Lab Assignments Computational Finance

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These assignments can be done in groups of two students. Reports with a *clear description of the assignment, the methods, the results and discussion* should be submitted *before the deadlines.* You are free to choose the programming language/environment in which you would like to write your computer programs. If you have questions about the assignments do not hesitate to contact the teaching assistant or the lecturer.

Assignment 2: Monte Carlo (MC) Methods in Finance

Part I: Basic Option Valuation

As we derived in the class, for the purpose of pricing options, we can assume that the stock price *S* evolves in the risk neutral world:

$$dS = rSdt + \sigma SdZ \quad (1)$$

where *r* is the risk free return, *σ* is the volatility, and *dZ* is the increment of a Wiener process. Let the expiry time of an option be *T*, and let

$$N = \frac{T}{\Delta t}$$

$$S^n = S(n\,\Delta t)$$

Then, given an initial price S^0, *M* realizations of the path of a risky asset are generated using the algorithm (Euler method)

$$S^{n+1} = S^n + S^n(r\Delta t + \sigma\varphi\sqrt{\Delta t})$$

where *φ* is a normally distributed random variable with mean zero and unit variance.

For special cases of constant coefficients, we can avoid time stepping errors for geometric Brownian motion, since we can integrate equation (1) exactly to get

$$S_T = S_0 \exp^{(r-0.5\sigma^2)T+\sigma\sqrt{T}Z} \quad (2)$$

The price of an option can be calculated by computing the discounted value of the average pay-off, i.e.

$$V(S^0, t=0) = e^{-rT} \frac{\sum_{m=1}^{M} payoff^m(S^N)}{M}$$

Write a computer program for the Monte Carlo method. Price European call and put options with (*T = 1 year, K = 99, r = 6%, S = 100 and σ = 20%*). Carry out convergence studies by increasing the number of trials. How do your results compare with the results obtained in assignment 1? Explain Equation 2. Perform numerical tests for varying values for the strike and the volatility parameter. What is the standard error of your estimate and what does this tell you about the accuracy?

Part II: Estimation of Sensitivities in MC

A. The hedge parameter δ in Monte Carlo can be estimated by the bump-and-revalue method. Calculate the delta by applying the following methods:
   - Use different seeds for the bumped and unbumped estimate of the value;
   - Use the same seed for the bumped and unbumped estimate of the value;

   Compare your results which the values obtained in Assignment I.

B. Consider a digital option which pays 1 euro if the stock price at expiry is higher than the strike and otherwise nothing. Calculate the hedge parameter δ using the method used in A. Explain your results and use the sophisticated methods discussed in the lectures to improve your results.

Part III: Variance Reduction

A major drawback of Monte Carlo simulation is that a large number of realizations are typically required to obtain accurate results. Therefore techniques to speed-up the simulations are quite useful. In this assignment students will work on different variance reduction techniques and quasi-Monte Carlo methods to accelerate numerical valuation of financial derivatives.

Assignment A: Variance Reduction by Antithetic Variables

The most straight-forward technique is to use antithetic variables.

A1. How does the error in MC depend on the number of simulation trails? Explain your result?

A2. Explain how variance reduction by antithetic variables works. Derive an expression for the variance of the point estimate when antithetic variables are used. Apply this technique to calculate the price of a European call option on a stock. Study the performance of this technique for different parameter settings (number of trails, volatility and strike).

A3. Consider an Asian option on a stock. In the case of Asian options based on *arithmetic* averages, the payoff depends on the (arithmetic) average price in a window of N observations, A_N, which is defined as,

$$A_N = \frac{\sum_{i=1}^{i=N} S_i}{N}$$

The stock price at observation time t_i is S_i. Typically, the interval between observations is one day.

The payoff of an Asian fixed strike call option is,

$$\mathrm{Max}(A_N - K, 0)$$

with K the strike.

What is the performance for different number of trials, volatility levels and N? Explain your results.

Assignment B: Variance Reduction by Control Variates

A description of the control variates technique is introduced in Ref. 1.

B1. Explain how this strategy works.

For the control variates technique an accurate estimate of the value of an option that is similar to the one that you would like to price is required. For valuation of an Asian option based on *arithmetic averages* one can use the value of an Asian option based on *geometric averages*. This case can be solved analytically.

B2. Derive an analytical expression for the price of an Asian option that is based on geometric averages.

Hint: First, recall that the geometric average is defined as:

$$\tilde{A}_N = \left(\prod_{i=1}^{N} S_i\right)^{1/N}$$

Use the following property:

$$\prod_{i=1}^{N} S_i = \frac{S_N}{S_{N-1}}\left(\frac{S_{N-1}}{S_{N-2}}\right)^2\left(\frac{S_{N-2}}{S_{N-3}}\right)^3 \cdots \left(\frac{S_2}{S_1}\right)^{N-1}\left(\frac{S_1}{S_0}\right)^N S_0^{\,N}$$

Show that the following identity is true:

$$\ln\left(\frac{\left(\prod_{i=1}^{N} S_i\right)^{1/N}}{S_0}\right) = Normal\_Distribution\left((r - \frac{1}{2}\sigma^2)\frac{(N+1)}{2N}T, \sigma^2\frac{(N+1)(2N+1)}{6N^2}T\right)$$

Note that this problem is very similar to derivation of the Black-Scholes formula for the price of an option on a stock. This has been derived in detail in the lectures. Using this similarity, it is straightforward to derive the analytical Black-Scholes formula for the Asian option? Check your analytical expression by comparing with values obtained by using Monte-Carlo simulations.

B3. Apply the control variates technique for the calculation of the value of the Asian option based on the arithmetic average. Study the performance of this technique for different parameter settings (number of trails, strike, number of time-point used in the averaging etc).