

## Some Motivation (I)

- Laplace (Bilateral/Unilateral) transform
- Efficient solution of differential equation

- Bilateral  $\mathcal{L}$ -Transform
  - Zero-state response, general inputs
- Unilateral  $\mathcal{L}$ -Transform
  - Total response for  $t \geq 0$  when init. cond. known

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} [\cos(\omega t) + j \sin(\omega t)]$$

- Frequency dependence hard to visualize, since  $\omega$  not an isolated variable

- Phasor analysis was better at showing frequency dependence
  - But, limited to sinusoidal excitation (input)

(Continued)

Fourier Transforms

## Some Motivation (2)

### ◦ Conceptual considerations / methods:

#### a) Extend phasor concept

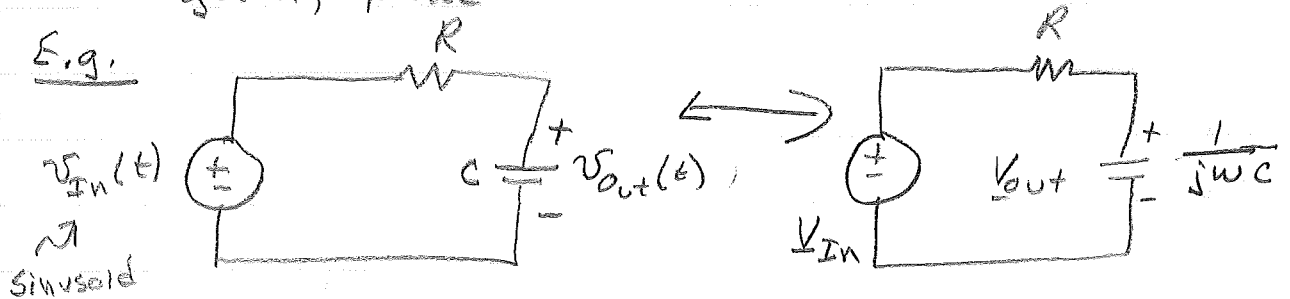
◦ Write excitation (input) as sum of sinusoids of different frequencies (often, continuous range!)

◦ Since sum, LTI system output is sum of outputs

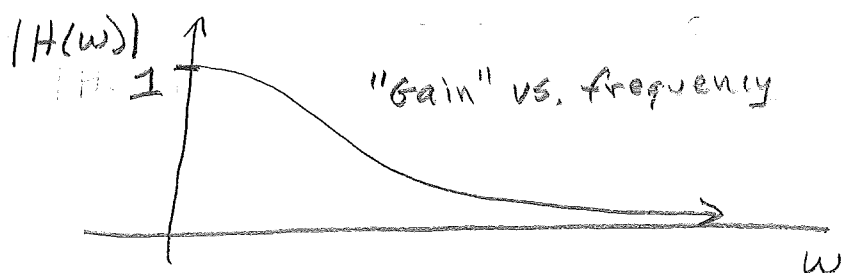
◦ Can study system response as function of  $\omega$

- LTI system, at each frequency, change gain, phase

E.g.



$$\underline{V}_{out} = \underline{V}_{in} \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \quad \text{or} \quad \frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{1}{1 + j\omega RC} \equiv H(\omega)$$



◦ Phasors  $\Rightarrow$  steady state response of sinusoid

(Continued)

Fourier Transforms

## Some Motivation

or

b) "Reduce" Laplace Transform coverage

$$s = \sigma + j\omega$$

◦ Get isolated frequency dependence if  $\sigma \rightarrow 0$

◦ Consequence:

◦ Only produce steady state analysis

⇒ Formalism  $\equiv$  Fourier Transform

If ROC of Laplace transform includes  $j\omega$  axis (i.e., includes  $\sigma = 0$  for all  $\omega$ ),

Then

$$\text{Fourier Transform} = X(s) \Big|_{s=j\omega} = \mathcal{F}[x(t)]$$

If Laplace ROC not include  $j\omega$  axis

Then

either a) must compute  $\mathcal{F}\{ \cdot \}$  by definition

or

b)  $\mathcal{F}\{ \cdot \}$  not exist!

Fourier Transforms

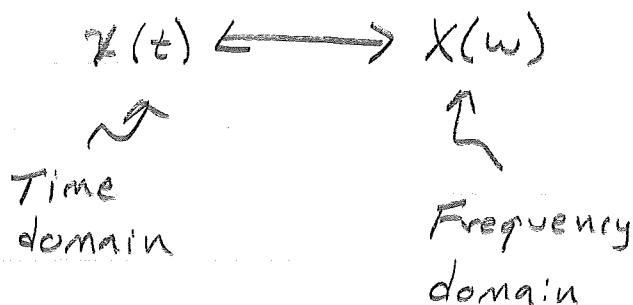
## Definition of Fourier Transform

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

• Notation:

$$X(\omega) = \mathcal{F}[x(t)]$$

ROC must  
be  $j\omega$ -axis  
→ Thus,  
not written.



Note:  $X(\omega)$  often written as  $X(j\omega)$

• Same as  $X(s) \Big|_{s=j\omega}$  IF  $\mathcal{L}$ -Transform ROC includes  $j\omega$  axis

• Inverse transform:

$$x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

• Ordinary integral.

• Invert via integral or transform table

Fourier Transform

# $\mathcal{F}$ -Transform of $\delta(t)$

o Recall:  $\mathcal{L}\{\delta(t)\} = 1$ , All  $s$ .

o ROC includes  $j\omega$ -axis

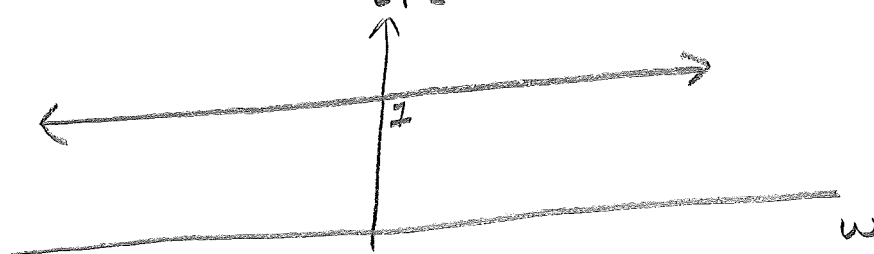
$$\Rightarrow \mathcal{F}\{\delta(t)\} = \mathcal{L}\{\delta(t)\} \Big|_{s=j\omega} = 1$$

o Formally:

$$\mathcal{F}\{\delta(t)\} = \int_{t=-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$$

$$= \underbrace{\int_{t=-\infty}^{0^-} \phi \cdot e^{-j\omega t} dt}_{\phi} + \underbrace{\int_{t=0^-}^{0^+} \delta(t) e^{-j\omega \cdot 0} dt}_{= 1} + \underbrace{\int_{t=0^+}^{\infty} \phi \cdot e^{-j\omega t} dt}_{\phi}$$

So,  $\mathcal{F}\{\delta(t)\} = 1$



Impulse: Equal sum of all frequencies

Fourier Transform

# $\mathcal{F}$ -Transform of $e^{-\lambda t} u(t)$ , $\lambda > 0$

• Recall:  $\mathcal{L}\{e^{-\lambda t} u(t), \lambda > 0\} = \frac{1}{s + \lambda}$ ,  $\text{Re}(s) > -\lambda$

• ROC includes  $j\omega$ -axis (for  $\lambda > 0$ )

$$\Rightarrow \mathcal{F}\{e^{-\lambda t} u(t)\} = \frac{1}{j\omega + \lambda}, \lambda > 0$$

• By definition:

$$\mathcal{F}\{e^{-\lambda t} u(t)\} = \int_{t=-\infty}^{\infty} e^{-\lambda t} u(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(j\omega + \lambda)t} dt = \frac{-e^{-(j\omega + \lambda)t}}{j\omega + \lambda} \Big|_{t=0}^{\infty} = \frac{-e^{-j\omega\infty - \lambda\infty} + e^{-j\omega\cdot 0 - \lambda\cdot 0}}{j\omega + \lambda}$$

Note:  $|e^{-j\omega\infty}| = 1$ ,  $e^{-\lambda\infty} = 0$  for  $\lambda > 0$ ,  $e^0 = 1$

$$\Rightarrow \mathcal{F}\{e^{-\lambda t} u(t)\} = \frac{1}{j\omega + \lambda}, \lambda > 0$$

## $\mathcal{F}$ - Transform of Step (1)

◦ Recall:  $\mathcal{L}\{u(t)\} = \frac{1}{s}$ ,  $\text{Re}(s) > 0$

◦ ROC NOT include  $j\omega$ -axis

$\Rightarrow$  Must use definition, if  $\mathcal{F}\{ \cdot \}$  exists!

◦ By definition:

$$\mathcal{F}\{u(t)\} = \int_{t=-\infty}^{\infty} u(t) e^{-j\omega t} dt = \int_{t=0}^{\infty} e^{-j\omega t} dt$$

$$= \left. \frac{-e^{-j\omega t}}{j\omega} \right|_{t=0}^{\infty} = \frac{-e^{-j\omega \infty} + e^{-j\omega 0}}{j\omega} = \frac{[-\cos(\omega \infty) - j \sin(\omega \infty)] + 1}{j\omega}$$

◦  $\cos(\cdot)$ ,  $\sin(\cdot)$  terms indeterminate

◦ Direct approach not usable

◦ Several functions require an indirect approach

(Continued)

Fourier Transform

## F-Transform of Step (2)

• Let  $u(t) = \lim_{\lambda \rightarrow 0} e^{-\lambda t} u(t)$

• Then,

$$\mathcal{F}\{u(t)\} = \lim_{\lambda \rightarrow 0} \mathcal{F}\{e^{-\lambda t} u(t)\} = \lim_{\lambda \rightarrow 0} \frac{1}{j\omega + \lambda}$$

• How do we take this limit?

Recall:  
Formula  
requires  $\lambda > 0$

1) Write in real, imag parts:

$$\mathcal{F}\{u(t)\} = \lim_{\lambda \rightarrow 0} \left[ \frac{\lambda}{\omega^2 + \lambda^2} - j \frac{\omega}{\omega^2 + \lambda^2} \right]$$

$$= \lim_{\lambda \rightarrow 0} \left[ \frac{\lambda}{\omega^2 + \lambda^2} \right] + \lim_{\lambda \rightarrow 0} \left[ \frac{-j\omega}{\omega^2 + \lambda^2} \right]$$

2)  $\lim_{\lambda \rightarrow 0} \left[ \frac{-j\omega}{\omega^2 + \lambda^2} \right] = \frac{-j\omega}{\omega^2} = \frac{-j}{\omega} = \frac{1}{j\omega}$

3) Limit for leftmost part:

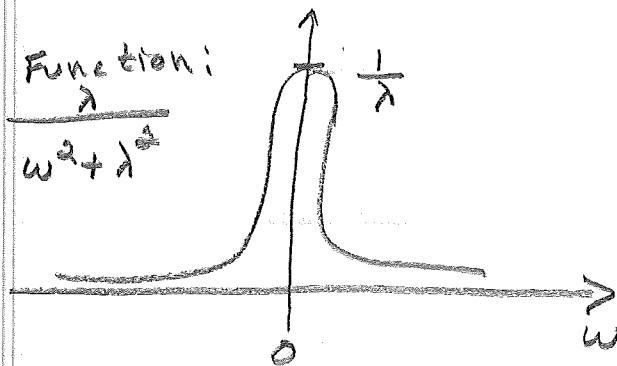
Note:  $\int_{-\infty}^{\infty} \frac{\lambda}{\omega^2 + \lambda^2} d\omega = \tan^{-1} \frac{\omega}{\lambda} \Big|_{-\infty}^{\infty} = \pi$

$\Rightarrow$  Area under curve equals  $\pi$ , regardless of  $\lambda$ .

(Continued)

Fourier Transform

## $\mathcal{F}$ -Transform of Step (3)



- As  $\lambda \rightarrow 0$ 
  - Function concentrates at  $\phi$
  - Height  $\rightarrow \infty$
  - Value at  $\omega \neq 0 \rightarrow 0$
  - Area remains  $\pi$



Impulse of area  $\pi$  at  $\omega = 0$

Thus,

$$\boxed{\mathcal{F}\{\mu(t)\} = \pi \delta(\omega) + \frac{1}{j\omega}}$$

## Existence of Fourier Transform

◦ Must exist if  $x(t)$ :

1) ... Absolutely integrable:

$$\int_{t=-\infty}^{\infty} |x(t)| dt < \infty$$

2) ... finite number of finite discontinuities on any finite interval

3) ... finite number of maxima, minima on any finite interval

◦ Can still exist otherwise

Note 1: Conditions 2, 3 satisfied by physical signals

Note 2: Condition 1 violated by periodic signals

But, periodic signals generally do have a  $\mathcal{F}\{ \}$ -Transform, but do not generally have a  $\mathcal{L}$ -Transform.

## Fourier Transform of Two-Sided Cosine (1)

- Does not have a Laplace Transform
- Direct approach is indeterminate

Consider:

If  $X(\omega) = \delta(\omega - \omega_0)$ , find  $x(t)$ :

By definition:  $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_0) e^{j\omega t} d\omega$

$$= \frac{1}{2\pi} \int_{\omega_0^-}^{\omega_0^+} \delta(\omega - \omega_0) e^{j\omega t} d\omega = \frac{e^{j\omega_0 t}}{2\pi}$$

Thus,  $\frac{e^{j\omega_0 t}}{2\pi} \longleftrightarrow \delta(\omega - \omega_0)$  or  $\boxed{e^{j\omega_0 t} \longleftrightarrow 2\pi \delta(\omega - \omega_0)}$

Similarly:  $e^{-j\omega_0 t} \longleftrightarrow 2\pi \delta(\omega + \omega_0)$

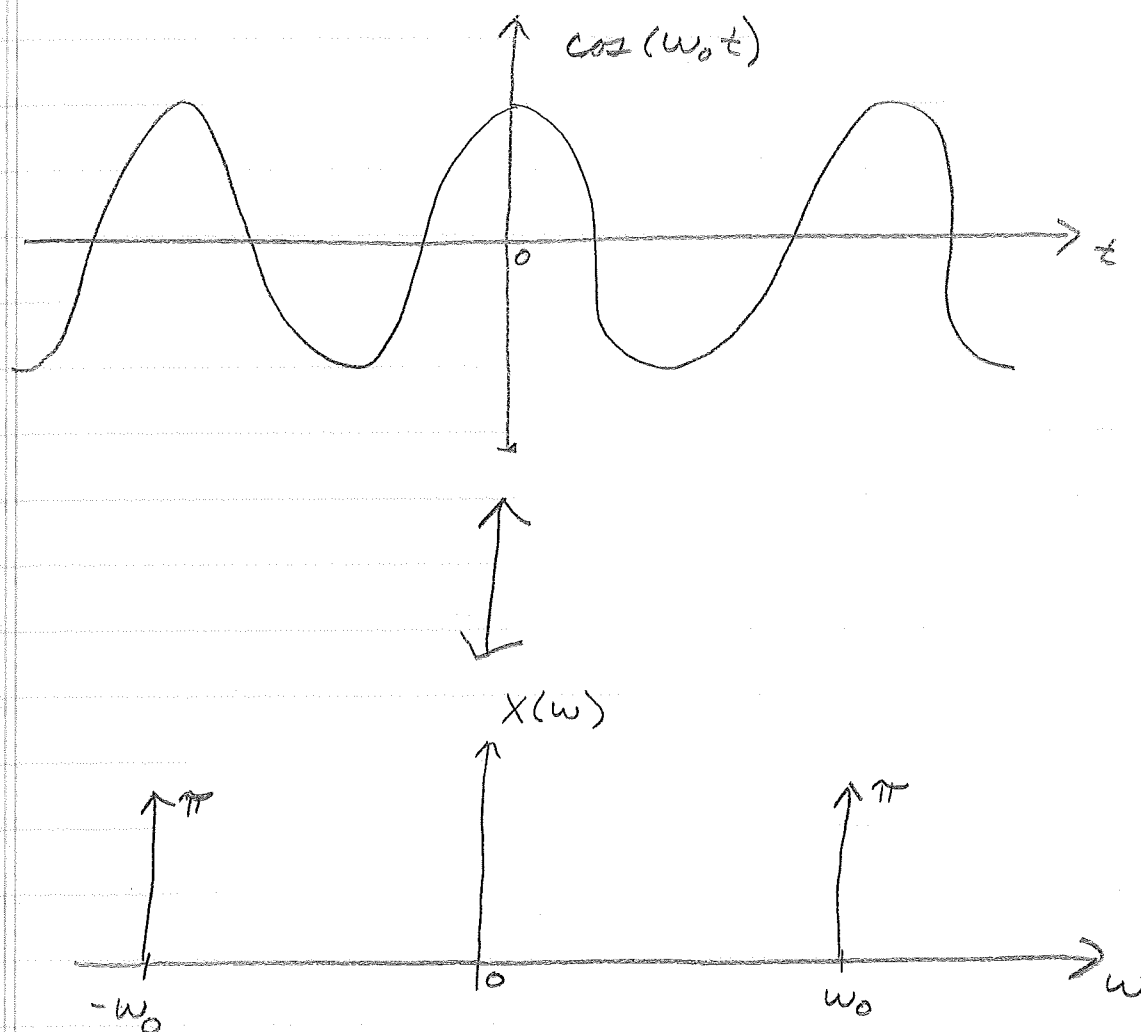
Now, write cosine as:  $\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

From above:

$$\boxed{\cos(\omega_0 t) \longleftrightarrow \pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]}$$

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# Fourier Transform of Two-Sided Cosine (2)



Sum two complex exponentials to form a cosine

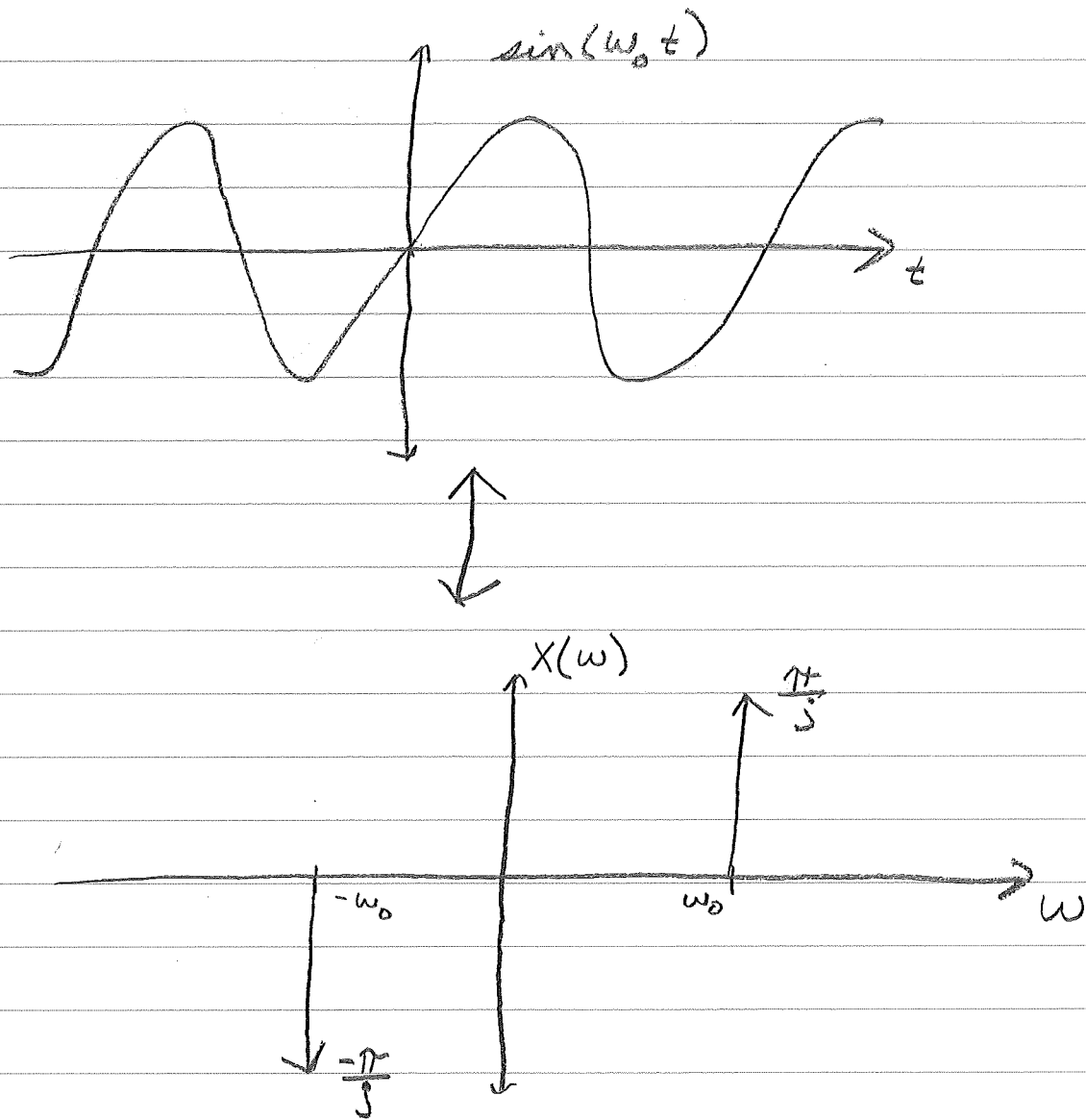
Pure frequency  $\longleftrightarrow$  Information at  
in time one frequency

Fourier Transform

## Fourier Transform of Two-Sided Sine

Similarly,

$$\sin(\omega_0 t) \xrightarrow{\mathcal{F}} \frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$



Fourier Transform

## Fourier Transform of Gate Function

◦ Unit gate/pulse function:  $g(t) = \begin{cases} 1, & |t| < \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$

[\* Formally,  $g(|t| = \tau/2) = 1/2$ ]

$\tau \rightarrow$  Pulse width

◦ By definition:  $G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$

$$= \int_{-\tau/2}^{\tau/2} e^{-j\omega t} dt = \frac{-e^{-j\omega \tau/2} + e^{j\omega \tau/2}}{j\omega} = \frac{\sin\left(\frac{\omega \tau}{2}\right)}{\left(\frac{\omega}{2}\right)}$$

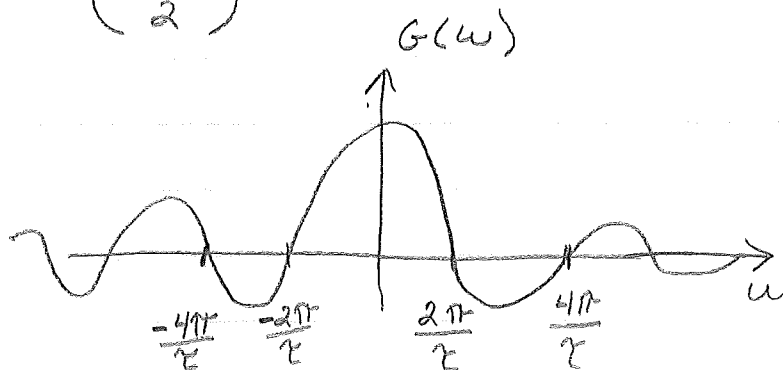
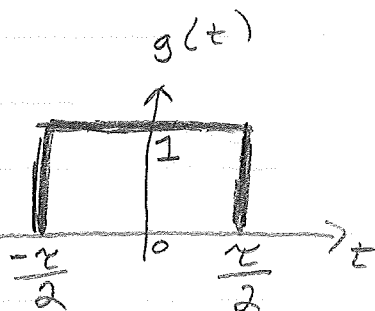
Define "sinc(x) over x":

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

Then,

$$G(\omega) = \tau \cdot \text{sinc}\left(\frac{\omega \tau}{2}\right)$$

$$* \text{sinc}(0) = 1$$



◦ Note: As  $\tau$  decreases, gate width of  $g(t)$  shrinks while main lobe of  $G(\omega)$  widens, and vice versa

Fourier Transform

# Common Fourier Transforms

**TABLE 7.1** Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12	$\text{sgn } t$	$\frac{2}{j\omega}$	
13	$\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14	$\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15	$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16	$e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17	$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi} \text{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

{Lathi}

Fourier Transforms

# Table of Fourier Transform Properties

**TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM**

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega - \theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^t x(t)dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\omega} X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \text{Re}\{X(j\omega)\} = \text{Re}\{X(-j\omega)\} \\ \text{Im}\{X(j\omega)\} = -\text{Im}\{X(-j\omega)\} \\  X(j\omega)  =  X(-j\omega)  \\ \angle X(j\omega) = -\angle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}\{x(t)\}$ [x(t) real] $x_o(t) = \mathcal{O}\{x(t)\}$ [x(t) real]	$\text{Re}\{X(j\omega)\}$ $j\text{Im}\{X(j\omega)\}$
4.3.7	Parseval's Relation for Aperiodic Signals		
	$\int_{-\infty}^{+\infty}  x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty}  X(j\omega) ^2 d\omega$		

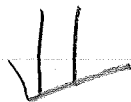
[Oppenheim, Willsky, Nawab. Signal and Systems 2/e. Prentice Hall]

Fourier Transform

Real-Valued Signals

If  $x(t)$  real-valued can show:

$$X(\omega) = X^*(-\omega)$$



$$\text{write } X(\omega) = \sigma(\omega) + j b(\omega)$$

$$\Rightarrow \operatorname{Re}\{X(\omega)\} = \operatorname{Re}\{X(-\omega)\}$$

$$\operatorname{Im}\{X(\omega)\} = -\operatorname{Im}\{X(-\omega)\}$$

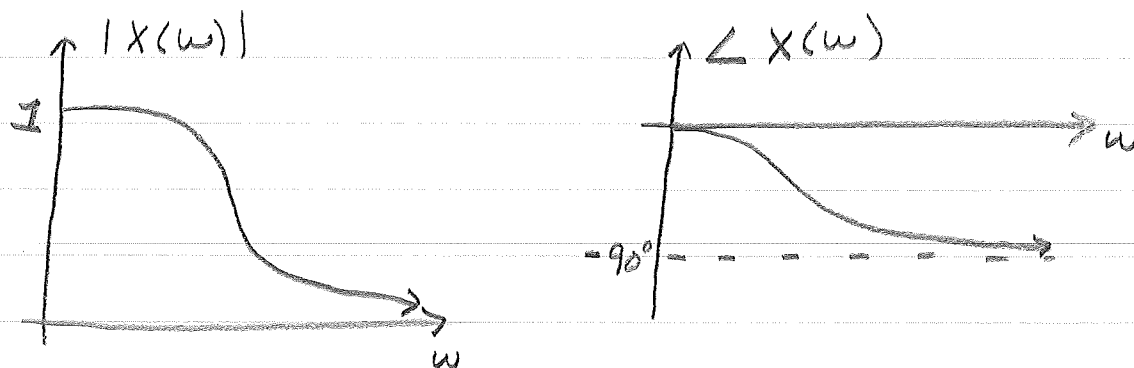
and

$$\Rightarrow |X(\omega)| = |X(-\omega)|$$

$$\angle X(\omega) = -\angle X(-\omega)$$

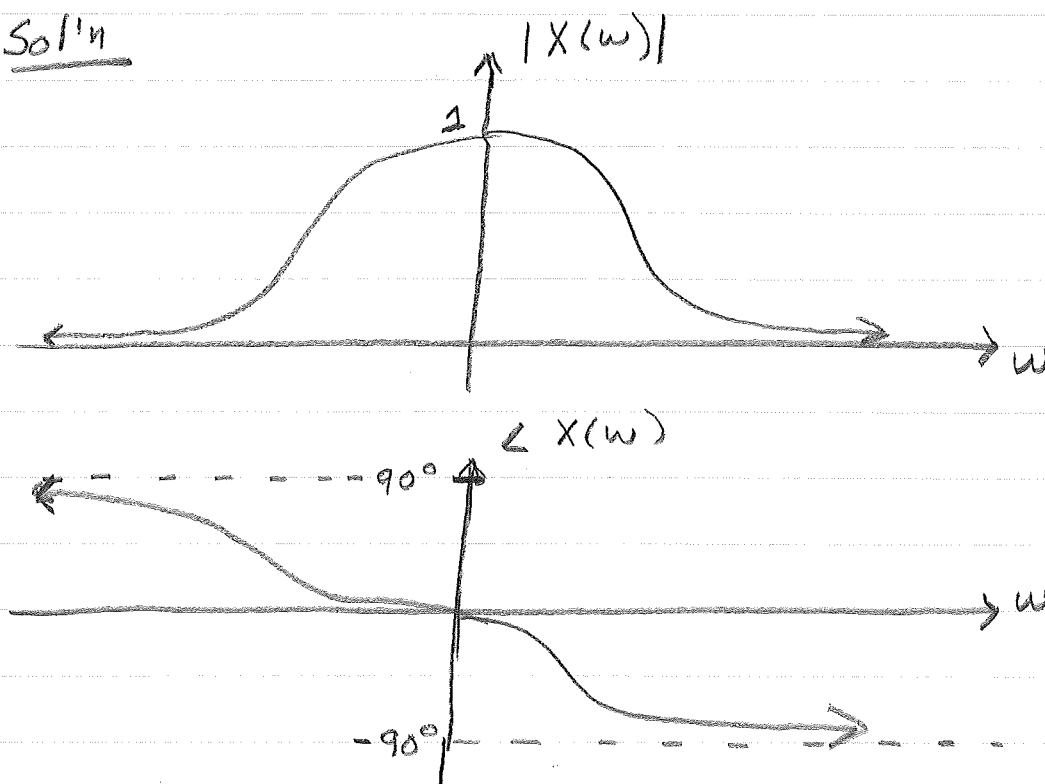
## Example: Real-Valued Signals

If  $x(t)$  real-valued and  $X(\omega \geq 0)$  is:



then draw  $|X(\omega)|$ ,  $\angle X(\omega)$  over all  $\omega$ .

Sol'n

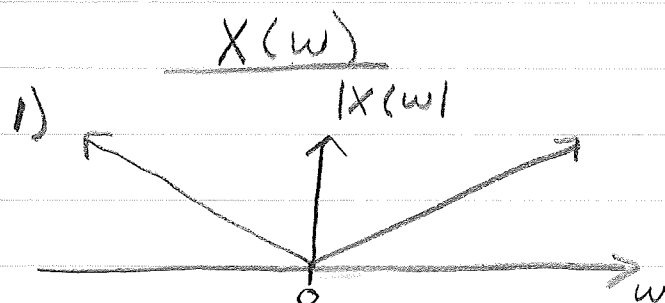


Often, only show  $X(\omega \geq 0)$  for real-valued signals.  
 → Negative frequencies = redundant information

Fourier Transform

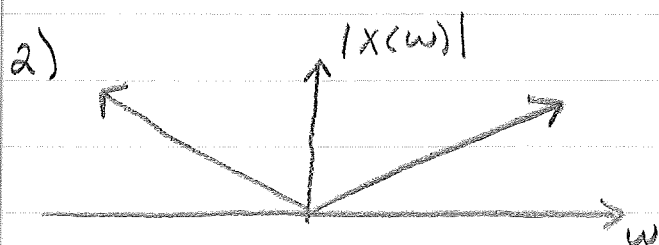
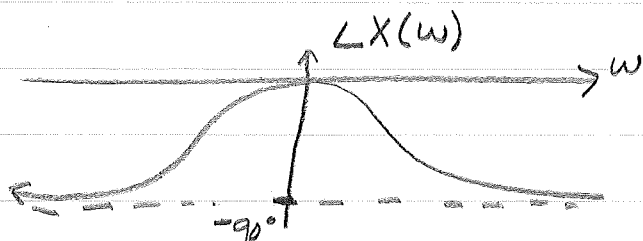
# Identifying Real- vs. Complex-Valued Signals

$x(t)$ : Real- or  
Complex-Valued?

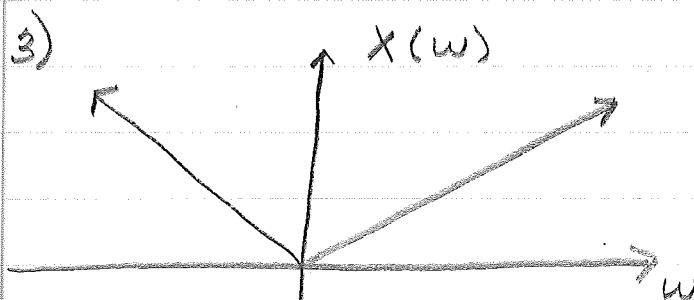
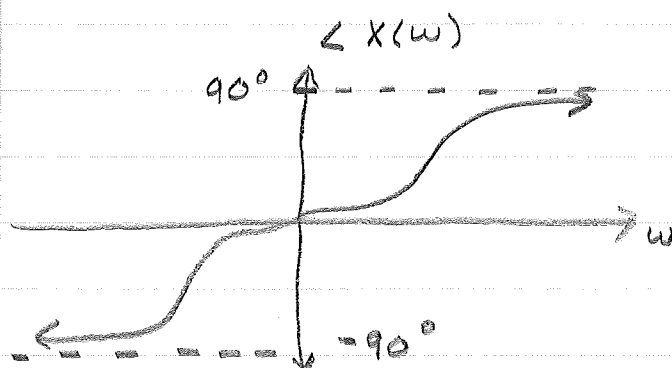


Complex

$$\angle X(\omega) \neq -\angle X(-\omega)$$



Real

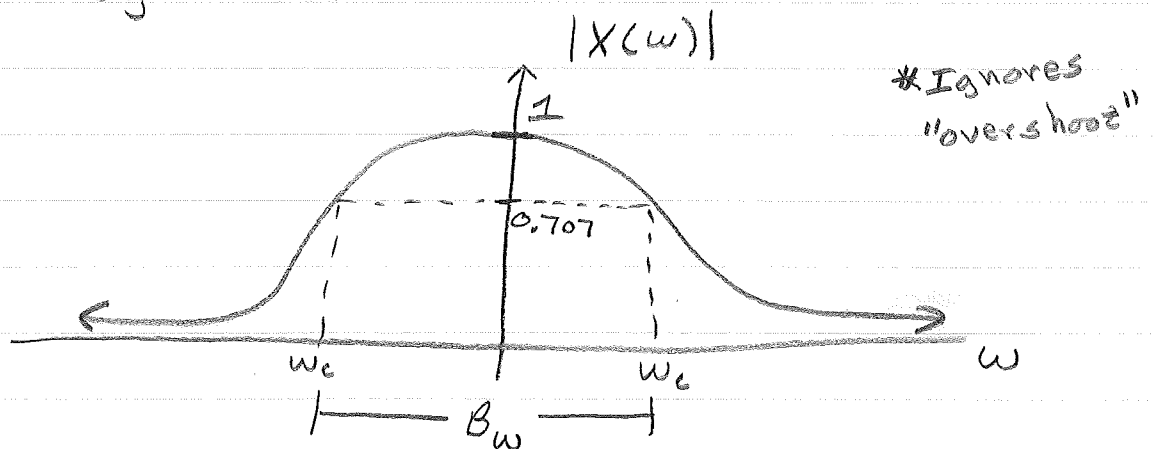


Real

$$[\angle X(\omega) = 0^\circ]$$

## Definition(s) of Bandwidth

- Frequency span of signals that are "passed" through a system (or utilized by)
  - Expressed in radians/sec or Hertz
  - Several different, but similar, mathematical definitions
- Narrow band: Passes <sup>utilizes</sup> relatively limited range of frequencies
- Broad band: Passes/utilizes relatively large range of frequencies
- 3 dB bandwidth of filter: Band extending from maximum gain\* plus and minus frequency location where gain reduced by 3 dB.

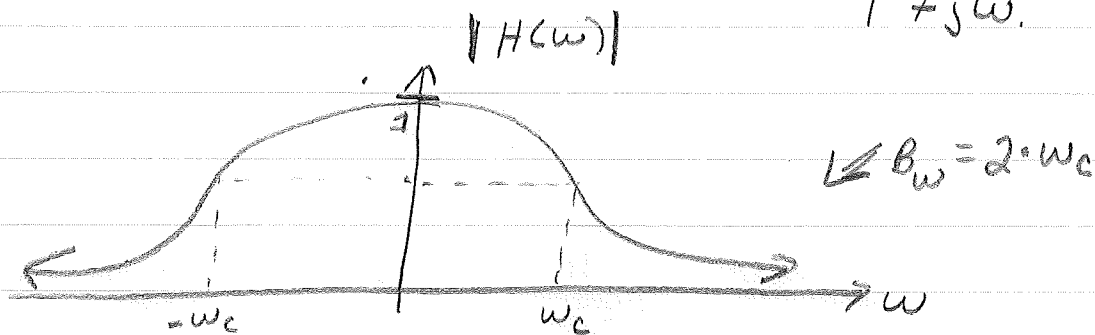


$$3 \text{ dB down} \Rightarrow \left( \frac{\sqrt{2}}{2} \right) \cdot \text{Max} = (0.707) \cdot \text{Max}$$

Fourier Transform

## Bandwidth Example

- Find 3dB bandwidth of low pass filter with frequency response  $H(\omega) = \frac{1}{1+j\omega}$ .



Sol'n

- Max value at  $\omega=0$  is:  $H(0) = 1$

$$|H(\omega)| = \frac{1}{|1+j\omega|} = \frac{1}{\sqrt{1+\omega^2}}$$

$$\Rightarrow |H(\omega)|^2 = \frac{1}{1+\omega^2} \quad \Leftarrow \text{Set equal to } \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$$

So,

$$\frac{1}{2} = \frac{1}{1+\omega_c^2} \Rightarrow 2-1 = \omega_c^2$$

$$\Rightarrow \omega_c = \pm \sqrt{1} = \pm 1 \frac{\text{rad}}{\text{s}} = \omega_c$$

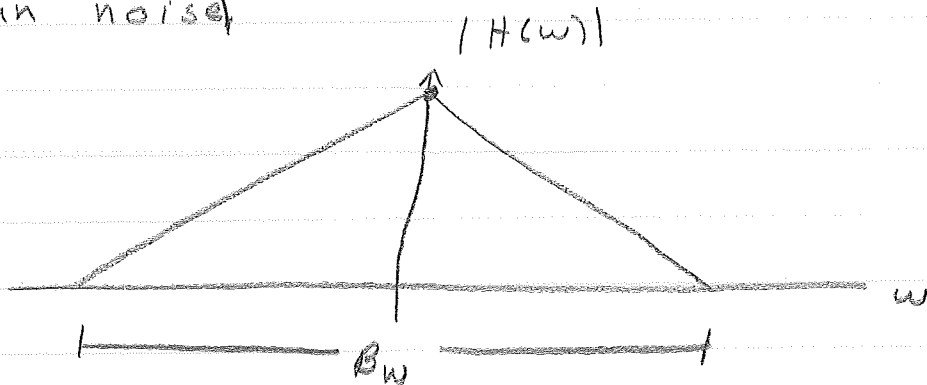
Thus,

$$B_w = 2 \frac{\text{rad}}{\text{s}} = \frac{1}{\pi} \text{ Hz}$$

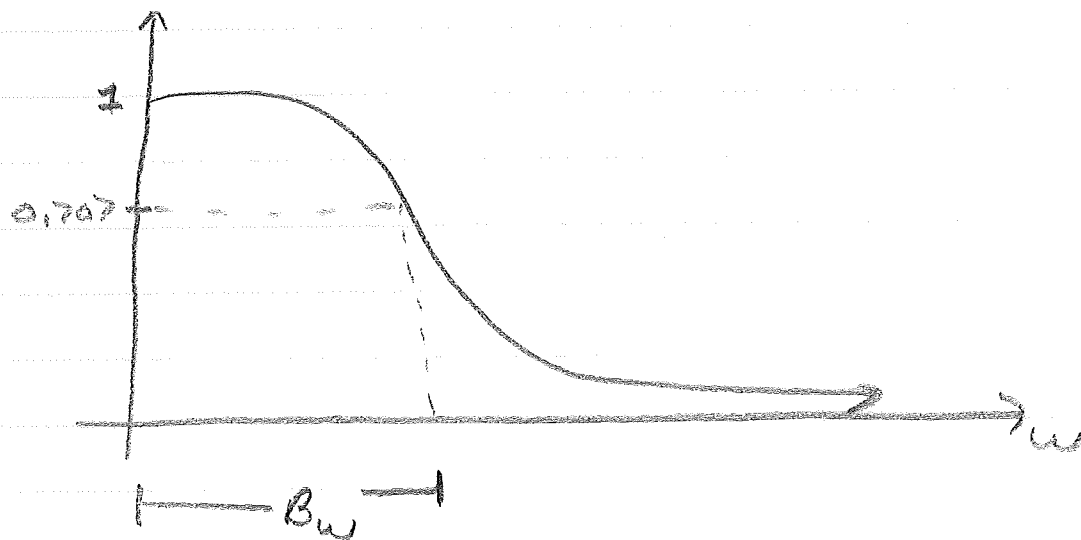
Fourier Transform

## Other Bandwidth Definitions

- Range of frequencies over which the signal power is non-zero (or, greater than noise)



- For real-valued signals, bandwidth sometimes refers only to positive-valued frequencies.



Fourier Transform

## Fourier Transform and Impedance

R:

$$v_R(t) = i_R(t) \cdot R$$

↕

$$V_R(\omega) = I_R(\omega) \cdot R \quad \text{or} \quad \frac{V_R(\omega)}{I_R(\omega)} = R$$

L:

$$v_L(t) = L \frac{di_L(t)}{dt}$$

↕

$$V_L(\omega) = L \cdot j\omega I_L(\omega) \quad \text{or} \quad \frac{V_L(\omega)}{I_L(\omega)} = j\omega L$$

C:

$$i_C(t) = C \frac{dv_C(t)}{dt}$$

↕

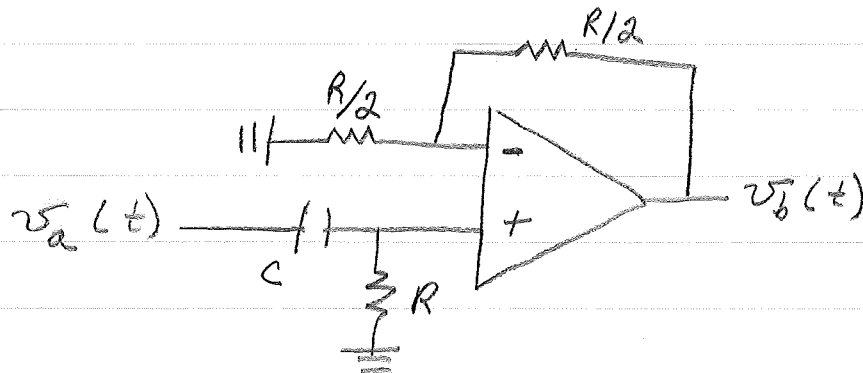
$$I_C(\omega) = C \cdot j\omega V_C(\omega) \quad \text{or} \quad \frac{V_C(\omega)}{I_C(\omega)} = \frac{1}{j\omega C}$$

Standard Impedances.  
Inputs must have a  
Fourier Transform.

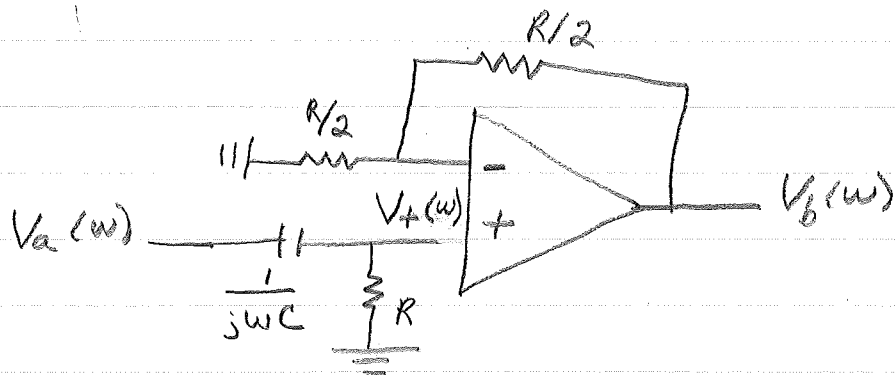
Fourier Transform

### Filter Example (1)

Find and draw the transfer function of the filter shown below for  $R = 1000 \Omega$ ,  $C = 1 \mu F$ .



Sol'n



$$(1): [V_+(w) - V_a(w)] jwC + \frac{V_+(w)}{R} = 0$$

$$\text{or } V_+(w) [jwRC + 1] = V_a(w) [jwRC]$$

$$(2): V_+(w) \cdot \frac{2}{R} + [V_+(w) - V_b(w)] \cdot \frac{2}{R} = 0$$

or

$$V_+(w) = \frac{V_b(w)}{2}$$

(Continued)

Fourier Transform

## Filter Example (2)

(Continued)

Substituting (2) into (1):

$$V_b(\omega) [j\omega RC + 1] = V_a(\omega) [j2\omega RC]$$

Thus,

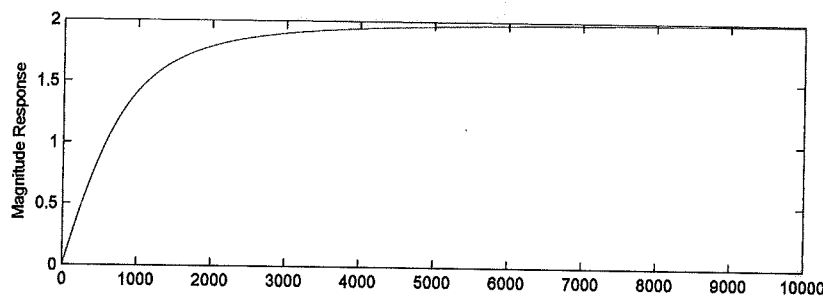
$$H(\omega) = \frac{V_b(\omega)}{V_a(\omega)} = \frac{j2\omega RC}{j\omega RC + 1}$$

Note:  $H(0) = 0$ ,  $H(\infty) = 2$

$$|H(\omega)| = \frac{|j2\omega RC|}{|j\omega RC + 1|} = \frac{2\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

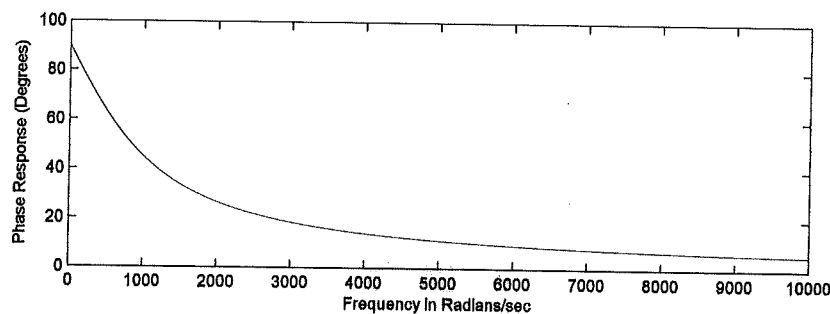
$$\angle H(\omega) = \angle(j2\omega RC) - \angle(j\omega RC + 1) = \frac{\pi}{2} - \tan^{-1}(\omega RC)$$

For  $R = 1000 \Omega$ ,  $C = 1 \mu F$ :



• High-pass filter

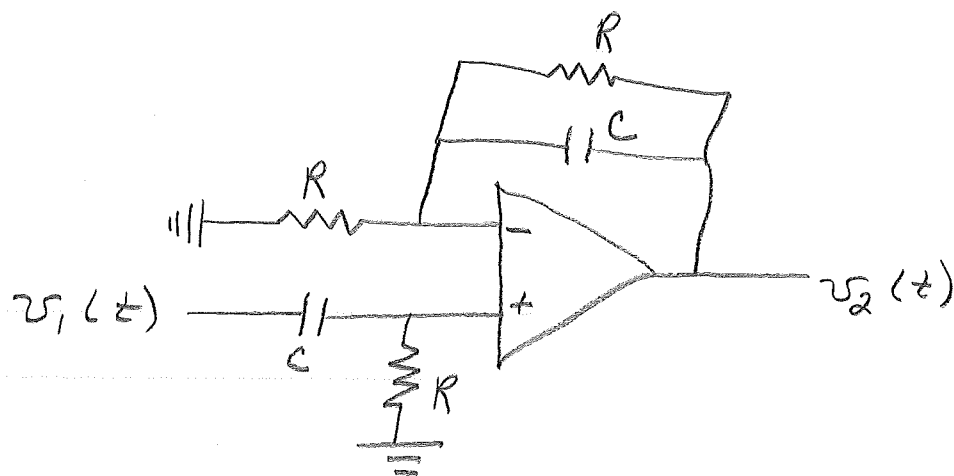
$$\omega_c \approx 1000 \frac{\text{rad}}{\text{sec}}$$



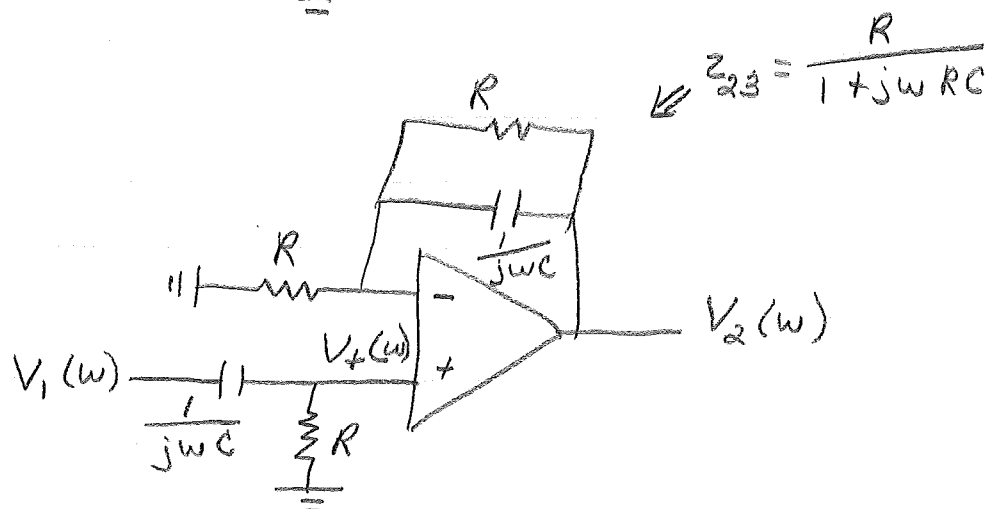
Fourier Transform

# Another Active Filter Example (1)

- Find and draw transfer function when  
 $R = 1000 \Omega$ ,  $C = 1 \mu F$ .



Sol'n



$$\underline{(1)}: [V_+(w) - V_1(w)] j\omega C + \frac{V_+(w)}{R} = 0$$

$$V_+(w) = V_1(w) \cdot \frac{j\omega RC}{1 + j\omega RC}$$

$$\underline{(2)}: \frac{V_+(w)}{R} + [V_+(w) - V_2(w)] \cdot \frac{[1 + j\omega RC]}{R} = 0$$

$$V_+(w) [2 + j\omega RC] = V_2(w) [1 + j\omega RC]$$

Fourier Transform

(Continued)

## Another Active Filter Example (2)

(Continued)

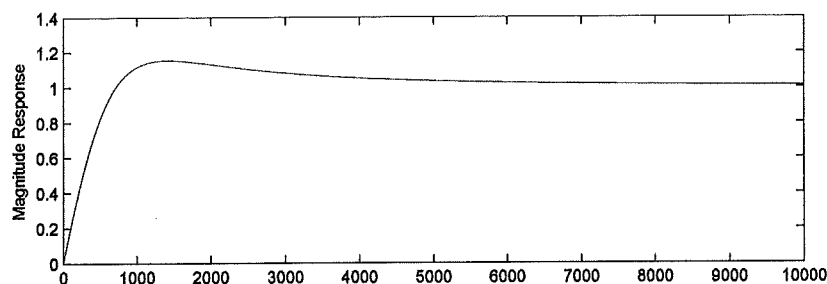
(1) into (2):  $V_1(\omega) \cdot \frac{(j\omega RC)(2 + j\omega RC)}{1 + j\omega RC} = V_2(\omega) \cdot [1 + j\omega RC]$

Then,

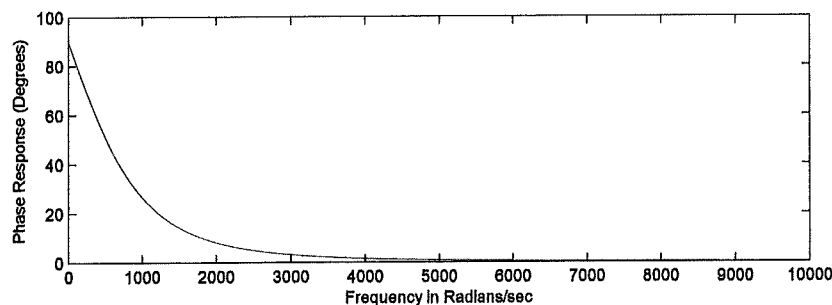
$$H(\omega) = \frac{V_2(\omega)}{V_1(\omega)} = \frac{(j\omega RC)(2 + j\omega RC)}{(1 + j\omega RC)^2}$$

Note:

$$H(0) = 0, \quad H(\infty) = 1$$



• High-pass  
filter  
(2<sup>nd</sup>  
order)



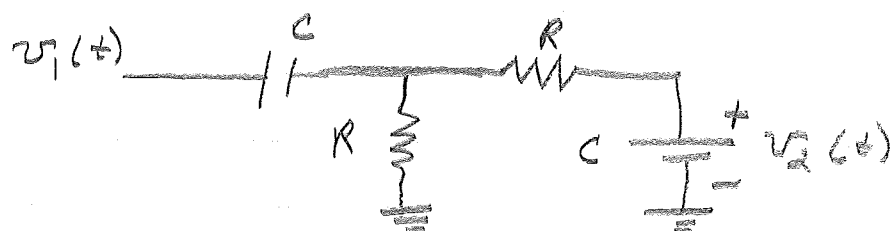
$$\omega_c \approx 400 \frac{\text{rad}}{\text{sec}}$$

Based on  
max value  
of  $H(\omega) = 1$

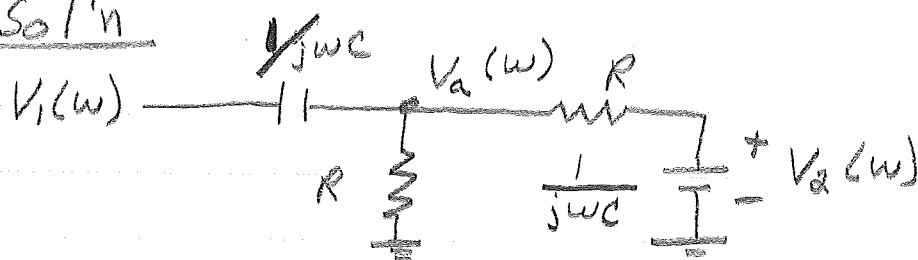
Fourier Transform

### Passive Filter Example (I)

- Find and draw transfer function between input  $v_1(t)$  and output  $v_2(t)$  for  $R = 1000 \Omega$ ,  $C = 1 \mu F$ .



Sol'n



$$(1): [V_a(w) - V_1(w)] j\omega C + \frac{V_a(w)}{R} + \frac{V_a(w) - V_2(w)}{R} = 0$$

$$V_a(w) [j\omega RC + 2] - V_2(w) = V_1(w) [j\omega RC]$$

$$(2): \frac{V_2(w) - V_a(w)}{R} + V_2(w) [j\omega C] = 0$$

$$V_2(w) [1 + j\omega RC] = V_a(w)$$

(2) into (1):

$$V_2(w) [1 + j\omega RC] [j\omega RC + 2] - V_2(w) = V_1(w) [j\omega RC]$$

$$V_2(w) \{ [1 + j\omega RC] [2 + j\omega RC] - 1 \} = V_1(w) [j\omega RC]$$

Fourier Transform

(Continued)

## Passive Filter Example (2)

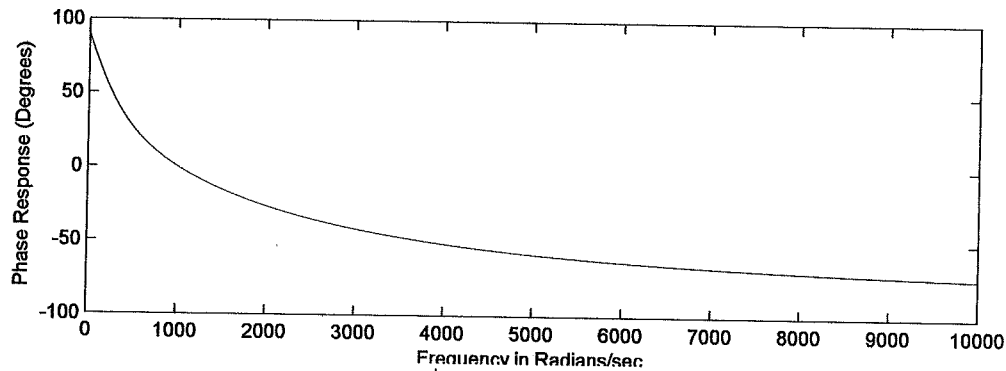
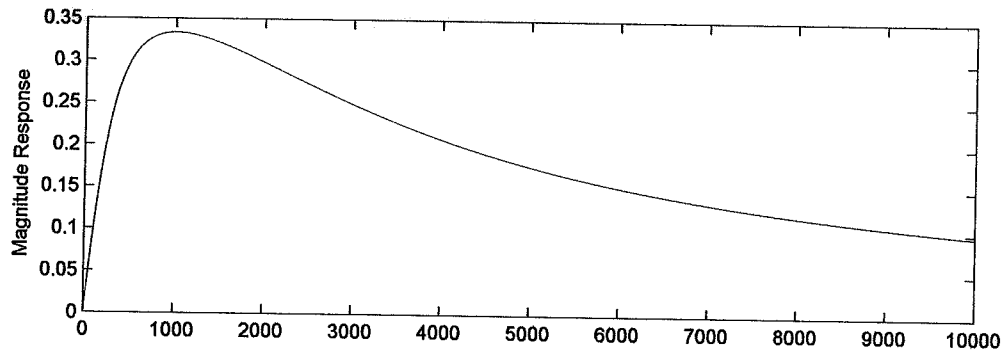
(Continued)

Giving the transfer function:

$$H(\omega) = \frac{V_2(\omega)}{V_1(\omega)} = \frac{j\omega RC}{(1+j\omega RC)(2+j\omega RC)-1}$$

Note:

$H(0) = 0$ ,  $H(\infty) = 0$ , peaks in between



• Bandpass filter

$$\omega_1 \cong 302 \text{ rad/s}$$

$$\omega_2 \cong 3310 \text{ rad/s}$$

Fourier Transform