AMATH 481 & 581 Fall 2018

Bose-Einstein Condensation in 3D

Submission open until 11:59:59pm Tuesday December 11, 2018

Consider the Gross-Pitaevskii system (nonlinear Schrödinger equation with potential) modeling a condensed state of matter

$$i\psi_t + \frac{1}{2}\nabla^2\psi - |\psi|^2\psi + [A_1 \sin^2(x) + B_1][A_2 sin^2(y) + B_2][A_3 sin^2(z) + B_3]\psi = 0 \tag{1}$$

where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ (you can google this to learn more). Consider periodic boundaries and using the **3D FFT** (`np.fft.fftn`) to solve for the evolution. Step forward using `solve_ivp`. VISUALIZE USING intersecting planes. WARNING: 3D problems involve working with vectors of size n^3, so pick n small to begin playing around.

ANSWERS:

(a) With $x, y, z \in [-\pi, \pi], n = 16,$ tspan from 0 to 4, including end points with step size .5, and parameters $A_i = -1$ and $B_i = -A_i$, with initial conditions

$$\psi(x, y, z) = \cos(x)\cos(y)\cos(z)$$

write out the solution of your numerical evolution from `solve_ivp` as A1 (real part) and A2 (imaginary part). The matrices will have size (9, 4096). To reshape it for the final answer, use order "F". (NOTE: your solution will be in the Fourier domain when you write it out).

(b) Now solve with initial conditions

$$\psi(x, y, z) = \sin(x)\sin(y)\sin(z)$$

write out the solution of your numerical evolution from `solve_ivp` as A3 (real part) and A4 (imaginary part). The matrices will have size (9, 4096). To reshape it for the final answer, use order "F". (NOTE: your solution will be in the Fourier domain when you write it out).