ECE4904 Lecture 5+6 Overview

Lecture 5 Lightning “Review”

Energy Band Diagram Review
- E_F flat in equilibrium

pn Junction
- Doping profile (5.1.1)
  - Step junction approximation
- pn Junction: Handwaving
- Qualitative Electrostatics (5.1.2, 3)
- Built-in Potential (5.1.4)
- Depletion Approximation (5.1.5)

Quantitative Electrostatics (5.2)
- Qualitative forward, reverse bias (5.2.4)

Lecture 6

Continuity Equation (3.4.1)
Minority Carrier Diffusion Equation (3.4.2)

Ideal Diode Equation (Ch. 6)

Handout

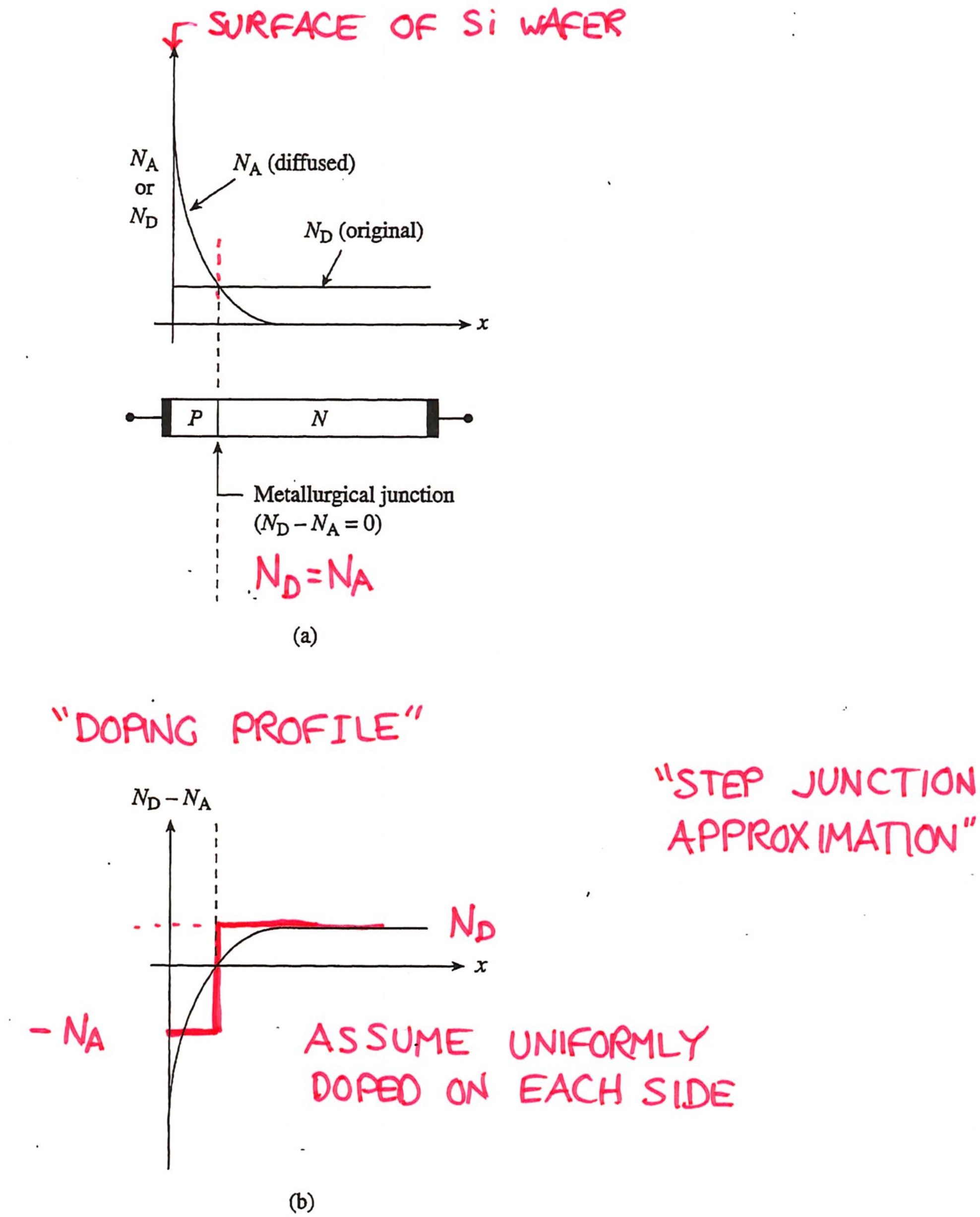

Figure 5.1 Junction definitions: (a) Location of the metallurgical junction, (b) doping profile—a plot of the net doping versus position.

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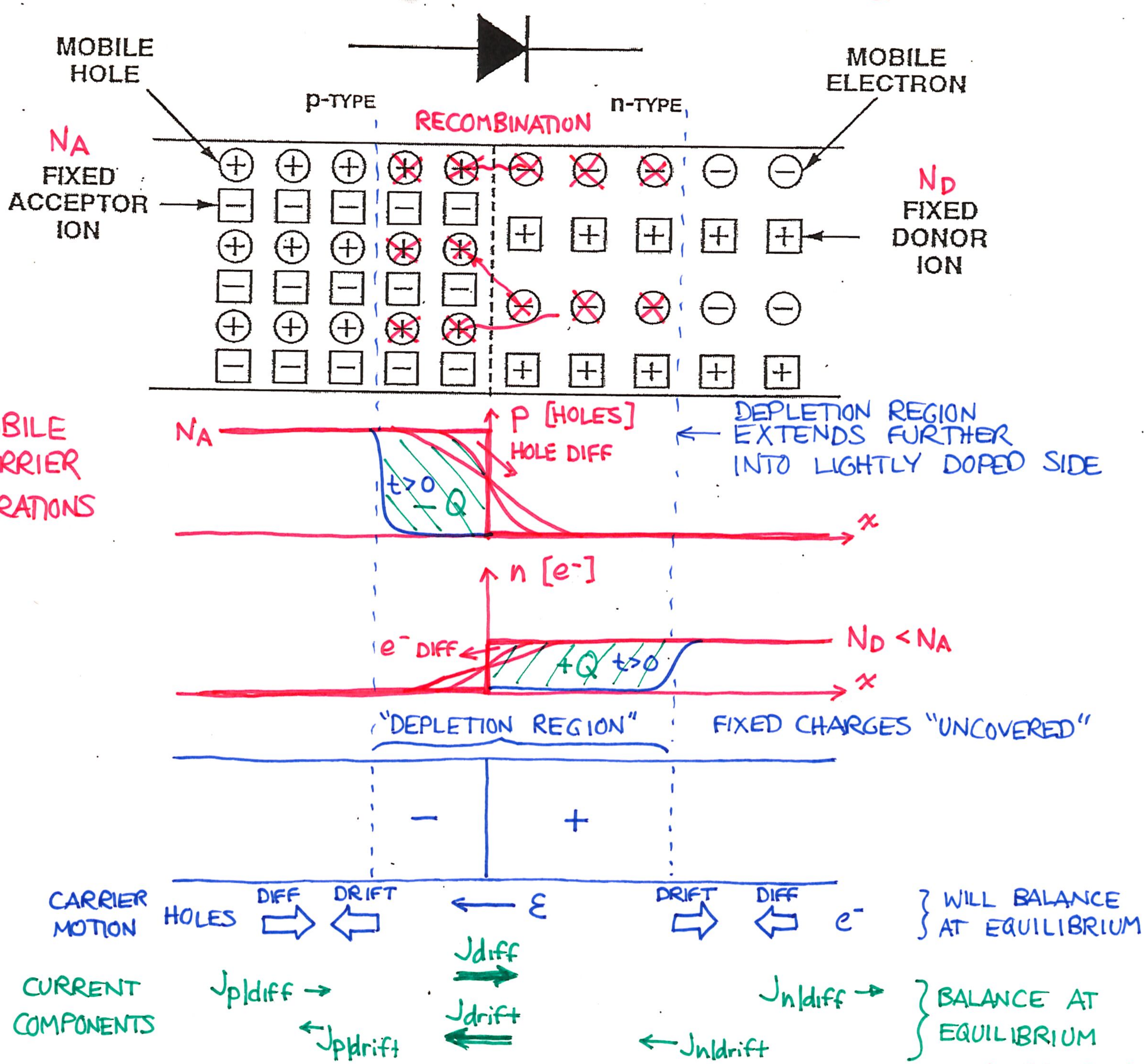
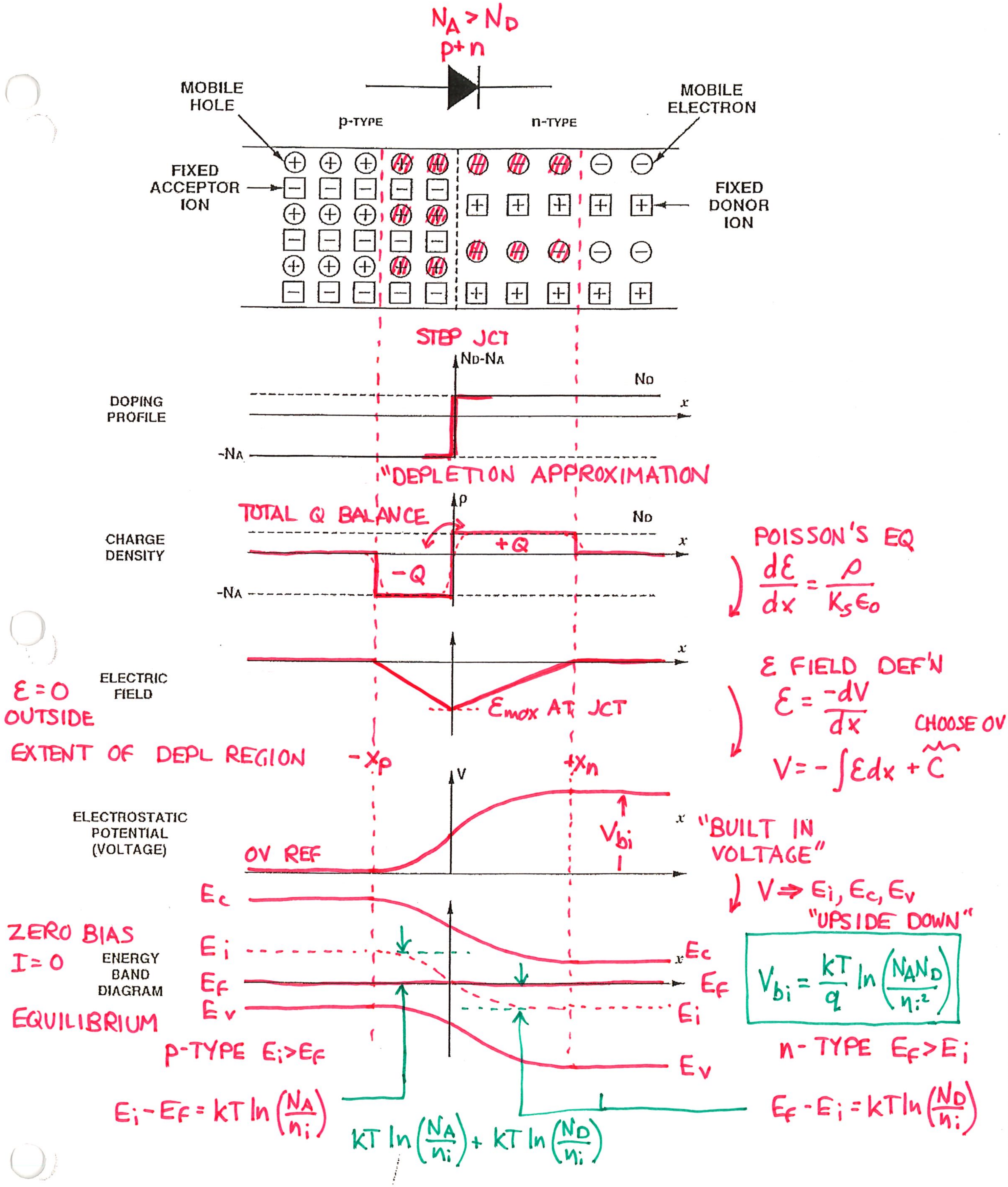
EVERYTHING ELECTRICALLY NEUTRAL



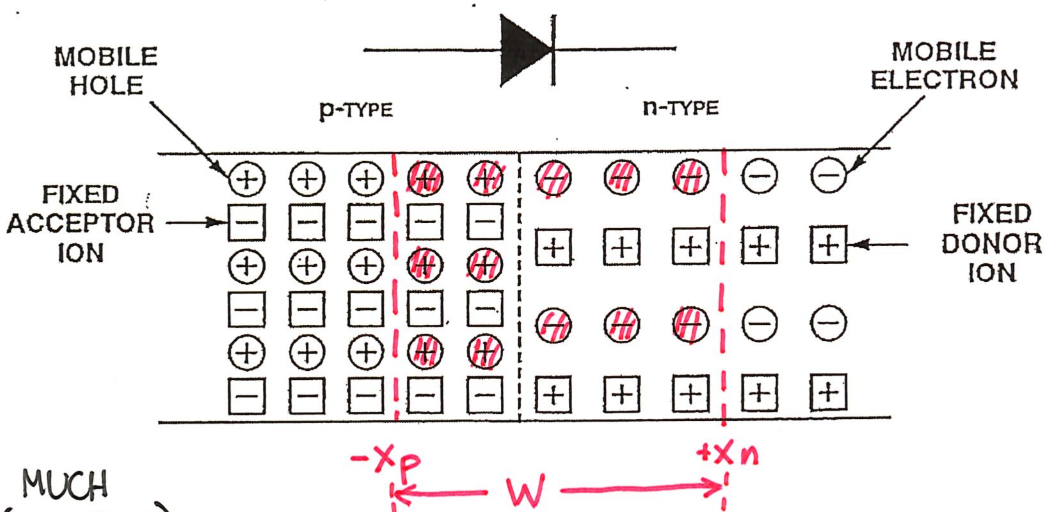
INITIALLY: $\mathcal{E} = 0$
EVERYWHERE
NO DRIFT

CARRIERS WILL MOVE DUE TO DIFFUSION
↓
e⁻ HOLE PAIR RECOMBINATION NEAR JUNCTION



$N_A > N_D$
$p^+ n$
MOBILE HOLE
p-TYPE
n-TYPE
MOBILE ELECTRON
FIXED ACCEPTOR ION
FIXED DONOR ION
STEP JCT
$N_D - N_A$
N_D
DOPING PROFILE
$-N_A$
"DEPLETION APPROXIMATION"
TOTAL Q BALANCE
ρ
$-Q$
$+Q$
CHARGE DENSITY
POISSON'S EQ
$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_s \epsilon_0}$
$\mathcal{E} = 0$ OUTSIDE
ELECTRIC FIELD
$\mathcal{E}_{max}$ AT JCT
$\mathcal{E}$ FIELD DEF'N
$\mathcal{E} = \frac{-dV}{dx}$
CHOOSE 0V
$V = -\int \mathcal{E}\, dx + C$
EXTENT OF DEPL REGION
$-x_p$
$+x_n$
V
ELECTROSTATIC POTENTIAL (VOLTAGE)
0V REF
V_{bi}
"BUILT IN VOLTAGE"
$V \Rightarrow E_i, E_c, E_v$ "UPSIDE DOWN"
E_c
E_i
E_F
E_v
ZERO BIAS
$I = 0$
ENERGY BAND DIAGRAM
EQUILIBRIUM
$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$
p-TYPE $E_i > E_F$
n-TYPE $E_F > E_i$
$E_i - E_F = kT \ln\left(\frac{N_A}{n_i}\right)$
$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$
$kT \ln\left(\frac{N_A}{n_i}\right) + kT \ln\left(\frac{N_D}{n_i}\right)$

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$



EXAMPLE

$N_A = 1E+17\ /cm^3$, $N_D = 1E+14\ /cm^3$ } $N_A >> N_D$

@ T = 300K

$$V_{bi} = \underbrace{\frac{kT}{q}}_{.0259V} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$.0259V \ \ln\left(\frac{1E+31}{1E+20}\right)$$

$V_{bi} = 0.659\ V$

AFTER MUCH MATH (see Ch.5)

APPROX: $N_A >> N_D$

$$x_p = \sqrt{\frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A}\frac{1}{N_A + \cancel{N_D}}V_{bi}}$$

$$x_n = \sqrt{\frac{2K_S\varepsilon_0}{q}\frac{\cancel{N_A}}{N_D}\frac{1}{\cancel{N_A} + \cancel{N_D}}V_{bi}}$$

$$W = \sqrt{\frac{2K_S\varepsilon_0}{q}\left(\frac{\cancel{N_A} + \cancel{N_D}}{\cancel{N_A}N_D}\right)V_{bi}}$$

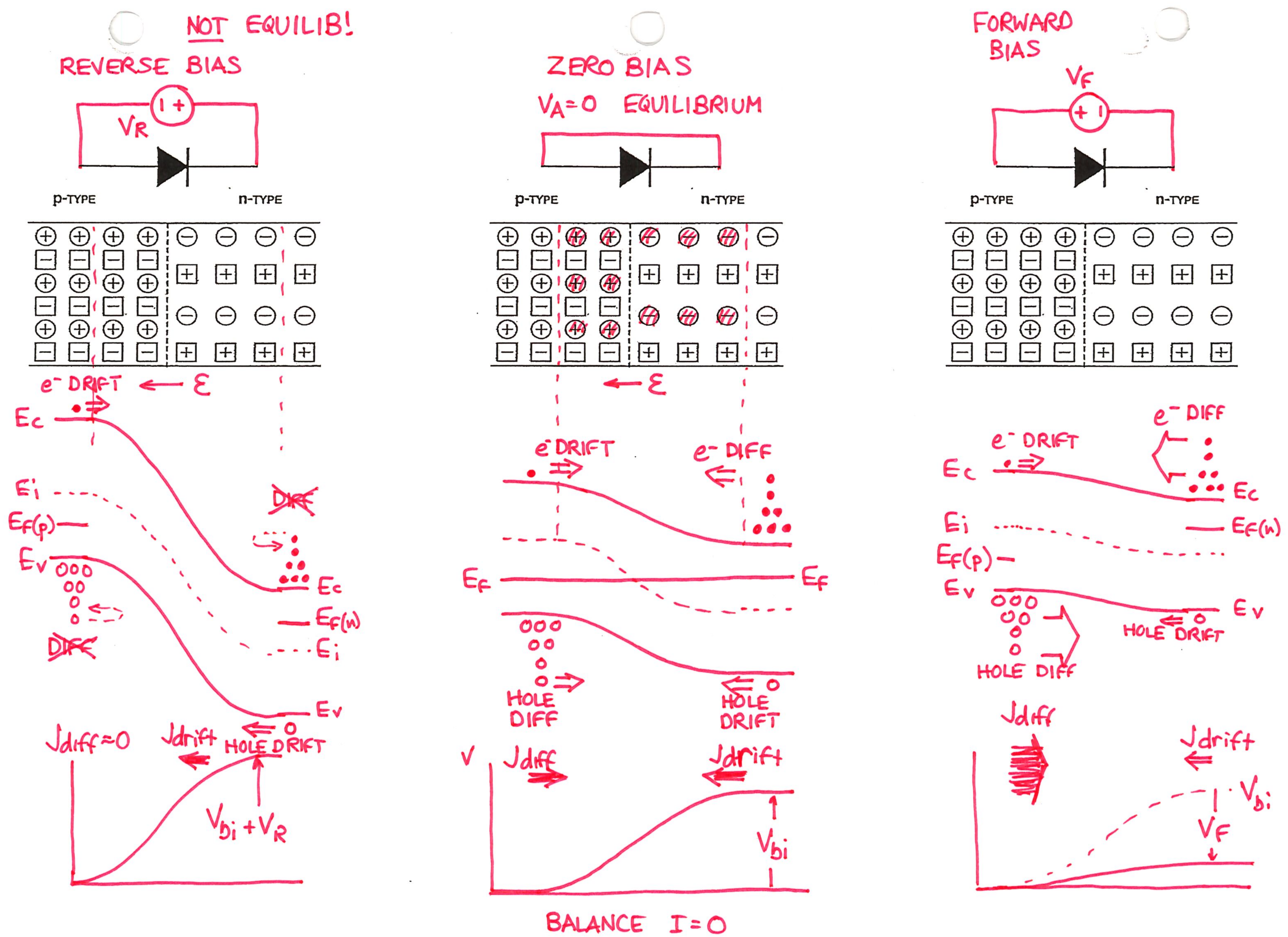
$$x_p \approx \sqrt{\frac{2K_S\epsilon_0}{q}\frac{N_D}{N_A^2}V_{bi}}$$

$$x_n \approx \sqrt{\frac{2K_S\epsilon_0}{q}\frac{1}{N_D}V_{bi}} \iff W \approx \sqrt{\frac{2K_S\epsilon_0}{q}\frac{1}{N_D}V_{bi}}$$

SAME!

DEPLETION REGION EXISTS MOSTLY IN LIGHTLY DOPED SIDE!

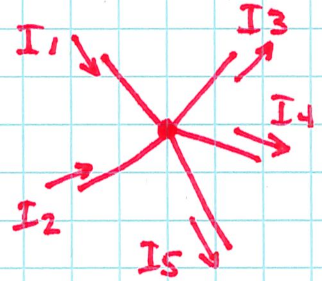
W DEPENDS "ONLY" ON LIGHTER DOPING


NOT EQUILIB!
REVERSE BIAS
V_R
p-TYPE
n-TYPE
e⁻ DRIFT
$\mathcal{E}$
E_c
E_i
$E_f(p)$
E_v
DIFF
DIFF
E_c
$E_f(n)$
E_i
E_v
HOLE DRIFT
Jdiff≈0
Jdrift
$V_{bi} + V_R$
ZERO BIAS
$V_A = 0$ EQUILIBRIUM
p-TYPE
n-TYPE
$\mathcal{E}$
e⁻ DRIFT
e⁻ DIFF
E_F
E_F
HOLE DIFF
HOLE DRIFT
V
Jdiff
Jdrift
V_{bi}
BALANCE I=0
FORWARD BIAS
V_F
p-TYPE
n-TYPE
e⁻ DIFF
e⁻ DRIFT
E_c
E_c
E_i
$E_f(n)$
$E_f(p)$
E_v
E_v
HOLE DRIFT
HOLE DIFF
Jdiff
Jdrift
V_{bi}
V_F

CONTINUITY EQUATION (3.4.1)

KCL

$\Sigma I_{IN} = \Sigma I_{OUT}$ $\quad I_1 + I_2 = I_3 + I_4 + I_5$



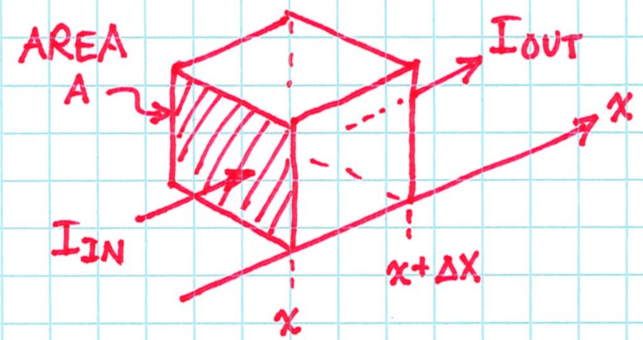
ΣI AT NODE $= 0$

$I_1 + I_2 - I_3 - I_4 - I_5 = 0$

~~CHARGE CAN'T "PILE UP" AT A NODE~~ } CIRCUITS

OH YES IT CAN! } DEVICES

LOOK AT SMALL VOLUME OF SEMICONDUCTOR



CURRENT FLOW PARALLEL TO x AXIS

HOW MUCH CHARGE Q IN THIS VOLUME DUE TO HOLES?

$$Q = \underbrace{q}_{\frac{\text{CHARGE}}{\cancel{\text{CARRIER}}}} \; \underbrace{p(x)}_{\frac{\cancel{\text{CARRIER}}}{\cancel{\text{VOL}}}} \; \underbrace{A\Delta x}_{\text{VOL}} \qquad [1]$$

HOW WILL THIS CHARGE VARY OVER TIME?

IF $I_{IN} \neq I_{OUT} \Rightarrow \frac{dQ}{dt} \neq 0$

$$\frac{dQ}{dt} = I_{IN} - I_{OUT} \quad [2]$$

[2] MACRO VIEW (CIRCUITS)

[1] MICRO VIEW (DEVICES)

TIME DERIVATIVE OF [1]

$$\frac{dQ}{dt} = q\,\underbrace{\frac{\partial p(x)}{\partial t}}\,A\,\Delta x \quad [3]$$

PARTIAL DERIVATIVE
p FUNCTION OF BOTH
x SPACE   t TIME

REARRANGE [3]

$$\frac{\partial p}{\partial t} = \frac{1}{q\,A\,\Delta x}\frac{dQ}{dt} \quad [4]$$

SUB [2] INTO [4]

$$\frac{\partial p}{\partial t} = \frac{1}{q\,A\,\Delta x}\left(I_{IN} - I_{OUT}\right) \quad [5]$$

DEVICES $J = I/A$ DENSITY

$$\frac{\partial p}{\partial t} = \frac{1}{q\,\Delta x}\left(\underbrace{\frac{I_{IN}}{A}}_{J_p(x)} - \underbrace{\frac{I_{OUT}}{A}}_{J_p(x+\Delta x)}\right)$$

CHANGE SIGN, MOVE Δx

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\underbrace{\left(\frac{J_p(x+\Delta x) - J_p(x)}{\Delta x}\right)}_{\lim\ \Delta x \to 0\quad \frac{\partial J_p}{\partial x}}$$

$$\boxed{\underbrace{\frac{\partial p}{\partial t}}_{\text{TIME RATE OF CHANGE FOR } p(x)} = \frac{-1}{q}\underbrace{\frac{\partial J_p}{\partial x}}_{\text{SPATIAL DERIVATIVE OF CURRENT DENSITY}}}$$

CONTINUITY
EQUATION
(1-D)

IN 3D
$$\frac{\partial p}{\partial t} = \frac{-1}{q}\underbrace{\nabla}_{\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}} \cdot \vec{J}_p$$

$\pi = 3.141592...$

WINDING
ROAD

"Muito
obrigado"

IS THIS THE
ONLY WAY $p(x)$
CAN CHANGE?
NO!

RECOMBINATION / GENERATION

$\underbrace{\text{RECOMBINATION}}_{e^- \text{ "FALLS INTO" HOLE}}$ / $\underbrace{\text{GENERATION}}_{e^- \text{ JUMP INTO CONDUCTION BAND LEAVES HOLE}}$

e^-HOLE PAIR

"R-G STATISTICS"

$$\underbrace{n}_{\text{TOTAL}} = \underbrace{n_o}_{\text{EQUILIB}} + \underbrace{\Delta n}_{\text{CHANGE FROM EQUILIB}}$$

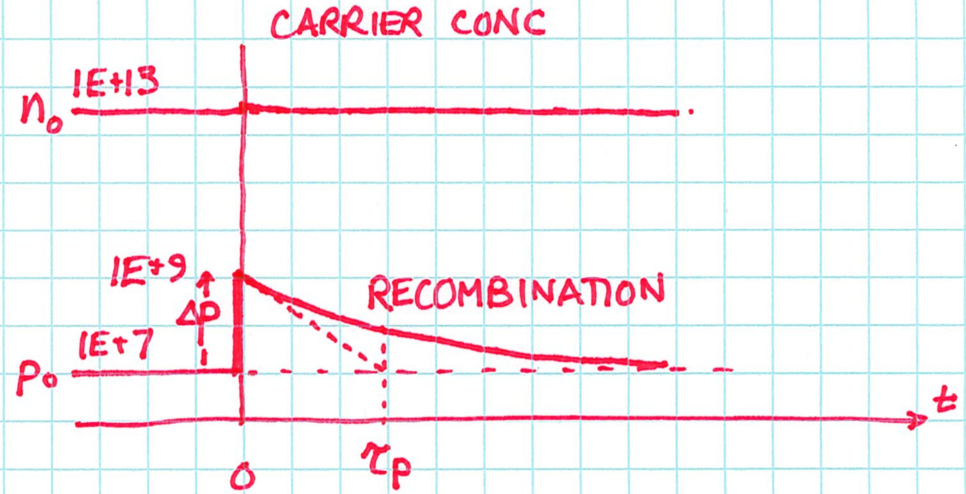
$$p = p_o + \Delta p$$

EXAMPLE: $N_D = 1E+13$ donor/cm^3

$$n = N_D = 1E+13 = n_o$$

$$p = \frac{n_i^2}{n} = \frac{(1E+10)^2}{1E+13} = 1E+7 = p_o$$

BLOCK OF SEMICONDUCTOR MATERIAL



AT TIME $t=0$: PULSE OF LIGHT CREATE $\underbrace{1E+9}$ e^- HOLE PAIRS /cm^3

Δn NEGLIGIBLE $\ll n_o$

p BIG DIFFERENCE: HOLES ARE MINORITY CARRIER

HOLES ↘ DUE TO RECOMBINATION

τ_p "MINORITY CARRIER LIFETIME"

$$\frac{\partial p}{\partial t} = \frac{-\Delta p}{\tau_p}$$

DEVELOP MINORITY CARRIER DIFF EQ: FOR e^-

START: 3-D CONTINUITY EQ FOR e^- (ALWAYS)

CHANGE IN MOBILE e^- CONCENTRATION

$$\frac{\partial n}{\partial t} = \frac{1}{q}\underbrace{\nabla \cdot J_N}_{} + \underbrace{\left.\frac{\partial n}{\partial t}\right|_{\substack{THERMAL \\ R-G}}}_{\text{RECOMB OR GENERATION}} + \underbrace{\left.\frac{\partial n}{\partial t}\right|_{other}}_{\text{LIGHT}} \qquad [1]$$

$\boxed{1 \mid \text{1-D}}$

$$\frac{\partial}{\partial x} \qquad \frac{1}{q}\frac{\partial J_N}{\partial x} \qquad [2]$$

USE TOTAL CURRENT EXPRESSION

$$J_N = \underbrace{\cancel{q\mu_n n \mathcal{E}}}_{\text{DRIFT}} + \underbrace{qD_N \frac{\partial n}{\partial x}}_{\text{DIFFUSION}} \quad \Big\} \text{ ALWAYS} \qquad [3]$$

small small

$\boxed{2 \mid \text{MINORITY CARR}}$ $\boxed{3 \mid \mathcal{E} \text{ SMALL; DIFFUSION DOMINATES}}$

$$\Rightarrow J_N \approx qD_N \frac{\partial n}{\partial x} \qquad [4]$$

DIFFUSION

CONC GRADIENT

SUB [4] INTO [2] INTO [1]

$$\frac{\partial n}{\partial t} = \frac{1}{\cancel{q}}\frac{\partial}{\partial x}\underbrace{\cancel{q}D_N\frac{\partial n}{\partial x}}_{J_N} + \ldots$$

$$\frac{\partial n}{\partial t} = \underbrace{D_N \frac{\partial^2 n}{\partial x^2}}_{} + \underbrace{\left.\frac{\partial n}{\partial t}\right|_{\substack{THERMAL \\ R-G}}}_{-\frac{\Delta n}{\tau_n}} + \underbrace{\left.\frac{\partial n}{\partial t}\right|_{other}}_{G_L} \qquad [5]$$

$n = \underbrace{n_o}_{\text{EQUILIBRIUM MINORITY CARR}} + \underbrace{\Delta n}_{\text{EXCESS CONC.}}$

$\boxed{4}$ n_o NOT A FUNCTION OF x. $\frac{\partial n_o}{\partial x} = 0$

$\boxed{5}$ LOW LEVEL INJECTION

$\boxed{6}$ R-G ACCORDING TO τ_n DOMINATES (MINORITY CARR LIFETIME)

$\boxed{7}$ GENERATION OF e^--HOLE PAIRS DUE TO LIGHT IS ONLY "OTHER" n-TYPE CARRIERS (e^-) — MINORITY CARRIERS

2

EXCESS — IN p-TYPE REGION

$$\underbrace{\frac{\partial \Delta n_p}{\partial t}}_{\substack{\text{CHANGE IN}\\ \text{MINORITY}\\ \text{CARRIER}\\ \text{CONC. VS TIME}}} = \underbrace{D_N \frac{\partial^2 \Delta n_p}{\partial x^2}}_{\text{DIFFUSION}} - \underbrace{\frac{\Delta n_p}{\tau_n}}_{\text{RECOMBINATION}} + \underbrace{G_L}_{\text{GENERATION}}$$

TEXT (3.54 a)

FOR HOLES

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

TEXT (3.54b)

ECE4904 IDEAL DIODE EQUATION DERIVATION (p. 1)

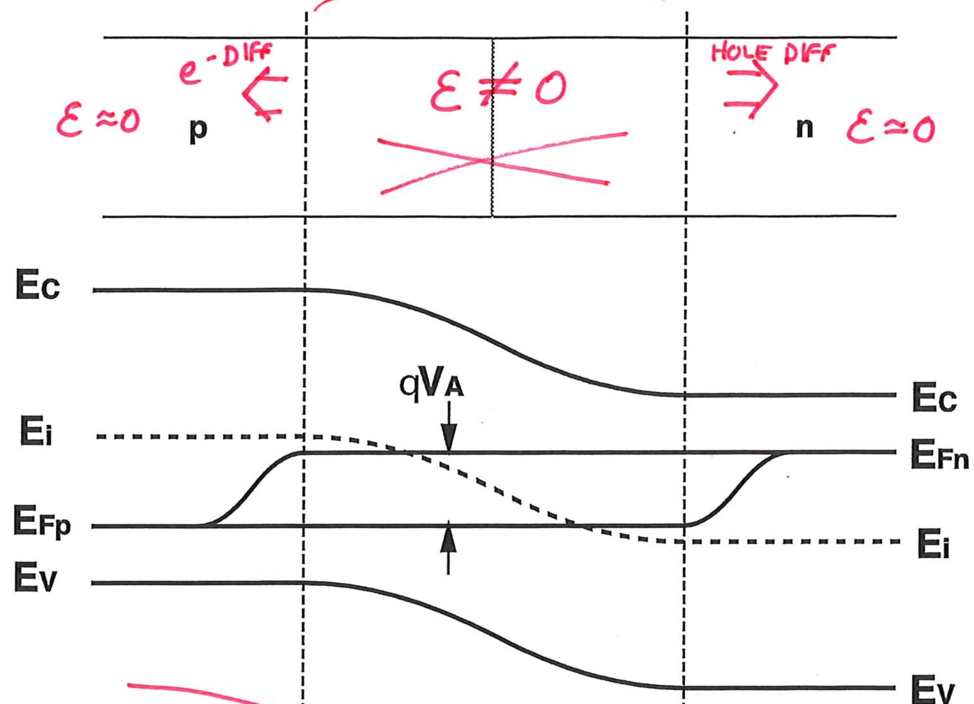
Minority carrier diffusion quations (valid in quasineutral region where E field ≈0)

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_N} \cancel{G_L} \to 0 \qquad \frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_P} \cancel{G_L} \to 0$$

Steady state (DC voltage, current: all d/dt = 0)

$$0 = D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_N} \qquad 0 = D_P \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_P}$$

DEPL REGION



"LAW OF THE JUNCTION" at edges of depletion region (assumes quasiFermi levels lat through depletion region)

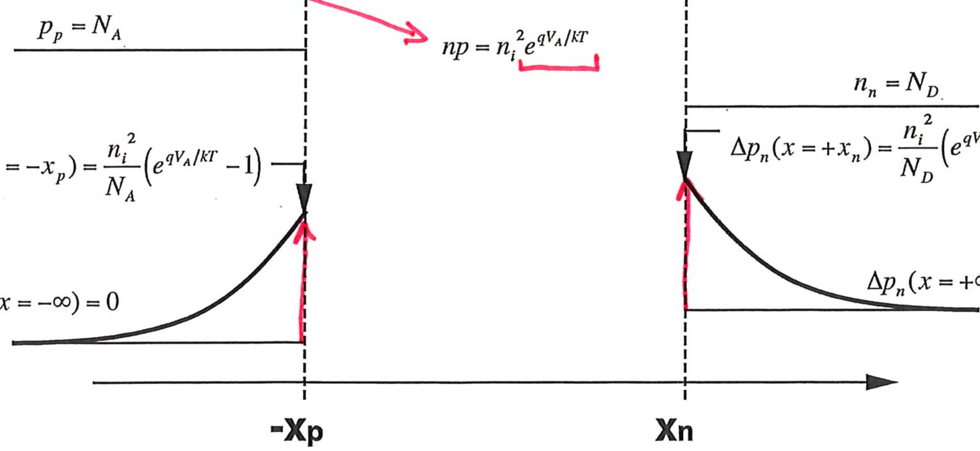
$p_p = N_A$ $\qquad np = n_i^2 e^{qV_A/kT}$ $\qquad n_n = N_D$

Boundary conditions t edges of depletion region (from law of the junction)

$$\Delta n_p(x=-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) \qquad \Delta p_n(x=+x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Boundary conditions far from depletion region eturns to equilibrium conc)

$$\Delta n_p(x=-\infty) = 0 \qquad \Delta p_n(x=+\infty) = 0$$



Solutions to differential equation subject to boundary conditions

$$\Delta n_p(x=-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{(x+x_p)/L_N} \qquad \Delta p_n(x=+x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{-(x-x_n)/L_P}$$

Current density equation

$$J_N = qD_N \frac{d\Delta n_p}{dx} \qquad J_P = -qD_P \frac{d\Delta p_n}{dx}$$

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'lug differential equation solution into current

$$J_N = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{(x+x_p)/L_N} \quad (x < -x_p) \qquad J_P = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{-(x-x_n)/L_P} \quad (x > +x_n)$$

ECE4904 IDEAL DIODE EQUATION DERIVATION (p. 2)

Current density equation

$$J_N = qD_N \frac{d\Delta n_p}{dx} \qquad J_P = -qD_P \frac{d\Delta p_n}{dx}$$

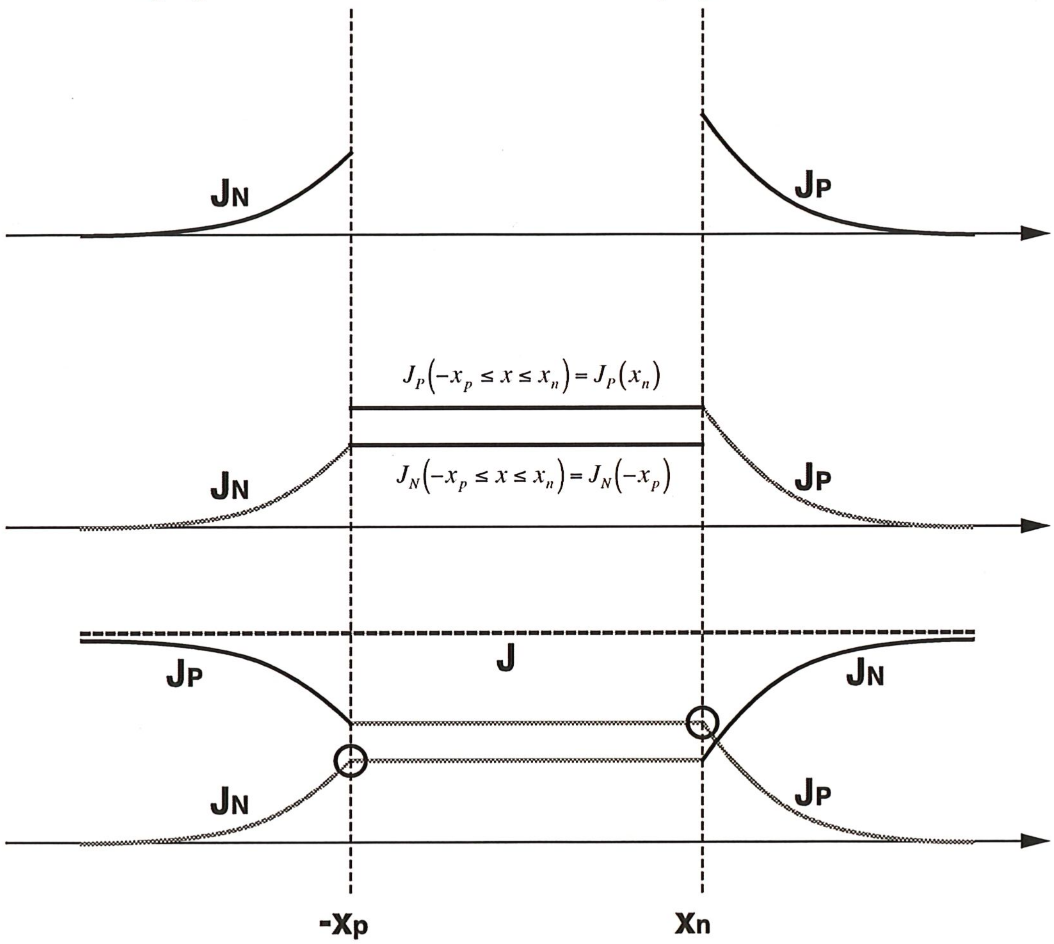
$$J_N = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}\left(e^{qV_A/kT}-1\right)e^{(x+x_p)/L_N} \quad (x \le -x_p) \qquad J_P = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT}-1\right)e^{-(x-x_n)/L_P} \quad (x \ge +x_n)$$

Plug differential equation solution into current density equation

Assume no R-G in depletion region

Steady state: total J constant throughout

J given by sum of hole. electron components at edges of depletion region



Evaluate current density expressions at edges of depletion region

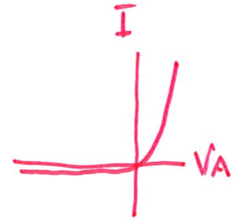
$$J_N(-x_p) = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}\left(e^{qV_A/kT}-1\right) \qquad J_P(x_n) = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT}-1\right)$$

Total current density

$$J = q\left[\frac{D_N}{L_N}\frac{n_i^2}{N_A}+\frac{D_P}{L_P}\frac{n_i^2}{N_D}\right]\left(e^{qV_A/kT}-1\right)$$

Total current I=JA Current density x junction area A

$$\boxed{I = qA\left[\frac{D_N}{L_N}\frac{n_i^2}{N_A}+\frac{D_P}{L_P}\frac{n_i^2}{N_D}\right]\left(e^{qV_A/kT}-1\right)}$$



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