Solution

Worcester Polytechnic Institute — Department of Electrical and Computer Engineering
ECE2311 — Continuous-Time Signal and System Analysis — Term B'17

Homework 11: Due Tuesday, 12 December 2017 (3:00 P.M.)

Write your name and ECE box at the top of each page.

General Reminders on Homework Assignments:

- Always complete the reading assignments *before* attempting the homework problems.
- Show all of your work. Use written English, where applicable, to provide a log or your steps in solving a problem. (For numerical homework problems, the writing can be brief.)
- A solution that requires physical units is *incorrect* unless the units are listed as part of the result.
- Get in the habit of underlining, circling or boxing your result.
- Always write neatly. Communication skills are essential in engineering and science. "If you didn't write it, you didn't do it!"

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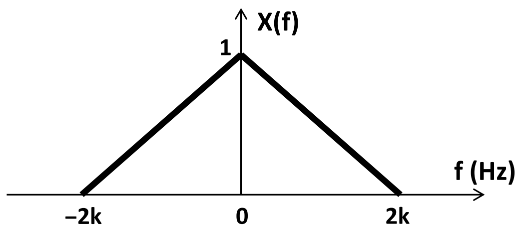
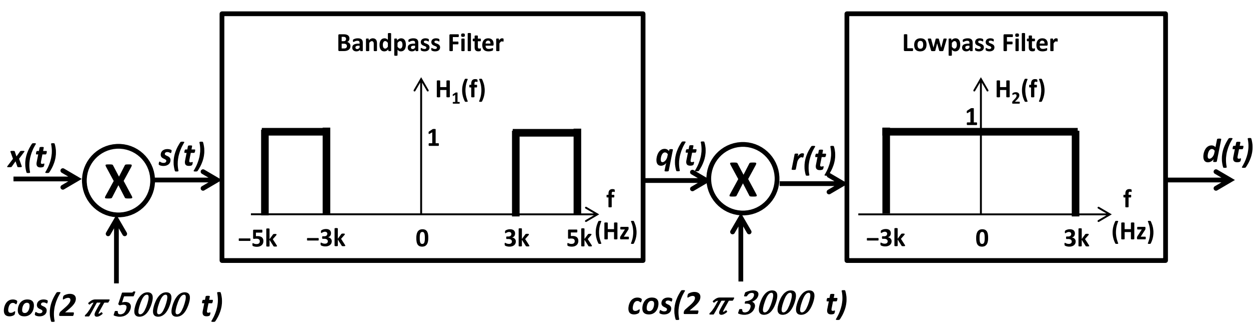
1) Sinusoidal Amplitude Modulation:

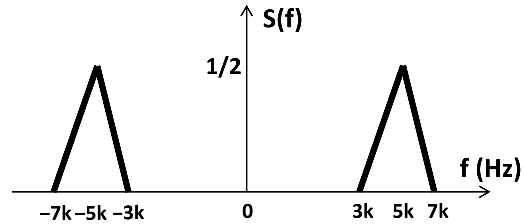
a) Consider the signal *x(t)* with magnitude spectrum shown below:



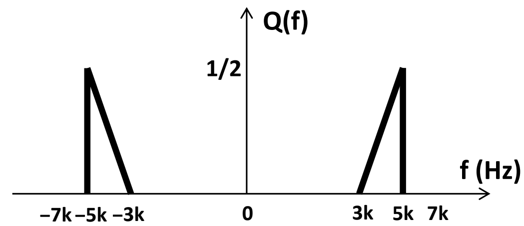
Let *x(t)* be processed through the system shown below to produce output signal *d(t)*. Draw the magnitude spectra of signals *s(t)*, *q(t)*, *r(t)* and *d(t)*.



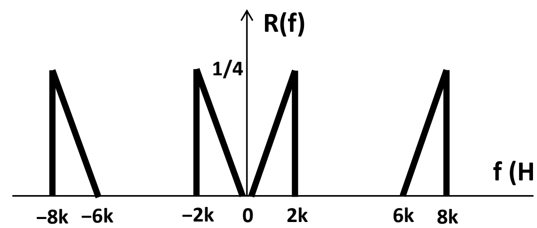
The first mixer creates two copies of the original magnitude spectrum, shifted by ±5000 Hz and reduced in magnitude by half. The resulting *S(f)* is shown to the right.



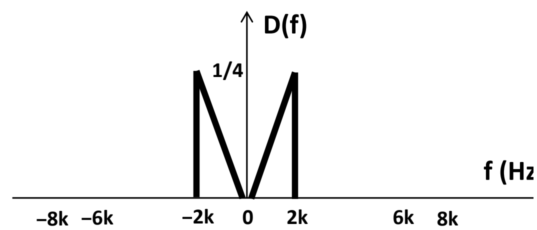
The bandpass filter removes the frequencies above 5k Hz in the modulated signal, producing *Q(f)* shown to the right.



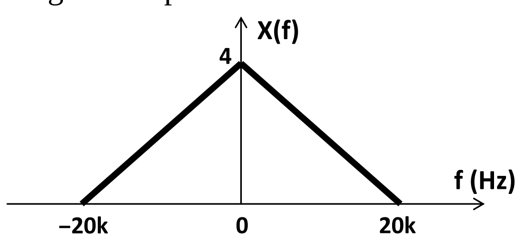
The second mixer shifts BOTH the positive and negative frequencies each by 3k Hz and reduces the magnitude in half. The resulting *R(f)* is shown to the right.



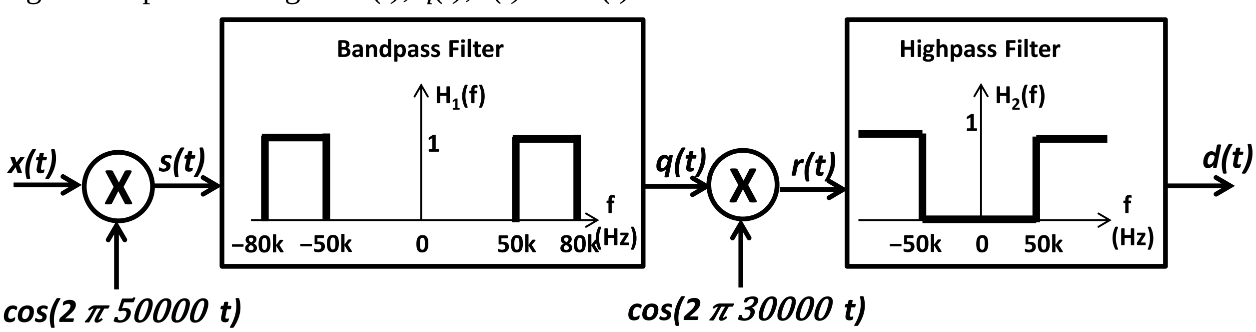
Finally, the lowpass filter eliminates the higher frequency portion, producing the final output *D(f)*.



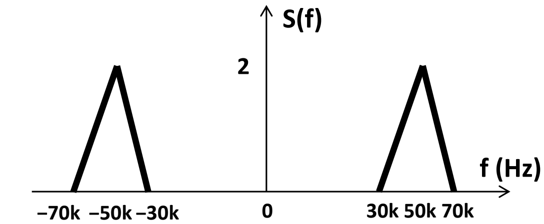
b) Consider the signal *x(t)* with magnitude spectrum shown below:



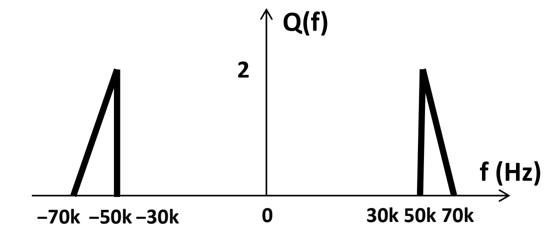
Let *x(t)* be processed through the system shown below to produce output signal *d(t)*. Draw the magnitude spectra of signals *s(t)*, *q(t)*, *r(t)* and *d(t)*.



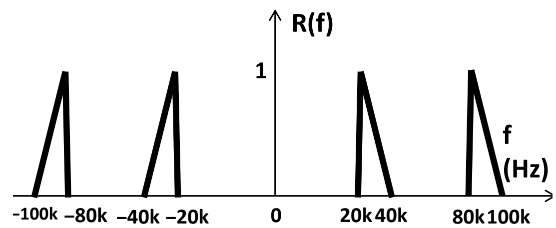
The first mixer creates two copies of the original magnitude spectrum, shifted by ±50k Hz and reduced in magnitude by half. The resulting *S(f)* is shown to the right.



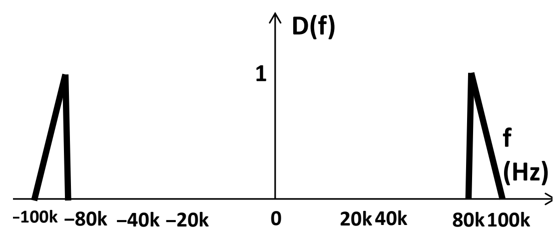
The bandpass filter removes the frequencies below 50k Hz in the modulated signal, producing *Q(f)* shown to the right.



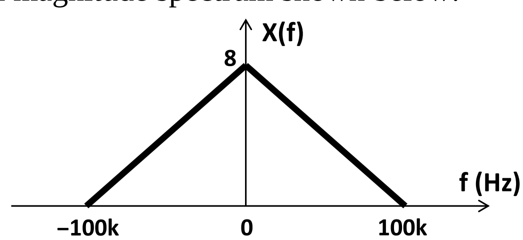
The second mixer shifts BOTH the positive and negative frequencies each by 30k Hz and reduces the power in half. The resulting *R(f)* is shown to the right.



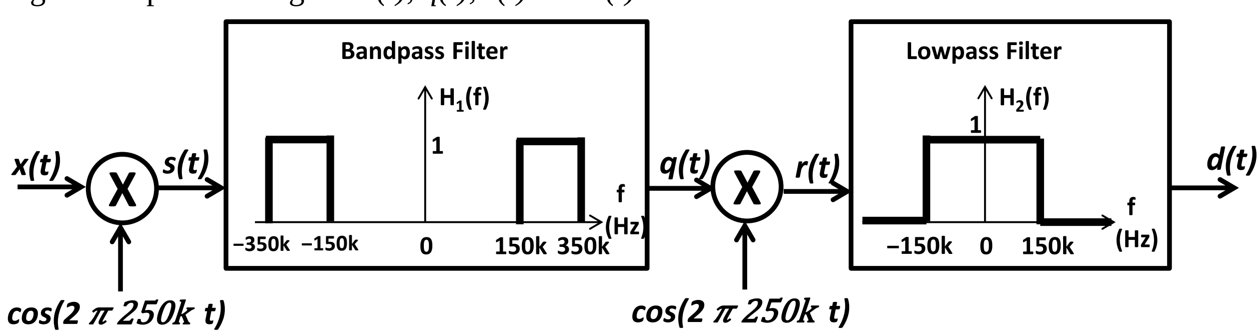
Finally, the highpass filter eliminates the lower frequency portion, producing the final output *D(f)*.

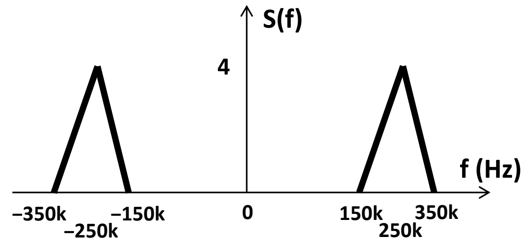
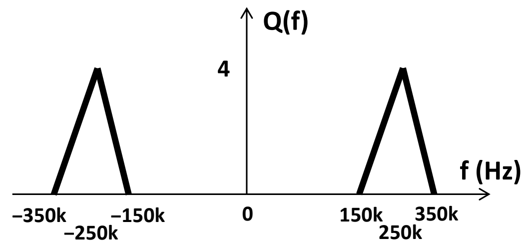
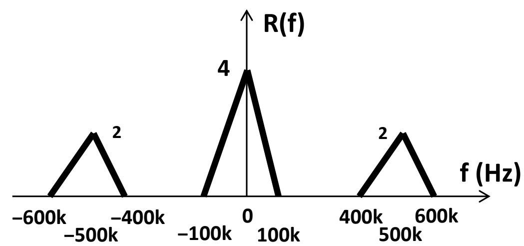
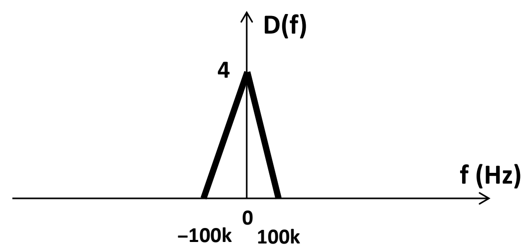


c) Consider the signal *x(t)* with magnitude spectrum shown below:



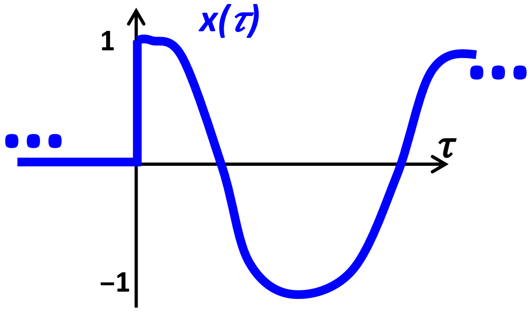
Let *x(t)* be processed through the system shown below to produce output signal *d(t)*. Draw the magnitude spectra of signals *s(t)*, *q(t)*, *r(t)* and *d(t)*.

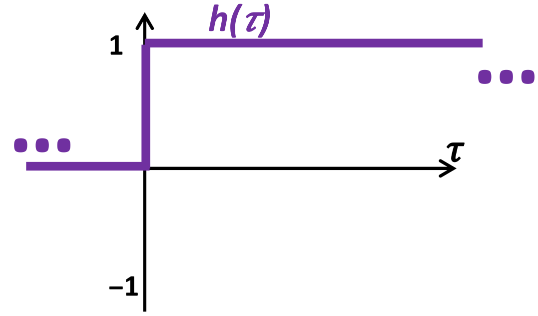


The first mixer creates two copies of the original magnitude spectrum, shifted by ±250k Hz (thus, centered at ±250k Hz) and reduced in magnitude by half. The resulting *S(f)* is shown to the right.	
The bandpass filter extends exactly over the bandwidth of the input signal (and has a pass band gain of 1), thus does not alter the signal. The magnitude spectrum of *Q(f)* is shown to the right.	
The second mixer shifts BOTH the positive and negative frequencies each by 250k Hz (so, each has one term re-centered at DC) and reduces the magnitude of each in half. Since the overlapping magnitudes at DC sum, the resulting *R(f)* is shown to the right.	
Finally, the lowpass filter eliminates the upper frequency portion, producing the final output *D(f)*.	

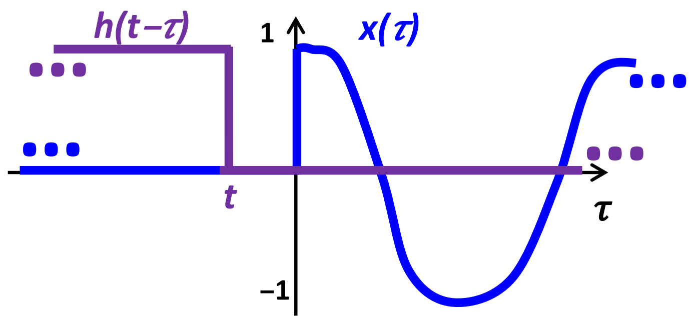
2) Convolution:

a) Use direct integration of the convolution integral/graphical convolution to determine the convolution $y(t)=x(t)*h(t)$ if $x(t)=\cos(t)\,\mu(t)$ and $h(t)=\mu(t)$.

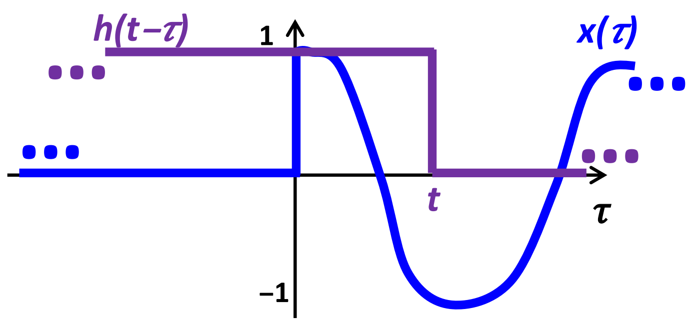




- For $t < 0$: No overlap. Convolution equals zero in this range.



- For $t \geq 0$: Partial overlap from the left.

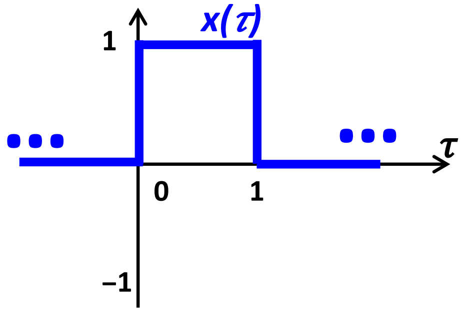


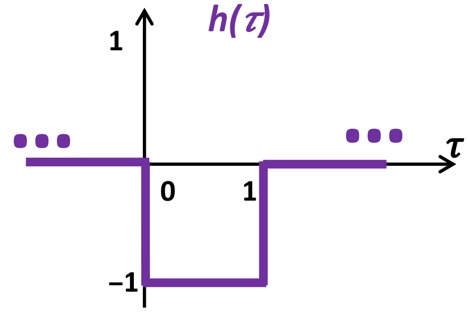
$$y(t) = \int_{\tau=0}^{t} \cos(\tau)\cdot 1\, d\tau = \sin(\tau)\Big|_{\tau=0}^{t} = \sin(t) - \sin(0) = \sin(t)$$

- <u>**Total Solution:**</u>

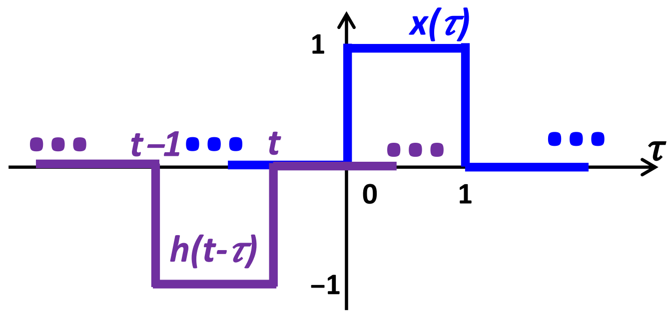
$$y(t) = x(t) * h(t) = \begin{cases} 0, & t < 0 \\ \sin(t), & t \geq 0 \end{cases} = \sin(t)\mu(t)$$

b) Use direct integration of the convolution integral/graphical convolution to determine the convolution $y(t) = x(t) * h(t)$ if $x(t) = \mu(t) - \mu(t-1)$ and $h(t) = -\mu(t) + \mu(t-1)$.

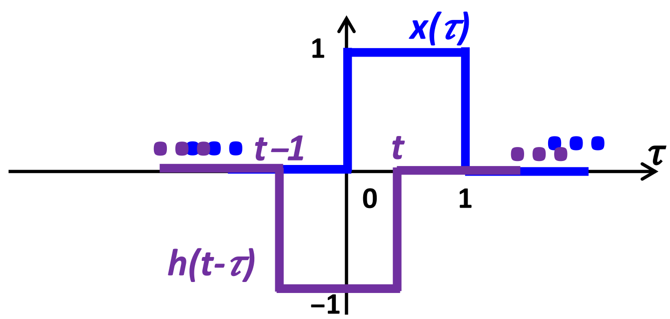




- For t < 0: No overlap. Convolution equals zero in this range.

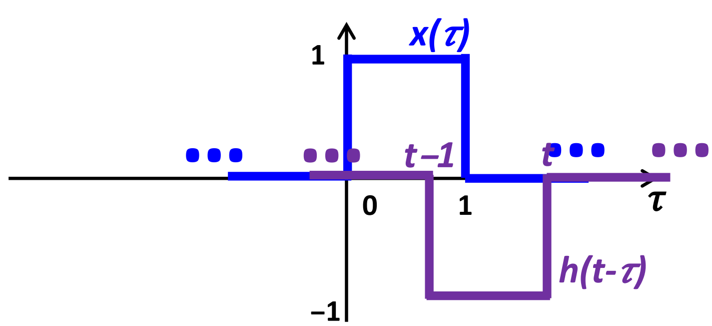


- For $0 \le t \le 1$: Partial overlap from the left.



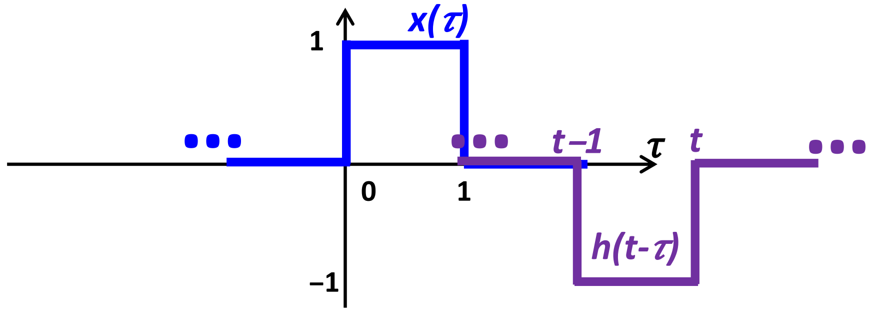
$$y(t) = \int_{\tau=0}^{t} 1 \cdot (-1)\, d\tau = -\tau\Big|_{\tau=0}^{t} = -t - -0 = -t$$

- For $1 \le t \le 2$: Partial overlap from the right.



$$y(t) = \int_{\tau=t-1}^{1} 1 \cdot (-1)\, d\tau = -\tau\Big|_{\tau=t-1}^{1} = -1 - -(t-1) = t-2$$

- For t > 2: No overlap. Convolution equals zero in this range.



- **<u>Total Solution:</u>**

$$y(t) = x(t) * h(t) = \begin{cases} 0, & t < 0 \\ -t, & 0 \le t \le 1 \\ t-2, & 1 < t \le 2 \\ 0, & t > 2 \end{cases}$$