Lab: Model Selection for Neural Data

Machine learning is a key tool for neuroscientists to understand how sensory and motor signals are encoded in the brain. In addition to improving our scientific understanding of neural phenomena, understanding neural encoding is critical for brain machine interfaces. In this lab, you will use model selection for performing some simple analysis on real neural signals.

Before doing this lab, you should review the ideas in the <u>polynomial model selection demo (./polyfit.ipynb)</u>. In addition to the concepts in that demo, you will learn to:

- Load MATLAB data
- Formulate models of different complexities using heuristic model selection
- Fit a linear model for the different model orders
- Select the optimal model via cross-validation

The last stage of the lab uses LASSO estimation for model selection. If you are doing this part of the lab, you should review the concepts in <u>LASSO demonstration (./prostate.ipynb)</u> on the prostate cancer dataset.

Loading the data

The data in this lab comes from neural recordings described in:

Stevenson, Ian H., et al. "Statistical assessment of the stability of neural movement representations." Journal of neurophysiology 106.2 (2011): 764-774 (http://jn.physiology.org/content/106/2/764.short)

Neurons are the basic information processing units in the brain. Neurons communicate with one another via *spikes* or *action potentials* which are brief events where voltage in the neuron rapidly rises then falls. These spikes trigger the electro-chemical signals between one neuron and another. In this experiment, the spikes were recorded from 196 neurons in the primary motor cortex (M1) of a monkey using an electrode array implanted onto the surface of a monkey's brain. During the recording, the monkey performed several reaching tasks and the position and velocity of the hand was recorded as well.

The goal of the experiment is to try to *read the monkey's brain*: That is, predict the hand motion from the neural signals from the motor cortex.

We first load the basic packages.

```
In [2]: import numpy as np
import matplotlib
import matplotlib.pyplot as plt
%matplotlib inline
```

The full data is available on the CRCNS website http://crcns.org/data-sets/movements/dream (http://crcns.org/data-sets/movements/dream). This website has a large number of great datasets and can be used for projects as well. To make this lab easier, I have pre-processed the data slightly and placed it in the file StevensonV2.mat, which is a MATLAB file. You will need to have this file downloaded in the directory you are working on.

Since MATLAB is widely-used, python provides method for loading MATLAB mat files. We can use these commands to load the data as follows.

```
In [3]: import scipy.io
mat_dict = scipy.io.loadmat('StevensonV2.mat')
```

The returned structure, mat_dict, is a dictionary with each of the MATLAB variables that were saved in the .mat file. Use the .keys() method to list all the variables.

We extract two variables, spikes and handVel, from the dictionary mat_dict , which represent the recorded spikes per neuron and the hand velocity. We take the transpose of the spikes data so that it is in the form time bins \times number of neurons. For the handVel data, we take the first component which is the motion in the x-direction.

```
In [5]: X0 = mat_dict['spikes'].T
    #print(X0)

#print(X1)

y0 = mat_dict['handVel'][0,:].T# this is ydat
    #y0 = mat_dict['handVel'][0,:]
    #y0 = np.transpose(y0)
    print(y0.shape[0])

print(y0)
    X0.shape

15536
[-0.0112006 -0.01074321 0.01767953 ..., 0.05812657 0.05378452
    0.04268675]

Out[5]: (15536, 196)
```

The spikes matrix will be a nt x neuron matrix where nt is the number of time bins and neuron is the number of neurons. Each entry spikes[k,j] is the number of spikes in time bin k from neuron j. Use the shape method to find nt and nneuron and print the values.

```
In [6]: nt, neuron = X0.shape
  print("num nt={0:d} num neuron={1:d}".format(nt,neuron))
  num nt=15536 num neuron=196
```

Now extract the time variable from the mat_dict dictionary. Reshape this to a 1D array with nt components. Each entry time[k] is the starting time of the time bin k. Find the sampling time tsamp which is the time between measurements, and ttotal which is the total duration of the recording.

```
In [7]: Time = mat dict['time']
        #time = Time.shape[0]
        Time = Time.reshape(nt)
        #print(Time)
        #for t in Time:
            print (t)
        #need to find the tsamp
        #first take the difference between the time samples
        # time iteration = (0.05, 0.1, 0.15...)
        tsamp= Time[2]-Time[1]
        print(tsamp)
        #need to find the ttotal
        #subtract the initial time t0 from the last time t15536
        #print(Time[nt-1])
        #ttotal = Time[15535] - Time[0]
        #print(ttotal +tsamp)
        ttotal = Time[15535] - Time[0] + tsamp
        print(ttotal)
```

0.05 776.8

Linear fitting on all the neurons

First divide the data into training and test with approximately half the samples in each. Let Xtr and ytr denote the training data and Xts and yts denote the test data.

```
In [8]: #Xtr = np.random.choice(Time, 7768) #random training set (15536/2)half
        of the training set Time
        Xtr = X0[:7768,:]#spikes
        print(Xtr)
        #ytr is a product of np.array(something) then i use a np.random.choice
        in the same range of samples as time
        ytr = y0[:7768]
        Xts = X0[7768:15536,:] #random test set
        yts = y0[7768:15536]
        #print(ytr)
        #print(Xts)
        #print(yts)
        #print(Xts)
        # yts = ...
        #yts = y[:7768]
        #plt.scatter(Xtr[:,1],ytr)
        #plt.xlabel('x')
        #plt.ylabel('y')
        #plt.grid()
        #plt.legend(['True (dtrue=3)', 'Data'], loc='upper left')
        #plt.show()
        [[1 \ 0 \ 2 \ \dots, \ 2 \ 0 \ 2]
         [3 1 1 ..., 0 0 0]
         [1 0 1 ..., 0 0 3]
          ...,
         [1 0 0 ..., 0 0 6]
         [0\ 0\ 0\ \dots,\ 0\ 0\ 5]
```

Now, we begin by trying to fit a simple linear model using *all* the neurons as predictors. To this end, use the sklearn.linear_model package to create a regression object, and fit the linear model to the training data.

 $[0 \ 0 \ 0 \ \dots, \ 0 \ 0 \ 2]]$

```
In [9]: import sklearn.linear_model
#from sklearn import linear_model
# TODO

regr = sklearn.linear_model.LinearRegression()
regr.fit(Xtr,ytr)
Out[9]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normaliz)
```

Measure and print the normalized RSS on the test data.

e=False)

```
In [10]: # fit isn't working
    y_ts_pred = regr.predict(Xts)
    RSS_rel_ts = np.mean((yts-y_ts_pred)**2)/(np.std(yts)**2)
    print("Normalized test RSS = {0:f}".format(RSS_rel_ts))

Normalized test RSS = 4999054779645876502528.000000
```

You should see that the test error is enormous -- the model does not generalize to the test data at all.

Linear Fitting with Heuristic Model Selection

The above shows that we need a way to reduce the model complexity. One simple idea is to select only the neurons that individually have a high correlation with the output.

Write code which computes the coefficient of determination, R_k^2 , for each neuron k. Plot the R_k^2 values.

You can use a for loop over each neuron, but if you want to make efficient code try to avoid the for loop and use <u>python broadcasting (../Basics/numpy axes broadcasting.ipynb)</u>.

```
In [11]: ym = np.mean(y0)
         syy = np.mean((y0-ym)**2)
         Rsq = np.zeros(neuron)
         beta0 = np.zeros(neuron)
         beta1 = np.zeros(neuron)
         for k in range(neuron):
             xm = np.mean(X0[:,k])
             #print(xm)
             sxy = np.mean((X0[:,k]-xm)*(y0-ym))
             sxx = np.mean((X0[:,k]-xm)**2)
             beta1[k] = sxy/sxx
             beta0[k] = ym - beta1[k]*xm
             Rsq[k] = (sxy)**2/sxx/syy
             print("{0:2d} Rsq={1:f}".format(k,Rsq[k]))
         #for d: 1->
         \# x temp = x1[1:10d,:]
         # for i: 1-10 #crossvalidaiton
         \# RSS = RSS[i,d]
         #RSS[19,10]
         #plot stem
         x = range (neuron)
         plt.stem(x,Rsq,'-.')
         plt.grid()
```

3 Rsq=0.014403 4 Rsq=0.013202 5 Rsq=0.000047 6 Rsq=0.026373 7 Rsq=0.000008 8 Rsq=0.000226

0 Rsq=0.006049 1 Rsq=0.017449

Rsq=0.017310

2

- 9 Rsq=0.000082
- 10 Rsq=0.000698 11 Rsq=0.000221
- 12 Rsq=0.000438
- 13 Rsq=0.000002
- 14 Rsq=0.003749
- 15 Rsq=0.000232
- 16 Rsq=0.000438
- 17 Rsq=0.000081
- 18 Rsq=0.000427
- 19 Rsq=0.000002
- 20 Rsq=0.005763

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- 21 Rsq=0.000008
- 22 Rsq=0.009351
- 23 Rsq=0.001622
- 24 Rsq=0.000000
- 25 Rsq=0.000202
- Rsq=0.00014926
- 27
- Rsq=0.000133
- 28 Rsq=0.00001129
- Rsq=0.01183830
- Rsq=0.006994
- 31 Rsq=0.000054
- 32 Rsq=0.000048
- 33 Rsq=0.000013
- 34 Rsq=0.000000
- 35 Rsq=0.005588
- 36 Rsq=0.011034
- 37 Rsq=0.000008
- 38 Rsq=0.000161
- 39 Rsq=0.001175
- 40 Rsq=0.000047
- 41 Rsq=0.000000
- 42 Rsq=0.001871
- 43 Rsq=0.018205
- 44 Rsq=0.006378
- 45 Rsq=0.001970
- 46 Rsq=0.000140
- 47 Rsq=0.000926
- 48 Rsq=0.000001
- 49 Rsq=0.000731
- 50 Rsq=0.001064
- 51 Rsq=0.000006
- 52 Rsq=0.004139
- 53 Rsq=0.000512
- 54 Rsq=0.003184
- 55 Rsq=0.002781
- 56 Rsq=0.000032
- 57 Rsq=0.000030
- 58 Rsq=0.084733
- 59 Rsq=0.001594
- 60 Rsq=0.001124
- 61 Rsq=0.024546
- 62 Rsq=0.000178
- 63 Rsq=0.000049
- 64 Rsq=0.001727
- 65 Rsq=0.002048
- 66 Rsq=0.008399
- 67 Rsq=0.057849
- 68 Rsq=0.002292
- 69 Rsq=0.001084
- 70 Rsq=0.000058

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- 71 Rsq=0.001408
- 72 Rsq=0.000004
- 73 Rsq=0.000852
- 74 Rsq=0.000006
- 75 Rsq=0.000751
- 76 Rsq=0.000086
- 77 Rsq=0.000532
- 78 Rsq=0.000016
- 79 Rsq=0.001977
- 80 Rsq=0.029364
- 81 Rsq=0.000013
- 82 Rsq=0.000000
- 83 Rsq=0.001138
- 84 Rsq=0.009751
- 85 Rsq=0.000007
- 86 Rsq=0.001297
- 87 Rsq=0.003312
- 88
- Rsq=0.000615
- 89 Rsq=0.000010
- 90 Rsq=0.000198
- 91 Rsq=0.000141
- 92 Rsq=0.000008
- 93 Rsq=0.000454
- 94 Rsq=0.000021
- 95 Rsq=0.000203
- 96 Rsq=0.000113
- 97 Rsq=0.001216
- 98 Rsq=0.041412
- 99 Rsq=0.000171
- 100 Rsq=0.013749
- 101 Rsq=0.000050
- 102 Rsq=0.000506
- 103 Rsq=0.007248
- 104 Rsq=0.000006
- 105 Rsq=0.000091
- 106 Rsq=0.000092
- 107 Rsq=0.000495
- 108 Rsq=0.000760
- 109 Rsq=0.001632
- 110 Rsq=0.004300
- 111 Rsq=0.006488
- 112 Rsq=0.000781
- 113 Rsq=0.000065
- 114 Rsq=0.005248
- 115 Rsq=0.001521
- 116 Rsq=0.000404
- 117 Rsq=0.005194
- 118 Rsq=0.000107
- 119 Rsq=0.000274
- 120 Rsq=0.043481

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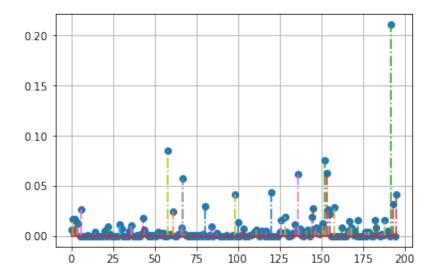
- 121 Rsq=0.000208
- 122 Rsq=nan
- 123 Rsq=0.000251
- 124 Rsq=0.000239
- 125 Rsq=0.003851
- 126 Rsq=0.015655
- 127 Rsq=0.002613
- 128 Rsq=0.018871
- 129 Rsq=0.004513
- 130 Rsq=0.000000
- 131 Rsq=0.000161
- 132 Rsq=0.001233
- 133 Rsq=0.002636
- 134 Rsq=0.011346135
- Rsq=0.003845
- 136 Rsq=0.062266
- 137 Rsq=0.007324
- 138 Rsq=0.000570
- 139 Rsq=0.000015
- 140 Rsq=0.000891
- 141 Rsq=0.005568
- 142 Rsq=0.006446
- 143 Rsq=0.000384
- Rsq=0.019351144
- 145 Rsq=0.027149
- 146 Rsq=0.005472
- 147 Rsq=0.008138
- 148 Rsq=0.002579

149

- Rsq=0.002082150
- Rsq=0.009285
- 151 Rsq=0.012662
- 152 Rsq=0.076110
- 153 Rsq=0.062699
- 154 Rsq=0.026915
- 155 Rsq=0.022048
- 156 Rsq=0.000025
- 157 Rsq=0.000176
- 158 Rsq=0.028348
- 159 Rsq=0.000003
- 160 Rsq=0.000199
- 161 Rsq=0.000240
- 162 Rsq=0.008190
- 163 Rsq=0.000027
- 164 Rsq=0.000006
- 165 Rsq=0.000050
- 166 Rsq=0.004791
- 167 Rsq=0.015360
- 168 Rsq=0.008925
- 169 Rsq=0.000566
- 170 Rsq=0.004922

```
171
    Rsq=0.000000
172
    Rsq=0.016245
173
    Rsq=0.000024
    Rsq=0.000001
174
175
    Rsq=0.000253
176
    Rsq=0.004500
177
    Rsq=0.000005
178
    Rsq=0.004516
179
    Rsq=0.002839
180
    Rsq=0.000009
181
    Rsq=0.000033
182
    Rsq=0.016409
183
    Rsq=0.008125
184
    Rsq=0.000704
185
    Rsq=0.000267
186
    Rsq=0.001975
187
    Rsq=0.001295
188
    Rsq=0.016119
189
    Rsq=0.001894
190
    Rsq=0.004888
191
    Rsq=0.000049
192
    Rsq=0.210846
193
    Rsq=0.031629
194
    Rsq=0.000219
195
    Rsq=0.041982
```

/anaconda/lib/python3.6/site-packages/ipykernel_launcher.py:11: Runt
imeWarning: invalid value encountered in double_scalars
 # This is added back by InteractiveShellApp.init_path()
/anaconda/lib/python3.6/site-packages/ipykernel_launcher.py:13: Runt
imeWarning: invalid value encountered in double_scalars
 del sys.path[0]



We see that many neurons have low correlation and can probably be discarded from the model.

Use the np.argsort() command to find the indices of the d=100 neurons with the highest R_k^2 value. Put the d indices into an array Isel. Print the indices of the neurons with the 10 highest correlations(meaning the highest Rsq value).

```
In [12]: d = 100 # Number of neurons to use
        Isel = np.zeros(d)
         ###################
         # the Rsq at 122 is undefined because the sxy is 0
         # therefore i am going to omit it from the data
         ###################
         Rsq[122] = 100.0000 ###need to omit it
         #print(Rsq)
         Indecies = np.argsort(-Rsq)
         ########returns the index of the values from low to high need to ne
         gate####
         #print(Indecies.shape)
         #for k in range(len(Indecies)):
             print("{0:2d} Highest Rsq={1:f}".format(k,Indecies[k]))
         #for k in range(d):
             Isel[k] = Rsq[Indecies[k]]
         #print("The neurons with the ten highest R^2 values = ...)
         #for k in range(96, neuron):
             #print("{0:2d} Highest 100 Rsq={1:f}".format(k,Rsq[Indecies[k]]))
         #Isel = Indecies[neuron-d:]
         Isel = Indecies[:d]
         for k in range(186, neuron):
            print("{0:2d} index with highest correlations {1:d}".format(k,Ind
         ecies[k]))
         print(Rsq[122])
```

```
index with highest correlations 13 index with highest correlations 19 index with highest correlations 174 index with highest correlations 48 index with highest correlations 82 index with highest correlations 82 index with highest correlations 41 index with highest correlations 34 index with highest correlations 24 index with highest correlations 171 index with highest correlations 171 index with highest correlations 130 100.0
```

Fit a model using only the d neurons selected in the previous step and print both the test RSS per sample and the normalized test RSS.

```
In [13]: XS = X0[:,Isel]
    #y_tr.shape

X_tr = XS[:7768,:]#spikes

#ytr is a product of np.array(something) then i use a np.random.choice
in the same range of samples as time

y_tr = y0[:7768]

X_ts = XS[7768:15536,:] #random test set

y_ts = y0[7768:15536]

reg = sklearn.linear_model.LinearRegression()
    #X_tr.shape
    #y_tr.shape
    reg.fit(X_tr,y_tr)

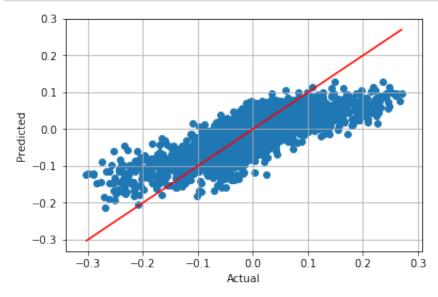
yts_pred = reg.predict(X_ts)

RSS_re_ts = np.mean((y_ts-yts_pred)**2)/(np.std(y_ts)**2)
print("Normalized test RSS = {0:f}".format(RSS_re_ts))
```

Normalized test RSS = 0.496102

Create a scatter plot of the predicted vs. actual hand motion on the test data. On the same plot, plot the line where yts hat = yts.

```
In [14]: ymin = np.min(y_ts)
    ymax = np.max(y_ts)
    plt.scatter(y_ts,yts_pred)
    plt.plot([ymin,ymax],[ymin,ymax],'r')
    plt.xlabel('Actual')
    plt.ylabel('Predicted')
    plt.grid()
```



Using K-fold cross validation for the optimal number of neurons

In the above, we fixed d=100. We can use cross validation to try to determine the best number of neurons to use. Try model orders with $d=10,20,\ldots,190$. For each value of d, use K-fold validation with 10 folds to estimate the test RSS. For a data set this size, each fold will take a few seconds to compute, so it may be useful to print the progress.

```
In [15]: # Create a k-fold cross validation object
         nfold = 10
         kf = sklearn.model selection.KFold(n splits=nfold,shuffle=True)
         # Model orders to be tested
         dtest = np.arange(10,200,10)
         nd = len(dtest)
         RSSts = np.zeros((nd,nfold))
         print(RSSts.shape)
         for i, d in enumerate(dtest):
             Isel d = Indecies[:d]
         ############### okay then we need to go from High RSS to low RSS(more
         accurate)###########
            X d = X0[:, Isel d]
            # print(X d.shape)
             for ifold, ind in enumerate(kf.split(X_d)):
                # Get the training data in the split
                Itr,Its = ind
                X \text{ tr } k = X \text{ d}[Itr,:]
                y tr k = y0[Itr]
                X_{ts_k} = X_d[Its,:]
                y ts k = y0[Its]
                #linear fit
                regd = sklearn.linear model.LinearRegression()
                reqd.fit(X tr k,y_tr_k)
                # Compute the prediction error on the test data
                y ts pred k = regd.predict(X ts k)
                RSSts[i,ifold] = np.mean((y ts k-y ts pred k)**2)
                #print(RSSts[i,ifold])
                #print(a)
         print(RSSts)
         (19, 10)
         016
            0.00188236 0.00190841 0.0019501
                                              0.00200372]
          [ 0.00181851 \ 0.00150285 \ 0.00180729 \ 0.00174197 \ 0.00177415 \ 0.001
         65565
            0.00194295 0.00174671
```

0.00174211

0.001713391

0.00166121 0.00166505 0.001

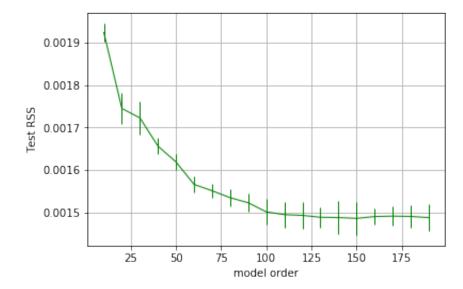
[0.00184177 0.00156146 0.00174427

| 81823 | | | | | |
|----------------------------|--------------------------|--------------------------|---------------------------|------------|-------|
| 0.00152341 | 0.00174115 | 0.00174781 | 0.00191888] | | |
| [0.00170323 | 0.00167819 | 0.00160337 | 0.00152952 | 0.00164699 | 0.001 |
| 61978 | | | | | |
| 0.00170973 | 0.0017102 | 0.00169622 | 0.00166184] | | |
| [0.00160432 | 0.00153974 | 0.00159406 | 0.00166341 | 0.00161284 | 0.001 |
| 74407 | | | | | |
| 0.00166364 | 0.00156219 | 0.00159218 | 0.00160957] | | |
| [0.0015602 | 0.00151301 | 0.00162653 | 0.00165851 | 0.00161355 | 0.001 |
| 53451 | | | | | |
| 0.00147122 | 0.00155615 | 0.00159796 | 0.00152553] | | |
| [0.00161859 | 0.00160553 | 0.00158628 | 0.00153694 | 0.0015526 | 0.001 |
| 47522 | | | | | |
| 0.001599 | 0.00149749 | 0.00152223 | 0.00151405] | | |
| [0.00143568 | 0.001599 | 0.0016082 | 0.00159808 | 0.00151819 | 0.001 |
| 56634 | | | | | |
| 0.00154199 | 0.00147545 | 0.00144041 | 0.0015614] | | |
| [0.00152143 | 0.00156836 | 0.0015163 | 0.00158606 | 0.00149052 | 0.001 |
| 57234 | | | | | |
| 0.00137592 | 0.00145804 | 0.00154095 | 0.00159581] | | |
| [0.00157843 | 0.00152698 | 0.00135927 | 0.00140451 | 0.00161589 | 0.001 |
| 55928 | | | | | |
| 0.00141174 | 0.00161509 | 0.00141015 | 0.00153024] | | |
| [0.00152043 | 0.00154753 | 0.00165524 | 0.0014226 | 0.00133273 | 0.001 |
| 5101 | | | | | |
| 0.00150537 | 0.00151892 | 0.00156187 | 0.00137167] | | |
| [0.00163753 | 0.00151859 | 0.00143113 | 0.00143511 | 0.00160018 | 0.001 |
| 59079 | | | | | |
| 0.00136518 | 0.00140504 | 0.00155646 | 0.00138889] | | |
| [0.00141685 | 0.00156237 | 0.00138891 | 0.00157003 | 0.00159697 | 0.001 |
| 51728 | | | | | |
| 0.00144979 | 0.00148237 | 0.00149834 | 0.00140135] | | |
| [0.00152335 | 0.00159732 | 0.00133782 | 0.0015604 | 0.00161291 | 0.001 |
| 48737 | 0 00107001 | 0 00163030 | 0 001433011 | | |
| | 0.00127091 | 0.00163928 | 0.00143291] | 0.00106501 | 0 001 |
| [0.00151061 | 0.00171378 | 0.00144116 | 0.00141336 | 0.00126501 | 0.001 |
| 46199 | 0 00140467 | 0 00151004 | 0 001501311 | | |
| 0.00163524 | 0.00140467 | 0.00151084 | 0.00150131] | 0 00142072 | 0 001 |
| [0.0014926 | 0.00150025 | 0.00138791 | 0.00148594 | 0.00143973 | 0.001 |
| 53854 | 0 00140712 | 0 00155526 | 0 001421041 | | |
| 0.00158165 [0.00140293 | 0.00148712 0.00145939 | 0.00155536 0.00150656 | 0.00143184] 0.00139797 | 0.0015645 | 0.001 |
| • | 0.00145939 | 0.00130636 | 0.00139797 | 0.0013643 | 0.001 |
| 55356 0.00148464 | 0.00162211 | 0.00148678 | 0.00143259] | | |
| [0.0015004 | 0.00162211 | 0.00148678 | 0.00143239 | 0.00147131 | 0.001 |
| 38487 | 0.00103090 | 0.00144004 | 0.00143047 | 0.0014/131 | 0.001 |
| 0.00151449 | 0.00156444 | 0.00140155 | 0.00153966] | | |
| [0.00136409 | 0.00136444 | 0.00140133 | 0.00153966 | 0.00157393 | 0.001 |
| 47918 | 0.0014130 | 0.00130303 | 0.00130024 | 0.0013/333 | 0.001 |
| 0.00144906 | 0.00166261 | 0.00134329 | 0.00151628] | 1 | |
| 0.00144000 | 0.00100201 | 0.00134327 | 0.00131020] | ı | |

Compute the RSS test mean and standard error and plot them as a function of the model order d using the plt.errorbar() method.

```
In [16]: # Compute the mean and standard deviation over the different folds.
    RSS_mean = np.mean(RSSts,axis=1)
    RSS_std = np.std(RSSts,axis=1) / np.sqrt(nfold-1)

# Plot the mean test RSS and test RSS standard error
    plt.errorbar(dtest,RSS_mean,fmt='g-',yerr=RSS_std,linewidth=1)
# the x axis should be log of alphas
    plt.grid()
    plt.xlabel('model order')
    plt.ylabel('Test RSS')
    plt.show()
```



Find the optimal order using the one standard error rule. Print the optimal value of d and the mean test RSS per sample at the optimal d.

```
In [17]: # Find the minimum RSS target
   imin = np.argmin(RSS_mean)
   RSS_tgt = RSS_mean[imin] + RSS_std[imin]

# Find the lowest model order below the target
   I = np.where(RSS_mean <= RSS_tgt)[0]
   print(I)
   iopt = I[0]
   dopt = dtest[iopt]

   print("the optimal value of d is",dopt)
   print(RSS_mean[I[0]])

[ 8    9    10    11    12    13    14    15    16    17    18]
   the optimal value of d is 90
   0.00152257348773</pre>
```

Using LASSO regression

Instead of using the above heuristic to select the variables, we can use LASSO regression.

warnings.warn(msg, _DataConversionWarning)

First use the preprocessing.scale method to standardize the data matrix x0. Store the standardized values in xs. You do not need to standardize the response. For this data, the scale routine may throw a warning that you are converting data types. That is fine.

```
In [18]: from sklearn import preprocessing

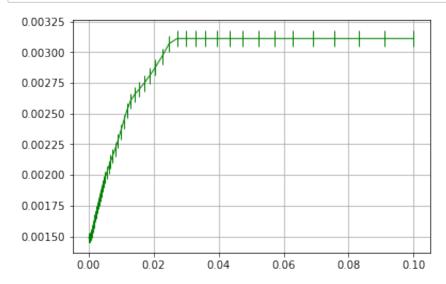
Xs = sklearn.preprocessing.scale(X0)
#y = sklearn.preprocessing.scale(y0)

/anaconda/lib/python3.6/site-packages/sklearn/utils/validation.py:42
9: DataConversionWarning: Data with input dtype uint8 was converted to float64 by the scale function.
```

Now, use the LASSO method to fit a model. Use cross validation to select the regularization level alpha. Use alpha values logarithmically spaced from 1e-5 to 0.1, and use 10 fold cross validation.

```
In [19]: # Create a k-fold cross validation object
         nfold = 10
         kf = sklearn.model selection.KFold(n splits=nfold,shuffle=True)
         # Create the LASSO model. We use the `warm start` parameter so that t
         he fit will start at the previous value.
         # This speeds up the fitting.
         model = sklearn.linear_model.Lasso(warm_start=True)
         # Regularization values to test
         nalpha = 100
         alphas = np.logspace(-5, -1, nalpha)
         # MSE for each alpha and fold value
         mse = np.zeros((nalpha,nfold))
         for ifold, ind in enumerate(kf.split(Xs)):
             # Get the training data in the split
             Itr,Its = ind
             X_tr_1 = Xs[Itr,:]
             y_tr_1 = y0[Itr]
             X \text{ ts } l = Xs[Its,:]
             y ts 1 = y0[Its]
             # Compute the lasso path for the split
             for ia, a in enumerate(alphas):
                 # Fit the model on the training data
                 model.alpha = a
                 model.fit(X tr l,y tr l)
                 # Compute the prediction error on the test data
                 y ts pred l = model.predict(X ts l)
                 mse[ia,ifold] = np.mean((y_ts_pred_l-y_ts_l)**2)
```

Plot the mean test RSS and test RSS standard error with the plt.errorbar plot.



Find the optimal alpha and mean test RSS using the one standard error rule.

```
In [21]: #We find the optimal alpha, by the following steps:
    #Find the alpha with the minimum test MSE
    #Set mse_tgt = minimum MSE + 1 std dev MSE
    #Find the least complex model (highest alpha) such that MSE < mse_tgt

# Find the minimum MSE and MSE target
    imin = np.argmin(mse_mean)
    mse_tgt = mse_mean[imin] + mse_std[imin]
    alpha_min = alphas[imin]

# Find the least complex model with mse_mean < mse_tgt
    I = np.where(mse_mean < mse_tgt)[0]
    #print(I)
    iopt = I[-1]
    alpha_opt = alphas[iopt]
    print("Optimal alpha = %f" % alpha_opt)</pre>
```

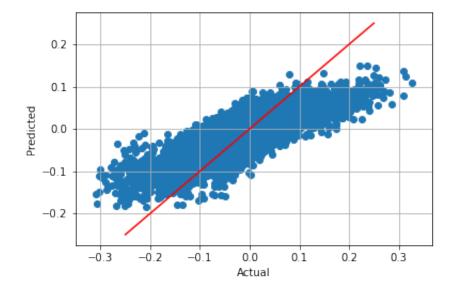
Optimal alpha = 0.000722

Using the optimal alpha, recompute the predicted response variable on the whole data. Plot the predicted vs. actual values.

```
In [22]: model.alpha = alpha_opt
    model.fit(Xs,y0)

    yts_pred_a = model.predict(Xs)

    plt.scatter(y0,yts_pred_a)
    plt.plot([-0.25,0.25],[-0.25,0.25],'r-')
    plt.xlabel('Actual')
    plt.ylabel('Predicted')
    plt.grid()
```



More Fun

You can play around with this and many other neural data sets. Two things that one can do to further improve the quality of fit are:

- Use more time lags in the data. Instead of predicting the hand motion from the spikes in the previous time, use the spikes in the last few delays.
- Add a nonlinearity. You should see that the predicted hand motion differs from the actual for high values of the actual. You can improve the fit by adding a nonlinearity on the output. A polynomial fit would work well here.

You do not need to do these, but you can try them if you like.

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| T11 []• | J • | |
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