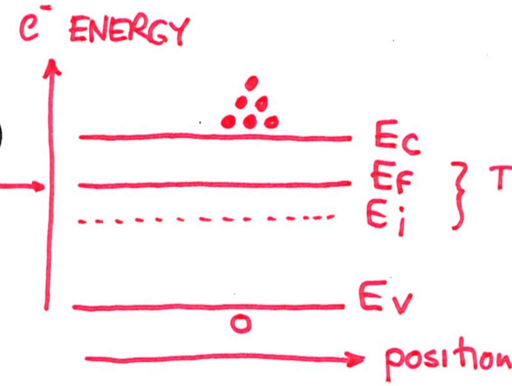
ECE4904 Lecture 3

Energy Band Diagram Review (Ch. 2)

E_F flat in equilibrium



} TELLS YOU n, p
ELECTRON, HOLE CONCENTRATION

IN EQUILIBRIUM
NONDEGENERATE

$$\left.\begin{aligned} n &= n_i e^{(E_F - E_i)/kT} \\ p &= n_i e^{(E_i - E_F)/kT} \end{aligned}\right\} \underbrace{np = n_i^2}_{\text{"np PRODUCT RELATIONSHIP"}}$$

Carrier Motion: Drift (3.1)

$\mathcal{E}$ FIELD

- Drift Current (3.1.2)
- Mobility (3.1.3)
- Resistivity (3.1.4)

Carrier Motion: Diffusion (3.2)

CONCENTRATION GRADIENT

- Diffusion current (3.2.3)
- Diffusion Coefficient (3.2.4)

Handouts

- 1-minute Quiz
- Textbook Figures 3.4,5,7,8

Hand In: HW 1

HW 2 (online)

ECE4904 "One Minute" Quiz: Energy Band Diagram

This quiz explores the relationship between carrier concentrations (n,p), doping concentrations (N_A, N_D), types of material (n-type, p-type, intrinsic), the energy band diagram, and the Fermi level E_F.

Each line in the table below describes a different sample of silicon semiconductor in equilibrium at T=300K. If the material is doped, it is doped uniformly and only one type of dopant (acceptor or donor) is used. ~~Using the~~ information provided in each line of the table, fill in the blanks in each of the other columns on each line.

Indicate the type of material:	Indicate the acceptor N_A and donor N_D concentrations		Indicate the hole p and electron n carrier concentrations		Draw the energy band diagram; indicate the Fermi level E_F relative to the intrinsic Fermi level E_i (with energy difference in eV)
n-TYPE p-TYPE INTRINSIC	**DOPING**		**CARRIER CONC**		**ENERGY BAND DIAGRAM**
	N_A	N_D	p	n	
n-TYPE ~~p-TYPE~~ ~~INTRINSIC~~ $n>n_i$	0	1E+14 atom/cm³	1E+6 /cm³ $< n_i$	1E+14 /cm³ $\frac{1E+20}{1E+6}$	E_C E_F — 0.239eV E_i E_V $E_F > E_i$; n TYPE 1.12 eV

np PRODUCT RELATIONSHIP $np = n_i^2$

$n(1E+6) = (1E+10)^2 \Rightarrow n = 1E+14$

$n = n_i e^{(E_F - E_i)/kT}$

KNOWN $E_F - E_i = kT \ln\left(\frac{n}{n_i}\right)$

$.0259\,eV \ln\left(\frac{1E+14}{1E+10}\right)$

$= 0.239$ eV

	N_A	N_D	p	n	
p-TYPE $E_F < E_i$	2.2E+13 $p = N_A$ IN ACCEPTOR DOPED REGION	0	2.2E+13 /cm³	4.5E+6 /cm³	E_C 0.20eV $E_i - E_F$ E_i E_F E_V

$np = (1E+10)^2$

$n = \frac{1E+20}{2.2E+13}$

$p = n_i e^{(E_i - E_F)/kT}$

$(1E+10)\, e^{(0.2eV/.0259eV)}$

$p = 2.2E+13$

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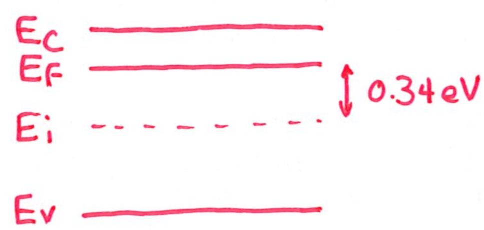
n-TYPE p-TYPE INTRINSIC	DOPING		CARRIER CONC		ENERGY BAND DIAGRAM
	N_A	N_D	p	n	
$N_D > N_A$ n-TYPE	0	5E+15 /cm^3	2E+4	5E+15	

$p = \frac{n_i^2}{n} = \frac{1E+20}{5E+15}$

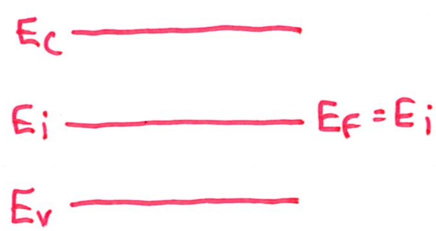
n TYPE: $E_F > E_i$

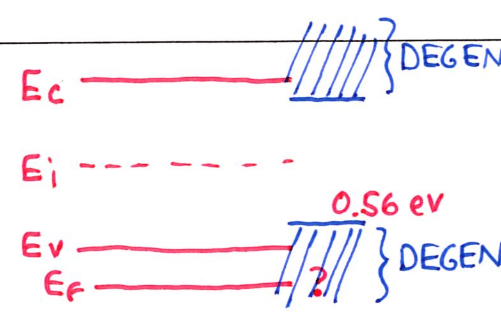
$E_F - E_i = KT \ln\left(\frac{n}{n_i}\right) = .0259 \ln\left(\frac{5E+15}{1E+10}\right)$

≈ 0.34 eV

T=300K

$p = n_i$ $n = n_i$

INTRINSIC (NOT DOPED)	0	0	1E+10 /cm^3	1E+10 /cm^3	

	N_A	N_D	p	n	
p-TYPE	5E+22 /cm^3 VERY HEAVILY DOPED DEGENERATE	0	5E+22	$\cancel{np = n_i^2}$	

$E_i - E_F = \cancel{.0259 \ln\left(\frac{5E+22}{1E+10}\right)}$

$= \cancel{0.757 \text{ eV}}$

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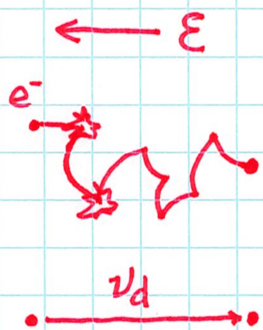
DRIFT

CARRIER IN SEMICONDUCTOR

NO $\vec{\mathcal{E}}$ FIELD

RANDOM
COLLISIONS
(THERMAL
MOTION)
ON AVERAGE:
NO NET
MOTION

$\vec{\mathcal{E}}$ FIELD



NET MOTION

AVERAGE
VELOCITY
v_d

"DRIFT
VELOCITY"

PLOT v_d VELOCITY VS. $\mathcal{E}$ FIELD

$$v_d = \mu \mathcal{E}$$

$$\left[\frac{cm}{sec}\right] \quad \left[\frac{cm^2}{V\cdot sec}\right] \left[\frac{V}{cm}\right]$$

MOBILITY: AVERAGE EFFECT
ON CARRIERS
ALSO EXPERIENCING
RANDOM THERMAL
MOTION

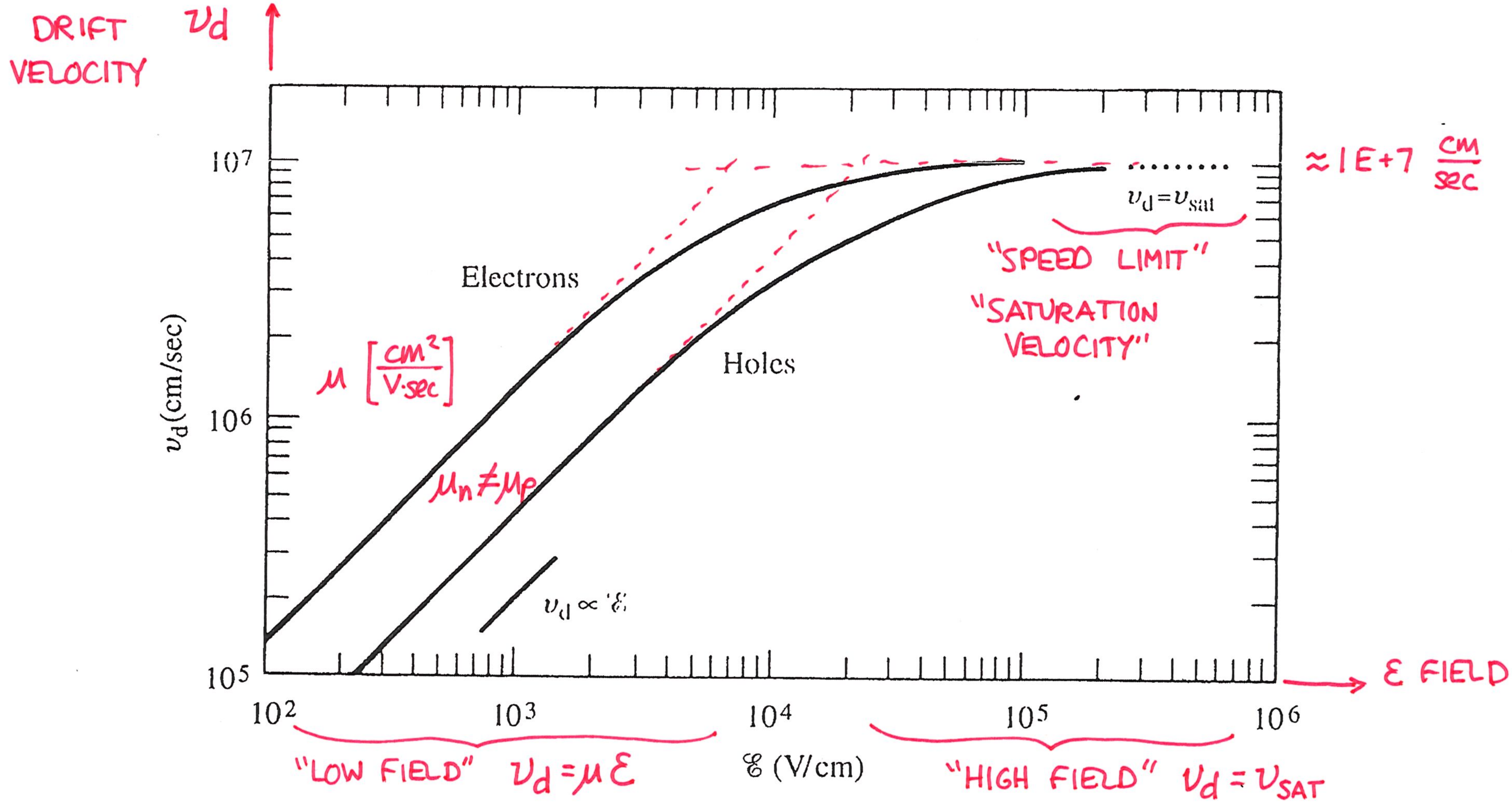


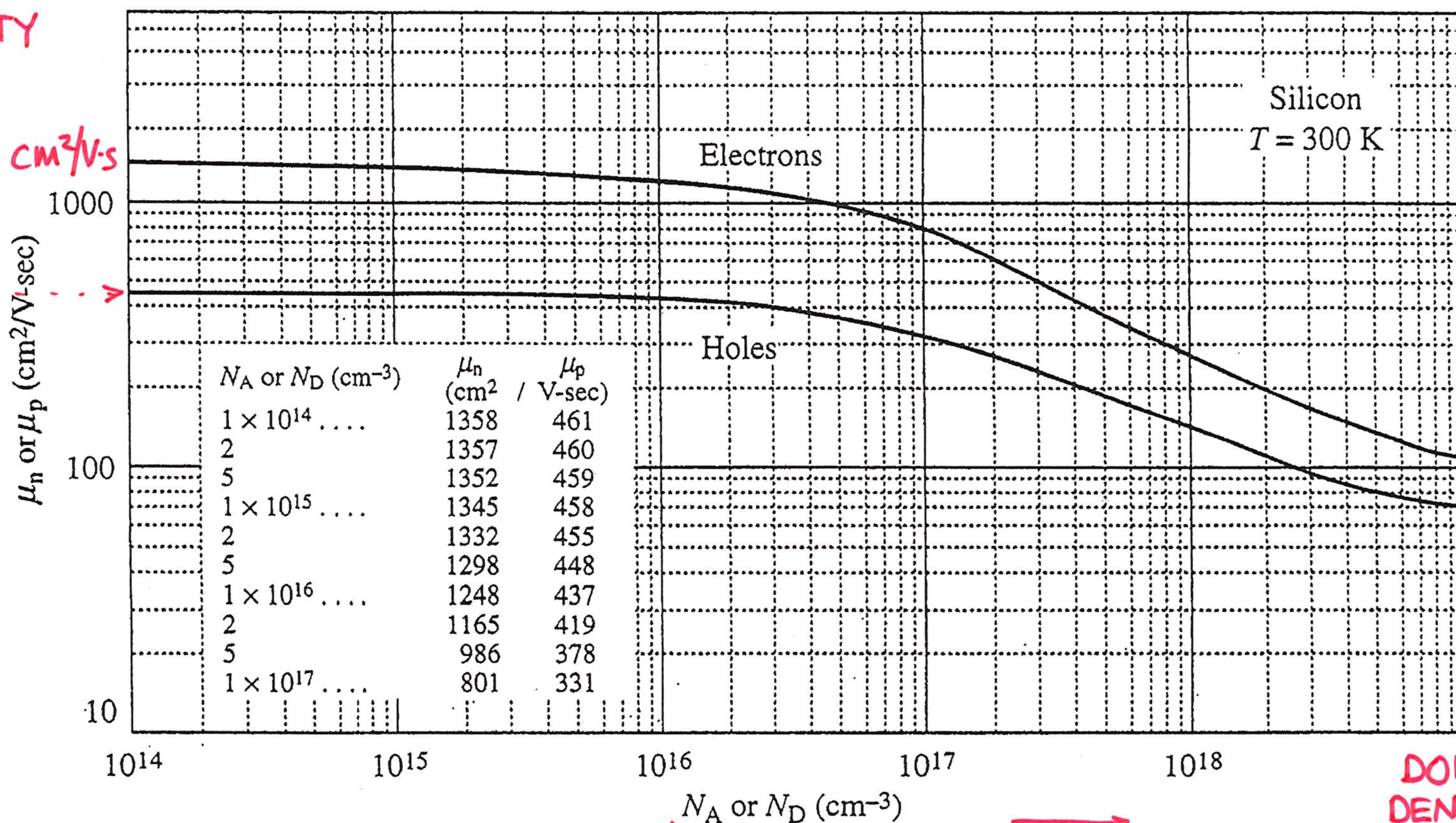
Figure 3.4 Measured drift velocity of the carriers in ultrapure silicon maintained at room temperature as a function of the applied electric field. Constructed from the data fits and the data respectively in Jacoboni et al.[4] and Smith et al.[5]

MORE LATTICE COLLISIONS (DOPANT IMPURITY ATOMS)

MOBILITY

$\mu_n = 1360\ cm^2/V{\cdot}s$

$\mu_p = 460\ \frac{cm^2}{V \cdot S}$



N_A or N_D (cm^{-3})	μ_n (cm^2	μ_p / V-sec)
1×10^{14}	1358	461
2	1357	460
5	1352	459
1×10^{15}	1345	458
2	1332	455
5	1298	448
1×10^{16}	1248	437
2	1165	419
5	986	378
1×10^{17}	801	331

EITHER ONE

HEAVIER DOPING

DOPING DENSITY

Figure 3.5 Room temperature carrier mobilities as a function of the dopant concentration in (a) Si

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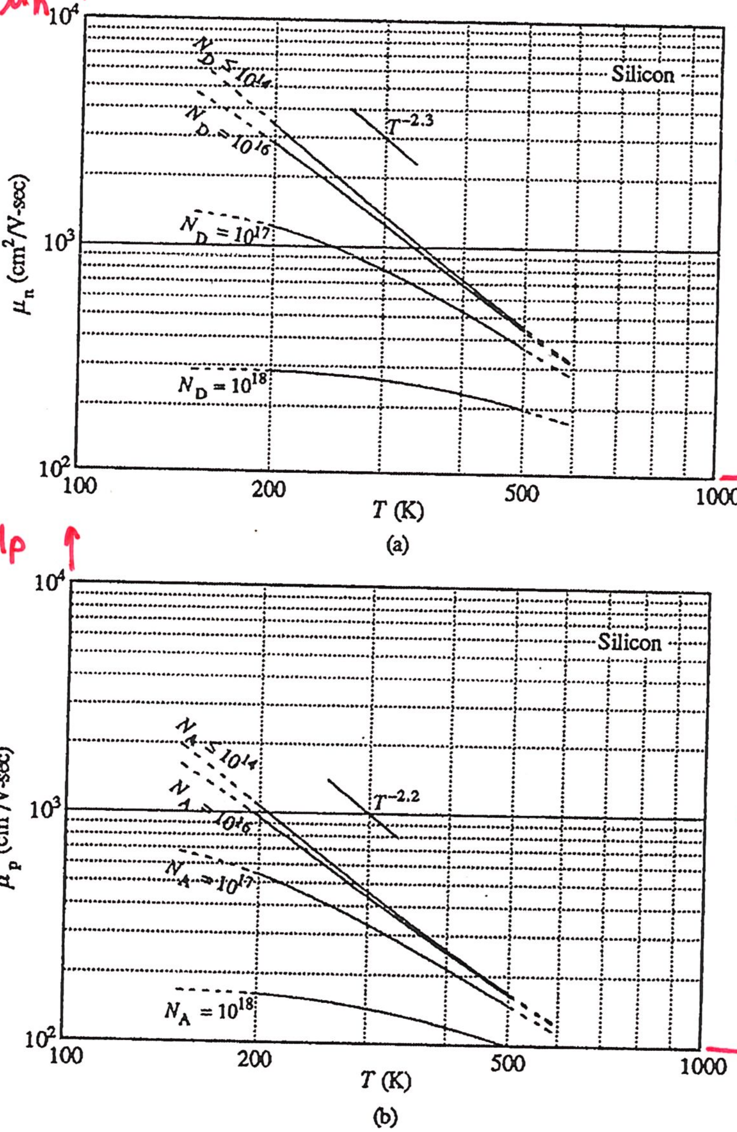
e^-

$\mu_n \uparrow$

HOLES

$\mu_p \uparrow$



$\propto T^{-2.3}$

$\rightarrow$ TEMP

INCREASING TEMP

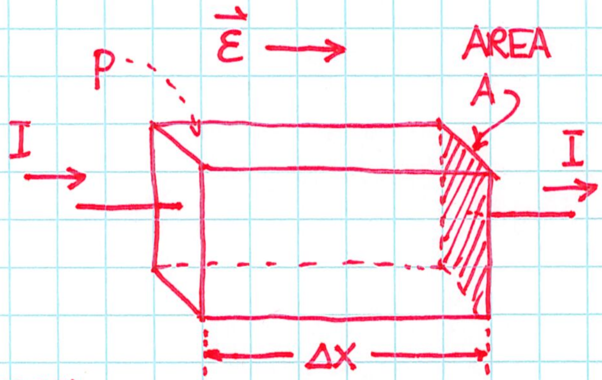
MORE THERMAL VIBRATION OF LATTICE

$\propto T^{-2.2}$

$\rightarrow$ TEMP

Figure 3.7 Temperature dependence of (a) electron and (b) hole mobilities in silicon for dopings ranging from $\leq 10^{14}/\text{cm}^3$ to $10^{18}/\text{cm}^3$. The curves were constructed using the empirical fit relationships and parameters presented in Exercise 3.1. The dashed line portion of the curves correspond to a slight extension of the fit beyond the verified $200\text{ K} \leq T \leq 500\text{ K}$ range of validity.

CURRENT $\quad \frac{\text{FLOW OF CHARGE}}{\text{PER UNIT TIME}} \quad \frac{dQ}{dt}$



DEFINITION

$$I = \frac{dQ}{dt} \xRightarrow{DC} \frac{\Delta Q}{\Delta t} \qquad [1]$$

ΔQ ALL CHARGE IN VOLUME

Δt TIME FOR ALL OF ΔQ TO CROSS THROUGH A

p HOLES/cm^3 (UNIFORM)

q e^- CHARGE 1.6 E-19 coul

TOTAL CHARGE

$$\Delta Q = \underbrace{q}_{\frac{\text{CHARGE}}{\text{CARRIER}}} \quad \underbrace{p}_{\frac{\text{CARRIER}}{\text{VOL}}} \quad \underbrace{A\Delta x}_{\text{VOL}} \qquad [2]$$

DRIFT VELOCITY
(ASSUME LOW FIELD)

$$v_d = \mu \mathcal{E} \qquad ★$$

FOR "LAST" CHARGE TO GET THROUGH A

$$v_d = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{v_d} \qquad [3]$$

SUB [2], [3] INTO [1]

$$I = \frac{q\, p\, A\, \cancel{\Delta x}\, v_d}{\cancel{\Delta x}}$$

$$I = q\, p\, A\, v_d$$

MORE SPECIFIC LABELING

$$I_{\underbrace{p}_{\text{HOLE}}|\underbrace{drift}_{\mathcal{E}\text{ FIELD}}} = q\, p\, A\, v_d \qquad [4]$$

NORMALIZE CURRENT TO CROSS SECTIONAL AREA

$$J = \frac{I}{A} \begin{matrix} \} [A] \\ \} [cm^2] \end{matrix} \Rightarrow \left[\frac{A}{cm^2}\right]$$

"CURRENT DENSITY"

$$I = JA$$

$$\vec{J}_{p|drift} = q\, p\, \vec{v}_d \qquad [5]$$

IN GENERAL $\vec{J}, \vec{v}$ VECTORS

LOW FIELD: USE ★ IN [5]

$$\underbrace{\vec{J}_{p|drift}}_{\frac{A}{cm^2}} = \underbrace{q\, p\, \mu}_{\frac{1}{\rho}} \underbrace{\vec{\mathcal{E}}}_{\frac{V}{cm}}$$

"OHM'S LAW FOR MATERIALS"

$$I = \frac{1}{R} V \quad \} \text{CIRCUITS}$$

$[\rho]$ RESISTIVITY
$[\Omega \cdot cm]$

CURRENT DUE TO BOTH HOLES, e^-

	HOLE	e^-
CARRIER	$\oplus$	$\ominus$
FIELD	$\xrightarrow{\mathcal{E}}$	
		opposite
MOTION	$v_d \rightarrow$	$\leftarrow v_d$
		opposite!
CURRENT COMPONENT	$\xrightarrow{J_p}$	$\xrightarrow{J_n}$

CURRENT COMPONENTS IN <u>SAME DIRECTION</u> AS $\mathcal{E}$ FIELD FOR BOTH HOLES, e^-

TOTAL DRIFT CURRENT

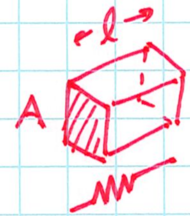
$$J_{drift} = J_p|_{drift} + J_n|_{drift}$$

$$= q p \mu_p \mathcal{E} + q n \mu_n \mathcal{E}$$

$$J_{drift} = q \underbrace{(\mu_p p + \mu_n n)}_{\frac{1}{\rho}} \mathcal{E}$$

OHM'S LAW $V = I R$

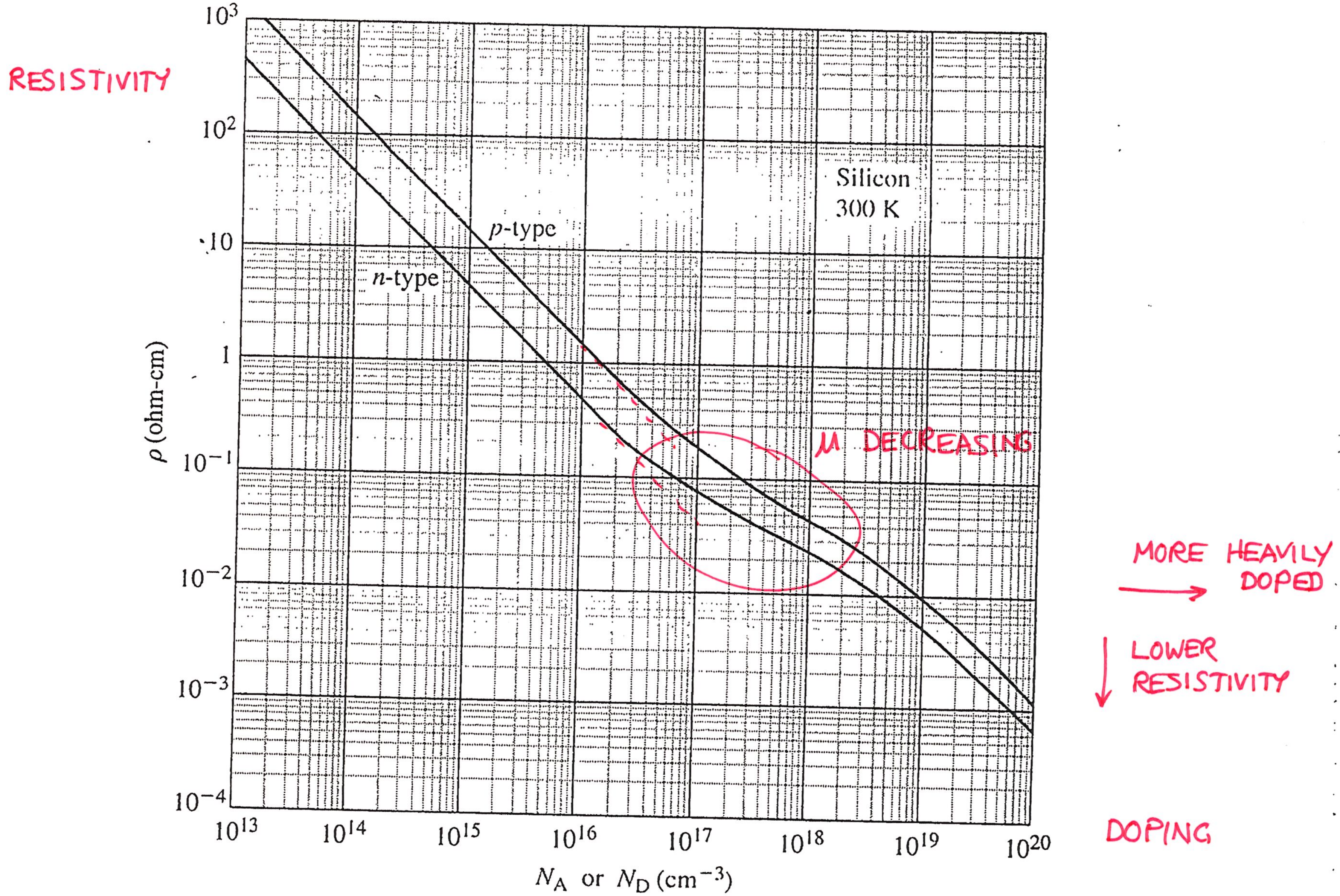
$\mathcal{E} = J \rho$ } RESISTIVITY



$R = \rho \frac{l}{A}$

RESISTIVITY

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)} \quad [\Omega \cdot cm]$$



(a)

Figure 3.8 Resistivity versus impurity concentration at 300 K in (a) Si and (b) other semiconductors. [(b) From Sze[2], © 1981 by John Wiley & Sons, Inc. Reprinted with permission.]

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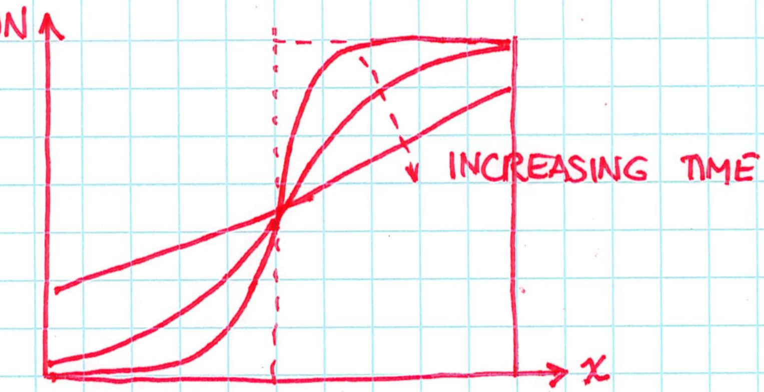
DIFFUSION

GENERAL: APPLIES TO ANY MOBILE PARTICLES

+

CONCENTRATION GRADIENT

WILL BE "FLATTENED OUT" (REDUCED) OVER TIME BY RANDOM THERMAL MOTION



DIFFUSION LAW

NOT NECESSARILY UNIFORM! FUNCTION OF SPATIAL POSITION

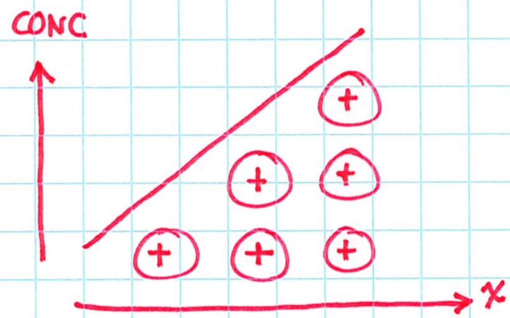
$$\underbrace{\mathcal{F}}_{\text{FLOW}} = \underbrace{-}_{\text{FLOW OPPOSES GRADIENT}} D \; \underbrace{\vec{\nabla}}_{\text{GRADIENT}} \; \underbrace{\rho(x)}_{\text{CONCENTRATION OF PARTICLES}}$$

FLOW: $\frac{\text{PARTICLES/SEC}}{\text{cm}^2}$

GRADIENT: $\frac{d}{dx}$ SLOPE

D: DIFFUSION COEFFICIENT $\left[\frac{\text{cm}^2}{\text{sec}}\right]$

DIFFUSION OF CHARGE CARRIERS

	HOLES	e^-
CARRIER CONCENTRATION		
GRADIENT	↗	↗
FLOW DUE TO DIFFUSION	←	←
CURRENT DENSITY J	←	→

$$J_p|_{diff} = -q D_p \vec{\nabla} p \qquad\qquad J_n|_{diff} = +q D_n \vec{\nabla} n$$

1D $$J_p|_{diff} = -q D_p \frac{dp}{dx} \qquad\qquad J_n|_{diff} = +q D_n \frac{dn}{dx}$$

HOLE, e^- J IN OPPOSITE DIRECTIONS!

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