CrossCorrelator

January 17, 2017

1 Running Spatial Correlations + options

The goal of this log is to show the API of the spatial correlations and the options available. With this code, it is possible to run the spatial correlations on masked and unmasked data.

Also, it is possible to apply a correction, called symmetric averaging, which is a derivative of a method by Schatzel (1988):

Schätzel, Klaus, Martin Drewel, and Sven Stimac. "Photon correlation measurements at large lag times: improving statistical accuracy." Journal of Modern Optics 35.4 (1988): 711-718.

Julien Lhermitte, jan 2017

The correlation function in 1 dimension is:

$$C = \frac{1}{N(k)} \sum_{j}^{N_t} I_j I_{j+k} M_j M_{j+k}$$

We may normalize it by its average intensity in two different ways:

1.1 1. Naive averaging:

the normalized correlation function is just divided by the average squared:

$$cc_{reg} = \frac{CC}{\bar{I}^2}$$

where:

$$\bar{I} = \frac{1}{N(k)} \sum_{j=1}^{N_t} I_j$$

is average intensity and where

$$N(k) = \sum_{j=1}^{N_t} M_j M_{j+k}$$

(Note that in the limit of no mask, $N(k) = N_t$ as it should, mask has effect of inducing a k dependence on the effective ''N_t")

1.2 2. Symmetric Averaging:

For symmetric averaging, we define two new averages, I_p and I_f (I 'past' and I 'future'):

$$I_p = \frac{1}{N(k)} \sum_j I_j M_j M_{j+k}$$

$$I_f = \frac{1}{N(k)} \sum_l I_{l+k} M_l M_{l+k}$$

we define symmetric averaging as:

$$cc_{sym} = \frac{CC}{\bar{I}_p \bar{I}_p}$$

Schatzel shows this averaging is superior for the case of a simple ''mask'': a 1D time series (data outside of time range is''masked'')

I'm happy to provide my derivation/example for the motivation of symmetric averaging.

In [13]:
```
# import libraries

# %load ../pyscripts/0937-test-spatialcorrelator.py
%matplotlib inline
import numpy as np
#from pyCXD.tools.CrossCorrelator import CrossCorrelator
from skbeam.core.correlation import CrossCorrelator
import matplotlib.pyplot as plt
from skbeam.core.roi import ring_edges, segmented_rings


# for some convolutions, used to smooth images (make spatially correlated
def convol2d(a,b=None,axes=(-2,-1)):
    ''' convolve a and b along axes axes
        if axes 1 element, then convolves along that dimension
        only works with dimensions 1 or 2 (1 or 2 axes)
    '''
    from numpy.fft import fft2, ifft2
    if(b is None):
        b = a
    return ifft2(fft2(a,axes=axes)*np.conj(fft2(b,axes=axes)),axes=axes).r
```

In [14]:
```
#this test is just meant to show an example
# for a test for nosetests, reduce number points to something manageable
# random seed for reproducibility
np.random.seed(123)
```

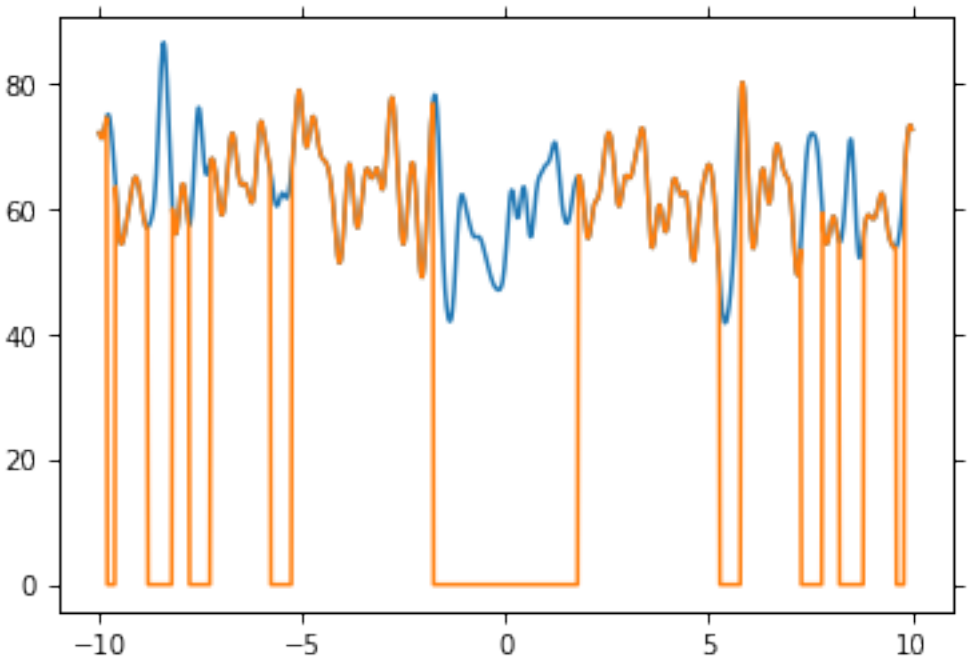
2 1. Try on 1D data

In [15]:
```
# test 1D data
sigma = .1
```

```
Npoints = 1000
x = np.linspace(-10, 10, Npoints)
y = convol2d(np.random.random(Npoints)*10, np.exp(-x**2/(2*sigma**2)),axes

mask_1D = np.ones_like(y)
mask_1D[10:20] = 0
mask_1D[60:90] = 0
mask_1D[111:137] = 0
mask_1D[211:237] = 0
mask_1D[411:537] = 0

mask_1D *= mask_1D[::-1]

y_masked = y*mask_1D

cc1D = CrossCorrelator(mask_1D.shape)
cc1D_symavg = CrossCorrelator(mask_1D.shape,normalization='symavg')
cc1D_masked = CrossCorrelator(mask_1D.shape,mask=mask_1D)
cc1D_masked_symavg = CrossCorrelator(mask_1D.shape, mask=mask_1D,normaliza

ycorr_1D = cc1D(y)
ycorr_1D_masked = cc1D_masked(y*mask_1D)
ycorr_1D_symavg = cc1D_symavg(y)
ycorr_1D_masked_symavg = cc1D_masked_symavg(y*mask_1D)
```

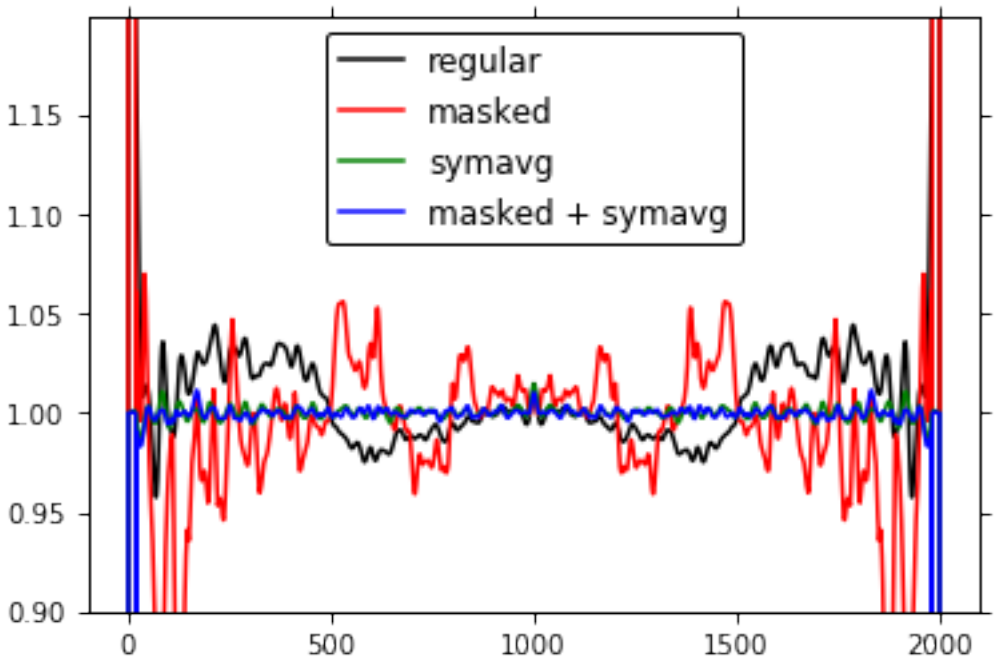
2.1 Plot the data

```
In [16]: # plot 1D Data
         plt.figure(0);plt.clf();
         plt.plot(x,y)
         plt.plot(x,y*mask_1D)

Out[16]: [<matplotlib.lines.Line2D at 0x7f0fc4431240>]
```



2.2 Correlations for different cases

```
In [17]: plt.figure(1);plt.clf();
         plt.plot(ycorr_1D,color='k',label='regular')
         plt.plot(ycorr_1D_masked,color='r',label='masked')
         plt.plot(ycorr_1D_symavg,color='g',label='symavg')
         plt.plot(ycorr_1D_masked_symavg,color='b',label='masked + symavg')
         plt.ylim(.9,1.2)
         plt.legend()

Out[17]: <matplotlib.legend.Legend at 0x7f0fc43f8eb8>
```



3 2. Try for 2D data

(In this case, even no mask has a strong effect on data. No mask still contains a ''mask" since at higher correlation lengths we are correlating less points. Symmetric averaging excels to overcome these effects here.)

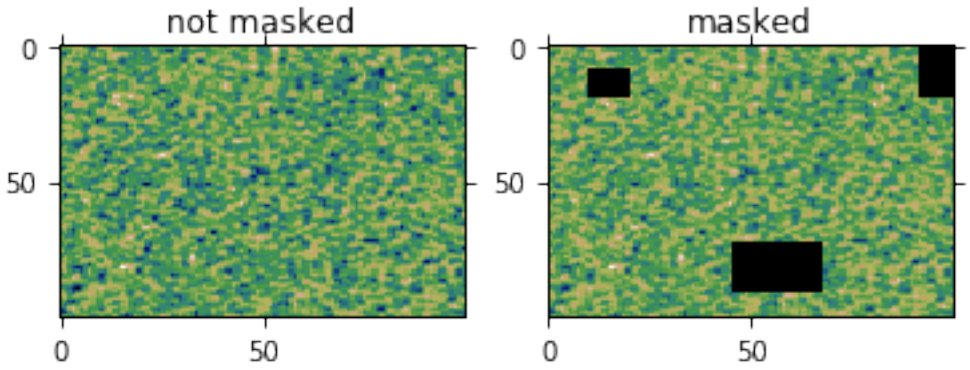
```
In [18]: # test 2D data
         Npoints2 = 100
         x2 = np.linspace(-10, 10, Npoints2)
         X, Y = np.meshgrid(x2,x2)
         Z = np.random.random((Npoints2,Npoints2))
         Z = convol2d(Z, np.exp(-(X**2 + Y**2)/2./sigma**2))

         mask_2D = np.ones_like(Z)
         mask_2D[10:20, 10:20] = 0
         mask_2D[73:91, 45:67] = 0
         mask_2D[1:20, 90:] = 0

         cc2D = CrossCorrelator(mask_2D.shape)
         cc2D_symavg = CrossCorrelator(mask_2D.shape,normalization='symavg')
         cc2D_masked = CrossCorrelator(mask_2D.shape,mask=mask_2D)
         cc2D_masked_symavg = CrossCorrelator(mask_2D.shape, mask=mask_2D,normaliza

         ycorr_2D = cc2D(Z)
```

```
ycorr_2D_masked = cc2D_masked(Z*mask_2D)
ycorr_2D_symavg = cc2D_symavg(Z)
ycorr_2D_masked_symavg = cc2D_masked_symavg(Z*mask_2D)
```

3.1 plot 2D data

```
In [19]: plt.figure(2);plt.clf();
         plt.subplot(2,2,1)
         plt.title("not masked")
         plt.imshow(Z)

         plt.subplot(2,2,2)
         plt.title("masked")
         plt.imshow(Z*mask_2D)
```

```
Out[19]: <matplotlib.image.AxesImage at 0x7f0fc46a6780>
```

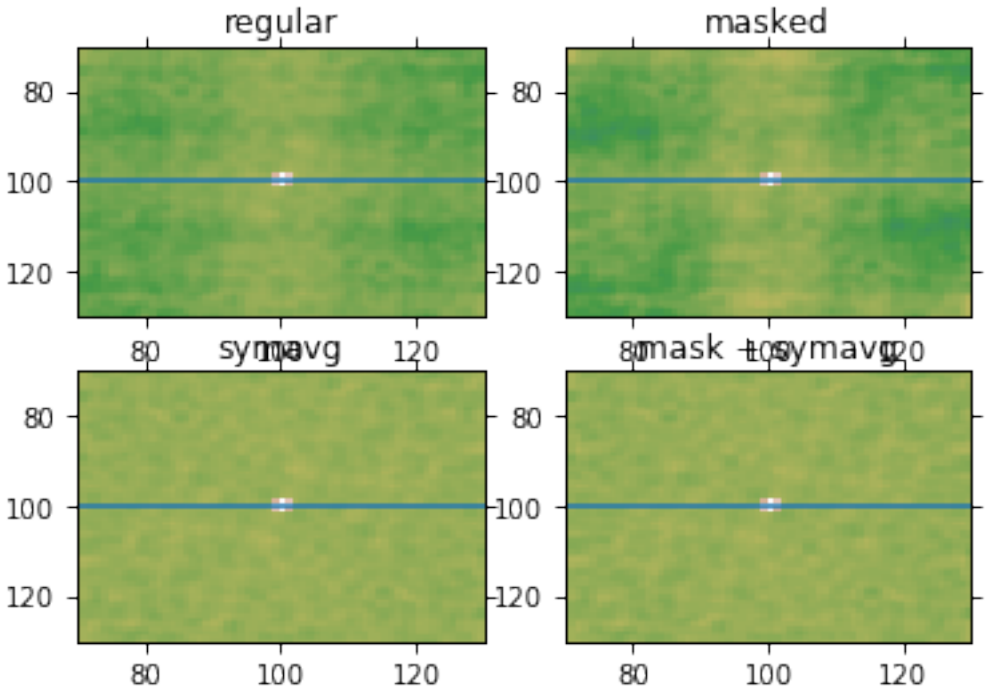


3.2 Correlations (2D)

```
In [20]: vmin=.95; vmax=1.03

         plt.figure(3);plt.clf();
         plt.subplot(2,2,1)
         plt.title("regular")
         plt.imshow(ycorr_2D)
         plt.axhline(ycorr_2D_masked.shape[0]//2)
         plt.clim(vmin,vmax)
         plt.xlim(70,130)
         plt.ylim(130,70)
         plt.subplot(2,2,2)
         plt.title("masked")
         plt.imshow(ycorr_2D_masked)
         plt.axhline(ycorr_2D_masked.shape[0]//2)
         plt.clim(vmin,vmax)
```

```
plt.xlim(70,130)
plt.ylim(130,70)
plt.subplot(2,2,3)
plt.title("symavg")
plt.imshow(ycorr_2D_symavg)
plt.axhline(ycorr_2D_masked.shape[0]//2)
plt.clim(vmin,vmax)
plt.xlim(70,130)
plt.ylim(130,70)
plt.subplot(2,2,4)
plt.title("mask + symavg")
plt.imshow(ycorr_2D_masked_symavg)
plt.axhline(ycorr_2D_masked.shape[0]//2)
plt.clim(vmin,vmax)
plt.xlim(70,130)
plt.ylim(130,70)
```

Out[20]: (130, 70)

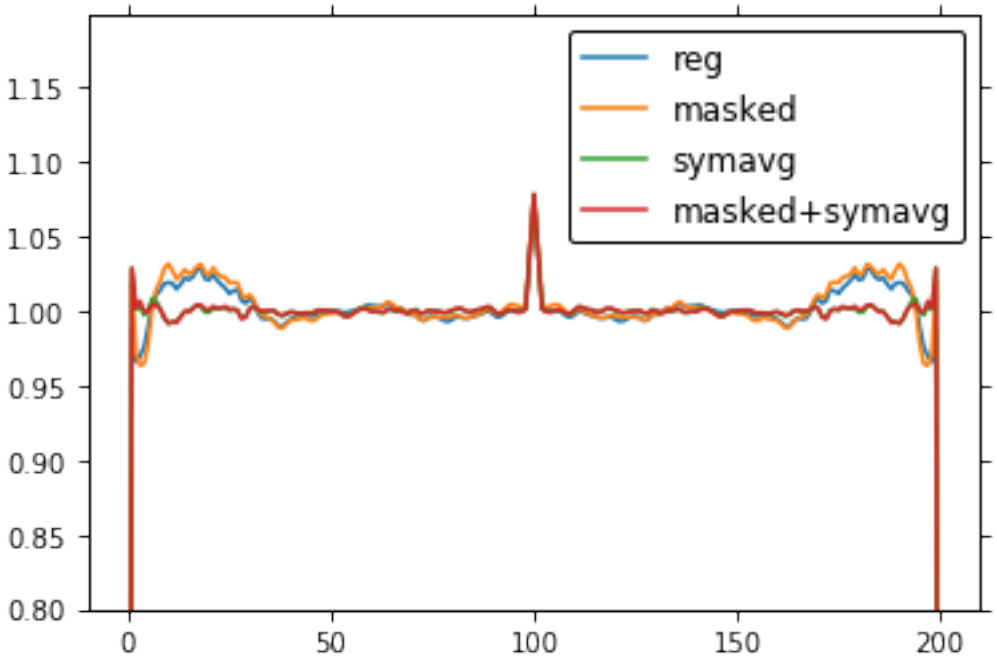


3.3 Correlation Cross sections

```
In [21]: plt.figure(4);plt.clf();
         plt.plot(ycorr_2D[ycorr_2D.shape[0]//2],label="reg")
         plt.plot(ycorr_2D_masked[ycorr_2D_masked.shape[0]//2],label="masked")
```

```
plt.plot(ycorr_2D_symavg[ycorr_2D_symavg.shape[0]//2],label="symavg")
plt.plot(ycorr_2D_masked_symavg[ycorr_2D_masked_symavg.shape[0]//2],label=
plt.ylim(0.8, 1.2)
plt.legend()
```

Out[21]: <matplotlib.legend.Legend at 0x7f0fc462f080>



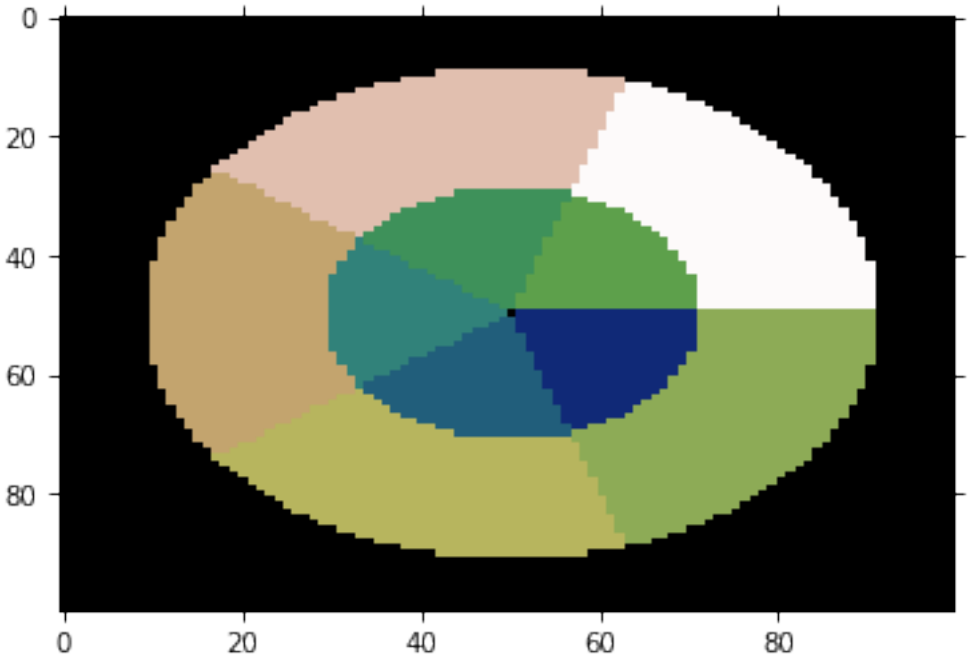
4 3. Try with different id's in different regions of image

```
In [22]: # make id numbers
         edges = ring_edges(1, 20, num_rings=2)
         segments = 5
         x0, y0 = np.array(mask_2D.shape)//2
         maskids = segmented_rings(edges,segments,(y0,x0),mask_2D.shape)

         cc2D_ids = CrossCorrelator(mask_2D.shape, mask=maskids)
         cc2D_ids_symavg = CrossCorrelator(mask_2D.shape,mask=maskids,normalizatior

         ycorr_ids_2D = cc2D_ids(Z)
         ycorr_ids_2D_symavg = cc2D_ids_symavg(Z)
```

4.1 Plot mask

```
In [23]: plt.figure(2);plt.clf();
         plt.imshow(maskids)
```

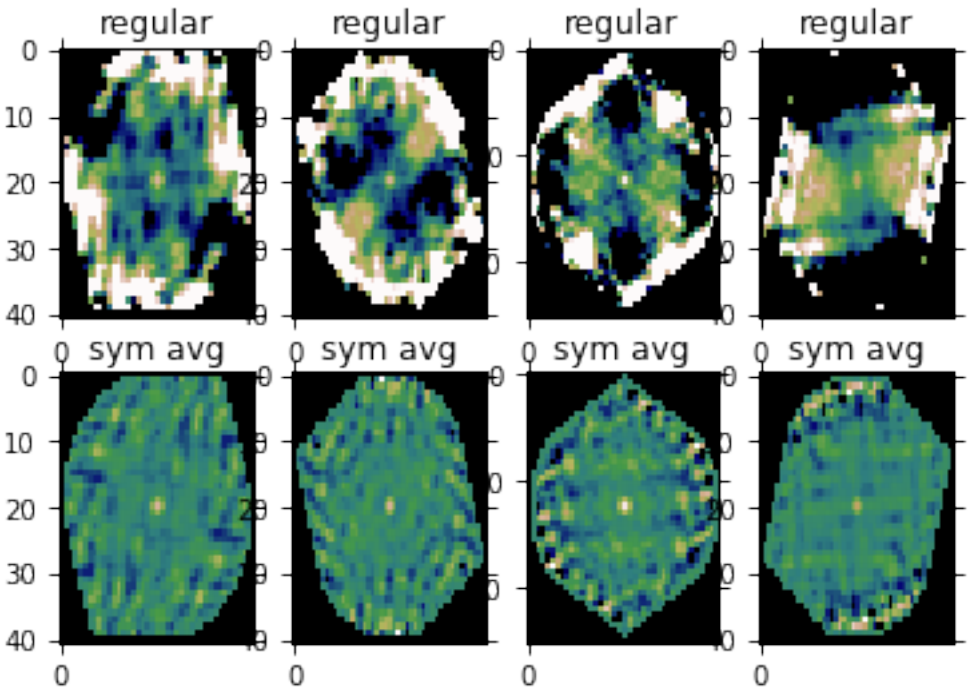
```
Out[23]: <matplotlib.image.AxesImage at 0x7f0fc46b6da0>
```



4.2 plot correlations

Here, we see that without symmetric averaging, the correlations quickly come back at values higher than the point of initial correlation, whereas with symmetric averaging, the result looks more as what is expected, a nice Gaussian like curve centered in image. (Center of image is zero correlation)

```
In [24]: vmin=.95; vmax=1.1

         fig, axes = plt.subplots(2,4)
         ax1 = axes[:len(axes)//2].ravel()
         ax2 = axes[len(axes)//2:].ravel()

         for i in range(len(ax1)):
             plt.sca(ax1[i])
             plt.title("regular")
             plt.imshow(ycorr_ids_2D[i])
             plt.clim(vmin,vmax)

             plt.sca(ax2[i])
             plt.title("sym avg")
             plt.imshow(ycorr_ids_2D_symavg[i])
             plt.clim(vmin,vmax)
```


regular
regular
regular
regular
sym avg
sym avg
sym avg
sym avg

In [ ]: