Solution

Worcester Polytechnic Institute — Department of Electrical and Computer Engineering
ECE2311 — Continuous-Time Signal and System Analysis — Term B'17

Homework 1: Due Friday, 27 October 2017 (3:00 P.M.)

Write your name and ECE box at the top of each page.

General Reminders on Homework Assignments:

- Always complete the reading assignments *before* attempting the homework problems.
- Show all of your work. Use written English, where applicable, to provide a log or your steps in solving a problem. (For numerical homework problems, the writing can be brief.)
- A solution that requires physical units is *incorrect* unless the units are listed as part of the result.
- Get in the habit of underlining, circling or boxing your result.
- Always write neatly. Communication skills are essential in engineering and science. "If you didn't write it, you didn't do it!"

---

0) MATLAB Tutorial(s): MATLAB is a higher-level software tool originally developed for matrix manipulation and signal processing. The tool has expanded greatly into many fields of numerical and symbolic analysis, and even embedded system prototyping and soft real-time data acquisition. We will be using MATLAB throughout this course (including this homework assignment). In general, we assume that you are NOT familiar with MATLAB when you begin this course. Thus, your first assignment is to BECOME familiar with the basic command-line operation of MATLAB. Choose one or more of the following options and become familiar with simple command-line (interactive) computation in MATLAB:

a) The MathWorks (developers of MATLAB) maintains a set of entry-level tutorials on their corporate site at http://www.mathworks.com/academia/student_center/tutorials/. I recommend "MATLAB Tutorial: Get started with the fundamentals of MATLAB."

b) The MATLAB software itself has written tutorials and examples. To use them, begin by launching MATLAB (on WPI computers, MATLAB is usually available from the MicroSoft "start" panel). Type "doc" into the MATLAB command window, then select "MATLAB" from the Documentation window.

c) Type "MATLAB tutorial" into your favorite Internet search engine. A plethora of academic sites and others have MATLAB tutorials publicly available.

NOTE: No specific work need be turned-in with problem 0 of this homework assignment.

1) Review of Complex Numbers and Phasors: Let $j = \sqrt{-1}$.

a) Let $c_1 = 2 - j4$ and $c_2 = -1 + j5$. Compute (by hand—no calculator) in rational, rectangular form: $c_1 \cdot c_2$.

$$c_1 \cdot c_2 = (2 - j4)(-1 + j5) = -2 + j4 + j10 + 20 = 18 + j14$$

(Rectangular form required)

b) Let $c_3 = -6 + j7$ and $c_4 = -5 + j2$. Compute (by hand—no calculator) in rational, rectangular form: $\dfrac{c_3}{c_4^*}$, where c* denotes the complex conjugate.

$$\frac{c_3}{c_4^*} = \frac{-6+j7}{-5-j2} \cdot \frac{-5+j2}{-5+j2} = \frac{30-j35-j12-14}{25+4} = \frac{16-j47}{29}$$

(Rectangular form required)

c) If $c_5 = -2 - j3$, write c_5 in polar form (magnitude and angle). Use a hand calculator for the magnitude and angle computation.

$c_5 = -2 - j3$ Note: 3rd quadrant.

$$c_5 = \sqrt{13}\angle\left(\tan^{-1}(3/2)+\pi\right) = 3.61\angle -123.7°$$

(Polar form required)

d) By hand (no calculator), determine $7\angle 270^o + 7\,e^{-j\pi}$ in polar form.

$$7\angle 270° + 7\,e^{-j\pi} = -j7 + (-7) = -7 - j7 = \sqrt{49+49}\,\angle 225°$$

$$= \sqrt{49 \cdot 2}\,\angle 225° = 7\sqrt{2}\,\angle 225°$$

(Polar form required)

e) By hand (no calculator), use phasors to find the following sum: $11\cos(50\,t) - 11\cos\left(50\,t - \dfrac{\pi}{2}\right)$.

(Be sure to express the result in the time domain.)

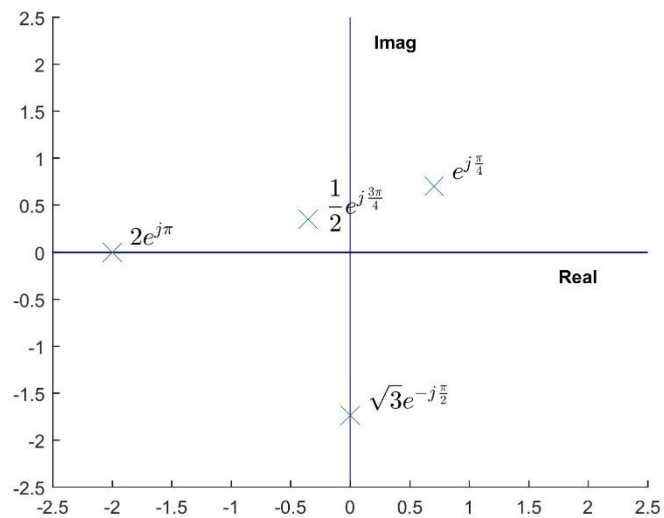
Since both cosines are of the same frequency, we can use phasors:

$$11\cos(50t) - 11\cos(50t - \pi/2)$$

$$\Rightarrow 11\angle 0° - 11\angle -90° = 11 - (-j11) = 11 + j11 = \sqrt{121 \cdot 2}\angle 45° = 11\sqrt{2}\angle 45°$$

Result: $\boxed{11\sqrt{2}\cos(50t + 45°)}$

f) Draw an axis in the complex plane. Label the following points in this one complex plane. Do so without any conversions. Just look at the expressions and draw them — a skill that will come in handy in this class and other ECE courses. The values to draw are: $2\,e^{j\pi}$, $\sqrt{3}\,e^{-j\frac{\pi}{2}}$, $e^{j\frac{\pi}{4}}$ and $\dfrac{1}{2}\,e^{j\frac{3\pi}{4}}$.



g) Show the following: $e^{\pm j2\pi n} = 1, \; for \, n = 0,1,2,\ldots$. (You need not provide a formal mathematical proof. Just provide appropriate manipulation of the expression to demonstrate the result.)

$$e^{\pm j2\pi n} = \cos(2\pi n) \pm j\sin(2\pi n)$$
$$\sin(2\pi n) = 0, \text{ for } n = 0,1,2,\ldots$$
$$\cos(2\pi n) = 1, \text{ for } n = 0,1,2,\ldots$$

Thus,

$$e^{\pm j2\pi n} = 1 \pm j\,0 = 1, \text{ for } n = 0,1,2,\ldots$$

2) Review of Partial Fraction Expansion: A partial fraction expansion re-writes a ratio of two polynomials as the sum of several simpler polynomial ratios. In particular, the expanded ratios each have first-degree denominators. For example, $\frac{6s^2 - 40s + 60}{s^3 - 11s^2 + 36s - 36} = \frac{1}{s-2} + \frac{2}{s-3} + \frac{3}{s-6}$. More review information about partial fraction expansion is available in (a) the appendix of Oppenheim *et al.*, (b) in the textbook "Linear Systems and Signals, 2nd edition" by B.P. Lathi, (3) as well as on-line (for example "http://mathworld.wolfram.com/PartialFractionDecomposition.html").

a) By hand (no calculator or MATLAB), find the partial fraction expansion for: $\frac{-s-10}{s^2 - s - 2}$.

$$\frac{-s-10}{s^2-s-2}=\frac{-s-10}{(s-2)(s+1)}=\frac{A}{s-2}+\frac{B}{s+1}$$

a) Multiply both sides by (s–2), evaluate at s = 2:

$$\frac{-s-10}{s+1}=A+\frac{s-2}{s+1}\cdot B$$

Evaluate at s = 2:

$$\frac{-2-10}{2+1}=A+0\Rightarrow A=-4$$

b) Multiply both sides by (s+1), evaluate at s = –1:

$$\frac{-s-10}{s-2}=\frac{s+1}{s-2}\cdot A+B$$

Evaluate at s = –1:

$$\frac{1-10}{-1-2}=0+B\Rightarrow B=3$$

So,

$$\boxed{\frac{-s-10}{s^2-s-2}=\frac{-4}{s-2}+\frac{3}{s+1}}$$

b) Use the residue() function in MATLAB to expand the fraction given in problem "a" above. [The command you should need is: "[r, p, k] = residue([-1 -10], [1 -1 -2])".] You need not hand in any MATLAB code with this problem. Write the partial fraction expansion.

Using "[r, p, k] = residue([-1 -10], [1 -1 -2])" leads to

$$\frac{-s-10}{s^2-s-2}=\frac{-4}{s-2}+\frac{3}{s+1}$$

c) Use the residue() function in MATLAB to expand: $\frac{2s^3-16s^2+40s-42}{s^3-11s^2+39s-45}$. You need not hand in any MATLAB code with this assignment. Write the partial fraction expansion.

NOTE: We will use partial fraction expansion later in the course. MATLAB (and similar computational software) are invaluable tools to reduce the time it takes to perform these factorizations as well as reduce the incidence of computational errors.

Using "[r, p, k] = residue([2 -16 40 -42], [1 -11 39 -45])" leads to

$$\frac{s^3-10s^2+35s-41}{s^3-7s^2+16s-12}=2+\frac{2}{s-5}+\frac{4}{s-3}+\frac{6}{(s-3)^2}$$