Solution

Worcester Polytechnic Institute — Department of Electrical and Computer Engineering
ECE2311 — Continuous-Time Signal and System Analysis — Term B'17

Homework 7: Due Tuesday, 21 November 2017 (3:00 P.M.)

Write your name and ECE box at the top of each page.

General Reminders on Homework Assignments:

- Always complete the reading assignments *before* attempting the homework problems.
- Show all of your work. Use written English, where applicable, to provide a log or your steps in solving a problem. (For numerical homework problems, the writing can be brief.)
- A solution that requires physical units is *incorrect* unless the units are listed as part of the result.
- Get in the habit of underlining, circling or boxing your result.
- Always write neatly. Communication skills are essential in engineering and science. "If you didn't write it, you didn't do it!"

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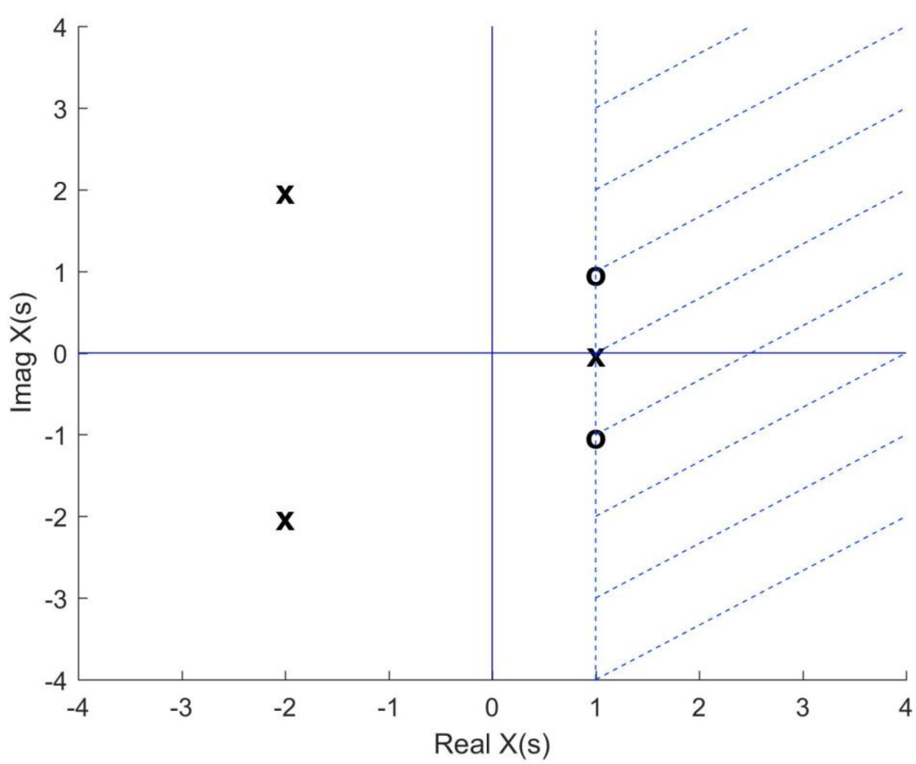
1) Rational Bilateral Laplace Transform: Use MATLAB for numerical computation, as needed, below. Useful functions include: residue (for partial fraction expansion and contraction) and roots (for factoring).

a) Draw the pole-zero plot for the system with system function: $H(s)=\dfrac{(s-1+j)(s-1-j)}{(s-1)(s+2+j2)(s+2-j2)}$, $\operatorname{Re}\{s\}>1$. Label the ROC on the same s-plane as the pole-zero plot. Is this the system function of a stable system?



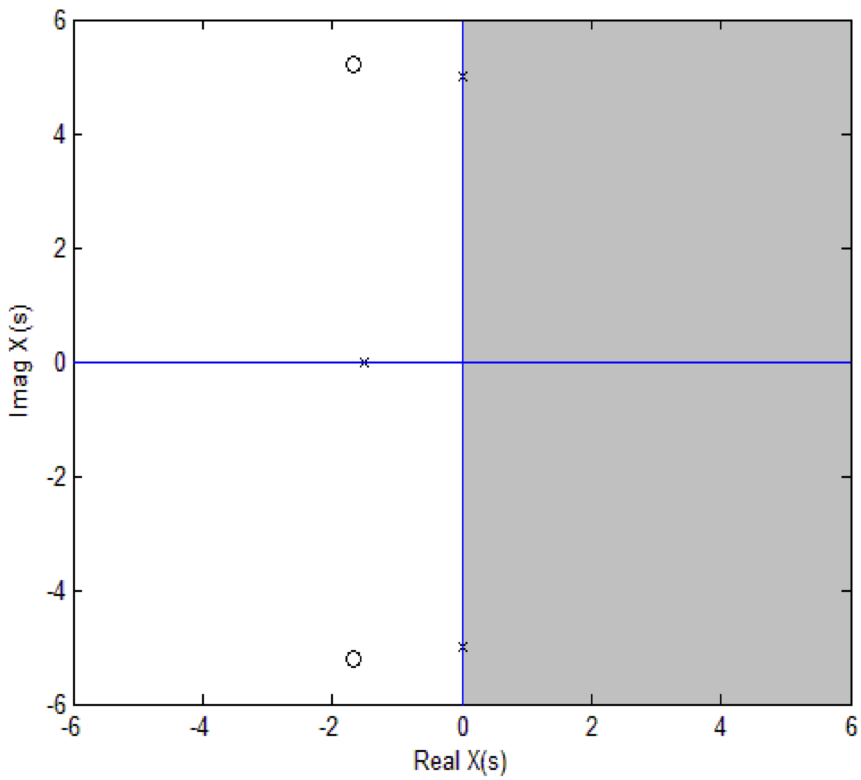
ROC shaded. Rational LTI system, but ROC NOT include $j\omega$ axis (s = 0 excluded)
$\Rightarrow$ **NOT stable**

b) Draw the pole-zero plot for the system with impulse response: $h(t) = \left[3e^{-1.5t} + 2\sin(5t)\right]u(t)$. Label the ROC on the same s-plane as the pole-zero plot. Is this the system function of a stable system? Hint: Transform *h(t)* to the frequency domain, then use the "[b, a] = residue(r, p, k)" form of residue() in MATLAB to contract the fractions; followed by "roots()" to factor them.

Using the MATLAB residue() and roots() functions:

$$H(s) = \frac{3}{s+1.5} + \frac{10}{s^2+25} = \frac{3}{s+1.5} + \frac{-j}{s-j5} + \frac{j}{s+j5} = \frac{3s^2+10s+90}{s^3+1.5s^2+25s+37.5}$$

$$= \frac{3\cdot(s+1.67-j5.22)(s+1.67+j5.22)}{(s-j5)(s+j5)(s+1.5)}, \quad \mathrm{Re}(s) > 0$$



ROC shaded. Rational LTI system, but ROC NOT include entire $j\omega$ axis (2 poles on axis, with ROC to the right of zero)

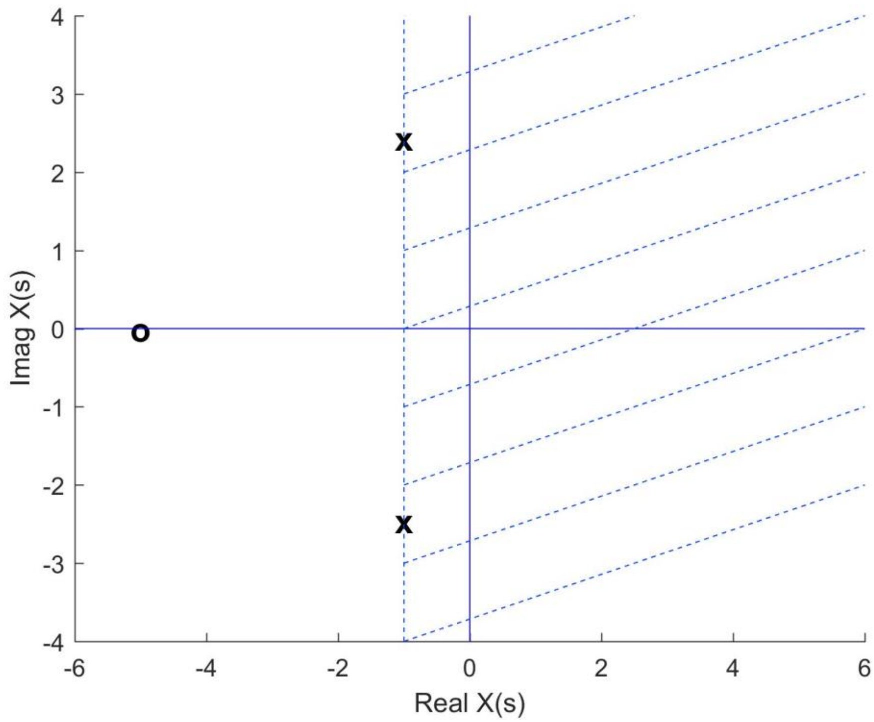
$\Rightarrow$ **NOT stable**

c) Draw the pole-zero plot for the system with system function: $H(s) = \frac{s+5}{s^2+2s+7}$, assuming that the impulse response of this system is right-sided. (Technically, this system function is incompletely specified, since its ROC is not listed. However, you can determine the ROC by knowing that the impulse response is right-sided.) Label the ROC on the same s-plane as the pole-zero plot. Is this the system function of a stable system?

Using the MATLAB roots() function:

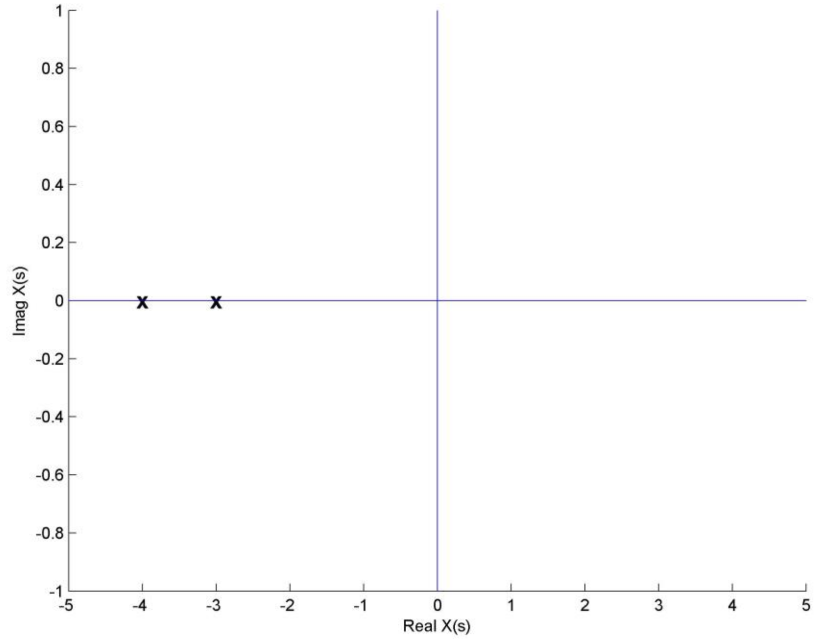
$$H(s) = \frac{s+5}{(s+1+j2.45)(s+1-j2.45)}$$

Since it is a right-sided sequence, ROC: $\mathrm{Re}(s) > -1$,



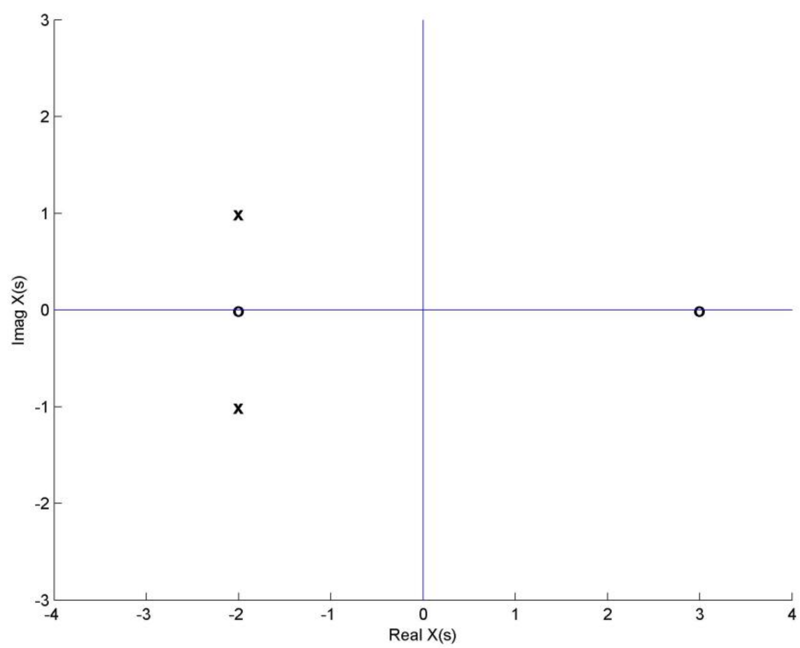
ROC shaded. Rational LTI system, and ROC INCLUDES entire $j\omega$ axis $\Rightarrow$ **STABLE**

d) Write $X(s)$ for the function shown in the pole-zero plot below. The ROC is: $Re(s) > -3$. You should be able to do so without computational aids.



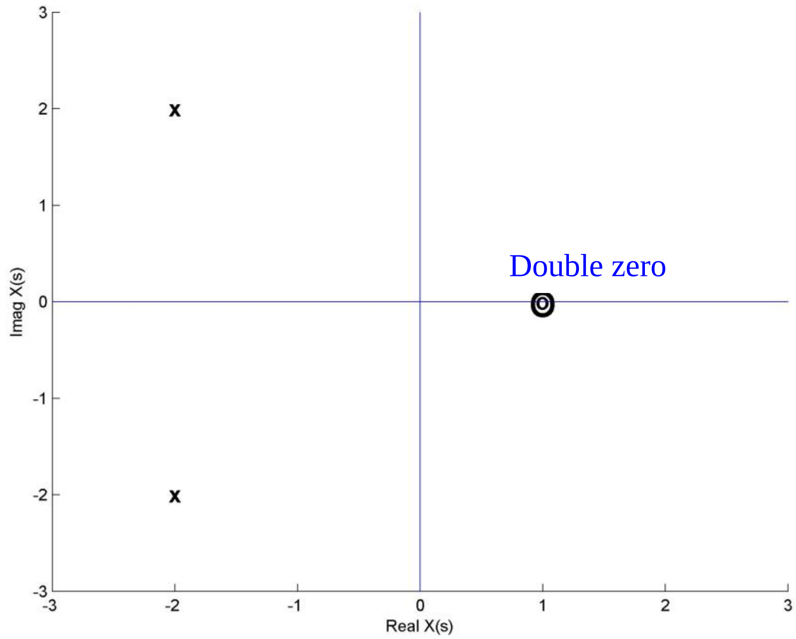
$$X(s) = G\frac{1}{(s+3)(s+4)}, \quad \text{Re}(s) > -3$$, where G is an unknown constant.

e) Write $X(s)$ for the function shown in the pole-zero plot below. The ROC is: $Re(s) > -2$. You should be able to do so without computational aids.



$$X(s) = G\frac{(s+2)(s-3)}{(s+2+j)(s+2-j)}, \quad \text{Re}(s) > -2$$, where G is an unknown constant.

f) Write *X(s)* for the function shown in the pole-zero plot below. The ROC is: *Re(s) > –2*. You should be able to do so without computational aids.



$$X(s) = G\frac{(s-1)^2}{(s+2-j2)(s+2+j2)}, \quad \mathrm{Re}(s) > -2$$, where G is an unknown constant.

2) Bilateral Laplace Transform Applications: Again, you are encouraged to use MATLAB for numerical computation.

a) Find the impulse response of the causal, LTI, zero-state system satisfying:

$$\frac{d^2\,y(t)}{dt^2} + 9\frac{d\,y(t)}{dt} + 14\,y(t) = x(t).$$

Transform to Laplace domain:

$$s^2\,Y(s) + 9\,s\,Y(s) + 14\,Y(s) = X(s)$$

Find system function:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 9s + 14}$$

Transform back to time domain:

$$H(s) = \frac{-1/5}{s+7} + \frac{1/5}{s+2}, \quad \mathrm{Re}(s) > -2$$

Then, via inverse *s*-transform:

$$h(t) = \left[\frac{-e^{-7t}}{5} + \frac{e^{-2t}}{5}\right]\mu(t)$$

b) Find the ***step*** response of the same causal, LTI, zero-state system from the prior problem. Note that the step response is equal to: $L^{-1}\left\{\frac{1}{s}\cdot H(s)\right\}$, where *H(s)* is the system function, $L^{-1}\{\bullet\}$ is the Inverse Bilateral Laplace Transform operator, and *1/s* is the Bilateral Laplace transform of the step function.

$$H(s)=\frac{Y(s)}{X(s)}=\frac{1}{s^2+9s+14}.$$

When the input is a step: $x(t)=\mu(t)\Leftrightarrow X(s)=\frac{1}{s}$ and (by convention) *y(t)* is denoted as *s(t)*.

So,

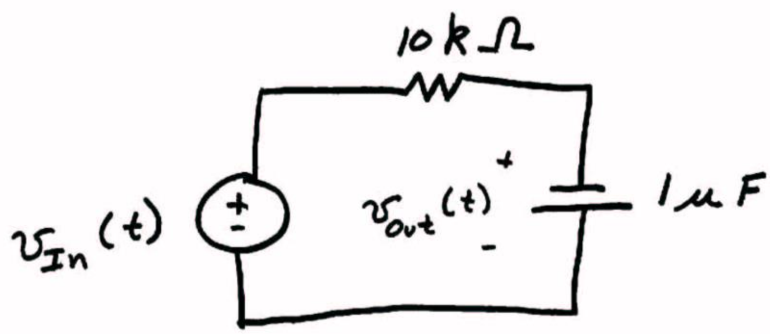
$$S(s)\equiv Y(s)=\frac{X(s)}{s^2+9s+14}=\frac{1/s}{s^2+9s+14}=\frac{1}{s^3+9s^2+14s}$$

$$=\frac{0.029}{s+7}-\frac{0.1}{s+2}+\frac{0.071}{s},\quad \mathrm{Re}(s)>0$$

Via the inverse *s*-transform, the Step Response is:

$$s(t)=\left[0.029e^{-7t}-0.1e^{-2t}+0.071\right]\mu(t)$$

c) Find the zero-state response for *t>0* for the circuit below, with input $v_{In}(t)=4u(t)$ *V* and output $v_{Out}(t)$:



In the Laplace domain:

$$V_{In}(s) = I(s)\cdot R + \frac{I(s)}{sC}$$

(1) $$I(s) = \frac{V_{In}(s)}{R + \frac{1}{sC}} = \frac{sCV_{In}(s)}{1 + sRC}$$

(2) $$V_{Out}(s) = \frac{I(s)}{sC} = \frac{1}{sC}\cdot\frac{sCV_{In}(s)}{1+sRC} = \frac{V_{In}(s)}{1+sRC}$$

Here,

$v_{In}(t) = 4\,\mu(t)$, thus via the Laplace transform: $V_{In}(s) = \frac{4}{s}$

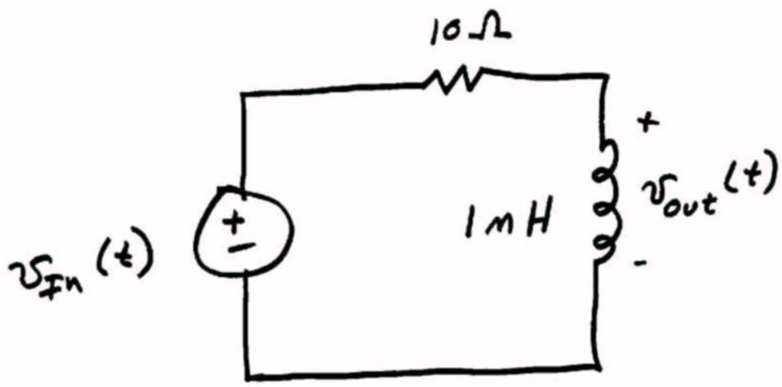
Substitute R = 10k, C = 1μ:

$$V_{Out}(s) = \frac{4}{s\,(1+0.01s)} = \frac{-4}{s+100} + \frac{4}{s}, \quad \mathrm{Re}(s) > 0$$

Then, via inverse *s*-transform:

$$v_{Out}(t) = \left[-4e^{-100t} + 4\right]\mu(t) \quad V$$

d) Find the zero-state response for *t>0* for the circuit below, with input $v_{In}(t) = e^{-4t}\,u(t)$ V and output $v_{Out}(t)$:



In the Laplace domain:

$$V_{In}(s) = I(s) \cdot R + I(s) \cdot sL$$

(1) $$I(s) = \frac{V_{In}(s)}{R + sL}$$

(2) $$V_{Out}(s) = I(s) \cdot sL = \frac{sLV_{In}(s)}{R + sL}$$

Here,

$v_{In}(t) = e^{-4t}\ \mu(t) \quad V$, thus via Laplace transform: $V_{In}(s) = \dfrac{1}{s+4}$.

Substitute R = 10, L = 1m:

$$V_{Out}(s) = \frac{10^{-3}s}{(s+4)(10 + 10^{-3}s)} = \frac{10^{-3}s}{10^{-3}s^2 + 10.004\,s + 40}$$

$$= \frac{1.0004}{s + 10000} - \frac{0.4m}{s+4}, \text{ and } \mathrm{Re}(s) > -4$$

Then, via inverse *s*-transform:

$$v_{Out}(t) = \left[1.0004\,e^{-10000t} - 0.4m\,e^{-4t}\right]\mu(t) \quad V$$

Note: the term $1.0004\,e^{-10000t}$ decays very fast and the term $0.4m\,e^{-4t}$ is rather insignificant.