Solution

Worcester Polytechnic Institute — Department of Electrical and Computer Engineering
ECE2311 — Continuous-Time Signal and System Analysis — Term B'17

Homework 2: Due Tuesday, 31 October 2017 (3:00 P.M.)

Write your name and ECE box at the top of each page.

General Reminders on Homework Assignments:

- Always complete the reading assignments *before* attempting the homework problems.
- Show all of your work. Use written English, where applicable, to provide a log or your steps in solving a problem. (For numerical homework problems, the writing can be brief.)
- A solution that requires physical units is *incorrect* unless the units are listed as part of the result.
- Get in the habit of underlining, circling or boxing your result.
- Always write neatly. Communication skills are essential in engineering and science. "If you didn't write it, you didn't do it!"

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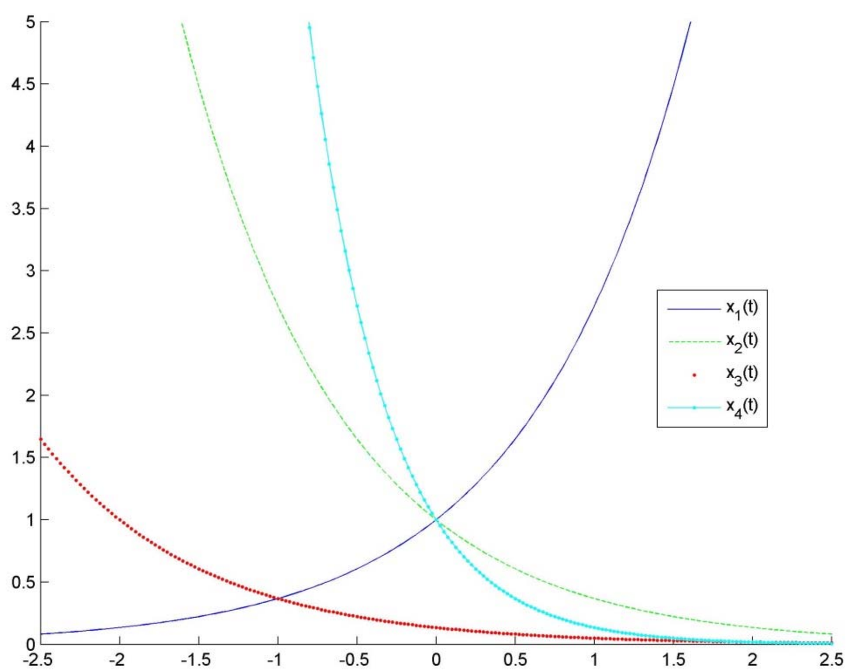
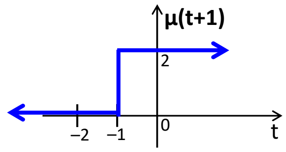
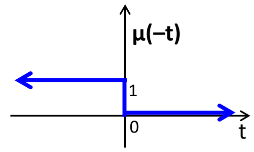
1) Basic Signals:

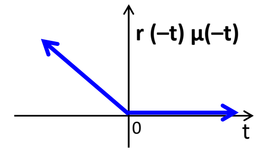
a) By hand (without using a graphing calculator, etc.), sketch and label on the same axes: $x_1(t) = e^t$, $x_2(t) = e^{-t}$, $x_3(t) = e^{-t-2}$ and $x_4(t) = e^{-2t}$. Express *t* in seconds.



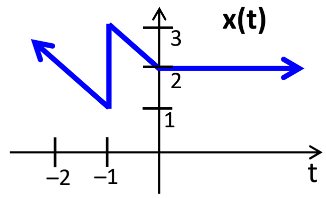
b) Sketch and label: $x(t) = 2u(t+1) + r(-t)u(-t)$, where $u(\cdot)$ is the unit step function and $r(\cdot)$ is the unit ramp function.







Thus, *x(t)*:



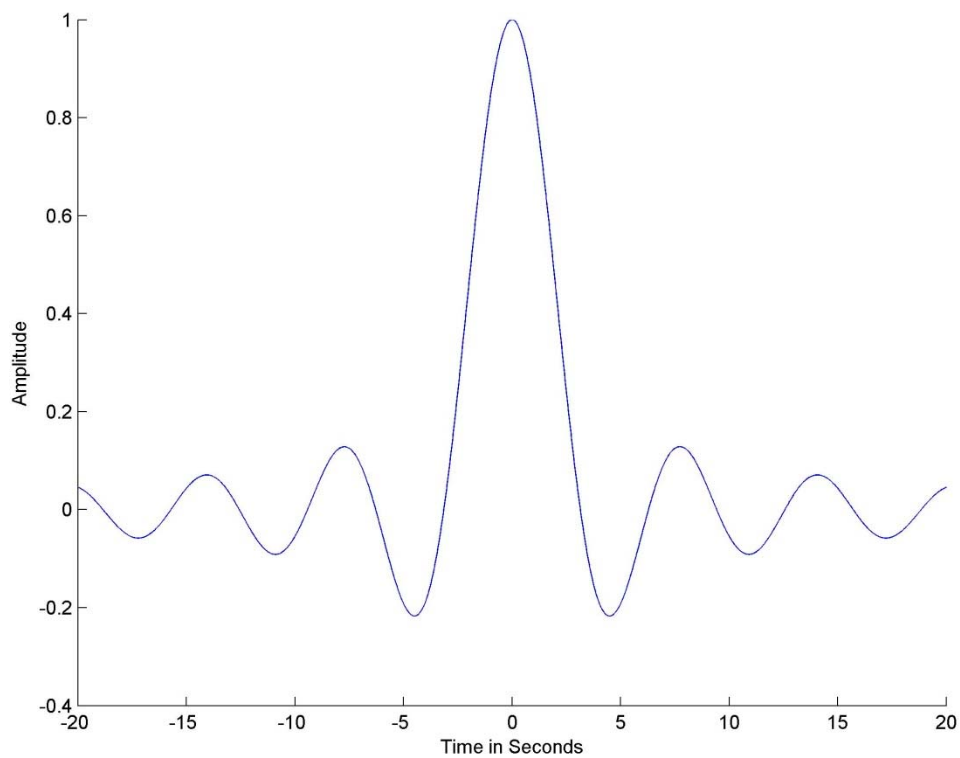
c) Use MATLAB's numerical software (***not*** its symbolic software) to plot $x(t) = \dfrac{\sin(t)}{t}$ over the range t = [–20, 20] seconds. Be sure (i) to use sufficient resolution to accurately show the waveform shape (the time values –20, –19, –18, …, 20 will *not* be adequate), and (ii) to generate the correct value at $t = 0$. Note that if you allow MATLAB to compute the value when $t = 0$, the division by 0 will result in the NaN value (NaN = "Not a Number"). The actual value at $t = 0$ can be found analytically by application of L'Hôpital's rule, with that value equaling *x(0) = 1*. Find a way to force MATLAB to plot this value at $t = 0$. **Hand-in printouts of your plot *and* your MATLAB code.** Be sure that all axes are properly labeled by using the MATLAB xlabel() and ylabel() commands.

Here is the printout of my MATLAB function. Of course, your code will vary:

```
function hw1_3c

t = -20:0.01:20;  % Time vector.  Lots of resolution.

x = sin(t)./t;  % Need element-wise division.  Will get warning when t=0.
Index = find(t==0);  % Not efficient; but easy.
if ~isempty(Index), x(Index) = 1; end  % Fixes value at t=0.

plot(t, x), box('off')
xlabel('Time in Seconds')
ylabel('Amplitude')
% print -djpeg -r300 hw1_3c.jpg

return
```

Note: Another (easier?) way to deal with $t = 0$ might be to create a time vector that does not have a zero-valued entry. For example: `t = -19.995:0.01:20;`

Here is my plot:



d) Determine the signal power in: $x(t) = 7\cos(t)$.

Signal is periodic with frequency $\omega_0 = 1$ rad/sec,

Or, $T_0 = \dfrac{2\pi}{\omega} = 2\pi$ sec.

So,

$$P_x = \frac{1}{2\pi}\int_{t=0}^{2\pi} 49\cos^2(t)\,dt = \frac{49}{2\pi}\left[\frac{t}{2}+\frac{\sin(2t)}{4}\right]\Bigg|_{t=0}^{2\pi}$$

$$= \frac{49}{2\pi}\left\{\left[\frac{2\pi}{2}+\frac{\sin(4\pi)}{4}\right]-\left[\frac{0}{2}+\frac{\sin(0)}{4}\right]\right\} = \frac{49}{2} \text{ unit}^2$$

e) Determine the signal energy in: $x(t) = 6e^{-|3t|}$.

$$E_x = \int_{t=-\infty}^{\infty}\left|6e^{-|3t|}\right|^2 dt = 36\int_{t=-\infty}^{0} e^{6t}\,dt + 36\int_{t=0}^{\infty} e^{-6t}\,dt = 36\left(\frac{e^{6t}}{6}\right)\Bigg|_{t=-\infty}^{0} + 36\left(\frac{e^{-6t}}{-6}\right)\Bigg|_{t=0}^{\infty}$$

$$= 36\left(\frac{e^{6\cdot 0}-e^{-6\cdot\infty}}{6}\right) + 36\left(\frac{e^{-6\cdot\infty}-e^{-6\cdot 0}}{-6}\right) = \frac{36}{6}+\frac{-36}{-6} = \frac{72}{6} = 12 \text{ unit}^2\cdot\text{sec}$$

2) The Impulse: Let $\delta(t)$ be the impulse function.

a) Simplify: $x(t) = \left[\dfrac{\cos(5t)}{t^3+3}\right]\cdot\delta(t)$.

Recall: $x(t)\cdot\delta(t-T) = x(T)\delta(t-T)$

So, $x(t) = \left[\dfrac{\cos(5\cdot 0)}{0^3+3}\right]\delta(t) = \dfrac{\delta(t)}{3}$

b) Simplify: $X(\omega) = \left[\dfrac{15}{j\omega+5}\right]\cdot\delta(\omega-5)$.

Here, impulse is non-zero at ω=5:

$$X(\omega) = \left[\frac{15}{j5+5}\right]\delta(\omega-5) = \left[\frac{3}{j+1}\right]\delta(\omega-5) = \left[\frac{3-3j}{2}\right]\delta(\omega-5)$$

c) Evaluate: $x(t) = \displaystyle\int_{\tau=-\infty}^{\infty}\delta(\tau-2)e^{-3\tau}\,d\tau$.

$$x(t) = e^{-3\tau}\Big|_{\tau=2} = e^{-6}$$

d) Evaluate: $x(t) = \displaystyle\int_{\tau=-\infty}^{\infty}\delta(\tau+4)e^{-3\tau}\,d\tau$.

$$x(t) = e^{-3\tau}\Big|_{\tau=-4} = e^{12}$$

3) Classification of Signals:

a) Determine if each of the following signals is periodic. If so, list its frequency in Hertz.

i) $x(t) = 5.411\sin\left(3.2t + \frac{4\pi}{3}\right)$.

Periodic.

$$\omega_0 = 3.2\,\text{rad/s} \;\text{ or }\; f_0 = \frac{\omega_0}{2\pi} = \frac{3.2}{2\pi} \approx 0.509 \;\text{ Hz}$$

ii) $x(t) = 4e^{j(\pi t + 2)}$.

Periodic.

$$4e^{j(\pi t+2)} = 4\cos(\pi t + 2) + j4\sin(\pi t + 2)$$

$$\omega_0 = \pi\,\text{rad/s} \;\text{ or }\; f_0 = \frac{\omega_0}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2} \;\text{ Hz}$$

iii) $x(t) = 7u(-t)$.

Not periodic.

b) For each below, let *y(t)* be the system output and *x(t)* be the system input. For each, determine if the system is static vs. dynamic *and* causal vs. non-causal.

i) $y(t) = 4x(t) + 3x(t-1) + 2x(t-2)$.

Dynamic, causal.

ii) $y(t) = 4x(t) + 3x(t+1) + 2x(t+2)$.

Dynamic, non-causal.

iii) $y(t) = 7x^2(t)$.

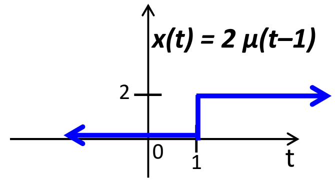
Static, causal.

iv) $y(t) = \int_{\tau=-\infty}^{t} x(\tau)\,d\tau$.

Dynamic, causal.

c) Determine and sketch the even and odd parts of: $x(t) = 2u(t-1)$, where $u(\cdot)$ is the step function.

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$



$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

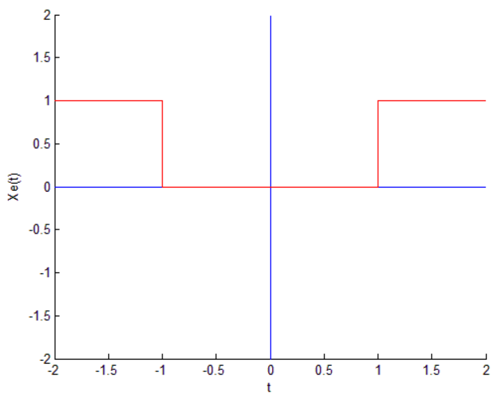
From the drawing of $x(t)$ and the definitions of $x_e(t)$ and $x_o(t)$:

$$x_e(t) = \begin{cases} 0, & -1 < t < 1 \\ 1, & \text{otherwise} \end{cases}$$

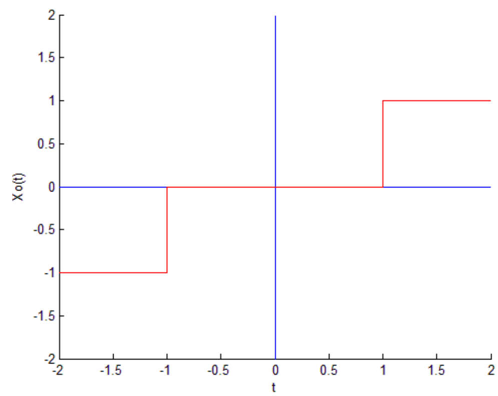
$$x_o(t) = \begin{cases} -1, & t \le -1 \\ 0, & -1 < t < 1 \\ 1, & t \ge 1 \end{cases}$$

Sketch the figures:

$x_e(t)$



$x_o(t)$



d) Determine and sketch the even and odd parts of: $x(t) = \sin(t)$.

$$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{\sin(t)+\sin(-t)}{2} = \frac{\sin(t)-\sin(t)}{2} = 0$$

$$x_o(t) = \frac{x(t)-x(-t)}{2} = \frac{\sin(t)-\sin(-t)}{2} = \frac{\sin(t)+\sin(t)}{2} = \sin(t)$$

Thus, *sin(t)* is an odd function.

Sketch of $x(t) = x_o(t)$: