

Hierarchical latent variable models:

Method, extensions, advantages, and educational policy implications.

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1 Topic justification and context

The short and long term benefits of effective teaching practices can be observed throughout the literature: improvements in student achievements (Rockoff, 2004; Rivkin et al., 2005; Duflo et al., 2009; Hanushek and Rivkin, 2012; Muralidharan and Sundararaman, 2013; Chetty et al., 2014a; Araujo et al., 2016); development of executive functions (Araujo et al., 2016), increased college attendance, higher salaries, lower possibility of premature parenthood (Chetty et al., 2014b), among others. Similarly, the literature has shown most of the negative impacts resulting from the presence of teacher shortages¹ (Duflo et al., 2009; Muralidharan and Sundararaman, 2013; Chetty et al., 2015; Ayala, 2017; Marotta, 2019) or ineffective teaching practices (Hanushek and Rivkin, 2012).

However, while the evidence have a solid methodological support, Hanushek and Rivkin (2006) have indicated that some of the proxy variables used, are not consistently related to either teacher effectiveness or quality of instruction, examples of such are: out of field teaching² (Ingersoll, 1998; Dee and Cohodes, 2008; Bertoni, Elacqua, Marotta, Martinez, Méndez, Montalva, Olsen, Santos and Soares, 2020); teaching hours (Bruns et al., 2015); years of experience or educational degree (Rockoff, 2004; Rivkin et al., 2005; Clotfelter et al., 2006, 2007; Hanushek and Rivkin, 2012); among others.

Given the lack of consistency of the effects that arises from using such proxies, Hanushek and Rivkin (2012) have pointed out that the analysis of teacher effectiveness has largely turned away, from attempts to identify the teacher’s specific characteristics, to focus its attention into measuring the direct relationship between them and the student outcomes³. For that reason, considerable uncertainty is still present in the literature, regarding exactly which aspects of teachers are key for the student’s learning and whether those qualities can be measured (Rockoff, 2004; Clotfelter et al., 2006).

However, because the evidence still largely supports the perception that teachers are the main driver behind the student’s learning processes, one of the main points in the agenda of any educational authority should be the design of an assessment system that can attract, select, develop, and retain the most effective ones (Elacqua et al., 2018). But first, the authority would have to define the Educational Performance Standards (EPS) that best agrees with the country’s context. With the EPS establishment, the authorities can set clear expectations about what a ”good” teacher should know and know to do (Cruz-Aguayo et al., 2020).

But because of the uncertainty surrounding such specific requirements, these conditions are not easy to define. Nevertheless, while the specifics are hard to determine, Cruz-Aguayo et al. (2020) has hinted that most of them can be largely grouped into two: (i)

¹Bertoni, Elacqua, Marotta, Martinez, Méndez, Montalva, Olsen, Santos and Soares (2020) defined it as the context in which the teacher’s supply, i.e. the number of available teachers in the system, is less than its demand. The authors further elaborate that one of the causes of these shortages is related to the applicants’ lower quality or due to their faulty initial training, implying that the shortage can also be conceived as the lack of good quality teachers. In this sense, the evidence of such shortage has been more prevalent, but not decisive, with temporary teachers, as they are usually associated with inferior attributes, compared to their contracted counterparts

²Medeiros et al. (2018) defines it as teachers teaching a subject in which they are not specialized or do not have the appropriate certificate.

³The method is known as value-added analysis, and it is based on the perspective that a good teacher is one who consistently gets higher achievement from students after other determinants of such are controlled for. For a more detailed explanation of the method refer to Scherrer (2011).

to have the disciplinary knowledge and pedagogical practices adequate to the classroom characteristics, context and teaching level, and (ii) to display such knowledge and practices in the classroom, using the appropriate material and technological resources available.

As one can infer from the previous general conditions, and the slew evidence, knowledge is a relevant observable factor that it is consistently associated with teacher effectiveness and growth in student's achievement (Santibañez, 2006; Clotfelter et al., 2006, 2007; Hanushek and Rivkin, 2006; Marshall, 2009; Rockoff et al., 2011; Kane et al., 2010; Kane and Staiger, 2012; Ome, 2012; Metzler and Woessmann, 2012; Kane et al., 2013; Araujo et al., 2016; Bietenbeck et al., 2018; Estrada, 2019); and in that sense, its measurement should be of interest for any educational authority.

The measurement of knowledge has a myriad of available tools, and while Bertoni, Elacqua, Méndez, Montalva, Munevar, Olsen and Román (2020) had advocated for the use of multiple instruments, it is important to keep in mind that any educational department are bounded by budgetary constraints. In this setting, and compared to other instruments, valid⁴ and reliable⁵ standardized tests⁶ stand out not only for its cost-effectiveness and a much simpler implementation (Cruz-Aguayo et al., 2020), but also because, they are one of tools with less subjective scoring processes and interpretations.

However, as no instrument is perfect, the teacher's subject knowledge scores will likely reflect measurement error (Metzler and Woessmann, 2012). As established by Angrist and Krueger (1999), measurement error in the explanatory variable could bias the estimated coefficients. This last result implies, that evidence based on test scores could be an attenuated reflection of the true effects. On the other hand, the use of one composite value, i.e. the score, does not allow to test which specific factors -if any- leads to better or worse teacher performance, making also difficult to know which teachers should be hired or what should be done to train them (Hanushek and Rivkin, 2012).

But beyond the use of test results as explanatory variables in modeling processes, there is one more pressing argument on why the issue of measurement error should be addressed: approximately 60% of the Caribbean and Latin American countries use standardized test scores as part of or as a main teacher selection tool (Cruz-Aguayo et al., 2020). In this setting, devoting effort to assess the issues related to measurements errors, could help the educational authorities to understand if the scores thresholds used for the selection are appropriately set, and ultimately, to know the what kind of teachers are being integrated into the public teaching staff.

In summary, teachers are one of the main drivers behind the student achievements. However, some of the evidence supporting this claim has been based on proxy variables that are not consistently related to the quality of instruction, or methods that are not concerned with the outline of the teaching factors responsible for the student's learning. Nevertheless, while the literature still reflects considerable uncertainty on what are the "ingredients for a good teacher", a good amount of evidence has supported the disciplinary and pedagogical knowledge as relevant components of the teacher effectiveness. Finally,

⁴the extend to which a measurement tool is well-founded and accurately corresponds to the real measure (Kelley, 1927)

⁵the overall consistency of a measure under consistent conditions.

⁶Assessment instrument in which the implementation, questions, scoring processes, and interpretations are consistent with a predetermined or typified way. The instrument is usually composed of questions or items that fulfill three conditions: (i) they are polytomous, i.e. they have multiple choices, (ii) the choice categories are nominal, i.e. do not present any specific order, and (iii) there is only one "correct" category or answer (Rivera, 2019)

the literature has shown that valid and reliable standardized tests are among the best tools to assess such factors, but also have emphasized that such scores could reflect the teacher’s abilities with considerable noise.

In that sense, this research plans to fill three main literature gaps. First, the researcher will use hierarchical latent variable models to obtain a noise-free score for the competencies of teachers. Second, the method would also help to obtain a dynamic multidimensional depiction of their disciplinary abilities. And lastly, the researcher will tests the real implications of the method in a data composed of repeated large standardized educational assessments from Peru.

Concerning the first two objectives of the research, the author expects to appraise: (i) if hierarchical latent variable models can provide a general framework that could serve multiple psychometric purposes; and (ii) what are the advantages or disadvantages of using such models.

For the last objective, the author expects to shed some lights about key policy decisions related to those large evaluation processes. To mention a few: (i) do the instruments guarantee a fair assessment of minority groups with different abilities?; (ii) are we screening the most knowledgeable teachers?; (iii) what are the general characteristics of the career applicants?; (iv) what differentiate a contract teacher from a temporary one?; (v) what is the main evolution of the disciplinary knowledge of the teachers?, and, is there any identifiable divergence from such pattern?; (vi) does initial training or socioeconomic status proxy variables explain different levels of disciplinary knowledge?; (vii) what specific factor of the disciplinary knowledge is consistently related to classroom observation scores.

In this sense, the researcher believes the present thesis contributes to the literature in two aspects: (i) in a the theoretical and methodological sense, as the research is focused on offering a exhaustive description and analysis of the models; and (ii) in a more practical sense, as it helps to provide evidence on some of key policy decisions that Latin America countries are currently facing.

2 Methods

Four measurements issues receive considerable attention in the research literature: (a) random measurement error, (b) the focus of test on particular portions of the achievement distribution, (c) cardinal versus ordinal comparisons of test scores, and (d) the multidimensionality of educational outcomes. Not only do the test measurements issues introduce noise into the estimates of the teacher effectiveness, but they also bias upwards estimates of the variance in teacher quality (Hanushek and Rivkin, 2012). While this was mentioned for the value-added measures it is equally valid for the standardized evaluation of teachers.

We address measurement error by correcting the estimated coefficients using a reliability ratio estimated on the basis of answers to all items on the teacher tests (see Section 5.3).

- Paragraph’s main point: what method are you using?

one can improve the value-added measures if we incorporate other measures of teacher quality, such as teacher characteristics (Chetty et al., 2014a)

- Paragraph’s idea 1: IRT and the focus on items

De esta forma, mientras que los modelos para respuestas dicotómicas, tales como Rasch (?), de uno, dos, tres parámetros (?) y cuatro parámetros (?), expresan la probabilidad de elegir la alternativa correcta en función de la “habilidad” del individuo; el **Modelo de**

Respuesta Nominal (NRM) y todas sus extensiones (Fajardo y Fajardo, capítulo 2), expresa la probabilidad de elegir cada alternativa de la pregunta en función de la misma “habilidad”.

A diferencia de los modelos de respuesta graduada (Fajardo y Fajardo, capítulo 5), el NRM no se sustenta sobre el concepto de la dicotomización de las alternativas, que derivan en los umbrales por categorías característicos de los modelos mencionados; por el contrario, la probabilidad correspondiente a cada alternativa es modelada directamente, implementando una generalización multivariada del modelo de rasgos latentes logístico (Fajardo, 2015).

- Paragraph’s idea 2: SEM and the focus on abilities
- Paragraph’s idea 3: IRT and SEM equivalence (evidence)

(Brown, 2015) The potential consequences of treating categorical variables as continuous variables in CFA are manifold: (1) They produce attenuated estimates of the relationships (correlations) among indicators, especially when there are floor or ceiling effects; (2) they lead to “pseudofactors” that are artifacts of item difficulty or extremeness; and (3) they produce incorrect test statistics and standard errors. ML can also produce incorrect parameter estimates, such as in cases where marked floor or ceiling effects exist in purportedly interval-level measurement scales (i.e., because the assumption of linear relationships does not hold).

Rasch Model with SEM

1. Requires to set the loadings = 1 in all items (there are no evidence that different items should load differently in all sub-factors, if that happen then we can say that an item does not behave good)

2. Thresholds can be transformed into difficulty parameters. They will be from the normal ogive model.

Evidence: It is well known that factor analysis with binary outcomes is equivalent to a two-parameter normal ogive IRT model (e.g., Ferrando Lorenza-Sevo, 2005; Glöckner-Rist Hoijtink, 2003; (Kamata and Bauer, 2008; Takane and de Leeuw, 1987).

Item difficulties have been alternatively referred to in the IRT literature as item threshold or item location parameters. In fact, item difficulty parameters are analogous to item thresholds (t) in CFA with categorical outcomes (Muthén et al., 1991).

Item discrimination parameters are analogous to factor loadings in CFA and EFA because they represent the relationship between the latent trait and the item responses.

Muthén (1988; Muthén et al., 1991) has shown that MIMIC models (see Chapter 7) with categorical indicators are equivalent to DIF analysis in the IRT framework (see also Meade Lautenschlager, 2004).

Muthén (1988; Muthén et al., 1991) notes that the MIMIC framework offers several potential advantages over IRT. These include the ability to (1) use either continuous covariates (e.g., age) or categorical background variables (e.g., gender); (2) model a direct effect of the covariate on the latent variable (in addition to direct effects of the covariate on test items); (3) readily evaluate multidimensional models (i.e., measurement models with more than one factor); and (4) incorporate an error theory (e.g., measurement error covariances). Indeed, a general advantage of the covariance structure analysis approach is that the IRT model can be embedded in a larger structural equation model (e.g., Lu, Thomas, Zumbo, 2005).

De esta forma, en el contexto de una evaluación estandarizada, suponemos que n sujetos responden p ítems de opción múltiple eligiendo **una sola** alternativa de m_j disponibles, las mismas que pueden variar de ítem a ítem y poseen un orden arbitrario. Entonces, el NRM define ***Funciones de Respuestas de las Categorías del ítem***

(ICRF, acorde con ?) o Curvas Características de la Alternativas del ítem (IOCC, acorde con ?) de la siguiente manera:

$$P_{jk}(\theta_i) = \frac{e^{z_{jk}(\theta_i)}}{\sum_{h=1}^m e^{z_{jh}(\theta_i)}} \quad (1)$$

Donde:

$$z_{jk} = a_{jk}\theta_i + c_{jk} \quad \forall \quad i = 1, \dots, n; \quad j = 1, \dots, p; \quad k = 1, \dots, m_j$$

El parámetro θ_i representa la “habilidad” del individuo i , a_{jk} corresponde al parámetro de discriminación de la alternativa k del ítem j y c_{jk} es proporcional a la “popularidad” de la alternativa k del ítem j . El vector compuesto por los vectores $z_{j1}, z_{j2}, \dots, z_{jm_j}$ es usualmente definido como el vector *logit multinomial*. La presente parametrización del modelo es expresada en términos del intercepto y la pendiente de las ICRFs; sin embargo, la literatura utiliza una parametrización que hace la estimación computacionalmente más eficiente.

- Paragraph’s idea 4: what can be gain from this merge

De la parametrización anterior se espera que, al igual que los modelos para respuestas dicotómicas, la ICRF de la alternativa “correcta” sea monotónicamente creciente respecto a la “habilidad”, mientras que la forma de las ICRFs de los distractores dependerá de como la alternativa sea percibida por el evaluado (?). De este modo, se plantea estudiar la formulación, supuestos, características y propiedades del NRM.

De manera complementaria al estudio del modelo, el presente proyecto plantea la estimación de los parámetros de interés a través de simulaciones de **Cadenas de Markov de Montecarlo (MCMC)**, perteneciente a los métodos de inferencia bayesiana. Se elige los métodos bayesianos debido a que: (i) elimina los problemas de no convergencia y estimación impropia de los parámetros encontrados en los procedimientos de máxima verosimilitud conjunta y/o marginal (?), (ii) bajo escenarios en los que la complejidad del modelo incrementa, el método se vuelve más atractivo, pues usa simulaciones en vez de métodos numéricos; (iii) los modelos MCMC se vuelven particularmente útiles cuando los datos son dispersos o cuando es poco probable que la teoría asintótica se mantenga (?); (iv) la flexibilidad y escalabilidad de las soluciones implementadas y (v) una mayor capacidad de recuperación de parámetros de interés, de los cuales existen muchos ejemplos (?, ?, entre otros).

- Paragraph’s idea 5: What are the difficulties

- Paragraph’s conclusion: SEM/IRT merge provides multiple benefits

3 Data

Con respecto a los requisitos generales, para ser docente en Perú es necesario poseer el título de profesor o licenciado en educación, otorgado por una institución de formación docente acreditada en el país o en el exterior (en este último caso, el título debe ser revalidado en el Perú) 16 Además de los requisitos generales también se deben cumplir requisitos específicos, por ejemplo: a) manejar fluidamente la lengua materna de los estudiantes y conocer la cultura local para postular a vacantes de instituciones educativas pertenecientes a educación intercultural bilingüe (EIB); b) acreditar la especialización

en la modalidad para postular a vacantes de instituciones educativas pertenecientes a educación básica especial (EBE); y c) se permite enseñar en inicial a los docentes con título de profesor o de licenciado en educación en la modalidad de educación básica regular (EBR) en el nivel primaria, con estudios concluidos de segunda especialidad en educación inicial y con experiencia mínima de dos (02) años lectivos en el nivel inicial.

- Paragraph's main point: What data do we have?

Finalmente, el modelo investigado será aplicado a un conjunto de datos reales pertenecientes al sector educativo.

- Paragraph's idea 1: Standardized MCQ in Peru for multiple purposes

En el actual escenario de la revalorización de la carrera magisterial⁷⁸⁹¹⁰, el Ministerio de Educación del Perú (MINEDU) aprobó en el año 2012 e inició la implementación en el año 2014 las evaluaciones a docentes con el propósito de: (i) evaluar las capacidades y/o competencias de los docentes nombrados en las especialidades que corresponden a su enseñanza y (ii) revalorizar las escalas salariales de los docentes nombrados. En este contexto, en el año 2015, el ministerio aplicó la evaluación de "Ingreso a la Carrera Pública Magisterial y Contratación Docente" (en adelante **Nombramiento 2015**), la cual permitió el ingreso de nuevos docentes a la primera de las siete escalas de la carrera magisterial.

- Paragraph's idea 2: Definition of the sample and variables

El presente proyecto optó por implementar el modelo investigado en 40 de los 90 ítems disponibles de Nombramiento 2015, aplicados a 11826 docentes de la especialidad de Matemática de la Modalidad de Educación Básica Regular Nivel Secundaria. El instrumento se encuentra diseñado para medir un *trazo latente unidimensional* que corresponde a las *competencias pedagógicas y de especialidad* que los docentes poseen. La elección del modelo se sustentó en que este no solo provee información acerca de la alternativa elegida (presuntamente "correcta"), sino también, permite conocer la "popularidad" con la que el individuo percibe el resto de categorías disponibles, información especialmente valiosa para el análisis de distractores y validez teórica de constructo de los ítems utilizados en el instrumentos de evaluación.

- Paragraph's idea 3: Composition of the exam - Paragraph's idea 4: Selection of factors and why - Paragraph's conclusion: The process can be performed in this data

En conclusión, el presente proyecto de tesis estudiará los supuestos, propiedades y características del Modelo de Respuesta Nominal (NRM) e implementará la estimación de sus parámetros desde el enfoque de la inferencia bayesiana. Entre los tópicos que adicionalmente serán presentados se encuentran: (i) estudios de simulación que comparan la recuperación de parámetros de interés entre el método clásico de estimación y el bayesiano y (ii) la aplicación a un conjunto de datos reales del sector educativo, acorde con lo detallado en párrafos previos.

⁷Ley N° 28044, Ley General de Educación

⁸Ley N° 29944, Ley de Reforma Magisterial

⁹Decreto Supremo N° 011-2012-ED, que aprueba el Reglamento de La Ley de Educación

¹⁰Decreto Supremo N° 004-2013-ED, que aprueba el Reglamento de la Ley de Reforma Magisterial, y sus modificaciones

4 Thesis objectives

El objetivo general de la tesis consiste en estudiar la formulaci3n, supuestos, caracter3sticas y propiedades del **Modelo de Respuesta Nominal (NRM)** en el contexto de la Teor3a de Respuesta al 3tem (IRT). Del mismo modo, se pretende realizar un estudio de simulaci3n que compare el m3todo cl3sico de estimaci3n del NRM frente a los m3todos bayesianos. Finalmente, se aplicar3 el modelo descrito a un conjuntos de datos reales del sector educativo, desde el enfoque de la inferencia bayesiana. De manera espec3fica:

- Se realizar3 una extensiva revisi3n de la literatura acerca del modelo de inter3s.
- Se estudiar3n los supuestos, caracter3sticas y propiedades del modelo, desde la perspectiva cl3sica y bayesiana.
- Se implementar3n m3todos de inferencia bayesiana para la estimaci3n de los par3metros de inter3s.
- Se realizar3n estudios de simulaci3n para comprobar la capacidad de recuperaci3n de los par3metros de inter3s por parte del m3todo cl3sico y bayesiano.
- Se aplicar3 el modelo de inter3s a un conjunto de datos reales pertenecientes al sector educativo.

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