Activity 2. Fourier Transformation Model of Image Formation

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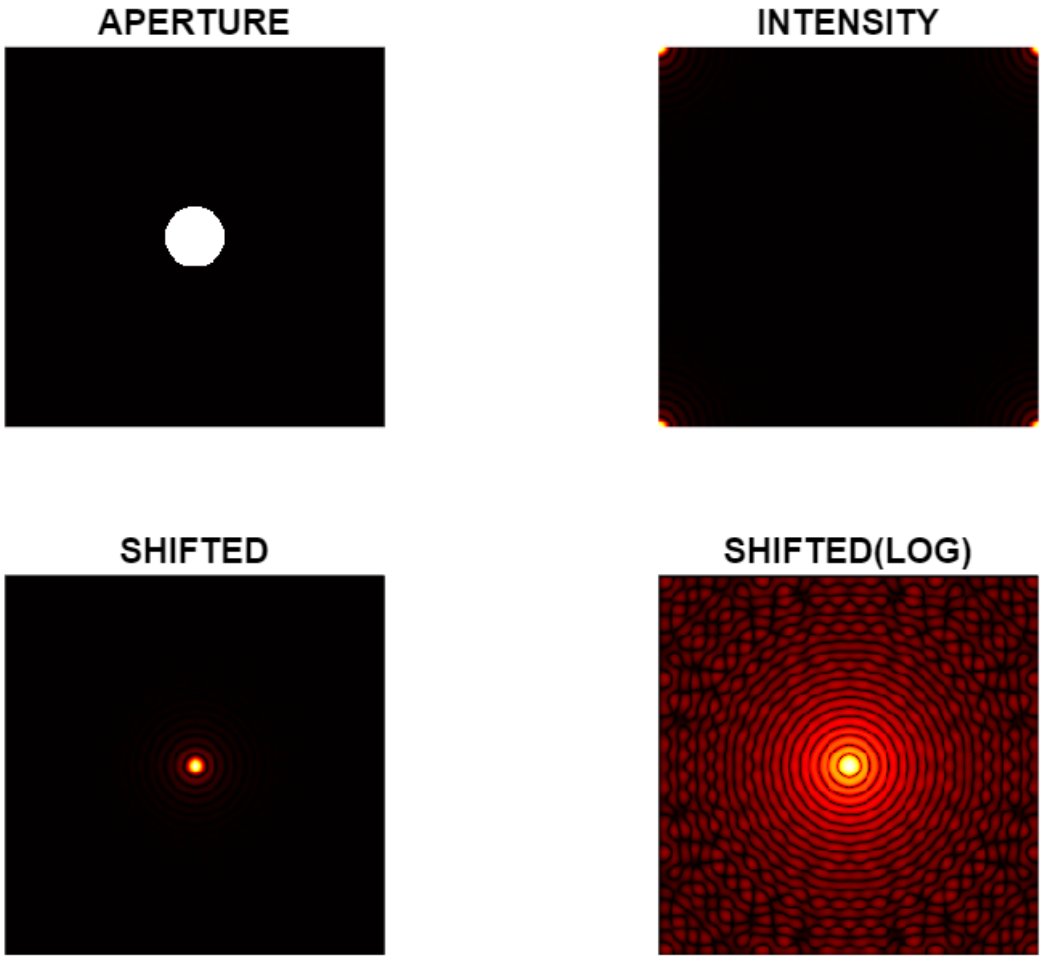
Activity 2.1. Familiarization with Discrete FT

Create an image of a white circle against a black background centered in a 400x400 pixel matrix. Apply fft2() on the image and compute the intensity values using abs(). Use fftshift to make the FFT2 output appear zero-centered then display. Use the "hot" colormap to make it look like a laser diffraction pattern.

Circle Aperture

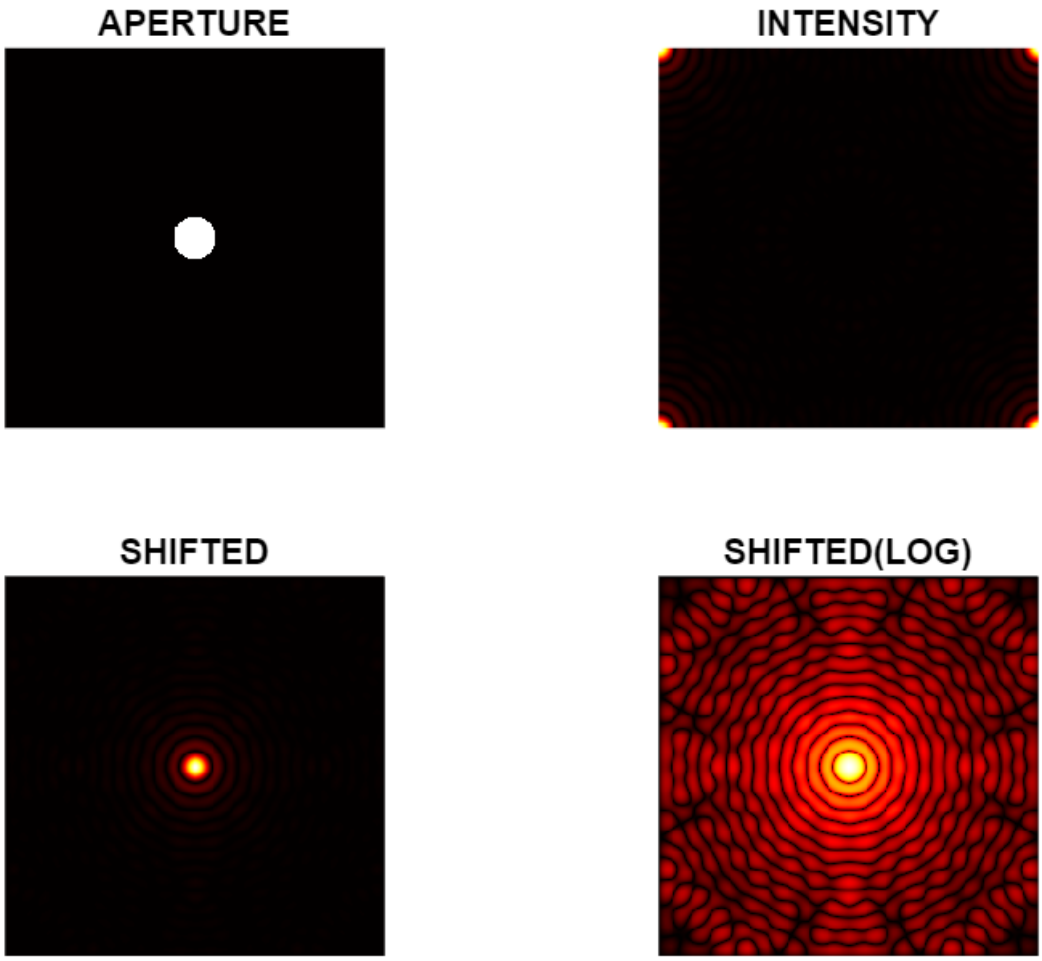
r = 0.1

```
clear; close;
FFT(Circle(0.1));
```



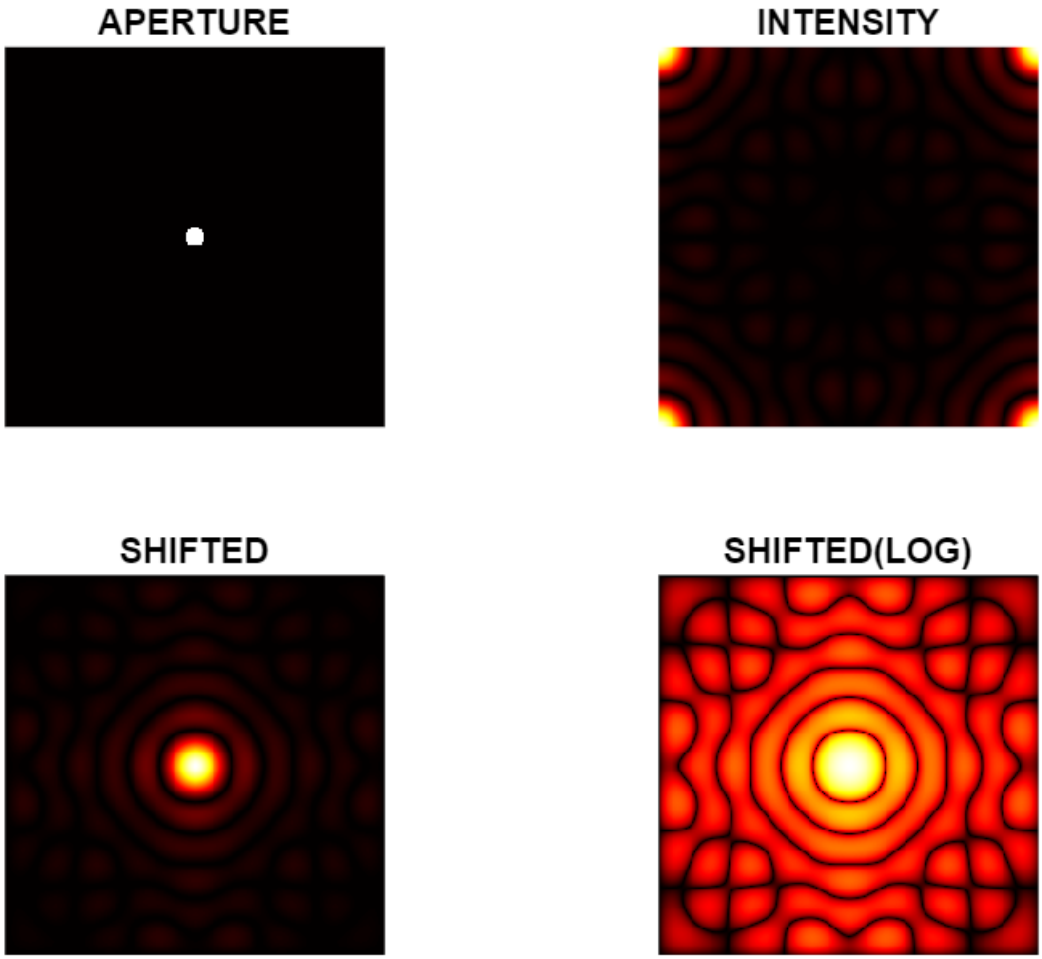
r = 0.05

```
FFT(Circle(0.05));
```



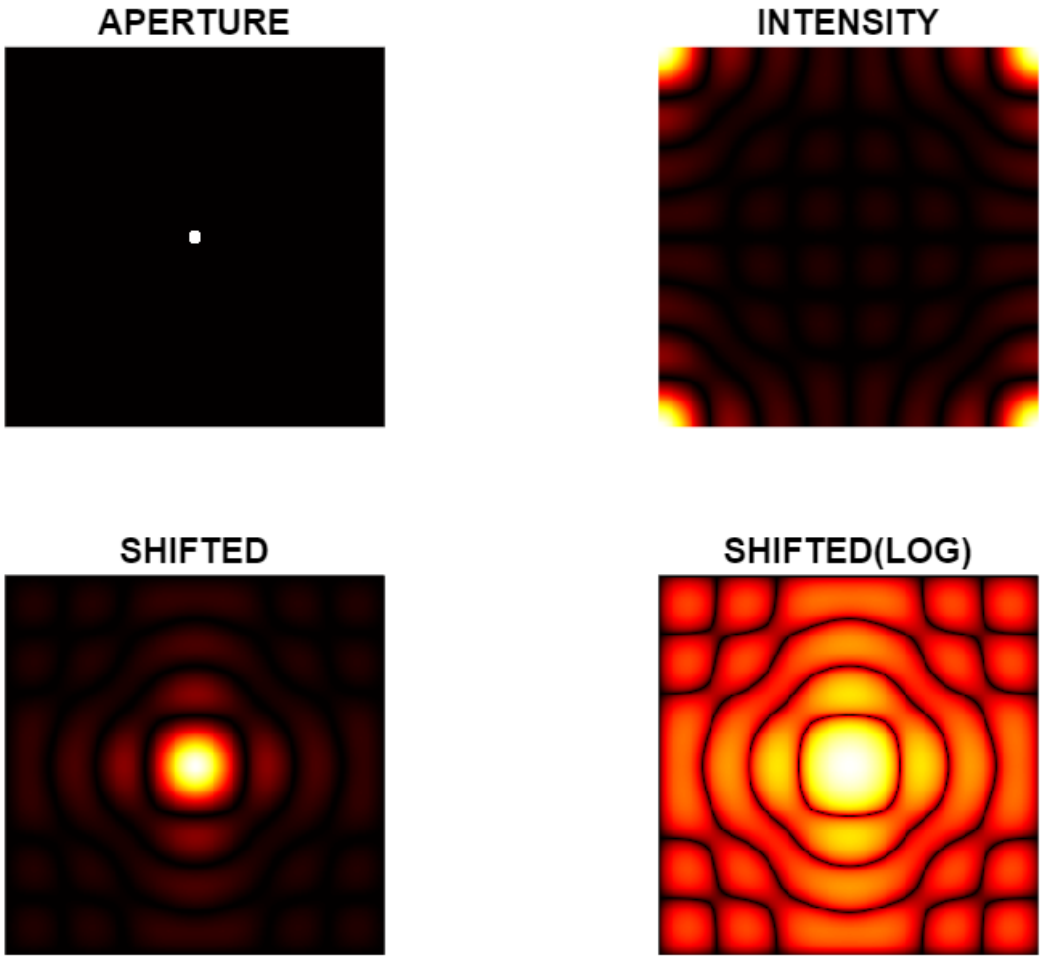
r = 0.01

```
FFT(Circle(0.01));
```



r = 0.005

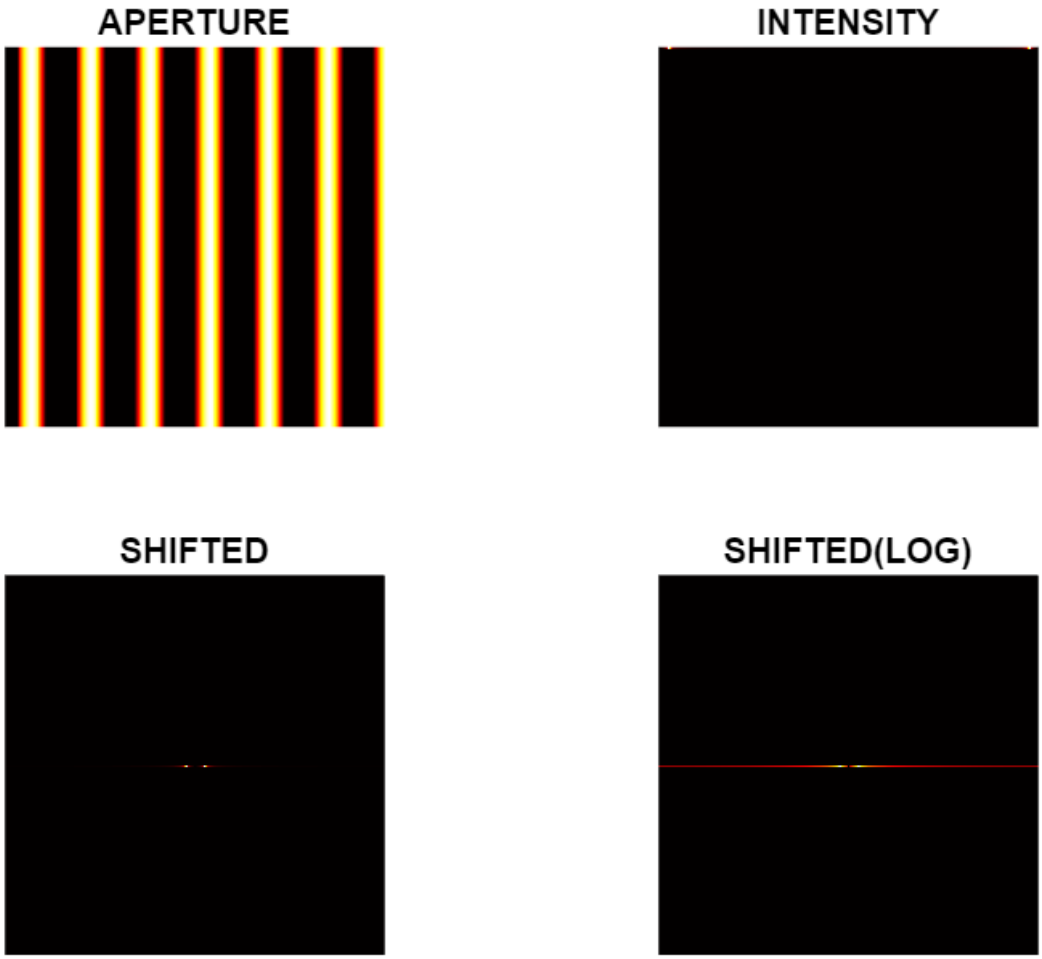
```
FFT(Circle(0.005));
```



Sinusoidal Aperture

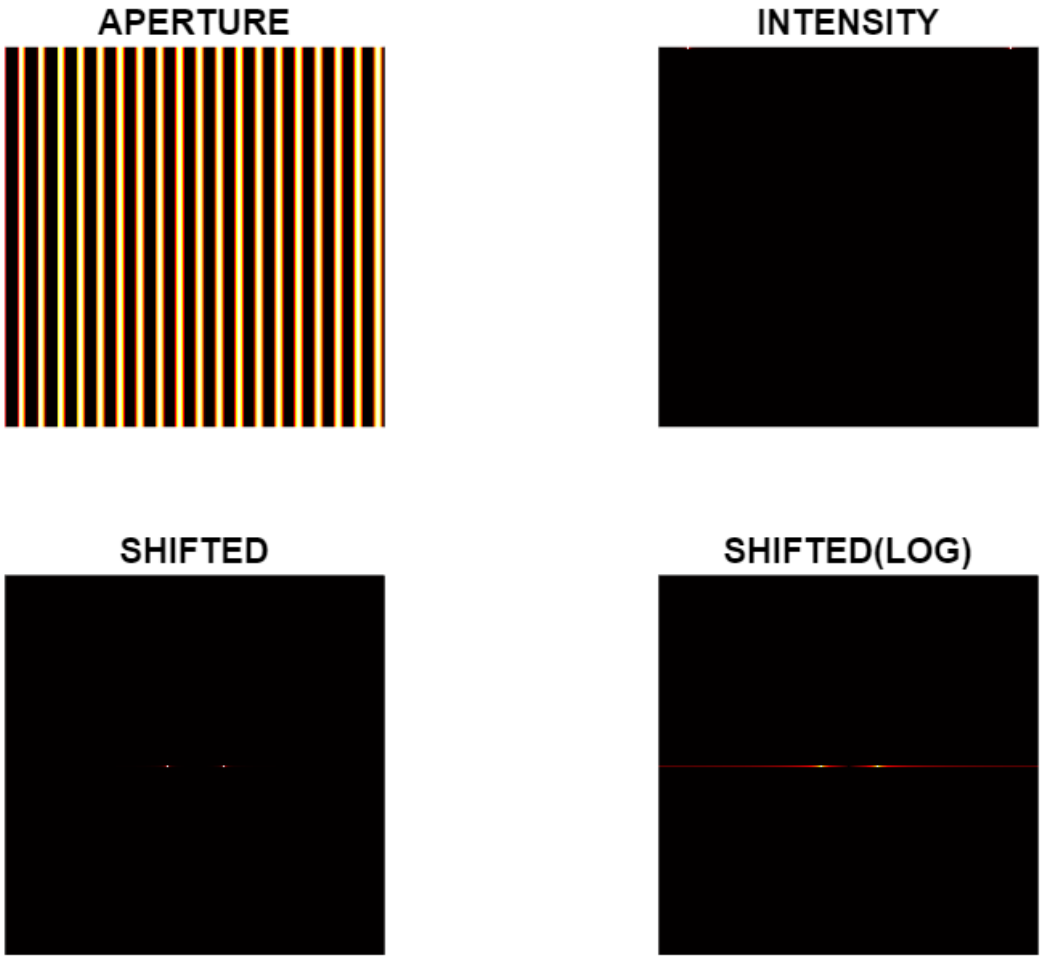
frequency = 5, angle of rotation = 0

```
FFT(Sinusoid(5,0));
```



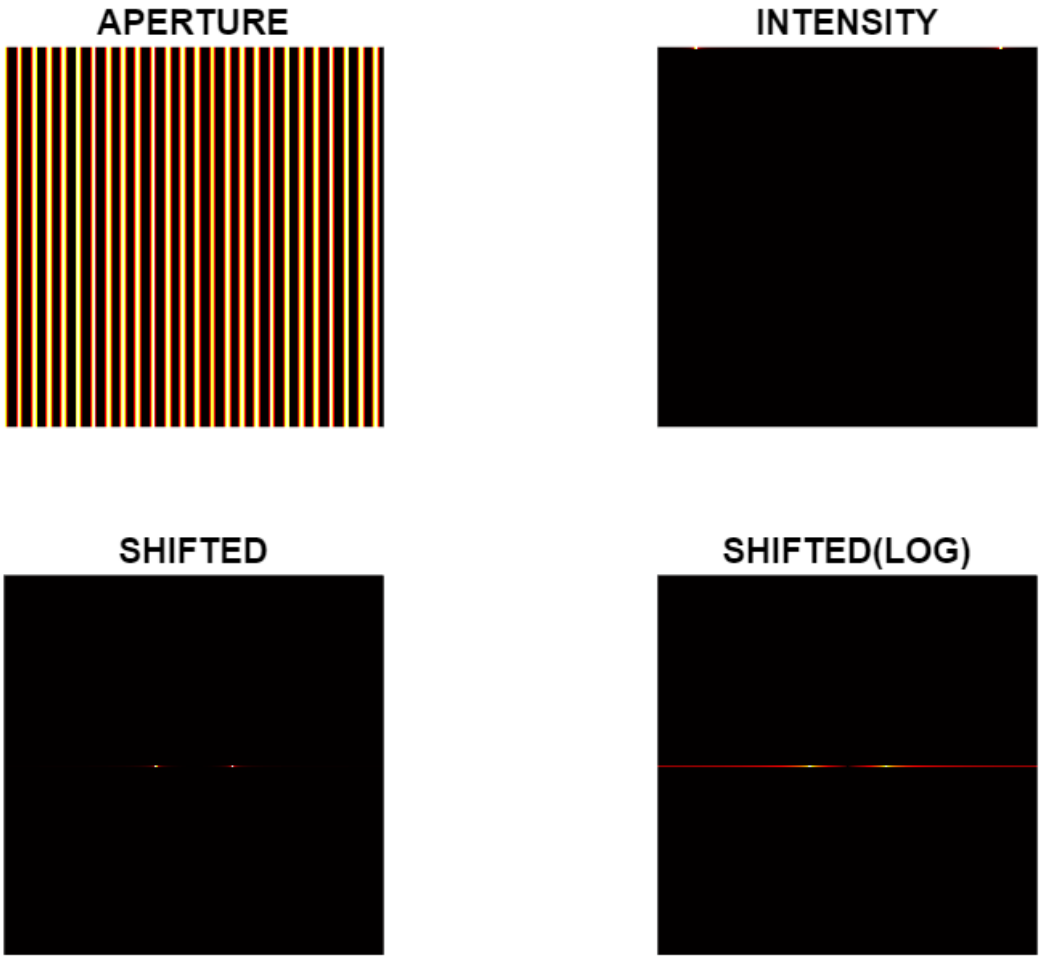
frequency = 15, angle of rotation = 0

```
FFT(Sinusoid(15,0));
```



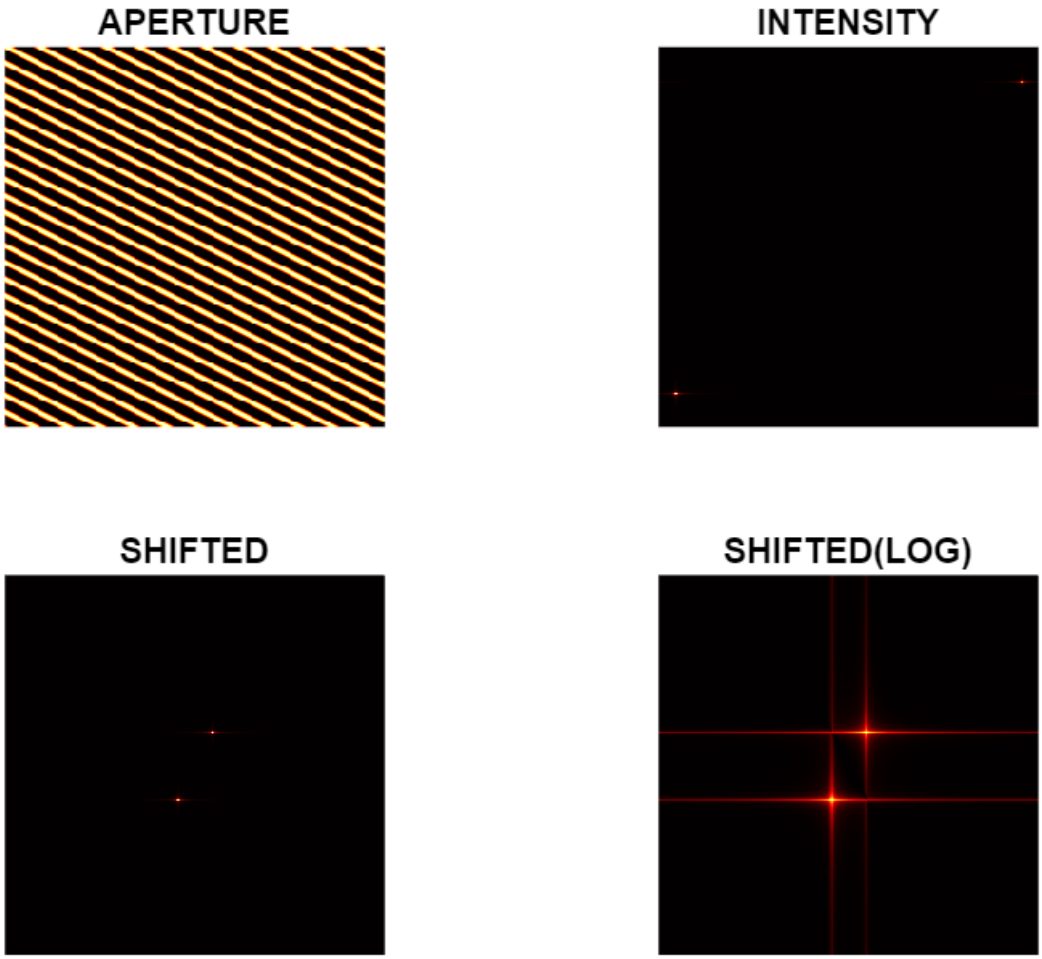
frequency = 20, angle of rotation = 0

```
FFT(Sinusoid(20,0));
```



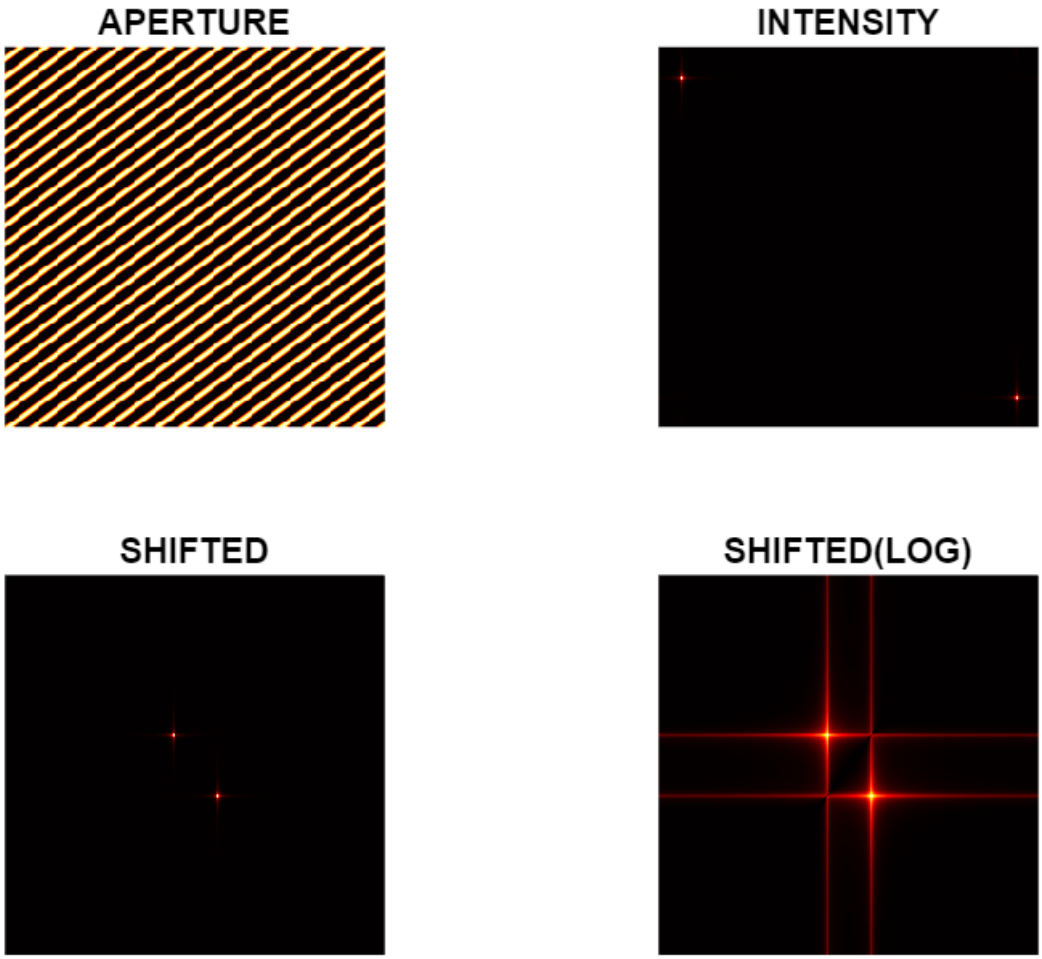
frequency = 20, angle of rotation = 90

```
FFT(Sinusoid(20,90));
```



frequency = 20, angle of rotation = 180

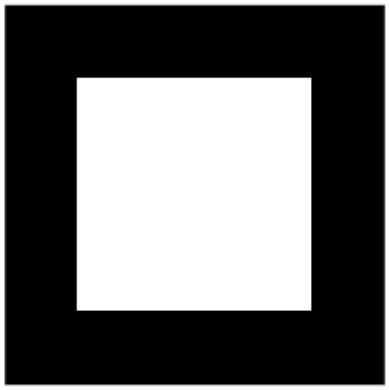
```
FFT(Sinusoid(20,180));
```



Rectangular aperture

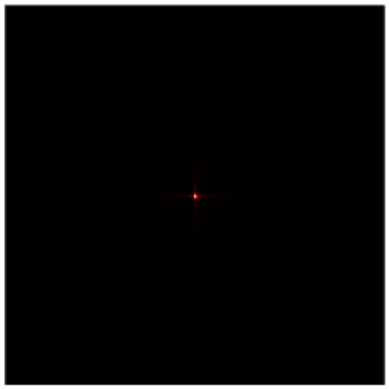
```
FFT(Rectangle(50,206,50,206));
```

APERTURE

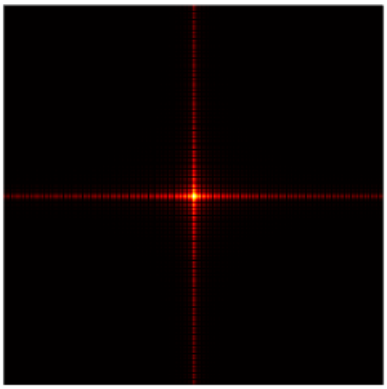


INTENSITY

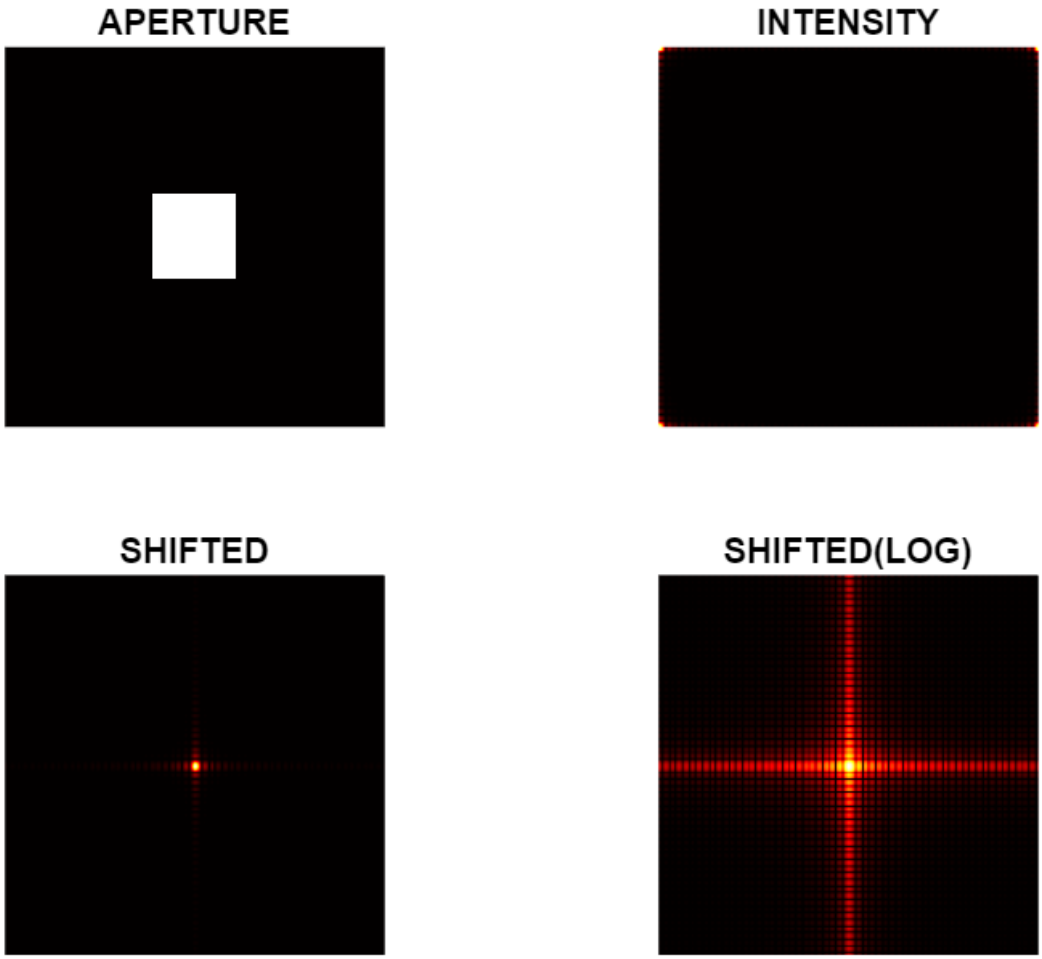
SHIFTED



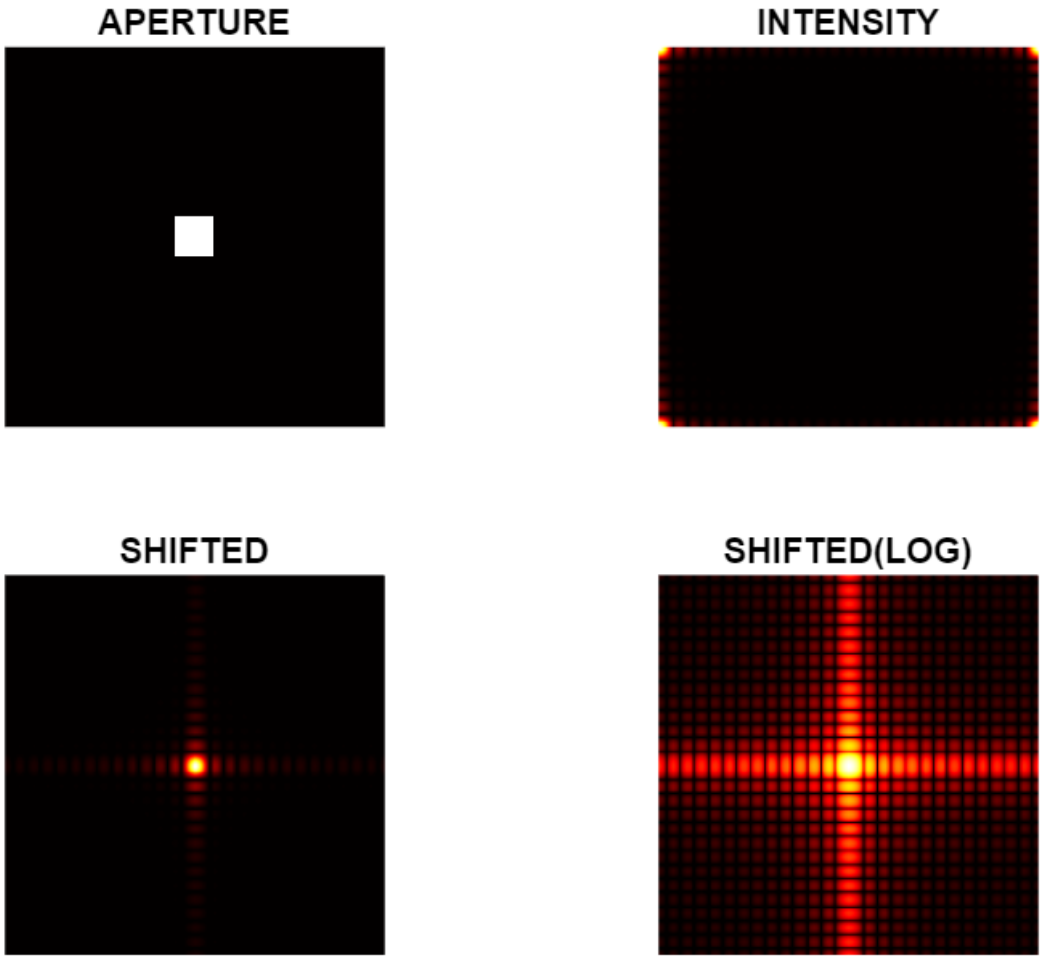
SHIFTED(LOG)



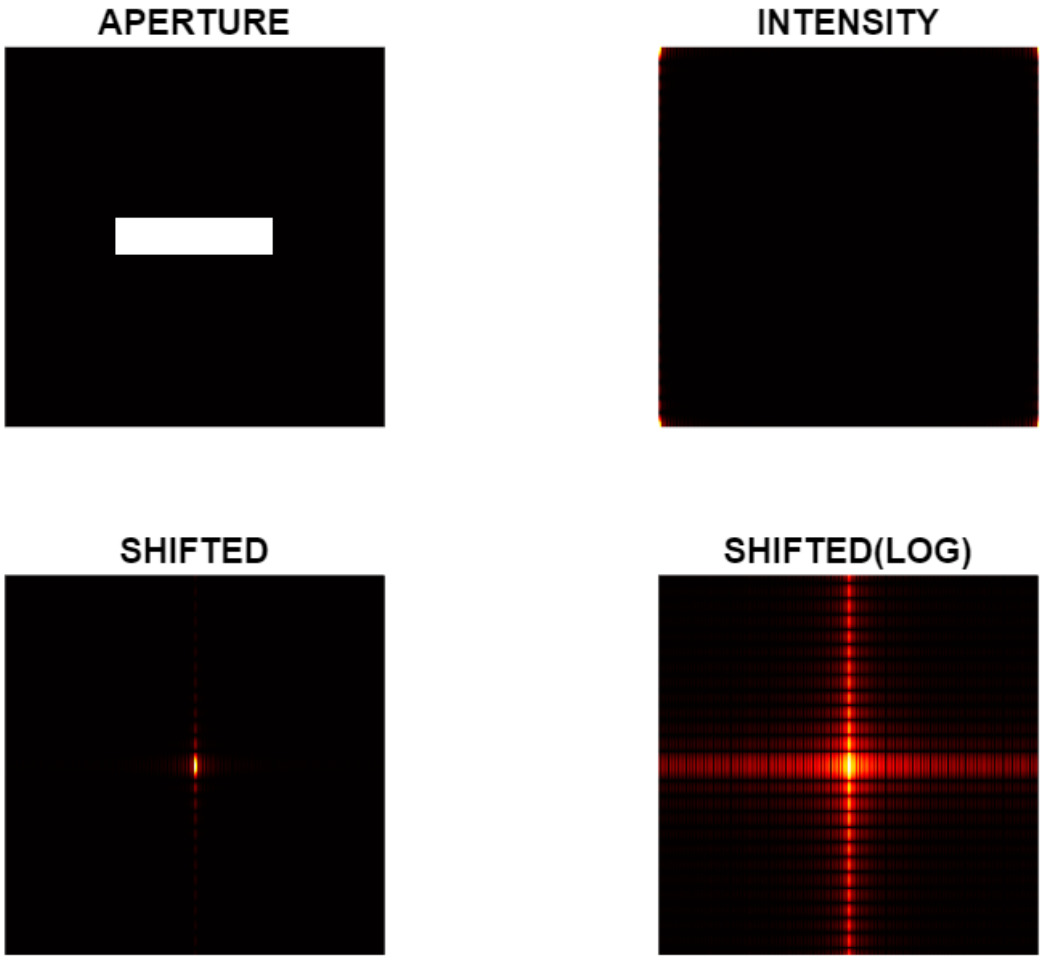
```
FFT(Rectangle(100, 156,100,156));
```



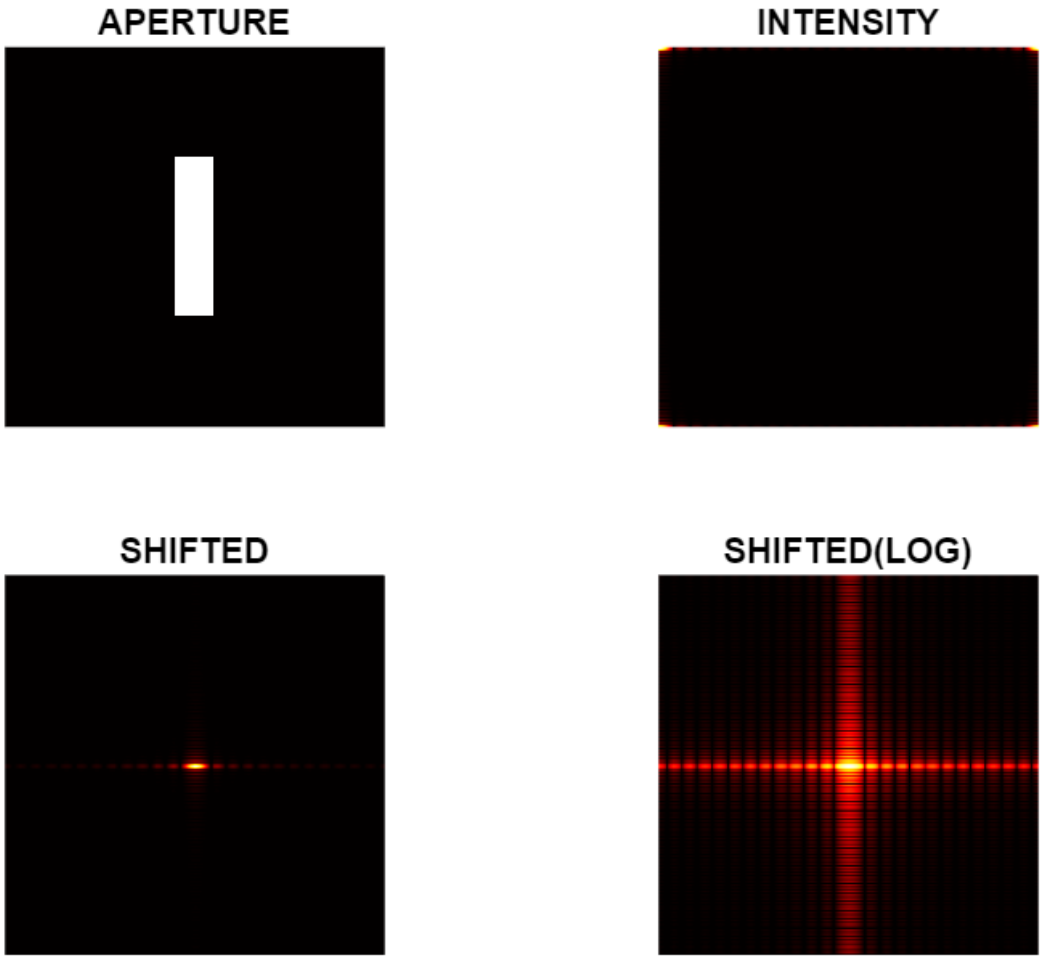

```
FFT(Rectangle(115,141,115,141));
```




```
FFT(Rectangle(116,140,75,181));
```

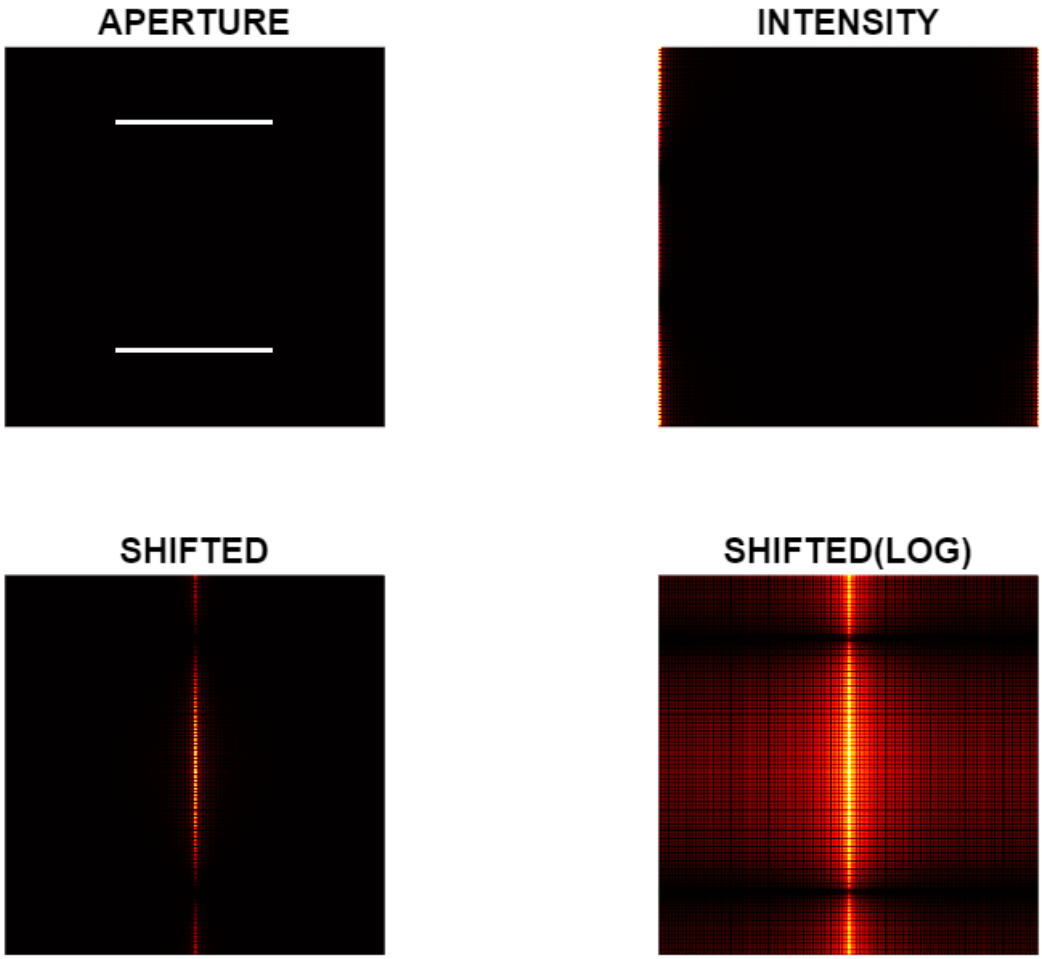



```
FFT(Rectangle(75,181,116,140));
```

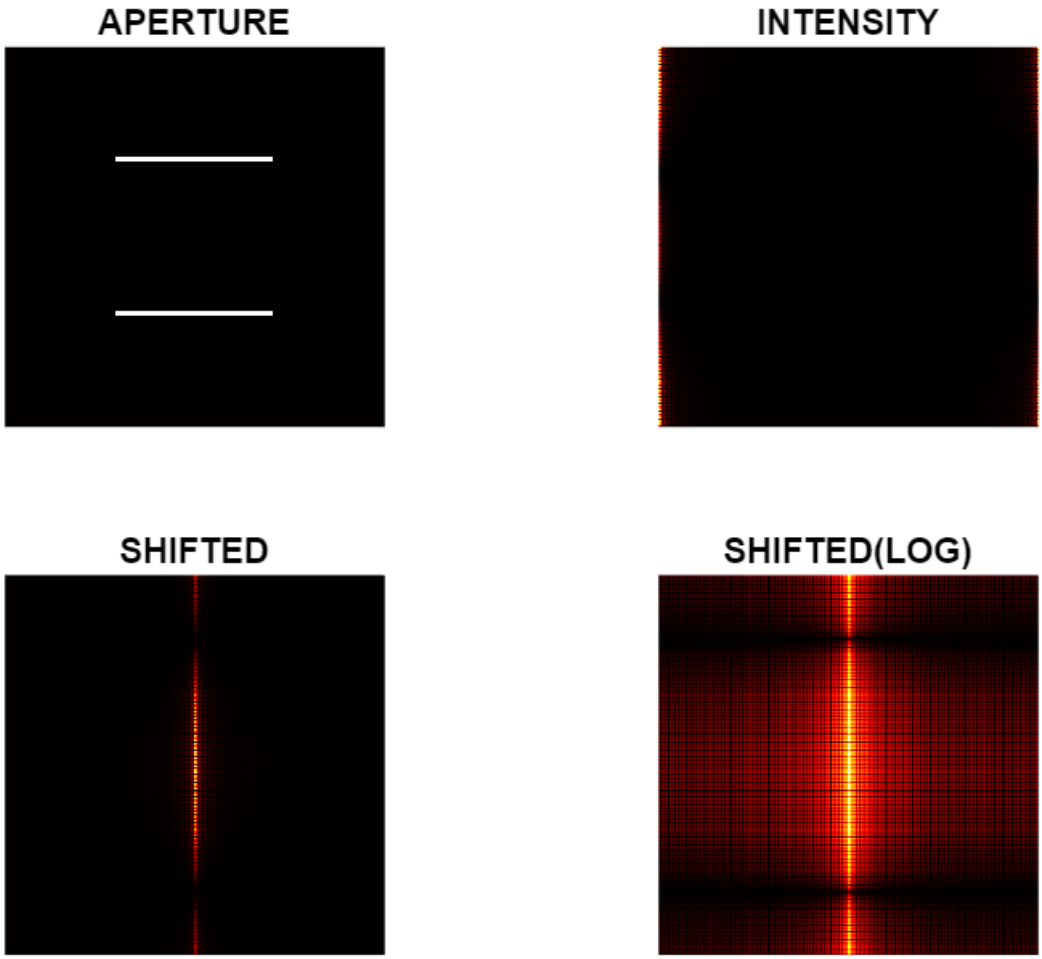


Double Slit Aperture

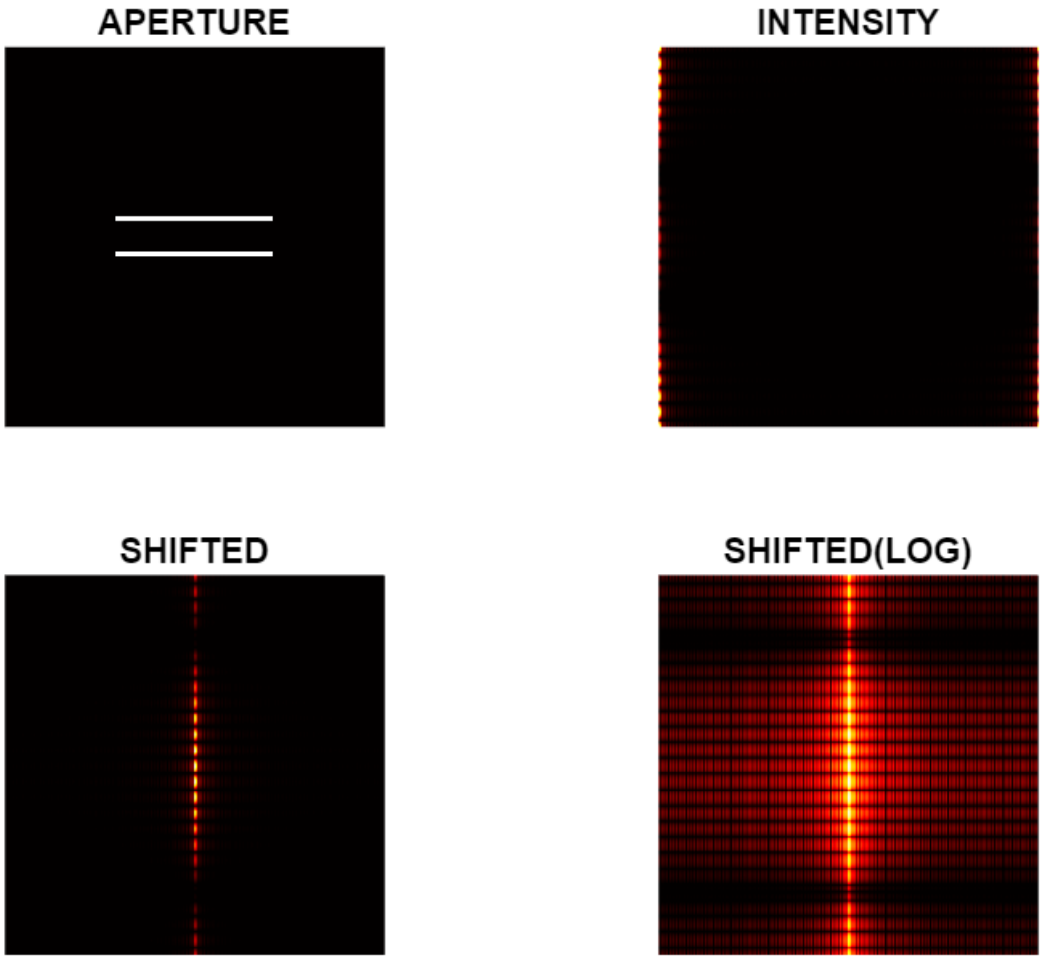
```
FFT(Slit(50, 52, 75,181, 204,206, 75,181));
```




```
FFT(Slit(75, 77, 75,181, 179, 181, 75,181));
```

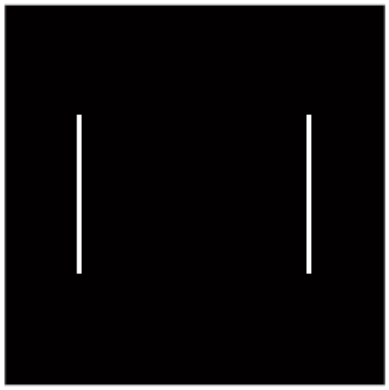



```
FFT(Slit(115, 117, 75,181, 139 ,141, 75,181));
```



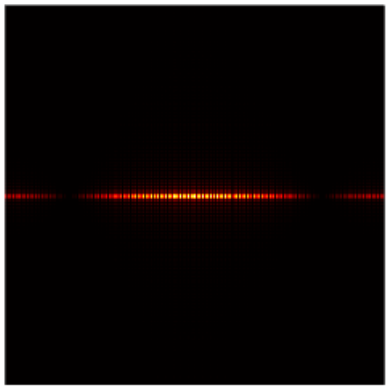

```
FFT(Slit(75,181,50, 52, 75,181, 204,206));
```

APERTURE

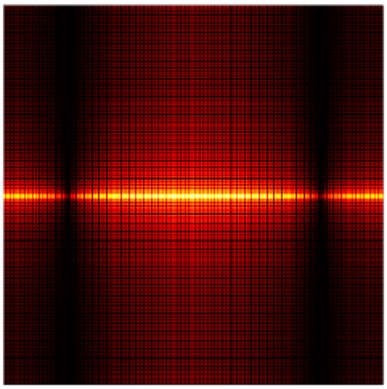


INTENSITY

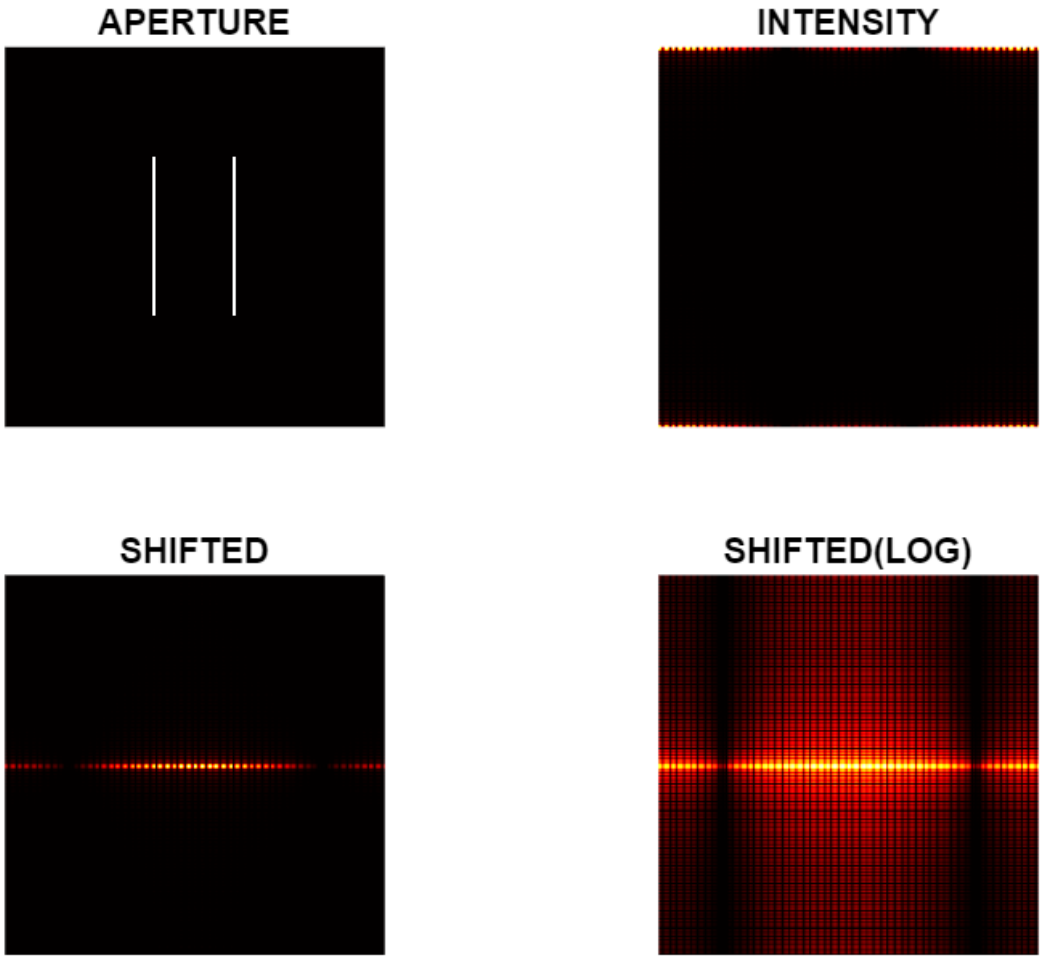
SHIFTED



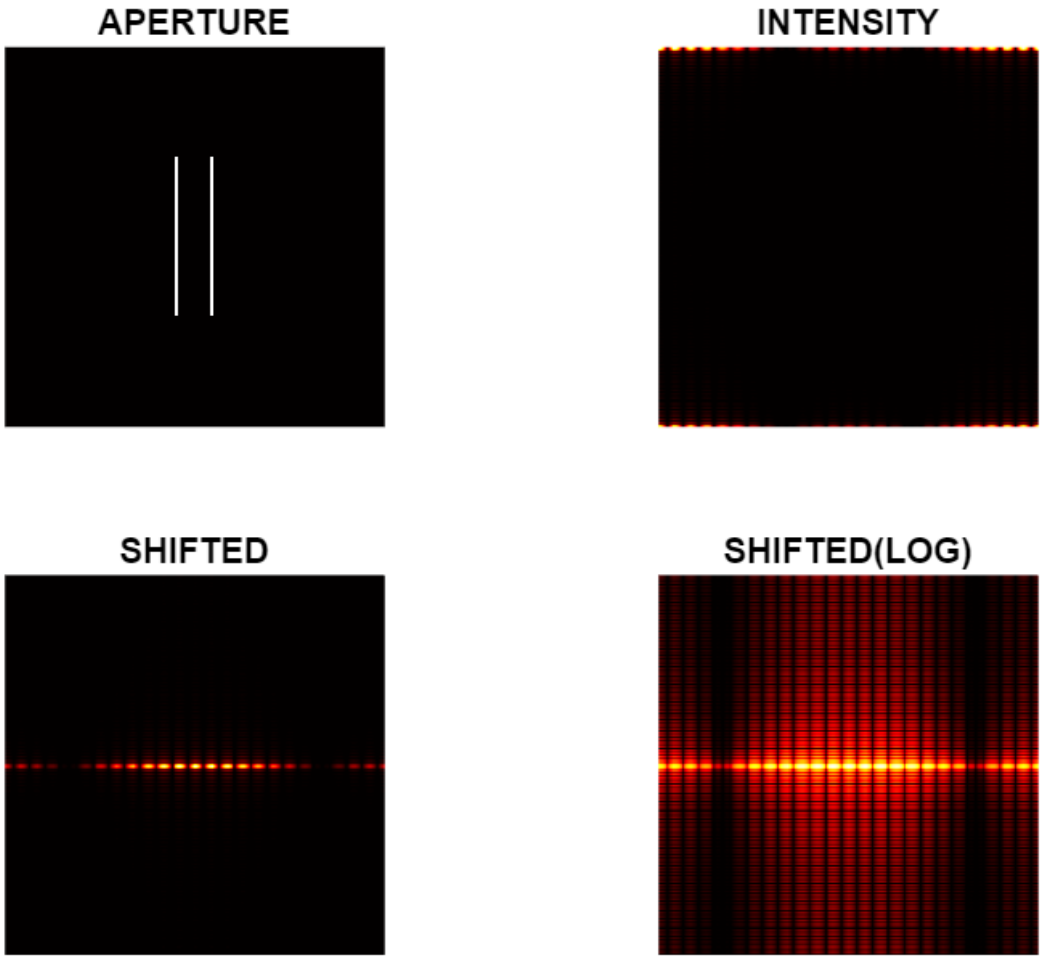
SHIFTED(LOG)



```
FFT(Slit(75,181,100, 102, 75,181, 154, 156));
```



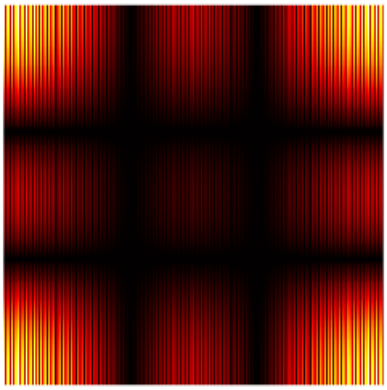

```
FFT(Slit(75,181,115,117,75,181,139,141));
```



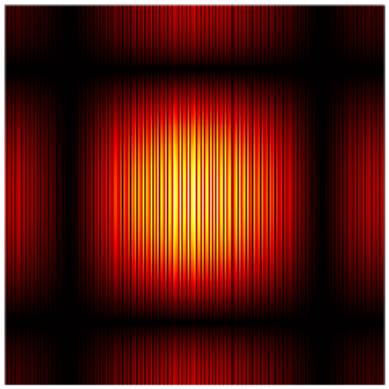

```
FFT(Slit(127, 129,50, 52, 127, 129, 204, 206));
```

APERTURE

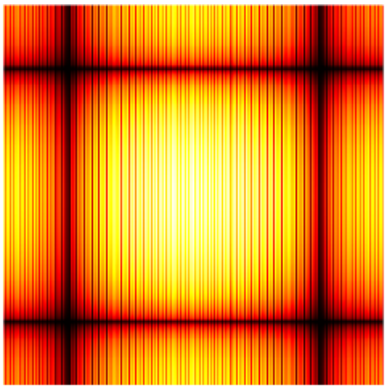
INTENSITY



SHIFTED

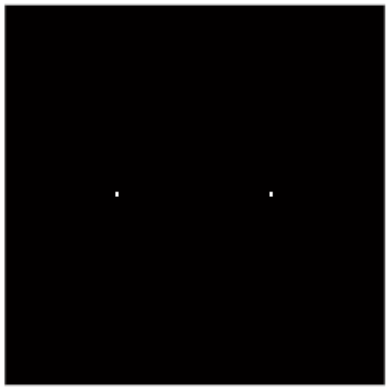


SHIFTED(LOG)

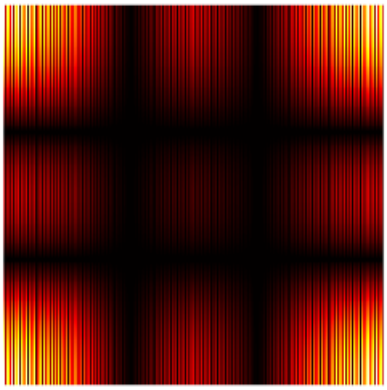


```
FFT(Slit(127, 129,75, 77, 127, 129, 179, 181));
```

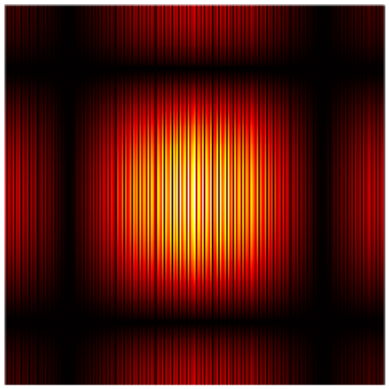
APERTURE



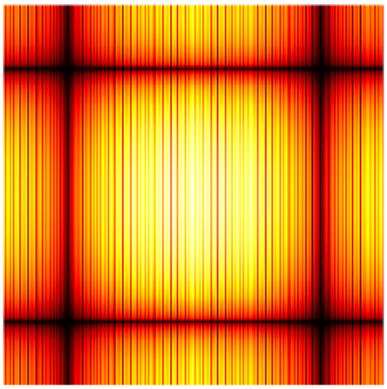
INTENSITY



SHIFTED

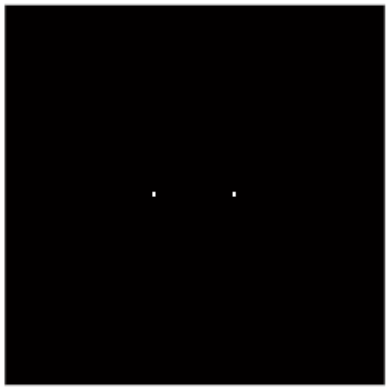


SHIFTED(LOG)

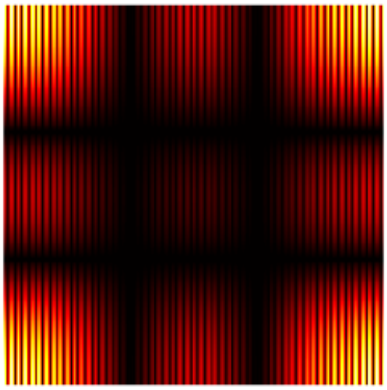


```
FFT(Slit(127, 129,100, 102, 127, 129, 154, 156));
```

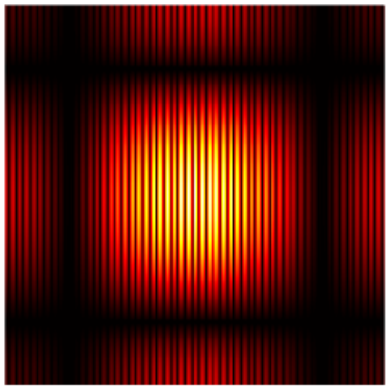
APERTURE



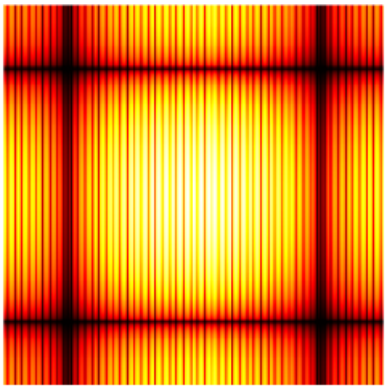
INTENSITY



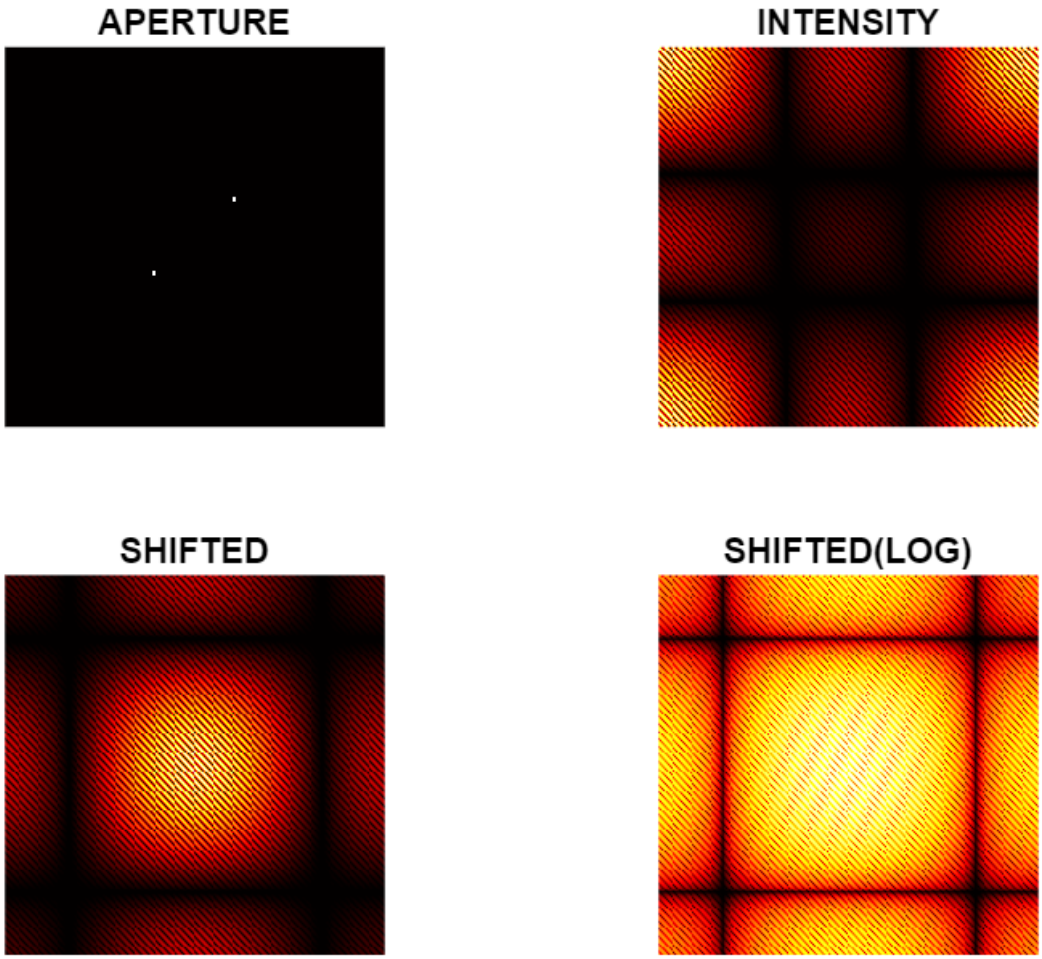
SHIFTED



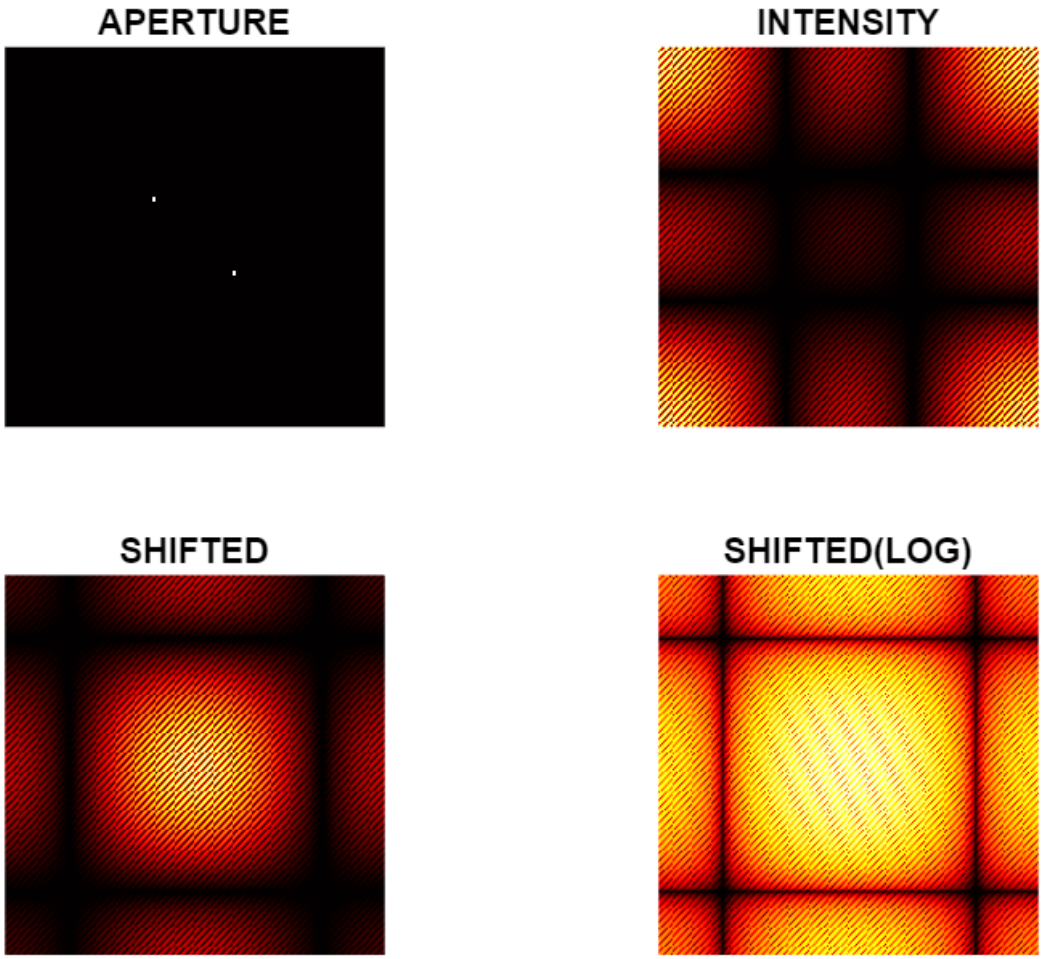
SHIFTED(LOG)



```
FFT(Slit(152, 154,100, 102, 102, 104, 154, 156));
```




```
FFT(Slit(102, 104,100, 102, 152, 154, 154, 156));
```



4. Load the image in your program and convert this image into grayscale. Apply fft2() twice on the image and display the reconstruction. Do it again, but this time apply fft() followed by ifft2(). Comment on the appearance of the reconstructed images.

```
clear; close all;
%Load image
I = rgb2gray(imread('squarejanen.jpg'));

%perfom fft2 twice
Irec = fft2(fft2(I));
Irecmag = abs(Irec);

%Apply fft2 then ifft2
Irec2 = ifft2(fft2(I));
Irec2mag = abs(Irec2);
```

```
%plotting
subplot (1,3,1); imshow(I); title("original image");
subplot(1,3,2); imagesc(Irecmag); colormap gray; axis image; axis off; title("fft2 twice");
subplot(1,3,3);imagesc(Irec2mag); colormap gray; axis image; axis off; title("fft2 then ifft2")
```

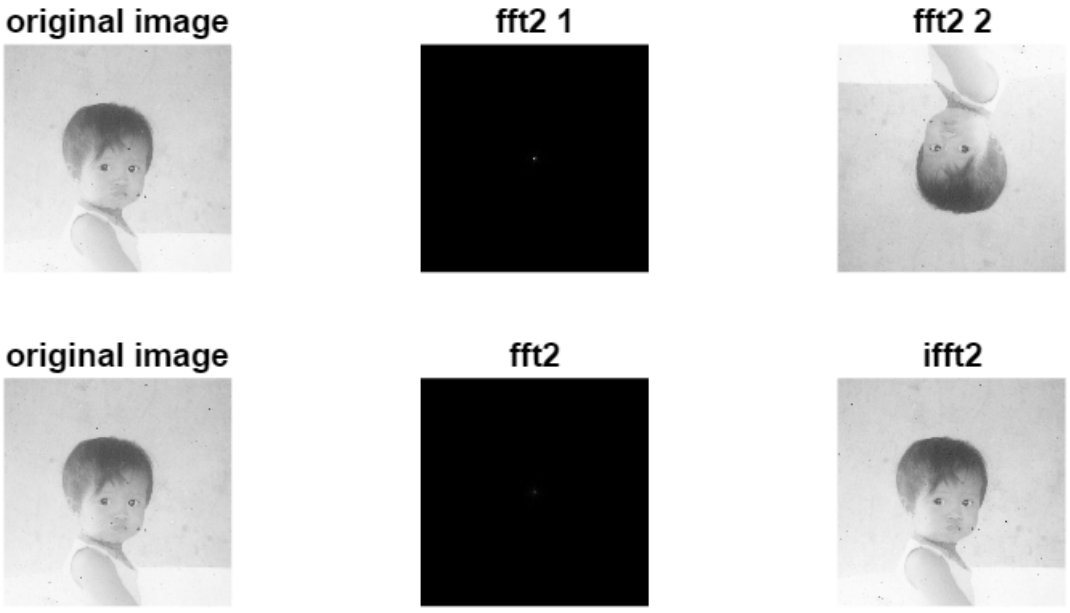
Trying to see what the FT of the image is.

```
clear; close all;
%Load image
I = rgb2gray(imread('squarejanen.jpg'));

%perfom fft2 twice
FFT1 = fft2(I);
FFTmag = abs(FFT1);
FFTshifted = fftshift(FFT1);
FFTmagshift = rescale(abs(FFTshifted), 0, 256);
FFTT =fft2(fft2(I));
```

```
%Apply fft2 then ifft2
FFT3 = fft2(I);
FFTmag1 = abs(FFT3);
FFTshifted1 = fftshift(FFT3);
FFTmagshift1 = rescale(abs(FFTshifted1), 0, 256);
FFTT1 = ifft2(fft2(I));

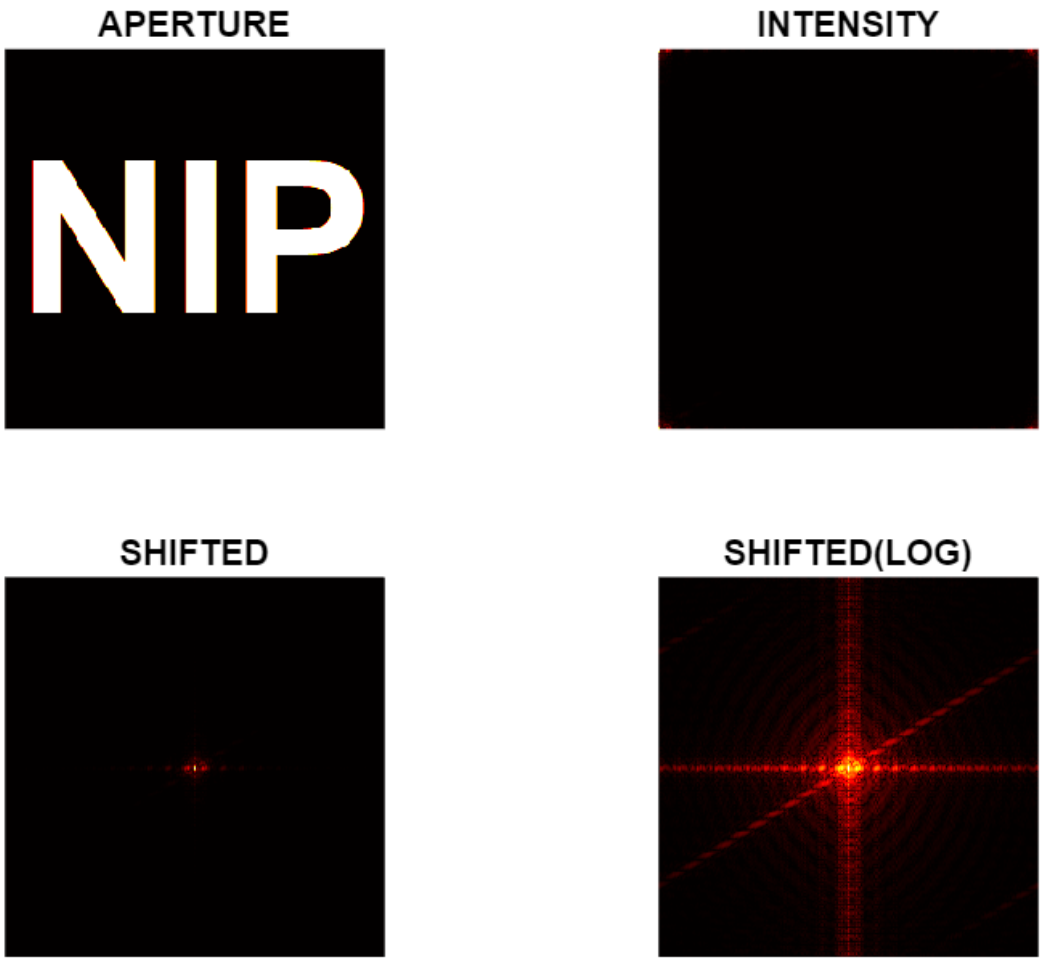
%plotting
subplot (3,3,1); imshow(I); title("original image");
subplot(3,3,2); imshow(FFTmagshift); colormap hot; axis image; axis off; title("fft2 1");
subplot(3,3,3);imagesc(abs(FFTT)); colormap gray; axis image; axis off; title("fft2 2");
subplot (3,3,4); imshow(I); title("original image");
subplot(3,3,5); imshow(FFTmagshift1); colormap hot; axis image; axis off; title("fft2");
subplot(3,3,6);imagesc(abs(FFTT1)); colormap gray; axis image; axis off; title("ifft2");
```



Activity 2.2. Simulation of an Imaging System

The fourier transform (FT) of the NIP image.

```
clear; close;
I = rgb2gray(imread('nip.tif'));
FFT(I);
```



circle = 10% of the 256x256 image

```
clear; close all;
Convolution('circle.tif');
```

circle = 25% of the 256x256 image

```
clear; close all;
Convolution('circle25.tif');
```

circle = 50% of the 256x256 image

```
clear; close all;
Convolution('circle50.tif');
```

circle = 75% of the 256x256 image

```
clear; close all;
Convolution('circle75.tif');
```

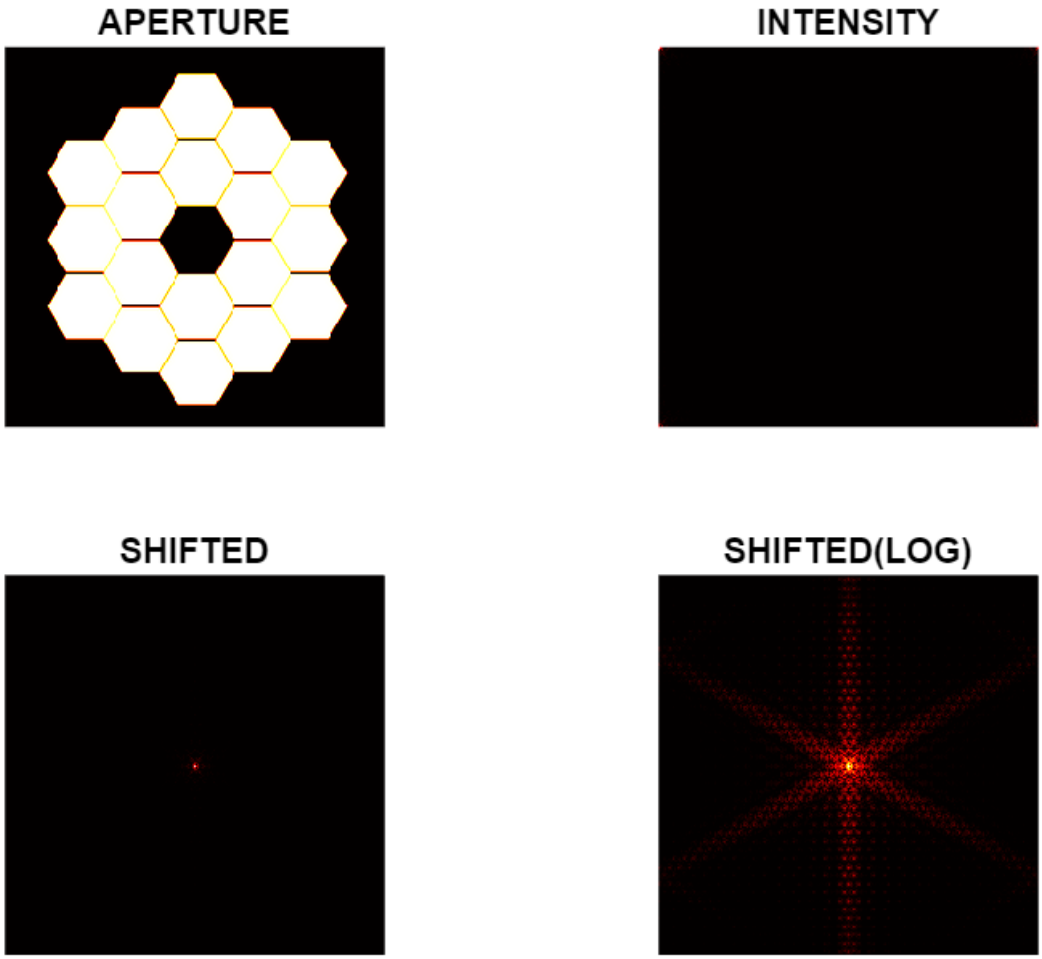
circle = 100% of the 256x256 image

```
clear; close all;
Convolution('circle100.tif');
```

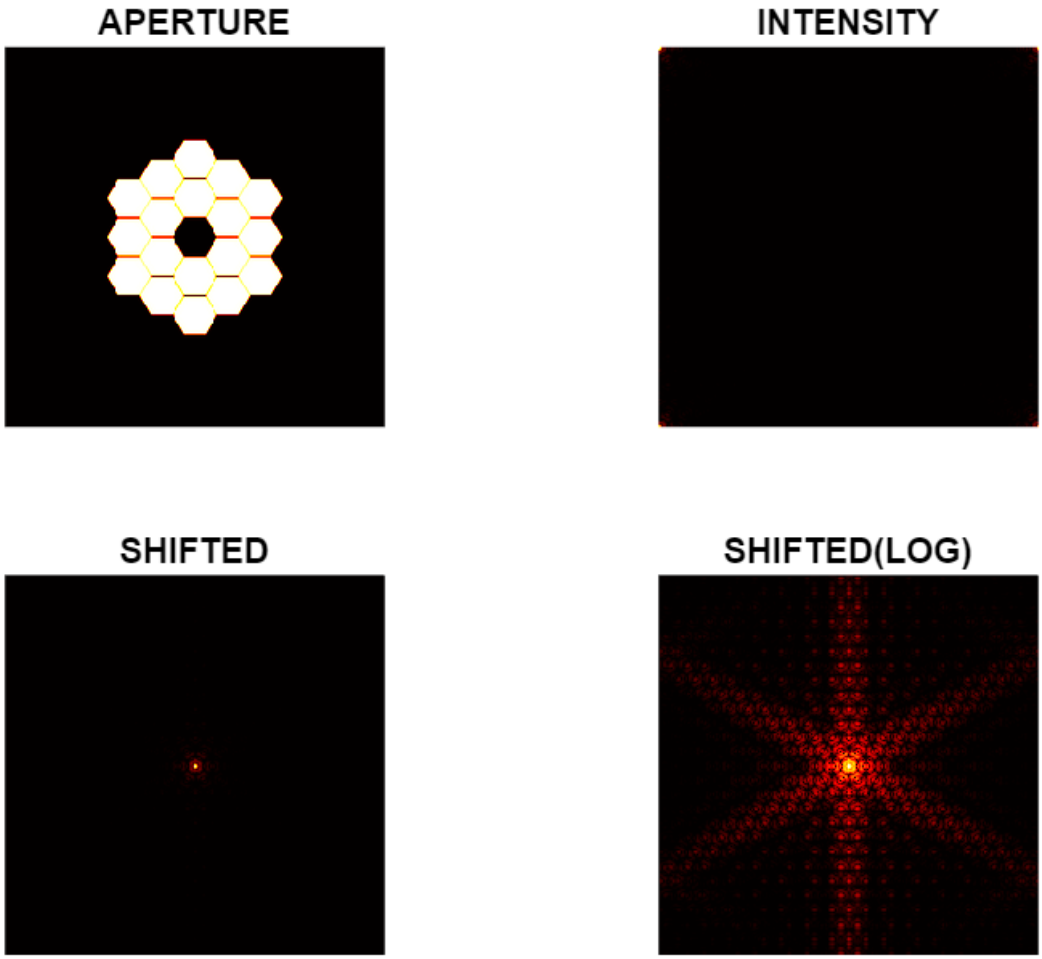
James Webb Space Telescope

Simulating the image of a start as seen by the James Webb Space Telescope.

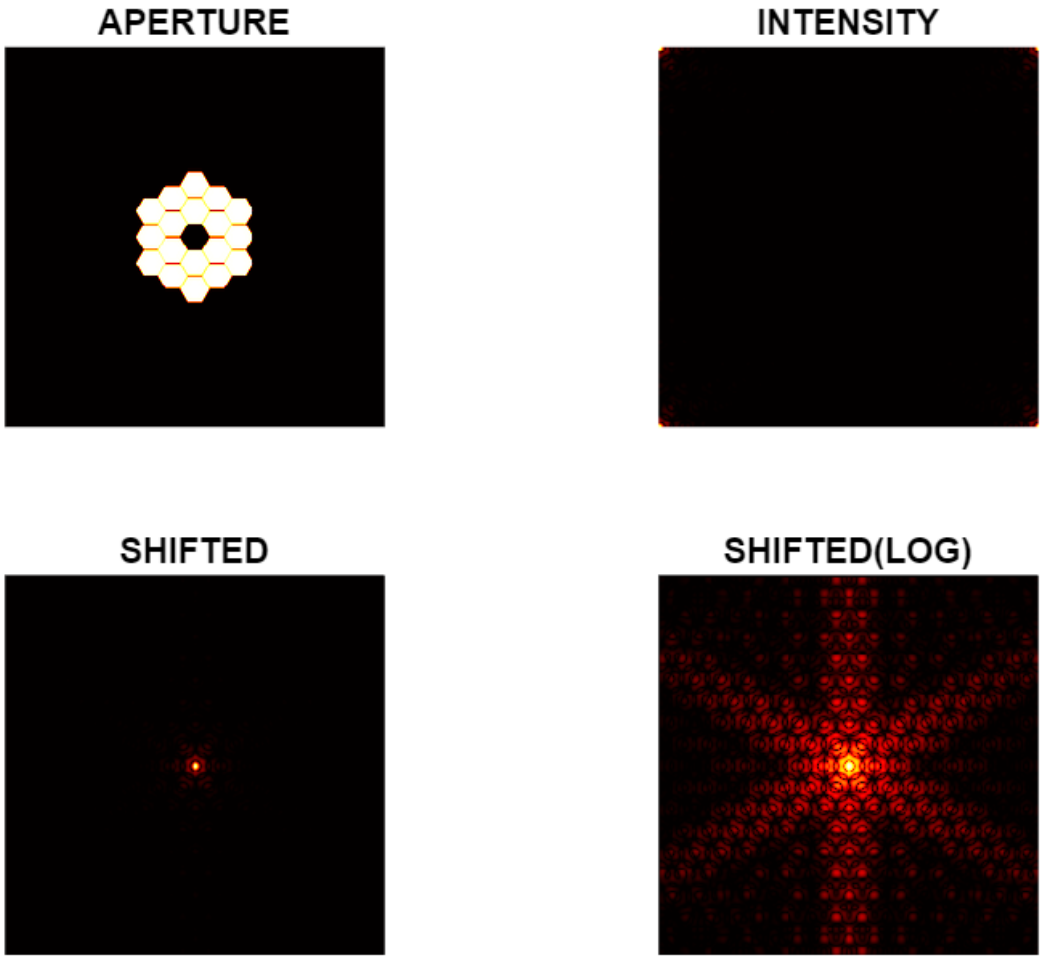
```
clear; close;
A = rgb2gray(imread('jwst1.png'));
FFT(A);
```




```
clear; close;
A = rgb2gray(imread('jwst2.png'));
FFT(A);
```

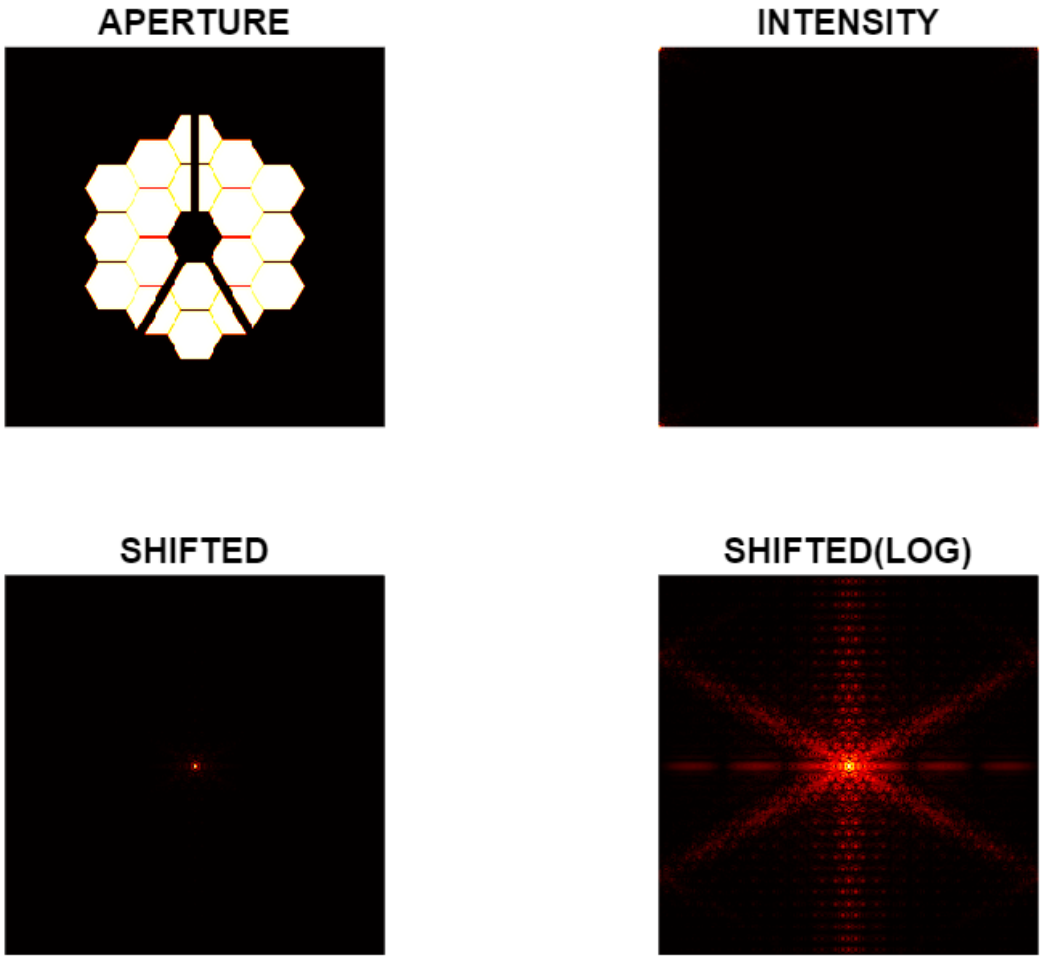



```
clear; close;
A = rgb2gray(imread('jwst3.png'));
FFT(A);
```



JWST with the secondary mirror.

```
clear; close;
A = rgb2gray(imread('jwstnew.png'));
FFT(A);
```



Using JWST as the aperture and NIP as the image.

```
clear; close all;
Convolution('jwst3.png');
```


```
clear; close all;
Convolution('jwst2.png');
```


```
clear; close all;
Convolution('jwst1.png');
```

Activity 2.3. Template matching using correlation

```
clear;close;
Correlation('A.tif', 'THERAIN.tif');
```


```
clear;close;
Correlation('I.tif', 'THERAIN.tif');
```

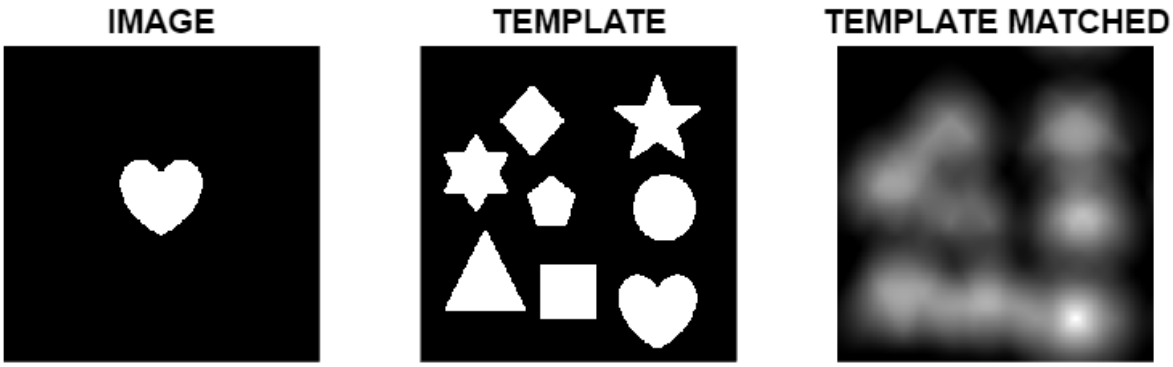
The famous Filipino tongue twister as the template.

```
clear;close;
Correlation('M.tif', 'MINIKANIKO.tif');
```

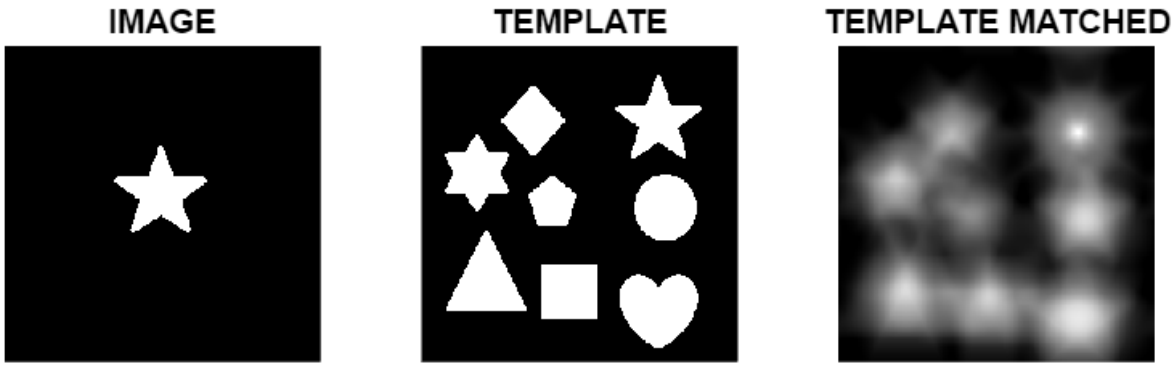

```
clear;close;
Correlation('O.tif', 'MINIKANIKO.tif');
```

Different shapes and sizes as the template.

```
clear;close;
Correlation('heart.tif', 'shapes.tif');
```




```
clear;close;
Correlation('star.tif', 'shapes.tif');
```



Functions

Here are the functions that were used for the simulations.

For Activity 2.1:

1. FFT(aperture)
2. Circle(r)
3. Sinusoid(f,d)
4. Rectangle(a,b,c,d)
5. Slit(a,b,c,d,e,f,g)

For Activity 2.2:

1. Convolution(aperture)

For Activity 2.3:

1. Correlation(template, I)

```
function [a,b, c,d] = FFT(aperture)
    FFT = fft2(aperture);
    FFTmag = abs(FFT);
    FFTshifted = fftshift(FFT);
    FFTmagshift = rescale(abs(FFTshifted), 0, 256);
    FFTlog = log(FFTmagshift+1);
    FFTres = rescale(FFTlog, 0, 256);
    a = subplot(2,2,1); imshow(aperture); title("APERTURE");
    b = subplot(2,2,2); image(rescale(FFTmag,0,256)); colormap hot; title("INTENSITY"); axis of
    c = subplot(2,2,3); image(rescale(FFTmagshift,0,256)); colormap hot; title("SHIFTED"); axis
    d = subplot(2,2,4); image(FFTres); colormap hot; title("SHIFTED(LOG)"); axis off; axis imag
end
%%%
function A = Circle(r)
    N = 256;
    x = linspace(-2,2,N);
    y = x;
    [X,Y] = meshgrid(x,y);
    R = (X.^2 + Y.^2);
    A = zeros(size(R));
    A(R<r) = 1;
end
%%%
function A = Sinusoid(f, d)
    N = 256;
    x = linspace(-2,2,N);
    y = x;
    [X,Y] = meshgrid(x,y);
    A = sin(2*f*(sin(d)*Y + cos(d)*X));
end
%%%
function A = Rectangle(a,b,c,d)
    N = 256;
    A = zeros(N,N);
    A(a:b, c:d) = 255;
end
%%%
function A = Slit(a,b,c,d,e,f,g,h)
    N = 256;
    A = zeros(N,N);
    A(a:b, c:d) = 255;
    A(e:f, g:h) = 225;
end
%%%
function [a,b,c] = Convolution(aperture)
    A = im2double(imread(aperture));
    Ashift = fftshift(A);
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```
    N = im2double(imread('NIP.tif'));
    Nfft = fft2(N);
    H = Ashift.*Nfft;
    h = ifft2(H);
    habs = abs(h);
    a = subplot(1,3,1); image(A); axis image; axis off; title('aperture');
    b = subplot(1,3,2); image(N); axis image; axis off; title('image');
    c = subplot(1,3,3);imagesc(habs); axis image; axis off; title('reconsructed');
end
%%%
function [a,b,c] = Correlation(template, I)
    I = im2gray(imread(I));
    A = im2gray(imread(template));
    Ifft2 = fft2(I);
    Afft2 = fft2(A);
    P = conj(Afft2).*Ifft2;
    Prev = ifft2(P);
    r = fftshift(abs(Prev));
    a = subplot(1,3,1);image(A); axis image; axis off; title('IMAGE');
    b = subplot(1,3,2);image(I); axis image; axis off; title('TEMPLATE');
    c = subplot(1,3,3);imagesc(r); colormap gray; axis image; axis off; axis equal; title('TEMP
end
%%%
```