# Fourier Transform Model of Image Formation (Part 1)

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Applied Physics 157 WFY-FX2

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# Objectives

01 Familiarize with Discrete FT

02 Display and describe the FT of different images

03 Use Convolution to simulate an imaging system

04 Show the image of a star captured by the James Webb Space Telescope

05 Apply correlation to template matching

# Activity 2.1 Familiarization with Discrete FT

## Background

Fourier transform (FT) has several applications in fields such as applied mathematics, physical sciences, and engineering. Moreover, because it is helpful in collection and reconstruction of data, it is a powerful image processing tool used in biomedicine, photonics, nuclear physics, etc [1]. Therefore, it is important to understand how it works and familiarize ourselves with the concepts behind it.

In this activity, we'll look at the Fourier transforms of different synthetic images and determine how FT and inverse FT works.

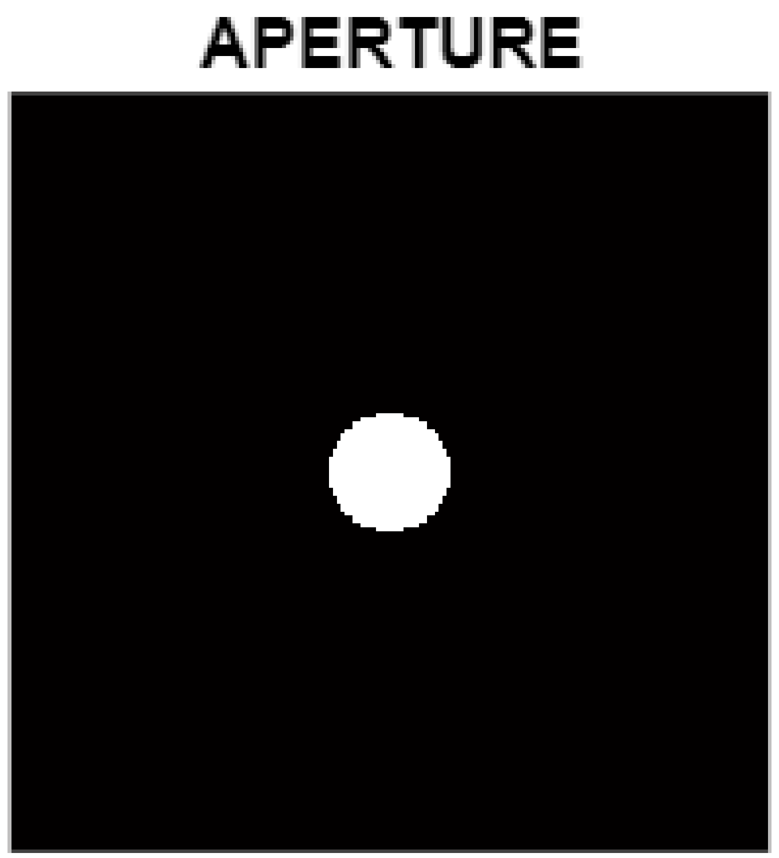
# Fourier Transform

## I created a function FFT(aperture) to implement the following:

| | | | |
|---|---|---|---|
| Apply fft2( ) on the image and compute the intensity values using abs( ). | Display the FFT magnitude as an intensity image. | Use fftshift( ) to make the output zero-centered. Display the magnitude of the shifted image. | Display the magnitude of the shifted image in logarithmic scale for better visualization. |

# Circular Aperture

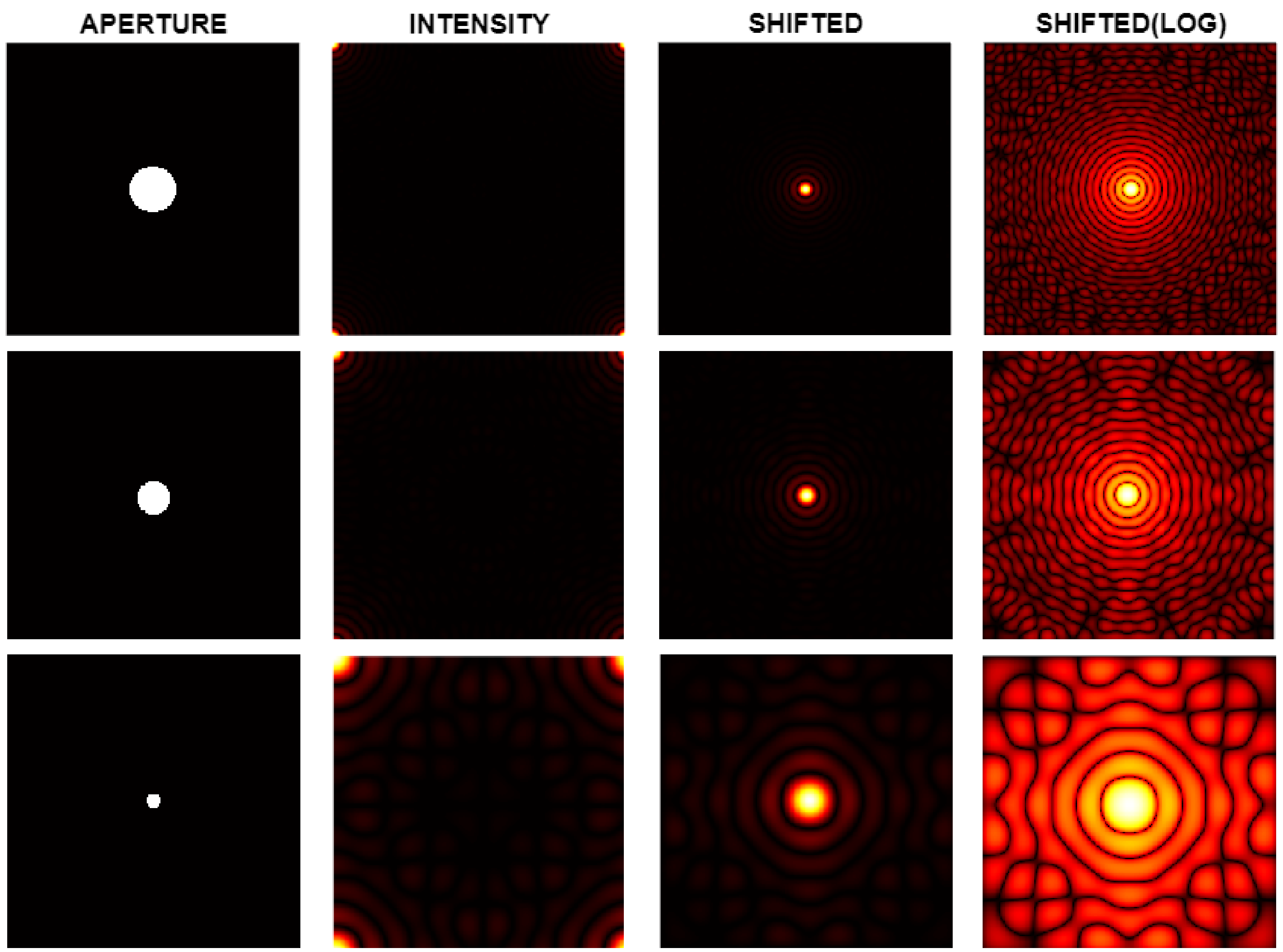
The first aperture that we'll investigate is the circle. Here, I reused the code that I have from Activity 1 to create an image of a white circle. Then, I created a function for it that takes the input r, which is the radius of the circle, and then returns the image. Here is an example of the aperture.




Let's investigate what happens when we change the radius of the circle.

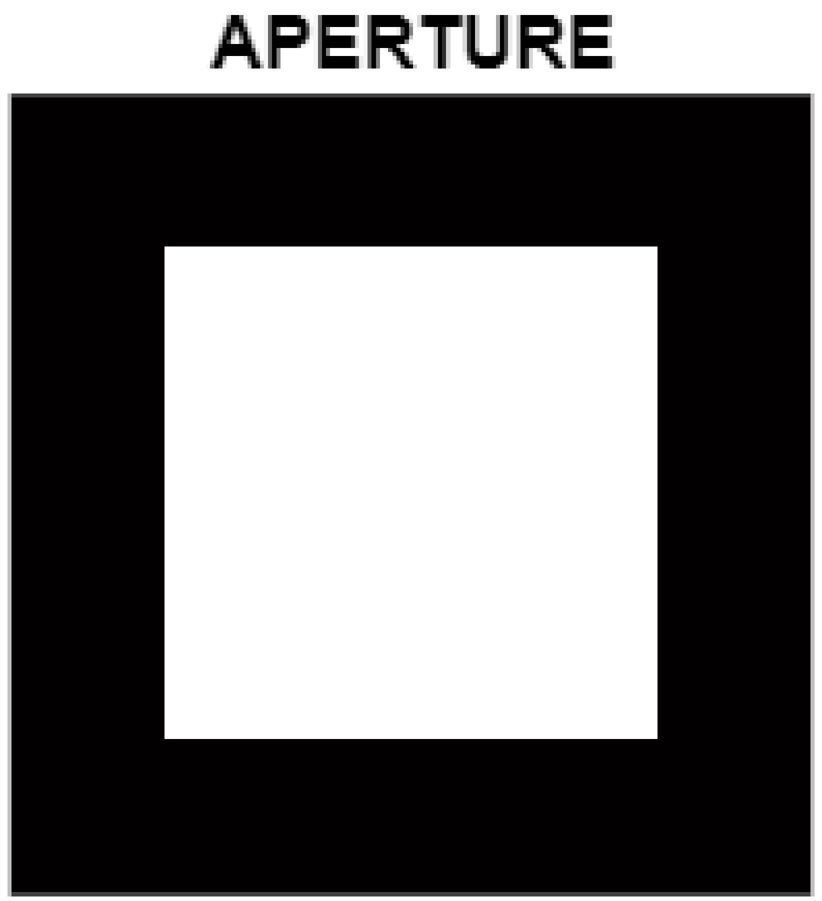
# Circular Aperture

As we can observe, the magnitude of the shifted intensity image of a circular aperture has a **circular symmetry**. As the radius is decreased, the center of the shifted image increased. This is consistent with the fact that the FT of a circular aperture is the **first-order Bessel function** [2]. This is more clearly seen in the logarithmic scale of the shifted image. The plotting of the magnitude of the shifted image in logarithmic scale shows details which can be displayed in a wider range, so we'll use it for visualization [3].



# Rectangular Aperture

To synthetically create a white rectangle (square in this case) in a black background, I created a function Rectangle(a,b,c,d) which takes the input of the range of length and width of the rectangle. The function returns the image of the rectangular aperture as shown below.

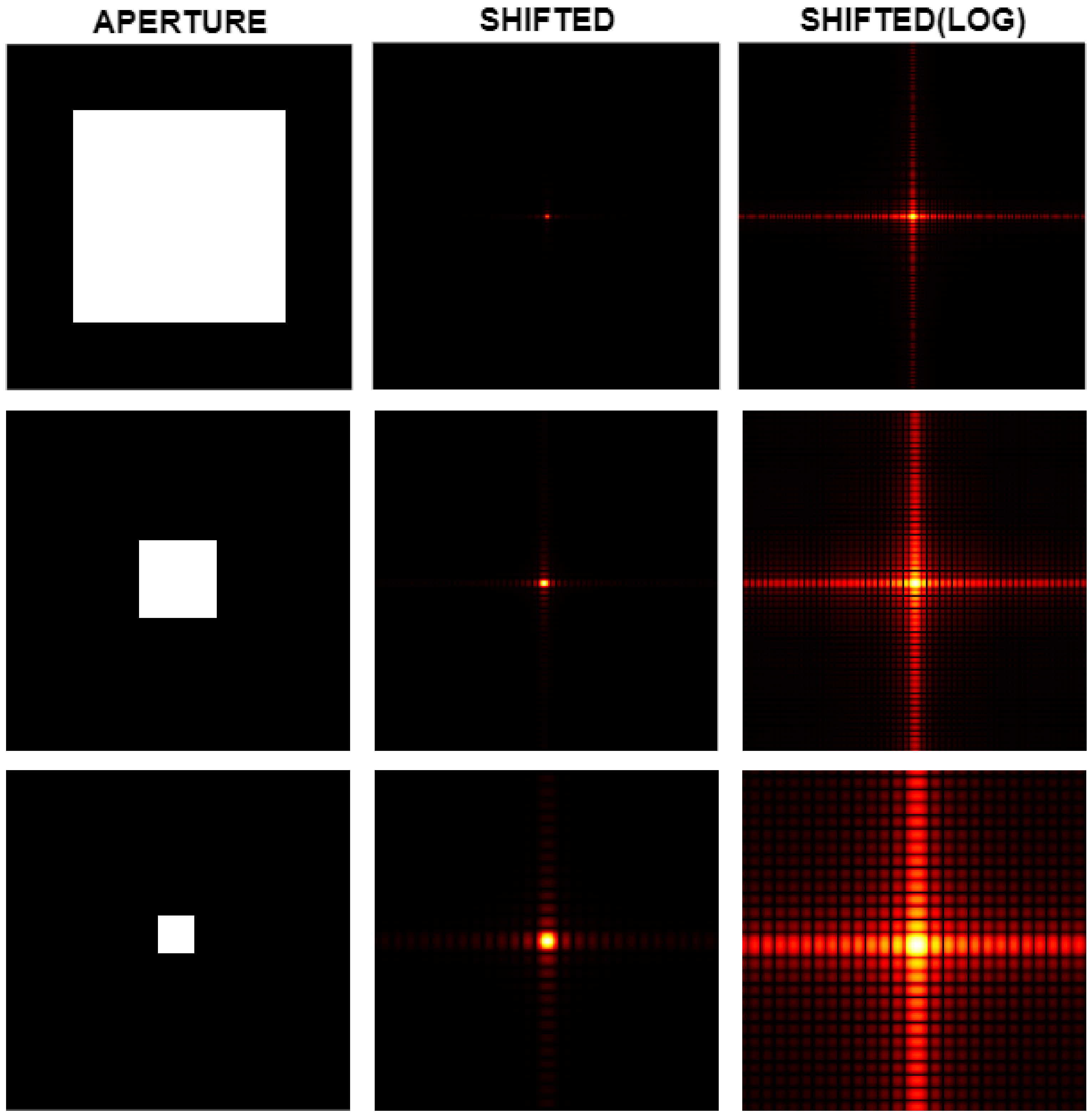



**Let's see what happens if we decrease the size of the rectangle.**

# Rectangular Aperture

**Varying rectangle size**

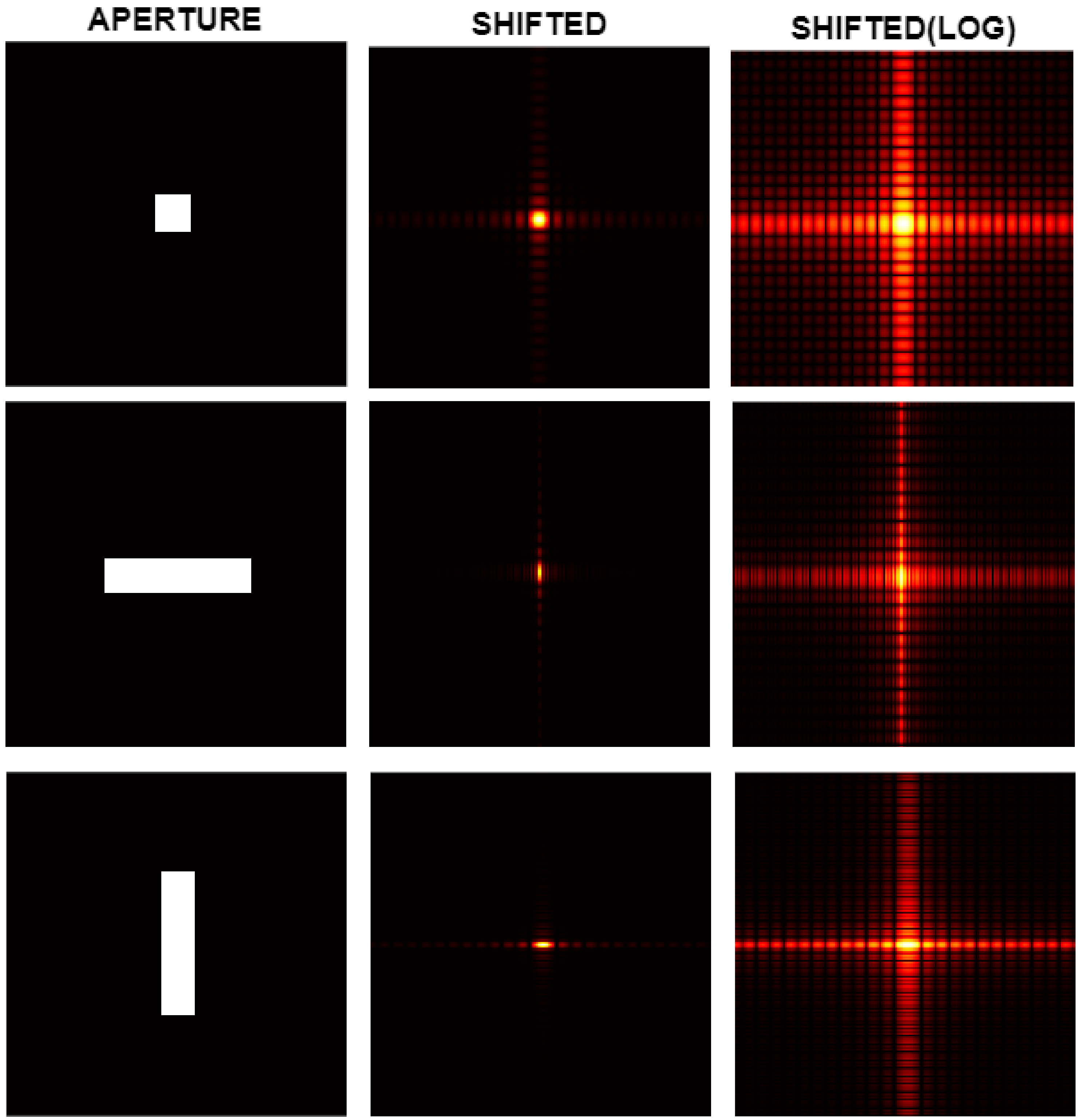
For the rectangular aperture, the shifted image looks like a combination of two lines intersecting in the middle. As the area of the aperture is decreased, the pattern looks more visible. The log scale gives a clearer view of the pattern. The FT of a rectangular aperture is a **product of two sine functions,** hence the pattern that we see[4].



# Rectangular Aperture

**Varying width and length**

The first aperture is a rectangle with equal width and length. This gives off an FT of a combination of 2 sine waves with the same intensity. Increasing the length or width of the rectangle, as shown, changes the intensity of the sine waves. If the horizontal side is longer, the horizontal sine wave also has a greater intensity. This can be explained by the amount of light that passes through the aperture.



# Sinusoidal Aperture

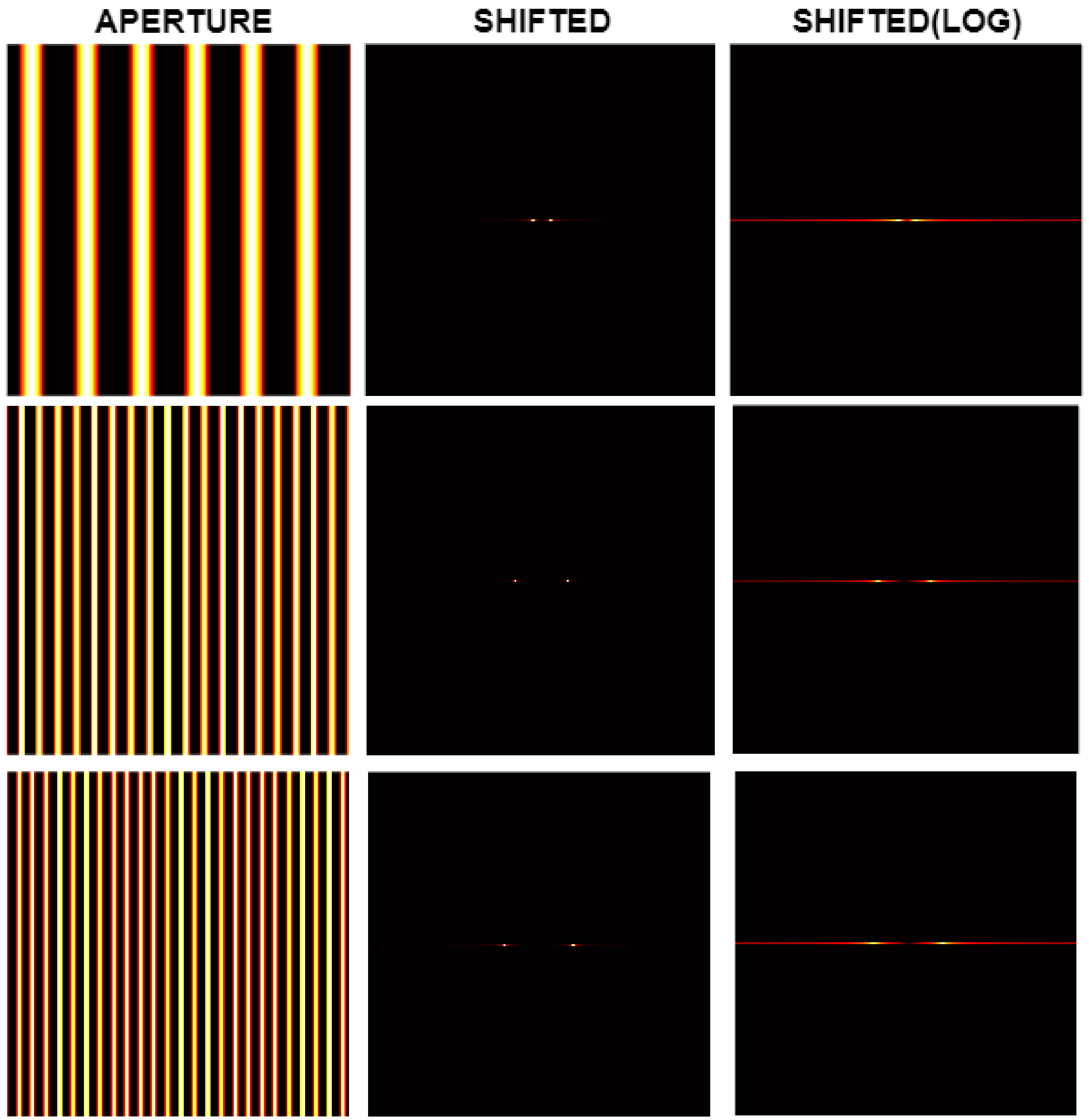
To synthetically create a sinusoid, I defined a function Sinusoid(f,d) where the inputs are f for the frequency and d for the degrees of its orientation. It will return an image that looks like a grating as shown below. This is a sinusoid of frequency = 5.


Let's investigate what happens if we increase the frequency of the sinusoid and what happens if we rotate it.

# Sinusoidal Aperture

## Increasing the frequency

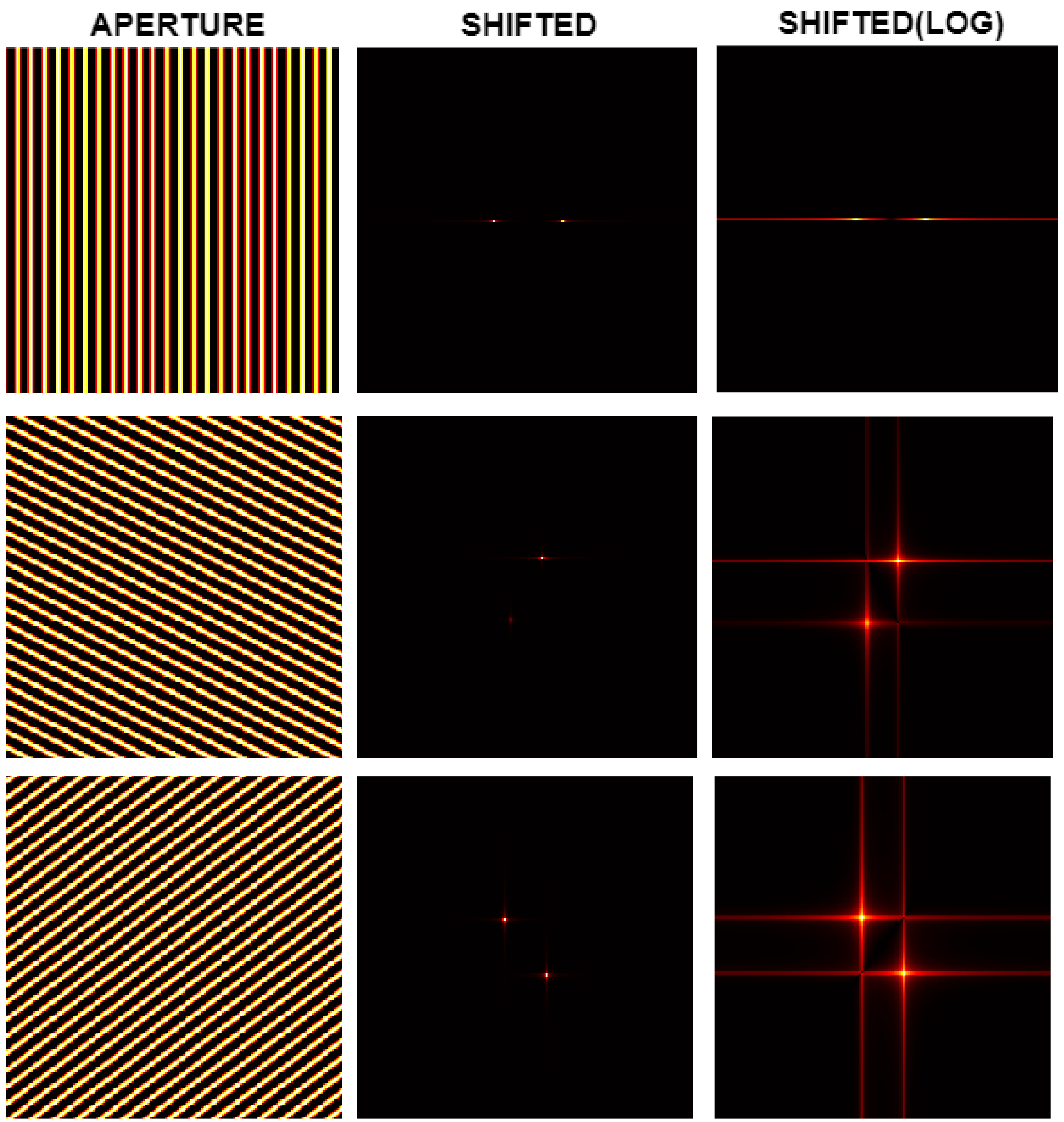
The frequency that I used is f = 5, 15, and 20. As we can see, the shifted image appears to be two dots equally spaced around the origin. These dots represents the **"positive" and "negative" frequencies** which is the FT of a sinusoid. In the log scale, it appears to be **two lines of intensity** equally spaced about the center position of the observing screen [5]. As we increase the frequency, the space between the dots also increases.



# Sinusoidal Aperture

## Rotating the Sinusoid

Here we investigate the effect when we rotate the sinusoid (frequency = 20). As we can see, when the sinusoid is rotated at a certain angle, say 90 degrees, the positive and negative frequencies (or the two dots) are also **rotated in the same angle.** This, however, does not affect their frequencies.



# Double Slit Experiment

One of the experiments that has been "designed to contain all of the mystery of quantum mechanics," is the Young's double slit experiment. This proves that light is made of waves. The experiment starts with a source of light shined through a screen with two narrow slits some distance apart. The light, as a wave, would pass through the slits then recombine to make a pattern: an alternating bright and dark stripes, or known as the **interference pattern [6,7].**

In this part, let's simulate the results of the double slit experiment by using the double slit as the aperture which we will Fourier transform.

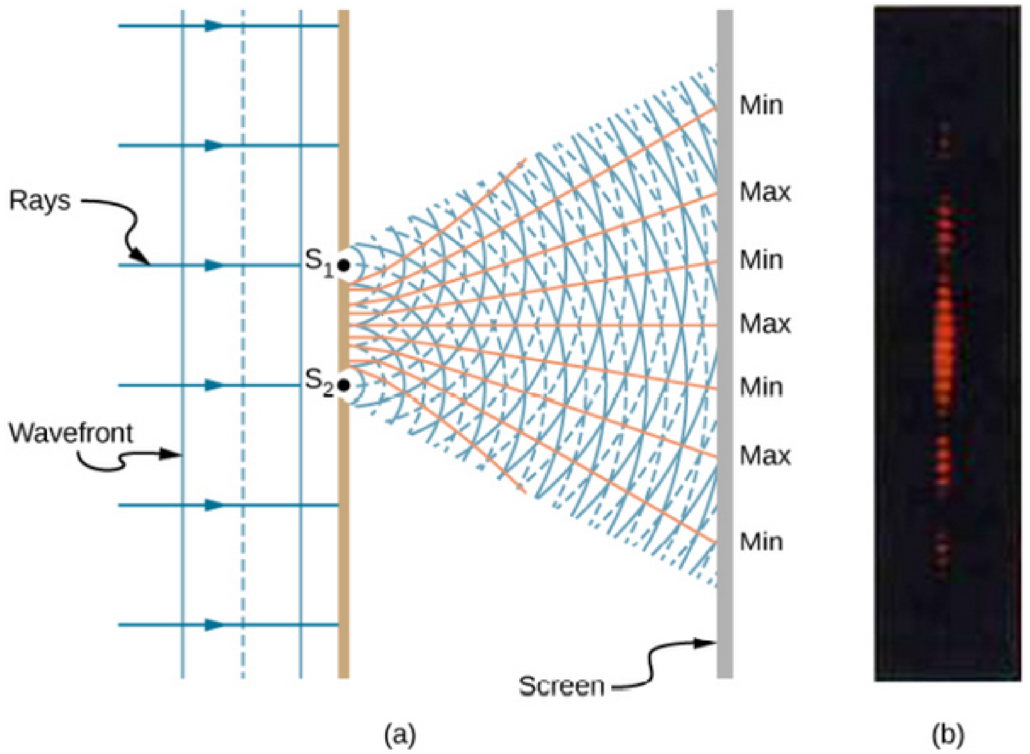
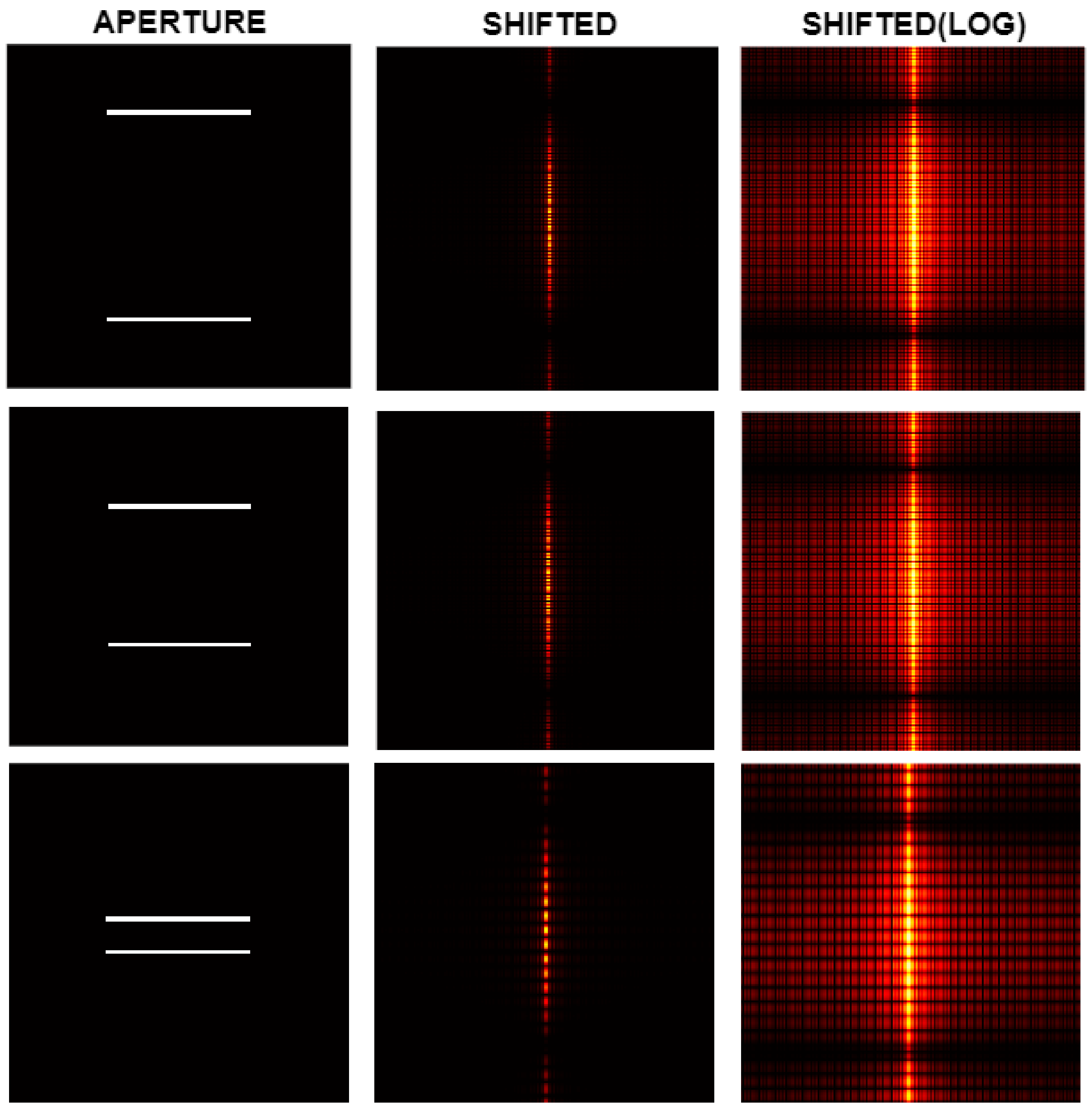



Figure 3.5 Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) When light that has passed through double slits falls on a screen, we see a pattern such as this.

# Double Slit Experiment

## Horizontal Varying Distance

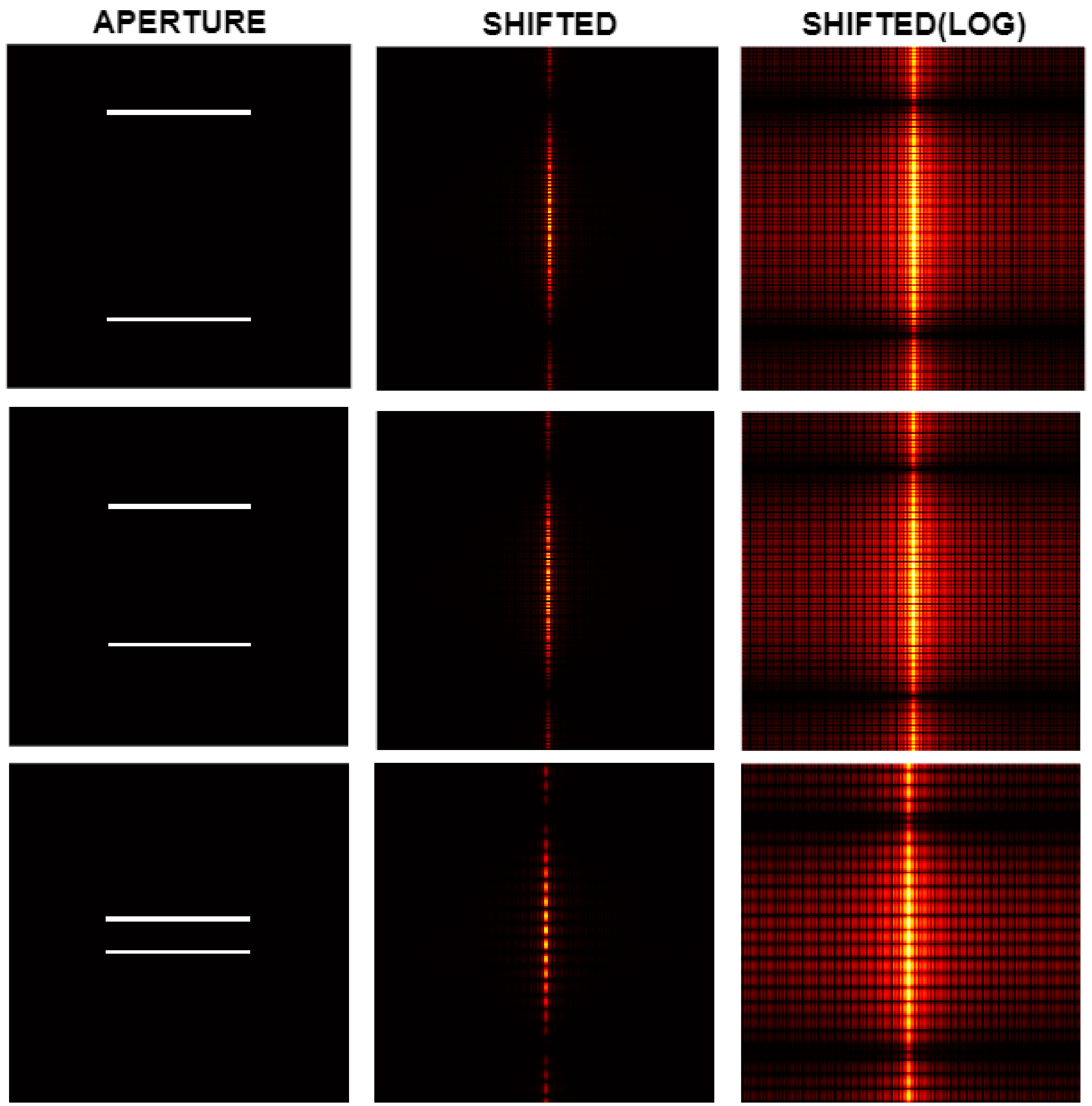
Here we two horizontal slits equally distanced from the center as our aperture. As we can see, the shifted image is similar to the **interference pattern** from the double slit experiment in the previous slide. The waves interfere constructively (**bright fringes**) and destructively (**dark fringes**) as we can observe in the pattern [7]. We can observe two things here: the **middle part is brighter** and the **distance affects the spacing of the fringes.**



# Double Slit Experiment

## Horizontal Varying Distance

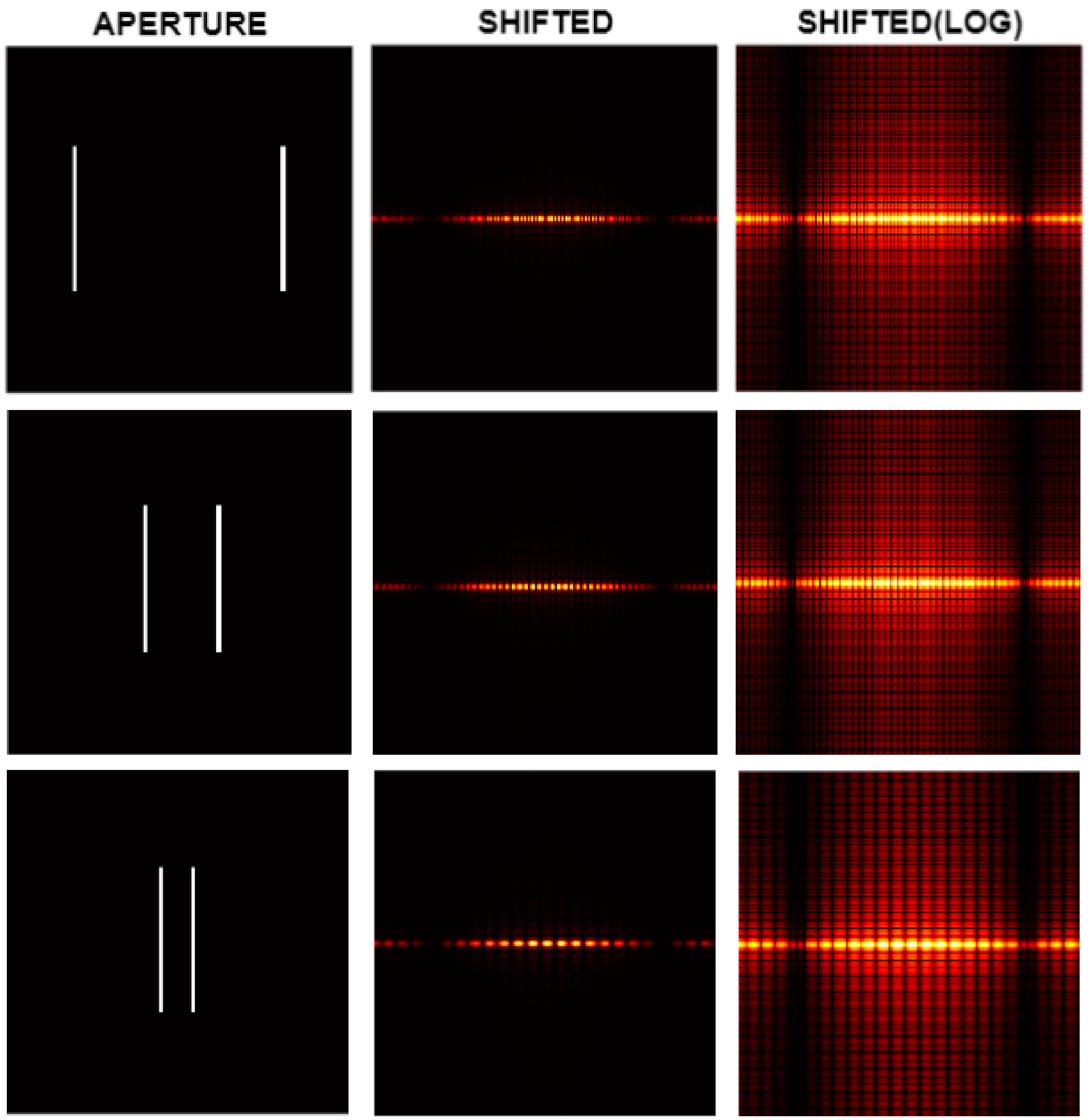
The center part appears to be brightest because it is the **closest region to the two slits**, as the fringes moves away from the slits, they gradually become darker [8]. Moreover, as we can see, if we decrease the distance between the slits, the spacing between the fringes increases. Hence, the spacing between the fringes and the distance between the slits are **inversely related** [9].



# Double Slit Experiment

## Vertical Varying Distance

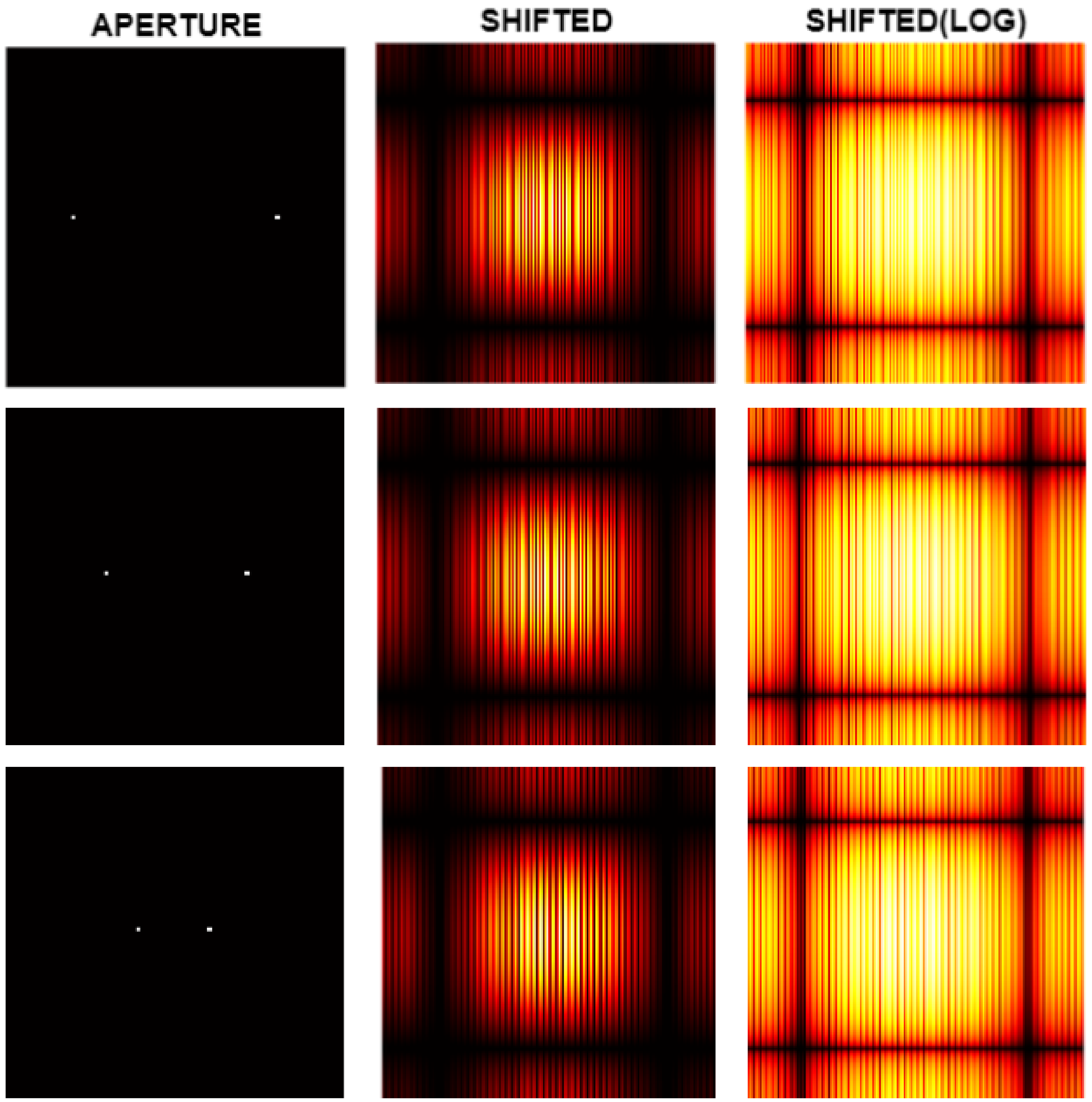
We see the same effects in the vertical slits. The center of the interference pattern is brighter, and the decreasing distance between the slits increases the spacing between the fringes.



# Two Dots Aperture

**Varying distance**

To make the two dots, I just used the function for the double slit and made the slit small enough to look like dots equally distanced about the center. The shifted image shows sinusoids with decreasing frequency as the distance of the dots are decreased. Hence, the FT of two dots is a sinusoid. It is interesting to note the FT of a sinusoidal aperture is two dots, as we previously seen, and now the FT of two dots is a sinusoid.

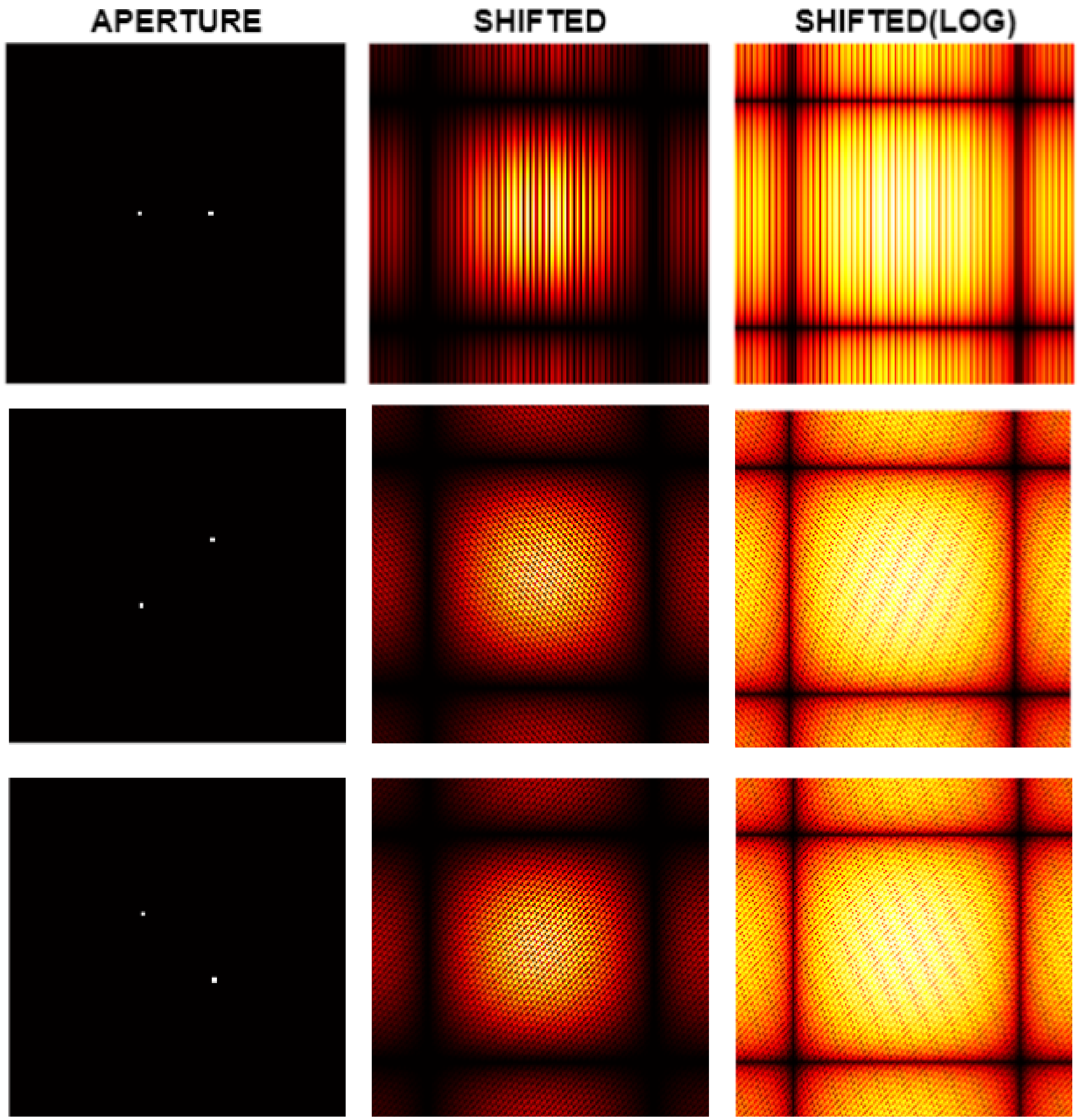


# Double Slit Experiment

## Rotating the dots

We expect to see sinusoids as the FT of two dots, but what happens if we rotate them?

The result, as we can see, is still a sinusoid. However, this time, they are rotated in an angle that follows the angle of rotation of the dots. Interesting, isn't it?!



# Double Slit Experiment

**<u>Rotating the dots</u>**

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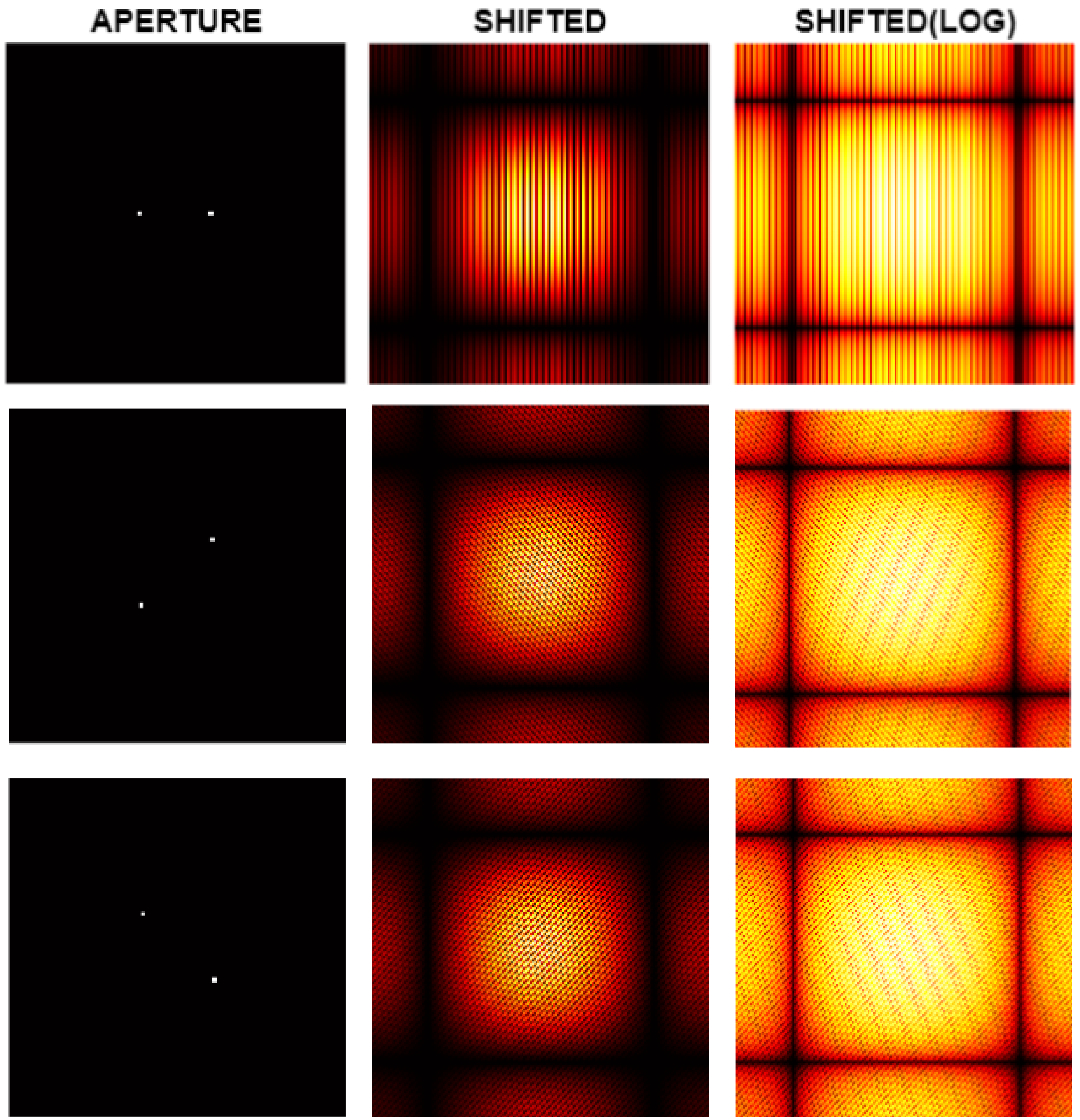
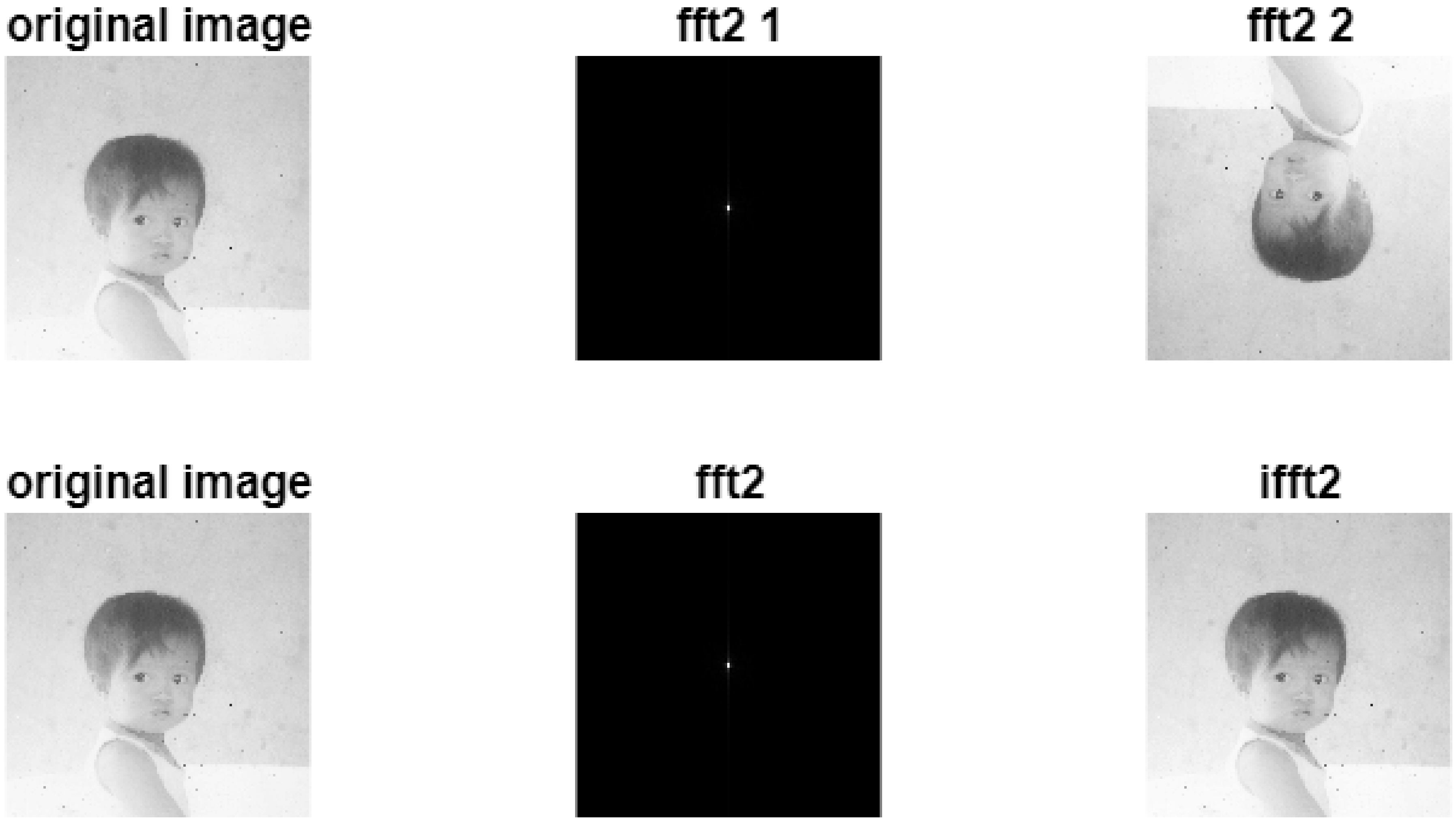


# FFT and IFFT

In this part, we tried to investigate what happens when (1) we apply fft2( ) twice and (2) fft( ) then ifft( ) on a square image. Here, I chose a throwback image of myself and cropped it. The original image looks like a dot in Fourier space. As we can see, the reconstructed image is inverted when we applied fft2( ) twice. On the other hand, when we applied the latter method, the resulting image is almost similar to the original image.

I Applying the FT twice yields a function (image) that is the reverse of the original function (image). Moreover, if you apply the inverse FT in the FT of a function(image), it returns the original function(image).



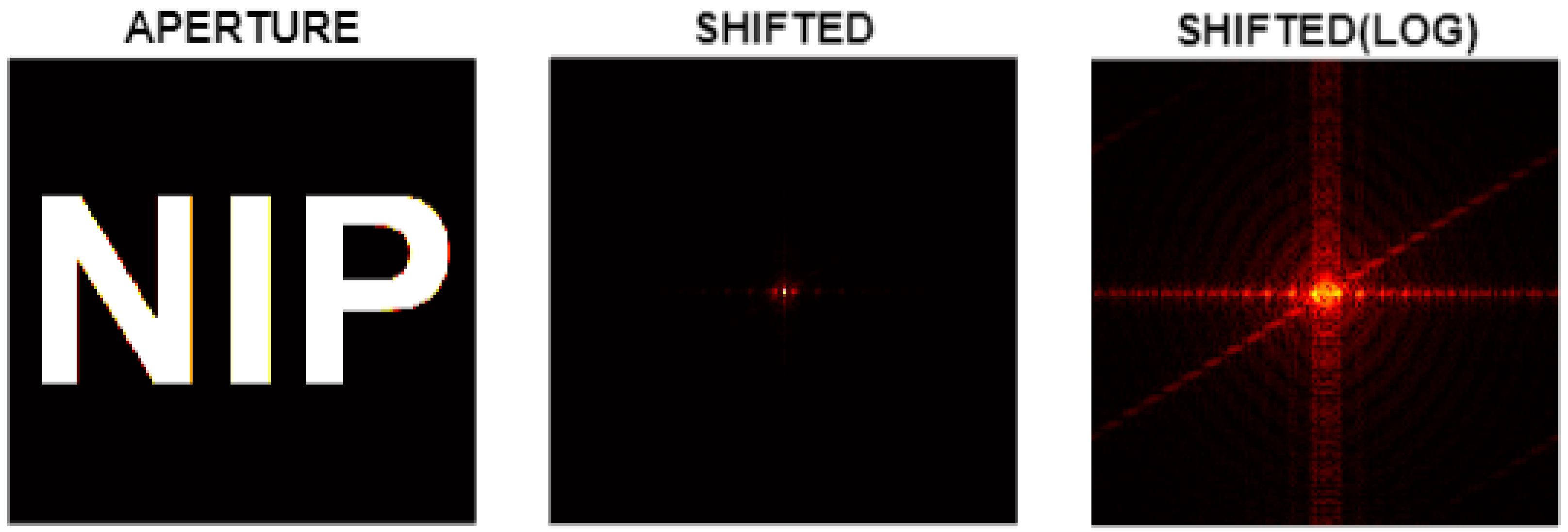
# Activity 2.2 Simulation of an Imaging System

## Background

The process of convolution in a 2D space is a multiplication in Fourier space. Thus, this is a method of "smearing" of one function against another producing a resulting function that is a little like both of the two functions convolved. We can use this method to model imaging systems. The convolution of the object and the impulse of an imaging system produces an image which may or may not be identical to the original object [10]. In this activity, we will do a simulation of an imaging system and use different apertures to see its effect on the resulting image.

# The FT of NIP Image

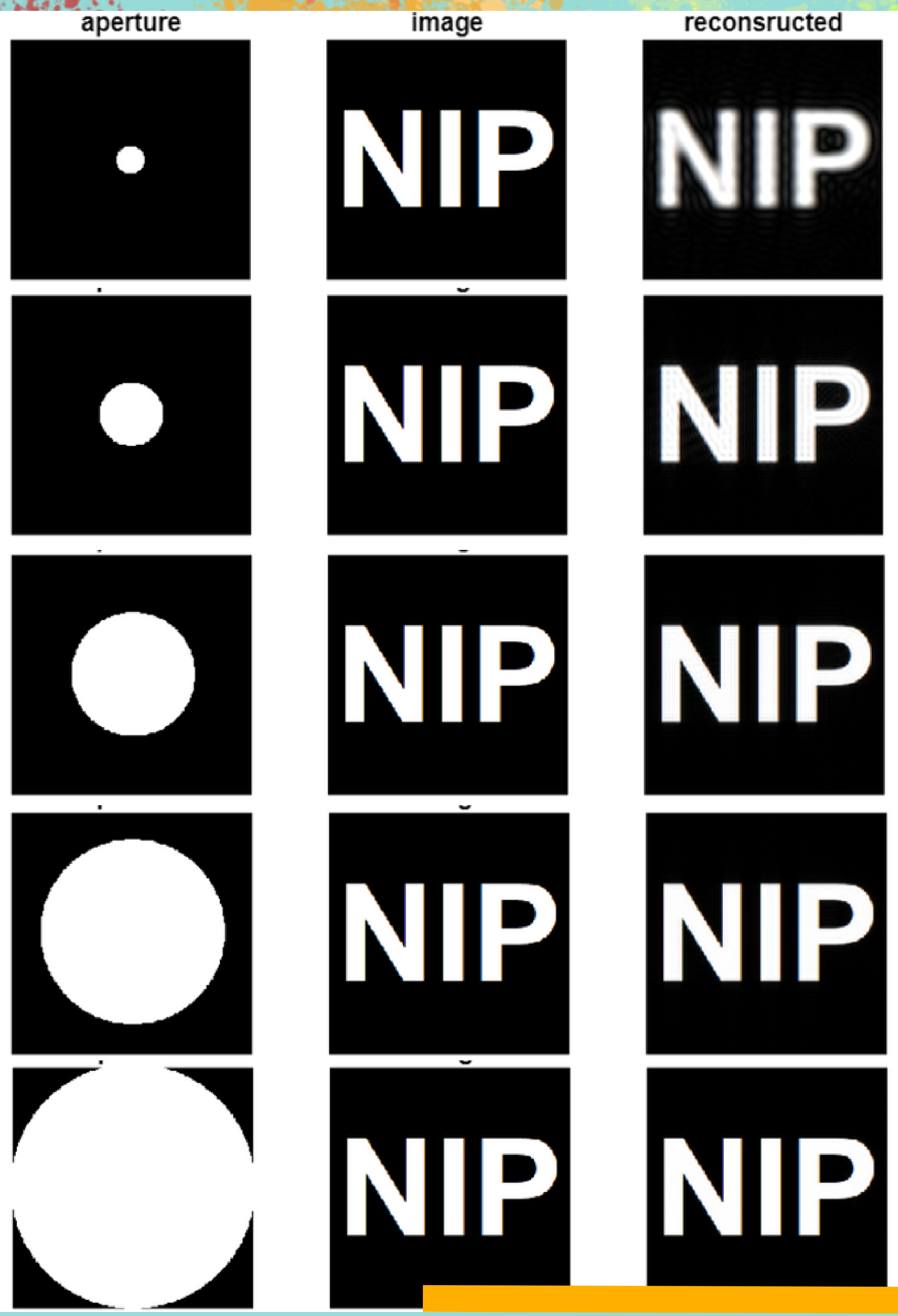
The convolution of the aperture and the NIP image is done in the Fourier space. Hence, we shall first see what the NIP image looks like in the Fourier space. The shifted image shows a faint dot in the middle. Using the log scale, we can see it clearly. Since the apertures that we will be using are already in the Fourier space, there is no need to apply fft2( ) to them. To implement this in matlab, I made a function Convolution(aperture) that takes the aperture as the input and returns the reconstructed image.



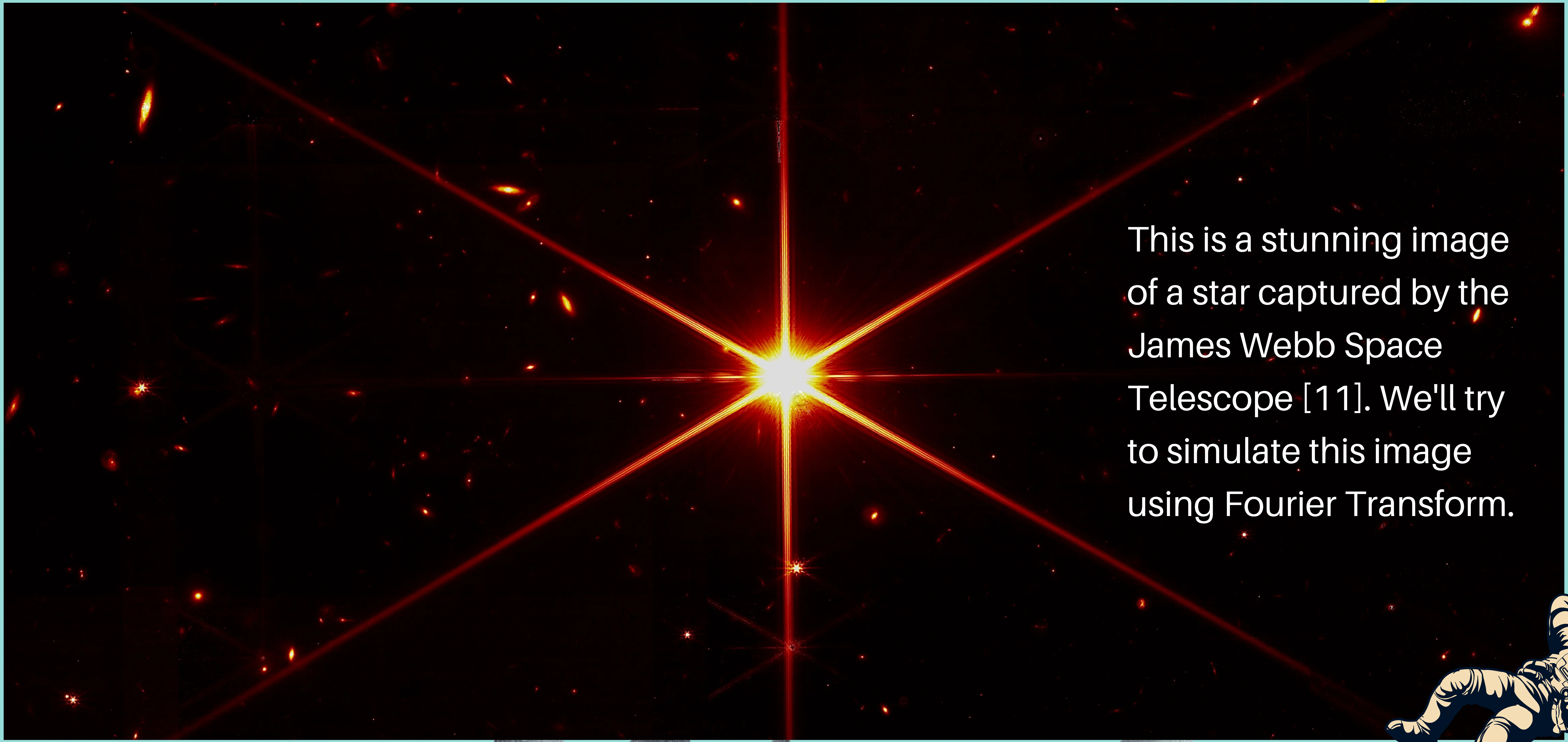
# Convolution

For this part, we used a circular aperture and we vary the size to see its effect when convolved with the 'NIP' image. Both the apertures and the image were made using the software paint. The circular aperture serves as the lens of an imaging system, and the NIP image serves as the object.

The reconstructed images are seen to be clearer as the aperture size increases. This is since for an imaging system that uses a circular aperture as lens, the size of the aperture affects the appearance of the image. Smaller apertures produce imperfect images as only a small bundle of reflected rays from the image can be gathered. More details will be seen as the aperture size increases [10].



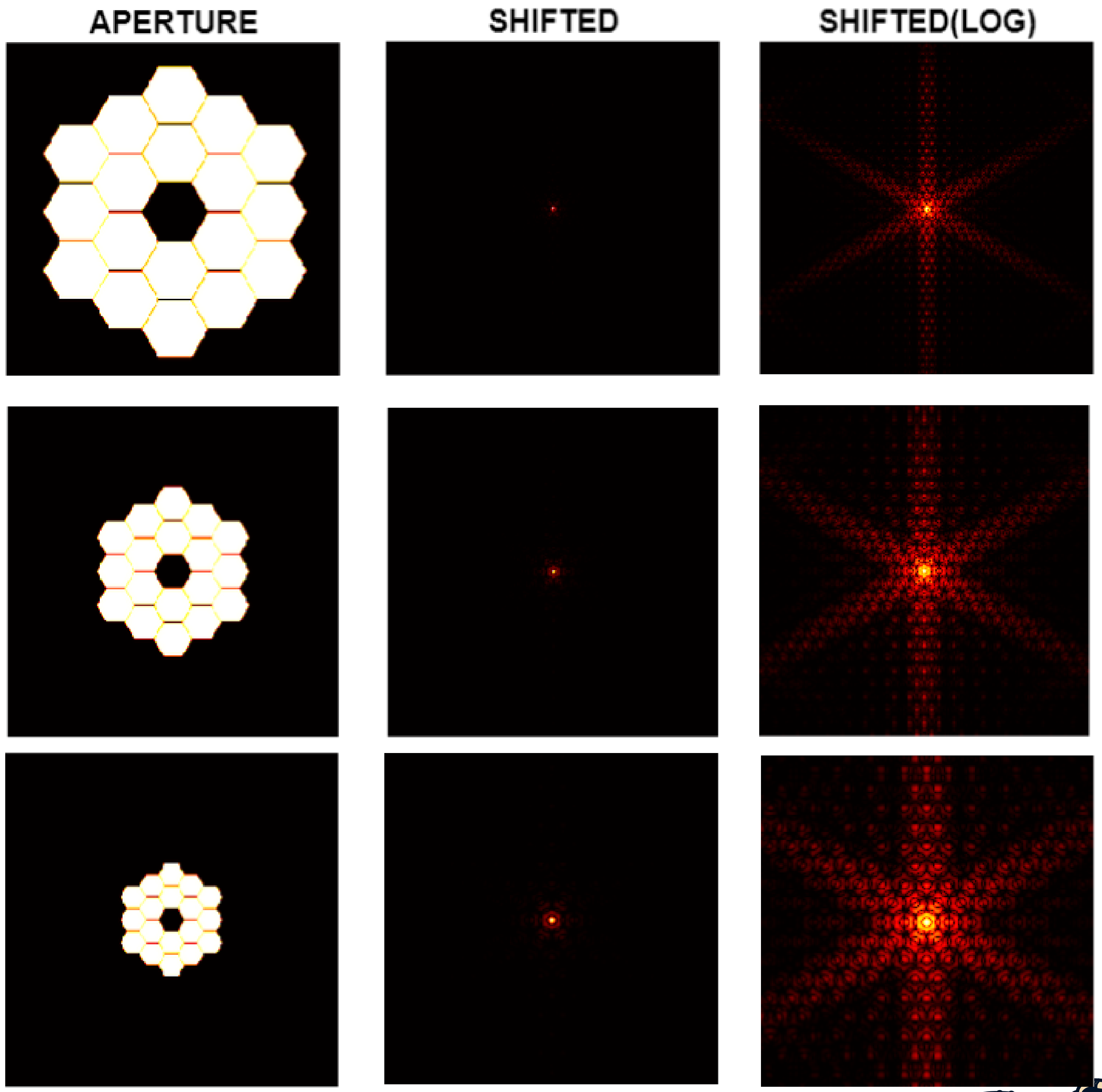
# JWST Aperture



This is a stunning image of a star captured by the James Webb Space Telescope [11]. We'll try to simulate this image using Fourier Transform.

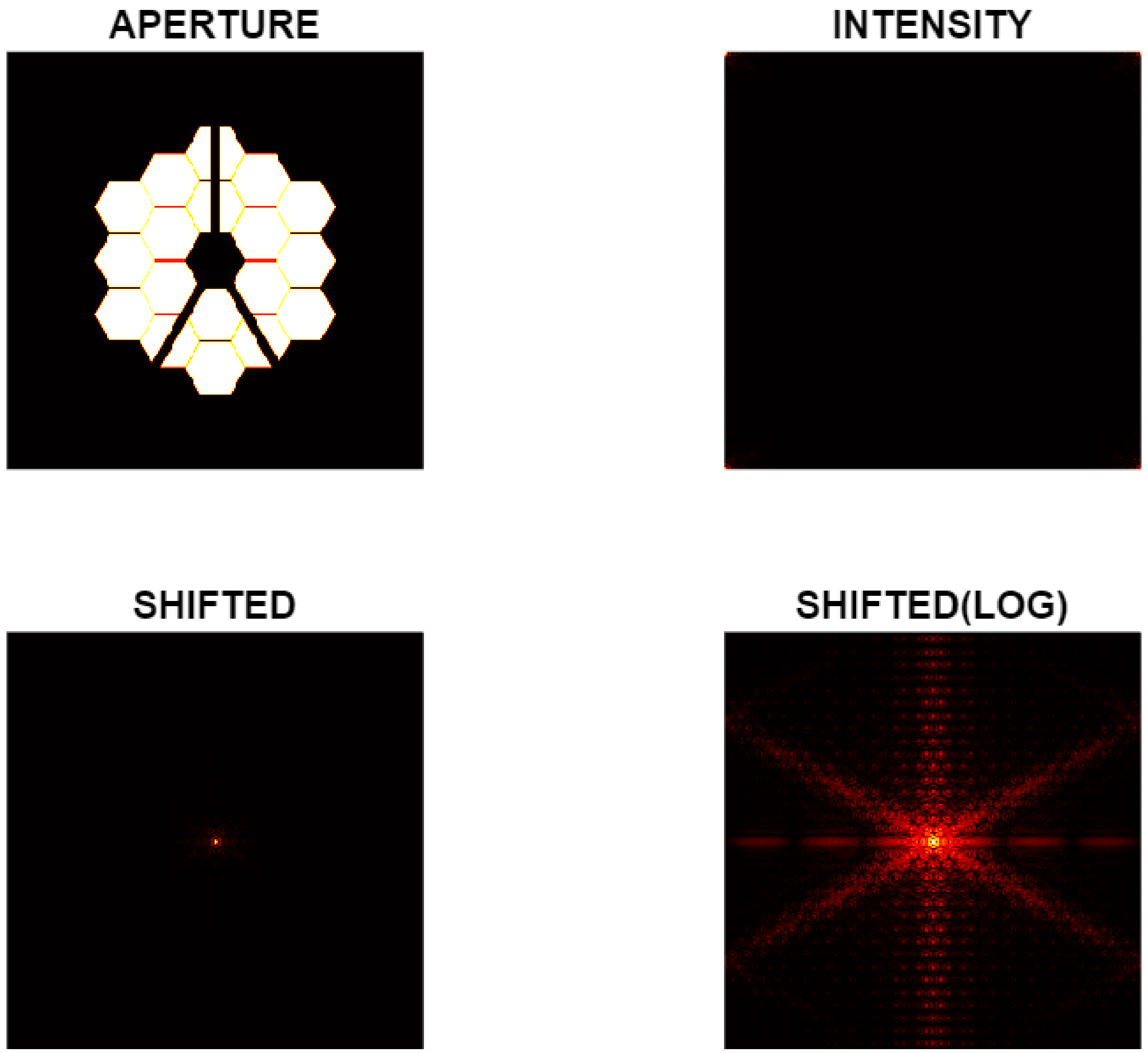
# JWST Aperture

To simulate the image of a star produced by the JWST, we simply get the FT of the JWST honeycomb mirror configuration. Here, I tried to use different sizes of the honeycomb aperture. As we can observe, the resulting image in the log scale resembles that of the image of a star from the previous slide. As the size of the aperture decreases, the image of the star increases in size and becomes brighter. However, the bigger aperture captures less noise and the image is sharper. This can be associated to how far the star is from the telescope.



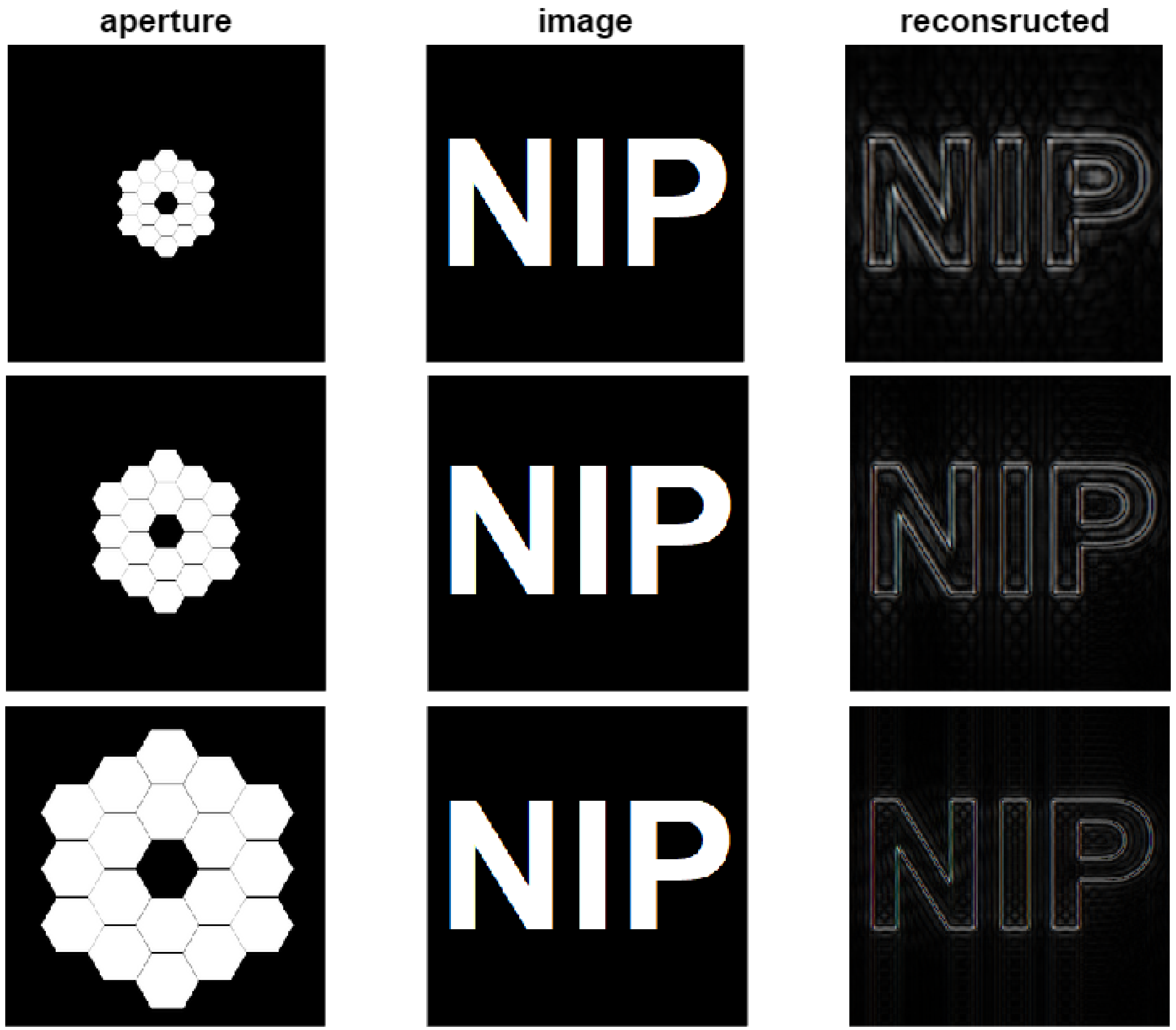
# JWST Aperture

This is the James Webb Space Telescope with the secondary mirror. As we can observe, the shifted log scale image of the star is brighter and sharper compared to the previous images.



# JWST Aperture

I also tried to apply the JWST aperture to the NIP image and convolve them. As we can observe, as the aperture becomes larger, the NIP image becomes clearer and sharper. However, compared to the circular aperture that produced a reconstructed image with NIP letters in white, the reconstructed image by the JWST only consists of the outline of the letters.



# Activity 2.3. Template matching using correlation

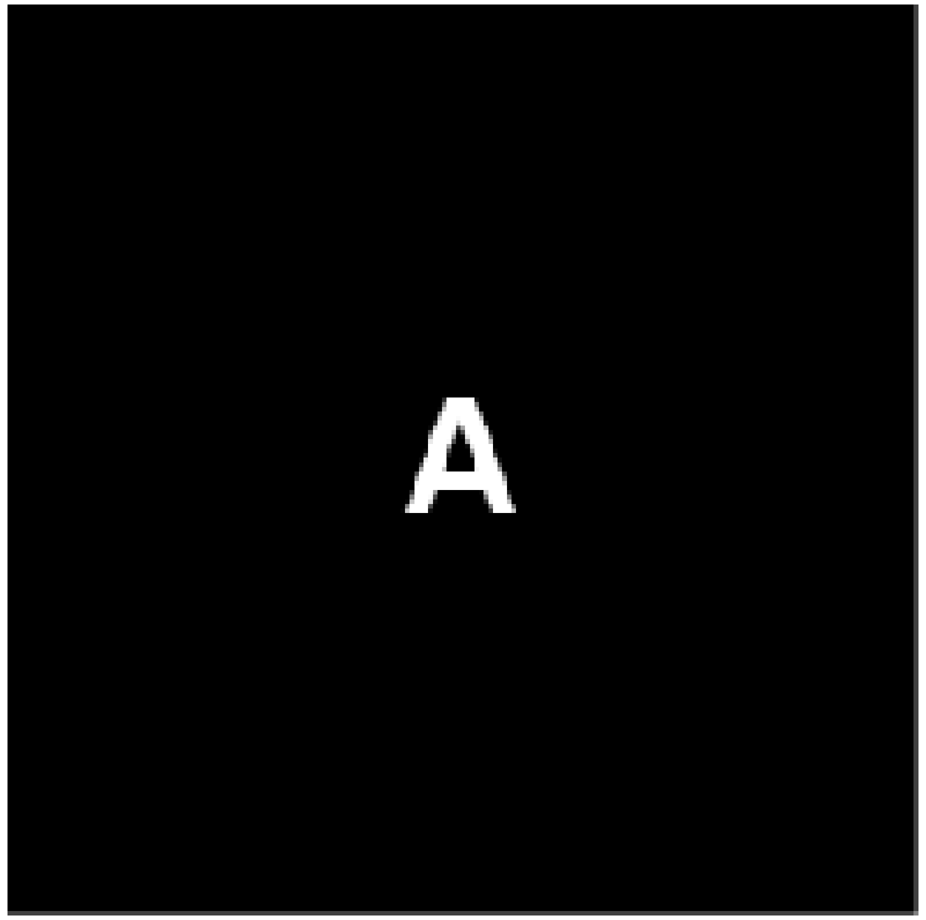
## Background

Template matching has several applications in computer vision systems ranging from quality control to object recognition systems [12]. It uses the correlation theorem that states that the correlation "measures the degree of similarity between two functions." The more identical these two functions are, the higher the correlation value that can be seen in that certain position [10].

In this activity, we'll apply the correlation theorem in template matching and observe what happens.

# Correlation

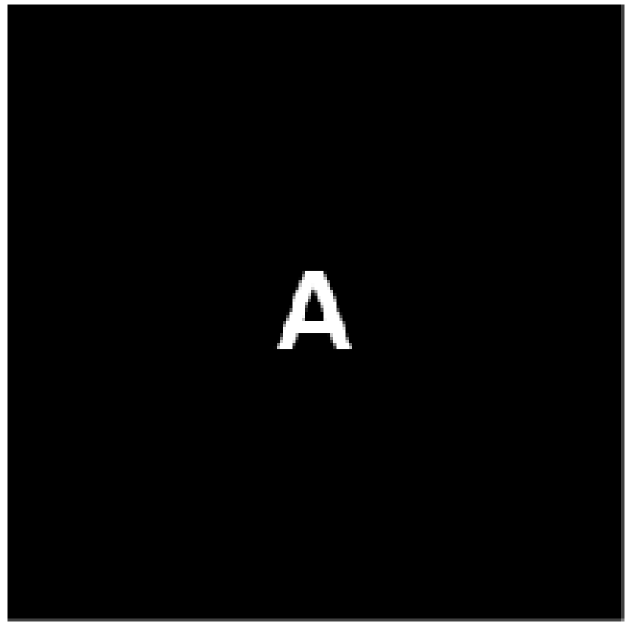
I created a 256x256 image in paint of the template with the phrase, "THE RAIN IN SPAIN STAYS MAINLY IN THE PLAIN." I used the Arial font. In the same font and size, I also created an image with only 'A' in the center.

# Correlation

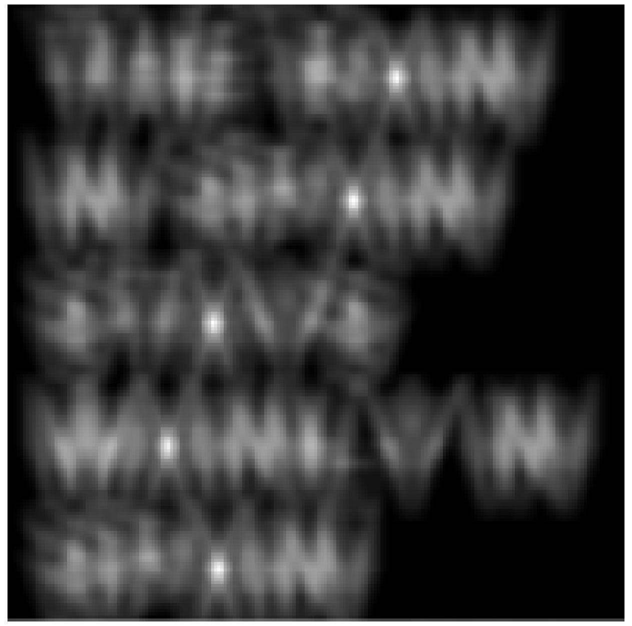
I applied the Correlation(template, I) function that I made which takes the input of the template and the image then returns the template matched image. As we can observe, the resulting image has brighter spots in the position where the template has the letter A. This means that these are the positions where the highest correlation value can be detected.

IMAGE




TEMPLATE

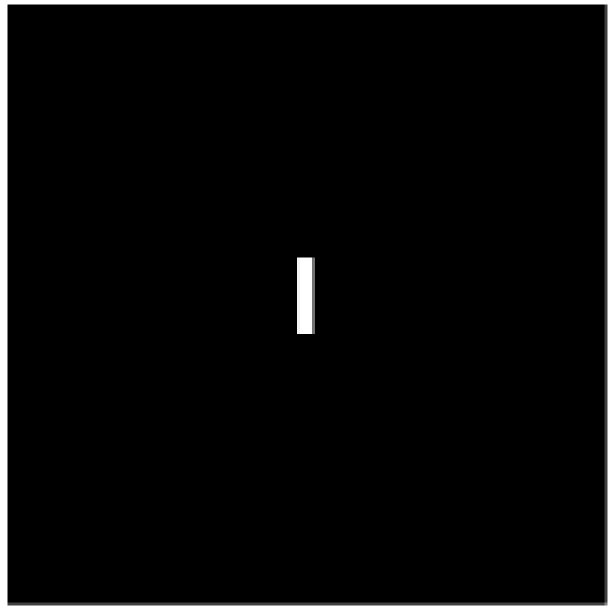

TEMPLATE MATCHED



# Correlation

I tried the same method but now I used the letter 'I'. From the resulting image, we can see that there are several positions where we can see the bright spots even if 'I' cannot be seen in those positions. This is because all of the letters except A, Y, and S contained in the template has a vertical line similar to the letter 'I'. The correlation function then sees these positions as a match for the letter 'I'.

IMAGE

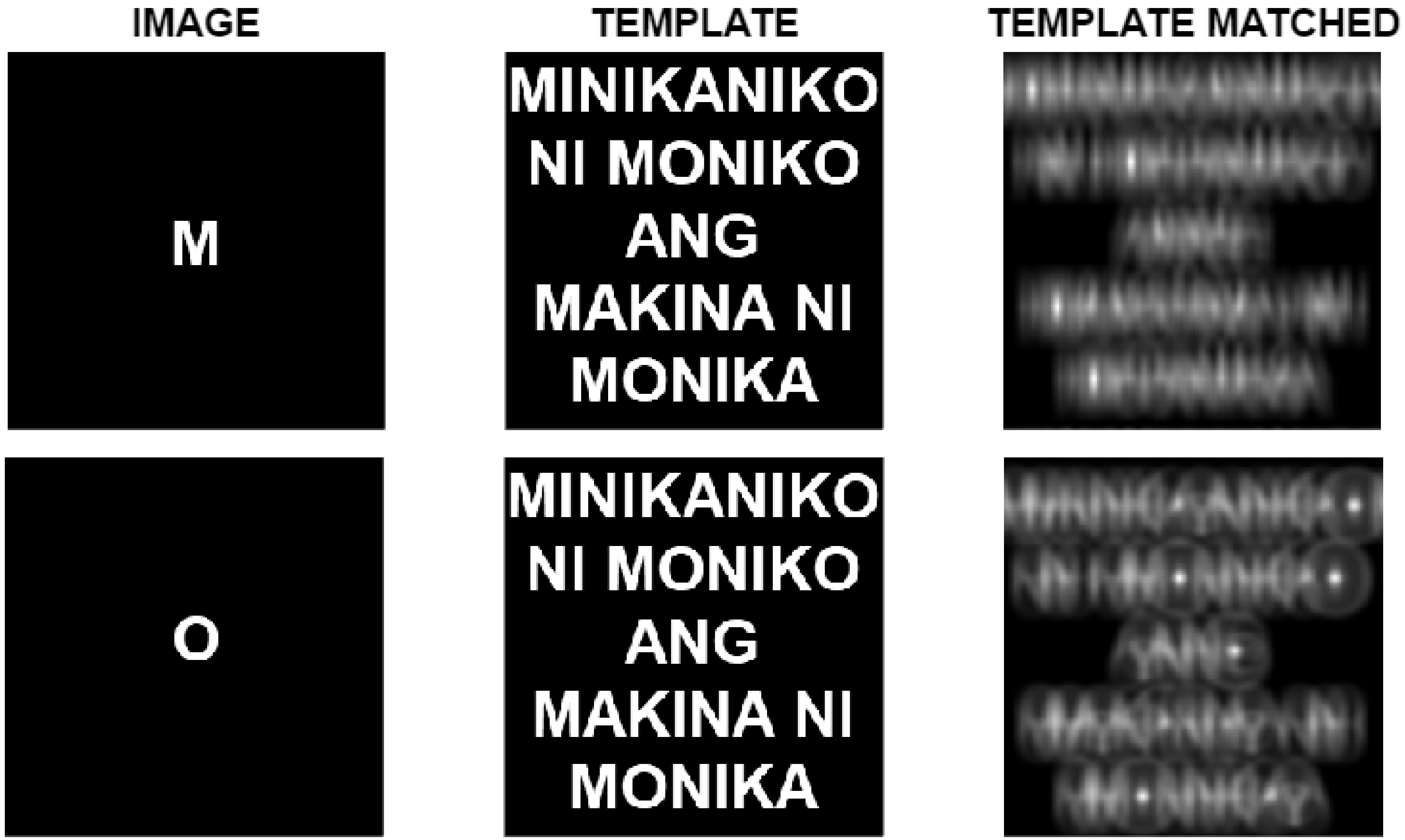



TEMPLATE


TEMPLATE MATCHED

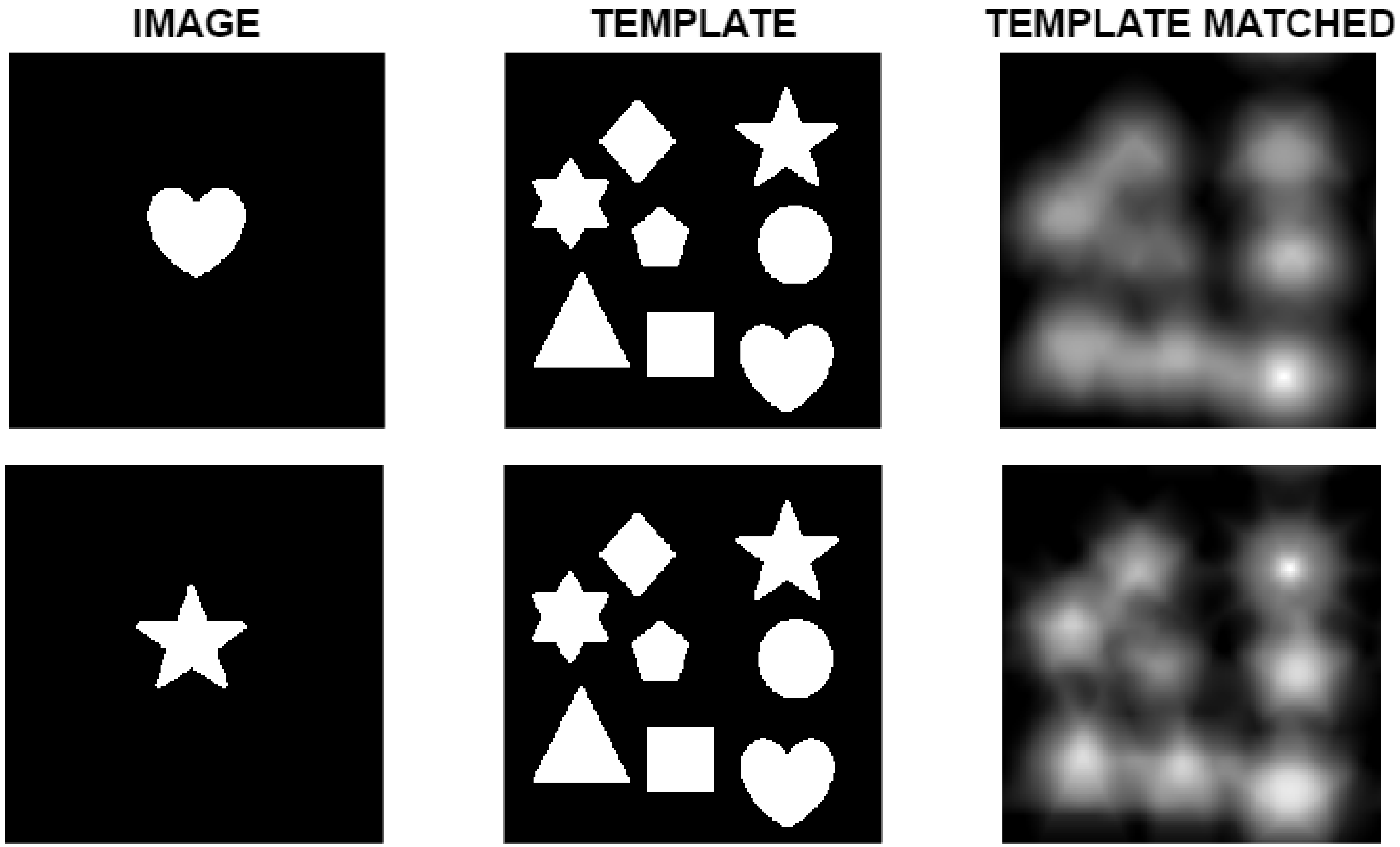
# Correlation

I tried to use a different template. The phrase is the famous tongue twister in Filipino. Similar to what we observed from the previous slides, the positions where the letters M and O can be seen have bright spots which means higher correlation value.



# Correlation (Shapes)

Now I tried it with shapes. I created a template with different shapes in different sizes. The position where the heart and star shown the brightest which indicates highest correlation value.



# Reflection

Similar with the Activity 1, this activity also involves working with images. It is a fun experience over all because I get to see how the images unfold. I already have a visualization of whether the code I made worked or did not work.

The coding part of this activity is not that cumbersome; as well as the report. However, the report part took much of my time because I wanted to find reasonable explanations from credible sources to confirm whether the resulting images that I got are theoretically correct. There is that "kilig" factor when I see that the results that I have matches the theory behind them.

Personally, the part that I particularly enjoyed is the simulation of the image of a star as seen by the James Webb Space Telescope. I saw a picture from an article online of the real image of a star that was captured by the JWST, and it matched the image of the star in my simulations. KILIG!!

# References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11. Howell, E. (2022, March 23). A stunning image shows stars aligned for the James Webb Space Telescope. Scientific American. Retrieved March 24, 2023, from https://www.scientificamerican.com/article/a-stunning-image-shows-stars-aligned-for-the-james-webb-space-telescope/
12. Brunelli, R. (2009). Template Matching Techniques in Computer Vision: Theory and Practice. Germany: Wiley.

# REPORT GRADE

| Criteria | Score |
|---|---|
| Technical Correctness | 35 |
| Quality of Presentation | 35 |
| Self Reflection | 30 |
| Initiative | 10 |
| TOTAL | 100 |

**EVALUATION**

Overall, I know that I have accomplished all the tasks included in this activity. Moreover, I did some extra experiments and made some comparisons in the resulting images. I also put a lot of effort in creating this presentation and cited a lot of sources for additional information in the report.