Lecture 4:
Spectral synthesis and star-formation indicators

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1 Introduction

In this lecture, we will cover how we actually measure the properties of galaxies. This is critical to our understanding of galaxy evolution since, if we can't measure their properties, we can't build up a theory of how they evolve.

2 Measuring galaxy properties

In astronomy, we can only study objects from a distance, normally via the electromagnetic radiation they emit (and, as of 2015, gravitational waves). We can't handle them, look at them from a different angle etc. etc. So how do we measure their physical properties such as mass, composition etc.?

In the case of stars, we have the luxury of binaries that can help us to measure their masses, while the fact that stars are "simply" spherical balls of gas also helps (though, I'm sure Profs. Crowther and Dhillon would disagree!). In the case of galaxies, we don't have such "ideal" scenarios, so we're simply limited to analysing the combined light from their populations of stars.

But, how do we take the light from a galaxy and convert it into a mass, metallicity, star-formation rate, etc.? One of the most important and widely-used means is by spectral synthesis.

2.1 Spectral synthesis

The basic idea of spectral synthesis is to build a model that represents the light from all the stars in a galaxy. If that model accurately reproduces the spectrum (ideally) or colours (more usually) of the galaxy, then we can infer various galaxy properties from that model. For example, if the spectrum of a galaxy was perfectly reproduced by combining the spectra of a billion identical, sun-like stars, then we'd be able to say that the mass of the galaxy is a billion solar masses. In reality, however, it is far more complicated than that since, as you'd expect, galaxies are made up of stars spanning a wide range of masses, ages and metallicities. So, to produce these "synthetic" spectra astronomers have to combine the spectra of lots of different types of stars. Thankfully, we have a recipe to help us constrain the range of stars we include in our models.

2.1.1 Initial mass function

The first part of the recipe requires us to choose an Initial Mass Function (IMF). As with most such "functions" we've met so far, it's really more of a histogram. It is the histogram of stellar masses that a cloud of gas produces the moment it collapses to form stars. It tells us that for every single 10 $M_\odot$ star, there are about a hundred 1 $M_\odot$ stars, and about ten thousand 0.1 $M\odot$ stars. Why is this important? Because the spectrum of a star (and how it evolves, see later) depends hugely on its mass, so it is important that our model includes the relative numbers of stars of different masses.

Finally, while there is some uncertainty in the shape of the IMF and whether it was different at earlier times in the Universe (indeed, whole conferences are dedicated to this topic), it seems that the IMF is fairly universal. As such, astronomers typically assume a constant IMF when doing extragalactic studies. However, since there are various different IMF (e.g., the Salpeter IMF, the Kroupa IMF, the Chabrier IMF), it is important that if you want to compare physical properties you use the same IMF throughout. This is also true if you want to compare your results to another study – it is often the case that astronomers need to "convert" their results to a different IMF. Usually, however, this is simply a case of multiplying the physical parameters (e.g., mass, star formation rate) by a constant factor.

2.1.2 Stellar evolutionary tracks, or isochrones

The IMF gives us the relative numbers of stars of different masses at time $t = 0$. However, all but a tiny fraction of stars in a galaxy will have an appreciable age (i.e., millions to billions of years old). So, to be able to model the stellar population of a galaxy, we need to know how the stars (whose masses are given by the IMF) evolve over time.

Thankfully, how a star evolves is largely dictated by its mass, with a lesser dependency on metallicity. Since we know the mass of every star in our model (from the IMF), we can model how it will evolve across the Herzsprung-Russel (HR) diagram. Of course, more massive stars will travel across the HR diagram more quickly than low mass stars, since they "live fast, die young". A line joining together all the different stars on the HR diagram at a given time, t, is called an *isochrone*; i.e., *iso* - meaning the same, *chrone* - meaning time. At this stage, we can also specify a metallicity dependence.

Provided we know our stellar evolutionary tracks, we will know the positions on the HR diagram of all the stars on our model at any given time.

2.1.3 Stellar spectra

At this stage, we can imagine our population of stars at time t as being represented by a whole load of points on the HR diagram. There will be lots and lots of low mass stars that will have barely evolved, and a handful of high mass stars that will have evolved very quickly (indeed, some of the most massive may even have undergone supernova). To generate the synthetic spectrum of the population, we simply have to sum the spectra of all the individual stars represented by those points.

Thankfully, we have observed spectra for stars across almost all the HR diagram, so we can use real, observed spectra in most cases. In some parts of the HR diagram where there are few stars, however, we sometimes have to resort to synthetic spectra due to a lack of observed spectra.

2.1.4 Star formation histories

Up to this point, we've only considered a single, instantaneous burst of stars. However, it is unlikely that a galaxy will form all its stars in one single burst. Instead, a galaxy will form its stars over a protracted period of time. This will result in a different synthetic spectrum. For example, if we have an extended period of star-formation, then massive young stars (which are hot and therefore emit strongly in the UV and blue part of the spectrum) will continue to be produced. Therefore, the synthetic spectrum of a continuous burst will stay bluer for longer than an instantaneous burst.

How do we model a continuous period of star-formation? Well, all we need to do is take lots of instantaneous bursts going off one after another. We can even modulate the size of each burst (i.e., how many stars are produced in a given burst) according to a *star formation history*. For example, we could model a constant, continuous episode of star formation by a set of bursts, one after another, which all produced the same number of stars. Alternatively, we could model an exponentially declining episode as a series of bursts, each containing:

$$N_{\text{Stars}}(t) = N_{\text{Stars}}(t=0)\exp\left(-\tau t\right) \tag{1}$$

stars at time t (where τ is a constant that described how quickly the rate of star formation falls away with time). Then, we simply treat each burst separately as described above.

2.1.5 A gardening analogy

Sometimes an analogy can really help to explain an idea; I like this gardening one I came up with for generating synthetic spectra (it doesn't contain caterpillars!).

Let's say I have a garden and some seeds. It's March, I'm just about to plant my seeds, and I want to have an idea of what my garden will look like in June, July and August.

We can think of the IMF as the relative numbers of different types of seeds. Say 100 daisies, 50 marigolds, 20 sunflowers. The equivalent of stellar evolutionary tracks would be how quickly each variety grows; sunflowers grow really quickly, marigolds more slowly. Finally, the stellar spectra would be the colours of the flowers: yellow sunflowers, orange marigolds, white daisies. Given the relative numbers of seeds, their growth rates and their colours, I can predict what colours I will have in my garden during each month of the summer. The whole, *fabulous* floral effect would be the synthetic spectrum.

Finally, if I want to, I can plant lots of different sets of seeds each week to prolong the flowering season. This would be the equivalent of the star-forming history.

2.2 Model fitting

With modern computers, we can genarate a population of stars and produce its synthetic spectrum extremely quickly (i.e., in a few milliseconds). As such, given an observed spectrum/photometry of a real galaxy, what astronomers typically do is model a whole range of different metallicities, ages, and star-formation histories and see what combination reproduces the data the best. This iterative trial-and-error is simply a form of model fitting. Indeed, astronomers often take a χ-squared minimisation approach to find the best-fitting synthetic spectrum to observed spectra or photometry.

2.3 Caveats to using spectral synthesis

In principle, spectral synthesis and model fitting is relatively straight-forward. As such, it's relatively easy to use spectral synthesis to measure galaxy properties. However, just getting a measurement is only part of the effort. We must also consider how reliable these measurements are. We must consider what could affect the reliability of results from spectral synthesis and model fitting. Here are considerations:

- **Dust**: Since blue light is more readily absorbed by dust, if there is intervening dust then it will change the shape of the observed spectrum. Dust can make a population of stars look older than it is, because the blue light from the young stars is absorbed.
- **Uncertainties in post-MS tracks**: Any uncertainties in the paths that stars take across the HR diagram as they evolve will result in uncertainties in the resultant synthetic spectrum. In particular, there is a huge debate on how post-asymptotic giant branch stars evolve (especially if they're pulsating) because they are *really* bright so can contribute a lot to a galaxy's total spectrum.
- **Incomplete spectral libraries**: While we have taken spectra for many millions of stars, there remains some parts of the HR diagram that remian poorly sampled. In particular, since most of the stars in our region of the Milky Way have similar metallicities, we only have a few spectra of non-solar metallicity stars (and these are biased to more luminous stars). Further, since very massive stars don't live very long, they tend to be rare, making it difficult to find ones to measure their spectra.
- **Uncertain IMF**: We've already touched upon this point. Uncertainties in the IMF introduce uncertainties in the range of stellar masses, which subsequently introduces uncertainties into measured physical properties.
- **Uncertain star-forming histories**: One can imagine that a galaxy can have multiple episodes of star-formation during its evolution, possible with different regions having different histories. While, in priciple, we could model such complex histories, in reality there is not enough data to discriminate between such complex models. As such, we typically assume simple models (single burst, continuous, exponentially falling), which introduces uncertainties.

3 Measuring integrated star formation rates

One of the most important measurements we can make of a galaxy is its star formation rate (SFR). This effectively tells us how quickly a galaxy is growing, and is thus a fundamental aspect to understanding how today's galaxies have formed. With SFRs, we can ask questions like: When did galaxies grow fastest? Are today's galaxies growing faster or slower than previously? What causes galaxies to grow more quickly?

By modelling the stellar content of a galaxy, spectral synthesis *will* give us a measure of the SFR of a galaxy. Often, however, we don't have to go to such lengths, as there are other ways to measure accurate SFRs that rely on observations in just one band (i.e., UV, infrared) or a single emission line.

All of the single-band/emission line measures of SFR rely on one key fact: **that the hottest, most massive stars die young**. The most massive stars ($> 5M_\odot$) will live for "only" around 100 million years (compared to 10 *billion* years for the Sun). So, if we can count the number of massive stars in a galaxy, then we can calculate the average rate of *massive* star formation in that galaxy over the past 100 million years (which is pretty instantaneous for a galaxy!). However, the average rate of massive star formation *is not* the same as the actual SFR; recall that the IMF tells us that for every massive star formed, there are many, many more lower mass stars produced. So, to calculate the true SFR, we take the number of massive stars, then multiply it by the ratio of the total mass of stars per the total number of stars with $> 5M_\odot$, i.e.:

$$\mathrm{SFR} = \frac{N_{\mathrm{M}>5\mathrm{M}_\odot}}{10^8 \mathrm{yr}} \times R_{\mathrm{Stars}}(\mathrm{IMF}) \tag{2}$$

where, $N_{\mathrm{M}>5\mathrm{M}_\odot}$ is the *observed* number of stars with masses greater than 5 $M_\odot$ and $R_{\mathrm{Stars}}(\mathrm{IMF})$ is given by:

$$R_{\mathrm{Stars}}(\mathrm{IMF}) = \frac{\text{Total mass of all stars}}{\text{Number of } > 5\mathrm{M}_\odot \text{ stars}} \tag{3}$$

which is calculated from our chosen IMF. As you should be able to see, the SFR is in units of $M_\odot$ yr^{-1}.

So, to calculate a galaxy's SFR (assuming a given IMF), all we need to do is measure how many stars with $M > 5$ $M_\odot$ it contains. How can we do that?

3.1 The UV luminosity of a galaxy

Massive stars are very hot, so they produce a lot of ultraviolet radiation. In fact, in the absence of an AGN, massive stars are pretty much the only sources of UV radiation in a galaxy. As such, measuring the UV luminosity of a galaxy provides a measure of the number of > 5 $M_\odot$ stars in a galaxy. With this in mind, astronomers have calculated a simple conversion from UV luminosity to total SFR:

$$\mathrm{SFR}\ (\mathrm{M}_\odot\ \mathrm{yr}^{-1}) = 4 \times 10^{-41} L(\mathrm{FUV})\ (\mathrm{erg\ s}^{-1}\ \mathrm{A}^{-1}) \tag{4}$$

As with all the other SFR indicators in this section, this takes into account the increased luminosity of high mass stars relative to the Sun, and for the corrections mentioned in §3 and summarised in Eqns. 2 and 3, i.e., it is the "true" rate of star formation in the galaxy, averaged over the past ~ 100 Myr.

The main problem with using the UV emission to measure SFRs is that it is readily affected by dust. Attempts can be made to correct for dust extinction, but they rely on various assumptions and even a small amount of dust can strongly affect measured UV flux.

3.2 The Hα luminosity of a galaxy

Because hot, massive stars emit strongly in the UV part of the spectrum, they emit a lot of photons with wavelengths shortward of 912 Å. Photons shortward of this wavelength are energetic enough to ionise Hydrogen. When this ionised hydrogen recombines with free electrons, the electrons cascade through the atomic levels, emitting photons as they drop to lower energyies. One of the strongest lines produced by this process is the Balmer-alpha (i.e., Hα) emission line at 6563 Å, which corresponds to an electron transition from $n = 3$ to $n = 2$. Measuring the luminosity of the Hα line therefore provides a measure of the number of young stars in a galaxy, from which we can calculate the SFR:

$$\mathrm{SFR}\ (\mathrm{M}_\odot\ \mathrm{yr}^{-1}) = 7.9 \times 10^{-42} L(\mathrm{H}\alpha)\ (\mathrm{erg\ s}^{-1}) \tag{5}$$

Ionising Hydrogen requires particularly energectic photons, which only the most massive, hottest stars are capable of producing. Indeed, stars with masses below about 10 $M_\odot$ are unable to produce sufficient numbers of high energy photons to ionise Hydrogen. This means that Hα traces only the very hottest, most massive stars that live for *only* around 10 Myr. As such, the Hα line gives a measure of the SFR averaged over only the past 10 million years. It is pretty much the most "instantaneous" measure of SFR that we can get.

There are a few of caveats that come with using Hα to measure SFR. Firstly, like the UV, Hα is affected by extinction due to dust. However, it is not as badly affected by the UV and is easier to correct-for using the "Balmer decrement" (the intrinsic ratio of the Hα to Hβ lines is 3:1, so if we measure, say, a 4:1 ratio, we know it is being extinguished by dust, and can correct for it). A second caveat is that some of the ionising photons may escape the gas cloud without hitting a H-atom, meaning we'll measure a lower SFR rate. Thirdly, since AGN are also prodigious producers of Hα, it is difficult to use this to measure the SFR of galaxies hosting AGNs (although these are the minority of galaxies).

3.3 The infrared luminosity of a galaxy

Most stars form out of gas and dust. When the massive stars start to shine, their heat warms the dust that surrounds them and the dust re-radiates this warmth in the form of infrared radiation. As with Hα and UV emission, it is only the most massive (> 5 M$_\odot$) stars that are hot enough to warm the surrounding dust to produce this infrared radiation. As such, by measuring the infrared luminosity arising from a galaxy, we can get a measure of the number of young stars it contains, and thus its SFR:

$$\mathrm{SFR}\ (\mathrm{M}_\odot\ \mathrm{yr}^{-1}) = 1.8 \times 10^{-44} L(\mathrm{FIR})\ (\mathrm{erg\ s}^{-1}) \tag{6}$$

Since stars with masses greater than about 5 M$_\odot$ are capable of heating the dust, the infrared provides a measure of a galaxy's SFR averaged over the past ~ 100 Myr.

The benefit of using infrared wavelengths is that, unlike UV and Hα is it largely unaffected by dust obscuration. However, since infrared wavelengths are readily absorbed by the Earth's atmosphere, we need to use expensive space missions to observe at these long wavelengths. The constraints on telescope size that this introduces, combined with the long wavelengths, mean that infrared observations typically suffer from poor spatial resolution.

4 Learning objectives

In this lecture, we have covered how we actually measure the physical properties of galaxies. This is crucial if we want to be able to build-up a coherent theory of galaxy exolution. The key learning objectives you should take from this lecture are:

- Have an understanding of how spectral synthesis modelling can be used to determine the masses, SF histories and metallicities of galaxies.
- Have an understanding of the main ingredients and uncertainties in spectral synthesis modelling.
- Appreciate of the main techniques used to determine the integrated SFR of galaxies and their pros and cons