypothesis 2:	15
4.3.3	Hypothesis 3:	15

5	Conclusion and Discussion	16
5.1	Discussion	16
5.2	Conclusions	16
5.3	Future development	16
A	Additional Theory	17
A.1	Special cases for the GLAMM	17
B	Code	18

List of Figures

List of Tables

Abbreviations

GLLAMM	Generalized Linear Latent and Mixed Model.
SEM	Structural Equation Model.
GLM	Generalized Linear Model.
EFA	Exploratory Factor Analysis.
CFA	Confirmatory Factor Analysis.
IRT	Item Response Theory.

Symbols

J	total number of subjects.
j	index of specific subject.
I	total number of items.
i	index for the specific item.
$M_{(l)}$	total number of latent variables at level l .
m	index for the specific latent variable.
L	total number of levels (clusters).
l	index for the specific level (cluster).
\mathbf{V}	vector of the linear predictors.
$\boldsymbol{\beta}$	vector of fixed effects, for the I items.
\mathbf{X}	design matrix for the $\boldsymbol{\beta}$ parameters.
$\eta_{mj}^{(l)}$	latent variable, at the respective indices.
$\boldsymbol{\lambda}_m^{(l)}$	vector of loadings, at the respective indices.
$\mathbf{Z}_m^{(l)}$	design matrix for the $\boldsymbol{\lambda}_m^{(l)}$ parameters.

Chapter 1

Introduction

1.1 Preliminar considerations

The short and long term benefits of effective teaching practices can be observed throughout the literature: improvements in student achievements (Rockoff; 2004; Rivkin et al.; 2005; Duflo et al.; 2009; Hanushek and Rivkin; 2012; Muralidharan and Sundararaman; 2013; Chetty et al.; 2014a; Araujo et al.; 2016); development of executive functions (Araujo et al.; 2016), increased college attendance, higher salaries, lower possibility of premature parenthood (Chetty et al.; 2014b), among others. Similarly, the literature has shown most of the negative impacts resulting from the presence of teacher shortages¹ (Duflo et al.; 2009; Muralidharan and Sundararaman; 2013; Chetty et al.; 2015; Ayala; 2017; Marotta; 2019) or ineffective teaching practices (Hanushek and Rivkin; 2012).

However, while the evidence have a solid methodological support, Hanushek and Rivkin (2006) have indicated that some of the proxy variables used, are not consistently related to either teacher effectiveness or quality of instruction, examples of such are: out of field teaching² (Ingersoll; 1998; Dee and Cohodes; 2008; Bertoni et al.; 2020); teaching hours (Bruns et al.; 2015); years of experience or educational degree (Rockoff; 2004; Rivkin et al.; 2005; Clotfelter et al.; 2006, 2007; Hanushek and Rivkin; 2012); among others.

Given the lack of consistency of the effects that arises from using such proxies, Hanushek and Rivkin (2012) have pointed out that the analysis of teacher effectiveness has largely turned away, from attempts to identify the teacher's specific characteristics, to focus its attention into measuring the direct relationship between them and the student outcomes³. For that reason, considerable uncertainty is still present in the literature, regarding exactly which aspects of teachers are key for the student's learning and whether those qualities can be measured (Rockoff; 2004; Clotfelter et al.; 2006).

¹Bertoni et al. (2020) defined it as the context in which the teacher's supply, i.e. the number of available teachers in the system, is less than its demand. The authors further elaborate that one of the causes of these shortages is related to the applicants' lower quality or due to their faulty initial training, implying that the shortage can also be conceived as the lack of good quality teachers. In this sense, the evidence of such shortage has been more prevalent, but not decisive, with temporary teachers, as they are usually associated with inferior attributes, compared to their contracted counterparts

²Medeiros et al. (2018) defines it as teachers teaching a subject in which they are not specialized or do not have the appropriate certificate.

³The method is known as value-added analysis, and it is based on the perspective that a good teacher is one who consistently gets higher achievement from students after other determinants of such are controlled for. For a more detailed explanation of the method refer to Scherrer (2011).

However, because the evidence still largely supports the perception that teachers are the main driver behind the student's learning processes, one of the main points in the agenda of any educational authority should be the design of an assessment system that can attract, select, develop, and retain the most effective ones (Elacqua et al.; 2018).

In that sense, an Educational Performance Standard (EPS) that best agrees with the country's context have to be defined. With the EPS establishment, the authorities can set clear expectations about what a "good" teacher should know and know to do (Cruz-Aguayo et al.; 2020). While the specific requirements are not easy to define, Cruz-Aguayo et al. have hinted that most of them can be largely grouped into two: (i) to have the disciplinary knowledge and pedagogical practices adequate to the classroom characteristics, context and teaching level, and (ii) to display such knowledge and practices in the classroom, using the appropriate material and technological resources available.

As one can infer from the previous general conditions, and the slew evidence, the disciplinary knowledge is a relevant observable factor that it is consistently associated with teacher effectiveness and growth in the student's achievement (Santibañez; 2006; Clotfelter et al.; 2006, 2007; Hanushek and Rivkin; 2006; Marshall; 2009; Rockoff et al.; 2011; Kane et al.; 2010; Kane and Staiger; 2012; Ome; 2012; Metzler and Woessmann; 2012; Kane et al.; 2013; Araujo et al.; 2016; Bietenbeck et al.; 2018; Estrada; 2019); and in that sense, its measurement should be of interest for any educational authority.

The measurement of knowledge has a myriad of available tools, however, given that any educational department are bounded by budgetary constraints, valid⁴ and reliable⁵ standardized tests⁶ stand out not only for its cost-effectiveness, and a much simpler implementation (Cruz-Aguayo et al.; 2020), but also because, they are one of tools with less subjective scoring processes and interpretations, compared to other instruments.

However, as no instrument is perfect, the teacher's subject knowledge scores will likely reflect measurement error (Metzler and Woessmann; 2012). As established by Angrist and Krueger (1999), measurement error in the explanatory variable could bias the estimated coefficients, which implies that evidence based on test scores could be an attenuated reflection of the true effects. On the other hand, the use of one composite value, i.e. the score, does not allow to test which specific factors -if any- leads to better or worse teacher performance, making also difficult to know which teachers should be hired or what should be done to train them (Hanushek and Rivkin; 2012).

But beyond the use of test results as explanatory variables in modeling processes, there is one more pressing argument on why the issue of measurement error should be addressed: approximately 60% of the Caribbean and Latin American countries use standardized test scores as part of or as a main teacher selection tool (Cruz-Aguayo et al.; 2020). In this setting, devoting effort to assess the issues related to measurements errors, could help the educational authorities to understand, for example: if the scores thresholds used for the selection processes are appropriately set, or ultimately, to know the characteristics of the teachers that are being integrated into the public teaching staff.

⁴the extend to which a measurement tool is well-founded and accurately corresponds to the real measure (Kelley; 1927)

⁵the overall consistency of a measure under consistent conditions.

⁶Assessment instrument in which the implementation, questions, scoring processes, and interpretations are consistent with a predetermined or typified way. The instrument is usually composed of questions or items that fulfill three conditions: (i) they are polytomous, i.e. they have multiple choices, (ii) the choice categories are nominal, i.e. do not present any specific order, and (iii) there is only one "correct" category or answer (Rivera; 2019)

In summary, teachers are one of the main drivers behind the student achievements. However, some of the evidence supporting this claim has been based on proxy variables that are not consistently related to the quality of instruction, or methods that are not concerned with the outline of the teaching factors responsible for the student's learning. Nevertheless, while the literature still reflects considerable uncertainty on what are the "ingredients for a good teacher", a good amount of evidence has supported the disciplinary and pedagogical knowledge as relevant components of the teacher effectiveness. Finally, the literature has shown that valid and reliable standardized tests are among the best tools to assess such factors, but also have emphasized that such scores could reflect the teacher's abilities with considerable noise.

1.2 Objectives

This research will have two main goals. First, to describe the method, estimation procedures, and advantages of the Generalized Linear Latent and Mixed Modeling framework (GLLAMM, Rabe-Hesketh et al.; 2004a,c; ?, 2012). Second, to test the real implications of the method, in a data composed of large repeated Teacher's standardized educational assessments from Peru.

Specifically, for the first objective of the research, the author expects to appraise:

1. If the method can provide a general framework that could serve multiple psychometric purposes, e.g. to analyze the quality of the items, to obtain a dynamical noise-free "score" for the disciplinary abilities of the teachers, among others; and
2. What are the advantages or disadvantages of such models, specially compared to factor, item-response theory and multilevel models.

For the second objective, the author expects to shed some lights about some key policy decisions related to those large evaluation processes, to mention a few:

- Are the educational authorities screening the teachers with higher disciplinary knowledge?, and in that sense, what differentiate a contract teacher from a temporary one?,
- What are the general characteristics of the teaching-career applicants?, What is the level of their disciplinary knowledge, and how it evolves?,
- Do the initial training or socioeconomic status help to explain the disciplinary knowledge profile of the applicants?
- What specific factor of the disciplinary knowledge is consistently related to a good performance in the classroom?
- Do the instruments guarantee a fair assessment of minority groups with different abilities?

In this sense, the researcher believes the master's thesis contributes to the literature in two aspects:

1. In a the theoretical and methodological sense, as the research is focused on offering an exhaustive description and analysis of the GLLAMM framework; and
2. In a more practical sense, as it helps to provide evidence on some of key policy decisions that Latin America countries are currently facing.

Finally, it is important to mention, that the computational implementation of the methods will be developed in **R** (R Core Team; 2015) and **WinBUGS** (Lunn et al.; 2000).

1.3 Organization

Chapter 2, The Generalized Linear Latent and Mixed Model, will describe the model, its components, characteristics, assumptions and properties, to finally assess its benefits against factor (EFA and CFA), IRT and Multilevel models.

Chapter 3, Estimation, will describe **two** of the methods that can be used to fit such models: **Likelihood and Bayesian methods**. The chapter will also present the computational implementation of the model.

Chapter 4, Application, will describe the instruments and the "dimensions" under analysis. Additionally, it will describe briefly the data collection process, the sample design, and the results of the analysis under the GLLAMM framework.

Finally, **Chapter 5, Conclusions**, will discuss the conclusion for the method, its estimation process, the policy implications derived from the implementation of the model in a large teacher's assessment process, and the path of future research that can be derived from the present effort.

Chapter 2

The Generalized Linear Latent and Mixed Model

The Generalized Linear Latent and Mixed Model (GLLAMM) is a framework that unifies a wide range of latent variable models. Developed by Rabe-Hesketh et al. (2004a,b); Skromdal and Rabe-Hesketh (2004a), the method was motivated by the need of a multilevel Structural Equation Model (SEM) that accommodates for unbalanced data, noncontinuous responses and the use of cross-level effects among latent variables.

This chapter presents the definition of such model, its characteristics, assumptions and properties.

2.1 Definition

Following Rabe-Hesketh et al. (2004a, 2012), we depart from the traditional multivariate framework for formulating factor and structural models, i.e. a "wide" data format, and adopt a univariate approach, i.e. "long" or vectorized format. In that sense, all the response (manifest) variables for each unit are "stacked" in a single response vector with different variables distinguished from each other by a design matrix. With this structure, we proceed to outline the three parts of the framework:

1. the response model,
2. the structural latent variable model, and
3. the distribution of the latent variables.

For a detailed description of some of the special cases of multilevel SEM, that can be derived with this framework, refer to Appendix A.

2.1.1 Response model

As outlined by the authors, conditional on the latent variables, the response model is a Generalized Linear Model (GLM) defined by a systematic and a distributional part. For the systematic part, a linear predictor and a link function are selected, accordingly to the characteristics of the manifest variables. On the other hand, for the distributional part, a distribution from the exponential family is also selected.

In the following sections, we proceed to describe the linear predictor, the link function and the distributions accommodated by the framework.

Linear predictor

For a model with L levels and M_l latent variables at $l > 1$ levels, the linear predictor takes the following form:

$$v = \mathbf{X}\boldsymbol{\beta} + \sum_{l=2}^L \sum_{m=1}^{M(l)} \eta_m^{(l)} \mathbf{Z}_m^{(l)} \boldsymbol{\lambda}_m^{(l)} \quad (2.1)$$

where \mathbf{X} is a design matrix that maps the parameter vector $\boldsymbol{\beta}$ to the linear predictor, $\eta_m^{(l)}$ the m th latent variable at level l ($m = 1, \dots, M(l)$ and $l = 1, \dots, L$), and $\mathbf{Z}_m^{(l)}$ a design matrix that maps the vector of loadings $\boldsymbol{\lambda}_m^{(l)}$ to the m th latent variable at level l .

Note that we do not use subscripts for the units of observation at different levels. This decision was made with the purpose of avoiding the use of mathematical definitions with large number of subscripts. However, a careful reader should consider that equation 2.1 rest on the assumption that each unit is identified at their appropriate level. For special cases of multilevel SEM, and their use of subscripts, refer to Appendix A.

Links and Distributions

As GLM, the model "links" the expectation of the conditional response, to the linear predictor, through a response function $h(\cdot)$, in the following form:

$$\mu = E[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] = h(v) \quad (2.2)$$

where equation 2.2 can be re-written in terms of the link function $g(\cdot) = h^{-1}(\cdot)$:

$$g(\mu) = g(E[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}]) = v \quad (2.3)$$

with $\boldsymbol{\eta} = [\eta^{(2)T}, \dots, \eta^{(L)T}]^T$ and $\mathbf{Z} = [\mathbf{Z}^{(2)T}, \dots, \mathbf{Z}^{(L)T}]^T$, as the "stacked" vector of latent variables, and the "stacked" design matrices of explanatory variables, for all L levels, respectively. Additionally, $\boldsymbol{\eta}^{(l)} = [\eta_1^{(l)}, \dots, \eta_{M(l)}^{(l)}]^T$ and $\mathbf{Z}^{(l)} = [\mathbf{Z}_1^{(l)T}, \dots, \mathbf{Z}_{M(l)}^{(l)T}]^T$, denotes the vector of latent variables, and the "stacked" design matrix of explanatory variables, at level l , respectively.

Finally, the response model specification is complete when we select an appropriate distribution from the family of exponential distributions. The types of responses that can be accommodated by the framework are the following:

1. Continuous:

It results from selecting an identity link function for the scaled mean response,

$$\begin{aligned} \mu^* &= E[y^*|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= v^* \end{aligned} \quad (2.4)$$

where $\mu^* = \mu\sigma^{-1}$, $y^* = y\sigma^{-1}$, and $v^* = v\sigma^{-1}$.

On the other hand, the distributional part is defined by a Standard Normal distribution $\phi(x) = (2\pi)^{-1/2} \exp(-x^2/2)$,

$$\begin{aligned} f(y^*|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}) &= \phi(\mu^*)\sigma^{-1} \\ &= \phi(v^*)\sigma^{-1} \end{aligned} \quad (2.5)$$

Notice that we decided to standardize the variables to make the estimation process easier. Additionally, it is important to mention that the same parametrization can be achieved considering $y^* = v^* + \epsilon^*$, and $\epsilon^* \sim N(0, 1)$.

2. Dichotomous:

It results from selecting an appropriate response function for the expected value of the manifest variable, which describe the probability of endorsing one of the two available categories,

$$\begin{aligned} \mu &= E[y = 1|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= P[y = 1|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \pi \\ &= h(v) \end{aligned} \quad (2.6)$$

where $h(\cdot)$ can be defined in three ways:

$$h(x) = \begin{cases} \exp(x)[1 + \exp(x)]^{-2} \\ (2\pi)^{-1/2} \exp(-x^2/2) \\ \exp(x - \exp(x)) \end{cases} \quad (2.7)$$

which corresponds to the logistic, standard normal $\phi(x)$, and Gumbel (extreme value type I) density distributions, respectively. In terms of link functions, the distributions corresponds to the well known logit, probit and complementary log-log link functions, respectively. Alternatively, the same parametrization can be achieved using the concept of an underlying latent variable in the form $y^* = v + \epsilon^*$, where ϵ^* can have a distribution as the ones defined in equation 2.7.

Finally, the distributional part is defined by a Binomial distribution,

$$\begin{aligned} f[y = 1|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] &= \binom{n}{k} \mu^k (1 - \mu)^{n-k} \\ &= \binom{n}{k} \pi^k (1 - \pi)^{n-k} \end{aligned} \quad (2.8)$$

where k denotes the number of successes in n independent Bernoulli trials.

3. Polytomous:

It results from selecting a generalized logistic response function (Bock; 1972) for the expected value of the response, which in this case, describe the probability of endorsing one of the S unordered available categories,

$$\begin{aligned} \mu_s &= E[y = y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= P[y = y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \pi_s \\ &= h(v_s) \end{aligned} \quad (2.9)$$

where v_s is the linear predictor for category s ($s = 1, \dots, S$), and $h(\cdot)$ is defined as:

$$h(x) = \exp(x) \left[\sum_{s=1}^S \exp(x) \right]^{-1} \quad (2.10)$$

It is important to note that, as in the dichotomous case, the same parametrization can be achieved using the concept of underlying continuous responses in the form $y_s^* = v_s + \epsilon_s^*$, where y_s denotes the random utility for the s category, and ϵ_s^* have a Gumbel (extreme value type I) distribution, as the one defined in equation 2.7.

Finally, the distributional part is defined by a Multinomial distribution,

$$\begin{aligned} f[y = \{y_1, \dots, y_S\} | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] &= \frac{n!}{y_1! \dots y_S!} \prod_{s=1}^S \mu_s^{y_s} \\ &= \frac{n!}{y_1! \dots y_S!} \prod_{s=1}^S \pi_s^{y_s} \end{aligned} \quad (2.11)$$

where y_s denotes the number of "successes" in category s .

4. Ordinal and discrete time duration:

It results from selecting an appropriate response function for the expected value of the manifest variable. For the ordinal case, the expected value corresponds to the probability of endorsing category s against all previous categories, in the following form:

$$\begin{aligned} \mu_s &= E[y = y_s | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= P[y \leq y_s | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] - P[y \leq y_{s-1} | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= H(\kappa_s - v_s) - H(\kappa_{s-1} - v_{s-1}) \end{aligned} \quad (2.12)$$

where κ_s denotes the thresholds for category s . For discrete time duration, the expected value corresponds to the probability that the survival time lies in the s th interval, as follows:

$$\begin{aligned} \mu_s &= E[t_{s-1} \leq T \leq t_s | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= P[T \leq t_s | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] - P[T \leq t_{s-1} | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= H(v_s + t_s) - H(v_{s-1} + t_{s-1}) \end{aligned} \quad (2.13)$$

where T is the unobserved continuous time, and t_s its observed discrete realization. Additionally, $H(\cdot)$ can be defined, for both type of responses, as the cumulative distributions for the logistic, standard normal, and Gumbel (extreme value type I) density distributions, respectively:

$$H(x) = \begin{cases} \exp(x)[1 + \exp(x)]^{-1} \\ \Phi(x) & \text{no closed form} \\ \exp(-\exp(x)) \end{cases} \quad (2.14)$$

It is important to note that for discrete time duration, the logit link corresponds to a **Proportional-Odds model**, while the complementary log-log link to a **Discrete**

Time Hazards model (Rabe-Hesketh et al.; 2001). Other models for ordinal responses, such as the **Adjacent Category Logit** or the **Baseline Category Logit** models can be specified as special cases of the generalized logistic response function, defined in equation 2.10. Similar to the dichotomous and polytomous case, the same parametrization can be achieved using the concept of underlying latent variables, often used for discrete choice models in econometrics.

Finally, the distributional part is defined by a Multinomial distribution, as the one defined in equation 2.11.

5. Counts and continuous time duration:

It results from selecting an exponential response function (log link function) for the expected value of the response,

$$\begin{aligned}\mu &= E[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \lambda \\ &= \exp(v)\end{aligned}\tag{2.15}$$

and a Poisson conditional distribution of the counts,

$$\begin{aligned}f[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] &= \exp(-\mu)\mu^y(y!)^{-1} \\ &= \exp(-\lambda)\lambda^y(y!)^{-1}\end{aligned}\tag{2.16}$$

It is important to mention that unlike the models for dichotomous, polytomous and ordinal responses, model for counts cannot be written under the random utility framework.

6. Rankings and pairwise comparisons:

Following Skrondal and Rabe-Hesketh (2003a), the parametrization for polytomous responses can serve as the building block for the conditional distribution of rankings. Selecting a "exploded logit" function (Chapaaan and Staelin; 1982) for the expected value of the response, which describes the probability of the full rankings of category s ,

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$$\begin{aligned}\mu_s &= P[\mathbf{R}_s = \{r_s^1, \dots, r_s^1\}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \pi_s \\ &= h(v_s)\end{aligned}\tag{2.17}$$

where v_s is the linear predictor for category s ($s = 1, \dots, S$), and $h(\cdot)$ is defined as:

$$h(x) = \prod_{s=1}^S \exp(x^s) \left[\sum_{s=1}^S \exp(x^s) \right]^{-1}\tag{2.18}$$

Again, as in specific previous cases, the same parametrization can be achieved using the concept of underlying latent variables.

Finally, the distributional part is defined by a Multinomial distribution,

$$\begin{aligned} f[y = \{y_1, \dots, y_S\} | \mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] &= \frac{n!}{y_1! \cdots y_S!} \prod_{s=1}^S \mu_s^{y_s} \\ &= \frac{n!}{y_1! \cdots y_S!} \prod_{s=1}^S \pi_s^{y_s} \end{aligned} \quad (2.19)$$

where y_s denotes the number of "success cases" in category s .

7. Mixtures:

Given the previous definitions, the framework easily lends itself to model three additional settings:

- (a) **Different links and distributions for different latent variables.** This can be easily achieved by setting different links and distributions for each of the M_2 latent variables located at level 2.
- (b) **Left- or right-censored continuous responses.** Common in selection models (e.g. Heckman; 1979), they can be achieved by specifying an identity link and Normal distribution for the uncensored scaled responses (equations 2.4 and 2.5); and a scaled probit link and a Binomial distribution otherwise (equations 2.7 and 2.8).
- (c) **zero-inflated count responses.** where a log link and a Poisson distribution is set for the count part (equations 2.15 and 2.16); and a logit link and a Binomial distribution can be specified to model the zero center of mass (equations 2.6 and 2.8).
- (d) **Measurement error in covariates.** when common models use variables, with measurement error, as covariates, e.g. a logistic regression with a continuous covariate that presents measurement error. For more details on this type of models see Rabe-Hesketh, Skrondal and Pickles (2003); Rabe-Hesketh, Pickles and Skrondal (2003), and Skrondal and Rabe-Hesketh (2003b).
- (e) **Composite links.** Useful for specifying proportional odds models for right-censored responses, for handling missing categorical covariates and many other model types (Skrondal and Rabe-Hesketh; 2004b).

Heteroscedasticity and over-dispersion

Under the GLM framework, the continuous response model assumes that the errors are homoscedastic, i.e. $\epsilon^* \sim N(0, 1)$, assumption that is often violated when, for example, the units of analysis are correlated or belong to a cluster. In similar fashion, for non-continuous response models (dichotomous, ordinal and counts), the framework fixes the modeled dispersion to the mean value, e.g. for the dichotomous case $\mu = \pi$, and $v(\mu) = \pi(1 - \pi)$; however in practice, this assumption is also often violated as the data can register over- or under-dispersion.

Given the restrictions of GLM, the Generalized Linear Mixed Model (GLMM) framework was developed. Under the aforementioned, heteroscedasticity and over-dispersion is modeled by the inclusion of random effects in the linear predictor. The GLLAMM framework borrows the same idea to model similar characteristics of the data:

1. **Continuous:**

We model **heteroscedasticity** in the following form:

$$\sigma = \exp(\boldsymbol{\alpha}^T \mathbf{Z}^{(1)}) \quad (2.20)$$

Notice that the previous formula implies that equation 2.5 can be re-written in the following form:

$$f(y^*|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}) = \phi(v^* + \boldsymbol{\alpha}^T \mathbf{Z}^{(1)}) \quad (2.21)$$

which effectively corresponds to a model that includes random intercepts at level 1, where σ denotes the standard deviation of the errors, and $\mathbf{Z}^{(1)}$ is the design matrix that maps the random effects $\boldsymbol{\alpha}$.

2. **Dichotomous:**

We model over- or under-dispersion by modifying equation 2.6 in the following form:

$$\begin{aligned} \mu &= P[y = 1|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= h(v + \boldsymbol{\alpha}^T \mathbf{Z}^{(1)}) \end{aligned} \quad (2.22)$$

where $\mathbf{Z}^{(1)}$, and $\boldsymbol{\alpha}$ are defined as in equation 2.21.

3. **Ordinal, and discrete time duration:**

By modifying equation 2.12, we can model over- or under-dispersion:

$$\begin{aligned} \mu_s &= P[y \leq y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] - P[y \leq y_{s-1}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= H(\kappa_s - v_s - \boldsymbol{\alpha}^T \mathbf{Z}^{(1)}) - H(\kappa_{s-1} - v_{s-1} - \boldsymbol{\alpha}^T \mathbf{Z}^{(1)}) \end{aligned} \quad (2.23)$$

where $\mathbf{Z}^{(1)}$, and $\boldsymbol{\alpha}$ are defined as in equation 2.21. Notice that the modeled over-dispersion is assumed to be the same for all the categories.

4. **Counts, and continuous time duration:**

Modifying equation 2.15 allow us to model over- or under-dispersion under a counts model:

$$\begin{aligned} \mu &= \lambda \\ &= \exp(v + \boldsymbol{\alpha}^T \mathbf{Z}^{(1)}) \end{aligned} \quad (2.24)$$

where $\mathbf{Z}^{(1)}$, and $\boldsymbol{\alpha}$ are defined as in equation 2.21.

2.1.2 Structural model for the latent variables

The structural model for the latent variables has the form:

$$\underset{(M \times M)(M \times 1)}{\boldsymbol{\eta}} = \underset{(M \times M)}{\mathbf{B}} \underset{(M \times 1)}{\boldsymbol{\eta}} + \underset{(M \times Q)(Q \times 1)}{\boldsymbol{\Gamma}} \underset{(M \times 1)}{\mathbf{W}} + \underset{(M \times 1)}{\boldsymbol{\zeta}} \quad (2.25)$$

where \mathbf{B} and $\boldsymbol{\Gamma}$ are parameter matrices that maps the relationship between the latent variables, and the vector of "stacked" covariates \mathbf{W} , respectively; $\boldsymbol{\zeta}$ is a vector of errors or disturbances, and $M = \sum_l M_l$. It is important to notice that while equation 2.25 resembles to single-level structural equation models, the main difference lies in the fact that the latent variables may vary at different levels.

Additionally, considering that the $\boldsymbol{\eta}$ is permuted appropriately and sorted in increasing order, according to the level of the latent variables, the framework does not allow latent variables to be regressed on latent or observed variables varying at a lower level, as such specifications does not appear to provide **valid** modeling options, e.g. **put some examples**. Furthermore, the framework does not permit feedback effects among the latent variables, as **provide reasons**. Together, the two restrictions imply that the \mathbf{B} is strictly upper diagonal.

2.1.3 Distribution of the latent variables

Finally, to fully specify the framework, we need to define the assumed distributions for the disturbances $\boldsymbol{\zeta}$ or the latent variables $\boldsymbol{\eta}$. If our interest lies in the structural equation model, it is more convenient to specify the distribution of the disturbances; otherwise, we set the distributions for the latent variables. Additionally, as in a multilevel setting, it is assumed that latent variables at different levels are independent, whereas latent variables at the same level may be dependent. In that sense, we consider that all latent variables at level l have a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}_l$, i.e. $\boldsymbol{\eta}^{(l)} \sim MVN(\mathbf{0}, \boldsymbol{\Sigma}_l)$. However, while the multivariate normal distribution is widely used, any other can be assumed, and can be even left unspecified by using non-parametric maximum likelihood estimation (Rabe-Hesketh, Skrondal and Pickles; 2003)

2.1.4 Model identification

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The structure of the latent variables is specified by the number of levels L and the number of latent variables M_l at each level. A particular level may coincide with a level of clustering in the hierarchical dataset. However, there will often not be a direct correspondence between the levels of the model and the levels of the data hierarchy.

2.2 Relationship with other modeling schemes

Rabe-Hesketh et al. (2012)

The motivation for multilevel regression models is to handle hierarchical data where elementary units are nested in clusters, such as students in schools, which in turn may be nested in higher-level clusters (e.g., school districts or states). The latent variables, often called "random effects" in this context, can be interpreted as the effects of unobserved covariates at different levels that induce dependence among lower-level units. In contrast, the motivation for structural equation models is to handle variables that cannot be measured directly, and are hence latent, and to model their relationships with each other and with observed or manifest variables. The latent variables, often called "common factors" in this context, are measured by manifest variables and induce dependence among them.

2.2.1 Factor Models

2.2.2 Item Response Theory and Generalized Latent Models

2.2.3 Multilevel Models

2.3 Advantages and Disadvantages

Chapter 3

Estimation

3.1 Likelihood methods

3.1.1 Likelihood function

3.1.2 Adaptive Quadrature

3.2 Bayesian methods

3.2.1 Prior distributions

3.2.2 Initial start

3.2.3 Posterior distributions

Chapter 4

Application

4.1 Instruments

4.2 Data

4.2.1 Collection

4.2.2 Sample scheme

4.3 Results

4.3.1 Hypothesis 1:

4.3.2 Hypothesis 2:

4.3.3 Hypothesis 3:

Chapter 5

Conclusion and Discussion

5.1 Discussion

5.2 Conclusions

5.3 Future development

Appendix A

Additional Theory

A.1 Special cases for the GLAMM

Appendix B

Code

Bibliography

- Angrist, J. and Krueger, A. (1999). Empirical strategies in labor economics, in O. Ashenfelter and D. Card (eds), *Handbook of Labor Economics*, Vol. 3, Elsevier, chapter 23, pp. 1277 – 1366.
URL: <http://www.sciencedirect.com/science/article/pii/S1573446399030047>
- Araujo, M., Carneiro, P., Cruz-Aguayo, Y. and Schady, N. (2016). Teacher quality and learning outcomes in kindergarten, *The Quarterly Journal of Economics* **131**(3): 1415–1453.
URL: <https://publications.iadb.org/publications/english/document/Teacher-Quality-and-Learning-Outcomes-in-Kindergarten.pdf>
- Ayala, M. (2017). *Efecto de los docentes provisionales sobre desempeño escolar - evidencia para la educación secundaria oficial en colombia*, Master’s thesis, Universidad de los Andes.
URL: <http://biblioteca.uniandes.edu.co/acepto201699.php?id=11802.pdf>
- Bertoni, E., Elacqua, G., Marotta, L., Martinez, M., Méndez, C., Montalva, V., Olsen, A., Santos, H. and Soares, S. (2020). Escasez de docentes en latinoamérica: ¿cómo se puede medir y que políticas están implementando los países para resolverlo?, *Technical report*, Banco Interamericano de Desarrollo.
- Bietenbeck, J., Piopiunik, M. and Wiederhold, S. (2018). Africa’s skill tragedy: Does teachers’ lack of knowledge lead to low student performance?, *Comparative Education Review* **53**(3): 553–578.
URL: <http://jhr.uwpress.org/content/53/3/553.abstract>
- Bock, R. (1972). Estimating item parameters and latent ability when responses are scored in two or more nominal categories, *Psychometrika* **37**(1).
- Bruns, B., Luque, J., De Gregorio, S., Evans, D., Fernández, M., Moreno, M., Rodriguez, J. Toral, G. and Yarrow, N. (2015). Great teachers: How to raise student learning in latin america and the caribbean, *Technical report*, World Bank Group.
- Chapaaan, R. and Staelin, R. (1982). Exploiting rank ordered choice set data within the stochastic utility model, *Journal of Marketing Research* **19**(3): 288–301.
- Chetty, R., Friedman, J. and Rockoff, J. (2014a). Measuring the impacts of teachers i: Evaluating bias in teacher value-added estimates, *American Economic Review* **104**(9): 2593–2632.
URL: <https://www.aeaweb.org/articles?id=10.1257/aer.104.9.2593>

- Chetty, R., Friedman, J. and Rockoff, J. (2014b). Measuring the impacts of teachers ii: Teacher value-added and student outcomes in adulthood, *American Economic Review* **104**(9): 2633–2679.
URL: <https://www.aeaweb.org/articles?id=10.1257/aer.104.9.2593>
- Chetty, R., Friedman, J. and Rockoff, J. (2015). School governance, teacher incentives, and pupil–teacher ratios: Experimental evidence from kenyan primary schools, *Journal of Public Economics* **123**: 92–110.
URL: <http://www.sciencedirect.com/science/article/pii/S0047272714002412>
- Clotfelter, C., Ladd, H. and Vigdor, J. (2006). Teacher-student matching and the assessment of teacher effectiveness, *Working Paper 11936*, National Bureau of Economic Research.
URL: <http://www.nber.org/papers/w11936>
- Clotfelter, C., Ladd, H. and Vigdor, J. (2007). How and why do teacher credentials matter for student achievement?, *Working Paper 12828*, National Bureau of Economic Research.
URL: <http://www.nber.org/papers/w12828>
- Cruz-Aguayo, Y., Hincapié, D. and Rodríguez, C. (2020). Profesores a prueba: claves para una evaluación docente exitosa, *Technical report*, Banco Interamericano de Desarrollo.
- Dee, T. and Cohodes, S. (2008). Out-of-field teachers and student achievement: Evidence from matched-pairs comparisons, *Public Finance Review* **36**(1): 7–32.
URL: <https://doi.org/10.1177/1091142106289330>
- Duflo, E., Dupas, P. and Kremer, M. (2009). Additional resources versus organizational changes in education: Experimental evidence from kenya.
- Elacqua, G., Hincapié, D., Vegas, E. and Alfonso, M. (2018). Profesión: profesor en américa latina ¿por qué se perdió el prestigio docente y cómo recuperarlo?, *Technical report*, Banco Interamericano de Desarrollo.
- Estrada, R. (2019). Rules versus discretion in public service: Teacher hiring in mexico, *Journal of Labor Economics* **37**(2): 545–579.
URL: <https://doi.org/10.1086/700192>
- Hanushek, E. and Rivkin, S. (2006). Teacher quality, in E. Hanushek and F. Welch (eds), *Handbook of the Economics of Education*, Vol. 2, Elsevier, chapter 18, pp. 1051 – 1078.
URL: <http://www.sciencedirect.com/science/article/pii/S1574069206020186>
- Hanushek, E. and Rivkin, S. (2012). The distribution of teacher quality and implications for policy, *Annual Review of Economics* **4**(1): 131–157.
URL: <https://doi.org/10.1146/annurev-economics-080511-111001>
- Heckman, J. (1979). Sample selection bias as a specification error, **47**(1): 153–161.
- Ingersoll, R. (1998). The problem of out-of-field teaching.
URL: https://repository.upenn.edu/gse_pubs/137

Kane, T., McCaffrey, D., Miller, T. and Staiger, D. (2013). Have we identified effective teachers? validating measures of effective teaching using random assignment, *Research paper*, Bill Melinda Gates Foundation.

URL: <https://files.eric.ed.gov/fulltext/ED540959.pdf>

Kane, T. and Staiger, D. (2012). Gathering feedback for teaching: Combining high-quality observations with student surveys and achievement gains, *Research paper*, Bill Melinda Gates Foundation.

URL: https://k12education.gatesfoundation.org/download/?Num=2678filename=MET_Gathering_Fee

Kane, T., Taylor, E., Tyler, J. and Wooten, A. (2010). Identifying effective classroom practices using student achievement data, *Working Paper 15803*, National Bureau of Economic Research.

URL: <http://www.nber.org/papers/w15803>

Kelley, T. (1927). *Interpretation of educational measurements*, Measurement and adjustment series, World Book Co.

Lunn, D., Thomas, A., Best, N. and Spiegelhalter, D. (2000). Winbugs - a bayesian modelling framework: Concepts, structure, and extensibility, *Statistics and Computing* (10): 325–337.

Marotta, L. (2019). Teachers' contractual ties and student achievement: The effect of temporary and multiple-school teachers in brazil, *Comparative Education Review* **63**(3): 356–376.

Marshall, J. (2009). School quality and learning gains in rural guatemala, *Economics of Education Review* **28**(2): 207–216.

URL: <http://www.sciencedirect.com/science/article/pii/S0272775708000745>

Medeiros, M., Gómez, C., Sánchez, M. and Orrego, V. (2018). Idoneidad disciplinar de los profesores y mercado de horas docentes en chile, *Calidad en la Educación* (48): 50–95.

URL: <https://doi.org/10.31619/caledu.n48.479>

Metzler, J. and Woessmann, L. (2012). The impact of teacher subject knowledge on student achievement: Evidence from within-teacher within-student variation, *Journal of Development Economics* **99**(2): 486–496.

URL: <https://ideas.repec.org/a/eee/deveco/v99y2012i2p486-496.html>

Muralidharan, K. and Sundararaman, V. (2013). Contract teachers: Experimental evidence from india, *Working Paper 19440*, National Bureau of Economic Research.

URL: <http://www.nber.org/papers/w19440>

Ome, A. (2012). The effects of meritocracy for teachers in colombia, *Research report*, Fedesarrollo.

URL: <https://ideas.repec.org/p/col/000124/010260.html>

R Core Team (2015). *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria.

URL: <http://www.R-project.org/>

- Rabe-Hesketh, S., Pickles, A. and Skrondal, A. (2003). Correcting for covariate measurement error in logistic regression using nonparametric maximum likelihood estimation, *Statistical Modelling* **3**(3): 215–232.
- Rabe-Hesketh, S., Skrondal, A. and Pickles, A. (2003). Maximum likelihood estimation of generalized linear models with covariate measurement error, *The Stata Journal* **3**(4): 386–411.
URL: <https://journals.sagepub.com/doi/pdf/10.1177/1536867X0400300408>
- Rabe-Hesketh, S., Skrondal, A. and Pickles, A. (2004a). Generalized multilevel structural equation modeling, *Psychometrika* **69**(2): 167–190.
- Rabe-Hesketh, S., Skrondal, A. and Pickles, A. (2004b). *GLLAMM Manual*, UC Berkeley Division of Biostatistics.
URL: <http://www.biostat.jhsph.edu/~fdominic/teaching/bio656/software-gllamm.manual.pdf>
- Rabe-Hesketh, S., Skrondal, A. and Pickles, A. (2004c). Maximum likelihood estimation of limited and discrete dependent variable models with nested random effects, *Journal of Econometrics* **128**(2): 301–323.
URL: <http://www.sciencedirect.com/science/article/pii/S0304407604001599>
- Rabe-Hesketh, S., Skrondal, A. and Zheng, X. (2012). Multilevel structural equation modeling, in R. Hoyle (ed.), *Handbook of Structural Equation Modeling*, The Guilford Press, chapter 30, pp. 512–531.
- Rabe-Hesketh, S., Yang, S. and Pickles, A. (2001). Multilevel models for censored and latent responses, *Statistical Methods in Medical Research* **10**(6): 409–427.
- Rivera, J. (2019). *El modelo de respuesta nominal: Aplicación a datos educacionales*, Master’s thesis, Pontificia Universidad Católica del Peru.
URL: <http://hdl.handle.net/20.500.12404/14600>
- Rivkin, S., Hanushek, E. and Kain, J. (2005). Teachers, schools, and academic achievement, *Econometrica* **73**(2): 417–458.
URL: <https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0262.2005.00584.x>
- Rockoff, J. (2004). The impact of individual teachers on student achievement: Evidence from panel data, *The American Economic Review* **94**(2): 247–252.
URL: <http://www.jstor.org/stable/3592891>
- Rockoff, J., Jacob, B., Kane, T. and Staiger, D. (2011). Can you recognize an effective teacher when you recruit one?, *Education Finance and Policy* **6**(1): 43–74.
URL: https://doi.org/10.1162/EDFP_a00022
- Santibañez, L. (2006). Why we should care if teachers get a’s: Teacher test scores and student achievement in Mexico, *Economics of Education Review* **25**(5): 510–520.
URL: <http://www.sciencedirect.com/science/article/pii/S0272775705000804>
- Scherrer, J. (2011). Measuring teaching using value-added modeling: The imperfect panacea, *NASSP Bulletin* **95**(2): 122–140.
URL: <https://doi.org/10.1177/0192636511410052>

- Skrondal, A. and Rabe-Hesketh, S. (2003a). Multilevel logistic regression for polytomous data and rankings, *Psychometrika* **68**: 267–287.
- Skrondal, A. and Rabe-Hesketh, S. (2003b). Some applications of generalized linear latent and mixed models in epidemiology: Repeated measures, measurement error and multilevel modeling, *Norsk Epidemiologi* **13**(2): 265–278.
- Skrondal, A. and Rabe-Hesketh, S. (2004a). *Generalized Latent Variable Modeling: Multilevel, Longitudinal, and Structural Equation Models*, Interdisciplinary Statistics, Chapman Hall/CRC Press.
- Skrondal, A. and Rabe-Hesketh, S. (2004b). Generalized linear latent and mixed models with composite links and exploded likelihoods, in BiggeriA., E. Dreassi, C. Lagazio and M. Marchi (eds), *Proceedings of the 19th International Workshop on Statistical Modeling*, Firenze University Press, Florence, Italy, pp. 27–39.
URL: http://www.gllamm.org/composite_conf.pdf

AFDELING
Straat nr bus 0000
3000 LEUVEN, BELGIË
tel. + 32 16 00 00 00
fax + 32 16 00 00 00
www.kuleuven.be

