Random Variation in Sex Ratios: population sizes and sex-ratio precision

Fairly large populations of infants are required if one or two percentage-point differences in infant sex-ratios are to be informative about living conditions, and to ensure random variation in infant sex ratios is not mistaken for variation in living conditions. For example, consider a fairly healthy population of children with a "true" underlying SR1 of 105.0 from year to year.[1] In a census counting 10,000 infants, there is a about a one-sixth chance that random variation would result in an observed SR1 as low as 103.[2] However, in a census counting 50,000 infants, there is less than a 2% chance of getting an SR1 as low as 103 from a population whose true (or underlying) SR1 is 105. More generally, when using the SR1 as a indicator of well-being, it is important to recognize the potential role of random variation, and to consider the likely amount of "noise" in view of population sizes. And given the possibility of measurement errors in historical census data, in addition to random variation, we view infant sex ratios as fairly blunt measures of well-being in the past.

For example, consider England in 1841, with an SR2 of just 98.0 (with a 95% confidence interval of just +/- 0.2 percentage points).[3] That sex ratio points to poor living conditions for England's children and their mothers. The industrial city of Manchester also featured an SR2 of 98.0 in the 1841 census, but with less than 11,000 children counted, the 95% confidence interval is a fairly broad 96.5 to 99.5; with that range, we cannot reject claims that Manchester was better or worse than England as a whole. But we can reject a claim (hypothesis) that Manchester's sex ratio is consistent with healthy living conditions. More ambiguous is a comparison of Manchester to the older port city of Liverpool, where the SR2 was 101.3 with a 95% confidence interval of 99.6 to 103.1; that provides evidence of somewhat better living conditions in Liverpool than in Manchester or England more generally, but even allowing for possibly sizable random negative variation in Liverpool's SR2, the sex ratio is well below that of a healthy population of the time (for example, Ireland with a SR2 over 104 in 1841 and 1851).

However, in some cases random variation in sex ratios is largely irrelevant to patterns of interest; such is the case for Ireland in 1841. The population under-2 was over 184 thousand and the SR2 was 104.3 with a 95% confidence interval of 103.4 to 105.3; here, the sex ratio data simply contradict any claim that maternal and infant living conditions were unhealthy. With infant populations this large, differences of 1 or more percentage-points are very meaningful -- for example, the probability of a random fluctuation in the SR2 as large as 1-percentage-point would be about 4 percent.

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1 By "true" or underlying sex ratio, we have in mind the sex ratio reflective of the probability that an infant is male, viewing the sex of any infant as random outcome, and supposing that an observed population is just one set of random outcomes (following a binomial distribution).

2 See below for our methods for probabilistic claims, which rely on a binomial distribution generating the observed proportion male in a census-population (and the central limit theorem).

3 We use the SR2 here, because male-biased relative age-heaping at age one is evident in the 1841 census of England; for example, more males were counted at age one than at age under one, the reverse holding for females (1849:17, 440).

Modelling Random Variation in Sex Ratios

We model the infant sex-ratio (SR) as a random variable, based on the binomial; with the observed SR being the sample mean from n draws from a binomial distribution with mean of "p" -- the probability an observed infant is male (and n being the number of infants). Working with census data, sample sizes are sufficiently large to rely on the central limit theorem to and the observed proportion male (call it "m") will be normally distributed with mean p and variance of p*(1-p)/n; relying on (using) confidence intervals or probability statements about the observed (sampled) proportion (m), we can easily extrapolate to inferences about the sex ratio (with sex ratios calculated as SR = 100*m/(1-m))

In the case of Ireland in 1841, we might suspect that the SR1 of 104.3 is overstating well-being in a famously "poor" place, so a one-sided hypothesis test is appropriate. With a little more than 100 thousand infants in Ireland in 1841, if the true underlying SR2 was as low as 101 (healthier than England at mid-century), the probability of observing values as large as 104 is negligible.

Assorted notes on magnitudes of SR variation ... suppose the "true" SR1 is 101.6 (p=0.504)

In a sample of 1000, a 5-male decrease in the sex-mix drops the SR1 by about 2 percentage-points. With P=0.504, Pr(m<504) is 48.7% and Pr(m<495) is 36.4% ...

In a sample of 2000, a 10-person change in the sex-mix boosts the SR1 by about 2 percentage points. With P=0.504, Pr(m<1008) is about 51% and Pr(m<998) is about 32% ...

In a sample of 10,000, a 50-person change in the sex-mix boosts the SR1 by about 2 percentage points. With P=0.504, Pr(m<5040) is about 49.6% and Pr(m<4990) is about 16% ...

In a sample of 50,000, a 250-person change in the sex-mix boosts the SR1 by about 2 percentage points. With P=0.504, Pr(m<25,200) is about 49.8% and Pr(m<24950) is about 1.3% ...

Lesson: once we get to 50,000 infants, a 2%-point swing in the SR1 is not sensibly ascribed to chance.

More interesting is variation from census to census in most cases, where large swings in the SR1 must be diagnosed in terms of changes in maternal/infant conditions versus counting errors.

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