Introduction
Financial Derivatives

Derivatives

A derivative is a financial product whose value depends on the values of other more basic underlying assets.

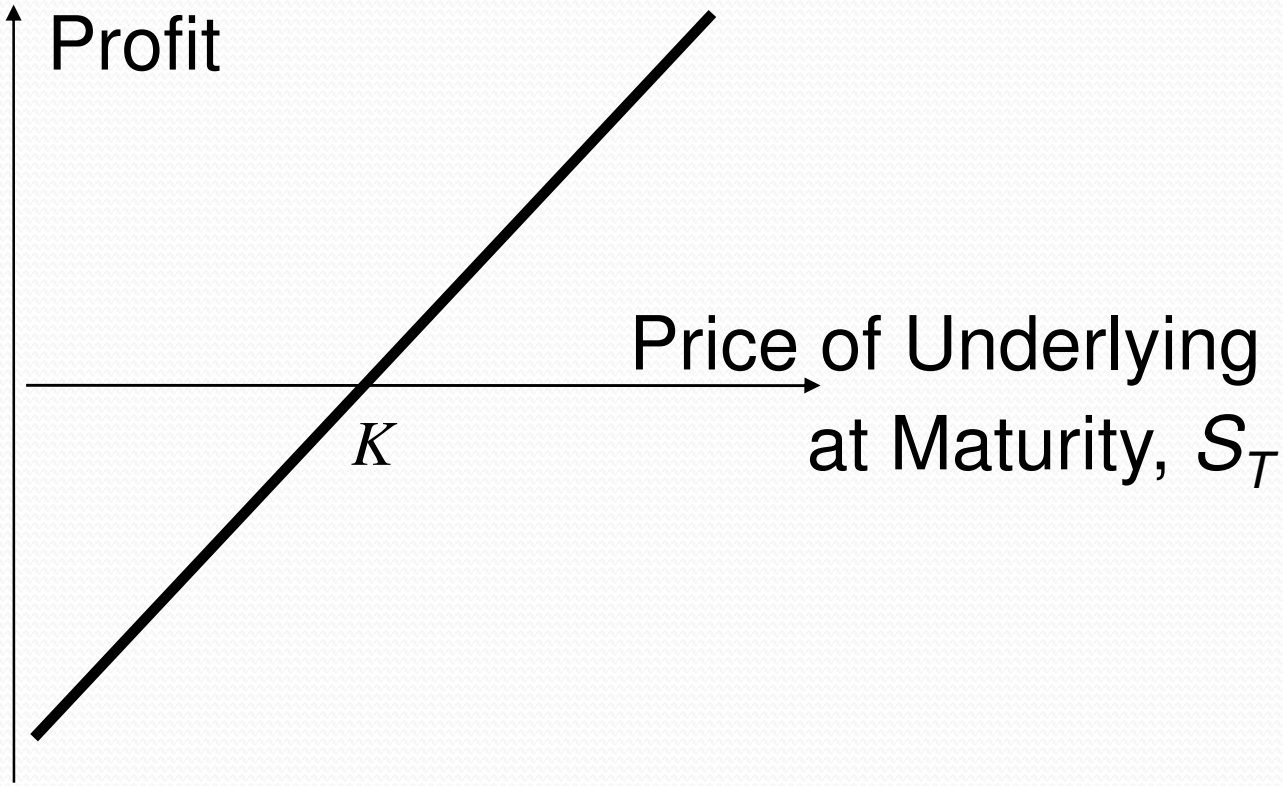
Examples:

- **Forward contract on a stock: a contract written today, to buy a stock *at time* = *T* at strike price K.**
- **Call option on a stock: a contract written today, which gives the holder the right but not the obligation to buy a stock at time t=T at strike price K.**

Terminology

- The party that has agreed to buy has what is termed a *long* position
- The party that has agreed to sell has what is termed a *short* position

Profit from a Long Forward Position



Forward Price

- The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero)

- The forward price may be different for contracts of different maturities

Arbitrage

- Derivative pricing models are based on no-arbitrage principle
- What is an arbitrage opportunity?

Arbitrage Example

- A stock price is quoted as £100 in London and $172 in New York
- The current exchange rate is 1.7500
- What is the arbitrage opportunity?

Gold: An Arbitrage Opportunity?

- Suppose that:
  - The spot price of gold is US$300
  - The 1-year forward price of gold is US$300
  - The 1-year US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

Gold: An Arbitrage Opportunity?

If you own Gold

- Sell Gold for the spot price
- Enter the Forward Contract
- Invest the proceeds at interest rate 5% for one year
- Buy gold back after 1 year

Pricing Forward Contracts

Determine the strike that will make the value of the forward contract equal to zero

- Hedge the forward contract : borrow S_0 today and buy the stock
- At time=T: Sell stock for K to holder of the forward contract and pay $S_0 e^{rT}$
- No arbitrage: K=$S_0 e^{rT}$

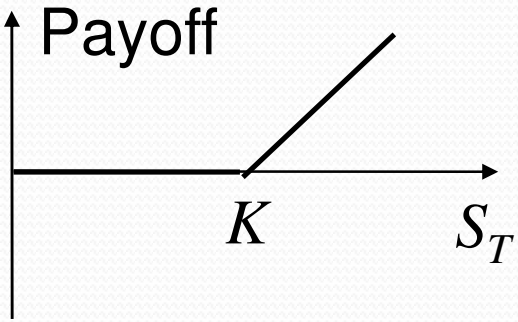
Options

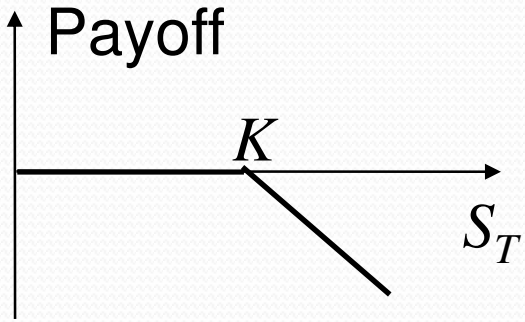
- A call is an option to buy a certain asset by a certain date for a certain price (the strike price)
- A put is an option to sell a certain asset by a certain date for a certain price (the strike price)

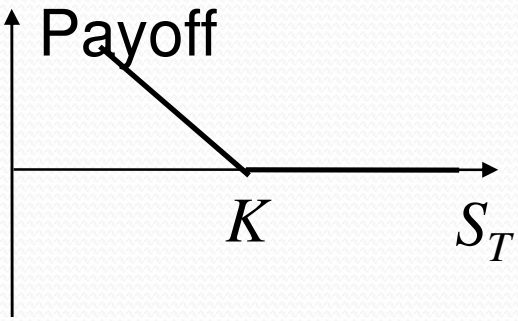
Payoffs from Options

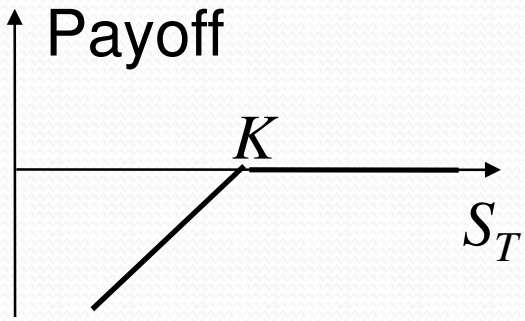
What is the Option Position in Each Case?

K = Strike price, S_T = Price of asset at maturity









Specification of Exchange-Traded Options

- Expiration date
- Strike price
- European or American
- Call or Put (option class)

Terminology

Moneyness :

- At-the-money option
- In-the-money option
- Out-of-the-money option

Put-Call Parity

- Consider the following 2 portfolios:
  - Portfolio A: European call on a stock + PV of the strike price in cash
  - Portfolio C: European put on the stock + the stock
- Both are worth MAX(S_T , K) at the maturity of the options
- They must therefore be worth the same today
  - This means that $c + Ke^{-rT} = p + S_0$

Ways Derivatives are Used

- To hedge risks
- To speculate (take a view on the future direction of the market)
- To lock in an *arbitrage* profit

American Options

- A European option can be exercised only at the end of its life
- An American option can be exercised at any time

American vs European Options

An American option is worth at least as much as the corresponding European

$$C \geq c$$
$$P \geq p$$

Notation

- c : European call option price
- p : European put option price
- S_0 :Stock price today
- K : Strike price
- T : Life of option
- σ: Volatility of stock price
- C : American Call option price
- P : American Put option price
- S_T :Stock price at option maturity
- D : Present value of dividends during option's life
- r : Risk-free rate for maturity T with cont comp

Effect of Variables on Option Pricing

Variable	c	p	C	P
S_0	+	−	+	−
K	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+