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Generalized Linear Latent and Mixed Models:

method, estimation procedures, advantages, and applications to educational policy.

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Dedication

To Manuel, for being my friend and father.
To Margarita, Susan, and Marysu, for their relentless encouragement.
To Ana, for showing me the value of family, here in this moorland.
To both of you, as you are always in my mind.
And to all that knowingly or not, help me to get here.
I am lucky due to all of you.
I hope I make you all proud.

A Manuel, por ser mi amigo y mi padre.
A Margarita, Susan y Marysu, por su incansable aliento.
A Ana, por mostrarme el valor de la familia, aquí en este páramo.
A ustedes dos, que siempre las tengo en mente.
Y a todos los que sabiendolo o no, me ayudaron a llegar aquí.
Soy un suertudo gracias todos ustedes.
Espero llenarlos de orgullo.

Acknowledgment

(in the works)

Abstract

(in the works)
Keywords:

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Abbreviations

GLLAMM	Generalized Linear Latent and Mixed Model.
SEM	Structural Equation Model.
GLM	Generalized Linear Model.
EFA	Exploratory Factor Analysis.
CFA	Confirmatory Factor Analysis.
IRT	Item Response Theory.

Symbols

J	total number of subjects.
j	index of specific subject.
I	total number of items.
i	index for the specific item.
$M_{(l)}$	total number of latent variables at level l.
m	index for the specific latent variable.
L	total number of levels (clusters).
l	index for the specific level (cluster).
$\mathbf{V}$	vector of the linear predictors.
$\boldsymbol{\beta}$	vector of fixed effects, for the I items.
$\mathbf{X}$	design matrix for the $\boldsymbol{\beta}$ parameters.
$\eta_{mj}^{(l)}$	latent variable, at the respective indices.
$\boldsymbol{\lambda}_m^{(l)}$	vector of loadings, at the respective indices.
$\mathbf{Z}_m^{(l)}$	design matrix for the $\boldsymbol{\lambda}_m^{(l)}$ parameters.

Chapter 1

Introduction

1.1 Preliminar considerations

The short and long term benefits of effective teaching practices can be observed throughout the literature: improvements in student achievements (Rockoff; 2004; Rivkin et al.; 2005; Duflo et al.; 2009; Hanushek and Rivkin; 2012; Muralidharan and Sundararaman; 2013; Chetty et al.; 2014a; Araujo et al.; 2016); development of executive functions (Araujo et al.; 2016), increased college attendance, higher salaries, and a lower possibility of premature parenthood (Chetty et al.; 2014b), among others. Similarly, the literature has shown most of the negative impacts resulting from the presence of teacher shortages[1] (Duflo et al.; 2009; Muralidharan and Sundararaman; 2013; Chetty et al.; 2015; Ayala; 2017; Marotta; 2019) or ineffective teaching practices (Hanushek and Rivkin; 2012).

However, while the evidence have a solid methodological support, Hanushek and Rivkin (2006) have indicated that some of the proxy variables, used in the methods, are not consistently related to either teacher effectiveness or quality of instruction, examples of such are: out of field teaching[2] (Ingersoll; 1998; Dee and Cohodes; 2008; Bertoni et al.; 2020); teaching hours (Bruns et al.; 2015); years of experience or educational degree (Rockoff; 2004; Rivkin et al.; 2005; Clotfelter et al.; 2006, 2007; Hanushek and Rivkin; 2012); among others.

Consequently, given that most of the measured teaching factors are proxies, and that the effects estimated from such variables lack consistency, Hanushek and Rivkin (2012) have pointed out that the analysis of teacher effectiveness has largely turned away, from attempts to identify the teacher's specific characteristics, to focus its attention into measuring the direct relationship between them and the student outcomes[3]. For that reason, considerable uncertainty is still present in the literature, regarding exactly which aspects

[1]Bertoni et al. (2020) defined it as the context in which the teacher's supply, i.e. the number of available teachers in the system, is less than its demand. The authors further elaborate that one of the causes of these shortages is related to the applicants' lower quality or due to their faulty initial training, implying that the shortage can also be conceived as the lack of good quality teachers. The evidence of such shortage has been more prevalent, but not decisive, with temporary teachers, as they are usually associated with inferior attributes, compared to their contracted counterparts

[2]Medeiros et al. (2018) defines it as teachers that are currently teaching a subject in which they are not specialized or do not have the appropriate certificate.

[3]The method is known as value-added analysis, and it is based on the perspective that a good teacher is one who consistently gets higher achievement from students after other determinants of such are controlled for. For a more detailed explanation of the method refer to Scherrer (2011).

of teachers are key for the student's learning and whether those qualities can be measured (Rockoff; 2004; Clotfelter et al.; 2006).

However, because the evidence still largely supports the perception that teachers are the main driver behind the student's learning processes, any educational authority need to have, among their main agenda points, the design of an assessment system that can attract, select, develop, and retain the most effective ones (Elacqua et al.; 2018), and in order to do so, the definition of an Educational Performance Standard (EPS) is a necessity. With an EPS, rooted in the country's context, the authorities can now set clear expectations about what a "good" teacher should know and know to do (Cruz-Aguayo et al.; 2020).

While the specific requirements for such definition are not easy to identify, the aforementioned authors have hinted that most of them can be largely grouped into two: (i) to have the disciplinary knowledge and pedagogical practices adequate to the classroom characteristics, context and teaching level, and (ii) to display such knowledge and practices in the classroom, using the appropriate material and technological resources available.

As one can infer from the previous general conditions, and the slew evidence, the disciplinary knowledge is a relevant observable factor, consistently associated with teacher effectiveness and growth in the student's achievement (Santibañez; 2006; Clotfelter et al.; 2006, 2007; Hanushek and Rivkin; 2006; Marshall; 2009; Rockoff et al.; 2011; Kane et al.; 2010; Kane and Staiger; 2012; Ome; 2012; Metzler and Woessmann; 2012; Kane et al.; 2013; Araujo et al.; 2016; Bietenbeck et al.; 2018; Estrada; 2019); and in that sense, its measurement should be of interest for any educational authority.

The measurement of knowledge has a myriad of available tools, nevertheless, given that any educational department are bounded by budgetary constraints, valid[4] and reliable[5] standardized tests[6] stand out, not only for its cost-effectiveness, and a much simpler implementation (Cruz-Aguayo et al.; 2020); but also because, compared to other instruments, they are one of tools with less subjective scoring processes and interpretations.

However, as no instrument is perfect, the subject's knowledge scores resulting from their use will likely have two main problems. First, they could manifest measurement error (Metzler and Woessmann; 2012), which would imply that the estimates obtained from them could be an biased reflection of the true effects (Angrist and Krueger; 1999). And second, as the score is a composite value, does not allow to test which specific factors leads to better or worse teacher performance.

These two issues has direct and important policy implications, and devoting effort to appropriately assess and control them, could help the educational authorities to understand, for example: (i) the characteristics of the applicants to the public teaching carrer, (ii) to identify which teachers should be hired, and finally, once they are inside, what the authorities should do to train them (Hanushek and Rivkin; 2012), (iii) if the scores thresholds used for the selection processes are appropriately set[7], to mention a few.

[4]the extend to which a measurement tool is well-founded and accurately corresponds to the real measure (Kelley; 1927)

[5]the overall consistency of a measure under consistent conditions.

[6]Assessment instrument in which the implementation, questions, scoring processes, and interpretations are consistent with a predetermined or typified way. The instrument is usually composed of questions or items that fulfill three conditions: (i) they are polytomous, i.e. they have multiple choices, (ii) the choice categories are nominal, i.e. do not present any specific order, and (iii) there is only one "correct" category or answer (Rivera; 2019)

[7]Approximately 60% of the Caribbean and Latin American countries use standardized test scores as

In summary, teachers are one of the main drivers behind the student achievements. However, some of the evidence supporting this claim has been based on proxy variables that are not consistently related to the quality of instruction, or methods that are not concerned with the outline of the teaching factors, responsible for the student's learning. Nevertheless, while the literature still reflects considerable uncertainty on what are the "ingredients for a good teacher", a good amount of evidence has supported the disciplinary and pedagogical knowledge as relevant components of the teacher effectiveness. Finally, the literature has shown that valid and reliable standardized tests are among the best tools to assess such factors, but also have emphasized that such scores could reflect the teacher's abilities with considerable noise.

1.2 Objectives

This research will have two main goals. First, to describe the method, estimation procedures, and advantages of the Generalized Linear Latent and Mixed Modeling framework (GLLAMM), developed by Rabe-Hesketh et al. (2004a,c); Skrondal and Rabe-Hesketh (2004a); Rabe-Hesketh et al. (2012). And second, tests the policy implications of the methods, and its results, in a data composed of large repeated Teacher's standardized educational assessments from Peru.

Specifically, for the first objective of the research, the author expects to appraise:

1. If the method can provide a general framework that could serve multiple psychometric purposes, e.g. to analyze the quality of the items, to obtain a dynamical noise-free "score" for the disciplinary abilities of the teachers, among others; and
2. What are the advantages or disadvantages of such models, specially compared to factor, item-response theory and multilevel models.

For the second objective, the author expects to shed some lights about some key policy decisions related to those large evaluation processes, to mention a few:

- Are the educational authorities screening the teachers with higher disciplinary knowledge?, and in that sense, what differentiate a contract teacher from a temporary one?,
- What are the general characteristics of the teaching-career applicants?, What is the level of their disciplinary knowledge, and how it evolves?,
- Do the initial training or socioeconomic status help to explain the disciplinary knowledge profile of the applicants?
- What factors of the disciplinary knowledge are consistently related to a good performance in the classroom?
- Do the instruments guarantee a fair assessment of minority groups with different abilities?

part of or as a main teacher selection tool (Cruz-Aguayo et al.; 2020).

Given the aforementioned goals, the researcher believes the master's thesis contributes to the literature in two aspects:

1. In a the theoretical and methodological sense, as the research is focused on offering an exhaustive description and analysis of the GLLAMM framework; and

2. In a more practical sense, as it helps to provide evidence on some of key policy decisions that most of Latin America countries are currently facing.

Finally, it is important to mention, that the computational implementation of the method will be developed in `R` (R Core Team; 2015) and `WinBUGS` (Lunn et al.; 2000).

1.3 Organization

Chapter 2, The Generalized Linear Latent and Mixed Model, will describe the model, its components, characteristics, assumptions and properties, to finally assess its benefits against factor (EFA and CFA), IRT and Multilevel models.

Chapter 3, Estimation, will describe two of the methods that can be used to fit such models: Likelihood and Bayesian methods. The chapter will also present the computational implementation of the model.

Chapter 4, Application, will describe the instruments and the "dimensions" under analysis. Additionally, it will describe briefly the data collection process, the sample design, and the results of the analysis under the GLLAMM framework.

Finally, **Chapter 5, Conclusions**, will discuss the conclusion for the research, in term of the method and the policy implications derived from its implementation in a large teacher's assessment process. Finally, it will outline the path of future research that can be derived from the present effort.

Chapter 2

The Generalized Linear Latent and Mixed Model

The Generalized Linear Latent and Mixed Model (GLLAMM) is a framework that unifies a wide range of latent variable models. Developed by Rabe-Hesketh et al. (2004a,c,b); Skrondal and Rabe-Hesketh (2004a); Rabe-Hesketh et al. (2012), the method was motivated by the need of a multilevel Structural Equation Model (SEM) that accommodates for unbalanced data, noncontinuous responses and the use of cross-level effects among latent variables.

This chapter presents the definition, characteristics, assumptions and properties of such framework.

2.1 Definition

Following Rabe-Hesketh et al. (2004a, 2012), we depart from the traditional multivariate framework for formulating factor and structural models, i.e. a "wide" data format, and adopt a univariate approach, i.e. "long" or vectorized format. In that sense, for each unit, the response variables are "stacked" in a single response vector, with different variables distinguished from each other, by a design matrix. With this structure, we proceed to outline the three parts of the framework:

1. The response model,
2. The structural latent variable model, and
3. The distribution of the latent variables.

For a detailed description of some of the special cases of multilevel SEM, that can be derived with this framework, refer to Appendix A.

2.1.1 Response model

As outlined by the authors, conditional on the latent variables, the response model is a Generalized Linear Model (GLM) defined by a systematic and a distributional part. For the systematic part, a linear predictor and a link function are selected, in accordance to the characteristics of the manifest variables. On the other hand, for the distributional part, a distribution from the exponential family is selected.

In the following sections, we proceed to describe the linear predictor, the link function and the distributions accommodated by the framework.

Linear predictor

For a model with L levels and M_l latent variables at $l > 1$ levels, the linear predictor takes the following form:

$$v = \mathbf{X}\boldsymbol{\beta} + \sum_{l=2}^{L} \sum_{m=1}^{M_{(l)}} \eta_m^{(l)} \mathbf{Z}_m^{(l)} \boldsymbol{\lambda}_m^{(l)} \tag{2.1}$$

where $\mathbf{X}$ is a design matrix that maps the parameter vector $\boldsymbol{\beta}$ to the linear predictor, $\eta_m^{(l)}$ the mth latent variable at level l ($m = 1, \ldots, M_{(l)}$ and $l = 1, \ldots, L$), and $\mathbf{Z}_m^{(l)}$ a design matrix that maps the vector of loadings $\boldsymbol{\lambda}_m^{(l)}$ to the mth latent variable at level l.

Note that wo do not use subscripts for the units of observation at different levels. This decision was made with the purpose of avoiding the use of mathematical definitions with large number of subscripts. However, a careful reader should consider that equation (2.1) rest on the assumption that each unit is identified at their appropriate level. For special cases of multilevel SEM, and their use of subscripts, refer to Appendix A.

Links and Distributions

As in the GLM framework, the model "links" the expectation of the conditional response, to the linear predictor, through a inverse-link function $h(\cdot)$, in the following form:

$$\mu = E[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] = h(v) \tag{2.2}$$

where equation (2.2) can be re-written in terms of the link function $g(\cdot) = h^{-1}(\cdot)$:

$$g(\mu) = g(E[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}]) = v \tag{2.3}$$

with $\boldsymbol{\eta} = \left[\eta^{(2)T}, \ldots, \eta^{(L)T}\right]^T$ and $\mathbf{Z} = \left[\mathbf{Z}^{(2)T}, \ldots, \mathbf{Z}^{(L)T}\right]^T$, as the "stacked" vector of latent variables, and the "stacked" design matrices of explanatory variables, for all L levels, respectively. Additionally, $\boldsymbol{\eta}^{(l)} = \left[\eta_1^{(l)}, \ldots, \eta_{M_{(l)}}^{(l)}\right]^T$ and $\mathbf{Z}^{(l)} = \left[\mathbf{Z}_1^{(l)T}, \ldots, \mathbf{Z}_{M_{(l)}}^{(l)T}\right]^T$, denotes the vector of latent variables, and the "stacked" design matrix of explanatory variables, at level l, respectively.

Finally, the response model specification is complete when we select an appropriate distribution from the family of exponential distributions. The types of responses that can be accommodated by the framework are the following:

1. **Continuous:**
   It results form selecting an identity link function for the scaled mean response,

$$\begin{aligned} \mu^* &= E[y^*|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= v \end{aligned} \tag{2.4}$$

   where $\mu^* = \mu\sigma^{-1}$, $y^* = y\sigma^{-1}$, and σ denotes the standard deviation of the errors.

On the other hand, the distributional part is defined by a Standard Normal distribution $\phi(x) = (2\pi)^{-1/2} exp(-x^2/2)$,

$$\begin{aligned} f(y^*|\mathbf{X},\mathbf{Z},\boldsymbol{\eta}) &= \phi(\mu^*)\sigma^{-1} \\ &= \phi(v)\sigma^{-1} \end{aligned} \tag{2.5}$$

Notice that the same parametrization can be achieved considering $y^* = v + \epsilon^*$, and $\epsilon^* \sim N(0,1)$. Additionally, the decision to standardize the response variables has been made with the purpose of making the estimation process easier, as such distribution is free of unknown parameters.

2. **Dichotomous:**
   It results from selecting an appropriate inverse-link function for the expected value of the manifest variable, which describe the probability of endorsing one of the two available categories,

$$\begin{aligned} \mu &= E[y = 1|\mathbf{X},\mathbf{Z},\boldsymbol{\eta}] \\ &= P[y = 1|\mathbf{X},\mathbf{Z},\boldsymbol{\eta}] \\ &= \pi \\ &= h(\kappa - v) \end{aligned} \tag{2.6}$$

   where κ is the decision threshold, and $h(\cdot)$ can be defined in three ways:

$$h(x) = \begin{cases} exp(x)[1 + exp(x)]^{-1} \\ \Phi(x) \quad \text{No closed form.} \\ exp(-exp(x)) \end{cases} \tag{2.7}$$

   which corresponds to the logistic, standard normal $\Phi(x)$, and Gumbel (extreme value type I) *cumulative distributions*, respectively. In terms of link functions, the distributions corresponds to the well known logit, probit and complementary log-log link functions, respectively.

   Alternatively, the same parametrization can be achieved using the concept of an underlying latent variable in the form $y^* = v + \epsilon^*$, where $y = 1$ if $y^* \geq \kappa$, and ϵ^* can have a distribution as the ones defined in equation (2.7). It is important to mention that under this parametrization, the threshold parameters κ and the $\boldsymbol{\beta}$ are confounded as they serve similar purposes, so only one would be estimated.

   Finally, the distributional part is defined by a Binomial distribution,

$$\begin{aligned} f[y = 1|\mathbf{X},\mathbf{Z},\boldsymbol{\eta}] &= \binom{n}{k}\mu^k(1-\mu)^{n-k} \\ &= \binom{n}{k}\pi^k(1-\pi)^{n-k} \end{aligned} \tag{2.8}$$

   where k denotes the number of successes in n independent Bernoulli trials.

3. **Polytomous:**
   It results from selecting a generalized logistic inverse-link function (Bock; 1972) for

the expected value of the response, which in this case, describe the probability of endorsing one of the S unordered available categories,

$$\begin{aligned}\mu_s &= E[y = y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= P[y = y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \pi_s \\ &= h(v_s)\end{aligned} \tag{2.9}$$

where v_s is the linear predictor for category s ($s = 1, \ldots, S$), and $h(\cdot)$ is defined as:

$$h(x) = exp(x)\left[\sum_{s=1}^{S} exp(x)\right]^{-1} \tag{2.10}$$

It is important to note that, as in the dichotomous case, the same parametrization can be achieved using the concept of underlying continuous responses in the form $y_s^* = v_s + \epsilon_s$, where $y = s$ if $y_s^* > y_k^*$ $\forall s, s \neq k$, ϵ_s have a Gumbel (extreme value type I) distribution, as the one defined in equation (2.7), and y_s denotes the random utility for the s category.

Finally, the distributional part is defined by a Multinomial distribution,

$$\begin{aligned} f[y = \{y_1, \cdots, y_S\}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] &= \frac{n!}{y_1! \cdots y_S!}\prod_{s=1}^{S}\mu_s^{y_s} \\ &= \frac{n!}{y_1! \cdots y_S!}\prod_{s=1}^{S}\pi_s^{y_s}\end{aligned} \tag{2.11}$$

where y_s denotes the number of "successes" in category s.

4. **Ordinal and discrete time duration:**
For the ordinal case, the linear predictor is "linked" to the probability of endorsing category s, against all previous categories, in the following form:

$$\begin{aligned}\mu_s &= E[y = y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= P[y \leq y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] - P[y \leq y_{s-1}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= h(\kappa_s - v_s) - h(\kappa_{s-1} - v_{s-1})\end{aligned} \tag{2.12}$$

where κ_s denotes the thresholds for category s. For discrete time duration, the linear predictor is "linked" to the probability of survival, in the sth time interval, as follows:

$$\begin{aligned}\mu_s &= E[t_{s-1} \leq T \leq t_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= P[T \leq t_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] - P[T \leq t_{s-1}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= h(v_s + t_s) - h(v_{s-1} + t_{s-1})\end{aligned} \tag{2.13}$$

where T is the unobserved continuous time, and t_s its observed discrete realization. Additionally, for both type of responses, $h(\cdot)$ can be defined as the logistic, standard normal, and Gumbel (extreme value type I) *cumulative distributions*, as in equation (2.7).

Similar to the dichotomous and polytomous case, the same parametrization can be achieved using the concept of underlying latent variables with $y_s^* = \upsilon_s + \epsilon_s$, where $y = s$ if $\kappa_{s-1} < y_s^* \leq \kappa_s$, $\kappa_0 = -\infty$, $\kappa_1 = 0$, $\kappa_S = +\infty$, ϵ_s has one of the distributions in equation (2.7), and y_s denotes the random utility for the s category.

It is important to note, for discrete time duration responses, the logit link corresponds to a *Proportional-Odds model*, while the complementary log-log link to a *Discrete Time Hazards model* (Rabe-Hesketh et al.; 2001). Other models for ordinal responses, such as the *Baseline Category Logit* or the *Adjacent Category Logit* models can be specified as special cases of the generalized logistic response function, defined in equation (2.10).

Finally, the distributional part is defined by a Multinomial distribution, as the one defined in equation (2.11).

5. **Counts and continuous time duration:**
It results from selecting an exponential inverse-link function (log link) for the expected value of the response,

$$\begin{aligned} \mu &= E[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \lambda \\ &= exp(\upsilon) \end{aligned} \tag{2.14}$$

and a Poisson conditional distribution for the counts,

$$\begin{aligned} f[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] &= exp(-\mu)\mu^y(y!)^{-1} \\ &= exp(-\lambda)\lambda^y(y!)^{-1} \end{aligned} \tag{2.15}$$

It is important to mention that unlike the models for dichotomous, polytomous and ordinal responses, model for counts cannot be written under the random utility framework.

6. **Rankings and pairwise comparisons:**
Following Skrondal and Rabe-Hesketh (2003a), the parametrization for polytomous responses can serve as the building block for the conditional distribution of rankings. Selecting a "exploded logit" inverse-link function (Chapaaan and Staelin; 1982) for the expected value of the response, which describes the probability of the full rankings of category s,

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$$\begin{aligned} \mu_s &= P[\mathbf{R}_s = \{r_s^1, \ldots r_s^1\}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \pi_s \\ &= h(\upsilon_s) \end{aligned} \tag{2.16}$$

where υ_s is the linear predictor for category s ($s = 1, \ldots, S$), and $h(\cdot)$ is defined as:

$$h(x) = \prod_{s=1}^{S} exp(x^s) \left[\sum_{s=1}^{S} exp(x^s) \right]^{-1} \tag{2.17}$$

Again, as in specific previous cases, the same parametrization can be achieved using the concept of underlying latent variables.

Finally, the distributional part is defined by a Multinomial distribution,

$$
\begin{aligned}
f[y = \{y_1, \cdots, y_S\}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] &= \frac{n!}{y_1! \cdots y_S!} \prod_{s=1}^{S} \mu_s^{y_s} \\
&= \frac{n!}{y_1! \cdots y_S!} \prod_{s=1}^{S} \pi_s^{y_s}
\end{aligned}
\tag{2.18}
$$

where y_s denotes the number of "success cases" in category s.

7. **Mixtures:**
   Given the previous definitions, the framework easily lends itself to model five additional settings:

   (a) **Different links and distributions for different latent variables**. This can be easily achieved by setting different links and distributions for each of the M_2 latent variables located at level 2.

   (b) **Left- or right-censored continuous responses**. Common in selection models (e.g. Heckman; 1979), they can be achieved by specifying an identity link and Normal distribution for the uncensored scaled responses, as in equations (2.4) and (2.5); and a scaled probit link and Binomial distribution otherwise, as in equations (2.7) and (2.8).

   (c) **zero-inflated count responses**. where a log link and a Poisson distribution is set for the counts, as in equations (2.14) and (2.15); and a logit link and Binomial distribution is specified to model the zero center of mass, as in equations (2.6) and (2.8).

   (d) **Measurement error in covariates**. this setting occurs when standard models use variables, with measurement error, as covariates, e.g. a logistic regression with a continuous covariate that presents measurement error. For more details on this type of setting see Rabe-Hesketh, Skrondal and Pickles (2003); Rabe-Hesketh, Pickles and Skrondal (2003), and Skrondal and Rabe-Hesketh (2003b).

   (e) **Composite links**. Useful for specifying proportional odds models for right-censored responses, for handling missing categorical covariates and many other model types. For more details on this type of settings see Skrondal and Rabe-Hesketh (2004b).

Heteroscedasticity and over-dispersion in the response

Much like the Generalized Linear Mixed Model framework (GLMM), the GLLAMM allows to model heteroscedasticity, and over- or under-dispersion by adding random effects to the linear predictor, at level 1. The types of responses, in which such characteristics can be modeled, are the following:

1. **Continuous:**
   We model **heteroscedasticity** in the following form:
   $$\sigma = exp(\boldsymbol{\alpha}^T \mathbf{Z^{(1)}}) \tag{2.19}$$
   Notice that the previous formula implies that equation (2.5) can be re-written in the following form:
   $$f(y^*|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}) = \phi(v + \boldsymbol{\alpha}^T \mathbf{Z^{(1)}}) \tag{2.20}$$
   where $\mathbf{Z^{(1)}}$ is the design matrix that maps the random effects $\boldsymbol{\alpha}$. Notice that equation (2.20) effectively corresponds to a model that includes random intercepts at level 1.

2. **Dichotomous:**
   In a more straightforward way, we model over- or under-dispersion by modifying equation (2.6), to include random intercepts at level 1, in the following form:
   $$\begin{aligned} \mu &= P[y = 1|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \pi \\ &= h(\kappa - v + \boldsymbol{\alpha}^T \mathbf{Z^{(1)}}) \end{aligned} \tag{2.21}$$

3. **Ordinal, and discrete time duration:**
   Similar to the dichotomous case, by including random intercepts at level 1 in equation (2.12), we can model over- or under-dispersion:
   $$\begin{aligned} \mu_s &= P[y \leq y_s|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] - P[y \leq y_{s-1}|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= h(\kappa_s - v_s + \boldsymbol{\alpha}^T \mathbf{Z^{(1)}}) - h(\kappa_{s-1} - v_{s-1} + \boldsymbol{\alpha}^T \mathbf{Z^{(1)}}) \end{aligned} \tag{2.22}$$
   A similar parametrization can be used for discrete time duration.

4. **Counts, and continuous time duration:**
   Finally, modifying equation (2.14) allow us to model over- or under-dispersion under a counts model:
   $$\begin{aligned} \mu &= E[y|\mathbf{X}, \mathbf{Z}, \boldsymbol{\eta}] \\ &= \lambda \\ &= exp(v + \boldsymbol{\alpha}^T \mathbf{Z^{(1)}}) \end{aligned} \tag{2.23}$$

2.1.2 Structural model for the latent variables

The structural model for the latent variables has the form:

$$\boldsymbol{\eta} = \underset{(M\times M)}{\mathbf{B}} \underset{(M\times 1)}{\boldsymbol{\eta}} + \underset{(M\times Q)}{\boldsymbol{\Gamma}} \underset{(Q\times 1)}{\mathbf{W}} + \underset{(M\times 1)}{\boldsymbol{\zeta}} \tag{2.24}$$

where $\mathbf{B}$ and $\boldsymbol{\Gamma}$ are parameter matrices that maps the relationship between the latent variables $\boldsymbol{\eta}$, and the vector of "stacked" covariates $\mathbf{W}$, respectively; $\boldsymbol{\zeta}$ is a vector of errors or disturbances, and $M = \sum_l M_l$. Notice that while equation (2.24) resembles to single-level structural equation models, the main difference lies in the fact that the latent variables may vary at different levels. Additionally, considering that $\boldsymbol{\eta}$ has no feedback effects, and it is permuted and sorted according to the levels, $\mathbf{B}$ is defined as a strictly upper triangular matrix. In this regard, it is important to mention that,

1. The absence of feedback loops implies that the method deals with non-recursive models, i.e. none of the latent variables are specified as both causes and effects of each other (Kline; 2012); this in turn allows the easy estimation of the model parameters.

2. The strictly upper triangular structure reveals that the framework does not allow latent variables to be regressed on lower level latent or observed variables, as such specification is more related to the use of formative, rather than reflective, latent variables. For a detail explanation on the topic refer to Edwards and Bagozzi (2000).

Notice, however, the previous restrictions does not hinder the ability of the method to model contextual effects, after controlling the lower level compositional effects. For examples of such refer to Appendix A.

2.1.3 Distribution of the latent variables

Finally, to fully specify the framework, and provide a scale for the latent variables, we have to make assumptions for either the distribution of the disturbances $\boldsymbol{\zeta}$ or the latent variables $\boldsymbol{\eta}$. If our research interest lies in the structural equation model, it is more convenient to make assumptions for the distribution of the disturbances; otherwise, we make assumptions for the distributions for the latent variables.

Furthermore, as in the hierarchical framework, it is assumed the latent variables at different levels are independent, whereas latent variables at the same level may present dependency. In that sense, we presume all latent variables at level l to have a multivariate normal distribution with zero mean and covariance matrix Σ_l, i.e. $\boldsymbol{\eta}^{(l)} \sim MVN(\mathbf{0}, \boldsymbol{\Sigma}_l)$. It is important to emphasize that, while the multivariate normal distribution is widely used in these settings, it is not the only distribution that can be assumed. Rabe-Hesketh, Skrondal and Pickles (2003) have provided evidence that it can be even left unspecified, by using non-parametric maximum likelihood estimation.

2.2 Model identification

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The structure of the latent variables is specified by the number of levels L and the number of latent variables Ml at each level. A particular level may coincide with a level of clustering in the hierarchical dataset. However, there will often not be a direct correspondence between the levels of the model and the levels of the data hierarchy.

2.3 Relationship with other modeling schemes

From section 2.1, it is evident that the GLLAMM framework shares some common ground, and even extends, some of the most important modeling schemes, such as the GLM, GLMM, SEM, and the Generalized Latent Model framework, from which the Item Response Theory Model (IRT) stand out.

2.3.1 Generalized Linear and Mixed Models

The Generalized Linear Model (GLM) framework, presented by Nelder and Wedderburn (1972), and further developed by McCullagh and Nelder (1989), was formulated with the purpose of expanding the linear regression model to other types of responses, like dichotomous, and counts. The scheme generalizes the linear model by "linking" the mean response variable to a linear predictor, and further allowing the magnitude of the variance, of each measurement, to be a function of its predicted value. Finally, the scheme is fully defined after selecting a distribution, from the exponential family, to model the distribution of the response variable.

As expressed in the previous paragraph, the GLM framework fixes the relationship of the modeled dispersion to the mean value, e.g. in the counts case $\mu = \lambda$, and $v(\mu) = \lambda$. However, in practice, this assumption is often violated as the data can present over- or under-dispersion. Even in the continuous response case, where the mean and variance function are not related, the model assumes that the errors are homoscedastic, identical and independently distributed. However, this assumption is often violated when the units of analysis are correlated or belong to a cluster, e.g. when students are nested in schools, and these are further nested in districts or states.

It is important to mention that, while the GLM framework can model heteroscedasticity, over- or under-dispersion, it does it in a way that does not allow them to be dependent on covariates, something that might be of interest for a researcher.

Given the restrictions of GLM, the Generalized Linear Mixed Model (GLMM) framework was developed. The method handled the hierarchical or clustered structure in the data, and in doing so, indirectly modeled the heteroscedasticity, over- or under-dispersion by adding latent variables, called "random effects", to the linear predictor. Under the framework, the random variables are often interpreted as the effects of unobserved covariates, at different levels, that induce dependence among lower-level units (Rabe-Hesketh et al.; 2012), and can be further explained by additional observed covariates.

From the previous description, it is easy to notice that the GLLAMM framework uses the same generalization and distributional assumptions, for the response variables, as the GLM; while it borrows the idea of modeling the hierarchical or clustered structure in the data, by including random effects; from the GLMM. However, it is clear that the GLLAMM further generalize both, by allowing the framework to model measurement error at different levels of the hierarchy in the data.

2.3.2 Structural Equation Models

Considering that, in practice, researchers are often faced with variables that cannot be measured directly or reflect measurement error, e.g. intelligence, depression, student abilities, among other; the statistical literature was instigated to develop methods that can handle such data characteristics.

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The disciplinary seeds of Structural Equation Models (SEM) were set by **?**, with a factor model on intelligence testing, passing through Wright (1920), with a path analysis in the context of genetic and biology, to finally land in the sociological field, with the work of Blalock (1961).

to include several features of the previous modeling scheme, i.e. generalized linear mixed models, the framework is characterized by the fact that it is a method that can impute relationships between unobserved factors or latent variables, and observable or manifest variables. Under this framework, it is assumed that such "common factors" are responsible for the variation and dependence in the manifest variables.

mention Factor Models, Item Response Theory and Generalized Latent Models, and Multilevel Structural Equation Models 2.1.3

multilevel structural equation models represent a synthesis between multilevel regression models and structural equation models. Considering that

2.4 Advantages and Disadvantages

Chapter 3

Estimation

3.1 Likelihood methods

3.1.1 Likelihood function

3.1.2 Adaptive Quadrature

3.2 Bayesian methods

3.2.1 Prior distributions

3.2.2 Initial start

3.2.3 Posterior distributions

Chapter 4

Application

4.1 Instruments

4.2 Data

4.2.1 Collection

4.2.2 Sample scheme

4.3 Results

4.3.1 Hypothesis 1:

4.3.2 Hypothesis 2:

4.3.3 Hypothesis 3:

Chapter 5

Conclusion and Discussion

5.1 Discussion

5.2 Conclusions

5.3 Future development

Appendix A

Additional Theory

A.1 Special cases for the GLAMM

Appendix B

Code

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