Solution

Worcester Polytechnic Institute — Department of Electrical and Computer Engineering
ECE2311 — Continuous-Time Signal and System Analysis — Term B'17

Homework 6: Due Friday, 17 November 2017 (3:00 P.M.)

Write your name and ECE box at the top of each page.

General Reminders on Homework Assignments:

- Always complete the reading assignments *before* attempting the homework problems.
- Show all of your work. Use written English, where applicable, to provide a log or your steps in solving a problem. (For numerical homework problems, the writing can be brief.)
- A solution that requires physical units is *incorrect* unless the units are listed as part of the result.
- Get in the habit of underlining, circling or boxing your result.
- Always write neatly. Communication skills are essential in engineering and science. "If you didn't write it, you didn't do it!"

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1) Drawing poles and zeros with MATLAB:

a) As we will see, a large class of LTI systems can be characterized by so-called rational Laplace Transforms. In the frequency domain, a Laplace transform is rational if it can be expressed as a polynomial in *s* divided by another polynomial in *s*:

$$H(s) = \frac{N(s)}{D(s)} = \frac{b_P\, s^P + b_{P-1}\, s^{P-1} + \cdots b_1\, s + b_0}{a_R\, s^R + a_{R-1}\, s^{R-1} + \cdots a_1\, s + a_0}.$$

If we factor the numerator and denominator polynomials (and divide out the coefficient on the highest numerator and denominator power), we can re-write the rational form as:

$$H(s) = \frac{b_P}{a_R} \cdot \frac{(s - z_1)(s - z_2)\cdots(s - z_P)}{(s - p_1)(s - p_2)\cdots(s - p_R)}.$$

If we were to evaluate *H(s)* at any of the values $s = \{z_1, z_2, \ldots z_P\}$, then *H(s)* would equal zero at these locations. These values (which can be real or complex) are known as "zeroes" and are the roots of *N(s)*. Similarly, if we were to evaluate *H(s)* at any of the values $s = \{p_1, p_2, \ldots p_R\}$, then *H(s)* would equal infinity. These values (which can be real or complex) are known as "poles" and are the roots of *D(s)*.

For your assignment, you are asked to develop a MATLAB function "my_pole_zero()" that plots poles and zeroes given a rational Laplace transform. You are NOT PERMITTED to use any MATLAB function that does this task for you directly. Use MATLAB's numerical computation, ***not*** symbolic. Proceed as follows:

- Let your function have two inputs, "N" and "D". "N" should specify the numerator polynomial, in standard MATLAB polynomial format, e.g., the polynomial "$10\, s^3 + 7\, s + 11$" would be specified by the MATLAB vector "N = [10 0 7 11];". "D" should specify the denominator polynomial, in standard MATLAB polynomial format.
- Numerically find the roots of the numerator and denominator polynomials. The most appropriate MATLAB tool for doing so would seem to be the "roots()" function.
- Plot each zero with the character "o" on an s-plane axis. Remember that the real part of the zero is the x-axis value and the imaginary part of the zero is the y-axis value.
- Similarly plot each pole with the character "x" in the same s-plane.

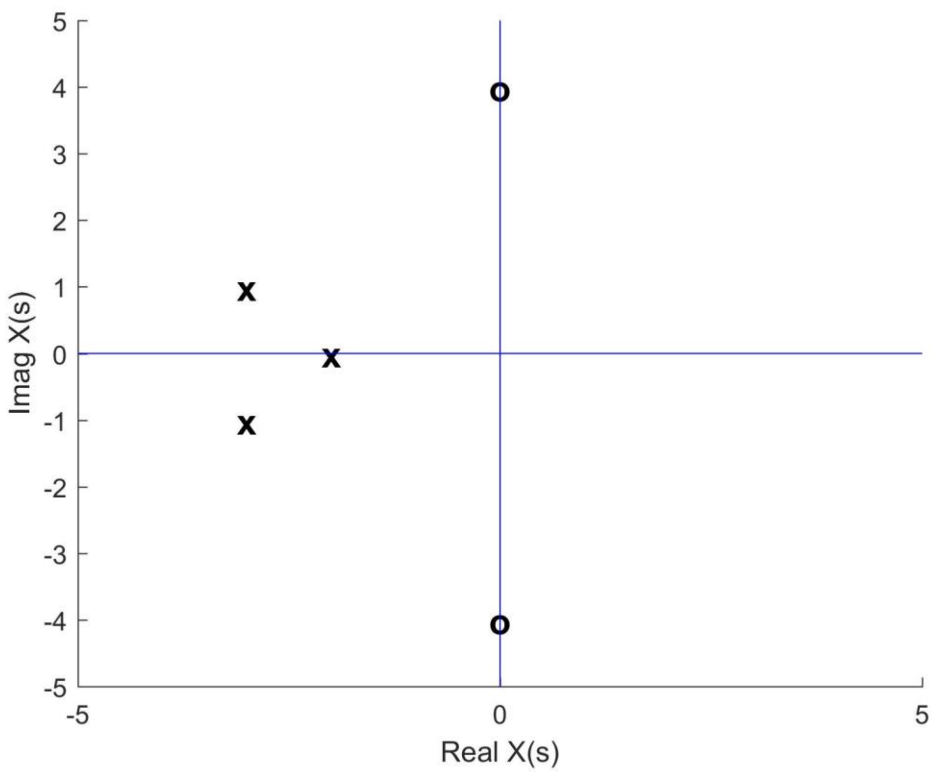
- Two useful commands for plotting an "o" or an "x" are *(i)* the plot() command, using the 'o' (or'x') marker specifier or *(ii)* the text() command. Note that subsequent calls to plotting commands, by default, erase the prior plot. To avoid this behavior, type the MATLAB command "hold on" prior to the second plot command. This command will tell MATLAB to plot subsequent plot commands via over-plotting. (When ALL plots commands are completed, it is customary to type "hold off" to return MATLAB to its default behavior.) Note that the plot() command will automatically set the *x*- and *y*-axis limits, but the text() command will not. Thus, if you use the text command, you will have to set the limits yourself [usually via the axis() command], else some of the text will fall outside of the plot limits (and won't be visible).
- Label your axes, be sure the plot is appropriately scaled, etc.
- We will keep this function simple, in that if two characters appear at the same location, it is OK to allow that second character to obscure the first.

Your function should work for general rational polynomials. For your homework, turn in two items:

- Your MATLAB code.
- The pole-zero plot for: $H(s) = \dfrac{s^2+16}{s^3+8s^2+22s+20}$.

i) Code will differ for each student

ii) $H(s) = \dfrac{s^2+16}{s^3+8s^2+22s+20} = \dfrac{(s-j4)(s+j4)}{(s+2)(s+3-j)(s+3+j)}$



2) Inverse Bilateral Laplace Transforms:

a) Find the inverse Bilateral Laplace Transform of the following functions by manually (without MATLAB or other automated tools) performing the partial fraction expansion, then writing the time-domain function:

i) $H(s)=\dfrac{-s-11}{s^2+7s+10}$, $\mathrm{Re}\{s\}>-2$.

$$H(s)=\frac{-s-11}{s^2+7s+10}=\frac{-s-11}{(s+2)(s+5)}=\frac{A}{s+2}+\frac{B}{s+5}$$

Multiply by $(s+2)(s+5)$:

$$-s-11=A(s+5)+B(s+2)$$

At $s=-2$:

$$-(-2)-11=A(-2+5)+B(-2+2)\Rightarrow A=-3$$

At $s=-5$:

$$-(-5)-11=A(-5+5)+B(-5+2)\Rightarrow B=2$$

Thus,

$H(s)=\dfrac{-3}{s+2}+\dfrac{2}{s+5}$, and $\mathrm{Re}(s)>-2$, since it is right-sided sequence.

Then, via inverse s-transform:

$$h(t)=-3e^{-2t}\mu(t)+2e^{-5t}\mu(t)$$

ii) $Y(s)=\dfrac{2s+11.5}{s+3}$, $\mathrm{Re}\{s\}>-3$.

This fraction is improper. So, divide:

$$\begin{array}{r|l} & \quad 2 \\ \hline s+3 & 2s+11.5 \\ & \underline{-(2s+6)} \\ & \qquad\quad 5.5 \end{array}$$

Hence, $Y(s)=\dfrac{2s+11.5}{s+3}=2+\dfrac{5.5}{s+3}$, and $\mathrm{Re}(s)>-3$, since it is right-sided sequence.

Then, via inverse s-transform:

$$y(t)=2\delta(t)+5.5e^{-3t}\mu(t)$$

b) Use MATLAB function "residue" (use numeric variables, ***not*** symbolic) to aid in finding the inverse Bilateral Laplace Transform of the following functions: (You need not hand in any MATLAB code. Rather, write the partial fraction expansion of the given rational function—use MATLAB as a tool for doing so—and then write the inverse transform.)

i) $X(s)=\dfrac{s^2+3s+4}{(s+3)(s+2)(s+1)}$, $\mathrm{Re}\{s\}>-1$.

ROC: $\mathrm{Re}(s) > -1 \Rightarrow$ Right-sided sequence.
Using MATLAB conv() and residue() functions:

$$X(s) = \frac{s^2+3s+4}{s^3+6s^2+11s+6} = \frac{2}{s+3} + \frac{-2}{s+2} + \frac{1}{s+1}$$

Then, via inverse *s*-transform:

$$x(t) = \left[2e^{-3t} - 2e^{-2t} + e^{-t}\right]\mu(t)$$

ii) $X(s) = \dfrac{-2s^2 - 20s - 53}{(s+3)(s+5)^2}$, $\mathrm{Re}\{s\} > -3$.

ROC: $\mathrm{Re}(s) > -3 \Rightarrow$ Right-sided sequence.
Using MATLAB conv() and residue() functions:

$$X(s) = \frac{-2s^2 - 20s - 53}{s^3 + 13s^2 + 55s + 75} = \frac{-2.75}{s+3} + \frac{0.75}{s+5} + \frac{1.5}{(s+5)^2}$$

Note: second two terms represent a repeated root.
Then, via inverse *s*-transform:

$$x(t) = \left[-2.75e^{-3t} + 0.75e^{-5t} + 1.5te^{-5t}\right]\mu(t)$$

iii) $H(s) = \dfrac{s^2 - 3s + 4}{(s-3)(s-2)(s-1)}$, $\mathrm{Re}\{s\} < 1$. Carefully note the ROC.

ROC: $\mathrm{Re}(s) < 1 \Rightarrow$ Left-sided sequence.
Using MATLAB conv() and residue() functions:

$$H(s) = \frac{s^2 - 3s + 4}{s^3 - 6s^2 + 11s - 6} = \frac{2}{s-3} + \frac{-2}{s-2} + \frac{1}{s-1}$$

Then, via inverse *s*-transform:

$$h(t) = \left[-2e^{3t} + 2e^{2t} - e^{t}\right]\mu(-t)$$

c) The following Bilateral Laplace Transforms can be converted using the transform tables, perhaps after some mathematical manipulation. (You should not need further fraction expansion or MATLAB help.) Each corresponding time function is real-valued for these examples, thus write each time function using only real-valued parameters.

i) $X(s) = \dfrac{1}{s+3-j4} + \dfrac{1}{s+3+j4} = \dfrac{2s+6}{s^2+6s+25}$, $\mathrm{Re}\{s\} > -3$.

ROC $\mathrm{Re}(s) > -3 \Rightarrow$ Right-sided sequence.

$$X(s) = \frac{2s+6}{s^2+6s+25} = \frac{2(s+3)}{(s+3)^2 + 4^2}, \quad \mathrm{Re}(s) > -3$$

Then, via inverse *s*-transform table:

$$x(t) = 2e^{-3t}\cos(4t)\,\mu(t)$$

ii) $Y(s)=\dfrac{13}{5s^5}$, $\mathrm{Re}\{s\}>0$.

$$Y(s)=\frac{13}{5s^5}=\frac{13}{5}\cdot\left(\frac{1}{s^5}\cdot\frac{4!}{4!}\right)=\frac{13}{5\cdot 24}\cdot\frac{4!}{s^5}=\frac{13}{120}\cdot\frac{4!}{s^5},\quad \mathrm{Re}(s)>0$$

Then, via inverse *s*-transform table:

$$y(t)=\frac{13}{120}t^4\,\mu(t)$$

iii) $Y(s)=e^{2s-3}$, All s.

$Y(s)=e^{2s-3}=e^{-3}\cdot e^{-s(-2)}$, All *s*.

Then, via inverse *s*-transform table:

$$y(t)=e^{-3}\delta(t+2)$$