Impulse Response

- System output (response) if input $= \delta(t-\tau)$

$$\delta(t-\tau) \longrightarrow \boxed{\quad \mathcal{T} \quad} \longrightarrow h(t,\tau)$$

- For general system:

$$h(t,\tau) = \mathcal{T}[\delta(t-\tau)]$$

Time index

Time at which impulse occurred

- If system is time-invariant

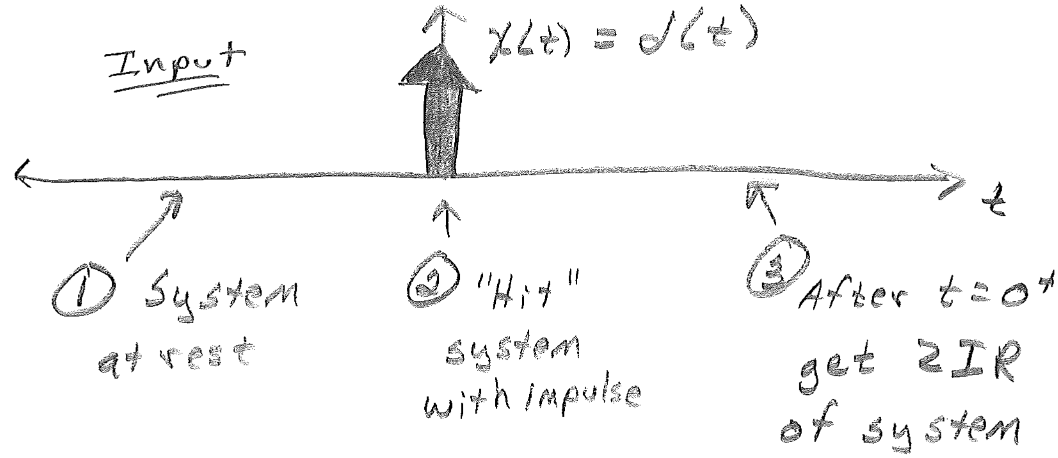
$$h(t-\tau) = \mathcal{T}[\delta(t-\tau)]$$

Function of one variable $\Rightarrow$ time difference between current time and when impulse occurred

> Note: $h(t)$ implies zero-state before $\delta(t)$

Convolution

Impulse Response Concept



- Impulse dumps energy into system, system responds with characteristic modes after $t=0^+$.

- For causal systems:

$$
\begin{cases}
h(t)=0, & \text{for } t<0^- \\
h(t) \text{ can contain impulse}, & \text{for } t=0 \\
h(t) \text{ has characteristic mode terms}, & \text{for } t \geq 0^+
\end{cases}
$$

- Can solve for $h(t)$ in time domain

  $\Rightarrow$ will find easier method later

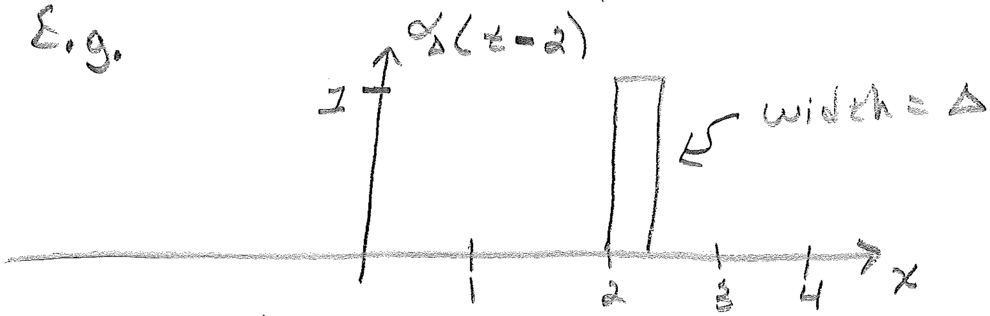
Convolution

Sifting Property of Impulse (1)

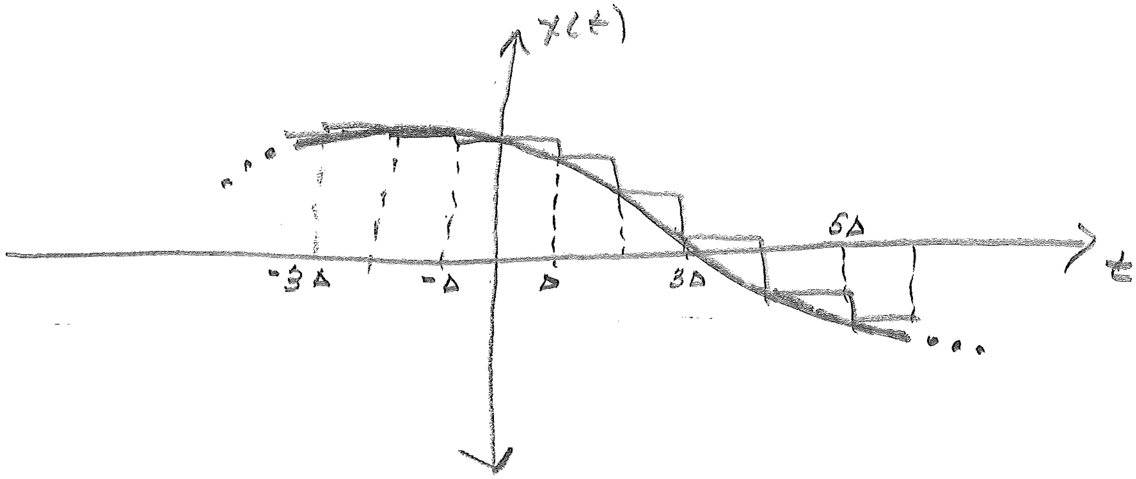
① Define rectangular pulse of unit height, width Δ, starting at location $t=T$:

$$\delta_\Delta(t-T) = \begin{cases} 1 & T \le t < T+\Delta \\ 0, & \text{otherwise} \end{cases}$$

E.g.



② Use many scaled $\delta_\Delta(\cdot)$ to staircase approximate a function $x(t)$:



$$\hat{x}(t) = \sum_{m=-\infty}^{\infty} x(m\Delta)\,\delta_\Delta(t-m\Delta)$$

At any given t_g only one term in sum is non-zero

Continued

Convolution

Sifting Property of Impulse (2)

- From above: $\hat{x}(t) = \sum_{m=-\infty}^{\infty} x(m\Delta)\, \delta_\Delta(t - m\Delta)$

- As $\Delta \to 0$, $\hat{x}(t) \to x(t)$:

$$x(t) = \lim_{\Delta \to 0} \sum_{m=-\infty}^{\infty} \underbrace{x(m\Delta)\, \delta_\Delta(t - m\Delta)}_{\text{Area of rectangle}}$$

$$\downarrow$$

$$\boxed{x(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau}$$

- Will use this property to develop convolution integral.

Derivation of LTI Convolution

- General Continuous-Time System: $y(t) = \mathcal{T}[x(t)]$

- Re-write $x(t)$ using sifting property:

$$y(t) = \mathcal{T}\left[\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right]$$

- Assume <u>Linearity</u>:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\cdot \underbrace{\mathcal{T}\left[\delta(t-\tau)\right]}_{\text{By definition = impulse response}} d\tau$$

Linear, so sum comes out of $\mathcal{T}[\cdot]$

Scales $\delta(\cdot)$, so scaling comes out of $\mathcal{T}[\cdot]$ if linear

- For linear system:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t,\tau)\,d\tau$$

- Assume <u>Time Invariance</u>:

$$\boxed{y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau \equiv x(t) * h(t)}$$

  - Convolution integral of LTI system
  - ∴ LTI system fully characterized by impulse response $h(t-\tau)$

Convolution

Interpretation of Convolution Integral

- **IF**
  - System is LTI

    AND
  - Impulse response known/measured

THEN

- Convolution integral gives

  output $\{y(t)\}$ due to **ANY** input $\{x(t)\}$

- No need to re-calculate homogeneous response or forced response

◦ But, not account for initial conditions $\Rightarrow$ $h(t)$ is found from zero-state system

Logical Consequence of Convolution

- If an LTI system has impulse response $h(t)$

  AND

  an input $x(t) = \delta(t)$ is applied.

THEN

the output <u>must</u> be the impulse response: $y(t) = h(t)$.

- Test with convolution integral:

$$y(t) = \int_{\tau=\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{\tau=\infty}^{\infty} \delta(\tau)\, h(t-\tau)\, d\tau$$

$$\boxed{\text{Recall:} \int_{t=-\infty}^{\infty} x(t) \cdot \delta(t-T)\, dt = x(T)}$$

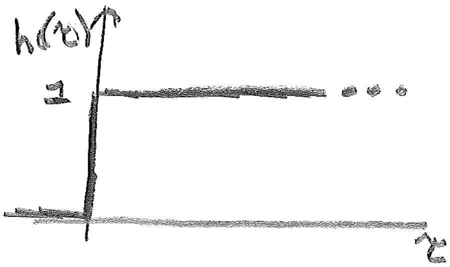
Above, impulse occurs at $\tau = 0$

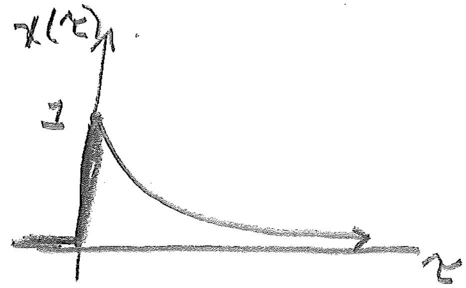
$$\Rightarrow \int_{\tau=-\infty}^{\infty} \delta(\tau)\, h(t-\tau)\, d\tau = \boxed{h(t) = y(t)}$$

Q.E.D.

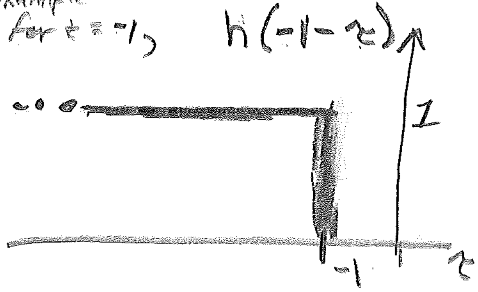
Convolution Example

- LTI system with: $h(t) = u(t)$
- **Find** $y(t)$ if $x(t) = e^{-3t}u(t)$





Example for $t=-1$,



① Value of t "shifts" $h(t-\tau)$

② For $t<0$, no overlap w/ $x(\tau)$
$\longrightarrow y(t) = 0$

③ For $t \geq 0$

$$x(\tau)\,h(t-\tau) = \begin{cases} 1 \cdot e^{-3\tau}, & 0<\tau<t \\ 0, & \text{otherwise} \end{cases}$$

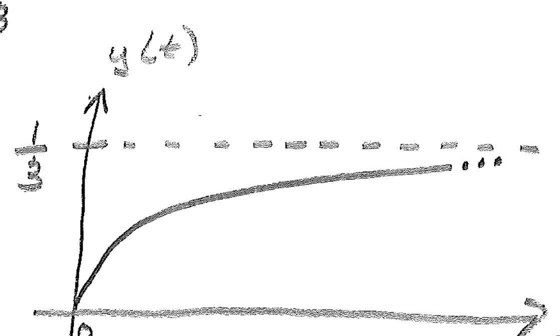
For $t>0$:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = \int_0^t e^{-3\tau}\,d\tau$$

$$= \left.\frac{-e^{-3\tau}}{3}\right|_{\tau=0}^{t} = \frac{1-e^{-3t}}{3}$$

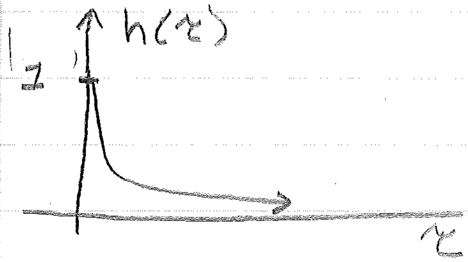
- Overall:

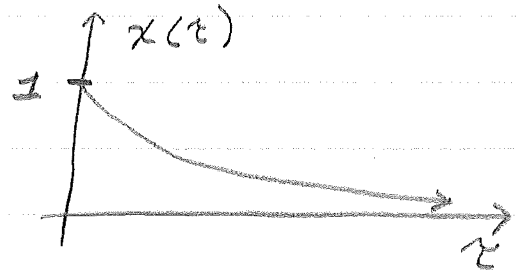
$$\boxed{y(t) = \frac{1-e^{-3t}}{3}\,u(t)}$$

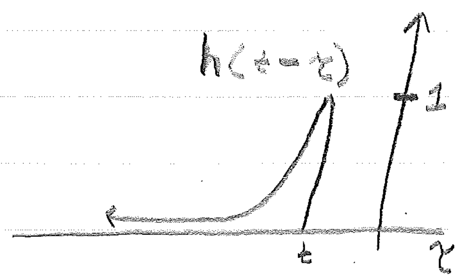


Example: Convolving One-Sided Exponentials

- LTI system with: $h(t) = e^{-2t} u(t)$.
- Find $y(t)$ if $x(t) = e^{-t} u(t)$







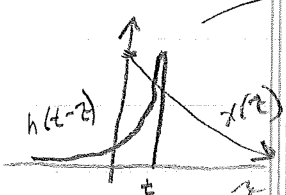
① Value of t "shifts" $h(t-\tau)$

② For $t<0$, no overlap $\Rightarrow y(t)=0$

③ For $t \geq 0$

$$x(\tau) \cdot h(t-\tau) = \begin{cases} e^{-\tau} e^{-2(t-\tau)}, & 0 < \tau < t \\ 0, & \text{otherwise} \end{cases}$$

For $t \geq 0$:



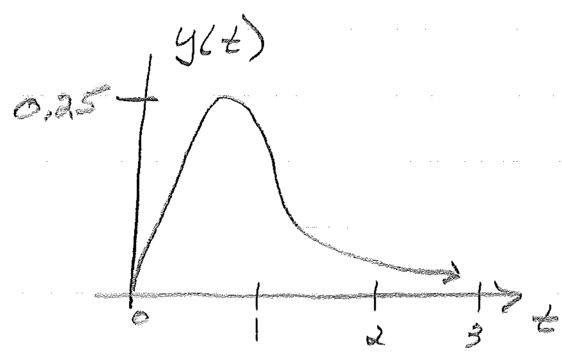
$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$= \int_{\tau=0}^{t} e^{-\tau} e^{-2(t-\tau)} d\tau = e^{-2t} \int_{\tau=0}^{t} e^{\tau} d\tau = e^{-2t} e^{\tau} \Big|_{\tau=0}^{t}$$

$$= e^{-2t}\left(e^{t} - 1\right)$$

- Overall:

$$\boxed{y(t) = \left(e^{-t} - e^{-2t}\right) u(t)}$$



Graphical Convolution: Method

- Useful to visualize convolution & set axis limits for integration

Method

① Fold/reflect $h(\tau)$ about $\tau = 0 \rightarrow h(-\tau)$

② Delay/advance $h(-\tau)$ by $t \rightarrow h(t-\tau)$

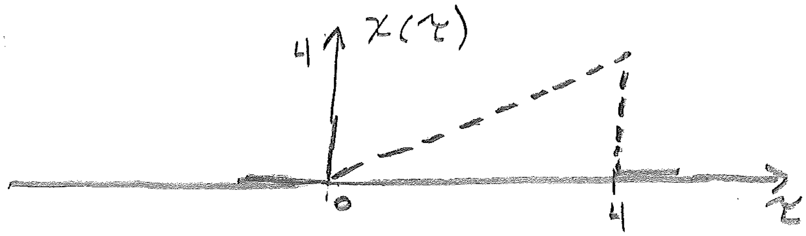
③ Multiply $x(\tau)$ by $h(t-\tau)$ where overlap

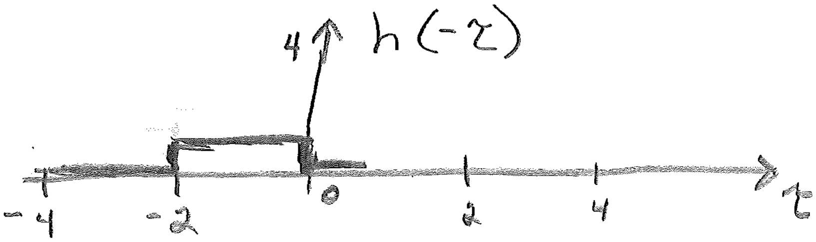
④ Integrate the product $\rightarrow y(t)$

shift to new t (or range of t), then repeat

Graphical Convolution: Example 1 (1)

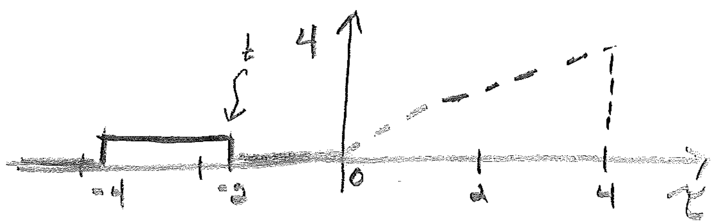
$$x(t) = \begin{cases} t, & 0 < t < 4 \\ 0, & \text{otherwise} \end{cases} \qquad h(t) = \begin{cases} 1, & 0 < t < 2 \\ 0, & \text{otherwise} \end{cases}$$





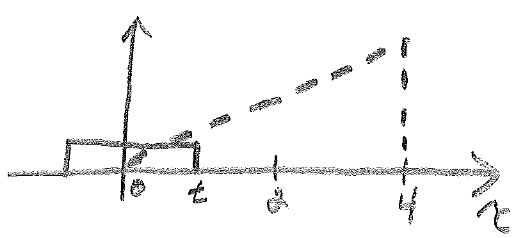
Note:
$t \to +$, slide right
$t \to -$, slide left

Region 1: $t < 0$



No overlap $\Rightarrow$ $y(t) = 0$

Region 2: $0 < t < 2$



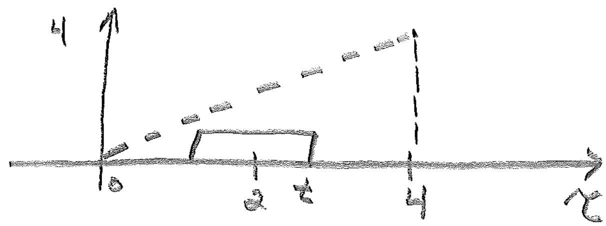
- Partial overlap from left

$$y(t) = \int_0^t \tau \cdot 1 \, d\tau = \frac{\tau^2}{2}\Big|_0^t = \frac{t^2}{2}$$

Continued

Graphical Convolution: Example 1 (2)

- **Region 3: $2 < t < 4$**



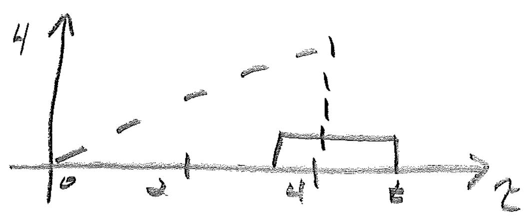
- Full overlap

$$y(t) = \int_{t-2}^{t} \tau \cdot 1 \, d\tau$$

$$= \frac{\tau^2}{2}\Bigg|_{t-2}^{t} = \frac{t^2}{2} - \frac{(t-2)^2}{2}$$

$$= 2t - 2$$

- **Region 4: $4 < t < 6$**



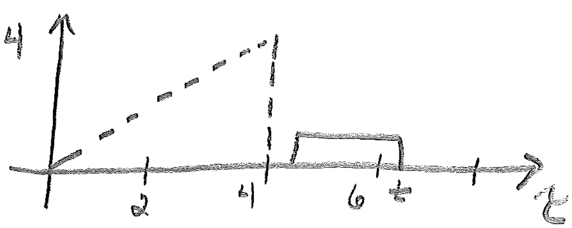
- Partial overlap from right

$$y(t) = \int_{t-2}^{4} \tau \cdot 1 \, d\tau$$

$$= \frac{\tau^2}{2}\Bigg|_{t-2}^{4} = \frac{16}{2} - \frac{(t-2)^2}{2}$$

$$= \frac{-t^2}{2} + 2t + 6$$

- **Region 5: $t > 6$**



No overlap

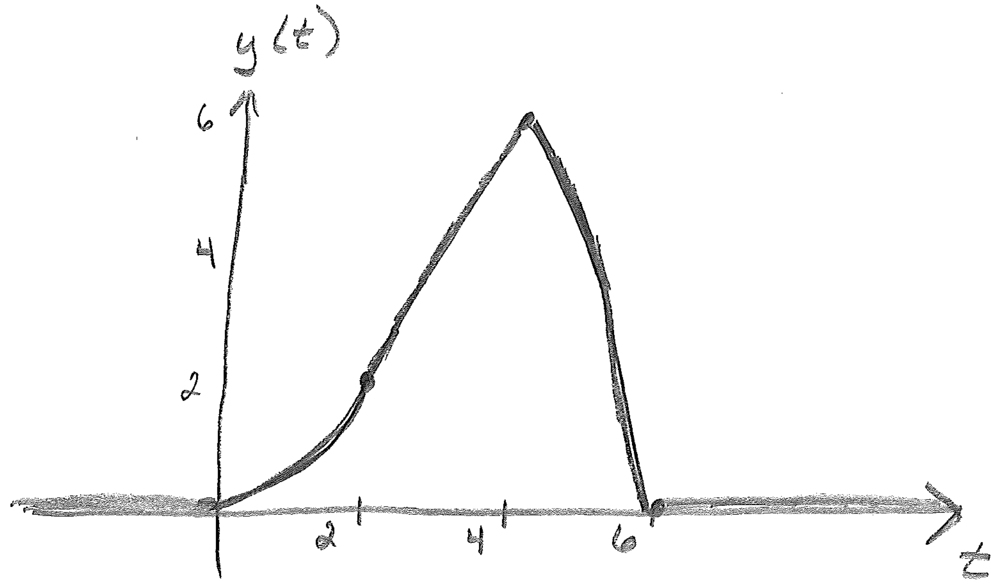
$$y(t) = 0$$

Continued

Graphical Convolution: Example 1 (3)

- Gather complete solution:

$$y(t) = \begin{cases} 0, & t<0 \\ \dfrac{t^2}{2}, & 0<t<2 \\ 2t-2, & 2<t<4 \\ \dfrac{-t^2}{2}+2t+6, & 4<t<6 \\ 0, & t>6 \end{cases}$$

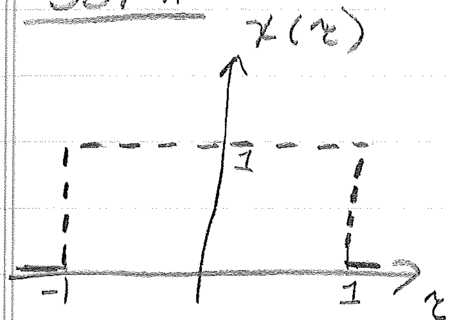


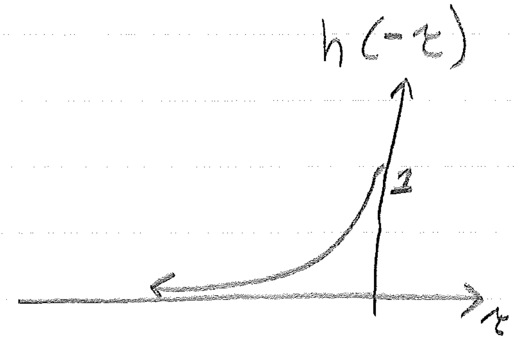
Graphical Convolution: Example 2 (1)

- **Find** $y(t) = x(t) * h(t)$ **if**

$$x(t) = \begin{cases} 1, & -1 < t < 1 \\ 0, & \text{otherwise} \end{cases} \qquad h(t) = e^{-4t}\, u(t)$$

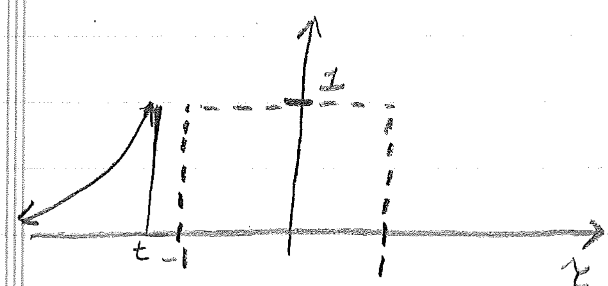
Sol'n





$\boxed{\text{shift: } h(t-\tau)}$

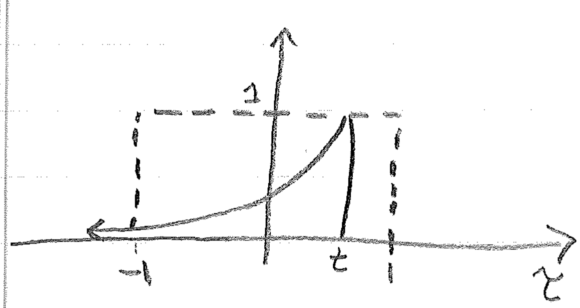
Region 1: $t < -1$



No overlap

$\Rightarrow y(t) = 0$

Region 2: $-1 \le t < 1$



- Partial overlap from left

$$y(t) = \int_{-1}^{t} 1 \cdot e^{-4(t-\tau)}\, d\tau$$

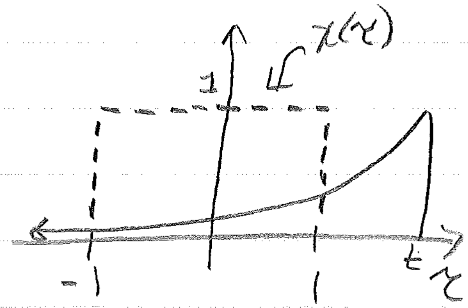
$$= e^{-4t} \cdot \left.\frac{e^{4\tau}}{4}\right|_{\tau=-1}^{t}$$

$$= \frac{e^{-4t}}{4}\left(e^{4t} - e^{-4}\right) = \underline{\frac{1 - e^{-4(t+1)}}{4}}$$

Continued

Graphical Convolution: Example 2 (2)

Region 3: $t > 1$



$$y(t) = \int_{-1}^{1} 1 \cdot e^{-4(t-\tau)}\, d\tau$$

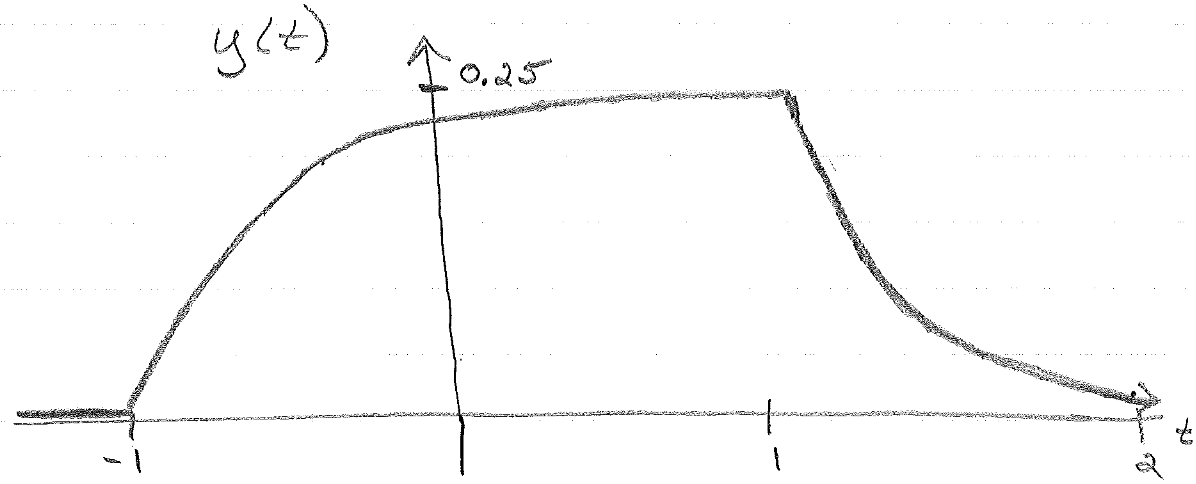
$$= e^{-4t} \cdot \left. \frac{e^{4\tau}}{4} \right|_{\tau=-1}^{1}$$

- Overlaps $x(t)$ completely

$$= \frac{e^{-4t}}{4}\left(e^{4} - e^{-4}\right)$$

- OVERALL:

$$y(t) = \begin{cases} 0, & t < -1 \\ \frac{1}{4} - \frac{e^{-4(t+1)}}{4}, & -1 \le t \le 1 \\ e^{-4t}\left(\frac{e^{4} - e^{-4}}{4}\right), & t > 1 \end{cases}$$



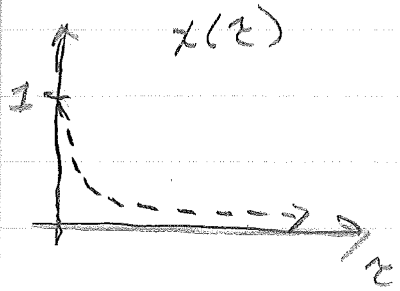
Graphical Convolution: Example 3 (1)

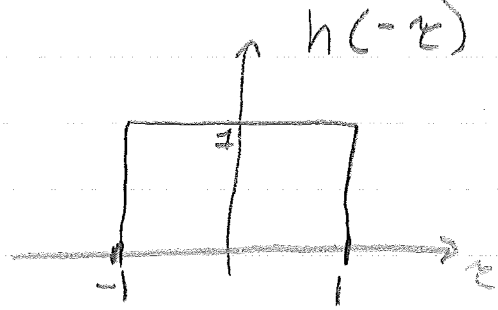
∘ Find $y(t) = x(t) * h(t)$ if

$x(t) = e^{-4t}\,u(t)$

$h(t) = \begin{cases} 1, & -1 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

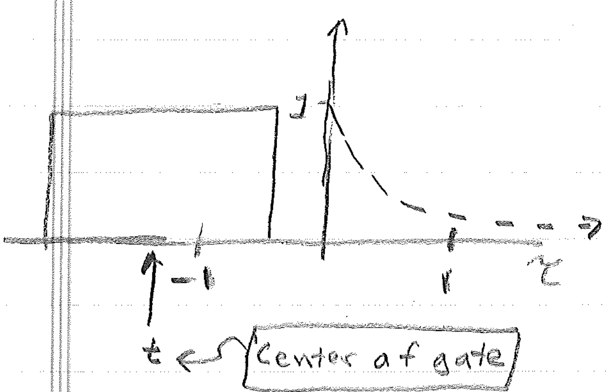
Sol'n





$\boxed{\text{shift: } h(t-\tau)}$

Region 1: $t < -1$



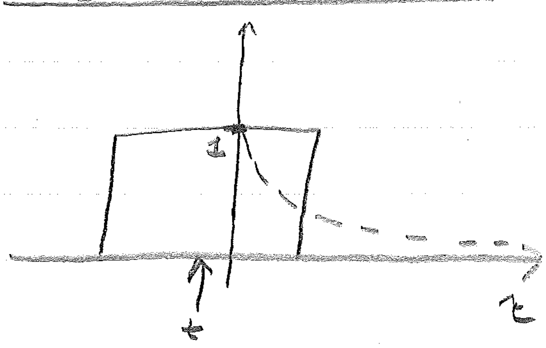
No overlap

$\Rightarrow y(t) = 0$

Region 2: $-1 \le t < 1$

$$y(t) = \int_{\tau=0}^{t+1} e^{-4\tau} \cdot 1 \, d\tau$$

$$= \left.\frac{e^{-4\tau}}{-4}\right|_{\tau=0}^{t+1} = \frac{-e^{-4(t+1)} + 1}{4}$$

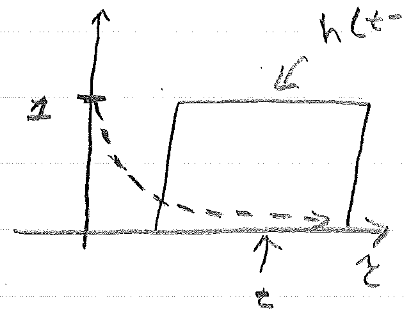


- Partial overlap from left

Continued

Graphical Convolution: Example 3 (2)

Region 3: $t > 1$



$$y(t) = \int_{\tau = t-1}^{t+1} e^{-4\tau} \cdot 1 \, d\tau$$

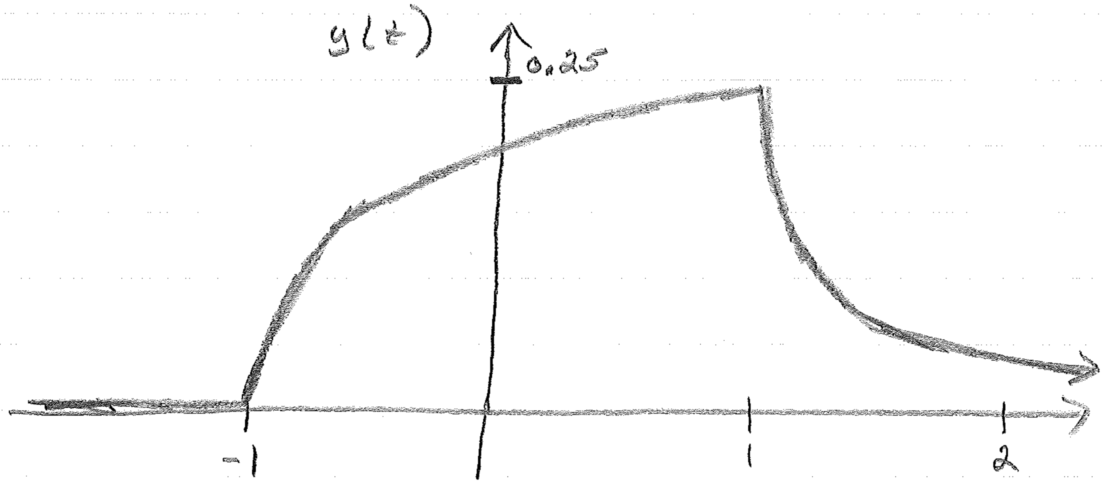
$$= \left. \frac{e^{-4\tau}}{-4} \right|_{\tau = t-1}^{t+1} = \frac{e^{-4(t+1)} - e^{-4(t-1)}}{-4}$$

$$= \frac{e^{-4t}}{4} \left(e^{4} - e^{-4} \right)$$

- Overlaps $h(t-\tau)$ completely

OVERALL:

$$y(t) \begin{cases} 0, & t < -1 \\ \dfrac{1 - e^{-4(t+1)}}{4}, & -1 \le t \le 1 \\ e^{-4t}\left(\dfrac{e^{4} - e^{-4}}{4}\right), & t > 1 \end{cases}$$

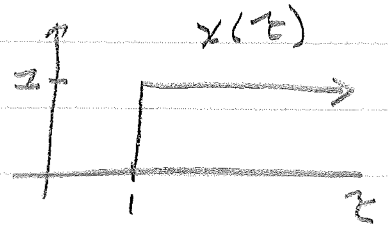


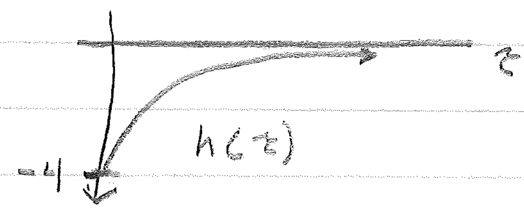
Convolution Example 4

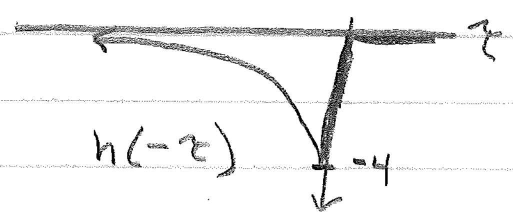
° Find $y(t) = x(t) * h(t)$ if

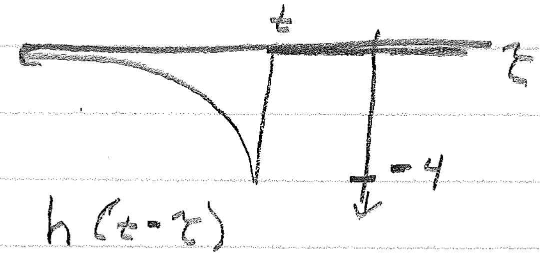
$$x(t) = u(t-1), \qquad h(t) = -4e^{-t}u(t)$$

Sol'n



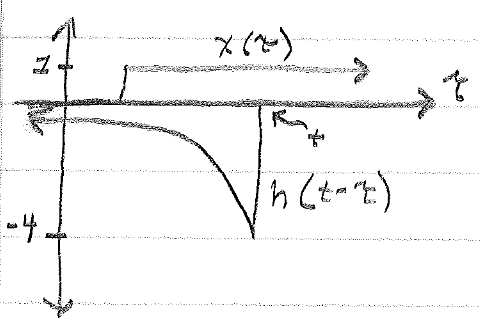






Region 1: $t < 1$ → No overlap ⇒ $y(t) = 0$

Region 2: $t \geq 1$



$$y(t) = \int_{\tau=1}^{t} 1 \cdot \left(-4e^{-(t-\tau)}\right) d\tau$$

$$= -4e^{-t}\int_{\tau=1}^{t} e^{\tau} d\tau = -4e^{-t} e^{\tau}\Big|_{\tau=1}^{t}$$

$$= -4e^{-t}\left(e^{t} - e^{1}\right) = -4\left(1 - e^{-t+1}\right)$$

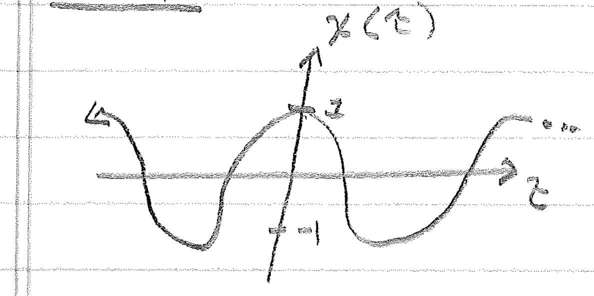
Overall: $\boxed{y(t) = -4\left(1 - e^{-t+1}\right)u(t-1)}$

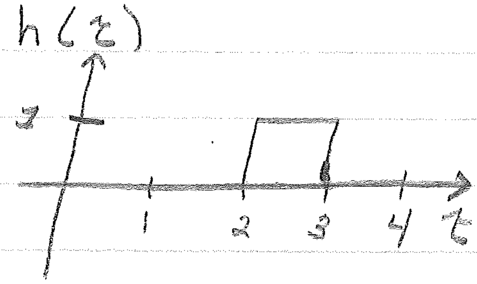
Convolution Example 5

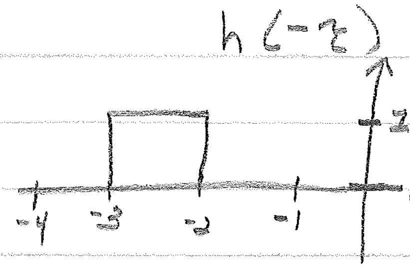
- Find $y(t) = x(t) * h(t)$ if:

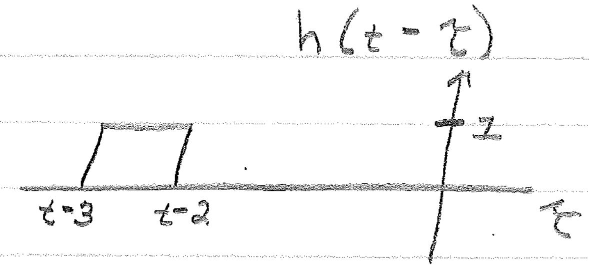
$x(t) = \cos(6t)$ $\qquad$ $h(t) = u(t-2) - u(t-3)$

Sol'n

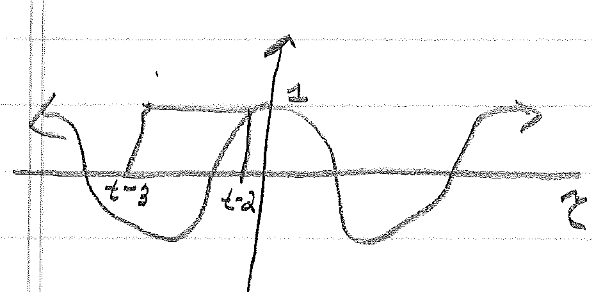








→ One region, extending from $-\infty \le t \le \infty$:



$$y(t) = \int_{\tau = t-3}^{t-2} \cos(6\tau) \cdot 1 \, d\tau$$

$$= \left. \frac{\sin(6\tau)}{6} \right|_{\tau = t-3}^{t-2}$$

$$= \frac{\sin(6(t-2)) - \sin(6(t-3))}{6} = \frac{\sin(6t-12) - \sin(6t-18)}{6}$$

$$\boxed{y(t) \cong 0.0470 \sin(6t - 0.85)}$$

Trig Identities

Duration of Convolved Signals

- Given: $y(t) = x_1(t) * x_2(t)$

- If $x_1(t)$ or $x_2(t)$ are infinite duration

  $\Rightarrow y(t)$ infinite duration

- If $x_1(t)$ of duration T_1,
  $x_2(t)$ of duration T_2

  $\Rightarrow y(t)$ of duration $T_1 + T_2$

  - Demonstrated from graphical convolution

$$\boxed{\begin{array}{l}\underline{\text{NOTE: In Practice}} \\ \circ\ \text{Use graphical convolution to set up integrals} \\ \circ\ \text{Use tables, software tools to perform integration}\end{array}}$$

Convolution is Cummutative

$$y(t)=x(t)*h(t) \equiv \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

- Let $l = t-\tau \Rightarrow \tau = t-l$, $dt = -dl$

$$y(t) = \int_{t-l=-\infty}^{\infty} -x(t-l)\, h(l)\, dl$$

$$= \int_{-l=-\infty}^{\infty} -h(l)\, x(t-l)\, dl$$

"t" fixed while evaluating integral

Negating limits ⇒ multiply by -1

$$y(t) = \int_{l=-\infty}^{\infty} h(l)\, x(t-l)\, dl = h(t) * x(t)$$

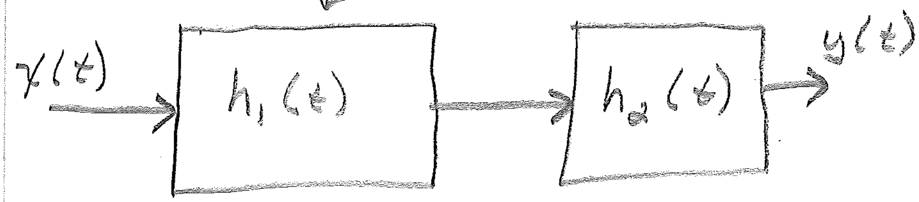
Therefore: $\boxed{x(t)*h(t) = h(t)*x(t)}$

°Commutative Property

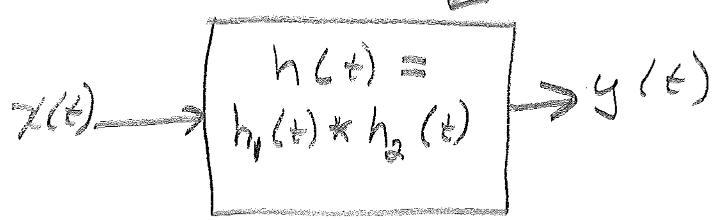
Convolution is Associative

- Can show:

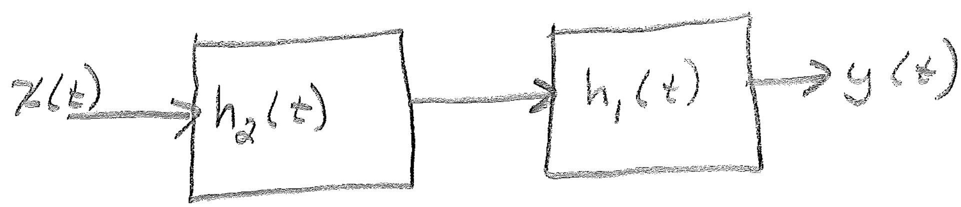
$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$



Same as:



- Also, by <u>commutative</u>, same as:

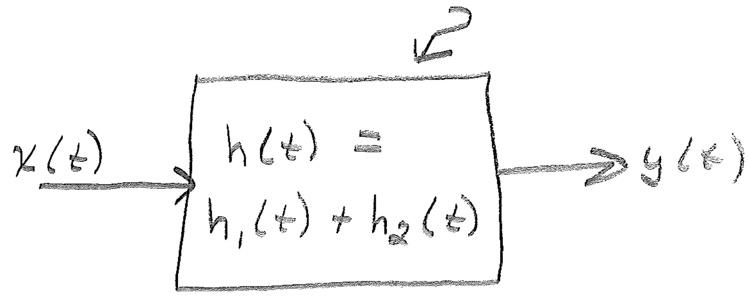


Remember: Need LTI systems
$\Rightarrow$ Convolution integral computes system action

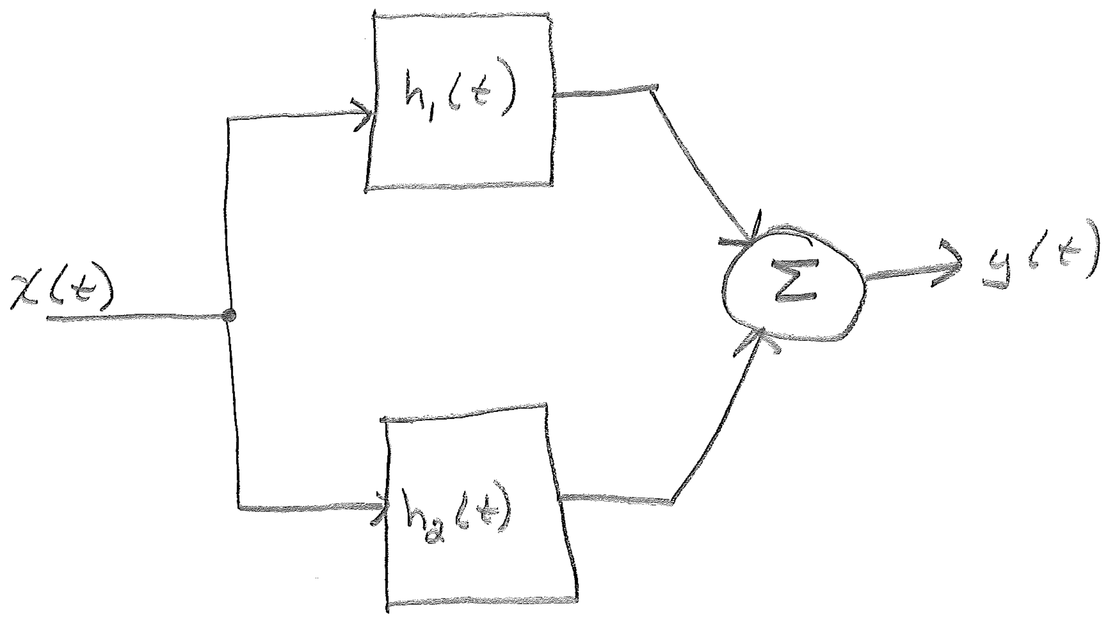
Convolution is Distributive

- Can show:

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$



Same as:



Convolution Shift Property

If $\quad y(t) = x_1(t) * x_2(t)$

THEN

$$x_1(t) * x_2(t-T) = x_1(t-T) * x_2(t) = y(t-T)$$

AND

$$x_1(t-T_1) * x_2(t-T_2) = y(t-T_1-T_2)$$

Convolving with an Impulse

$$x(t) * \delta(t) = x(t)$$

Deriving Shift Property

If $y(t) = x(t) * h(t)$

Show that

$$x(t) * h(t-T) = y(t-T)$$

Sol'n

By definition,

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Let $t \to t-T$ (substitute $t-T$ for t)

$$\Downarrow$$

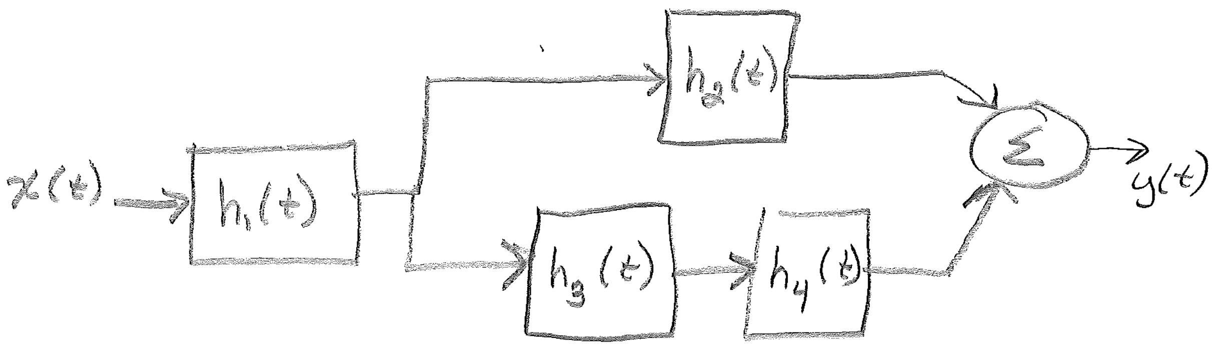
$$y(t-T) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-T-\tau)\, d\tau$$

which says:

$$\boxed{y(t-T) = x(t) * h(t-T)}$$

Convolution Properties; Example 1

- Given the LTI systems below, find the overall impulse response as a function of $h_1(t)$, $h_2(t)$, $h_3(t)$, $h_4(t)$.



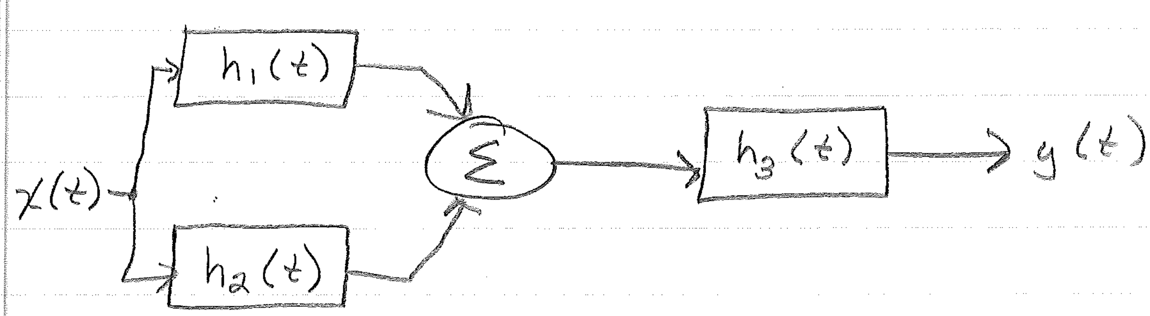
Soln.

$$h(t) = h_1(t) * [h_2(t) + h_3(t) * h_4(t)]$$

$$y(t) = h(t) * x(t)$$

Convolution Properties: Example 2

- **Given** the LTI systems below, **find** the overall impulse response as a function of the $h_i(t)$.



Sol'n

$$h(t) = \left[h_1(t) + h_2(t)\right] * h_3(t)$$

Consequence

- Associative & distributive combination of LTI systems

$\Downarrow$

Overall system is LTI

Causal LTI Systems

- LTI system causal iff $h(t<0)=0$

- To see:

$$y(t) = \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$y(t) = \underbrace{\int_{\tau=-\infty}^{0} h(\tau)\, x(t-\tau)\, d\tau}_{\text{Negative } \tau\text{'s}} + \underbrace{\int_{\tau=0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau}_{\text{Positive } \tau\text{'s}}$$

Negative τ's
$\Rightarrow$ Input times $> t$
$\Rightarrow$ Future inputs

So, need $h(\tau)=0$ for $-\infty < \tau < 0$

Positive τ's
$\Rightarrow$ Input times $< t$
$\Rightarrow$ Past inputs

OK ✓

- If causal,

$$y(t) = \int_{\tau=0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{\tau=-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

Stable LTI Systems

- LTI systems BIBO stable

  iff $$\int_{t=-\infty}^{\infty} |h(t)|\,dt < \infty$$

- E.g., consider integrator (/accumulator):

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

Find $h(t)$ by letting $x(t) \to \delta(t)$:

$$h(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$$

Then, $$\int_{-\infty}^{\infty} |u(\tau)|\,d\tau = \int_{0}^{\infty} 1 \cdot d\tau = \tau \Big|_{0}^{\infty} = \infty$$

$$\boxed{\text{Thus, integrator } \underline{\underline{\text{NOT}}} \text{ BIBO stable}}$$

First-Order Low-Pass Stability

- Consider first-order low-pass system (e.g., RC filter) of form:

$$h(t) = e^{-at} u(t), \quad a > 0$$

- BIBO stability:

$$\int_{t=-\infty}^{\infty} |e^{-at} u(t)| \, dt$$

$$= \int_{t=0}^{\infty} e^{-at} \, dt = \left. \frac{-e^{-at}}{a} \right|_{t=0}^{\infty} = \frac{-e^{-a \cdot \infty} - -e^{0}}{a}$$

For $a > 0$,

$$= \frac{0+1}{a} = \frac{1}{a}$$

Since $\frac{1}{a} < \infty \Rightarrow$ $\boxed{\text{Always BIBO stable}}$

Step Response

- System output (response) if input $= u(t-T)$
  - For simplicity, let $T=0$

$$u(t) \longrightarrow \boxed{\;\mathcal{T}\;} \longrightarrow s(t)$$

- By convolution (LTI system)

$$s(t) = h(t) * u(t)$$

$$\equiv \int_{\tau=-\infty}^{\infty} h(\tau)\, \underbrace{u(t-\tau)}_{\begin{cases} 0 & \text{for } t-\tau < 0 \\ 1 & \text{for } t-\tau \geq 0 \end{cases}}\, d\tau$$

$$s(t) = \int_{\tau=-\infty}^{t} h(\tau)\, d\tau$$

$$\boxed{\text{Unit step response} \equiv \text{running integral of impulse response}}$$

Step Response Example

- If the impulse response of an LTI system is:

$$h(t) = \left(2e^{-t} + e^{-3t}\right) u(t)$$

Find the step response of the system.

Sol'n

For $t<0$: $h(t<0)=0 \Rightarrow \int_{\tau=-\infty}^{t<0} h(\tau)\,d\tau = 0$

For $t \geq 0$

$$s(t) = \int_{\tau=-\infty}^{t} h(\tau)\,d\tau = \int_{\tau=0}^{t} \left(2e^{-\tau} + e^{-3\tau}\right) d\tau$$

$$= \left(-2e^{-\tau} - \frac{e^{-3\tau}}{3}\right)\Bigg|_{\tau=0}^{t} = -2e^{-t} - \frac{e^{-3t}}{3} - \left(-2 - \frac{1}{3}\right)$$

$$= \frac{7}{3} - 2e^{-t} - \frac{e^{-3t}}{3}$$

Giving:

$$\boxed{s(t) = \left(\frac{7}{3} - 2e^{-t} - \frac{e^{-3t}}{3}\right) u(t)}$$

LTI Response to Complex Exponentials

- Let $x(t) = e^{st}$ for $s = \sigma + j\omega$

  - Recall: e^{st} sinusoidal, in general

$$y(t) = \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} \cdot \underbrace{\left[\int_{\tau=-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau \right]}$$

- Not function of t
- Just a complex number
- Depends on value of $s = \sigma + j\omega$
- Denote as $H(s)$.

So, $y(t) = H(s) \cdot e^{st}$

where $H(s) = \int_{\tau=-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$

Note: Output is <u>scaled</u> input

Eigen function

Scaling = Eigen value

$$\boxed{\begin{array}{l}\text{Analogy: Sinusoids in linear circuits} \\ \quad \rightarrow \text{Phasors simplify analysis}\end{array}}$$

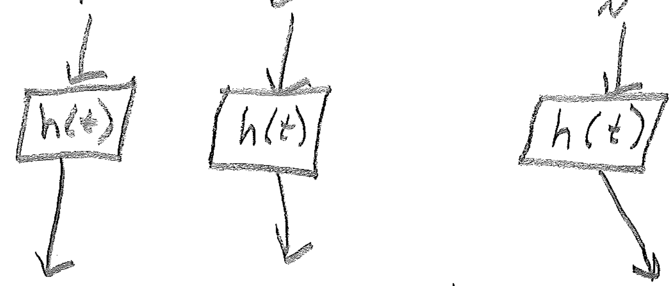
Convolution

LTI Systems with Complex Exponentials

- Decompose inputs into sum of complex exponentials

$$x(t) = a_1 e^{s_1 t} + a_2 e^{s_2 t} + \cdots a_N e^{s_N t}$$

- Determine each response



$$a_1 H(s_1) e^{s_1 t} \qquad a_2 H(s_2) e^{s_2 t} \qquad a_N H(s_N) e^{s_N t}$$

- Linear →

$$y(t) = a_1 H(s_1) e^{s_1 t} + a_2 H(s_2) e^{s_2 t} + \cdots + a_N H(s_N) e^{s_N t}$$

- For electric circuits w/ one sine frequency

Time Domain ⟷ Phasor Domain
(Sinusoidal steady-state response)

- For many frequencies

Laplace Transform
(Total response)

Time Domain ⟷ Fourier Transform
(Zero-state response, aperiodic signals)

Fourier Series
(Zero-state response, periodic signals)

Convolution

Linear Differential System is LTI

- System defined by:

$$a_R \frac{d^R y(t)}{dt^R} + a_{R-1} \frac{d^{R-1} y(t)}{dt^{R-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t)$$

$$= b_P \frac{d^P x(t)}{dt^P} + b_{P-1} \frac{d^{P-1} y(t)}{dt^{P-1}} + \cdots + b_1 \frac{dx(t)}{dt} + b_0 x(t)$$

$\boxed{\text{This system is LTI}}$

- E.g.: • RLC circuit
  • Linear amplifiers

Convolution