

Some Motivation (I)

- Laplace (Bilateral/Unilateral) transform
 - Efficient solution of differential equation

- Bilateral \mathcal{L} -Transform

- Zero-state response, general inputs

- Unilateral \mathcal{Z} -Transform

- Total response for $t \geq 0$ when init. cond. known

$$e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t} [\cos(\omega t) + j \sin(\omega t)]$$



- Frequency dependence hard to visualize, since ω not an isolated variable

- Phasor analysis was better at showing frequency dependence

- But, limited to sinusoidal excitation (input)

(Continued)

Some Motivation (2)

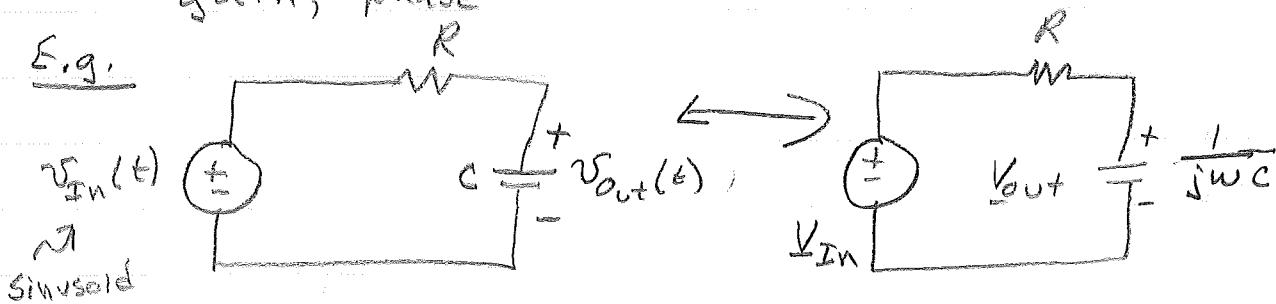
- o Conceptual considerations / methods:

- a) Extend phasor concept

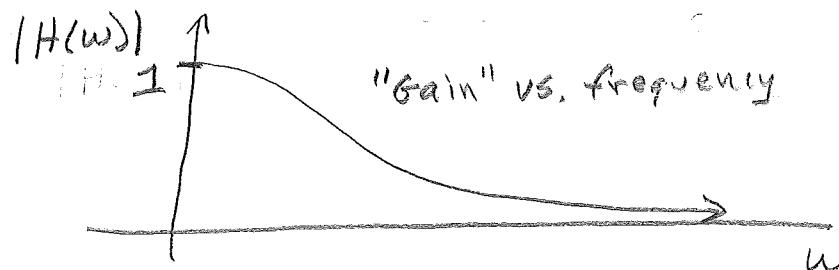
- o Write excitation (input) as sum of sinusoids of different frequencies (often, continuous range!)
- o Since sum, LTI system output is sum of outputs

- o Can study system response as function of ω
 - o LTI system, at each frequency, change gain, phase

E.g.



$$\frac{V_{out}}{V_{in}} = \frac{1}{R + \frac{1}{j\omega C}} \quad \text{or} \quad \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC} \equiv H(\omega)$$



- o Phasors \Rightarrow steady state response of sinusoid

(Continued)

Some Motivation

or

b) "Reduce" Laplace Transform coverage

- $s = \sigma + j\omega$



- Get isolated frequency dependence if $\sigma \rightarrow 0$

- Consequence:

- Only produce steady state analysis

⇒ Formalism ≡ Fourier Transform

If ROC of Laplace transform includes
jw axis (i.e., includes $\sigma = 0$ for all w),

Then

$$\text{Fourier Transform} = X(s) \Big|_{s=jw} = \mathcal{F}[x(t)]$$

If Laplace ROC not include jw axis

Then either a) must compute $\mathcal{F}[x(t)]$ by definition
or

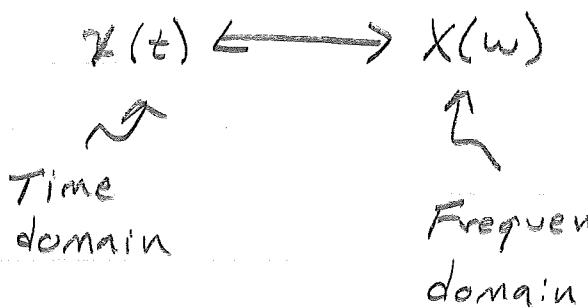
b) $\mathcal{F}[x(t)]$ not exist!

Definition of Fourier Transform

$$X(w) = \int_{t=-\infty}^{\infty} x(t) e^{-jw t} dt$$

- Notation:

$$X(w) = \mathcal{F}[x(t)]$$



Note: $X(w)$ often written as $X(jw)$

- Same as $X(s)$ / IF L-Transform ROC includes jw axis

- Inverse transform:

$$x(t) = \frac{1}{2\pi} \int_{w=-\infty}^{\infty} X(w) e^{jwt} dw$$

- Ordinary integral.

- Invert via integral or transform table

\mathcal{F} -Transform of $\delta(t)$

- Recall: $\mathcal{L}\{\delta(t)\} = 1$, All s.

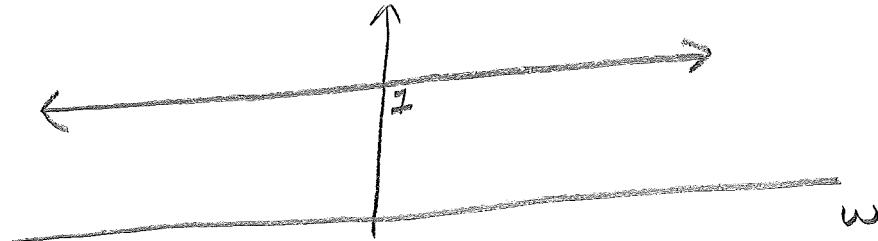
ROC includes $j\omega$ -axis

$$\Rightarrow \mathcal{F}\{\delta(t)\} = \mathcal{L}\{\delta(t)\} \Big|_{s=jk\omega} = 1$$

Formally:

$$\begin{aligned} \mathcal{F}\{\delta(t)\} &= \int_{t=-\infty}^{\infty} \delta(t) e^{-j\omega t} dt \\ &= \underbrace{\int_{t=-\infty}^{0^-} \phi \cdot e^{-j\omega t} dt}_{\phi} + \underbrace{\int_{t=0^+}^{0^+} (\delta(t)) e^{-j\omega \cdot 0} dt}_{\equiv 1} + \underbrace{\int_{t=0^+}^{\infty} \phi \cdot e^{-j\omega t} dt}_{\phi} \end{aligned}$$

So, $\mathcal{F}\{\delta(t)\} = 1$



Impulse: Equal sum of all frequencies

\mathcal{F} -Transform of $e^{-\lambda t} u(t)$, $\lambda > 0$

- Recall: $\mathcal{L}\{e^{-\lambda t} u(t), \lambda > 0\} = \frac{1}{s + \lambda}$, $\text{Re}(s) > -\lambda$

- Roc includes jw-axis (for $\lambda > 0$)

$$\Rightarrow \mathcal{F}\{e^{-\lambda t} u(t)\} = \frac{1}{jw + \lambda}, \lambda > 0$$

- By definition:

$$\mathcal{F}\{e^{-\lambda t} u(t)\} = \int_{t=-\infty}^{\infty} e^{-\lambda t} u(t) e^{-jw t} dt$$

$$= \int_0^{\infty} e^{-(jw + \lambda)t} dt = \frac{-e^{-(jw + \lambda)t}}{jw + \lambda} \Big|_{t=0}^{\infty} = \frac{-e^{-jw\infty - \lambda\infty} - e^{-jw0 - \lambda 0}}{jw + \lambda}$$

Note: $|e^{-jw\infty}| = 1$, $e^{-\lambda\infty} = 0$ for $\lambda > 0$, $e^{-jw0} = 1$

$$\Rightarrow \mathcal{F}\{e^{-\lambda t} u(t)\} = \frac{1}{jw + \lambda}, \lambda > 0$$

\mathcal{F} -Transform of Step (1)

• Recall: $\mathcal{L}\{u(t)\} = \frac{1}{s}$, $\text{Re}(s) > 0$

• ROC not include jw -axis

→ Must use definition, if $\mathcal{F}\{f\}$ exists!

• By definition:

$$\begin{aligned}\mathcal{F}\{u(t)\} &= \int_{t=-\infty}^{\infty} u(t) e^{-j\omega t} dt = \int_{t=0}^{\infty} e^{-j\omega t} dt \\ &= \left[-\frac{e^{-j\omega t}}{j\omega} \right]_{t=0}^{\infty} = \frac{-e^{-j\omega 0} + e^{j\omega 0}}{j\omega} = \frac{[-\cos(\omega 0) - j \sin(\omega 0)] + 1}{j\omega}\end{aligned}$$

• 'cos()', 'sin()' terms indeterminate

• Direct approach not usable

• Several functions require an indirect approach

(Continued)

F_t-Transform of Step (2)

- Let $u(\epsilon) = \lim_{\lambda \rightarrow 0} e^{-\lambda t} u(t)$

- Then,

$$\mathcal{F}\{u(t)\} = \lim_{\lambda \rightarrow 0} \mathcal{F}\{e^{-\lambda t} u(t)\} = \lim_{\lambda \rightarrow 0} \frac{1}{jw + \lambda}$$

- How do we take this limit?

- Write in real, imag parts.

$$\mathcal{F}\{u(t)\} = \lim_{\lambda \rightarrow 0} \left[\frac{\lambda}{w^2 + \lambda^2} - j \frac{w}{w^2 + \lambda^2} \right]$$

$$= \lim_{\lambda \rightarrow 0} \left[\frac{\lambda}{w^2 + \lambda^2} \right] + \underbrace{\lim_{\lambda \rightarrow 0} \left[\frac{-jw}{w^2 + \lambda^2} \right]}$$

$$2) \lim_{\lambda \rightarrow 0} \left[\frac{-jw}{w^2 + \lambda^2} \right] = \frac{-jw}{w^2} = \frac{-j}{w} = \frac{1}{jw}$$

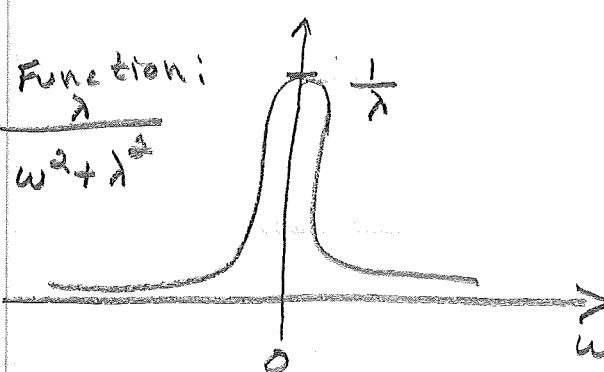
- Limit for leftmost part:

Note: $\int_{-\infty}^{\infty} \frac{d}{w^2 + \lambda^2} dw = \tan^{-1} \frac{w}{\lambda} \Big|_{-\infty}^{\infty} = \pi$

\Rightarrow Area under curve equals π , regardless of λ .

(Continued)

F-Transform of Step (3)



- As $\lambda \rightarrow 0$

- Function concentrates at ϕ

- Height $\rightarrow \infty$

- Value at $w \neq 0 \rightarrow 0$

- Area remains π

∴

Impulse of area π at $w=0$

Thus,

$$\boxed{F\{u(t)\} = \pi S(\omega) + \frac{1}{j\omega}}$$

Existence of Fourier Transform

• Must exist if $x(t)$:

1) ... Absolutely Integrable:

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

2) ... finite number of finite discontinuities
on any finite interval

3) ... finite number of maxima, minima on
any finite interval

• Can still exist otherwise

Note 1: Conditions 2,3 satisfied by physical
signals

Note 2: Condition 1 violated by periodic signals

But, periodic signals generally do
have a $\mathcal{F}\{.\}$ -Transform, but do not
generally have a \mathcal{L} -Transform.

Fourier Transform of Two-Sided Cosine (I)

- Does not have a Laplace Transform }
 ◦ Direct approach is indeterminate }

- Consider:

If $X(w) = \mathcal{S}(w - w_0)$, find $x(t)$!

◦ By definition: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{S}(w - w_0) e^{jw_0 t} dw$

$$= \frac{1}{2\pi} \int_{w=w_0^-}^{w_0^+} \mathcal{S}(w - w_0) e^{jw_0 t} dw = \frac{e^{jw_0 t}}{2\pi}$$

Thus, $\frac{e^{jw_0 t}}{2\pi} \longleftrightarrow \mathcal{S}(w - w_0)$ or $e^{jw_0 t} \longleftrightarrow 2\pi \mathcal{S}(w - w_0)$

◦ Similarly: $e^{-jw_0 t} \longleftrightarrow 2\pi \mathcal{S}(w + w_0)$

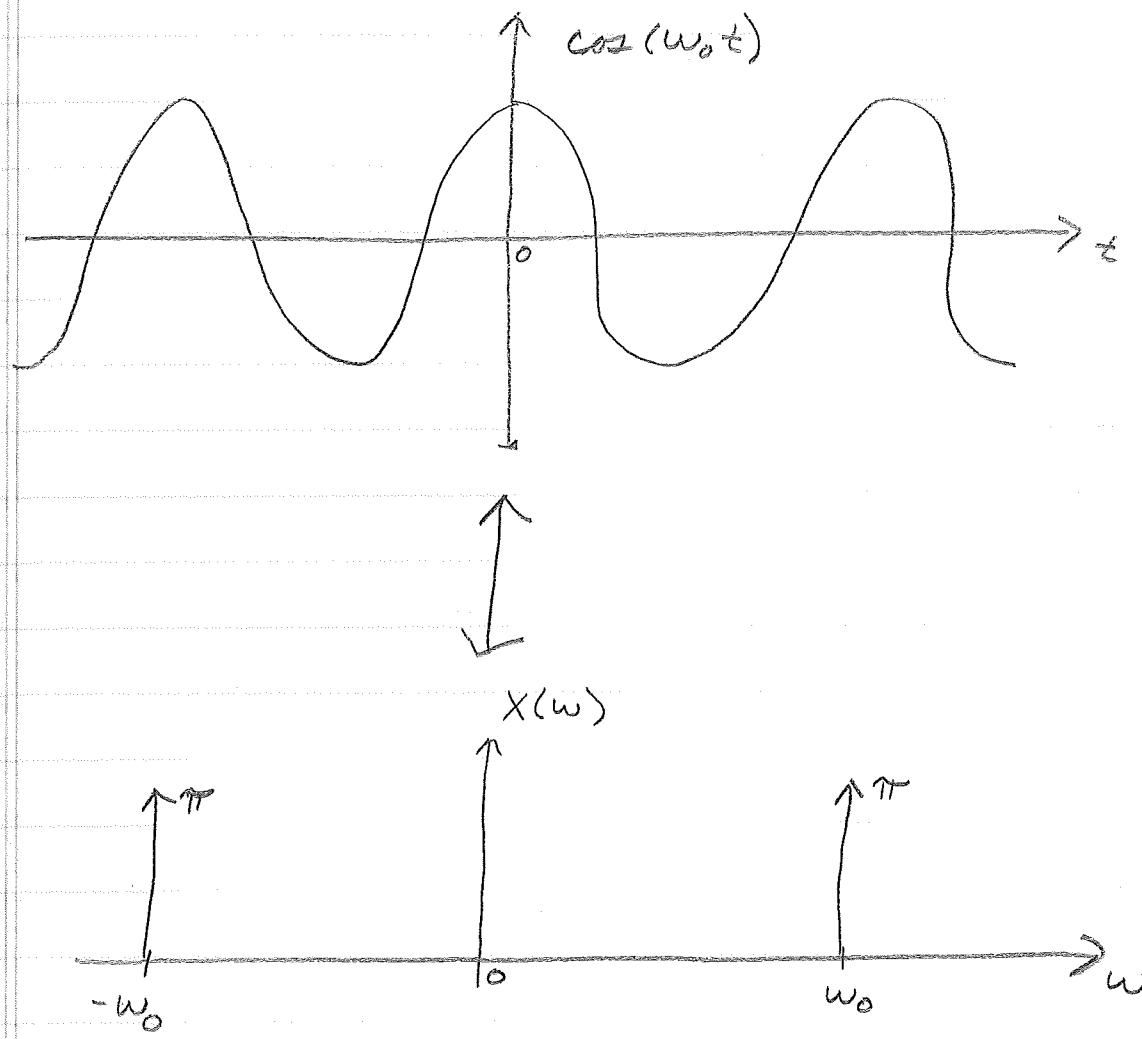
◦ Now, write cosine as: $\cos(w_0 t) = \frac{e^{jw_0 t} + e^{-jw_0 t}}{2}$

From above:

$$\boxed{\cos(w_0 t) \longleftrightarrow \pi [\mathcal{S}(w + w_0) + \mathcal{S}(w - w_0)]}$$

(Continued)

Fourier Transform of Two-Sided Cosine (2)



Sum two complex exponentials to form a cosine

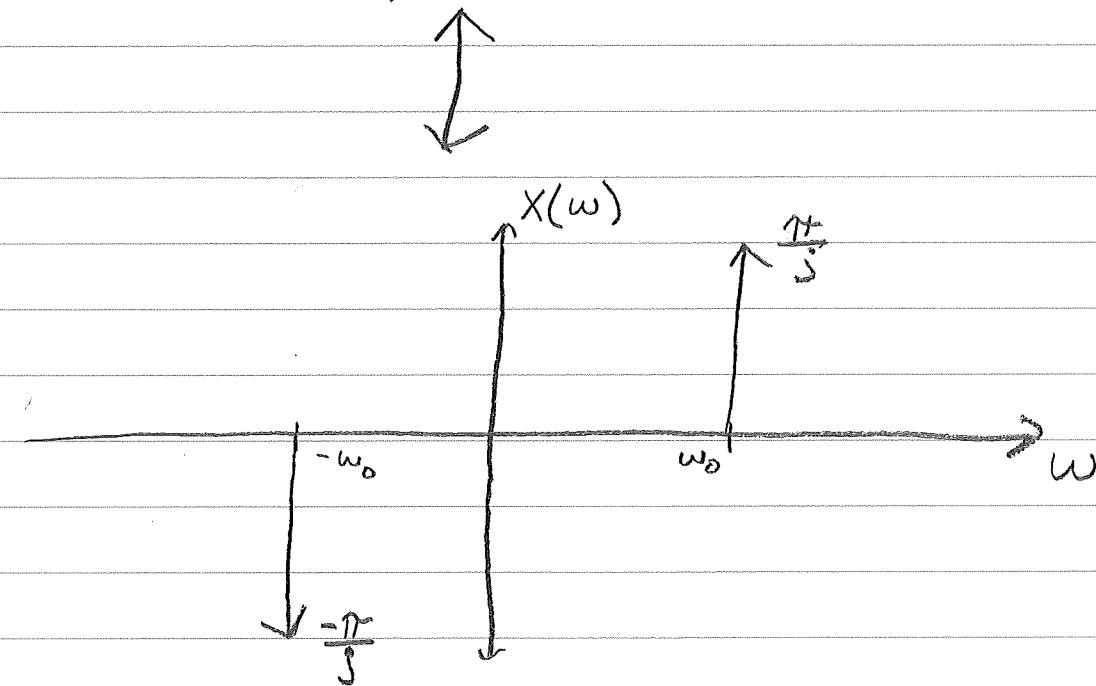
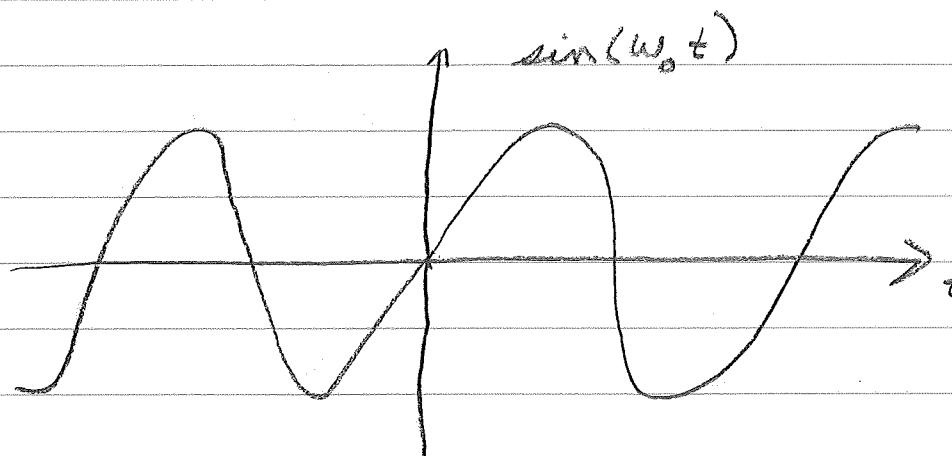
Pure frequency \Rightarrow Information at
in time one frequency

Fourier Transform

Fourier Transform of Two-Sided Sine

Similarly,

$$\sin(\omega_0 t) \xrightarrow{\mathcal{F}} \frac{\pi}{j} [\delta(w - \omega_0) - \delta(w + \omega_0)]$$



Fourier Transform of Gate Function

Unit gate/pulse function: $g(t) = \begin{cases} 1, & |t| < \pi/2 \\ 0, & |t| > \pi/2 \end{cases}$

* Formally, $g(1t) = \pi/2 = 1/2$

$\tau \rightarrow$ Pulse width

By definition: $G(w) = \int_{-\infty}^{\infty} g(t) e^{-jw t} dt$

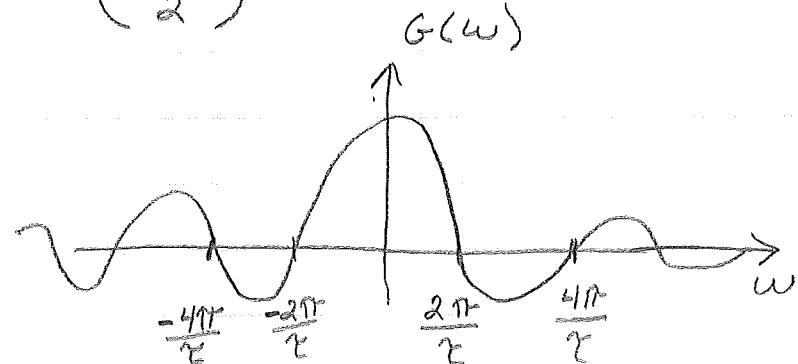
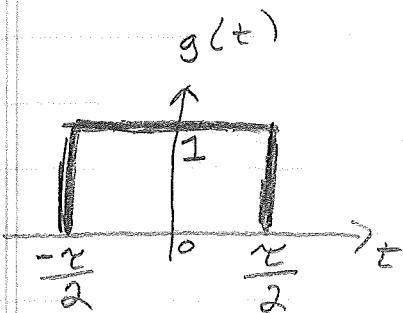
$$= \int_{-\pi/2}^{\pi/2} e^{-jw t} dt = \frac{-e^{-jw \frac{\pi}{2}} + e^{jw \frac{\pi}{2}}}{jw} = \frac{\sin(\frac{w\pi}{2})}{(\frac{w}{2})}$$

Define "sin(x) over x": $\text{sinc}(x) = \frac{\sin(x)}{x}$

Then,

* $\text{sinc}(0) = 1$

$$G(w) = \tau \cdot \text{sinc}\left(\frac{w\tau}{2}\right)$$



Note: As τ decreases, gate width of $g(t)$ shrinks while main lobe of $G(w)$ widens, and vice versa

Common Fourier Transforms

TABLE 7.1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14	$\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15	$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16	$e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi} \operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} \operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi} \operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

{Latihis}

Table of Fourier Transform Properties.

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega_0 t} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega - \theta)) d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt} x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^t x(t) dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\omega} X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re\{X(j\omega)\} = \Re\{X(-j\omega)\} \\ \Im\{X(j\omega)\} = -\Im\{X(-j\omega)\} \\ X(j\omega) = X(-j\omega) \\ \angle X(j\omega) = -\angle X(-j\omega) \end{cases}$ $X(j\omega)$ real and even
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \Re\{x(t)\}$ [$x(t)$ real] $x_o(t) = \Im\{x(t)\}$ [$x(t)$ real]	$\Re\{X(j\omega)\}$ $j\Im\{X(j\omega)\}$
4.3.7	Parseval's Relation for Aperiodic Signals	$\int_{-\infty}^{+\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) ^2 d\omega$	

[Oppenheim, Willsky, Nawab. Signal and Systems 2/e, Prentice Hall]

Real-Valued Signals

If $x(t)$ real-valued can show:

$$x(\omega) = x^*(-\omega)$$

✓

$$\text{write } x(\omega) = \sigma(\omega) + j b(\omega)$$

$$\Rightarrow \text{Re}\{x(\omega)\} = \text{Re}\{x(-\omega)\}$$

$$\text{Im}\{x(\omega)\} = -\text{Im}\{x(-\omega)\}$$

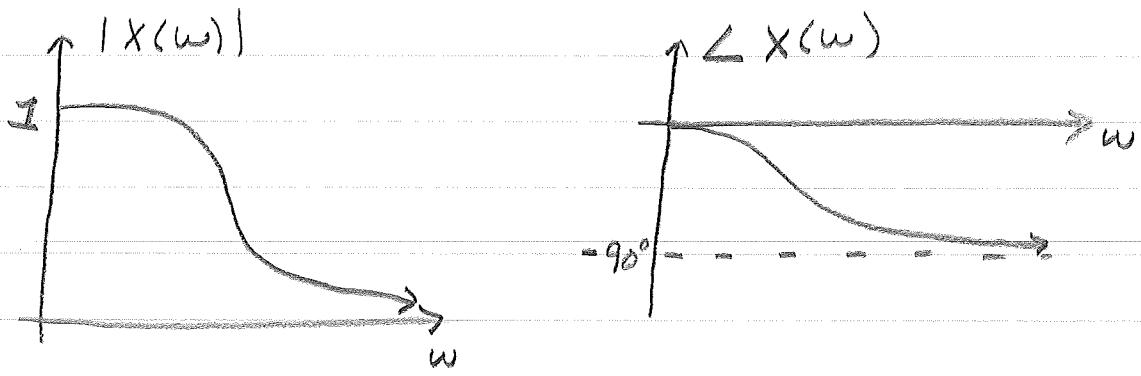
and

$$\Rightarrow |x(\omega)| = |x(-\omega)|$$

$$\angle x(\omega) = -\angle x(-\omega)$$

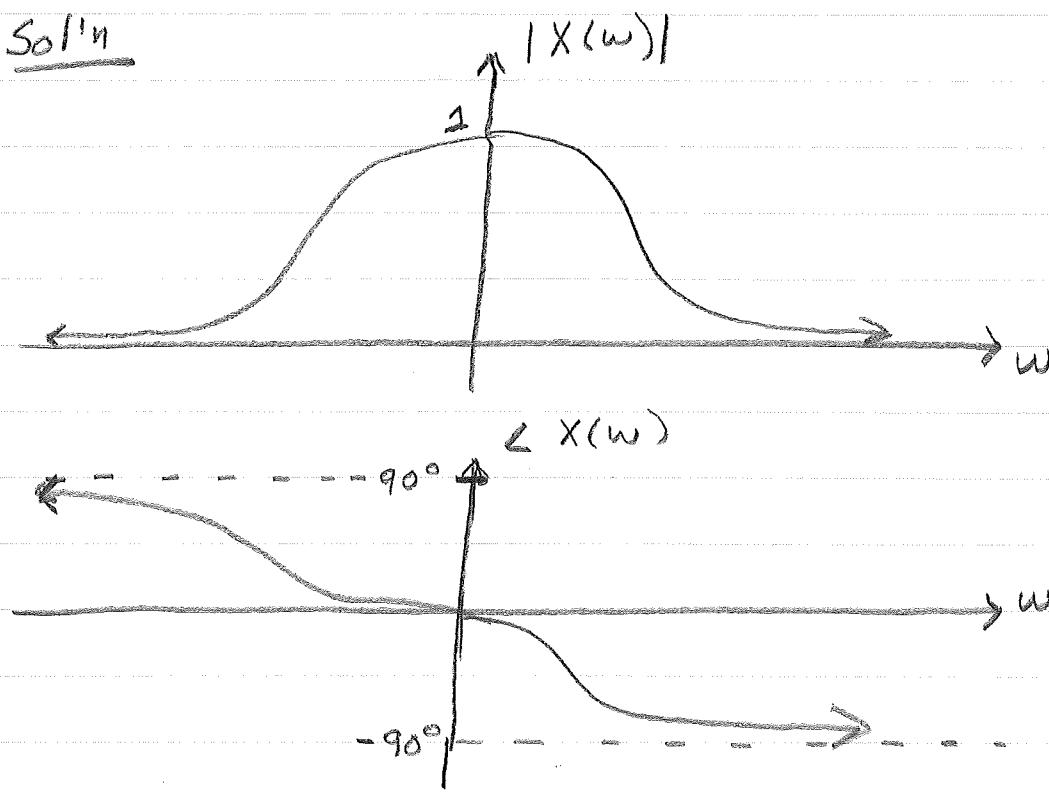
Example: Real-Valued Signals

If $x(t)$ real-valued and $X(\omega_0)$ is:



then draw $|X(w)|$, $\angle X(w)$ over all w .

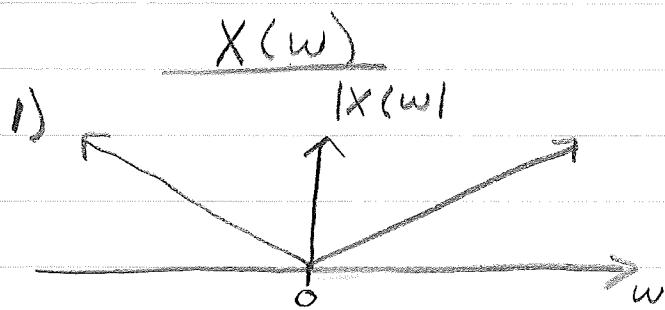
Sol'n



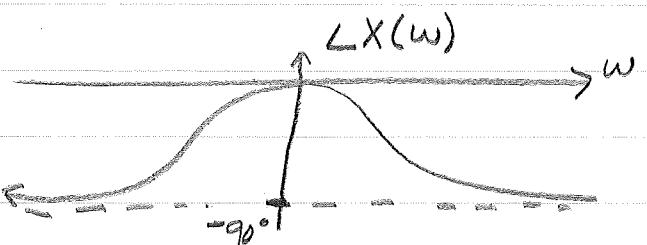
Often, only show $X(\omega \geq 0)$ for real-valued signals.
 → Negative frequencies = redundant information

Identifying Real- vs. Complex- Valued Signals

$x(t)$: Real- or
complex-Valued?

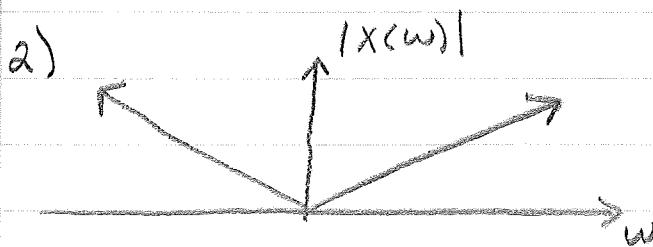


Complex

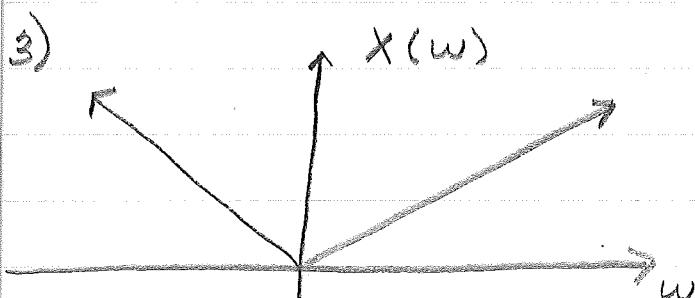
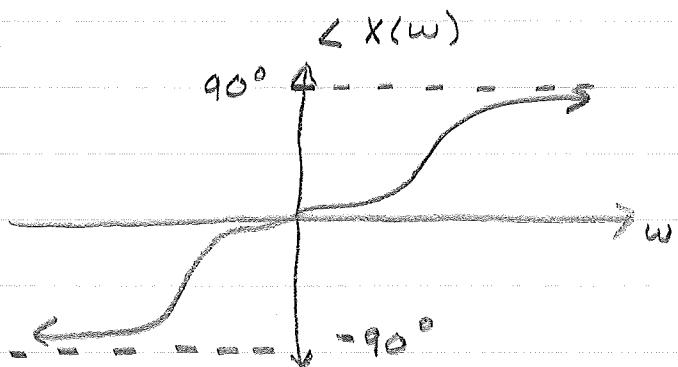


$\angle X(w)$

$$\neq -\angle X(-w)$$



Real

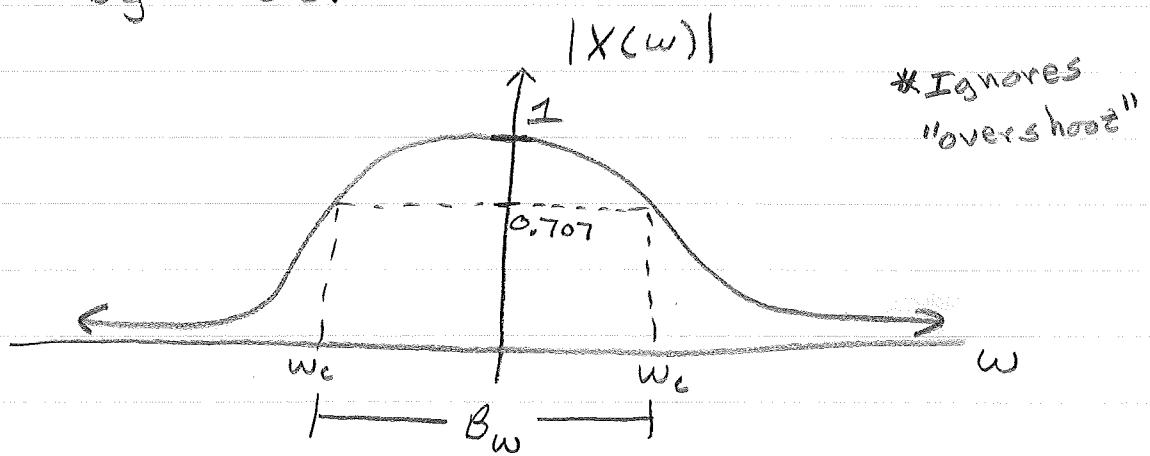


Real

$$[\angle X(w) = 0^\circ]$$

Definition(s) of Bandwidth

- Frequency span of signals that are "passed" through a system (or utilized by)
 - Expressed in radians/sec or Hertz
 - Several different, but similar, mathematical definitions
- Narrow band: Passes / utilizes relatively limited range of frequencies
- Broad band: Passes / utilizes relatively large range of frequencies
- 3 dB bandwidth of filter: Band extending from maximum gain* plus and minus frequency location where gain reduced by 3 dB.

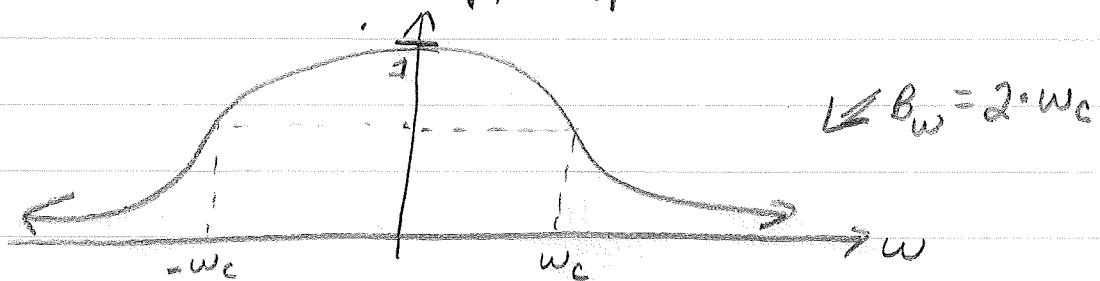


$$3 \text{ dB down} \Rightarrow \left(\frac{\sqrt{2}}{2}\right) \cdot \text{Max} = (0.707) \cdot \text{Max}$$

Fourier Transform

Bandwidth Example

- Find 3dB bandwidth of low pass filter with frequency response $H(\omega) = \frac{1}{1+j\omega}$

Sol'n

- Max value at $\omega=0$ is: $H(0)=1$

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2}} = \frac{1}{\sqrt{1+\omega_c^2}}$$

$$\Rightarrow |H(\omega)|^2 = \frac{1}{1+\omega^2} \quad \text{Set equal to } \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$$

So,

$$\frac{1}{2} = \frac{1}{1+\omega_c^2} \Rightarrow 2 - 1 = \omega_c^2$$

$$\Rightarrow \omega_c = \pm \sqrt{1} = \pm \frac{1}{\sqrt{2}} \text{ rad} = \omega_c$$

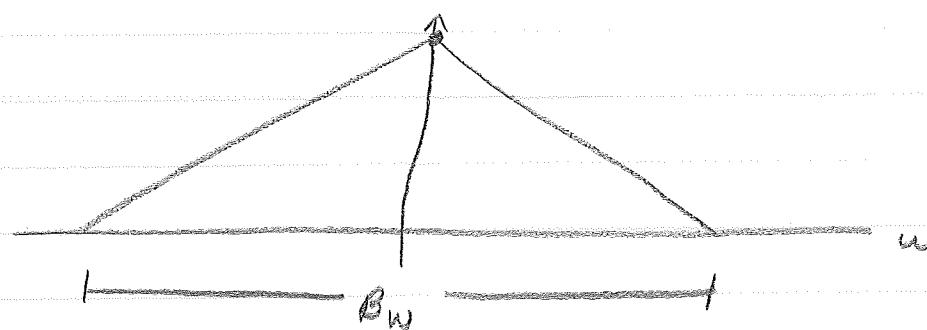
Thus,

$$\boxed{B_\omega = 2 \frac{\text{rad}}{\text{s}} = \frac{1}{\pi} \text{ Hz}}$$

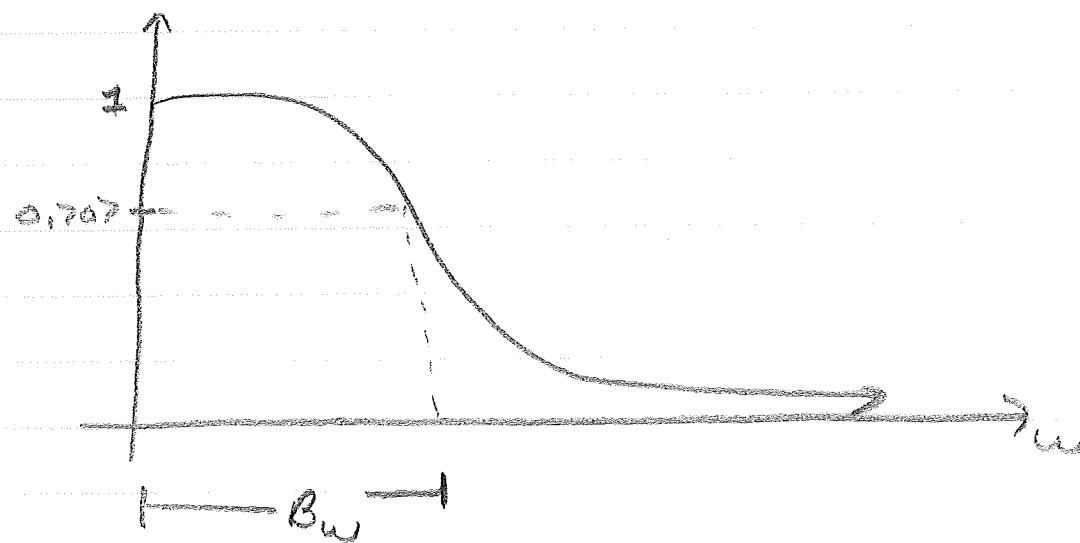
Other Bandwidth Definitions

- Range of frequencies over which the signal power is non-zero (or, greater than noise)

$$|H(\omega)|$$



- For real-valued signals, bandwidth sometimes refers only to positive-valued frequencies.



Fourier Transform

Fourier Transform and Impedance

R :

$$v_R(t) = i_R(t) \cdot R$$

$$V_R(w) = I_R(w) \cdot R \quad \text{or} \quad \frac{V_R(w)}{I_R(w)} = R$$

L :

$$v_L(t) = L \frac{di_L(t)}{dt}$$

$$V_L(w) = L \cdot jw I_L(w) \quad \text{or} \quad \frac{V_L(w)}{I_L(w)} = jwL$$

C :

$$i_C(t) = C \frac{dv_C(t)}{dt}$$

$$I_C(w) = C \cdot jw V_C(w) \quad \text{or} \quad \frac{V_C(w)}{I_C(w)} = \frac{1}{jwC}$$

Standard Impedances

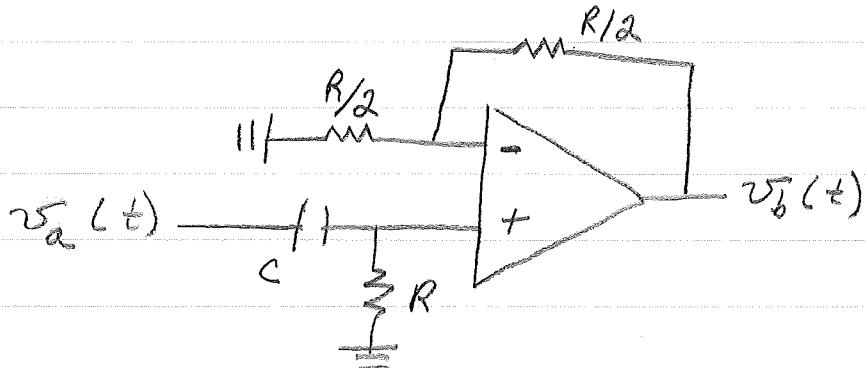
Inputs must have a

Fourier Transform

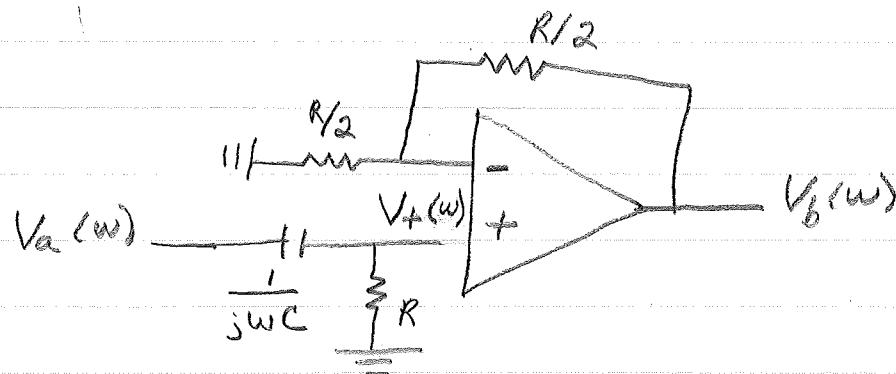
Fourier Transform

Filter Example (1)

Find and draw the transfer function of the filter shown below for $R = 1000 \Omega$, $C = 1 \mu F$.



Sol'n



$$(1): [V_+ (w) - V_a (w)] \frac{1}{jwC} + \frac{V_+ (w)}{R} = 0$$

$$\text{or } V_+ (w) [jwRC + 1] = V_a (w) [jwRC]$$

$$(2): V_+ (w) \cdot \frac{2}{R} + [V_+ (w) - V_b (w)] \cdot \frac{2}{R} = 0$$

or

$$V_+ (w) = \frac{V_b (w)}{2}$$

(Continued)

Filter Example (2)

(Continued)

Substituting (2) into (1) :

$$V_b(w) \{ jwRC + 1 \} = V_a(w) \{ j\omega wRC \}$$

Thus,

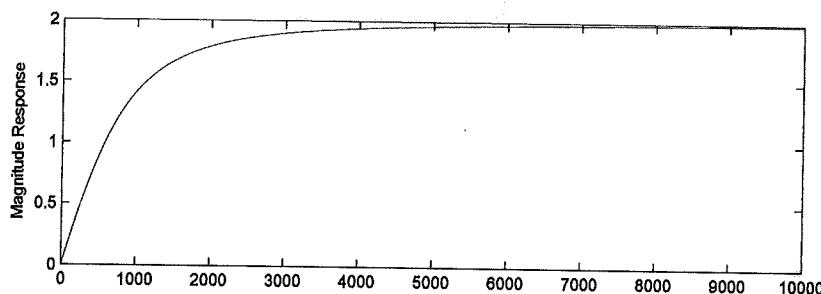
$$H(w) = \frac{V_b(w)}{V_a(w)} = \frac{j\omega wRC}{jwRC + 1}$$

Note: $H(0) = 0$, $H(\infty) = 2$

$$|H(w)| = \frac{|j\omega wRC|}{|jwRC + 1|} = \frac{\omega wRC}{\sqrt{1 + w^2 R^2 C^2}}$$

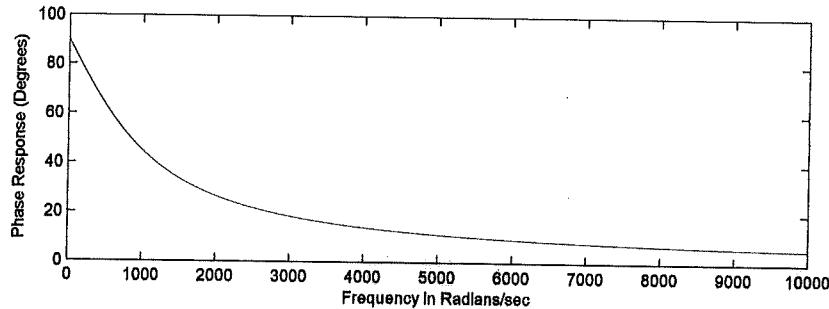
$$\angle H(w) = \angle(j\omega wRC) - \angle(jwRC + 1) = \frac{\pi}{2} - \tan^{-1}(wRC)$$

For $R = 1000 \Omega$, $C = 1 \mu F$:



- High-pass filter

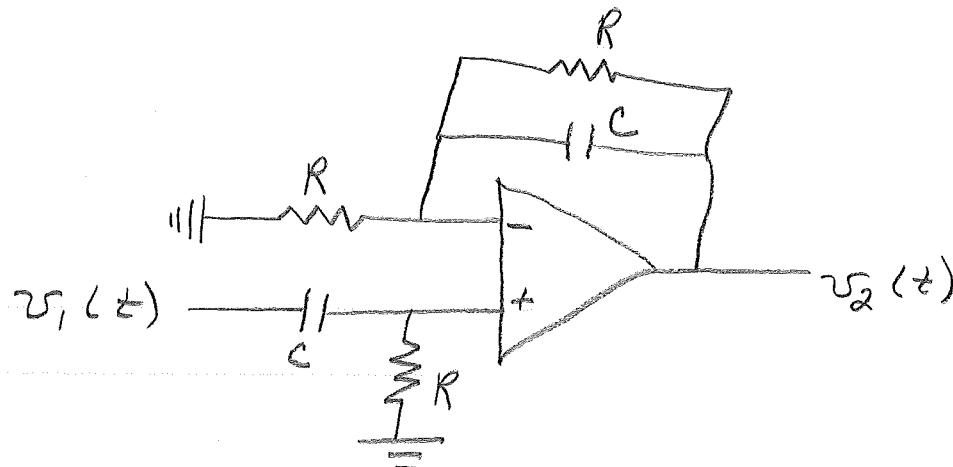
$$\omega_c \approx 1000 \frac{\text{rad}}{\text{sec}}$$



Another Active Filter Example (1)

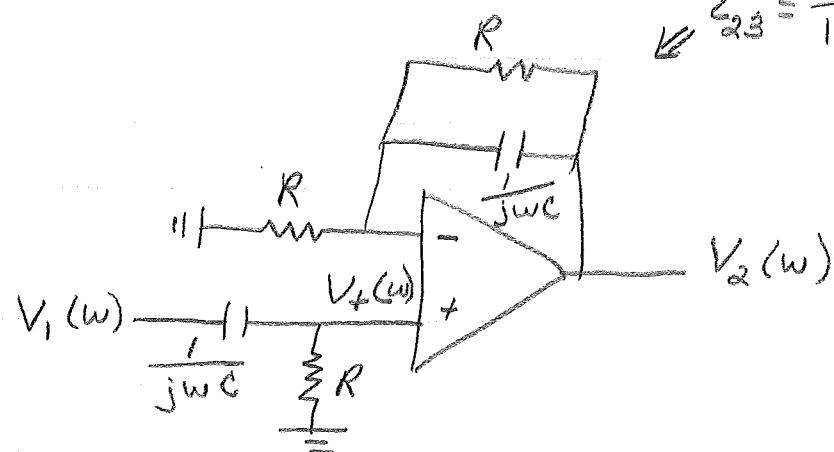
* Find and draw transfer function when

$$R = 1000 \Omega, C = 1 \mu F.$$



Sol'n

$$\Rightarrow Z_{23} = \frac{R}{1 + j\omega RC}$$



$$(1) \stackrel{!}{=} [V_+(w) - V_1(w)] j\omega C + \frac{V_+(w)}{R} = 0$$

$$V_+(w) = V_1(w) \cdot \frac{j\omega RC}{1 + j\omega RC}$$

$$(2) \stackrel{!}{=} \frac{V_+(w)}{R} + [V_+(w) - V_2(w)] \cdot \frac{1 + j\omega RC}{R} = 0$$

$$V_+(w) [2 + j\omega RC] = V_2(w) [1 + j\omega RC]$$

Fourier Transform

(continued)

Another Active Filter Example (2)

(Continued)

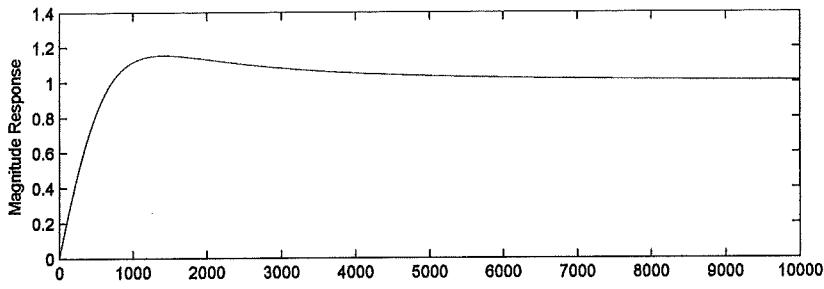
$$(1) \text{ into (2)}, V_1(w) \cdot \frac{[jwRC](2+jwRC)}{1+jwRC} = V_2(w) \cdot [1+jwRC]$$

Then,

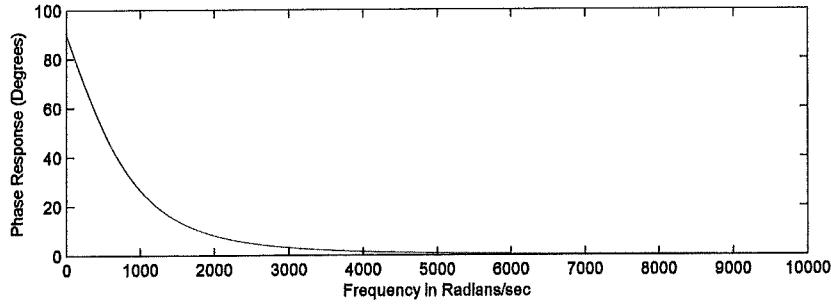
$$H(w) = \frac{V_2(w)}{V_1(w)} = \frac{(jwRC)(2+jwRC)}{(1+jwRC)^2}$$

Note:

$$H(0) = 0, H(\infty) = 1$$



• High-pass
filter
(2nd
order)



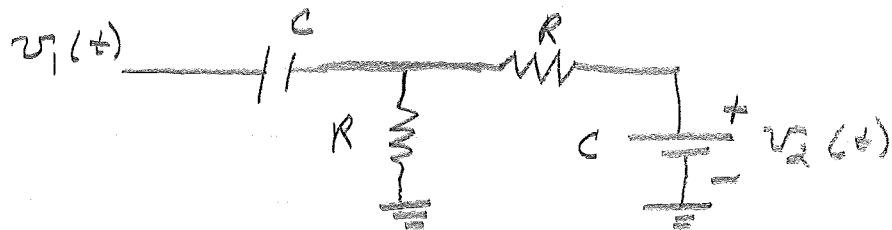
$$\omega_c \approx 400 \frac{\text{rad}}{\text{sec}}$$

↑
Based on
max value
of $H(w)=1$

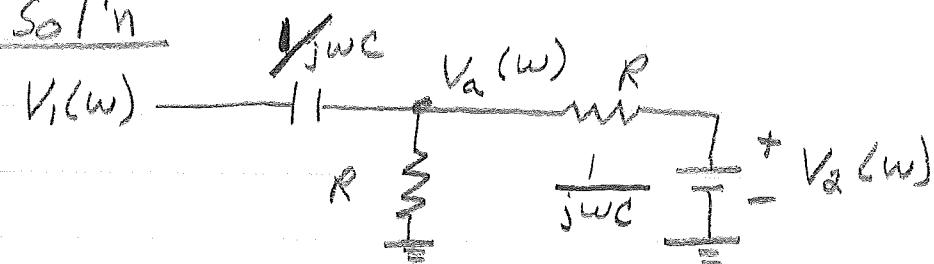
Fourier Transform

Passive Filter Example (2)

- Find and draw transfer function between input $v_1(t)$ and output $v_2(t)$ for $R = 1000 \Omega$, $C = 1 \mu F$.



Sol'n



$$(1): [V_a(w) - V_1(w)] jwC + \frac{V_a(w)}{R} + \frac{V_a(w) - V_2(w)}{jwC} = 0$$

$$V_a(w)[jwRC + 1] - V_2(w) = V_1(w)[jwRC]$$

$$(2): \frac{V_2(w) - V_a(w)}{R} + V_2(w)\{jwC\} = 0$$

$$V_2(w)\{1 + jwRC\} = V_a(w)$$

(2) into (1):

$$V_2(w)\{1 + jwRC\}\{1 + jwRC + 1\} - V_2(w) = V_1(w)\{jwRC\}$$

$$V_2(w)\{1 + jwRC\}\{2 + jwRC\} - V_2(w) = V_1(w)\{jwRC\}$$

Fourier Transform

(continued)

Passive Filter Example (2)

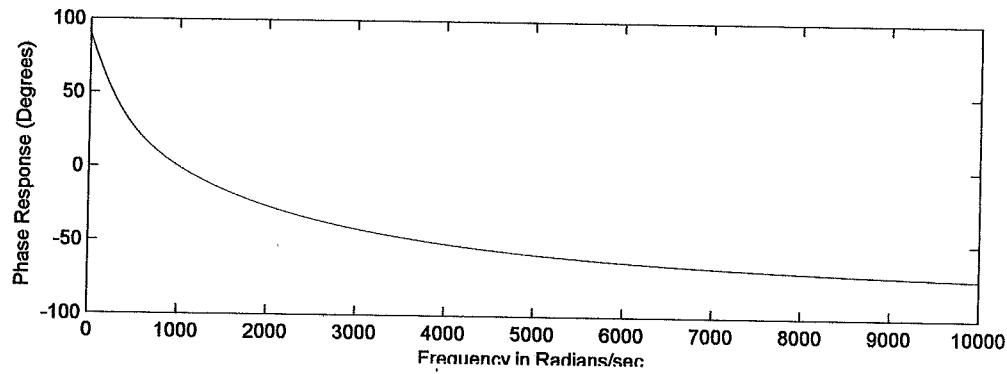
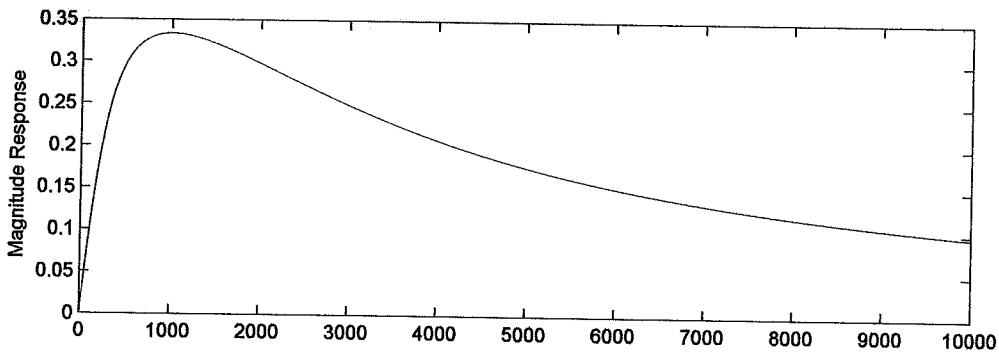
(Continued)

Giving the transfer function:

$$H(\omega) = \frac{V_2(\omega)}{V_1(\omega)} = \frac{j\omega RC}{(1+j\omega RC)(\omega+j\omega RC)-1}$$

Note:

$H(0) = 0$, $H(\infty) = 0$, peaks in between



• Bandpass filter

$$\omega_1 \approx 302 \text{ rad/s}$$

$$\omega_2 \approx 3310 \text{ rad/s}$$

Fourier Transform