

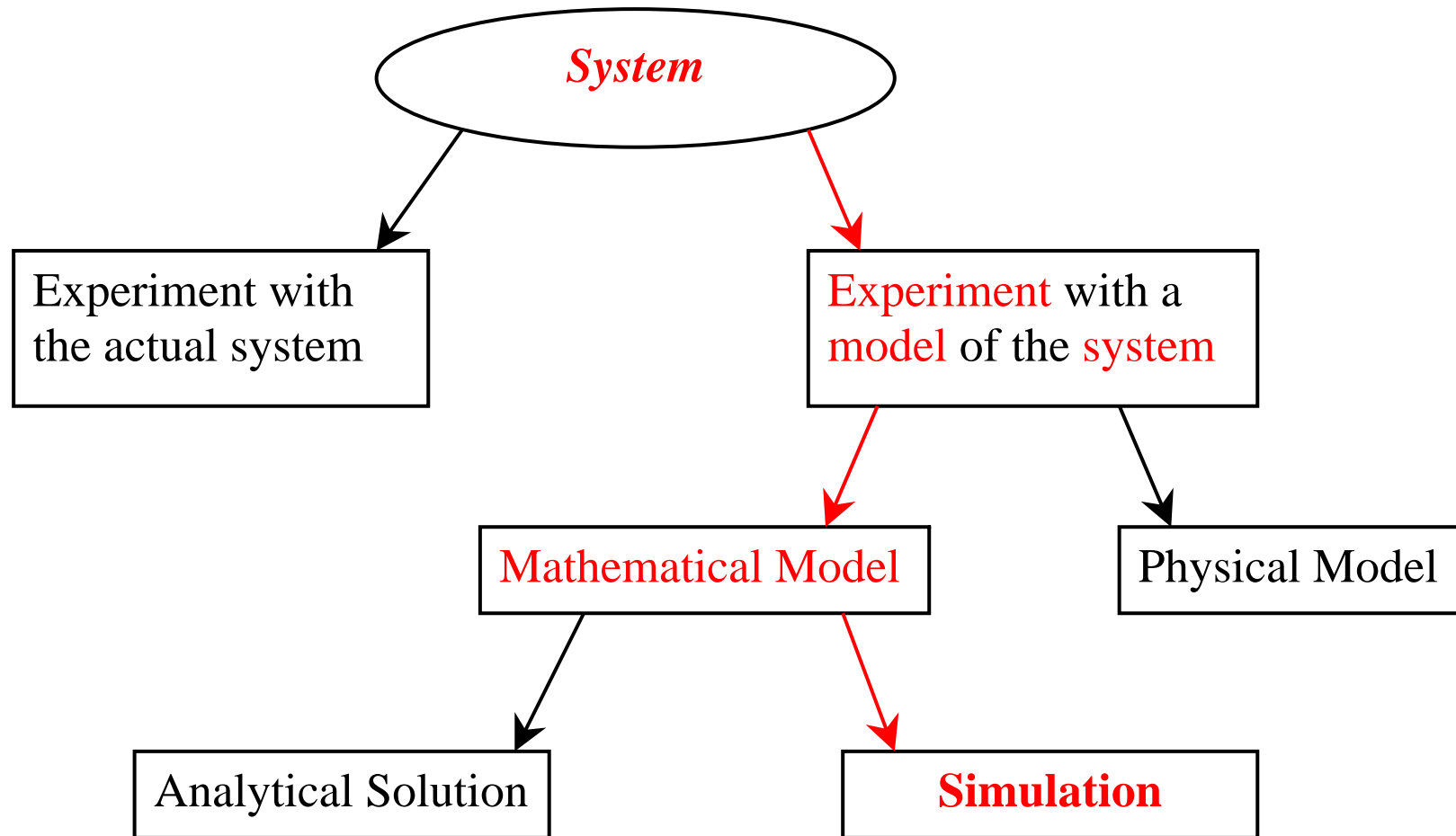


Modeling and Simulation / Distributed Computing

- A quick overview



Ways to Study a System



Definitions - 1

What is a **SYSTEM** ?

“A system is a potential source of data”

Bernard Siegler, *Theory of Modeling and Simulation*, 1976.

What is an **EXPERIMENT** ?

“An experiment is the process of extracting data from a system by exerting it through its inputs.”

François Cellier, *Continuous System Modelling*, 1990.

Definitions - 2

What is a **MODEL**?

“ A model (M) for a system (S) and an experiment (E) is anything to which E can be applied in order to answer questions about S . ”

Marvin Minski, *Models, Minds, Machines*, 1965.

Note that ...

- A model is not necessarily a computer program !
- Here we concentrate on models that can be expressed as computer programs, the so-called *Mathematical Models*.
- By definition, a model can be qualified as a system, which allows to cut out smaller pieces to generate a new model (*hierarchy of models*)
 - Jack Kleijnen, “Concept of Meta-Models,” in *Progress in Modelling and Simulation*, 1982.
- A model is always related to the tuple *System* and *Experiment*.
- Model validation *always* relates to an experiment to be performed on a system.



Definitions - 3

What is a **SIMULATION**?

“A simulation is an experiment performed on a model.”

Granino Korn and John Wait, *digital continuous system simulation*, 1978.



Note that ...

- A mathematical simulation is a coded description of an experiment with a reference to the model to which this experiment is applied.
- Note the (important) separation between model description and experiment description.
- Also note the potential danger of this separation; it is very easy to apply an experiment to a model for which the model is not *valid*.

The Dangers of Simulation

- Both it's strength and weakness: **generality** and **ease of its application...**

“All too often, simulation is a love story with an unhappy ending. We create a model of a system, and then fall in love with it. Since love is usually blind, we immediately forget all about the experimental frame, we forget that this is *not* the real world, but that it represents the world only under a very limited set of experimental conditions (we become ‘model addicts’).”

F. Cellier, *Continuous System Modeling*, 1990

Why is Modeling Important ?

“Modeling means the process of organizing knowledge about a given system”

Bernard Ziegler, *Multifaceted Modeling and Discrete Event Simulation*, 1984

“... it can thus be said that modeling is the single most central activity that unites *all* scientific and engineering endeavors. While the scientist is happy to simply *observe* and *understand* the world, i.e., create a model of the world, the engineer wants to *modify* it to his advantage. While science is all *analysis*, the essence of engineering is *design*..”

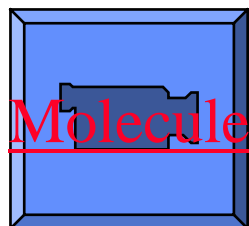
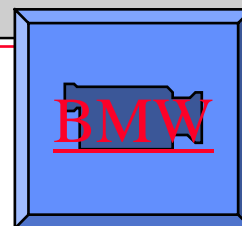
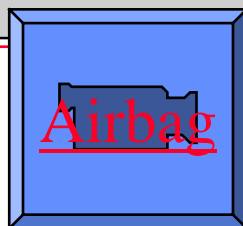
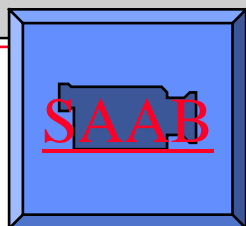
“... *simulation* can be used not only for analysis (direct problems) but also for design (inverse problems).”

François Cellier, *Continuous System Modelling*, 1990

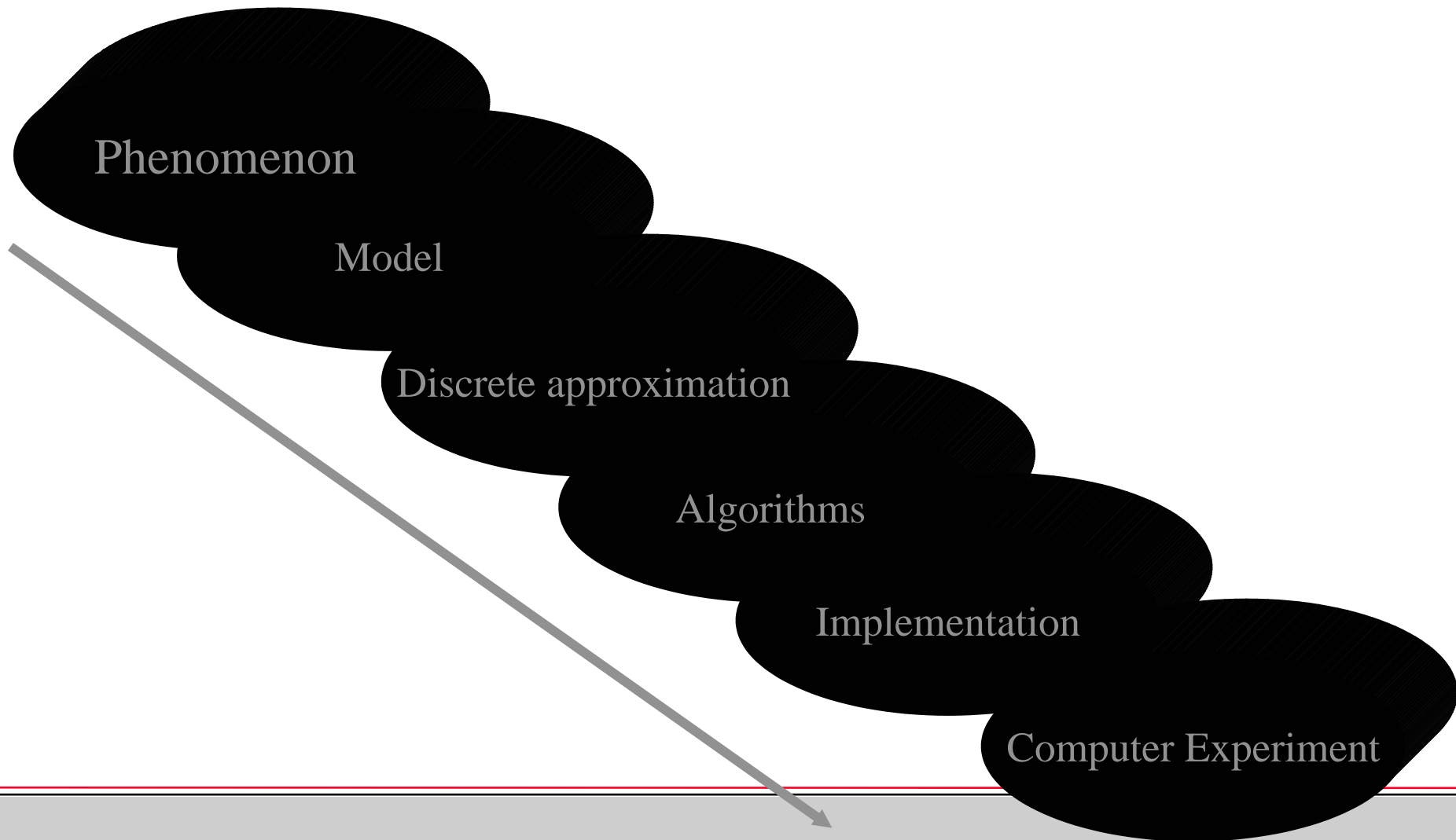


Reasons to use Simulation

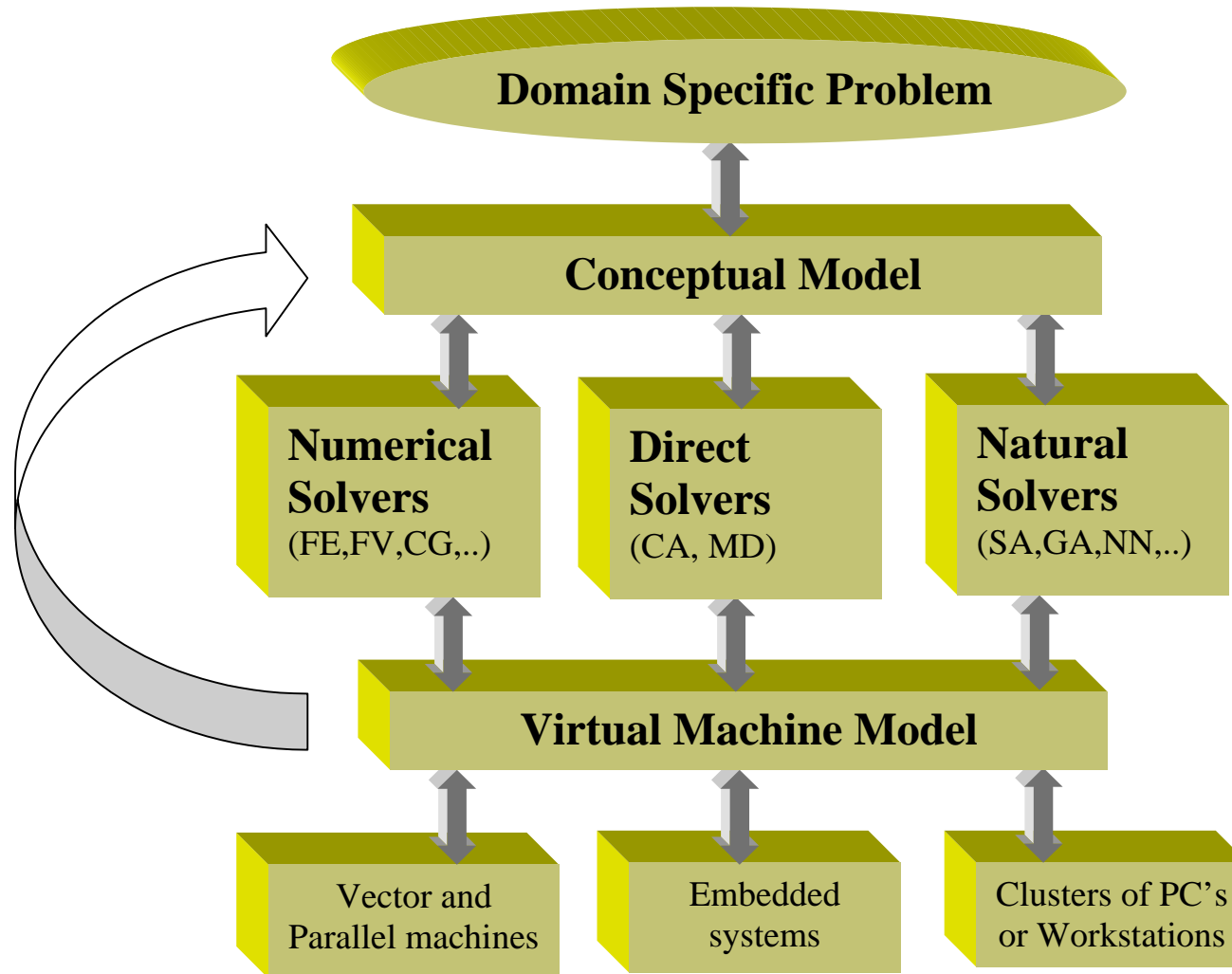
- The physical system is not available
- The experiment may be dangerous
- The cost of the experiment is too high
- The time constants of the system are not compatible with those of the experimenter
- Control variables may be inaccessible
- Suppression of disturbances
- Suppression of second-order effects



The computer experiment I



Modeling and Simulation Cycle



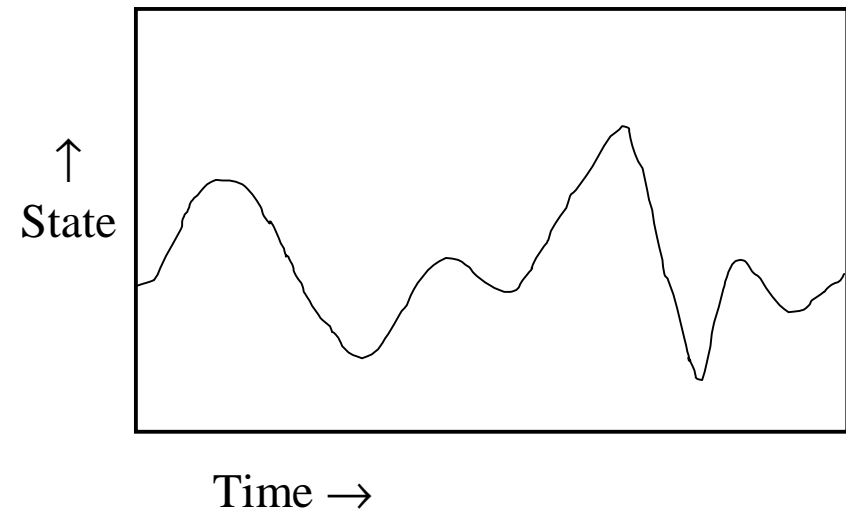


Validation and Verification

- Validation
 - Check the **validity** of the model.
- Verification
 - Check the **correctness** of the simulation.

Types of Mathematical Models I

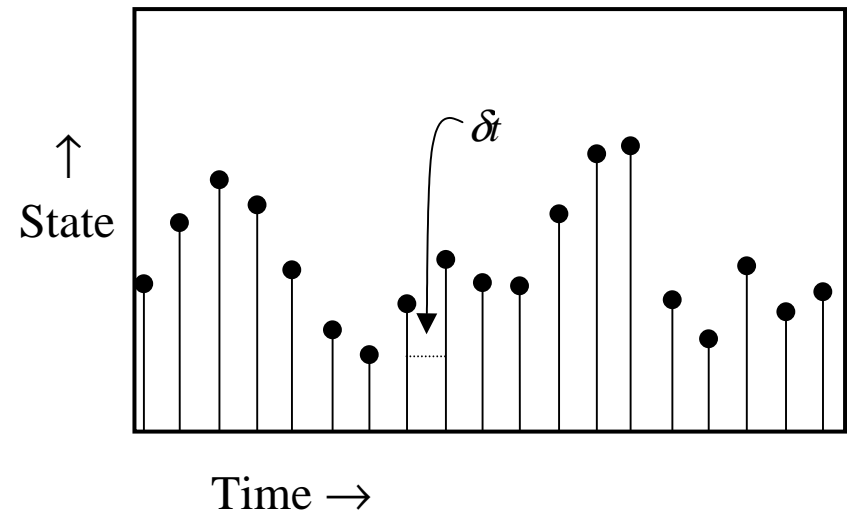
- Continuous Time model
 - State variables change continuously over time.
 - Sets of differential equations.



Types of Mathematical Models II

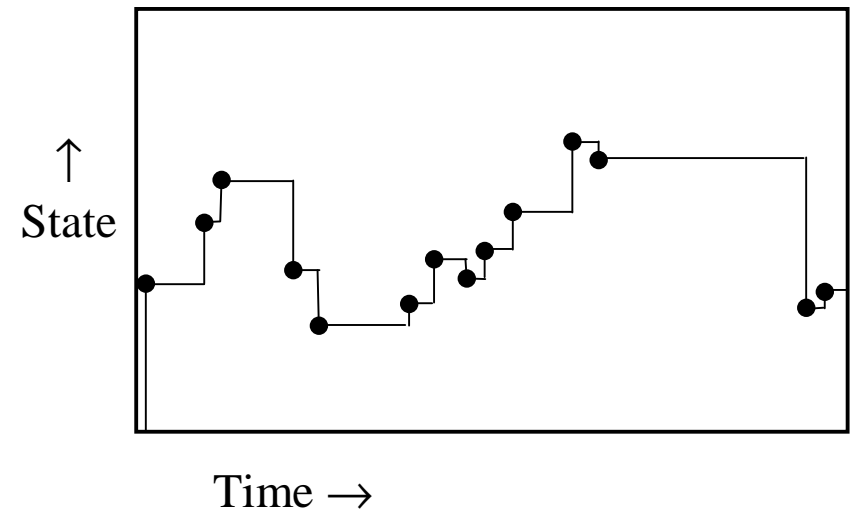
- **Discrete Time model**

- State variables change at discrete time intervals.
- Sets of difference equations.
- Can occur naturally (e.g. population dynamics), or in engineering.
- Discretized versions of continuous models.



Types of Mathematical Models III

- **Discrete Event** model
 - State variables change at discrete events.
 - Paradoxically, both the time axis and the state axis in Discrete Event Models are continuous.
 - In a finite time span only a finite number of changes may occur.



Overview Mathematical Models

Mathematical Models

Continuous Time

Example population dynamics.

Discrete Time

In many cases discretized versions of continuous time models.

But also inherent discrete time models like Cellular Automata

Discrete Event

Example waiting queue problems



- Within a *finite* time span, the state variables change their values *infinitely* often
- Represented by: Set of differential Equations
 - Lumped parameter models: ODE
 - Distributed parameter models: PDE

Time Driven

- the simulation clock is advanced in increments of exactly Δt time units
- the time step Δt is small enough to capture every event in the system



Event Driven

- **Step 1: The simulation clock is initialized to zero and the times of occurrence of future events are determined.**
- **Step 2: The simulation clock is advanced to the time of the occurrence of the most imminent (i.e. first) of the future events.**
- **Step 3: The state of the system is updated to account for the fact that an event has occurred.**
- **Step 4: Knowledge of the times of occurrence of future events is updated and the first step is repeated.**



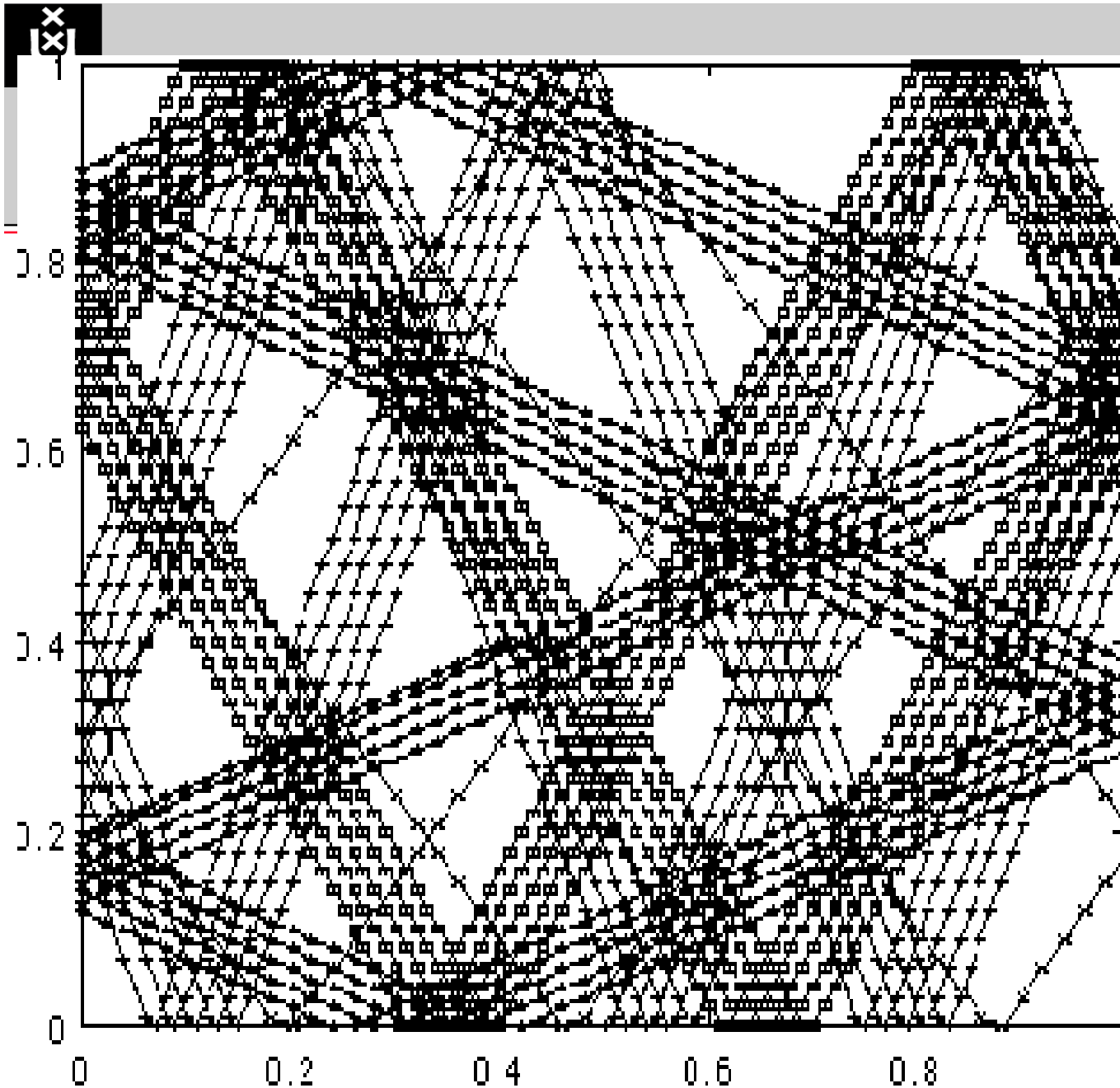
Billiard-ball example

- Gas dynamics of 4 gas molecules, $v=\text{const.}$
- 2D kinetic energy conservation
- Consider Time and Event driven:



Billiard-ball Time Driven Simulation

```
input stop, simulation time,  $\Delta t$ ;  
initialize balls with x,y position and velocity vx,vy  
in x,y direction;  
time = 0.0;  
while (time < stop simulation)  
{  
  for each ball do {  
    x +=  $\Delta t$  * vx;  
    y +=  $\Delta t$  * vy;  
    if (x == 0) vx = -vx;  
    if (y == 0) vy = -vy;  
  }  
  time +=  $\Delta t$ ;  
}
```



Result Time Driven

$t = 0.001$

1000 state evaluations



Billiard Balls event driven I

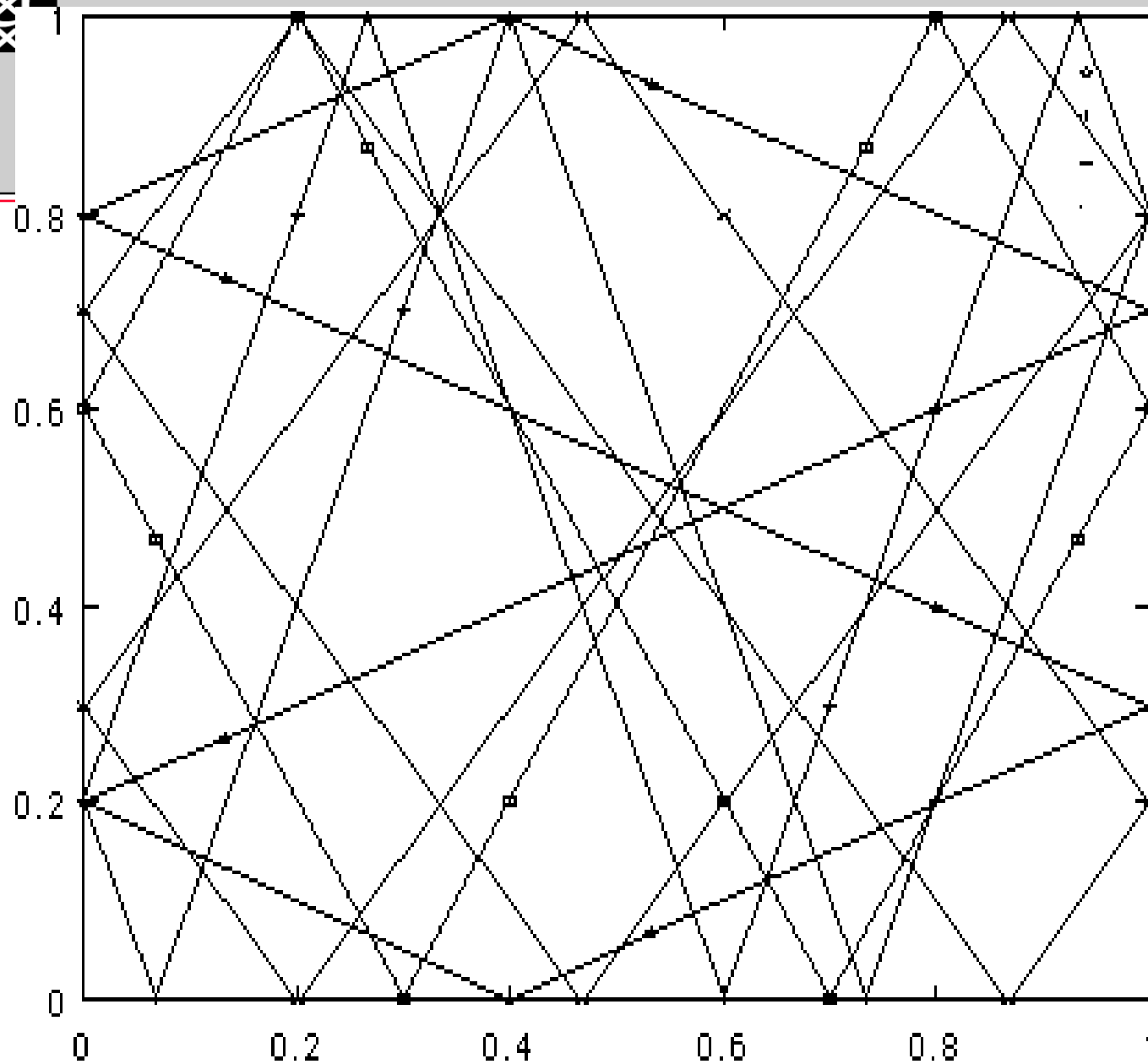
```
input stop_simulation_time;
initialize balls with x,y position and velocity vx,vy in
x,y direction;
for each ball do {
    impact_time = collision_time(ball);
    schedule(MOVE, impact_time, ball);
}
prev_time = 0.0;
while (time() < stop_simulation_time) {
    next_event(&event, &ball);
    switch(event) {
    case MOVE:
        update_positions(time() - prev_time);
        impact_time = collision_time(ball);
        schedule(MOVE, impact_time, ball);
        prev_time = time();
        break;
    }
```



```
collision_time(ball)
{
    if (vx >= 0) {
        t0 = (1 - x) / xv;
    }
    else {
        t0 = -x / xv;
    }
    if (vy >= 0) {
        t1 = (1 - y) / yv;
    }
    else {
        t1 = -y / yv;
    }
    return min(t0, t1);}
```



```
update_positions( $\Delta t$ )
{
    for each ball do {
         $x += \Delta t * vx$ ;
         $y += \Delta t * vy$ ;
        if ( $x == 0 \mid\mid x == 1$ )
             $vx = -vx$ ;
        if ( $y == 0 \mid\mid y == 1$ )
             $vy = -vy$ ;
    }
}
```



Result Event Driven

60 state evaluations



Some Observations

- If the time between the succeeding events becomes small, time driven is preferred. This is the *frequency* parameter.
- The overhead per event (i.e., the state update) is larger with event-driven simulation. This is referred to as the *event overhead*.
- The amount of work between any two events plays an important role. This we call the *granularity* of the system.