Verify Homework Problem 17.3 Part 2

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17.3 Part 2: Integrating Factors

If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact, it may be possible to make it exact by multiplying by an integrating factor.

If $\frac{1}{N(x,y)}[M_y(x,y) - N_x(x,y)] = h(x)$ is a function of x alone, then $e^{\int h(x)\,dx}$ is an integrating factor.

If $\frac{1}{M(x,y)}[N_x(x,y) - M_y(x,y)] = k(y)$ is a function of y alone, then $e^{\int k(y)\,dy}$ is an integrating factor.

Example 4 page 1053
Find the general solution of $(y^2 - x)dx + 2ydy = 0$

Solution

M: $(y^2 - x)dx$ N: $2ydy$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\frac{\partial(y^2-x)}{\partial y} - \frac{\partial 2y}{\partial x}}{2y} = \frac{2y - 0}{2y} = 1$$

Multiply **every term** by the integrating factor.

$$(y^2e^x - xe^x)dx + 2ye^xdy = 0$$

$$\int y^2e^x - xe^xdx = y^2e^x - xe^x + e^x + g(y) + C$$

Hint: Larson Integration Table Number 82 A18

$$\int ue^udu = (u-1)e^u + C$$

$$\int 2y^2e^xdy = y^2e^x + g(x) + C$$

Now we reconcile

$$g(x) = xe^x + e^x$$

$$g(y) = \text{None} = 0$$

So...

$$y^2e^x - xe^x + e^x = C$$