

# **Three Essays on Conflict and Financial Markets, and Political Methodology**

by

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## Biographical Sketch

The author was born in Pasadena, California. He attended Dartmouth College, and graduated with a Bachelor of Arts degree in Economics and Government in 2004. He worked as a research assistant and economist in the Money and Payments Studies group in the Economic Research division at Federal Reserve Bank of New York between 2004 and 2007. He began doctoral studies in Political Science at the University of Rochester in 2007. He was awarded a Sproull Fellowship in 2004 and received the Master of Arts degree in 2011 from the University of Rochester. He pursued his research in Political Science under the direction of Randall Stone.

# Abstract

3 sentences on each part

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<sup>1</sup>The source code and documents for this dissertation can be found at [http://github.com/jrnold/phantasmic\\_dirigible](http://github.com/jrnold/phantasmic_dirigible).

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# Contents

Biographical Sketch	ii
Abstract	iii
Contributors and Funding Sources	iv
List of Tables	viii
List of Figures	x
<b>1 Bonds and Battles: Financial Market Reactions to Battlefield Events in the Amer-</b>	
<b>ican Civil War</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Financial Markets and the Study of War . . . . .	4
1.2.1 How the Prices of Financial Assets Relate to War Outcomes . . . . .	5
1.2.2 How Financial Markets Can Improve the Study of War . . . . .	9
1.2.3 The Means and Mechanisms of War . . . . .	14
1.3 Why The American Civil War? . . . . .	17
1.4 U.S. Government Bond Yield Data . . . . .	19
1.4.1 The Fives of 1874 . . . . .	19

1.4.2	U.S. Government Bond Yields are Primarily a Function of the Ex- pected Cost of the War . . . . .	24
1.5	Battle Data . . . . .	29
1.6	Statistical Models of the Effects of Battles on Bond Yields . . . . .	37
1.6.1	Average Effects of Union and Confederate Victories . . . . .	43
1.6.2	Individual Battle Effects . . . . .	45
1.6.3	Model Comparisons . . . . .	50
1.7	Conclusion and Future Work . . . . .	52
<b>2</b>	<b>Financial Markets and the Onset of the American Civil War</b>	<b>54</b>
2.1	Financial Market Prices and War Initiation . . . . .	56
2.2	Data . . . . .	60
2.3	Yields of Government and State Bonds, March 1855–April 1861 . . . . .	62
2.4	South-North Yield Spread . . . . .	71
2.5	Probability of War . . . . .	76
2.6	Conclusion . . . . .	80
<b>3</b>	<b>Bayesian Change Points and Linear Filtering in Dynamic Linear Models using Shrinkage Priors</b>	<b>85</b>
3.1	Introduction . . . . .	85
3.2	Change points as a Variable Selection and Shrinkage Problem . . . . .	88
3.3	Estimation and Implementation in Stan . . . . .	95
3.4	Example: Annual Nile River Flows . . . . .	99
3.5	Change Points in Levels and Trends . . . . .	103
3.6	Example: George W. Bush Approval Ratings . . . . .	107
3.7	Conclusion . . . . .	111

**Bibliography****115**

## List of Tables

1.1	List of the 43 militarily significant battles of the American Civil War included in this analysis. . . . .	35
1.2	Summary statistics of the posterior distribution of the average effects of Confederate and Union victories on log-yields. . . . .	43
1.3	Comparison of models using WAIC and RMSE . . . . .	51
1.4	The posterior mean of the average effect of Confederate and Union victories relative to the standard deviation of the innovations, $\eta$ . . . . .	52
2.1	Yields to maturity of U.S. government and state bonds in the week before and after news of the surrender of Fort Sumter. . . . .	66
2.2	Summary of data used to calculate the probability of war initiation. . . . .	80
3.1	Model comparison statistics for models of the Nile Rive annual flow data. .	103
3.2	Model comparison statistics for models of President George W. Bush's approval rating. . . . .	108



## List of Figures

1.1	Causal Diagram of combat and war termination . . . . .	16
1.2	Price and yield of the Fives of 1874 from its issue in 1858 until the end of the Civil War in 1865 . . . . .	30
1.3	The Fives of 1874 compared with other government and state issued bonds and currency. . . . .	31
1.4	Dates of major battles of the American Civil War and their outcomes . . .	33
1.5	Battle effects, $\omega_b$ , for all battles for all models . . . . .	47
1.6	Battle effects, $\omega_b$ , for model $\mathcal{M}_3$ . . . . .	48
1.7	Battle effects, $\omega_b$ , for all battles for model $\mathcal{M}_4$ , by battle result. . . . .	49
2.1	Yields on U.S. government and state bonds, July 1, 1855–June 1, 1861 . . .	63
2.2	Yields on U.S. government and state bonds around the Presidential Elec- tion of 1860 . . . . .	64
2.3	Yields to maturity of U.S. government and state bonds before and after the Battle of Fort Sumter. . . . .	65
2.4	Yields of U.S. government and state bonds, by region, July 1, 1855 to June 1, 1861. . . . .	72
2.5	Yield spread between Southern and Northern state bonds . . . . .	73

2.6	Implied probabilities of the initiation of war calculated from each Southern and Border state bond, March 5, 1858 to April 13, 1861. . . . .	81
2.7	Implied probabilities of the initiation of war calculated from a latent variable model of the prices U.S. government and Southern and border state bonds, March 5, 1858 to April 13, 1861, using the model described in section 2.5 . . . . .	82
3.1	Comparison of the density functions of normal, Cauchy, Laplace, horseshoe, and horseshoe+ distributions. . . . .	94
3.2	Annual flow of the Nile River, 1871–1970 . . . . .	99
3.3	Posterior distributions of $\mu_t$ for models of the Nile River annual flow data. . . . .	104
3.4	Posterior distributions of $\omega_t$ for models of the Nile River annual flow data . . . . .	105
3.5	Approval ratings of President George W. Bush . . . . .	108
3.6	Posterior distribution of $\mu$ for Normal and Intervention models. . . . .	109
3.7	Posterior distribution of $\mu$ for Horseshoe and Horseshoe+ models. . . . .	110

## Chapter 1

# Bonds and Battles: Financial Market Reactions to Battlefield Events in the American Civil War

### 1.1 Introduction

Understanding what leads to the termination of war can help to understand the causes of war (Blainey 1988, p. x). But understanding Gartner (1998) calls “opening up the black box of war.” Carl von Clausewitz famously described war as “simply a continuation of political inter-course, with the addition of other means (Clausewitz 1989).” This statement has traditionally been used to remind the reader that war is not an in itself, but a means to a political end. However, that insight is well integrated into current international relations theories of war. War is thought to consist of both bargaining and fighting, and the theoretical puzzle is why, given that war is costly, then why do states use this ex post inefficient method to come to an agreement. How do the events within war, such as battles, lead to the termination of war?

Opening the “black box of war” has been difficult for quantitative analysis since the relevant data on intra-war is sparse (Reiter 2003)Reiter2009. Data on battles is not readily available for large numbers of wars. While there are some implications of the bargaining model of war that can be tested with within-case data, such as those regarding the offers

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made throughout the war, the use of within-case data prohibits the use of several key dependent variables of interest in these models—the cost, duration, and outcome of a war—since there is no variation in these variables within a single war.

The approach of using financial markets to better understand war termination is a general one that can be used in the study of conflict. Understanding theories of war initiation and termination requires an understanding of the means of war, military combat.

However, the empirical study of military combat within war has lagged behind the study of war initiation that treats war as a black box

How do financial markets respond to war events? The approach taken in this work, using the prices of financial assets as a proxy for expectations of war termination, provides a method that helps deal with two problems in the study of military combat within war. First, by providing a within-case time-varying measure of the expected war termination, it allows for the quantitative analysis of small numbers of cases. This is useful because quality data on intra-war events is rare, it is difficult if not infeasible to conduct analysis that requires many wars. Due to the lack of multi-war intra-war data, the preferred approach to the empirical study of the bargaining theory of war with intra-war data has been qualitative case studies (Reiter 2003; Reiter 2009, Chapter 9). While there are some implications of the bargaining model of war that can be tested with within-case data, such as those regarding the offers made throughout the war, the use of within-case data prohibits the use of several key dependent variables of interest in these models—the cost, duration, and outcome of a war—since there is no variation in these variables within a single war. Financial market data provides an alternative method with which to analyze the intra-war events using the small number of cases for which data is available. Financial asset price data, when it can be shown to be a prediction of the war outcome, allows the researcher to estimate the effects of events on war outcomes by employing the prices of carefully chosen financial assets as a time-varying expectation of those war outcomes.

The second application is as a measure of surprising events. Surprising events are particularly important in private information theories of war, but it is difficult to know what events were surprising to contemporaries (Shirkey 2009). Since prices of financial assets only respond to new information, they provide a natural measure of surprising events, which are particularly important in informational theories of war (Shirkey 2009).

I estimate the effects of militarily significant battles on U.S. government bonds yields. The yields of U.S. government bonds were almost certainly primarily driven by expectations of the cost of the war. Thus, the bond yields of U.S. government can act as a proxy for expectations of the war cost given the available information at the time. Modeling the effects of battles on these yields estimates how Confederate and Union victory affected expectations about the cost and duration of the war.

I estimate several models of incorporating the effects of battles on bond yields and find the following. First, the effect of battles on bond yields, and, by extension expectations of the war cost, was asymmetric. On average, Confederate victories increased yields by approximately 5%, while, on average, Union victories had little to no effect. This is consistent with the asymmetric strategic environment of the American Civil War: the U.S. had to win battles to recapture the seceding states, while the Confederacy had to win to keep its territory. It also suggests that U.S. victories were expected, while Confederate victories were surprising.

Second, the battles with the largest estimated effects on bond yields were Confederate victories in May–June, 1864. These battles were part of the Overland and Richmond–Petersburg campaigns in which U.S. forces advance on the Confederate capital of Richmond was met with heavy resistance.

Third, although including battles in model of the bond yields improves the model fit, the improvement is surprisingly small. This is consistent with many events within a war, not just major battles, having an influence on war expectations.

This work adds to an economic history literature on the effects of war events in the American Civil War on currencies and other financial assets (Mitchell 1903; Mitchell 1908; Schwab 1901; Roll 1972; Calomiris 1988; Davis and Pecquet 1990; Willard, Guinnane, and Rosen 1996; McCandless 1996; Smith and Smith 1997; Brown and Burdekin 2000; Weidenmier 2000; Weidenmier 2002; Haber et al. 2014). This work contributes to that literature by using the yields of U.S. government bonds, which unlike Greenbacks (U.S. currency), provide a time series that spans the entire war. This work estimates the effects of both Confederate and Union victories, as well as that of individual major battles. It also connects that literature to political science theories of war, and shows how its insights can be used not just to understand how markets work, but also to better understand how war works. This work builds on a large and growing literature which measures impact or identifies important political events using financial (North and Weingast 1989; North and Weingast 2000; Frey and Kucher 2000a; Sussman and Yafeh 2000; Wells and Wills 2000; Herron 2000; Eldor and Melnick 2004; Chen and Siems 2004; Greenstone 2007) or prediction markets (Wolfers and Zitzewitz 2004; Arrow et al. 2008; Wolfers and Zitzewitz 2009).

## 1.2 Financial Markets and the Study of War

Financial markets relate to the study of war in several ways. First, they provide a measure of the the costs of war (Schneider and Troeger 2006; Guidolin and La Ferrara 2010). Second, financial markets are themselves a key actor or mechanism in several theories of war. This includes the “Capitalist Peace” (Gartzke 2007; Dafoe and Kelsey 2014), in which markets can provides signals of the costs of war to actors, and Slantchev (2012), in which debt financing creates a commitment problem. Third, in some cases, the prices of financial assets act similarly to a prediction market for the expected onset or outcome of war. The

implied expectations of war outcomes derived from the prices of these financial assets can be used to assess theories of war initiation and termination.<sup>1</sup> In this work, I focus on the third case, and use the yields of U.S. government bonds as a measure of expectations of the outcome of the American Civil War.<sup>2</sup>

### 1.2.1 How the Prices of Financial Assets Relate to War Outcomes

A financial asset is a claim on a stream of future, possibly uncertain, cash flows.<sup>3</sup> For example, a coupon bond pays a set number of coupons at specified times and its face value on maturity. A stock pays dividends at potentially uncertain times. Both because these cash flows are in the future and because their payment may be uncertain, these cash flows are discounted. The current price of a financial asset,  $P_t$ , is the value of those discounted cash flows,

$$P_t = \sum_{j=1}^H \frac{E_{t+j}(C_{t+j}|\mathcal{I}_t)}{1 + E_t(\delta_{t+j}|\mathcal{I}_t)} \quad (1.1)$$

where  $H$  is the number of cash flows,  $C_{t+j}$  is the cash flow at time  $t + j$ , and  $\delta_{t+j}$  is the discount rate for time  $t + j$ , and  $\mathcal{I}_t$  is the information available to the market at the current time  $t$ . The discount rate often consists of a risk free interest rate and an asset-specific premium, both of which may be time-varying. The riskiness of the asset can also be incorporated in  $E(C_t|\mathcal{I}_t)$ . For example, if there is a positive probability of default,  $E(C_t)$  would be a function of the probability of default and the amounts received in default and non-default states. The key feature of Equation (1.1) is that all of the price inputs are

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<sup>1</sup>See Chapter 2 for an analysis of how financial markets assessed the probability of the onset of the American Civil War.

<sup>2</sup>I use the term “outcome” to mean the state of the world at the end of the war, including the duration, cost, and victor of the war. I use the term “result” to refer to the specific outcome of who “won” or “lost” the war, which can be context specific.

<sup>3</sup>This discussion largely follows the discussion in Guidolin and La Ferrara (2010, p. 673); refer to them for more detail. See also Haber et al. (2015). See any introductory finance textbook or course notes for more information. See Chan-Lau (2006) for an overview of market-based measures of sovereign risk.

expectations conditioned on current information. Thus, events change the price through changing the market's information set.

Since the prices of financial assets are a function of expectations about future cash flows and risk premia, assets in which those cash flows or risk premia are largely contingent on some outcome of a war are themselves a proxy for expectations about that war outcome. For this purpose, the ideal asset would be a binary option, which would pay out a non-zero amount if the war outcome of interest occurs and zero otherwise. Then, with only some assumptions about the risk-preferences of the market, the probability of the war outcome is easily calculated from the price of the asset. Binary options are often used in "prediction markets" for events. There are several examples of prediction markets for political events. The Iowa Electronic Market is a prediction market for U.S. presidential and Congressional elections.<sup>4</sup> Intrade, until its closure in 2013, had prediction markets for multiple political and foreign policy events. These prediction markets included U.S. and other elections, whether Saddam Hussein would be removed as the president of Iraq, when Osama Bin Laden would be either killed or captured, and when there would be a U.S. or Israeli airstrike against Iran.<sup>5</sup>

Unfortunately for the researcher of conflict, these prediction markets in political events are not widespread at present, and not available historically for wars. However, financial assets in which the cash flows or risk premia are contingent on a war outcome are effectively prediction markets for that war outcome. In particular, the sovereign bonds issued by the belligerents in a war are likely to be highly sensitive to war outcomes. These bonds respond to two features of the war outcome. First, the riskiness of a belligerent's

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<sup>4</sup> See Wolfers and Zitzewitz (2004) for an overview of prediction markets. Rhode and Strumpf (2004) discusses historical betting markets on U.S. presidential elections in the late nineteenth-early twentieth century, which unfortunately for me were not formalized until 1884. See <http://tippie.uiowa.edu/iem/> for the Iowa Electronic Market.

<sup>5</sup> The Saddam contract issued by Tradesports is used in Leigh, Wolfers, and Zitzewitz (2003). See <http://intrade-archive.appspot.com/event.jsp?event=4272> and <http://intrade-archive.appspot.com/event.jsp?event=37985> for the Intrade contracts on Osama bin Laden and an airstrike against Iran, respectively.



sovereign bond is a function of the expected cost of the war. A more costly war almost directly implies either or both more debt issued or higher taxes, that cannot be used to service existing debt. Both of these imply a higher probability of default for the bond, and a lower price of that bond. Wars are also costly to the belligerents in more ways than just direct government expenditures. The destruction of human capital, military and civilian casualties, and physical capital affects on the expected economic growth of that country, and the future resources with which to pay off the debt.<sup>6</sup> Note that these expectations of the cost of war incorporate both expectations about both the intensity of the war and its duration. Second, the riskiness of a belligerent's sovereign bond is a function of whether the belligerent is expected to win or lose the war. Victory in war can gain the belligerent more territory and greater resources to pay off debt; a loss can mean the converse. In some cases, a defeat can lead to a loss of sovereignty and a default on their debt. This is relatively rare in inter-state wars, but more common in civil wars in which the rebel side issued debt (Haber et al. 2015). A loss of sovereignty followed by a repudiation was the case for holders of the debt and currency of the Confederate States.<sup>7</sup> More generally, losing states may tend to default on their debt with a higher probability (Slantchev 2012). The price of a bond is generally some weighted function of all of these expectations about what the outcome of the war will be and how those outcomes will affect the ability of the belligerents to repay their debt. For some bonds, it may be easy to back out the probability of a specific war outcome. Haber et al. (2015) focus on cases in which rebels issued bonds in civil wars (American Civil War, Chinese Civil War, and Spanish Civil War) and estimate the probability that the rebel side is victorious. But it may not be possible to disentangle expectations about the cost and outcomes of the war for the belligerents. However, there is some advantage to the way in which financial assets weight these war outcomes. By

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<sup>6</sup>See Goldin and Lewis (1975) for a calculation of the costs of the American Civil War.

<sup>7</sup>Section 4 of the Fourteenth Amendment, passed after the American Civil War, explicitly repudiates Confederate debt.

putting all war outcomes on the same scale, namely, their fiscal effect on that belligerent, the prices of sovereign bonds are able to provide a single, if incomplete, measure of the war outcome for each belligerent that incorporates both the costs and benefits of the war to that belligerent. Additionally, because interest rates are expectations and are generally available at a high frequency, they provide a real-time measure war expectations. Sovereign bonds are not the only financial asset which could be used as a proxy for war outcomes, stocks Chen and Siems (2004), Schneider and Troeger (2006), and Wolfers and Zitzewitz (2009), exchange rates Hall (2004), and commodities Wolfers and Zitzewitz (2009), may respond to conflict in some cases and may be able to be used as proxies for expectations about war outcomes. However, while there are commonalities, the specifics of how a financial asset relates to war outcomes are not universal, and in each case the researcher should carefully consider how the financial asset relates to the war outcome of interest. When using a financial asset price as a proxy for an expected war outcome, it is also necessary that its price is primarily influenced by that war outcome and not other factors. For example, when studying the Iraq War, it would be implausible to use the interest rates of U.S. Treasury bonds or the S&P 500 index since the effect of the war one way or another is likely a small influence relative to other economic factors.<sup>8</sup> It would be plausible to use bonds issued by Iraq, if they had issued in 2003, because the outcome or initiation of a war with the U.S. would plausibly be the most important factor influencing them. This is not out of concern of confounding, but a need for the the signal (war outcome) to noise (other factors that cannot be controlled for) ratio of the price needs to be large for it to be plausible to use the price as a proxy for a war outcome. Using sovereign bonds, or most financial assets, to proxy for war expectations only works for war outcomes so long

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<sup>8</sup> It is still possible to ask questions such as how much influence did events related to the Iraq War have on the stock market, as in Wolfers and Zitzewitz (2009). It would just be difficult to ask the inverse question which may be of more interest to political science research; given that the stock market is proxying expectations of the Iraq War, what events had the largest influence on it.

as those outcomes has some influence on either cash flows or risk premia. Some policy objectives of a war, e.g. national pride or the abolition of slavery in the American Civil War, may be important to the belligerents and of interest to the researcher, but if they do not affect future cash flows or risk premia, then they will have little effect on the prices of financial assets.

In this work, instead of prices, I use the yields to maturity (yields) of coupon bonds. The yield to maturity of a coupon bond can be found by taking a known price and cash flow schedule in Equation (1.1) and solving for a discount rate. For example, assuming continuous compounding for simplicity, the yield to maturity,  $y$ , of a bond is the solution to

$$P_t = \sum_{j=1}^H C_{t+j} e^{-yj}. \quad (1.2)$$

Equation (1.2) shows that the interest rate moves inversely to the price: a higher yield corresponds to a lower price, and vice-versa. An increase in the risk of a bond lowers its price and raises its yield.

### 1.2.2 How Financial Markets Can Improve the Study of War

Since Fearon (1995), the dominant theory of war has been the bargaining theory of war.<sup>9</sup> While there is a large theoretical literature that has formalized much of the theory (Filson and Werner 2002; Slantchev 2003; Smith and Stam 2004; Powell 2004; Leventoglu and Slantchev 2007; Langlois and Langlois 2009; Wolford, Reiter, and Carrubba 2011), direct empirical evidence of the theory is still limited (Reiter 2009). Financial asset prices can improve the discipline's understanding of war in two ways: expanding the number of cases which can be used for analysis by allowing for intra-case quantitative analysis, and providing a measure of surprising events.

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<sup>9</sup>See Reiter (2003), Powell (2006), and Reiter (2009) for discussions.

The bargaining theory of war is essentially a formalization of Clausewitz's famous dictum: "War is merely the continuation of policy by other means (Clausewitz 1989, p. 87)." The bargaining theory of war sees war as part of a bargaining process. The central puzzle of war is, given that war is costly, why did the sides not reach an agreement without incurring the costs of war. In particular, at the end of a war, some agreement is reached, even if that involves the complete capitulation of one of the sides. So why was that agreement not reached prior to the war and without incurring the costs of the war. Fearon (1995) and Powell (2006) propose two frictions that would prevent an *ex ante* agreement: private information and commitment problems.<sup>10</sup> But this leads to a second puzzle: given that these frictions can lead to conflict instead of a peaceful bargain, how does the actual fighting in the conflict resolve that friction so that the sides can come to an agreement and return to peace. Leventoglu and Slantchev (2007, p. 757) describe theories which answer this question as *complete* and *coherent* theories of war: "[A theory of war] must be complete, which means it must account for war's outbreak and termination; and it must be coherent, which means that its account of termination must explain how fighting has resolved the cause." While some evidence for the bargaining theory of war can come from studying war onset, testing the "core of the theory" requires intra-war data: estimates of capabilities, estimates of resolve, exchange of offers, and combat data (Reiter 2003). Since these data are difficult to acquire, there have been few empirical studies that attempt this (Reiter 2003; Ramsay 2008; Reiter 2009; Weisiger 2015).

Financial asset price data is useful to this research agenda in two ways: it provides a method for quantitative analysis of a single war, and it provides a measure of surprising events. First, financial asset price data allows for within-case quantitative analysis of war

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<sup>10</sup> War can also occur due to the sides the being risk loving or due to domestic politics. Fearon (1995) dismisses risk lovingness as implausible. Many of the domestic politics explanations are explanations that generate a private information or commitment problem that allows for conflict as a solution to the bargaining problem.

termination. Instead of using the cross-sectional variation in war outcomes from multiple wars, financial asset prices provide measures of expected war outcomes that are time-varying within each war. Being able to use intra-case variation is useful to researchers, because, apart from controlling for case specific attributes, the data demands of the bargaining theory of war means that few cases have the requisite data. While data on the sides' estimates of resolve and capabilities and (often secret) bargaining offers are unsurprisingly scarce, even data on battles is also of surprisingly poor quality and quantity. While there exists a Correlates of War for wars, there does not exist a similar dataset for battles, and the available data on battles is limited in both quantity and quality. The most commonly used and comprehensive battle dataset for inter-state wars is the CDB90 or HERO dataset (HERO et al. 1984; CAA 1991), but that data is both dated and of questionable quality (Reiter 2003, p. 32; Biddle and Long 2004). This lack of available data was one motivation for the use of the American Civil War in this work, since it is a war for which data on battles is available, and of relatively high quality and comprehensiveness.<sup>11</sup> Even if battle data were more plentiful, the ability to compare features of battles across wars may be fundamentally limited due to the heterogeneity of combat across time and space (Reiter 2009). As such, Reiter (2003) and Reiter (2009) suggest that the best methodology for the empirical analysis of the bargaining theory of war is qualitative analysis on limited cases. But in within-case analysis, some important dependent variables—war outcome, *e.g.* duration—cannot be used in the analysis, since only one war outcome is observed for each war. But, as discussed in Section 1.2.1, the prices of some financial assets may act as time-varying expected values of the war outcome. This allows researchers to conduct within-case quantitative analysis using cases with the requisite battle data, and use the prices of carefully-chosen financial assets as an outcome variable that proxies for

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<sup>11</sup> More recently work by Weisiger (2015) and Cochran and Long (2014) have offered multi-war data. See CAA (1991) and Helmbold (1995) for a discussion of the difficulties in compiling battle data.

war outcomes, such as the duration, cost or result.

Second, financial asset price changes naturally provide a measure of surprising events and new information. Surprising events are particularly important in private information theories of war, as only new information should affect beliefs. However, it is difficult for the researcher to know what was surprising to contemporaries; see Shirkey (2009) for an attempt to code surprising events. Prices provide a natural measure of surprising events, since as discussed in Section 1.2.1, prices are expectations of future events given the current information available to the market, and changes in prices correspond to new information available to the market,

The prices of financial assets have several other characteristics that make them a useful addition to the conflict researcher's toolbox for analyzing war.<sup>12</sup> First, they incorporate only *ex ante* information. This distinguishes price data from sources written after the fact, including participant memoirs fact or secondary sources (Willard, Guinnane, and Rosen 1996, p. 1001; Frey and Kucher 2000b, p. 188). Those sources can suffer from hindsight bias because the author is aware of future events. Second, investors have strong incentives to be truthful in their assessments because the payoffs from their investments are contingent on the accuracy of their assessments. This distinguishes prices from other sources, such as surveys or diaries of decision makers, which, like prices, have only the information available at the time, but in which the author may have strategic incentives to deceive (Reiter 2009, p. 57).<sup>13</sup> Finally, price data is often available at high frequencies, often daily.

<sup>12</sup>See Willard, Guinnane, and Rosen (1996), North and Weingast (2000), and Frey and Kucher (2000a) for other discussions on the use of price data to analyze historical events.

<sup>13</sup> Although the investors in the American Civil War had their own preferences on the war, their investments seemed motivated by their preference for profits. Cornwallis (1879, pp. 5,7) writes,

Sectional feeling often entered largely into the bull and bear contests in the Gold Room, and Union men and rebel sympathizers fought their battles sometimes, as much to gratify this as to make money.

The heaviest speculative orders were sent from Washington and Baltimore, and next to these, from Louisville, Kentucky, owing to these cities being in close communication to the seat of war and the rebel lines; and the operators there, almost to a man, were "bulls" in feeling, and

This is in contrast with documentary evidence, which is often either produced at lower frequency or missing for large periods of time (Reiter 2009, p. 57). The high frequency of price data provides more data points to the researcher, and expands the set of questions that can be answered.

This is not to claim that prices of financial assets dominate other data sources. They simply provide another tool for conflict researchers. Financial asset prices have several shortcomings when studying conflict, and are not appropriate in all cases. First, these prices do not directly measure the beliefs of the belligerents. Prices of financial data generally incorporate public information, and do not incorporate the private information available to decision makers. However, the causal mechanism in private information theories of war is the convergence of the beliefs of decision makers. Yet, in those theories the means by which beliefs converge is that conflict reveals private information, making that private information public. It may be possible that many private information models of war could restated so that they have implications for changes in the expected outcome of the war conditional on public information, for which financial asset prices can server as a measure. Second, an appropriate financial asset may not be available in many conflicts. In many areas in which researchers are most interested in understanding conflict, *e.g.* civil wars in developing countries, a financial asset that is closely tied to the war outcome may not

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strong Secessionists. But though seldom or never found selling “short” they were quick to sell out their “long” gold — that is the gold they were carrying — whenever the Confederate arms met with a reverse, and as quick to buy it back again when the market seemed to “touch bottom”.

Mitchell (1903, p. 210) writes,

Viewed in a broad way, it is therefore a serious mistake to look on the gold market as a place where a few gamblers were tossing the premium about to suit their selfish schemes; a much saner view is that it was the place where the community’s estimate of the government’s credit was visibly recorded. Here, as in other markets, those operators succeeded who forecast the future correctly, and men who tried to advance the price of gold when public confidence was increasing, or to depress it when confidence was on the wane, learned to their cost that they were not masters of the situation.

exist. Even now, not all developing countries issue sovereign bonds. Additionally, those countries prone to war may not have liquid markets, so the prices of available assets may not be accurate assessments of market beliefs. Even states with liquid and well-developed markets in peace-time often intervene in their financial markets during war. This distorts the price signals from those assets (Haber et al. 2015, p. 12). Finally, in any case in which prices are used, they cannot be used without domain specific knowledge. The researcher will need to apply their substantive knowledge of the assets, market, and conflict in order to find a financial asset that is related to expected war outcomes, and to understand how that asset relates to the war outcomes which are of interest to the researcher.

### 1.2.3 The Means and Mechanisms of War

While I motivate the use of financial asset prices to study war from the perspective of the bargaining theory of war, I do not attempt to directly test the private information and commitment theories of war in this work.<sup>14</sup> Instead, I focus on the effects of military combat — victories and defeats in major battles — on the expected cost of the war. I do not attempt to distinguish whether these victories and defeats influence the expected war termination through the revelation of private information or through resolving commitment problems. I do this because distinguishing between private information and commitment theories of war using only battle data is harder than previously appreciated. Observing only battle outcomes, such as victories or casualties, is insufficient to identify the causal mechanism of the war without relying on a well-specified model of the war process. These models do not exist at the moment. Models of that kind would require a deeper study of military processes. By estimating the effects of battles on war termination, and identifying which battles seemed to have the largest effect, this work may in small part contribute.

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<sup>14</sup>See Ramsay (2008), Weisiger (2015), and Reiter (2009) for works which attempt to do so.



This section explains the reasoning behind this claim.

In the bargaining theory of war, war combines a political bargaining process with military combat process.<sup>15</sup> These processes are distinct yet interconnected. For example, in costly-process formal models of war, war is modeled as alternating rounds of “bargaining” and “fighting”, *e.g.* Slantchev (2003). The bargaining and combat are distinct, but the outcome of the game depends on both. As discussed in the previous section, a complete and coherent theory of war needs to explain how fighting resolves the friction—private information or a commitment problem—that prevented an agreement without war. The means of war is military combat, the immediate goals of which are the destruction of military forces, the destruction of civilian assets, or the control of territory (Reiter 2003, p. 30).<sup>16</sup> These means of war must lead to war termination by resolving the private information or commitment problems. But there may exist multiple immediate means of combat and multiple channels by which each of those means influence war termination. Figure 1.1 illustrates the relationship between means, mechanisms and war outcomes. In one view, resolution of private information or the commitment problem are causes of war termination. In another view, military combat is a cause of war termination and resolution of private information and commitment problems are mediators. And in general, it is difficult to identify the causal effects of mediators (Keele 2015).

With regard to war, identifying the channels through which combat influences the war termination is empirically quite difficult. Almost any observable feature of military combat alters the actual state of the war, and potentially resolves a commitment prob-

<sup>15</sup>“Combat is a violent clash between at least two politically distinct groups organized to wield force. War consists of sustained and substantial episodes of combat.” (Reiter 2003)

<sup>16</sup> While formal models of war can be categorized by whether they are models of the private information mechanism, commitment problem, or both, they can also be categorized by how they model military combat within the war. In some models battles are important primarily because they influence the probability of war victory (Powell 2004; Wagner 2000; Leventoglu and Slantchev 2007; Slantchev 2003; Smith and Stam 2004). In other models, battles are important because they impose costs on the sides (Filson and Werner 2002; Powell 2004; Leventoglu and Slantchev 2007).

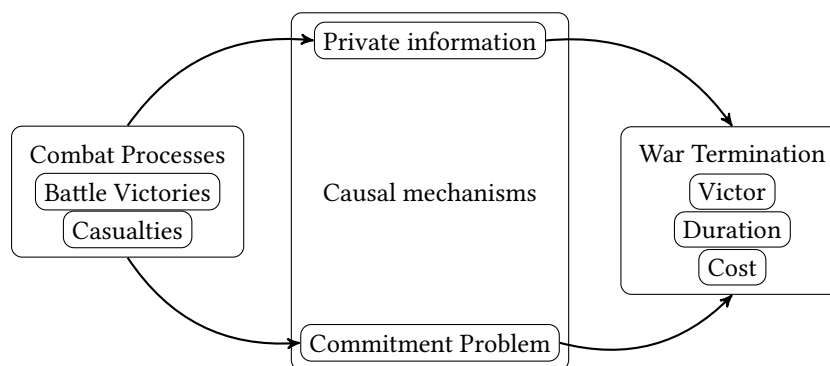


Figure 1.1: Causal Diagram of combat and war termination

lem, and also alters the beliefs of the participants about the combat process in which they are involved, and potentially resolves a private information problem. There are a couple ways in which this effects might be identified. First, if the beliefs of the participants were observed, then the effects of war events on the beliefs of the participants could be measured. A problem with this approach is that it requires data on the beliefs of the participants, which is difficult to acquire. Even if first-hand accounts are available, it is not clear whether these are accurate indicators of those beliefs. Another problem is that it is insufficient to simply observe changes in the beliefs of one participant over time, as in Reiter (2009). The causal mechanism in informational theories of war is the convergence of beliefs of the belligerents to a common distribution, which requires knowing the beliefs of both sides. Second, it could be identified if commitment problem that started the war were known and changes it were observable. Third, it could be identified through a structural model of how military combat influences war outcomes. Then it may be possible to back out how belligerents would update their beliefs and how combat could result in military victory or resolve commitment problems. However, this would require more work which focus on generating models of military combat, even if these models are not related tied to political variables. To understand how fighting resolves the frictions in bargaining, either changes in those frictions must be observed, or it is necessary to understand the process

that underlies the fighting. One difficulty in the later is that military combat may be extremely heterogeneous, both across time and space (Reiter 2003). But that is not a reason to ignore the study of military combat, but a call to describe and explain those patterns. A better understanding of the the military combat process in war may have different implications for these models, and improve the ability to distinguish between these causal mechanisms.<sup>17</sup>

### 1.3 Why The American Civil War?

The American Civil War has two features that make it particularly suited for this analysis. It is a war with plentiful, and high quality battle-level data, and in which there are financial assets which proxy expected war outcomes well. In terms of its battle data, the American Civil War is one of the best documented wars, and the battle data for it is discussed in Section 1.5.<sup>18</sup> In Section 1.4, I discuss why financial assets issued by the belligerents of the American Civil War may be good proxies for its war outcome.

Like any case, its generalizability is limited by its similarity to other observations. But despite its age, the American Civil War has several similarities to contemporary civil wars. First, the American Civil War is, if not the first modern war, a that marked the transition to modern warfare. This war introduced several of the technologies which came to prominence in World War I and later wars: rifled artillery, telegraph, machine guns, barbed wire, ironclad warships, and submarines (89 Fuller 1956; Weiss 1966, p. 760). Second, the American Civil War is primarily a conventional war. This places it in the plurality

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<sup>17</sup>For example, Walter (2009) conjectures that insurgency is slower to reveal information than other forms of war. Rigorously evaluating theories in this manner would require better theories of the actual military rationale between insurgency versus conventional war, the characteristics of each, and when belligerents choose to use each.

<sup>18</sup>For example, *The Official Records of the Union and Confederate Armies* (U.S. War Dept. 1880-1901), published between 1880 and 1901, consists of 128 books in 70 volumes totaling 139 thousand pages.

of post-Cold War civil wars, which are more often conventional wars than insurgencies (Kalyvas and Balcells 2010, p. 423). Finally, by the Civil War was U.S. was wealthy enough that the power parity adjusted GDP per capita of the U.S. and Confederate States in 1860 would classify them as lower-middle income countries today.<sup>19</sup>

The American Civil War is also an interesting case for study in its own right. Its importance to American history is obvious, and need not be stated.<sup>20</sup> But, given the voluminous literature on the subject, it is surprisingly understudied in at least two regards. First, the international relations literature has largely ignored this conflict. While the inter-state war literature considers it a civil war, and the civil war literature focuses on the post-1945 era (Reiter 2009, pp. 140-141; Poast 2012, p. 2). Exceptions to this are Reiter (2009) and Poast (2012). Second, almost none of the existing work on the American Civil War conflict, including the few examples in international relations, use quantitative methodologies.<sup>21</sup> In applying international relations theory and quantitative methods to the study of this war, this work contributes a droplet to the ocean that is the literature on the American Civil War.

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<sup>19</sup>The GDP per capita of the US in 1860 was \$2,241 in 1990 GK international dollars. In 2010, Cambodia had a GDP per capita of \$2,450, Pakistan \$2,494, and Ghana \$1,922 (Bolt and Zanden 2013). The southern states were poorer than the northern states, with a per capita consumption of about 70 percent of the overall U.S. level (Goldin and Lewis 1975, p. 324). The GDP per capita of the pre-war Confederacy is similar to that of Angola, Iraq and Senegal in 2010. The estimated GDP per capita of the southern states in 1860 at 70% that of the northern states was \$1,568. In 2010, the following countries had approximately the same real GDP per capita: Angola \$1,600, Iraq \$1,610, and Senegal \$1,507 (Bolt and Zanden 2013).

<sup>20</sup>See McPherson (2003) among too many others to list.

<sup>21</sup>Weiss (1966) for an example from operations research.

## 1.4 U.S. Government Bond Yield Data

### 1.4.1 The Fives of 1874

I use the yields of long-run U.S. government bonds as a measure of the expected outcome of the American Civil War. While the yields U.S. government bonds were almost certainly primarily driven by war expectations, they were plausibly influenced by both expectations of U.S. victory and expectations of war costs. However, it is likely that expectations of war costs is the more prominent of these two factors. In this work, I use the the yields of the Fives of 1874, a U.S. government coupon bond. It is the only U.S. government bond that was both issued prior to the war and quoted throughout the war, meaning that it is the only time series of a single asset that spans the war.

The Fives of 1874 were a coupon bond authorized by the Act of June 14, 1858 to pay for a budget deficit due to the Panic of 1857, a financial panic and recession.<sup>22</sup> As their name suggests, the Fives of 1874 were a coupon bond with a coupon rate of 5 percent per-annum paid semi-annually, and redeemable in 1874. Thus, during the period considered here, 1861–1865, the Fives had a maturity of 13–9 years. Like most U.S. government bonds of this period, the Fives of 1874 paid interest and principal in specie (gold dollars). They continued to pay interest in specie even after the U.S. Treasury ceased the redemption of Greenbacks (currency) in gold.<sup>23</sup>

Data on the prices of the Fives of 1874 come from tables in the two leading monthly financial magazines of the era, the *Bankers' Magazine and Statistical Register* and the *The*

<sup>22</sup> See Bayley (1882, p. 76), De Knight (1900, pp. 78–79), U.S. Treasury (1863, pp. 42–43), and Homer and Sylla (2005, pp. 300–301, 305).

<sup>23</sup> Greenback refers to several non-interest bearing notes issued by the U.S. as the currency during the war. Greenbacks were originally redeemable in gold dollars at par on demand. However, the Treasury ceased redemption of these notes for gold on January 1st, 1862. U.S. currency floated against the gold dollar from January 1st, 1862 until the resumption of convertibility on January 1st, 1879 (Mitchell 1908; Dewey 1918; Willard, Guinnane, and Rosen 1996).

*Merchants' Magazine and Commercial Chronicle* (Mitchell 1903, p. 186). In these publications the prices of bonds, not their yields, were quoted. The prices were quoted in currency, and not in gold dollars, even though they paid interest in gold dollars. I converted the prices to gold dollars using the current exchange rate of Greenbacks to gold dollars in New York, using the Mitchell (1908) data.<sup>24</sup> The yields of the bonds were calculated by converting prices to gold dollars accounting for the current exchange rate and the expected depreciation of the gold dollar.<sup>25</sup> Calculating the price and yield in gold dollars rather than currency is a meaningful decision, because while the price and yield in gold dollars fluctuated throughout the war, the price and yield in currency were stable.<sup>26</sup> This suggests that investors were less worried about a complete default than an implicit default in which government would redeem interest or principal payments in depreciated currency rather than specie. Both sources quote the bonds at irregular, but approximately weekly frequencies.<sup>27</sup> For the analysis, data from these two sources are combined to form a single series.

I collected and provide the financial data used in this work, as well as additional financial and economic from the American Civil War period in the American Civil War Era Financial Data Collection (ACWFD) (Arnold 2015b).<sup>28</sup> This data collection contains bond

<sup>24</sup> Available in digital form at <http://eh.net/wp-content/uploads/2013/11/greenback.txt> from Willard, Guinane, and Rosen (1996). I made some corrections to it where there were incorrect transcriptions from the original source.

<sup>25</sup> Since the price of bonds was quoted in currency but it paid interest and principal in currency, calculating the yields is difficult since they also incorporated beliefs about the future prices of gold dollars (Macaulay 1938, Appendix A; Roll 1972; Calomiris 1988; Homer and Sylla 2005, pp. 302-303). Most bonds, including the Fives of 1874, were priced in U.S. currency but paid specie and often principal in gold dollars. This work accounts for this using a simple method to calculate the future price path of the dollar; it assumes that currency will appreciate at the risk free interest rate of 5 percent until it reaches parity with a gold dollar. This follows from an assumption that the risk free rate is constant over the period, and the gold dollar is priced as a risk-free zero-coupon bond with an unknown redemption date McCandless 1996. Roll (1972) and Calomiris (1988) estimate the expected depreciation of gold dollars in Greenbacks using differentials in bonds, however those methods cannot be applied to the period considered here.

<sup>26</sup> See Roll (1972) for an analysis of this phenomena.

<sup>27</sup> *Bankers' Magazine*: 3–15 days, with a median of 10 days; *Merchants' Magazine*: 4–23 days, with a median of 7 days.

<sup>28</sup> Also available at The dataset is at [http://github.com/jrnold/civil\\_war\\_era\\_findata](http://github.com/jrnold/civil_war_era_findata).

prices and exchange rates published in the *Bankers' Magazine* and *Merchants' Magazine*, bid-level auction data for U.S. government bond auctions, and miscellaneous financial and economic data for this period from a variety of primary and secondary sources.

Figures 1.2a and 1.2b plot the price in gold dollars and the yield to maturity of the Fives of 1874. Prior to mid-1860, the yield was low with little variation, fluctuating between 4.5 and 5 percent.<sup>29</sup> After Lincoln's election in November 1860, the interest rate rose to 6 percent. After the initiation of the war with the Battle of Sumter (April 12–14, 1861), the yield rose to 8.2 percent. During the war, between April 1861 and April 1865, the yield fluctuated between 5.2 and 14.7 percent, reaching its peak on July 30, 1864. After the end of conflict in April 1865, the interest rate returned to stability at 6.5 percent, a rate higher than its pre-war level.

The Fives of 1874 were one of several bonds issued by the U.S. government during this period, but it has several advantages over other bonds for the purpose of this analysis. The Fives of 1874 were the only coupon bond regularly quoted for the entire war, April 1861–May 1865, in the sources considered here. The reason it is the only such bond, is that while the U.S. issued much debt during the Civil War, it had issued little debt in the antebellum period, and thus there were few outstanding bond issues at the start of the war.<sup>30</sup> The Sixes of 1848–1868 was the only other bond widely traded at the start of the war, but *Bankers' Magazine* stops quoting these shortly after the start of the war, in September 1861. Another bond, the Sixes of 1861–1881 were issued under acts in February and July 1861, just prior to and just after the start of the war. They are not regularly quoted in the *Bankers' Magazine* until September 1861, five months after the start of the

<sup>29</sup> This is actually below was generally considered risk-free rate in the U.S. for that period, 5 percent, which Elder (1863, p. 29) calls the “natural rate of interest on [government] Loans”. During this period British 3 percent consols had a yield of between of 3.10 and 3.28 (Homer and Sylla 2005, p. 193).

<sup>30</sup> After paying off the debt issued during the War of 1812, the U.S. government had no outstanding debt between 1835 and 1842 (Homer and Sylla 2005, p. 297). The debt as a percentage of GDP rose from 1.9 percent in 1860, before the war, to 31.4 percent in 1866, its peak just after the war (CBO 2012a; CBO 2012b).

war. The U.S. issued several other bonds of various maturities and rates throughout the war, but they provide even shorter time series. In many cases these loans also have exotic features relative to government bonds issued today. They often contained options to exchange the bond for other bond issues or were callable by the government. Both of these features make it difficult to calculate yields for these bonds.<sup>31</sup> Using the Fives of 1874 instead of these other bonds does not make a substantive difference for the analysis, since as Figure 1.3b shows, the patterns of the yields of the Fives of 1874 is similar to the yields of all the other U.S. government bonds. Willard, Guinnane, and Rosen (1996), McCandless (1996), and Smith and Smith (1997) use the exchange rate of Greenbacks to gold dollars in their analyses. Since the U.S. did not issue Greenbacks until after the start of the war, and did not suspend the redemption of Greenbacks for gold until January 1, 1862, any analysis using only Greenback prices cannot cover the first eight months of the war, April–December 1861. This is the period which should have the highest informational content in an informational theory of war, and includes battles such as the First Battle of Bull Run, and the Battle of Wilson’s Creek.<sup>32</sup> Apart from the loss of observations, using Greenbacks instead of the Fives of 1874 would not make a substantive difference, since as Figure 1.3a shows, the price of Fives of 1874 in gold dollars is similar in both level and trend as the the price of Greenbacks in gold dollars.

The American Civil War is an ideal case for which to use prices to infer expectations of war termination for three reasons. First, The war was of such severity that the nearly the entire budget of the U.S. government was war-related military spending, and thus fiscal expectations corresponded to military spending expectations. The War and Navy

<sup>31</sup> The 5-20’s and 10-40’s were bonds callable by the government after 5 (10) years and redeemable in 20 (40) years. The 7-30’s were bonds that had coupon rates of 7.30 percent (2 cents per day), were redeemable in 3 years, but included the option to be exchanged for either the Sixes of 1881 or 5-20’s. See Bayley (1882), De Knight (1900), and Homer and Sylla (2005, pp. 297–309) for overviews of U.S. debt issues during this period.

<sup>32</sup>Poast (2012) argues that the First Battle of Bull Run was pivotal in avoiding British recognition of the Confederate States.



Departments accounted for 76 percent (in 1862) to 61 percent (in 1865) of the expenditures (U.S. Treasury 1861b; U.S. Treasury 1861a; U.S. Treasury 1862; U.S. Treasury 1863; U.S. Treasury 1864; U.S. Treasury 1865). The fraction of the budget spent directly on the military by the Union fell over the course the war because the fraction spent on paying the principal and interest on its debt increased from 20 (1862) to 36 (1865) percent of the budget. The overwhelming the majority of that debt had been issued to fund the war, so debt service payments can be considered delayed military spending. As McCandless (1996, p. 668) states, “[d]uring the war, the single most important indicator of future government expenditures is the process of the war itself.”

Second, there is extensive economic history literature establishing that war events are the single best determinant of bond and currency prices during the American Civil War: (Mitchell 1903; Mitchell 1908; Calomiris 1988; Willard, Guinnane, and Rosen 1996; McCandless 1996; Smith and Smith 1997), graybacks (Schwab 1901; Weidenmier 2002), southern prices (Burdekin and Langdana 1993), Confederate bonds (Davis and Pecquet 1990; Brown and Burdekin 2000; Oosterlinck and Weidenmier 2007), and Union bonds (Roll 1972).<sup>33</sup>

Third, statements by and the actions of contemporary investors indicated the importance that war events had on the prices. Members of the government, military, and reporters all engaged in investment speculation based on their knowledge of war events (Cornwallis 1879, pp. 5-7; Mitchell 1903; Willard, Guinnane, and Rosen 1996, p. 1004). Investment firms even had their own correspondents stationed in cities close to the war fronts to relay the latest news (Cornwallis 1879, pp. 5-7). Finally, magazines and newspapers often cited war events as the cause of recent price movements. For example, the issue of *Bankers' Magazine* after Gettysburg and Vicksburg (Jul 23, 1863), cites these battles as

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<sup>33</sup>One notable exception is Burdekin and Weidenmier (2001), which shows that the prices of graybacks in Richmond and Houston diverged due to differential application of monetary reform in late 1864.

the cause of the recent changes in market prices, =

The month of July has been a very active one with numerous fluctuations, almost daily, in the market values of gold, with unusual changes in the current values of stocks. The advices as to the war movements are of a most satisfactory kind, leading to a fall in gold from 146 1/2 at the close of June, to 123 1/2, the 20th inst., at which dates the public had learned the capitulation of Vicksburgh and of Port Hudson, the defeat of Lee in Maryland and Pennsylvania, and of successful results at other points for the Union forces. (*The Bankers Magazine* 1864, p. 159)

#### 1.4.2 U.S. Government Bond Yields are Primarily a Function of the Expected Cost of the War

Since the cash flows of U.S. government bonds are not directly related to a war outcome, in order to use yields as proxy for the expectation of a war outcome, war outcomes need to influence future cash flows, and war expectations need to be the primary factor influencing the yield. U.S. government bond yields are plausibly related to both expectations about whether the U.S. would win the war, and as the cost of the war to the U.S. government. These two expectations have possibly cross-cutting implications on bond yields. An increasing probability of U.S. victory would make bonds less risky (higher price; lower yield), while a increasing expected war cost would make bonds more risky (lower price; higher yield). Unfortunately, it is not possible to separate these effects using only U.S. government bond yield data. However, it is likely that U.S. government bond bond yields mostly responded to and can be considered a proxy for the expected war costs to the U.S.

The risk on U.S. government bonds depended on the war cost for several reasons. The war was funded primarily through debt, and thus, expectations of a longer or more

vigorously fought war directly implied an increase in government debt. The proportions of the U.S. government revenues for each source of funding was as follows: 72–89% from loans, 11–25% from taxes, and 0–4% from miscellaneous sources (tariffs, land sales). In its analysis of the future fiscal situation, the 1863 *Annual Report of the Treasury* notes that taxes and other revenue sources covered non-war expenses and debt payments in its budget, but debt issued were needed to pay for the military expenditures needed to wage the war (U.S. Treasury 1863, pp. 10–13).<sup>34</sup> The budgetary effect of the war is clear from comparing forecasts of the U.S. budget for the 1866 fiscal year made before and after the war had ended. In *The Annual Report of the Treasury* issued on December 6, 1864, the forecast deficit for FY 1866, assuming that the war continued, was \$470 million, with \$1,168 million in expenditures (U.S. Treasury 1864, p. 13). In the following year's *Annual Report*, issued in December, 1865, after the conclusion of the war, the Treasury estimated a surplus of \$112 million, with expenditures of only \$396 million dollars (U.S. Treasury 1865). This forecast was close to what would be the actual surplus, \$132 million (U.S. Treasury 1866, p. 2). Thus, both expectations of the government military expenditures and the length of the war should have directly affected investors' expectations of the eventual debt level of the U.S. government. This increased debt would increase the possibility of default or the temptation of the government to repay its debt in currency rather than specie.

The future resources of the U.S. to pay off its debt also depended on the result of the war, namely, whether the seceding states would be reincorporated into the U.S. and increase the available resources to pay the costs incurred in the war. The size and wealth of the seceding Southern states was much smaller than that of the Northern and Western states. In 1860, the states that would make up the Confederacy accounted for only 29

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<sup>34</sup>See also Godfrey (1976, p. 14).

percent of the overall U.S. population,<sup>35</sup> and 25 percent of the total wealth.<sup>36</sup>

The default on U.S. government bonds could have taken several forms. The government could stop paying interest payments, as the Southern states did at the start of the war (*The Bankers Magazine* 1862, p. 159)[810,947]BankersMagazine1860. After the U.S. started issuing greenbacks which were not redeemable on demand in gold, it could have paid off the There was a minor controversy among investors regarding a different bond issue, the 5-20's which were explicit on repaying the interest in specie but were ambiguous in how the interest would be repaid (Roll 1972). Another example is that the Act of March 3, 1863 (12 Stat 710) declared that the interest on certificates of indebtedness issued under the Act of March 1, 1862 (12 Stat 352) was payable in currency (Bayley 1882, p. 81). Since these were issued after the suspension of convertability of Greenbacks to gold this may have not been a surprise to investors, but it suggests that this was a possibility. That the yields on U.S. government bonds when calculated in gold rose during the war, while those calculated in currency remained low and steady near 5 percent (HomerSylla20015), suggests that to the extent that investors viewed U.S. government bonds as risky it was that they would be repaid in depreciated currency rather than specie.If investors feared a complete default, then the yields calculated in currency should have also risen. This is also an important reason for using yields calculated in gold, which did respond to war risk, rather than yield calculated in currency as in Homer and Sylla (2005), which were much less sensitive to that risk.

Some statements by government officials and market participants suggest that they

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<sup>35</sup> Southern states included VA, TN, GA, NC, AL, MS, LA, SC, TX, AR, FL Eicher (2001, p. 5). Wealth is used instead of GDP, since GDP was not invented until later. Thus the total wealth of individuals, as reported in the Census, is a better measure of what contemporaries knew, since that is how they measured it.

<sup>36</sup> Elder (1865, p. 12). Wealth is defined as the value personal property excluding slaves, which was asked on the 1860 Census. It may be a better measure of how contemporaries would have viewed the value of the South than GDP because GDP had not yet been invented. The closest contemporaries had to GDP was an approximation that the value of yearly production was about 25-27 percent of total wealth (Elder 1865, p. 7; U.S. Treasury 1865, p. 24).

were more concerned with the cost and duration of the war than with the the reincorporation of the seceding states. William Elder, a leading economist of the era, in an 1863 analysis of the ability of the U.S. to repay its debt, stated “The Rebellion leaves our capital in real and personal property just where it was before the secession. ... So far now this is only a loss of that which we have not had, and at best or worst, a very small one in any time of need (Elder 1863, p. 19).”<sup>37</sup> The 1863 *Annual Report of the Treasury Department* described this relationship between war costs and the interest rate it paid on its bonds:

It will not escape observation that the average [interest] rate is now increasing, and it is obvious that it must continue to increase with the increase of the proportion of the interest-bearing to the non-interest-bearing debt. And as the amount of the latter, consisting of United States notes and fractional currency, cannot be materially augmented without evil consequences of the most serious character, the rate of interest must increase with the debt, and approach continually the highest average. That must be greater or less in proportion to the duration and cost of the war. (U.S. Treasury 1863, p. 13)

The striking difference in the trend of the prices of U.S. bonds and that of the Graybacks (Confederate currency) in Figure 1.3 is also evidence that U.S. bond yields were less driven by expectations of U.S. victory than by the war cost. That figure shows that the price of Graybacks in gold dollars decreases over the entire course of the war. Although the price of Graybacks is partially determined by the money supply (Burdekin and Weidenmier 2001), the price of Confederate bonds issued in Amsterdam and payable in gold followed a similar pattern, decreasing from mid-1863 onward (Haber et al. 2015). The prices of Confederate assets were likely primarily determined by whether the U.S.

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<sup>37</sup> However, Elder (1863) does emphasize the importance of the resources of the Western territories, which perhaps strengthens the argument of Weingast (1998) that the U.S. was concerned that Southern states seceding could lead to Western states seceding.

would win the war and repudiate their debt (Haber et al. 2015). Thus, if the U.S. bonds primarily reflected the probability of U.S. victory, then the expected trend of its prices (yields) would increase (decrease) immediately after the start of the war, and almost monotonically increase (decrease) throughout the war, or at least from mid-1863 onward. However, Figure 1.2b shows that while the yields of U.S. government bonds fluctuate throughout the war, they do not decrease over the entire course of the war. Instead, they spike in mid-1864 at a point at which U.S. victory should have been highly probable. This suggests that U.S. government bond yields do not primarily reflect expectations of U.S. victory, and instead reflect some other factor, which is likely war costs, by elimination.

Finally, the financial benefit of victory to the U.S. and war costs are inversely related. The longer and more destructive the war, the less the ability of the Southern states to assist in the repayment of the debt issued by the U.S. to pay war. Immediately after the war, the U.S. government attempted to collect direct taxes from the Southern states, but it soon suspended that collection after it became clear that the Southern states could not pay them. Treasury Secretary Hugh McCullough justified this suspension by noting that the inhabitants of the Southern states “had been subject to heavy taxation by the government which was attempted to be established in opposition to that of the United States, and had been greatly exhausted by the ravages of war.” (U.S. Treasury 1865, p. 29). In analyses of the repayment of debt after the war by the Treasury Department and others (Elder 1865; U.S. Treasury 1865; Walker 1865), the wealth of the Confederate states was assumed to remain constant between 1860 and 1870 due to the destruction of the war, and it was assumed that they would not be able to contribute to the repayment of the debt until at least after 1870.

While the precise relationship between these bond yields and outcomes of the American Civil War is unclear, it is clear that these bond yields almost completely reflect war expectations. As noted earlier in this section, military expenses accounted for the over-

whelming majority of U.S. government expenditures between 1861 and 1865, so there are no other budgetary concerns that could plausibly have been affecting investor's expectations. The importance of the war is also clear from comparing the level and variance of U.S. government bond yields during the war with the preceding period. During 1856–1860, yields on U.S. government bonds were stable at below 5 percent. At the height of the the Panic of 1857, the yield of U.S. government bonds (Sixes of 1868) only reached 5.1 percent.<sup>38</sup> Since a major financial crisis had little effect on bond yields prior to the war, there are certainly no non-war omitted factors affecting the yields. This is not to say that the battles in the war are the only factor affecting the bond yields. Government policies, elections, and other events could and almost certainly did have some affect on bond yields. But these events were affecting government bond yields through their effects on investors' expectations of the war, and not through other channels.

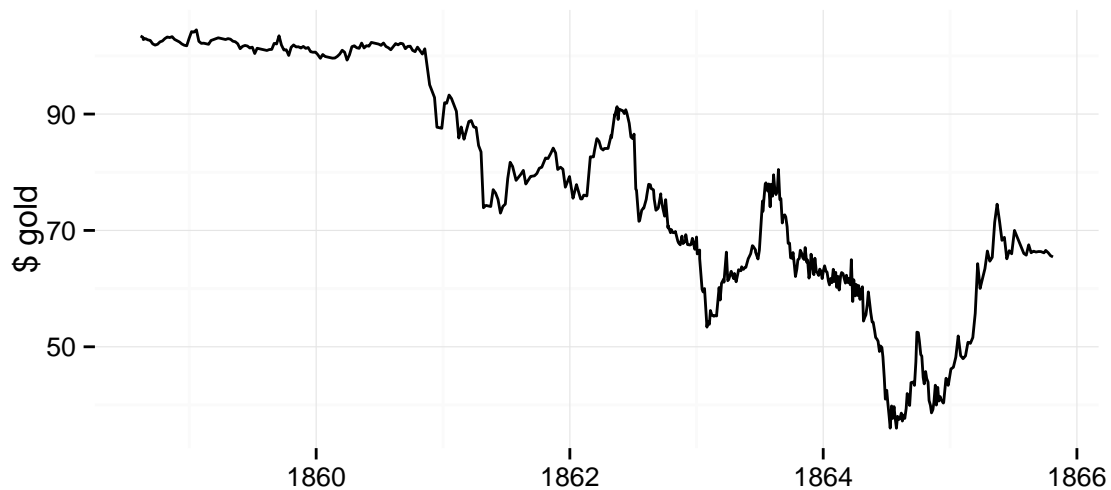
In summary, the U.S. government bond yields reflected some combination of expectations of the cost and the victor of the war, weighted by how each of these would affect the ability of the U.S. government to repay its debts. However, it is more likely that these yields are largely reflecting expectations about the cost and duration of the war.

## 1.5 Battle Data

A theoretical and practical issue in the empirical study of war is how to operationalize military combat within wars.<sup>39</sup> There are two major issues in the operationalization of military combat. The first is “what the unit of observation?” Some work uses events, *e.g.* battles or campaigns, as the unit of observation (Reiter and Stam 1998; Reiter and Stam 2002; Ramsay 2008; Reiter 2009, pp. 59-60; Pilster and Böhmelt 2011; Grauer and

<sup>38</sup>Data from the *Bankers' Magazine* and my calculations. Also see Homer and Sylla (2005).

<sup>39</sup> See the discussions in Reiter and Stam (1998), Biddle and Long (2004), Reiter (2003), Grauer and Horowitz (2012), and Weisiger (2015).



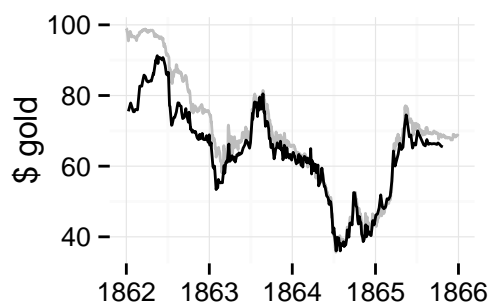
(a) Price in gold dollars. Par value is \$100.



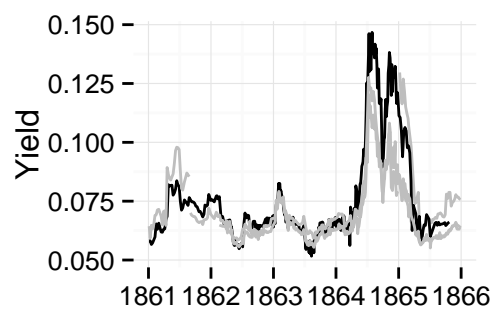
(b) Yield to maturity.

Figure 1.2: Price and yield of the Fives of 1874 from its issue in 1858 until the end of the Civil War in 1865. The yield is steady at near 5 percent until the election of Lincoln and the secession of the Southern States in late 1860-early 1861. During the American Civil War, the yields are variable, spiking in the middle of 1864.

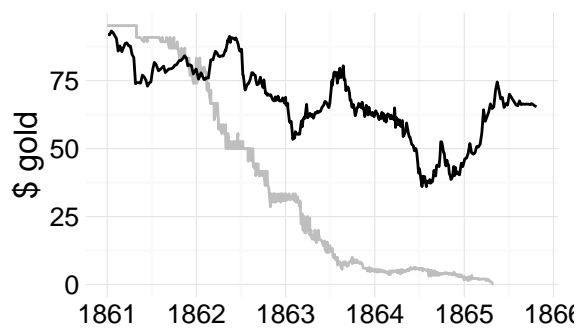




(a) Price in gold dollars for the Fives of 1874 compared to the price of Greenbacks, 1862-1865.



(b) Yield to maturity of the Fives of 1874 compared to other U.S. bonds.



(c) Price in gold dollars of the Fives of 1874 compared to \$100 in Graybacks (Confederate currency).

Figure 1.3: The Fives of 1874 compared with other government and state issued bonds and currency.

Horowitz 2012), while other work uses time-period aggregated values, *e.g.* monthly casualties (Weisiger 2015). The second is “how to measure military success?” Some work uses victory or defeat in battle (Reiter and Stam 1998; Reiter and Stam 2002; Grauer and Horowitz 2012), while other work uses casualty-based measures, such as loss-exchange ratios (Biddle 2004; Biddle and Long 2004; Pilster and Böhmelt 2011). In this work, I quantify military combat in the American Civil War by focusing my analysis on a set of important battles, and define success in combat by the battle result: Union or Confederate victory. An advantage of this method is that it allows for the estimation of individual battle effects with fewer assumptions on which features of those battles relate to war outcomes.

The battles used in this analysis are listed in Table 1.1. This set of battles consists of those which the National Park Service’s Civil War Sites Advisory Commission (CWSAC) classified as the most important of the war (CWSAC 1993a). These data come from the 1993 CWSAC report, *Civil War Sites Advisory Commission Report on the Nation’s Civil War Battlefields* (CWSAC 1993a; CWSAC 1993b). The CWSAC was a commission established in accordance with a 1990 law to preserve Civil War battlefields and was “asked to identify the nation’s historically significant Civil War sites; determine their relative importance; determine their condition; assess threats to their integrity; and recommend alternatives for preserving and interpreting them (CWSAC 1993b).” As part of that process they identified 383 battles of the over 10,500 military engagements listed in the *Official Records of the American Civil War*, as the “principal battles” of the war based on their “military significance.” As to the comprehensiveness of the list, CWSAC (1993a) states,

These [battle] sites encompass virtually all of the principal land battles that were of special strategic, tactical, or thematic importance to local operations, campaigns, theaters, or to the war as a whole ... The more than 10,500 conflict sites excluded from our inventory were relatively unimportant as individ-

ual military actions. These conflicts were the venues and actions that implemented the war between and beyond the dramatic major engagements.

These principal battles were further classified into four categories of military significance: “A”, “having a decisive influence on a campaign and a direct impact on the course of the war”, to “D”, “having a limited influence on the outcome of their campaign or operation but achieving or affecting important local objectives”. This work uses 43 battles with an “A” military significance classification.<sup>40</sup> While this is a subset of the battles in the American Civil War, it is still a relatively large number of battles by most war standards and similar in number and composition as the list of battles provided by other sources.<sup>41</sup> Livermore (1900) includes 63 battles, Bodart (1908) includes 50, and CAA (1991) includes 49. That this set of battles is a subset of battles in the American Civil War is largely due to the completeness of the records and richness of the data on the American Civil War.

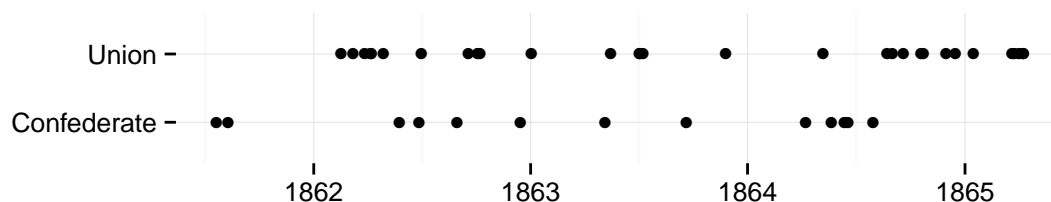


Figure 1.4: Dates of major battles of the American Civil War and their outcomes

The CWSAC (1993b) assigns a result to each battle, which is one of “Union victory”,

<sup>40</sup> Three CWSAC class “A” battles are omitted from the analysis. The Battle of Fort Sumter (SC, April 12–14, 1861) is omitted because it marks the start of the war and the analysis starts after this battle. Given the small size of Fort Sumter, no casualties, the effects of the battle can be attributed to marking the start of conflict. The Battle of Fort Blakely (AL, April 2–9, 1865) is omitted because news of battle does not reach New York City until after news of the surrender of Gen. Robert E. Lee’s forces at Appomattox Court House on Apr 10, 1865. The Battle of Five Forks (VA, April 1, 1865) is omitted because I could not find reports of it in the *New York Times*; news of it was overshadowed by the Third Battle of Petersburg on April 2nd, which ended the Siege of Petersburg and started the Appomattox Campaign, which ended in the surrender of Robert E. Lee.

<sup>41</sup>In CAA (1991) the only wars with more battles than this are World War I, World War II, and the Napoleonic Wars.

“Confederate victory”, or “Inconclusive”. While CWSAC (1993a) does not explicitly describe the methodology used to classify the battle outcomes, the classifications largely agree with other sources (Fox 1898; Livermore 1900; Bodart 1908; CAA 1991), several of which are themselves cited as sources of the report. Two of these sources, Fox (1898) and Livermore (1900), provide more detail on how they assigning the winner of a battle. Generally, the victorious side of a battle is considered to be the side that controls the battlefield at the end of battle. The data used in this work includes 30 Union victories, 13 Confederate victories, and no “Inconclusive” battles.<sup>42</sup> Figure 1.4 plots the date and outcomes of these battles over time. In the analysis, I will not explicitly include casualty or force size data. By selecting on the most militarily significant battles, I have already effectively selected those battles with the highest casualties and force sizes.

In order to estimate the effects of these battles on bond yields, I need the days on which the New York market received news about these battles. These are not the same as the dates of the battle, because news reaches the market with a lag. In general, the New York market received news about battles relatively quickly:

Members of both Houses [of Congress], and of all political creeds, resident bankers, the lobby agents, clerks, and secretaries, haunted the War Department for the latest news from the seat of war. The daily registry of the Gold Room was a quicker messenger of successes or defeats than the tardier telegrams of the Associated Press. A private secretary of a high official, with no capital at all save his position, which gave him authentic information of every shaping of the chess game of war a full twenty hours in advance of the public, simply flashed the words “sell, buy” across the wires, and trusted to the honor

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<sup>42</sup> In the original CWSAC data, there were two “Inconclusive” battles: The Wilderness, and Spotsylvania Court House. In the analysis, these were recoded as a Union victory and a Confederate victory, respectively, since with only two battles, I cannot estimate an average effect of “Inconclusive” battles. The Wilderness was reclassified as a Union victory, and Spotsylvania Court House as a Confederate victory based on how the results were reported in the *New York Times*.

	State	Theater	Dates	Result
First Bull Run	VA	Eastern	Jul 21, 1861	Confed
Wilson's Creek	MO	Trans-Mississippi	Aug 10, 1861	Confed
Fort Donelson	TN	Western	Feb 11–16, 1862	Union
Pea Ridge	AR	Trans-Mississippi	Mar 6–8, 1862	Union
Glorieta Pass	NM	Trans-Mississippi	Mar 26–28, 1862	Union
Shiloh	TN	Western	Apr 6–7, 1862	Union
Island Number Ten	MO	Western	Feb 28–Apr 8, 1862	Union
Forts Jackson & St. Philip	LA	Lower Seaboard	Apr 16–28, 1862	Union
First Winchester	VA	Eastern	May 25, 1862	Confed
Gaines's Mill	VA	Eastern	Jun 27, 1862	Confed
Malvern Hill	VA	Eastern	Jul 1, 1862	Union
Second Bull Run	VA	Eastern	Aug 28–30, 1862	Confed
Antietam	MD	Eastern	Sep 16–18, 1862	Union
Second Corinth	MS	Western	Oct 3–4, 1862	Union
Perryville	KY	Western	Oct 8, 1862	Union
Fredericksburg	VA	Eastern	Dec 11–15, 1862	Confed
Stones River	TN	Western	Dec 12, 1–Jan 12, 1	Union
Chancellorsville	VA	Eastern	Apr 30–May 6, 1863	Confed
Champion Hill	MS	Western	May 16, 1863	Union
Gettysburg	PA	Eastern	Jul 1–3, 1863	Union
Vicksburg	MS	Western	May 18–Jul 4, 1863	Union
Port Hudson	LA	Lower Seaboard	May 22–Jul 9, 1863	Union
Chickamauga	GA	Western	Sep 18–20, 1863	Confed
Missionary Ridge	TN	Western	Nov 23–25, 1863	Union
Mansfield	LA	Trans-Mississippi	Apr 8, 1864	Confed
the Wilderness	VA	Eastern	May 5–7, 1864	Union
Spotsylvania Court House	VA	Eastern	May 8–21, 1864	Confed
Cold Harbor	VA	Eastern	May 31–Jun 12, 1864	Confed
Second Petersburg	VA	Eastern	Jun 15–18, 1864	Confed
the Crater	VA	Eastern	Jul 30, 1864	Confed
Mobile Bay	AL	Western	Aug 2–23, 1864	Union
Jonesborough	GA	Western	Aug 31–Sep 1, 1864	Union
Opequon	VA	Eastern	Sep 19, 1864	Union
Cedar Creek	VA	Eastern	Oct 19, 1864	Union
Westport	MO	Trans-Mississippi	Oct 23, 1864	Union
Franklin (1864)	TN	Western	Nov 30, 1864	Union
Nashville	TN	Western	Dec 15–16, 1864	Union
Second Fort Fisher	NC	Eastern	Jan 13–15, 1865	Union
Bentonville	NC	Western	Mar 19–21, 1865	Union
Fort Stedman	VA	Eastern	Mar 25, 1865	Union
Third Petersburg	VA	Eastern	Apr 2, 1865	Union
Fort Blakely	AL	Western	Apr 2–9, 1865	Union
Appomattox Court House	VA	Eastern	Apr 9, 1865	Union

Table 1.1: List of the 43 militarily significant battles of the American Civil War included in this analysis.

of his broker for the rest. (Medbery 1870, p. 245)<sup>43</sup>

However, there was still a lag between the occurrence of a battle, and when news of it reached the market. More importantly for the analysis, this lag varied with the distance of the battle from New York and Washington, as well as idiosyncratic reasons. For example, the market knew nothing of Sherman's army after he moved north from Savannah until he arrived in Mitchell (1903, p. 204). As a measure of when information of the battle reaches the New York market, I coded when news about a battle was first reported in the *New York Times*. While battles in the Eastern theater were often reported the next day (Antietam, Gettysburg), news on battles in other theaters may not reach the market for days. The battle with the longest lag in news was the Battle of Glorietta Pass in New Mexico; it ended on March 28, 1862, but news of the battle did not reach New York city until April 15. For battles which lasted several days of approximately equal intensity, *e.g.* Chattanooga, I coded news as reaching the market on several days. For sieges which could last many days, *e.g.* Vicksburg, Port Hudson, I only coded when the news of the end of the siege was reported. A data driven approach to identifying the timing of this news is important since in an event-study analysis like this, the timing of the events is of key importance in identifying their effects.<sup>44</sup>

In using a small set of battles based on a *ex post* expert assessment of their military importance, it is important to note that this is not selecting on the dependent variable. In this case, that would be including explanatory variables by selecting periods of time that experienced large changes in yields. Instead, I am selecting a set of covariates, *i.e.* battles, which are expected to have large effects on the response variable based on prior information. This is no different than any other regression model, in which the researcher selects a relatively small set of covariates of all possible variables in the universe, because

<sup>43</sup>Originally quoted in Willard, Guinnane, and Rosen (1996).

<sup>44</sup> Measurement error due to how these dates are coded will be mitigated somewhat since prices are not observed daily, so usually the change in prices reflects the sum of the effects of several days.

those covariates are *a priori* expected to plausibly have an effect on the response variable.

For this project, I created a public data collection of American Civil War battles, the American Civil War Battle Database (ACWBD) (Arnold 2015a).<sup>45</sup> The ACWBD combines, cleans, cross-references battle data from multiple primary and secondary sources, including Phisterer (1883), Livermore (1900), Bodart (1908), Dyer (1908), Kennedy (1998), CWSAC (1993a), and NPS (2012). It is the most complete, machine-readable data on American Civil War battles of which I am aware.

## 1.6 Statistical Models of the Effects of Battles on Bond Yields

In this section, I present several models of the effects of battles on U.S. government bond yields. In all these models, the response variable is the yields of the Fives of 1875, as discussed in Section 1.4.1. The Fives data consist of 314 observations between 1861-04-27, the first observation after the Battle of Fort Sumter and the official start of the war, and 1865-05-04. These occur at irregular frequencies, with gaps of 1–1 days between observations. In these models, I use the logarithm of the yield since Figure 1.2 suggests that the variance of yields is increasing in its level.

All of these models are of the form,

$$\log y_t = y_t^* + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(0, \tau^2) \quad \text{if } y_t \text{ not missing} \quad (1.3)$$

$$y_t^* = y_{t-1}^* + \delta_t + \eta_t \quad \eta_t \sim \mathcal{T}_5(0, \sigma) \quad (1.4)$$

for days  $t = 1, \dots, T$ . Equation (1.3) states that the observed log-yields consist of a latent log-yield ( $y_t^*$ ) and measurement error ( $\epsilon_t$ ). Equation (1.4) states that the first difference in the latent log-yields,  $y_t^* - y_{t-1}^*$ , is the sum of  $\delta_t$  and a mean-zero innovation,  $\eta_t$ . The models

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<sup>45</sup>The data and source code of the data collection are available from [https://github.com/jrnold/acw\\_battle\\_data](https://github.com/jrnold/acw_battle_data).

differ in the definition of  $\delta_t$  and this is where battle effects are included. The innovation  $\eta_t$  is distributed Student's  $t$  to allow for large events that are unmodeled, *i.e.* any non-battle events. If  $\tau^2 = 0$ , then Equations (1.3) and (1.4) would be equivalent to simply estimating a model with the first difference of  $\log y_t$  as the dependent variable. With a non-zero  $\tau$ , and supposing  $\eta$  was distributed normal, this model would be equivalent to an ARIMA(0, 1, 1) model in which the MA parameter is constrained to be negative (Petris, Petrone, and Campagnoli 2009, p. 91).

In this section, I estimate four models which differ in how  $\delta_t$  is defined:

- Model  $\mathcal{M}_1$  is benchmark model that does not include any battle effects.
- Model  $\mathcal{M}_2$  includes variables for Confederate and Union victories, but no individual battle effects.
- Model  $\mathcal{M}_3$  is like  $\mathcal{M}_2$ , but also includes random effects for each battle.
- Model  $\mathcal{M}_4$  is like  $\mathcal{M}_2$ , but the effects of Confederate and Union victories are smoothly varying over time.

Model  $\mathcal{M}_1$  includes no battle effects,  $\delta_t = 0$  for all  $t$ . In the other models,  $\delta_t$  is non-zero and time-varying, and is a function of battle outcomes, but these models differ how battles are



modeled. In model  $\mathcal{M}_2$ ,  $\delta_t$  includes the effects of Confederate and Union victories.

$$\delta_t = \alpha + X_t \begin{bmatrix} \beta_C & \beta_U \end{bmatrix}' \quad (1.5)$$

$$X_{t,1} = \sum_{b \in B} W_{t,b} \quad \text{if } b \text{ is a Confederate victory} \quad (1.6)$$

$$X_{t,2} = \sum_{b \in B} W_{t,b} \quad \text{if } b \text{ is a Union victory} \quad (1.7)$$

$$W_{b,t} = \begin{cases} \frac{1}{n_b} & \text{If news of battle } b \text{ reached the market on day } t. \\ 0 & \text{otherwise} \end{cases} \quad (1.8)$$

where  $B$  is the set of all battles in the data, see Table 1.1. In this model, all Confederate victories have the same effect on the yield, and all Union victories have the same effect on the yield. In other words, given their result, battles are exchangeable. The parameters  $\beta_C$  and  $\beta_U$  are the average effects of Confederate and Union victories. The matrix  $W$  is a  $T \times |B|$  matrix, in which each column is a variable indicating if there was news about a battle on that day. News of battles is assumed to reach the market on the days the first news of the battle is reported in the *New York Times* and the following 5 days. For a given battle, all days with news are weighted equally, and the weights for each battle sum to one.  $n_b$  is the total number of days for which news of battle  $b$  reached the market. For example, the end of Gettysburg was reported in the news on July 2nd–4th, 1863, so July 2, 1863 through July 9, 1863 (July 2 plus 5 days) are equal to  $\frac{1}{8}$  in  $W_{\cdot, \text{Gettysburg}}$ , and all other days in  $W_{\cdot, \text{Gettysburg}}$  are equal to 0.  $X_t$  is an  $T \times 2$  matrix containing the sum of the news weights of Confederate and Union victories occurring on day  $t$ . Since each column of  $W_b$  sums to one,  $\beta_C$  is interpreted as the average effect of a single Confederate victory, and  $\beta_U$  as the average effect of a single Union victory. Finally,  $\alpha$  is a parameter representing a

constant trend in yields not due to the battles.

In model  $\mathcal{M}_3$ ,  $\delta$  also includes random effects for each battle,

$$\delta_t = \alpha + X_t \begin{bmatrix} \beta_C & \beta_U \end{bmatrix}' + W_t \gamma \quad (1.9)$$

$$\gamma_b \sim \mathcal{T}_7(0, \zeta^2) \quad (1.10)$$

where  $\alpha$  and  $\beta$  have the same meaning as in  $\mathcal{M}_2$ , and  $\gamma$  is a  $|B| \times 1$  vector with random effects for each battle. The random effects,  $\gamma$ , are given a Student's  $t$  distribution since it is expected that a few battles will have particularly large effects. The effect of an individual battle is the sum of its result (Confederate or Union victory) and random effect,

$$\omega_b = \begin{cases} \beta_C + \gamma_b & \text{if } b \text{ is a Confederate victory} \\ \beta_U + \gamma_b & \text{if } b \text{ is a Union victory} \end{cases} \quad (1.11)$$

In model  $\mathcal{M}_4$ , the effect of each battle,  $\omega_b$ , is shrunk towards both the average effect of its result and the effect of the previous battle of the same result. This is equivalent to a model in which the effects of Confederate and Union victories are smoothly time-varying with an  $AR(1)$  process.<sup>46</sup>

$$\delta_t = \alpha + W_{C,t} \omega_C + W_{U,t} \omega_U \quad (1.12)$$

$$\omega_{C,b} \sim \mathcal{N}((1 - \rho)\beta_C + \rho\omega_{C,b-1}, \rho\zeta^2) \quad \text{for } b \in \{2, \dots, C\} \quad (1.13)$$

$$\omega_{C,1} \sim \mathcal{N}(\beta_C, \zeta^2) \quad (1.14)$$

$$\omega_{U,b} \sim \mathcal{N}((1 - \rho)\beta_U + \rho\omega_{U,b-1}, \rho\zeta^2) \quad \text{for } b \in \{2, \dots, U\} \quad (1.15)$$

$$\omega_{U,1} \sim \mathcal{N}(\beta_U, \zeta^2) \quad (1.16)$$

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<sup>46</sup>  $\omega_{C,t} \sim N(\beta_{C,t}, \zeta^2)N(\omega_{C,t-1}, \xi^2) = N((1 - \rho)\beta_C + \rho\omega_{C,t-1}, \rho\zeta^2)$ , where  $\rho = \frac{\xi^2}{\xi^2 + \zeta^2}$ .

Where  $C$  is the number of Confederate victories,  $U$  is the number of Union victories. The Confederate and Union victories are ordered by their dates. For Confederate victories, the first battle was First Bull Run (July 21, 1861), and the last was the Crater (July 30, 1864). For Union victories, the first battle was Fort Donelson (February 11–16, 1862), and the last was Appomattox Court House (April 9, 1865). As an example, in this model the effect of the Battle of Gettysburg,  $\omega_{13}$ , is shrunk towards both the average effect of Union victories,  $\beta_U$ , and the effect of the previous battle,  $\omega_{12}$  (Champion Hill on May 16, 1863).

In all these models, since  $y^*$  represents log-yields, the parameters  $\beta$ ,  $\gamma$  and  $\omega$  can be interpreted as approximately the percent change in the yield associated with the result of a battle.

All models are estimated using MCMC methods to sample from the posterior distribution. Samples are drawn using the NUTS-HMC algorithm (Hoffman and Gelman 2014) as implemented in the probabilistic programming language, Stan (Stan Development Team 2015). I found that the sampling efficiency of state space models, such as those estimated here, could be improved with a two step method. In the first step, I marginalize over the latent states of the state space model ( $y^*$  in this case) and use HMC-NUTS to sample the other parameters. In the second step, given the values of the parameters, I use the forward-filter backwards-sample algorithm (Frühwirth-Schnatter 1994; Carter and Kohn 1994) to sample the latent states. Together, these steps constitute a partially collapsed Gibbs sampler (van Dyk and Park 2008). I describe this method and its implementation in Stan in more detail in Chapter 3 of this dissertation. For all models, four chains were run, and convergence assessed using multiple-chain  $\hat{R}$  statistics and the number of effective samples, as recommended in (Stan Development Team 2015) and Gelman, Carlin, et al. (2013).

In these models, I do not include indicators of other events in the war. While changes in policy, leadership, foreign policy events, negotiations and other events certainly in-

fluence war expectations, I do not have a coherent method for choosing a set of events. McCandless (1996) and Smith and Smith (1997) both include indicator variables for a few military, fiscal, and political events in the war. However, neither of these have a clear rule as to what constitutes major non-battle events in the war, and what does not. Smith and Smith (1997) includes indicator variables selected on the dependent variable, using the events mentioned in Mitchell (1903) as corresponding to large daily movements in the gold market. In Section 1.7, I propose a more coherent method for identifying war and non-war events.

These models do not include measures of inflation or the risk-free interest rate. Inflation is not included for several reasons. First, while inflation measured in U.S. currency (Greenbacks) rose during this period, inflation calculated in gold dollars did not (Mitchell 1903; Mitchell 1908). The price of gold closely tracked the price of goods during this period, so there was little inflation when the prices of goods were measured in gold dollars. Since the bonds considered here paid interest in gold dollars, and the bond yields are calculated with the cash flows converted to gold dollars, the inflation of currency is not explicitly a problem. Second, the influence of the war was such that inflation is plausibly a post-treatment variable that, like bond yields, responded to war expectations. Third, data on inflation is only available at frequencies much longer than used in the analysis; the Warren-Pearson Wholesale Price Index is only available at the monthly level (Warren and Pearson 1933). The risk free interest rate during this period is not included because there is not a good measure of the the risk free interest rate during this period. Before and after the American Civil War, the yields of U.S. government bonds could be considered approximately risk-free assets (Homer and Sylla 2005). But this work uses the yields of U.S. government bonds precisely because those bonds are risky during the American Civil War. Macaulay (1938) and Homer and Sylla (2005) consider indexes of railroad bonds and New England municipal and state bonds as risk free assets during the late nineteenth

Model	Result	Mean	2.5%	50%	97.5%
$\mathcal{M}_2$	Confederate	0.050	0.013	0.050	0.085
$\mathcal{M}_3$	Confederate	0.052	0.015	0.053	0.088
$\mathcal{M}_4$	Confederate	0.057	0.022	0.057	0.092
$\mathcal{M}_2$	Union	-0.015	-0.035	-0.015	0.005
$\mathcal{M}_3$	Union	-0.015	-0.036	-0.015	0.004
$\mathcal{M}_4$	Union	-0.018	-0.037	-0.018	0.002

Table 1.2: Summary statistics of the posterior distribution of the average effects of Confederate and Union victories on log-yields.

century. But railroad bonds were not yet safe assets at the time of the American Civil War; before the war they had much higher yields than U.S. government bonds. New England municipal and state bonds are the other candidate for risk-free assets, but when their yields are calculated in gold dollars, they are similar to those of U.S. government bonds for this period. Moreover, most New England states did not issue debt prior to the Civil War, and issued bonds during the Civil War to pay for their war expenditures (Martin 1871, pp. 86–87). Thus, yields of these bonds were also plausibly linked to war expectations. On a practical level, the currently available comprehensive source of these interest rates, Macaulay (1938), only provides them at monthly (railroad) or quarterly (New England state and municipal bonds) frequencies. Other work in 19th century economic history uses the interest rate of British consols as a risk free rate (Bordo and Rockoff 1996). The rate of consols is inappropriate for this analysis because that risk free interest rate corresponded to the London market, and this analysis uses the New York market.

### 1.6.1 Average Effects of Union and Confederate Victories

For these models, the first question to ask is “what is the average effect of a Confederate or Union victory on the bond yield?” The average effects of Confederate and Union victories are defined slightly differently for the models. For model  $\mathcal{M}_2$ , the average effect of a Confederate victory is the parameter  $\beta_C$ , and the average effect of a Union victory is

the parameter  $\beta_U$ .<sup>47</sup> For models  $\mathcal{M}_3$  and  $\mathcal{M}_4$ , the average effect is the average of the battle effects. For these models, the average effect of a Confederate victory is the average of the effects of the individual Confederate victories,  $\frac{1}{C} \sum_{b \in C} \omega_b$ , and, similarly, the average effect of a Union victory is  $\frac{1}{U} \sum_{b \in U} \omega_b$ .

Table 1.2 shows the summary statistics of the posterior distribution of the average effects of Confederate and Union victories for the models. The estimated average effect of a Confederate victory on yields is positive, meaning that Confederate victories were associated with an increase in the expected cost of the war. A posterior mean of the average effect of a Confederate victory is estimated to have increased the yield approximately 5–5.7 percent, with the 95% central credible intervals including the range of 1.3% to 9.2%. The posterior mean of the average effect of a Union victory is negative, meaning that Union victories were associated with a decrease in the expected cost of the war. However, the posterior mean is small in magnitude, -1.5% to -1.8%, and the central 95% percent credible intervals of all models contain zero. There are two explanations for this asymmetry in effects of victories on the expected cost of the war. First, individual Union victories had a smaller effect on the overall cost and duration of the war than Confederate victories. This is because the strategic situation of the war was itself asymmetric. As military historian, J.F.C. Fuller described it, “to reestablish the Union, the North must conquer the South, and to maintain the Confederacy and all it stood for, the South must resist invasion (Fuller 1942, p. 177).” Since the U.S. had to win victories and take territory while the Confederacy had to only delay and resist implies a differential value of Union and Confederate victories. Second, the market may have been expecting Union victories, so they were less surprising on average than Confederate victories.

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<sup>47</sup>Model  $\mathcal{M}_1$  does not include battles.

## 1.6.2 Individual Battle Effects

While model  $\mathcal{M}_2$  assumes that all Confederate and Union victories have the same effect, Models  $\mathcal{M}_3$  and  $\mathcal{M}_4$  estimate effects for individual battles in addition to the average effects of Confederate victories. The second set of questions to ask is about the variation in the effects of individual battles. How much variation was there in the effects of battles? How did the effects of Union and Confederate victories change over time? Which battles had the largest effects?

Figure 1.5 plots the battle effects,  $\omega_b$ , for each battle, for each model. Figure 1.6 plots the battle effects for model  $\mathcal{M}_3$ , for each battle. Since the battle effects in model  $\mathcal{M}_4$  can be considered time-varying values of effect of Confederate and Union victories, Figure 1.7 plots the battle effects for Confederate and Union victories over time. One thing to note is that the estimates of battle-level effects are highly uncertain; in all models most battles have 95 percent central credible intervals which include zero.

The battles with the largest effects are the Confederate victories in the summer of 1864 in the Overland Campaign (Spotsylvania Courthouse, Cold Harbor) and the start of the Richmond-Petersburg Campaign (Second Petersburg). In the battles of Spotsylvania Courthouse and Cold Harbor, U.S. forces commanded by Ulysses S. Grant suffered large casualties while advancing towards the Confederate capital, Richmond. In the of Battle of Second Petersburg, the U.S. failed break the Confederate lines and decisively capture Richmond. This defeat began the Siege of Petersburg which would last from June 1864 until the U.S. broke the Confederate lines at the Third Battle of Petersburg on April 2, 1865. Reiter (2009, pp. 151-152) cites this period as a turning point in the war, during which the U.S. “neared exhaustion”.

The next largest effect is the Confederate victory at the Battle of Gaine’s Mill. This was the major Confederate victory in the Seven Days Battles and Peninsula Campaign in June

1862. In the Peninsula Campaign, the U.S. Army of the Potomac, commanded by General George McClellan, moved to almost within artillery distance of Richmond. However, in the Seven Days battles, Confederate forces commanded by Robert E. Lee fought a series of battles, largely Confederate victories, which pushed U.S. forces back from Richmond. This was cited as a turning point of the war by Fuller (1942): “[The Peninsula Campaign] campaign was certainly one of [the Civil War’s] most important [campaigns], because, in spite of numerous blunders, the Army of Northern Virginia, under the guidance of Lee, saved Richmond. Had Richmond fallen into Federal hands, the whole course of the war would have been changed (Fuller 1942, p. 206).” The Seven Day’s battles are cited by Reiter (2009, p. 145) as one of the turning points in the war, when “it was becoming increasingly apparent that the war would not end quickly.” In this campaign, both sides had opportunities to bring the war to an end, the U.S. by capturing Richmond, and the Confederacy by destroying the Army of the Potomac, but neither was able to (Fuller 1942, Chapter6).

The estimates of the effects of Union battles are smaller in magnitude and 95 percent central credible intervals of all Union victories contain zero. However, the results are plausible in that the battles of Gettysburg (July 1–3, 1863) and Vicksburg (July 4, 1863) are estimated as having among the largest battle effects. This is despite these two battles occurring almost simultaneously, which makes it hard to disentangle their effects. The Union victory with the largest estimated effect is the Second Battle of Fort Fisher which blockaded the Wilmington, N.C. , the last unblockaded Confederate seaport. Reiter (2009, p. 154) cites the capture of Fort Fisher as final turning point of the war.



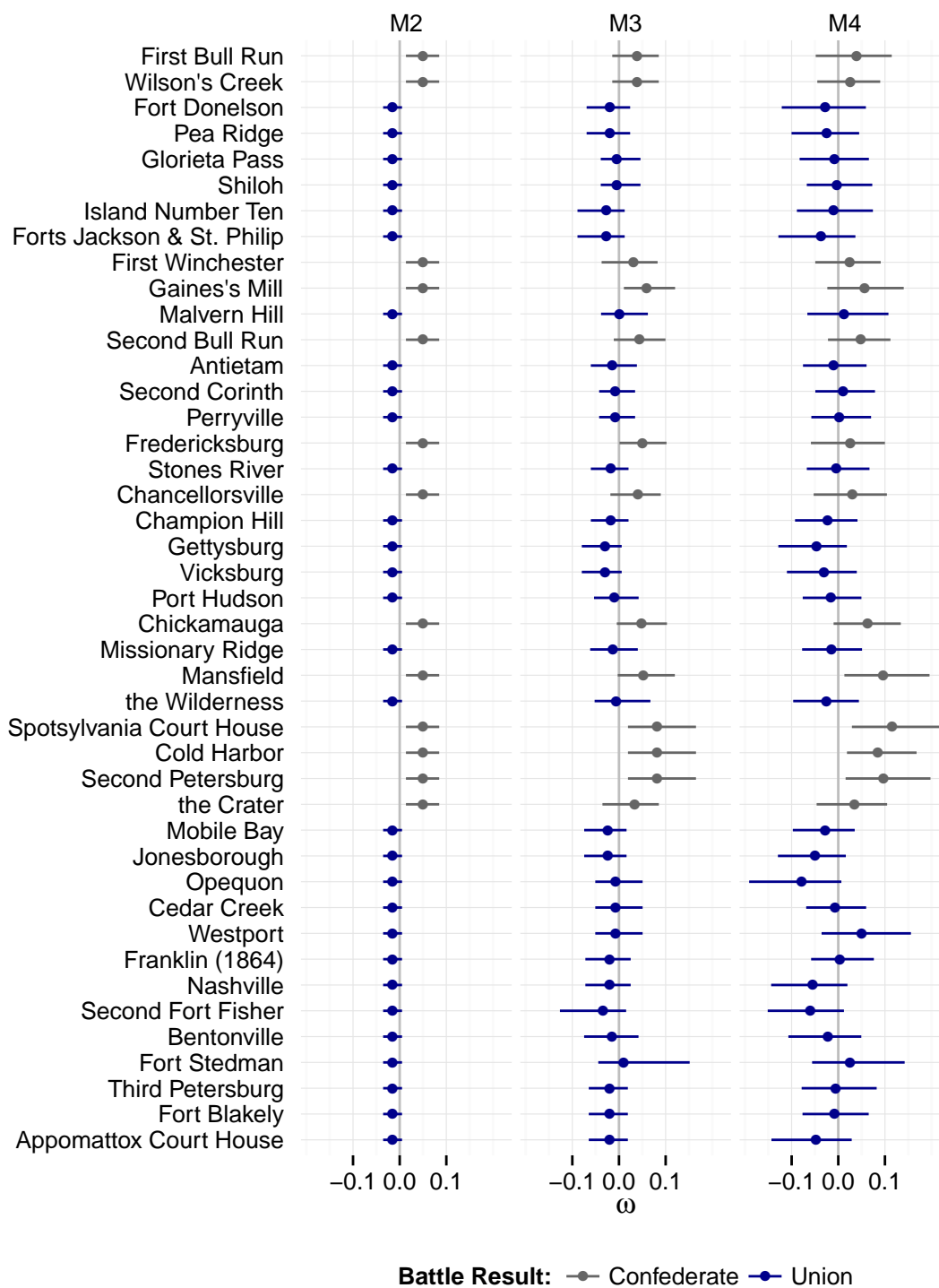


Figure 1.5: Battle effects,  $\omega_b$ , for all battles for all models. These are ordered by end date of the battle. Union victories are in dark blue. Confederate victories are in gray.

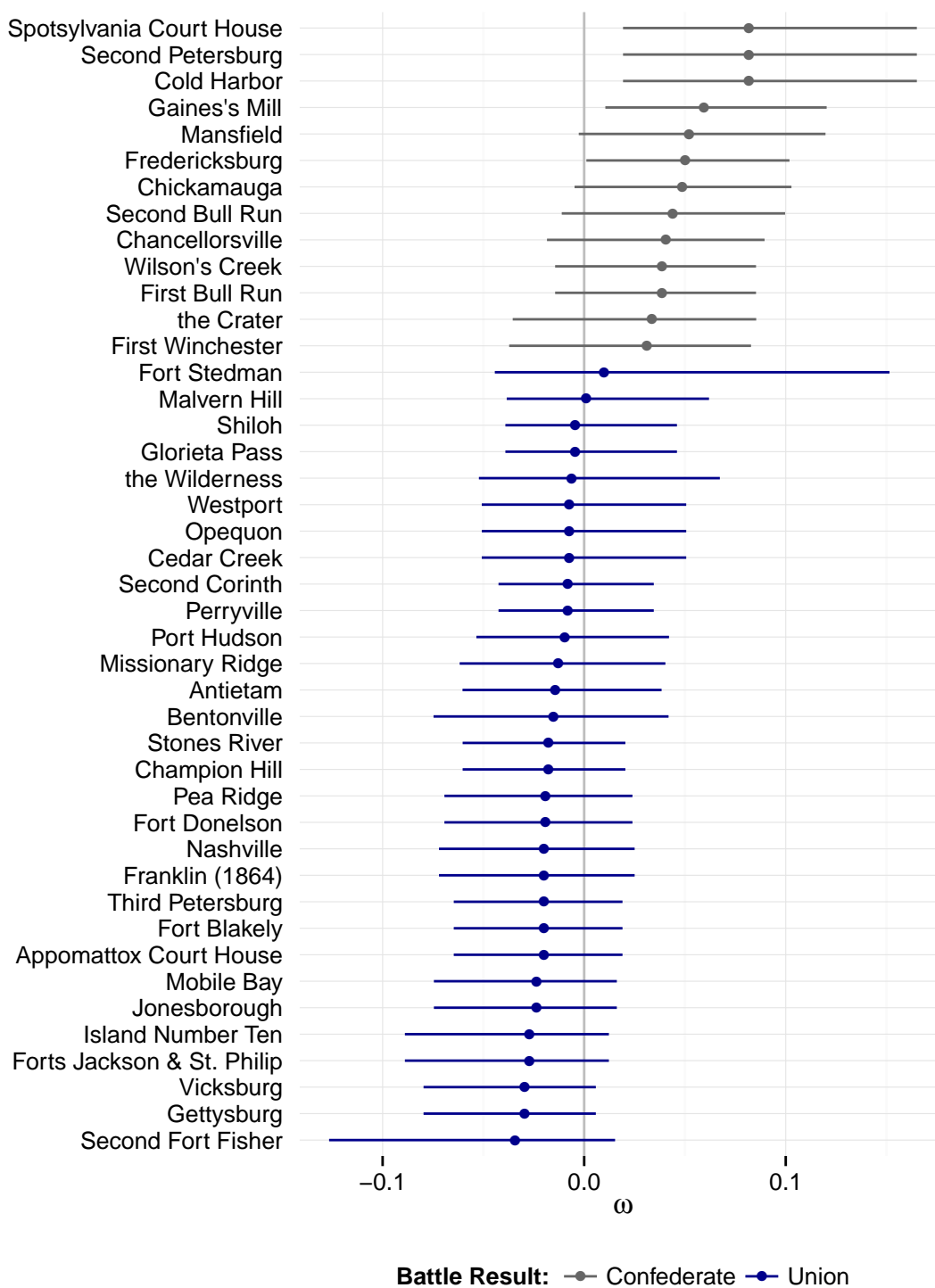


Figure 1.6: Battle effects,  $\omega_b$ , for model  $\mathcal{M}_3$ . These are ordered from largest to smallest. Union victories are in dark blue. Confederate victories are in gray.

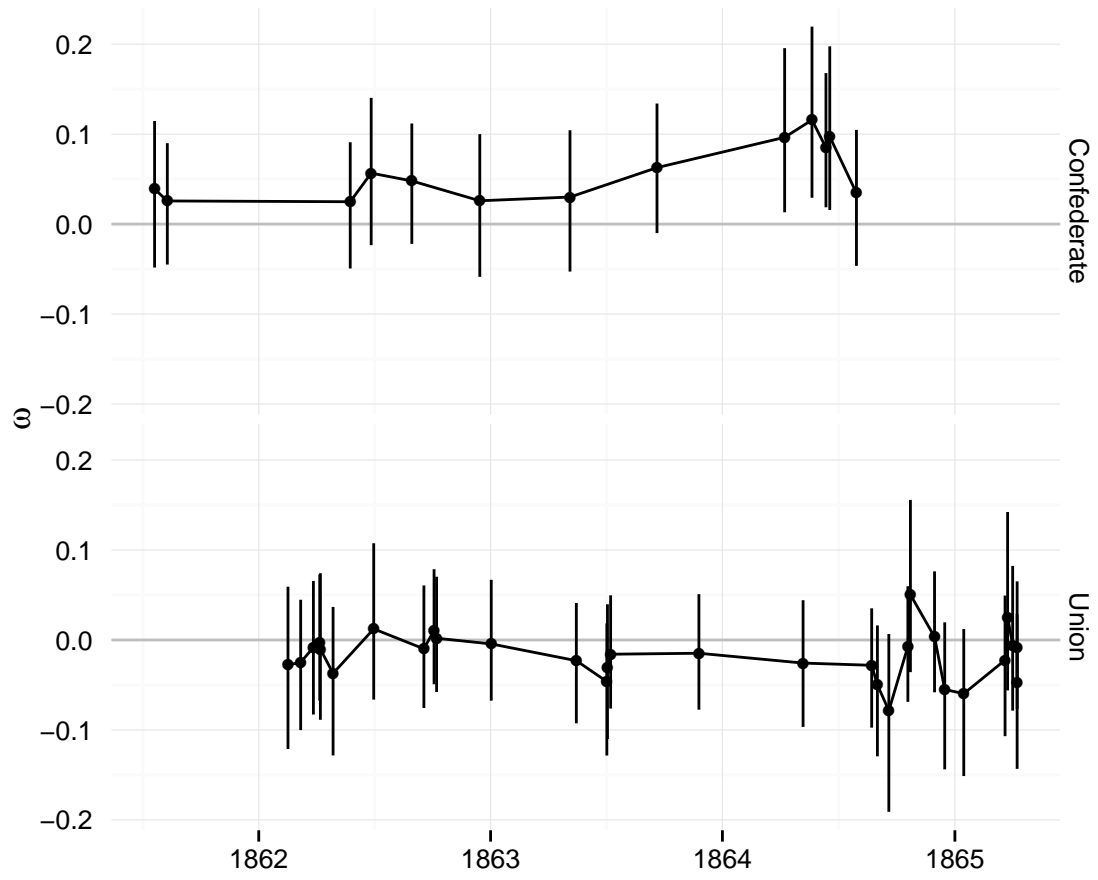


Figure 1.7: Battle effects,  $\omega_b$ , for all battles for model  $\mathcal{M}_4$ , by battle result.

### 1.6.3 Model Comparisons

The third question is “how well do battles explain the variation in bond yields during the period considered?” If the yield movements are taken as a proxy of war expectations, this is equivalent to asking “How well do major battle results (as operationalized here) explain the expected cost and duration of the war?”

To answer this, I compare the models using the root mean squared error (RMSE) and the WAIC, which approximates the expected log predictive density or leave-one-out cross-validation. Table 1.3 shows the RMSE, WAIC, and expected log predictive density (elpd) of all the models. The Widely Applicable Information Criterion (WAIC) (Watanabe 2010) is an information criteria similar to AIC or the Deviance Information Criterion (DIC). Unlike either AIC or DIC is it fully Bayesian, and is calculated by integrating over the posterior distribution rather than using point estimates. The expected log predictive density (elpd) is the expected log probability of a new (out-of-sample) data point. WAIC and other information criteria approximate the elpd by adjusting the log-likelihood with a penalty for the complexity of the model. See Gelman, Carlin, et al. (2013, Ch. 7), Gelman, Hwang, and Vehtari (2014), and Gelman and Vehtari (2014) for more on Bayesian information criteria for model comparison.

The RMSE of the models that include battle effects is lower than that of the benchmark model  $\mathcal{M}_1$ . However, the decrease in RMSE is relatively small.<sup>48</sup> The WAIC and elpd also show that the models with battles improve the predictive ability of the models, but that the improvement is small. Although models  $\mathcal{M}_2$ – $\mathcal{M}_3$  all improve the WAIC (lower values) and elpd (higher values), the improvement is less than the standard errors of these in model  $\mathcal{M}_1$ .<sup>49</sup> As operationalized in this work, battles appear to improve the predictive and

<sup>48</sup>The  $R^2$  of model  $\mathcal{M}_4$  using  $\mathcal{M}_2$  as the null model is only about 0.17.

<sup>49</sup>See Gelman and Vehtari (2014) for method to calculate standard errors for WAIC, and the R package `loo` for an implementation.

explanatory power of the model of bond yields, but that improvement is small.

The relatively small magnitude of the effects of individual battles is seen in Table 1.4. The average effect of a single Union victory is less than 1 standard deviation of the standard deviation of the system error  $\eta$ ; Confederate victories are larger than than 1 sd of  $\eta$ , but less than 2 standard deviations. Considering that there are only 43 battles, but 1469 values of  $\eta_t$ , neither of those magnitudes seems particularly large.

There are several interpretations for the limited explanatory power of battles in this analysis. First, major battles may be the key factor in war, but information diffuses slowly over time, while my model assumes that all effects of a battle occur within a short time of its conclusion. Second, smaller battles and non-battle events may not be as influential as large battles, but cumulatively they may have a large influence on the war outcome. Finally, perhaps improvements in the models employed in this analysis would produce different results. Perhaps I could better model the time-series processes, lag structure of battles, or operationalize combat differently. While I could make the models in this analysis more complicated, a better approach to distinguish between these interpretations is not better models, but better data. I conclude this work by discussing one such approach in Section 1.7.

Model	RMSE	elpd <sub>WAIC</sub>	se(elpd <sub>WAIC</sub> )	WAIC	se(WAIC)
$\mathcal{M}_1$	0.052	785.829	16.479	-1571.658	32.959
$\mathcal{M}_2$	0.051	788.187	15.814	-1576.374	31.628
$\mathcal{M}_3$	0.050	788.828	15.961	-1577.657	31.922
$\mathcal{M}_4$	0.048	791.867	15.640	-1583.733	31.280

Table 1.3: Comparison of models using WAIC and RMSE. RMSE is the root mean squared error of each model. elpd<sub>WAIC</sub> is the expected log predictive density implied by the Widely Applicable Information Criterion (WAIC). The standard errors for WAIC and elpd<sub>WAIC</sub> are calculated as in Gelman and Vehtari (2014).

Model	$\text{sd}(\eta)$	$E(\tilde{\beta}_C)$	$E(\tilde{\beta}_U)$
$\mathcal{M}_2$	0.040	0.050	-0.015
$\mathcal{M}_3$	0.039	0.052	-0.015
$\mathcal{M}_4$	0.037	0.057	-0.018

Table 1.4: The posterior mean of the average effect of Confederate and Union victories relative to the standard deviation of the innovations,  $\eta$ .

## 1.7 Conclusion and Future Work

This work has contributed On its own, its results are limited mainly to questions of this particular war. Combined with similar analyses across multiple wars, it may make clear patterns and

I conclude with some thoughts on future directions for this research. There are several methodological issues that this analysis had to confront. They are “What information about battles did the market receive?”, “When did the market receive that information?”, and “What other non-battle information did the market receive?” In this work, I handled the the first by assuming that only a small set of battles were important enough to have any effect, and that the market only cared about the results (Union or Confederate victory) of those battles. The assumption was that contemporaries would have also seen that same set of battles as the most important, and that they would classify the results of battles similarly to how they were coded in the data. As a measure of when the market received information about battles, I used the day on which the battle was first reported in newspapers. Since I had no data on the non-battle information that would be important to the market, I only modeled it with a fat-tailed distribution in the error term. A means to improve on this methodology is suggested by the way in which I handled the problem of when news reached the market; I consulted contemporary newspapers to determine when information about battles was publicly known. The way in which I used newspaper data to identify when the market received information about the battles could

be extended to address all three questions. A rough sketch of the analysis would be to use computational natural language processing tools, *e.g.* topic models, to extract events or topics from the text of contemporary newspapers and regress those topics on market movements. The information that the market received about battles would correspond to battle-related topics inferred from the newspaper articles. When the market received that information would simply be the days on which those topics appeared in newspapers. The same method would also provide measures of non-battle events. These would be the topics that do not correspond to This approach would provide a set of control variables for non-battle events, without the researcher needing to make arbitrary decisions about which events were important. This sort of analysis, which combines natural language processing to generate events with market movements as measures of war expectations seems a promising approach to understanding the effects of combat on war outcomes where the events are not always either clearly identified or discrete.

## Chapter 2

# Financial Markets and the Onset of the American Civil War

To what extent did financial markets anticipate the start of the American Civil War? How did the market's assessment of war risk change leading up to the war? Which events were associated with these changes? This work answers these questions using data on the prices and yields of U.S. government and state bonds between 1855 and the start of the war in April 1861. Three measures are used to assess the market's beliefs about the risk of war: bond yields, the spread between Northern and Southern state bonds, and the implied default probability of the bonds. These measures show that the financial market largely did not anticipate the conflict, even on the eve of the Battle of Fort Sumter. The financial market assigned a negligible probability of war prior to the election of Lincoln in 1860. After the election, it assigned some probability to a war, but that probability was quite small. Bond prices on April 13, 1861, immediately prior to news of the Battle of Fort Sumter reaching New York City implied only a 4.1 (3.3, 4.9) percent chance of war.<sup>1</sup>

The results in this paper speak to two important international relations theories: the bargaining theory of war (Fearon 1995) and the capitalist peace (Gartzke, Li, and Boehmer 2001; Gartzke and Li 2003). In rationalist theories of war, there are two fundamental explanations as to why two parties would fight a costly conflict rather than come to an

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<sup>1</sup>Interpret "4.1 (3.3, 4.9)" as "mean (2.5% quantile, 97.5% quantile)"



agreement: private information and commitment problems (Fearon 1995; Powell 2006). Although some progress has been made, it is notoriously difficult to test these mechanisms (Reiter 2003). Financial markets may allow some progress in testing these theories. Prices of certain assets whose payoffs are contingent on war outcomes, e.g. sovereign bonds, can be used as estimates of war initiation or outcome conditional on the public information available at the time. From this, the researcher can calculate an implied probability of the occurrence of war. This is useful because different theories of war have different implications as to how uncertain wars are *ex ante*. Wars due to private information should be unanticipated (Gartzke 1999). Since commitment problem wars can occur with complete information nothing requires them to be unanticipated. The point is that many models of war have implications of the *ex ante* probability of war, and prices of some assets can be used to as a measure of this *ex ante* probability. Thus, financial markets may provide another means to test rationalist theories of war initiation. Additionally, the inability of markets to assess the *ex ante* probability of war casts doubt on their ability to signal to the cost of war to potential belligerents as in the capitalist peace theory.

This paper contributes to an economic history literature on the reactions of financial markets to events within the American Civil War. This includes multiple papers on reactions of the financial markets on both sides and Europe to the American Civil War (Schwab 1901; Mitchell 1903; Willard, Guinnane, and Rosen 1996; McCandless 1996; Smith and Smith 1997; Brown and Burdekin 2000; Weidenmier 2002). This paper adds to that literature by analyzing the initiation of the war and by using Northern and Southern state bond data. This paper also adds to the growing international relations literature on the American Civil War, which has often been overlooked in the international relations literature as most work on civil wars focuses on civil wars after 1945 (Poast 2012; Reiter 2009). This work employs methods similar to Haber et al. (2015), which estimates the implied probability of victory by the Confederacy in the American Civil War and the Chinese

government in the Chinese Civil War from bond prices by treating defeat of the issuing side as a default. More broadly, this paper adds to the literature that analyzes the reactions of financial markets to conflict and political events. Other papers which have analyzed the responses of financial markets to conflicts include: Bueno de Mesquita (1990), Hall (2004) and Ferguson (2006) (World War I), and Waldenström and Frey (2008) and Waldenström and Frey (2008) (World War II). Schneider and Troeger (2006) analyze the reactions of major stock market indices to events in Iraq, Israel, and the former Yugoslavia from 1990-2000. Guidolin and La Ferrara (2010) analyzes market reactions to militarized interstate disputes. Several papers look at reactions of markets to the Iraq War (Rigobon and Sack 2005; Leigh, Wolfers, and Zitzewitz 2003; Wolfers and Zitzewitz 2009). Other papers estimate response of financial markets to elections or political change (Jayachandran 2006; Herron 2000).

## 2.1 Financial Market Prices and War Initiation

The objective of this paper is to use the prices and yields of bonds to infer market beliefs about the *ex ante* probability of war in the period leading up to the American Civil War. As such, it is important to establish the theoretical relationship between the observed market prices and yields and these beliefs about war. The price of generic bond is the present discounted value of future cash flows. Since the timely payment of these cash flows is uncertain, the price is an expectation of receiving these flows given current information. Thus the current price of a bond is a quantitative summary of investor's beliefs about the future with regard to payments from the bond. Since war, for reasons discussed below, can have large effects on the repayment of some bonds, the prices of those bonds are effectively predictions given current information of the risk of war. To be able to use a bond, or other financial asset, to make inferences about the risk of war requires that (1)

future payments from that asset must be contingent directly or indirectly on the war, and (2) confounding factors that could affect the future payments are controlled for. This section discusses the how the data and methods used in this work address those criteria.

To the extent that a war would change investors' expectations about the timely payment of future cash flows of these bonds, changes in the yields of bonds can be used to infer changes in beliefs about the probability of war. It is certainly case that war would influence expectations about the cash-flows of the U.S. government and state bonds considered here. A war would influence the credit of the issuers by increasing debt, decreasing revenue and possibly leading to default, and contemporary investors were aware of these risks.

For investors holding U.S. government bonds, secession and war would reduce the ability of the U.S. government to pay its debts both by reducing potential revenue due to the loss of the Southern states and increasing debt to pay war expenditures. The revenue of the U.S. would be reduced by the secession of Southern states due to the loss of tax revenue. In 1860, the Southern states accounted for 25 percent of the non-slave wealth of the United States (Elder 1865, p. 11).<sup>2</sup> The *Merchants' Magazine* wrote in January 1861,

The news [of secession] was also of a character to affect disastrously American credit, since it showed a decline of 13 per cent in United States government stocks, solely from fear of disunion, which event would reduce the States to the condition of the bankrupt, "disorderly houses" of South America, and leave the separate States a prey to every spoiler. (Homans and Dana 1861, p. 79)

A war would also increase the debt of the United States as military expenses would greatly increase government expenditures, and those additional expenditures would almost cer-

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<sup>2</sup>GDP was not invented until WWII. During the civil war era, the census estimates of personal and property wealth were used as a measure of the total economic activity and also explicitly used to assess the ability of the U.S. to repay the debt (Elder 1863; Elder 1865).

tainly be mostly funded by debt. While the U.S. government only infrequently issued debt during the 19th century, the exception was to fund wars. The U.S. government issued debt to fund the War of 1812 and the Mexican-American War of 1846-1848. Although the war would continue much longer than initial estimates, even the initial estimated costs of war represented a large increase in government debt. The amount of expenditures estimated for the fiscal year from July 1, 1861 to June 30, 1862 in the July 4, 1861 *Report of the Secretary of the Treasury* was 319 million (U.S. Treasury 1861b, p. 5), an increase of 369% (251 million) over the estimated expenditures from the previous years' report (U.S. Treasury 1860, p. 6). The *Annual Report* also suggested that the revenue to pay for expenditures be split as 80 million in taxes, and 240 million in the issue of new loans (U.S. Treasury 1861b, p. 6). The Act of July 17, 1861, the first act passed by Congress to fund the war, and it authorized the issue of \$250 million in debt (U.S. Treasury 1863, p. 44), an increase of 178%.<sup>3</sup> By comparison, the debt at the end of the War of 1816 was 127 million (Elder 1863, p. 29) and only 49 million in debt was issued to fund the Mexican American War.<sup>4</sup>

For Northern investors holding Southern state bonds, the immediate risk was that Southern states would default on interest payments. After the initiation of the war, Southern states immediately ceased interest payments to Northern creditors, starting with coupon payments due on July 1st, 1861 (*The Bankers Magazine* 1860, p. 947; *The Bankers Magazine* 1862, p. 159). Investors almost certainly anticipated this as there was concern that the Southern states would default on their January 1861 payments: "the decline [in the price of Virginia, Tennessee and Missouri bonds] ...was further stimulated by the political condition of the country and doubts as to the prompt payment of interest by these States." (*The Bankers Magazine* 1860, p. 756) After the initiation of the war, the concern over default on interest payments was the primary reason cited for the decline in the

<sup>3</sup>The debt on July 1, 1861 was 90 million (U.S. Treasury 1861b, p. 23).

<sup>4</sup>During the Mexican-American War, the Loans of 1846, 1847 and 1848 used to fund the war totaled \$49 million (U.S. Treasury 1863, p. 42).

prices of Southern state bonds: “These low values [prices of Southern state bonds] have partly been caused by the fear that these States will not be disposed to provide for the payment of their semi annual interest in July next.” (*The Bankers Magazine* 1860, p. 947) After Southern states had stopped paying interest, Northern investors almost certainly believed that as long as the war would continue, they not receive interest payments. In November 1861, the *Bankers’ Magazine* wrote, “...but faint hopes are entertained of their [Southern states] paying their interest for some years to come.” (*The Bankers Magazine* 1862, p. 559) After the war, most Southern states repaid the debt issued before the war, but defaulted on all debt issued while part of the Confederacy and some defaulted on debt issued by reconstruction governments (Randolph 1931; Ratchford 1941). How the Southern state governments would have handled the repayment of debt if the Confederacy had won the war is unclear. At best, the Southern states would resume paying interest and principal soon after the end of the war, possibly with compensation for missed payments. At worst, they would completely default on their obligations to Northern creditors. That the prices of Southern state bonds never came close to zero, suggests that investors’ beliefs put positive probability on those states being reincorporated into the Union or that they would resume payments on their debt after the war even in the event of a Confederate victory. An additional means by which the war could affect the creditworthiness of Southern states is, although, at first the ability of the states to pay their debts if they so choose was not much in doubt, an extended war would result in both more debt issued by Southern states and wealth destroyed, both of which would affect the ability of the states to eventually pay the debt they had issued. Thus, holders of Southern state bonds were particularly exposed to war risk, and almost certain to suffer losses immediately at the onset of a conflict.

In order to be able to confidently make inferences about the risk of war given bond prices, alternative confounding effects need to be controlled for. This work accounts for

these confounding effects in several ways. First, it focuses on the period close to the start of the war. That the effects are due to changes in war risk factors is likely higher in the period closer to the start of war, than in periods further removed from it. For example, a large price change in the week before the Battle of Fort Sumter is more likely to be related to changes in the expectation of war than a price movement in 1849, for example. Second, multiple measures are used: the yield spread between Southern and Northern state bonds and the implied probability of war from the bond prices. Third, within each of those measures multiple bonds prices are used to average over idiosyncratic shocks to bonds. Additionally, the south-north yield spread also controls for shocks which effect all bonds. Fourth, periods which have probable non-war explanations for changes in bond risk, e.g. the Panic of 1857, are either not included in the analysis, or the changes in bond risk are not attributed to changes in war beliefs.

Although U.S. government and state bonds were not direct prediction markets of the onset of war, since their expected payment of their future cash flows would be heavily influenced by the onset of war, they can be used to make inferences about investors' beliefs about the onset of war.

## 2.2 Data

The financial market data used in this work are the prices and yields of U.S. government and state bonds, both Northern and Southern, traded in New York City from 1855-1861.<sup>5</sup> The data consist of bonds issued by nine states (Georgia, Indiana, Kentucky, Louisiana, Missouri, North Carolina, Ohio, Pennsylvania and Virginia) and two bonds issued by the U.S. government (Sixes of 1868, Fives of 1874).<sup>6</sup> The set of bonds included here are those

<sup>5</sup> The data used here is available at [https://github.com/jrnold/civil\\_war\\_era\\_findata](https://github.com/jrnold/civil_war_era_findata). The particular dataset used in this section is `bankers_magazine_govt_state_loans_yields_2.csv`.

<sup>6</sup>In the financial jargon of that era, Fives or 5's refers to coupon bonds paying 5 percent interest, and Sixes or 6's to bonds paying 6 percent interest.

that were regularly quoted in *The Bankers' Magazine and Statistical Register*, a monthly business magazine.<sup>7</sup> Prices of these bonds are available at approximately a weekly frequency from July 1, 1855 onward.<sup>8</sup> All of these bonds are coupon bonds paying 5–6 percent per annum semi-annually with longer maturities. The U.S. government bonds matured in 1868 and 1874, while the state bonds variously matured between 1869 and 1892. The data only include two U.S. government bonds because these were the only two large debt issues of the U.S. government outstanding at the start of the the civil war. Between the War of 1812 and the Civil War, the U.S. government only infrequently issued debt. The two government bonds in the data were only issued in extraordinary circumstances: the Sixes of 1868 were issued in 1847 and 1848 to pay for the Mexican-American War, while Fives of 1874 were issued in 1858 due to a funding shortfall as a result of a financial crisis, the Panic of 1857 (Homer and Sylla 2005, pp. 297-302; De Knight 1900, pp. 72-73,76) The state bonds in the data were issued by Midwest and Southern states which issued debt to fund infrastructure projects such as railroads and canals. During this period, almost no New England states issued debt, with the notable exception of Massachusetts, whose bonds were the essentially the risk-free asset of the period, but they were not regularly quoted in the *Bankers' Magazine* (Martin 1871, p. 86; Homer and Sylla 2005; Macaulay 1938; Officer 2003, pp. 80-85).

The *Bankers' Magazine* quotes the price of bonds, but this work will primarily use the yield to maturity.<sup>9</sup> Yields to maturity are calculated for each bond using the quoted prices

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<sup>7</sup>After constructing this data I became aware that Dwyer, Hafer, and Weber (1999) constructed a dataset of state bonds from the source. The data used here expands upon the price data presented in the original source, by matching each quoted price in the source with metadata about the bond(s) it represents. This dataset removed discrepancies, converts prices to gold dollars after 1862, includes data on bond cash flows, accrued interest, and calculates yield to maturity and other statistics for each bond.

<sup>8</sup>The data come from Volumes 10-20. The full dataset has price data through 1865.

<sup>9</sup>It is unclear whether the prices are bid or ask. In most cases, the prices quoted are dirty (including the next coupon payment). The data were cleaned to account for the inclusion or removal of the coupon price from the data.

and the maturities and cash flows.<sup>10</sup> One difficulty in calculating yields to maturity for these bonds is that for most state bonds, the *Bankers' Magazine* would quote a price for bonds of a state without reference to specific issue or redemption date for state which had multiple issues of bonds with various redemption dates. For example, a price is given for “Virginia Sixes”, but Virginia issued multiple coupon bonds paying 6 percent semi-annually with redemption dates between 1885 and 1890. Using several sources, the prices are matched to a set of plausible bond issues.<sup>11</sup> Yields to maturity are calculated for all issues, and the yield to maturity corresponding to the bond price series is calculated by averaging over the yields for the issues.

## 2.3 Yields of Government and State Bonds, March 1855–April 1861

This section discusses the yield data of the U.S. government and state bonds discussed in the previous section. The purpose of this section is to provide the reader and understanding of the data and the context to interpret the more specific measures of the risk of war, the spread between Southern and Northern state bonds and the implied probability of war, which will be presented in the following sections.

Figure 2.1 plots the yields of all the bonds in the data from July 1, 1855 to 1861-06-01. By visual inspection, these time series can be divided into five periods: July 1855 to October 1857, November 1857 to January 1858 (the Panic of 1857), February 1858–October 1860, November 1860–April 14, 1861, and after the start of the war with Fort Sumter (April 14, 1861).

From July 1855 through October 1857, yields were low and stable. The yields of the

<sup>10</sup> For simplicity, yields to maturity are calculated using continuous compounding and actual/actual day count convention with no holidays. Thus the yield to maturity for a bond with price  $p$ , cash flows  $\{c_i\}_{i=1}^n$  at times  $\{t_i\}_{i=1}^n$  is  $y$  such that  $p = \sum_{i=1}^n c_i e^{-y t_i}$ .

<sup>11</sup> See the data for details.



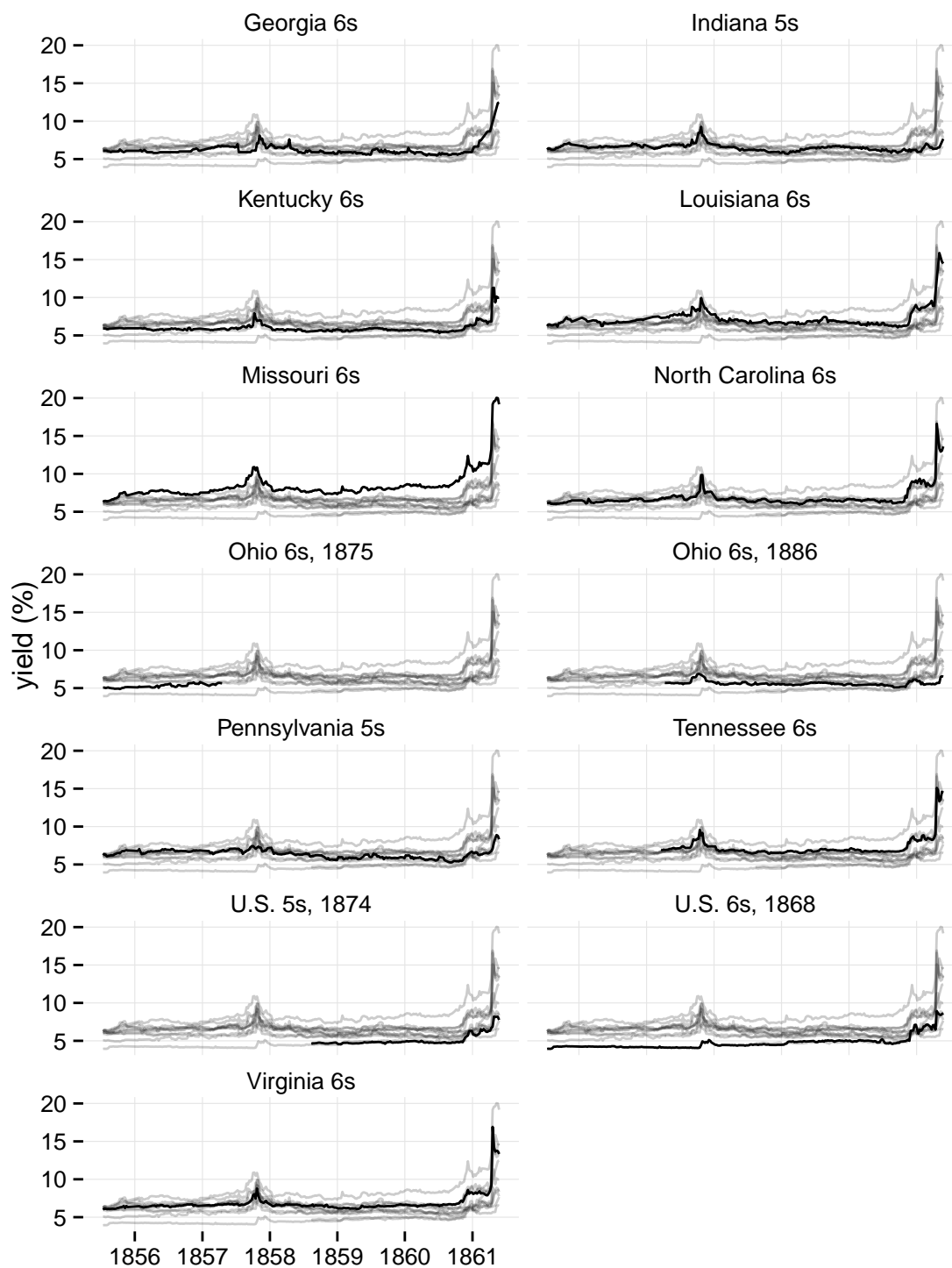


Figure 2.1: Yields on U.S. government and state bonds, July 1, 1855–June 1, 1861

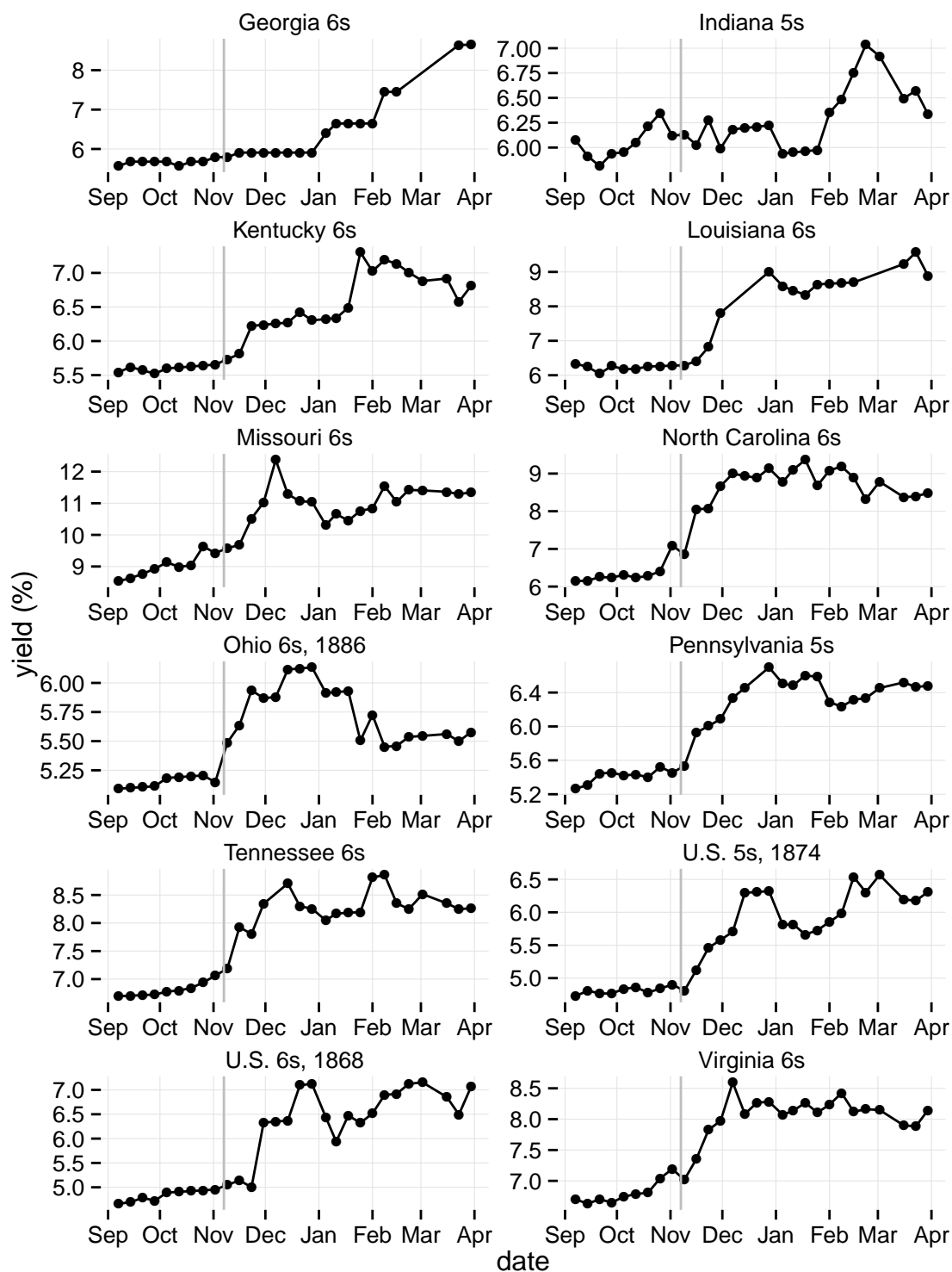


Figure 2.2: Yields on U.S. government and state bonds around the Presidential Election of 1860. The vertical lines indicate the date of the election, November 6, 1860.

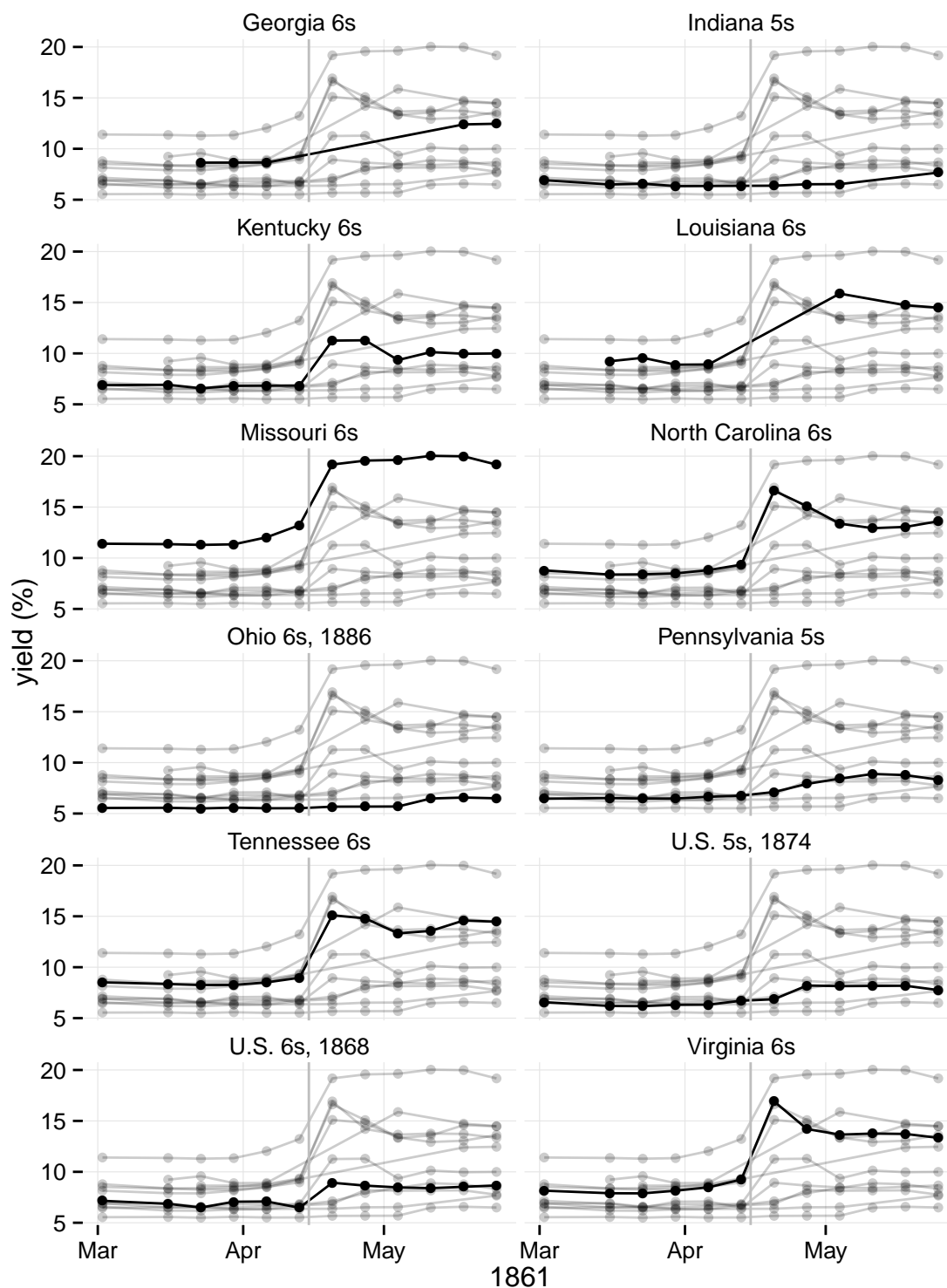


Figure 2.3: Yields to maturity of U.S. government and state bonds before and after the Battle of Fort Sumter. The vertical lines indicate the date when news of the surrender of Fort Sumter reaches New York, April 15, 1861.

	Apr 13, 1861	Apr 20, 1861	Change	% Change
Georgia 6s	8.65	12.39	3.75	30.23
Indiana 5s	6.36	6.38	0.01	0.21
Kentucky 6s	6.85	11.26	4.41	39.18
Louisiana 6s	8.91	15.87	6.96	43.85
Missouri 6s	13.23	19.18	5.95	31.04
North Carolina 6s	9.35	16.61	7.27	43.74
Ohio 6s, 1886	5.52	5.67	0.15	2.64
Pennsylvania 5s	6.76	7.11	0.35	4.91
Tennessee 6s	8.97	15.10	6.13	40.61
U.S. 5s, 1874	6.71	8.20	1.49	18.21
U.S. 6s, 1868	6.47	8.93	2.46	27.54
Virginia 6s	9.24	16.92	7.68	45.40

Table 2.1: Yields to maturity of U.S. government and state bonds in the week before and after news of the surrender of Fort Sumter. For most bonds the before and after columns correspond to April 13 and April 20, 1861, the last price before and the first price after news of the surrender of Fort Sumter reached the market. Prices for Georgia Sixes and Louisiana Sixes had missing data around Fort Sumter, so the closest available prices are used. The response of the prices of U.S. 5s of 1874 was lagged, so the after column is the price on April 27.

U.S. Sixes of 1868 were between NA and NA percent. These were the lowest yields on U.S. government bonds up to that point (Homer and Sylla 2005, pp. 282-283). The yields were even lower than the traditionally safe Massachusetts and Boston city bonds, and less than one percentage point higher than the yield on 3% British consols, which were considered the safest global asset during that period.<sup>12</sup> State bond yields were also relatively stable, although having higher yields than government bonds; with average yields between 5.29 (10) and 7.57 (8). State bonds had higher yields than U.S. government because were perceived to be relatively riskier assets, since between 1841 and 1842 nine states or territories defaulted on their debts (English 1996).<sup>13</sup> Overall, the relatively low and stable yields suggest that the market did not assign much, if any, risk to war.

<sup>12</sup>The average annual yields on British consols 1855-1857 were between 3.27 and 3.31 (Homer and Sylla 2005, p. 193).

<sup>13</sup>Of the states with bonds in the data, Indiana, Pennsylvania and Louisiana defaulted in the 1840s (English 1996, p. 265).

From October 1857 to January 1858, yields spiked during the Panic of 1857. The Panic of 1857 was a recession and financial crisis that included the failure of a major insurance company, the failures of banks and railroads, and the suspension of specie payments by banks in New York City. The causes of the panic were not directly related to political risks of secession or war, but were likely a decline in European demand for U.S. trade or insufficient specie supply to meet demand due to a decline in gold production (Dewey 1918, pp. 263-265; Homer and Sylla 2005, pp. 277,299; *The Bankers Magazine* 1857, p. 337). The *Bankers' Magazine* "Notes on the Money Market" sections of the time do not mention domestic political issues during the Panic of 1857, attributing it to a decline in foreign trade and bad banking (*The Bankers Magazine* 1857, p. 337). Thus, while yields reached high levels the panic, these high yields are almost certainly not attributable to an increase in war risk.

After the Panic of 1857, from February 1858 to November 1860, yields of bonds returned to levels similar to the period prior to the Panic of 1857. While the yields of U.S. government bonds were also stable during this period, their yields were approximately 0.5 percentage points higher than they had been before the panic. The yields of state bonds were also similar to or in some cases lower than in the period prior to the panic.

Lincoln's victory in the presidential election on November 6, 1860 precipitated a rise in yields, especially for Southern state bonds.<sup>14</sup> The yields of bonds before and after the election are plotted in Figure 2.2. The market partially anticipated the political issues that would arise with Lincoln's election, as there was a slight increase in yields in October 1860. This rise was partially due to a monetary contraction as the economy went into recession (*The Bankers Magazine* 1860, p. 413), but also the anticipation of a Republican

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<sup>14</sup>Lincoln's victory was known the next day: "It is too certain to speak confidently of detailed results of yesterday's elections. But there can be no reasonable doubt that the Republicans have elected their president." ("The Election Lincoln Triumphant" 1860)

victory in the election.<sup>15</sup> The *Bankers' Magazine* described the market,

The market for Government and State loans during the month has been dull. Prices have in most instances tended downward and a material decline is perceptible in some of the most prominent securities. The principal reasons for this seem to be the political excitement. (*The Bankers Magazine* 1860, p. 414)

The market reaction was likely in response the Republican victory in the Pennsylvania gubernatorial election on October 9:

The election in Pennsylvania early in October and the prospect which it gave of the election of the Republican candidate for President in November caused the timid to pause. ... The political complications of the day were the sole cause relied on by the bears and totally ignoring every other element in the market the pressure to sell under the influence of this sudden fear of some dreadful calamity became irresistible. ... The partisan presses aided the decline as far as they could and did their best to make it a panic by spreading abroad the most gloomy picture of the state of feeling at the South and directly prophesying ruin complete and full in the event of Mr. Lincoln's election. (*The Bankers Magazine* 1860, pp. 476-77)

While the market believed it was likely that Lincoln would win the election and aware that Lincoln's election cause political risk, the response observed in the yields was minimal.

It was not until after Lincoln's election that yields increased dramatically. However, this increase did not occur immediately after Lincoln's election victory, as would occur with Fort Sumter, but yields rose over the course of the following month, with some variation between bonds in the timing of the rise. However, by the end of January 1861, Southern and border state yields had increased 1–3 percentage points, and the yields on U.S.

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<sup>15</sup>The U.S. was in a slight recession prior to the start of the American Civil War, from October 1860 to June 1861, according to the [NBER](#).

government bonds had increased 1–1.5 percentage points. The initial increase in yields was due a small financial crisis that started when Southern banks started denying credit to Northern clients and withdrew specie from Northern banks (Homans and Dana 1861, p. 76). However, the response of banks mitigated the severity and duration of the crisis, and it was largely resolved by November (*The Bankers Magazine* 1860, pp. 539-542). However, political risk remained a concern:

The money market during December recovered somewhat from the worst effects of the panic. The action of the banks had been prompt and energetic, and the relief afforded by their largely expanded line of loans and discounts averted the worst consequences of the panic. The want of confidence in the political future prevented however a complete recovery and the results of prostrated credit and destroyed confidence were everywhere discernible. (*Bankers' Magazine*, January 1861: *The Bankers Magazine* 1860, p. 541)

The only speck in the horizon is the threat of secession in the South. We would be the last to give countenance to the libel upon the South, that the majority of any one State assented to the proposition of secession. The permanent social, commercial and financial interest of the various sections of the whole country are too strongly identified—too strongly bound together—to induce the loss of any one State. (*Bankers' Magazine*, December 1860: *The Bankers Magazine* 1860, p. 419)

While yields increased during this period, the yields on most bonds were less than their peak during the Panic of 1857.

Many of the bond series appear continue rising until dates near the ordinance of secession by South Carolina on December 20, 1860. In the months between the election and

the start of the war, yields remained high, but also variable as they responded to political events (Hunt et al. 1860, p. 718; Homans and Dana 1861, pp. 78,198,416,677; *The Bankers Magazine* 1860, pp. 413,481,669,756,838).

Comparing the individual series of the Southern state bonds, the market appeared to have treated the slave-holding states similarly, regardless of whether they had formally secede prior to the start of the war. Although Louisiana did not secede until January 26, its yield had already peaked near 9 percent at the start of January. Virginia, Tennessee and North Carolina did not secede until after the start of the war, yet the yields on all of these states rose between November and January.<sup>16</sup>

Although bond yields had risen after the election of Lincoln as the risk of civil war increased, the formal start of the war with the Battle of Fort Sumter was followed by a jump in yield much larger than had been observed after the election or during the Panic of 1857. The start of the war resulted in a “ depreciation in Southern State stocks exceeding any decline ever witnessed at the stock board of this city” (*The Bankers Magazine* 1860, p. 919). Unlike the election of 1860, in which bond yields rose over a period of weeks, prices fell and yields increased immediately after news of Fort Sumter reached the market. Figure 2.3 plots the yields around the Battle of Fort Sumter (March–June 1861). Table 2.3 tabulates yields in the week immediately before and after news of the surrender of Fort Sumter reached the New York market. Southern state yields rose between 6.1 (Tennessee) and 7.7 (Virginia) percentage points (41 and 45 percent) between April 13th and 20th. The yields of U.S. sixes also rose, but only by 2.5 percentage points, while Northern state bond yields were almost unchanged.<sup>17</sup> Since the yield of the bond incorporates expectations about future payoffs and war would certainly affect those payoffs, the extreme rise in

<sup>16</sup>Georgia is an exception; its yields noticeably increase after the election until January 1861 near is declaration of secession (January 19, 1861). This may be because the Georgia Sixes were illiquid; in the data, there are long sequences in which the price of the Georgia bonds does not change, and it is often traded in multiples of 5 or 10.

<sup>17</sup>The price of the U.S. Fives of 1874 did not change noticeably until the following week, April 27, 1861



yields after Fort Sumter imply that the probability the market assigned to war was low. It is likely that even on the eve of the war, the market anticipated some sort of peaceful settlement to the crisis. In April 1860, *The Bankers' Magazine* wrote, "The general condition of commercial and financial affairs still turns upon the uncertain political future. The fears of civil war, that at one time were entertained in certain quarters, have subsided, if not altogether disappeared, under the influence of passing events." (Homans and Dana 1861, p. 413) while *The Merchants' Magazine* wrote,

The securities of the Border States fluctuate from day to day as the tone of dispatches from Washington leans toward peace or war. There is undoubtedly a great want of confidence as to the final course which those States will pursue and it is the strong hope which is entertained of their remaining loyal which alone keeps up their price. (*The Bankers Magazine* 1860, p. 838)

The magnitude and speed of the reaction of yields with respect to the Battle of Fort Sumter implies that the market's hope that Southern states would remain loyal, must have been very strong. The reaction of the market to the Battle of Fort Sumter also implies that the market considered it the start of hostilities, even though large-scale engagements would not occur for months with the First Battle of Bull Run

## 2.4 South-North Yield Spread

For the case of the American Civil War, the spread (difference) between the average yield of the Southern and Northern state bonds is a measure of the market's expectation of war.<sup>18</sup> An increased risk of war increases the spread between Southern and Northern bond yields, because war would make Southern state bonds relatively more risky than North-

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<sup>18</sup> The spread between Southern and Northern bonds is approximately the difference in default risk between the bonds, when the recovery rate is 0. This follows from Equation (2.4).

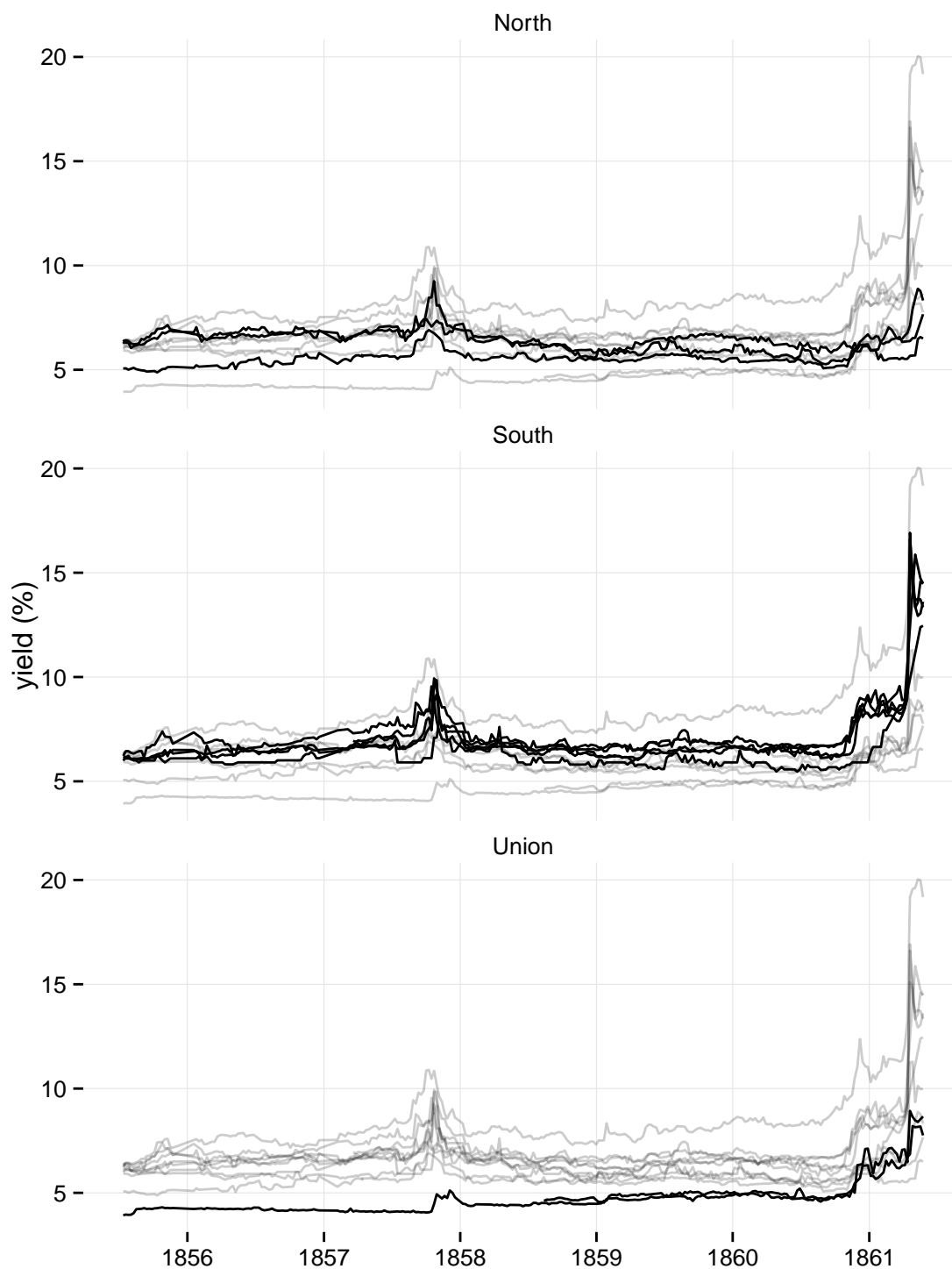


Figure 2.4: Yields of U.S. government and state bonds, by region, July 1, 1855 to June 1, 1861.

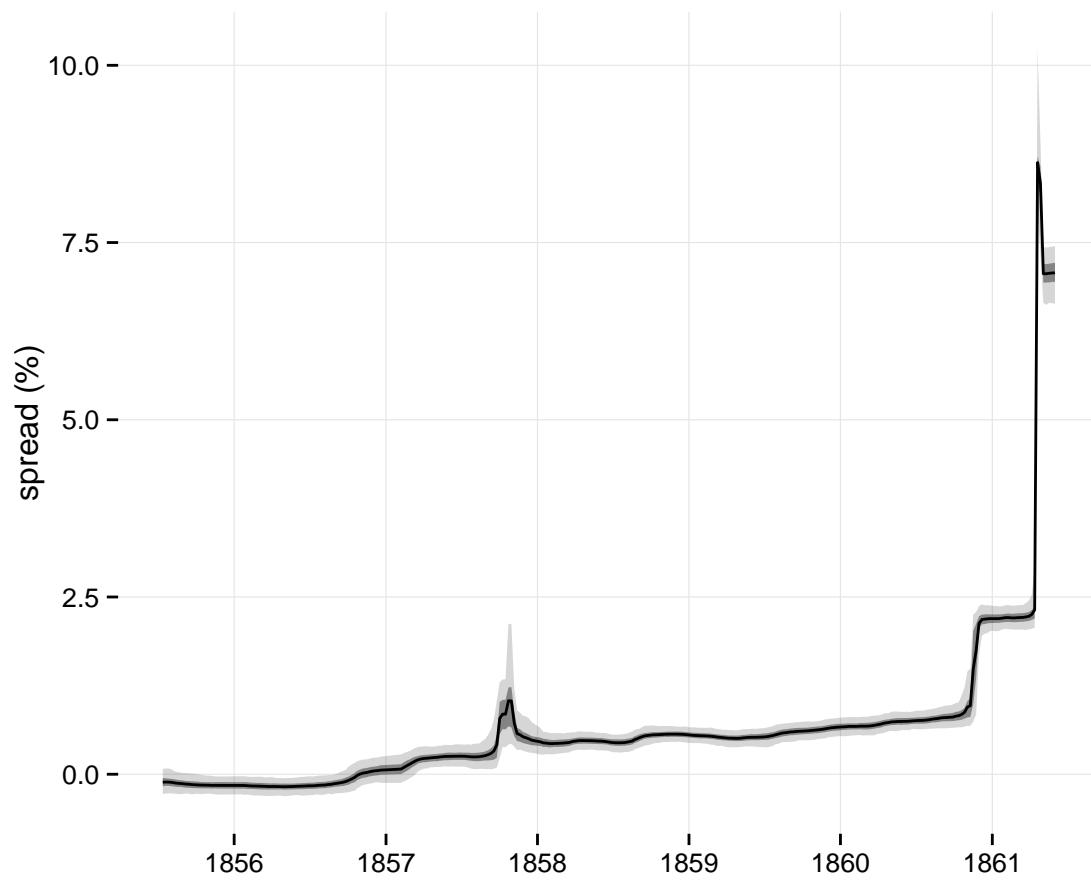


Figure 2.5: Yield spread between Southern and Northern state bonds. The line is the posterior mean, the dark ribbon is the 50% credible interval, the light ribbon is the 95% credible interval.

ern state bonds. Although war could increase the riskiness of all state bonds, there are at least two ways in which war would increase the risk of Southern bonds relative to Northern state bonds. First, and most immediately, Southern states would immediately stop the payment of interest to Northern investors at the start of conflict. As discussed in section 2.1, the suspension of interest did happen and was anticipated by investors. Second, less immediately, it is likely that a war would be fought primarily in Southern states, and thus Southern states would incur more destruction that could hinder their ability to repay debt in the future.<sup>19</sup> The differential effect the war on the risk of Southern and Northern bonds was observed in the changes in the yields of these bonds after the Battle of Fort Sumter, as shown in Table 2.1. In the week after Fort Sumter, the yields of Southern states increased 3–8 percentage points, while the yields of Northern states increased only 0.01 to 0.35 percentage points. An advantage of using the spread rather than the yields of each bond, is that it controls for shocks affecting all state bonds as well as averaging over idiosyncratic shocks to individual states. There can still be regional differences in the risk of Southern and Northern state bonds due to factors other than war risk. However, focusing on changes in the spread rather than the level of the spread controls for any of these regional factors which are constant over the time period considered.

The data used to calculate the south-north yield spread consists of the yields of Northern (Indiana, Ohio and Pennsylvania) and Southern (Georgia, Louisiana, North Carolina, Tennessee and Virginia).<sup>20</sup> Let  $y_{j,t}$  be the yield for bond  $j$  at time  $t$ . The yield for each bond is modeled as,

$$y_{j,t} \sim \mathcal{T}_v(\theta_t + \psi_t \text{south}_j, \sigma) \quad (2.1)$$

where  $\mathcal{T}_a(b, c^2)$  is the Student's  $t$ -distribution with degrees of freedom  $a$ , location  $b$  and

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<sup>19</sup>In the estimates of the economic cost of the American Civil War in Goldin and Lewis (1975), physical capital loss was the largest component of the direct cost of the war to the Confederacy (Goldin and Lewis 1975, p. 308)

<sup>20</sup>The border states of Kentucky and Missouri are not included in these calculations.

scale  $c$ , and  $\text{south}_j$  is a binary variable indicating whether the bond was issued by a Southern state.

Since the number of bonds in each period is relatively small, the parameters  $\theta$  and  $\psi$  are smoothed over time with random walk priors. However, the method used to pool rates over time cannot overly smooth the data since, otherwise the method would obscure the effect of important events on the spread. In order to achieve behavior similar to a change-point model, this paper uses a shrinkage prior for the first difference of  $\psi$  and  $\eta$ . Shrinkage priors are the Bayesian equivalent to penalized estimate like lasso (Tibshirani 1996; Park and Casella 2008). The particular shrinkage prior employed horseshoe prior distribution is used (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2009). The horseshoe prior has properties similar to a spike-and-slab prior while being continuous. This is similar to estimating change points in the parameter, while estimating the number of change points from the data.<sup>21</sup>

$$\theta_t \sim \mathcal{N}(\theta_{t-1}, \sigma^2 \tau_\theta^2 \lambda_{\theta,t}^2) \quad \lambda_{\theta,t} \sim \mathcal{C}^+(0, 1) \quad (2.2)$$

$$\psi_t \sim \mathcal{N}(\psi_{t-1}, \sigma^2 \tau_\psi^2 \lambda_{\psi,t}^2) \quad \lambda_{\psi,t} \sim \mathcal{C}^+(0, 1) \quad (2.3)$$

where  $\mathcal{N}(N)a, b^2$  is the normal distribution with mean  $a$  and scale  $b$ ,  $\mathcal{C}^+(0, a)$  is the half-Cauchy distribution with scale  $a$ .<sup>22</sup> In summary, this set of priors method attempts to estimate the parameters, e.g.  $\theta$ , where the first differences,  $\theta_t - \theta_{t-1}$ , are sparse (mostly zero) while not imposing assumptions about the sparsity of the differences. The model is estimated using Bayesian methods. The posterior distribution is sampled with the No-U-Turn Sampler implemented in Stan (Stan Development Team 2014; Hoffman and Gelman

<sup>21</sup>This approach is similar to a Bayesian version of the fused lasso Tibshirani et al. (2005) using the horseshoe prior distribution rather than a Laplace distribution for the reasons discussed in (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2009).

<sup>22</sup>Note that variable and parameter names are only unique for this section; variables and parameters with the same name in different sections do not necessarily correspond to the same thing.

2014).

Figure 2.5 plots estimates from the posterior distribution of the south-north spread ( $\psi_t$ ) for all periods. For most of the period, the spread was positive, meaning that Southern state bonds were riskier on average than Northern state bonds. The spread is increasing over most of the period before settling down immediately before the election. Between the start of the period (July 1, 1855) and prior to the election of 1860 (November 2, 1860), the spread increased by 1.06 (0.73, 1.45), from -0.11 (-0.28, 0.08) to 0.95 (0.68, 1.45). The spread increased even though the yields of Southern bonds remained largely stable or even slightly declined, because the yields of Northern bonds, especially Indiana and Pennsylvania, declined much more. The Panic of 1857 has some effect on the spread, but less than for the yields, and the uncertainty in the estimated spread increases during that period. The spread increases sharply after the election of Lincoln, from November 2, increasing by 1.23 (0.7, 2.4) percentage points to 2.18 (1.95, 2.4) on December 7, 1860. The spread is stable until the the Battle of Fort Sumter, when it increases 6.31 (5.47, 7.88) percentage points to 2.32 (2.07, 2.91).

Thus the risk of Southern bonds relative to Northern bonds was increasing over the period, perhaps due to increasing assessments of the risk of war. The market's beliefs about the risk of war jumps after Lincoln's election, but the jump in spreads is about 5 times smaller than the spread that is observed after war did start.

## 2.5 Probability of War

While the bond yields and spreads between Northern and Southern bonds can indicate changes in war risk, they are not directly interpretable as the probability of war. However, the market's beliefs about the probability of war can be inferred from bond prices and

yields by treating war as a default event (Haber et al. 2015).<sup>23</sup> A simple approximation for the probability of risk-neutral default probability per year of a risky bond given its yield is,

$$\text{Pr}(\text{default}) = \frac{y - r}{1 - R}, \quad (2.4)$$

where  $y$  is the yield of the risky bond,  $r$  is the yield of a risk-free bond with comparable cash flows, and  $R$  is the the recovery value of the bond in the event of the default as a proportion of the face value of the bond (Hull, Predescu, and White 2004).<sup>24</sup> Equation (2.4) can be used to estimate the probability of war if  $r$  and  $R$  are properly defined.

When estimating the probability of war, the risk-free interest rate  $r$  should be defined to be the yield of a comparable bond with no risk of war. In this case,  $r$  will not necessarily be risk-free — it should include any risk not due to war, but exclude risk due to war. This work will use is the yield of the same bond for an earlier period in which the risk of war is approximately as  $r$ .

The recovery rate  $R$  is the amount, as a proportion of par value, that the investor receives in the event of a war. If war does occur, then the value of  $R$  to use is straightforward because it is known — it is the price of the bond observed at the start of the war. method The implied probability of war is calculated for all Southern and border state bonds (Georgia, Kentucky, Louisiana, Missouri, Tennessee and Virgie) For each bond  $j$ , the implied probability of war at time  $t$ ,  $\pi_t$ , is

$$\pi_t = \frac{y_{j,t} - r_j}{1 - R_{j,t}} \quad (2.5)$$

<sup>23</sup>See Fons (1987), Merrick (2001), and Chan-Lau (2006) for derivations of default probabilities from bond prices.

<sup>24</sup>Waldenström and Frey (2008, p. 115) propose a similar method, that when the war is observed the probability of war is the proportion of the yield after war to the yield prior to war. That method is not directly derived from a cash flow model and produces higher estimates of the probability of war. Haber et al. (2015) use a cash flow method to calculate the probability of victory in wars (Confederate victory in the American Civil War, and government victory in the Chinese Civil War) using a cash flow pricing model of bonds, but assuming a recovery rate of 0.

For each bond,  $r_j$  is its average yield from July 1, 1855 to September 1, 1857.<sup>25</sup> Although there may have been some risk of war during this period, the yields were low (see section 2.3), and thus it is unlikely that war risk could not have been high. While this estimate of the war-risk free rate accounts for the risk heterogeneity between the bonds, it makes the strong assumption that  $r_j$  is that it is constant over time. Since it does not allow the war-risk free rate to vary over time, the estimates of the probability of war can be confounded by any factors which affect  $r$ . In order to reduce this possibility, I only consider the period between the Panic of 1857 and the Battle of Fort Sumter, when there are no major non-war financial crises and changes in war risk are most likely.

For each bond, the value of  $R_{j,t}$  is the approximately the price of the bond immediately after the Battle of Fort Sumter. More precisely, the  $R_{j,t}$  is the present value of bond  $j$  at time  $t$  calculated using the average yield of the bond April 15, 1861 to May 18, 1861.<sup>26</sup> The value of  $R$  incorporates expectations about the financial risk posed by expected course of the war, which is key to allowing separating the war risk due to the probability of war and the risk war will have on the bond, conditional on war occurring.<sup>27</sup>

Figure 2.6 displays the implied probabilities of war for each bond calculated with Equation (2.4). There are three things to note about the implied probabilities in that figure. First, the implied probability of war is approximately 0 for all bonds prior to the election of Lincoln in 1860. Second, although the implied probabilities increase to between 3 and 5 percent. Third, the implied probabilities largely agree, but there are differences in the probability of war implied by each bond.

<sup>25</sup> The Panic of 1857 (August 1857 through January 1858) is excluded from calculations because as discussed earlier in this paper, while the yields increased during this period, it was due to a financial crisis and not war risk.

<sup>26</sup> The average yield is used since the prices of most bonds spiked after Sumter before falling somewhat in the weeks after it. Using the average yield rather than the first observation after Sumter gives a less conservative estimate of war.

<sup>27</sup> An implicit assumption in these calculations is that expectations about the costliness of the war were constant prior to the start of the war and only beliefs about the probability of a war was changing in the periods leading up to the war. In the case of the American Civil War, this seems like a reasonable assumption.



By noting that all the implied probability of war should be the same for all bonds, rearranging (2.4) and adding uncertainty, the probability of war in each time period can be estimated as,

$$\log y_{j,t} \sim \mathcal{N}((1 - R_j)\pi_t + r_j, \sigma^2) \quad (2.6)$$

where  $\pi_t$  is the probability of war at time  $t$ , Equation (2.6) is a regression model with time-varying coefficients ( $\pi_t$ ). The inverse logit transformation of  $\pi$  evolves as a random walk with a horseshoe prior on the innovations to allow for change-points, as described in section 2.4:

$$\pi_t = \text{logit}^{-1}(\theta_t) \quad (2.7)$$

$$\phi_t \sim \mathcal{N}(\phi_{t-1}, \tau^2 \lambda_t^2) \quad (2.8)$$

$$\lambda_t \sim \mathcal{C}^+(0, 1) \quad (2.9)$$

The estimated values of  $\pi_t$  from the previous are plotted in Figure 2.7. The probability of war is estimated to be approximately 0 prior to Lincoln's election. After the election, the probability of war rose to 2.5 (2.1, 2.8) percent by January 5, 1861, to 3 (2.6, 3.4) percent on February 8, and to 4.1 (3.3, 4.9) on April 13, days before news of the outcome of the Battle of Fort Sumter reached the market.

These results are largely consistent with the results in section 2.4. The market estimated that war was almost certainly not likely prior to the election of Lincoln. Only after the election did the market adjust its estimates of the risk of war. However, even immediately before the war, the probability of war was estimated to be highly unlikely; while the measures of the previous section did not provide a direct estimate of that probability, this measure in this section does. The results differ in that although both measures show

	Avg. yield 1858-02-01– 1860-11-07	Avg. yield 1860-11-07– 1861-04-15	$r$ Avg. yield 1855-07-01– 1857-09-01	$R \cdot 100$ Avg. price 1861-04-15– 1861-05-18
Georgia 6s	5.92	6.72	6.15	62.52
Kentucky 6s	5.68	6.59	5.86	74.60
Louisiana 6s	6.70	8.31	6.90	48.00
Tennessee 6s	6.76	8.30	7.13	43.85
Virginia 6s	6.50	8.12	6.48	43.65

Table 2.2: Summary of data used to calculate the probability of war initiation.

little risk of war prior to the election of Lincoln, the estimates south-north yield spread show an increasing risk of war over time, while the implied probability of war from this section show that risk constant at 0. This difference likely arises from the use of a constant war-risk free rate in equation (2.6).<sup>28</sup> Nevertheless, the overall pattern between the results is similar.

## 2.6 Conclusion

This work estimates the Although the seeds of the American Civil War were preceded the start of the war for many years, the onset of war was unexpected by financial markets. Financial markets did not respond to war risk until after the election of Lincoln in November 1860, and then only after a financial crisis due to Southern banks withdrawing specie and discussion of secession became serious. Despite the awareness of the risk of war, yields spiked dramatically after the Battle of Fort Sumter, implying that the event was unanticipated. The implied probability of war based off the Southern state bonds was

<sup>28</sup>This could be avoided if a good time-varying estimate of the war-risk free interest rate were available. Currently available data of the risk free interest rate of the era are only available at low frequencies: prior to 1857, the yields of Massachusetts Fives and Boston City bonds are only available at a yearly frequency prior; after 1857, an index of New England state and municipal bonds is available at a quarterly frequency (Officer 2003). Using the average of Northern state bonds, as in section 2.4, is likely inappropriate because they also include some risk of war.

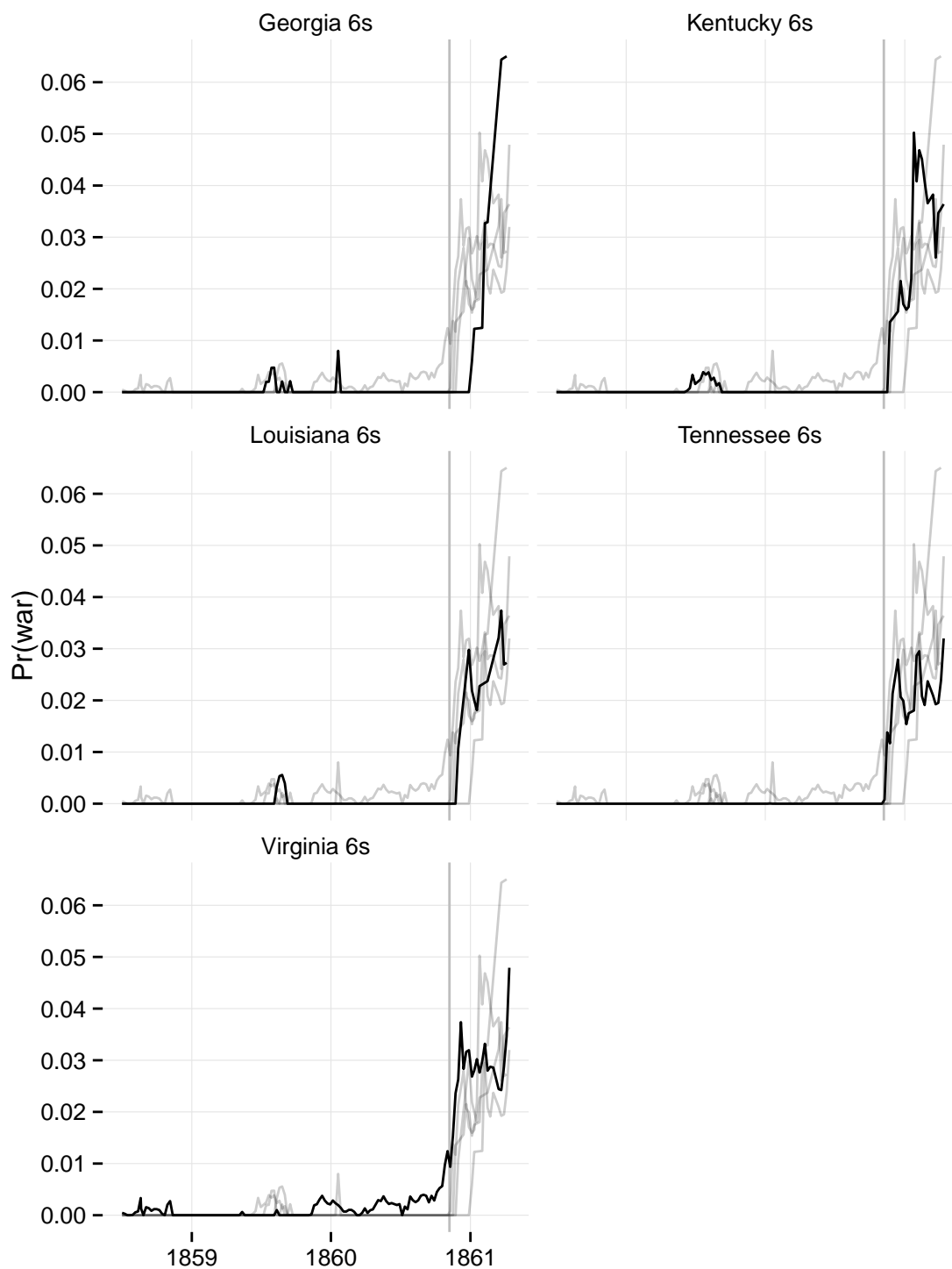


Figure 2.6: Implied probabilities of the initiation of war calculated from each Southern and Border state bond, March 5, 1858 to April 13, 1861.

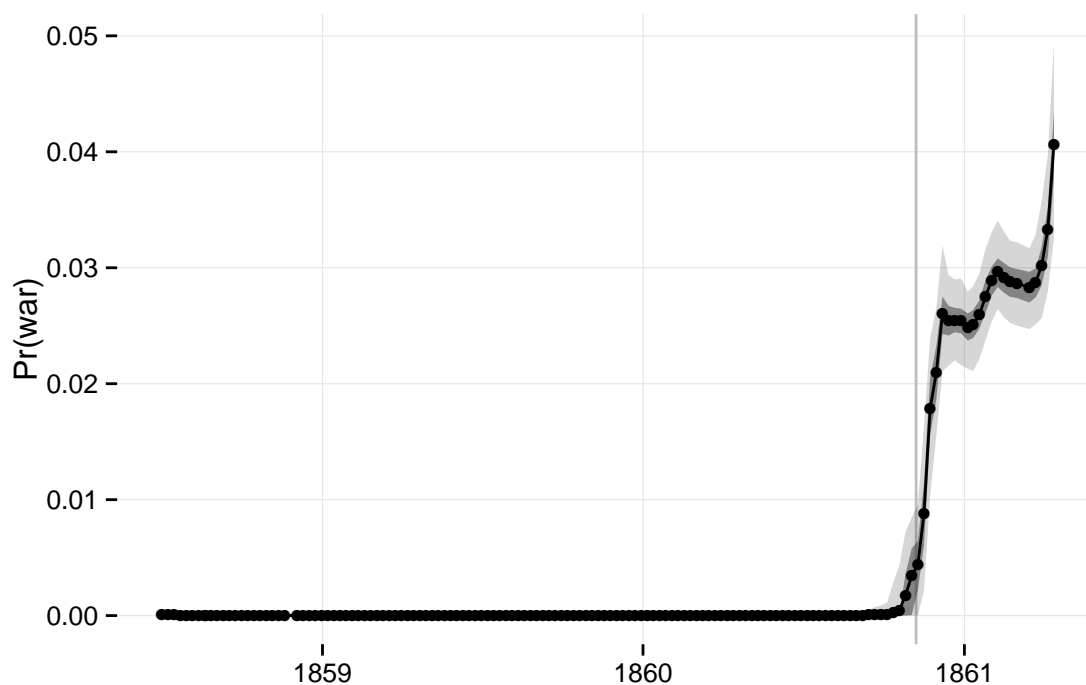


Figure 2.7: Implied probabilities of the initiation of war calculated from a latent variable model of the prices U.S. government and Southern and border state bonds, March 5, 1858 to April 13, 1861, using the model described in section 2.5. The line is the posterior mean, the dark ribbon is the 50% credible interval, the light ribbon is the 95% credible interval. The vertical line is the date of Abraham Lincoln's election.

only about 5 percent.

The results presented here are specific to the case of the American Civil War, but they are consistent with other work on how financial markets have responded to war initiation. Ferguson (2006) show that although World War I was preceded by a series of crises, European markets did not anticipate the onset of WWI. Frey and Kucher (2000a) and Waldenström and Frey (2008) show that Nordic and Swiss markets had large reactions to the start of World War II although this was known. Guidolin and La Ferrara (2010) find large abnormal returns at MIDs. Many of those works emphasize the efficiency of markets in responding to political events, or use the size of the market reaction to understand characteristics of the political events. However, the presence of strong market reactions to these events is itself interesting, as it is an indication that the onset of hostilities was unexpected. Overall, These cases suggest in in many cases financial markets do not effectively price in the the risk of war. If markets are efficient and incorporate public information, this suggests that given public information, many wars are not anticipated. Alternatively, markets do a poor job of assessing the *ex ante* theory of war. Either of these scenarios have important implications for prominent theories of war.

The *ex ante* public information assessments of the probability of war are important because they may provide methods for distinguishing between theories and mechanisms of war. The two rationalist theories of war, commitment problems and private information (Fearon 1995; Powell 2006), have different implications as to what the *ex ante* probability of war occurring will be. In the simplest version of war due to a commitment problem, given the conditions, war will either occur or not occur with certainty depending on the parameters of the model. However, in a private information model of war, if it is possible for war to occur, it can occur with a probability of less than one. Not only that, the private information model of war implies that many wars should be unanticipated (Gartzke 1999). In general, the prevalence of unanticipated wars would provide evidence for the preva-

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lence of some sort of private information model. Other more specific theories of war may offer different implications as to what the *ex ante* probability of war would be. However, the point remains that market predictions of war probability are both something to be explained by models of war, and a means to assess those models. In the specific case here, the low probability of war implied by financial markets for the American Civil War is surprising given that several political science explanations of the American Civil War emphasize commitment problems (Reiter 2009; Weingast 1998).

The second possibility is that financial markets maybe doing a poor job assessing and signaling the risk of war. This would pose serious problems for the mechanism in the *capitalist peace*.

This paper estimates the *ex ante* risk of the American Civil War as implied by financial markets. It is a single case, although its results of a low implied probability of war are largely consistent with other work on the reactions of financial markets to war. The reactions of financial markets provide another empirical pattern that should be assessed with respect to theories of war. In particular, if financial markets can serve as predictions given civil war, they provide a means to assess bargaining theories of war.

## Chapter 3

# Bayesian Change Points and Linear Filtering in Dynamic Linear Models using Shrinkage Priors

### 3.1 Introduction

Political and social processes are rarely, if ever, constant over time, and as such researchers have a need to model that change (Büthe 2002; Lieberman 2002). Moreover, these political process are often marked by both periods of stability and periods of large changes (Pierson 2004). If the researcher has a strong prior about or wishes to test a specific location of a change, they may include indicator variables, an example in international relations being the ubiquitous Cold War dummy variable. Other approaches estimate locations of these change using change point or structural break models (Calderia and Zorn 1998; Western and Kleykamp 2004; Spirling 2007b; Spirling 2007a; Park 2010; Park 2011; Blackwell 2012). This offers a different Bayesian approach to modeling change points. I combine a continuous latent state space approach (Beck 1989), with recent advances in Bayesian shrinkage priors (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2010; Polson and Scott 2010). These sparse shrinkage priors are the Bayesian analog to the advances in sparse regularization such as the LASSO (Tibshirani 1996). For example, to model change

points the mean, I use the following model of observations with a time-varying mean.

$$\begin{aligned} y_t &= \mu_t + \epsilon_t & \epsilon &\text{iid, } E(\epsilon) = 0 \\ \mu_t &= \mu_{t-1} + \omega_t \end{aligned} \tag{3.1}$$

To model change points, I model the change in the mean,  $\omega_t$ , with a shrinkage prior distribution that has most of its mass at zero but has wide tails. This will generate estimates of  $\omega$  in which most times are close to zero, but a few times, the change points, can be large. The estimated posterior distribution of the series of  $\mu$  will resemble a step function. The particular shrinkage priors that will be used in this work are the the horseshoe and horseshoe+ distributions (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2010; Bhadra et al. 2015).

Modeling change points using sparse shrinkage prior distributions has several advantageous features. First, it does not require specifying the number of change points *ex ante*. The change point method of Chib (1998) requires choosing a number of features, and methods that estimate the The sparsity of the changes in the parameter,  $\omega$ , can be estimated from the data.

Second, although this method will not produce exact values of change points, it is likely more representative of the data generating processes in the social science domain. The prior distribution that will be applied to  $\omega_t$  are continuous, meaning that it will not produce exact zeros. However, many social science processes are likely better thought of as always changing, but having many periods of small changes with a few periods of large changes.

Third, this method is flexible and extensible in that it can be adapted to a variety of models. This work considers the cases of change points in the level and in both the level and trend of a single parameter. But it can be applied to linear regression with time varying



parameters, changes in seasonality, variance, and a variety of other models.

Fourth, this method is computationally efficient. The shrinkage prior distributions considered here are representable as scale-mixtures of normal distributions, which allows them to be expressed as a Gaussian dynamic linear models (GDLMs). This means that they can be sampled using specialized methods for GDLMs, e.g. forward-filter backwards-sampling algorithm.

Fifth, this method is implementable and implemented efficiently in a popular Bayesian probabilistic programming language, Stan. Stan does not directly sample discrete variables, meaning that implementing standard Bayesian change point methods are either difficult, requiring marginalizing over the discrete latent states, or impossible. Since in this approach all parameters are continuous, it can be directly implemented in Stan. However, computational improvements can be gained if the change point problem can be represented as a GDLM, and a methods and an implementation is provided to take advantage of that.

A complementary contribution of this work is that it provides a method to efficiently estimate Gaussian dynamic linear models (GDLMs) using a general purpose Bayesian programming language, Stan. Gaussian dynamic linear models are a class of models that incorporates many common time series models, including ARIMA, structural time series, and linear regression with time-varying parameters. These had previously been difficult to sample in general purpose Bayesian programming languages, such as JAGS (Jackman 2009, p. 477). This work provides a complete set of functions to perform Kalman filtering, smoothing, and backward sampling within Stan. This allows for efficiently estimating DLMs in Stan using a partially collapsed Gibbs sampler in which Stan's standard algorithms are used to estimate parameters after marginalizing over the latent states, and then sample the latent states of the DLM using backwards sampling.

This work presents two implementations and examples of this approach to change

points. The first is calculating change points in the level (mean) of a time series, using the example of the annual flow of the Nile River, 1870-1970. The second is calculating change points in both the level and trend of a time series, using the example of approval ratings for George W. Bush.

### 3.2 Change points as a Variable Selection and Shrinkage Problem

For simplicity, I start with the mode of change points in the mean, and later generalize to other cases. In this case, there are  $n$  ordered observations,  $y_1, \dots, y_n$ , with a time-varying mean,  $\mu_t$ .

$$y_t = \mu_t + \epsilon_t \quad \epsilon_t \text{ are iid with } E(\epsilon) = 0. \quad (3.2)$$

Suppose that there are  $M$  change points, with ordered change-point locations,  $\tau_1, \dots, \tau_m$ , with the convention that  $\tau_0 = 0$  and  $\tau_{M+1} = n$ . This splits the mean into  $M$  segments, with values of the mean  $\mu_1^*, \dots, \mu_M^*$ , such that

$$\mu_t = \mu_m^* \quad \text{if } \tau_m \leq t \leq \tau_{m+1} \quad (3.3)$$

There are a variety of approaches to the change point problem in both classical (frequentist) (Page 1954; Hinkley 1970; Bai and Perron 2003; Olshen et al. 2004; Bai and Perron 1998; Killick, Fearnhead, and Eckley 2012) and Bayesian statistics (Yao 1984; Barry and Hartigan 1993; Chib 1998; Fearnhead 2006; Fearnhead and Liu 2007).

An alternative approach to the change point problem is to rewrite the problem in Equation (3.3) to focus on the changes in the mean (system errors),  $\omega_t = \mu_t - \mu_{t-1}$ , rather

than the locations of the change points:

$$\mu_t = \mu_{t-1} + \omega_t \quad (3.4)$$

In a change point model, the system errors,  $\omega_t$ , are sparse, meaning that most values of  $\omega_t$  are zero, and only a few are non-zero. In this formulation, although times of the change points are not directly estimated, the change points are those times at which  $\omega_t$  are non-zero. This formulation turns the problem from one of segmentation, to one of variable selection, in which the goal is to find the non-zero values of  $\omega_t$  and estimate their values. The natural approach to this variable selection problem is to explicitly model the values of  $\omega$  as a discrete mixture between a point mass at zero for the non-change points, and an alternative distribution (Mitchell and Beauchamp 1988; Efron 2008):

$$\omega_t = \rho g(\omega) + (1 - \rho)\delta_0 \quad (3.5)$$

where  $g$  is a distribution of the non-zero  $\omega$ , and  $\delta_0$  is a Dirac delta distribution (point mass) at 0. Equation (3.5) is often called a “spike and slab” prior (Mitchell and Beauchamp 1988). Since it explicitly models the two groups (zeros and non-zeros), Efron (2008) calls this the two-group answer to the two-groups problem. This prior is convenient because it directly provides for each observation a posterior probability that it is a change point (non-zero). Giordani and Kohn (2008) propose using the representation in Equation (3.4) with the a discrete mixture distribution for  $\omega$ , as in Equation (3.5), to estimate change points.

Recent work in Bayesian computation has focused on one-group solutions to the variable selection problem, which combine shrinkage and variable selection through the use of continuous distribution with a large spike at zero and wide tails (Polson and Scott 2010). Numerous sparse sparse shrinkage priors have been proposed, but this paper will

look at the Student's  $t$  (Tipping 2001), the Laplace or double exponential (Park and Casella 2008)Hans2009, the horseshoe (Carvalho, Polson, and Scott 2010), and horseshoe+ distributions (Bhadra et al. 2015).

**Student's  $t$**  The Student's  $t$  distribution for  $x \in \mathbb{R}$  with scale  $\tau \in \mathbb{R}^+$  and degrees of freedom  $\nu \in \mathbb{R}^+$ ,

$$p(\omega_t | \tau, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{\omega_t^2}{\nu}\right) \quad (3.6)$$

The Student's  $t$  distribution can also be expressed as a scale mixture of normals,<sup>1</sup>

$$\begin{aligned} \omega_t | \tau, \nu, \lambda_t &\sim \mathcal{N}(0, \tau^2 \lambda_t^2) \\ \lambda_t^2 | \nu &\sim \mathcal{IG}\left(\frac{\nu}{2}, \frac{\nu}{2}\right) \end{aligned} \quad (3.8)$$

(Tipping 2001) uses the Student's  $t$  for sparse shrinkage by letting the degrees of freedom  $\nu \rightarrow 0$ .

**Double Exponential** The double exponential (Laplace) with scale  $\tau$ ,

$$p(\omega_t | \tau) = \frac{1}{2\tau} \exp\left(-\frac{|\omega_t|}{\tau}\right) \quad (3.9)$$

The double exponential distribution can also be expressed as a scale-mixture of nor-

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<sup>1</sup>  $\mathcal{IG}(x|\alpha, \beta)$  is an inverse-gamma distribution for  $x \in \mathbb{R}^+$  with shape  $\alpha \in \mathbb{R}^+$  and inverse-scale  $\beta \in \mathbb{R}^+$ ,

$$\mathcal{IG}(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} \exp\left(-\beta \frac{1}{x}\right) \quad (3.7)$$

mal distributions,<sup>2</sup>

$$\begin{aligned}\omega_t | \tau, \lambda_t &\sim \mathcal{N}(0, \tau^2 \lambda_t^2) \\ \lambda_t^2 &\sim \mathcal{E}\left(\frac{1}{2}\right)\end{aligned}\tag{3.11}$$

The double exponential distribution is the distribution that corresponds to the  $\ell_1$  penalty used in LASSO (Park and Casella 2008; Hans 2009). The LASSO achieves sparsity due to the spike in the double exponential distribution at zero places also places the mode of estimates at zero. However, the although the double exponential distribution has a spike at zero, it does not produce sparse posterior means since it has relatively little mass close to zero. Additionally, the tails of the double exponential are narrow, so it tends to excessively shrink large signals (Carvalho, Polson, and Scott 2010).

**Horseshoe** The horseshoe distribution (Carvalho, Polson, and Scott 2009)CarvalhoPolsonScott2010 is defined hierarchically as a scale-mixture of normals

$$\begin{aligned}\omega_t | \lambda_t, \tau &\sim \mathcal{N}(0, \tau^2 \lambda_t^2) \\ \lambda_t &\sim \mathcal{E}^+(0, 1)\end{aligned}\tag{3.12}$$

where  $\mathcal{E}^+(x|0, s)$  denotes half-Cauchy distribution with a scale parameter  $s$ , and density

$$p(x|s) = \frac{2}{\pi x \left(1 + \left(\frac{x}{s}\right)^2\right)}\tag{3.13}$$

The Horseshoe distribution has some theoretically attractive properties for shrinkage and variable selection (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and

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<sup>2</sup>  $\mathcal{E}(x|\beta)$  is the exponential distribution for  $x \in \mathbb{R}^+$  with inverse-scale (rate) parameter  $\beta \in \mathbb{R}^+$ ,

$$\mathcal{E}(x|\beta) = \beta \exp(-\beta x)\tag{3.10}$$

Scott 2010; Datta and Ghosh 2012; Pas, Kleijn, and Vaart 2014).

**Horseshoe+** The horseshoe+ distribution Bhadra et al. (2015) is similar to the Horseshoe distribution, but with an additional term,

$$\omega_t | \lambda_t, \eta_t, \tau \sim \mathcal{N}(0, \tau^2 \lambda_t^2) \quad (3.14)$$

$$\lambda_t \sim \mathcal{C}^+(0, \eta_t)$$

$$\eta_t \sim \mathcal{C}^+(0, 1)$$

The shrinkage distributions considered here also global-local scale mixtures of normal, with a global variance component  $\tau$ , and local variance components  $\lambda_t$  (Polson and Scott 2010). The global variance component,  $\tau$ , concentrates the distribution around zero, while the local variance components,  $\lambda_t$ , allow individual values to be far from zero. The choice of the prior distribution for the global variance component,  $\tau$ , is particularly important in these shrinkage distributions as it effectively controls the sparsity of the estimates, number of change points in this example. As per the suggestion in Bhadra et al. (2015) and Pas, Kleijn, and Vaart (2014), throughout this work, I will use the prior distribution

$$\tau \sim \mathcal{C}^+\left(0, \frac{1}{n}\right) \quad (3.15)$$

where  $n$  is the number of observations.<sup>3</sup> That these are distributions can all be expressed as normal distributions, conditional the values of  $\lambda_t$  and  $\tau$ , is useful computationally, as is discussed in Section 3.3.

These Bayesian sparse shrinkage priors are analogous to the sparse regularization in frequentist statistics of which LASSO (Tibshirani 1996) and its the numerous variations

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<sup>3</sup> Other suggestions have been  $\tau \sim \mathcal{C}^+(0, 1)$ ,  $\tau \sim \mathcal{U}(0, 1)$  where  $\mathcal{U}()$  is the uniform distribution, and a plug-in value of  $p/n$ , where  $p$  is the expected number of non-zero parameters. See Polson and Scott (2012), Pas, Kleijn, and Vaart (2014), and Bhadra et al. (2015).

is the most prominent example. Several papers have proposed using LASSO-like penalties and maximum likelihood to estimate change-points (Tibshirani et al. 2005; Harchaoui and Lévy-Leduc 2010; Chan, Yau, and Zhang 2014). This is a Bayesian extension of that approach.

To recap, the proposed model for change points is

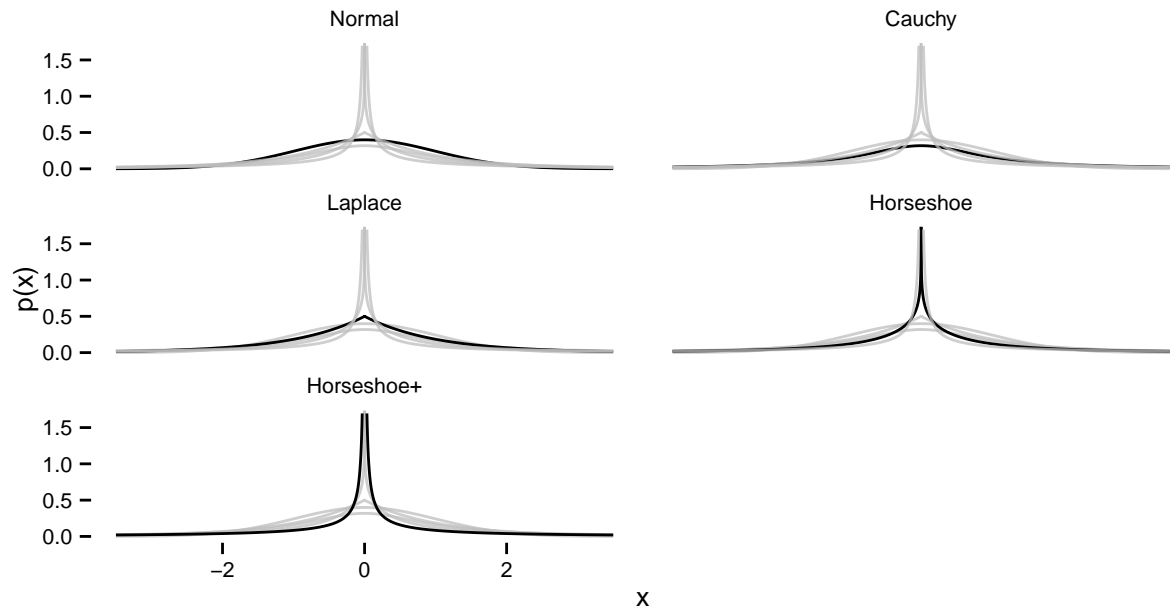
$$y_t \sim \epsilon_t \quad \epsilon_t \text{ iid, } E(\epsilon) = 0, \text{Var}(\epsilon) = \sigma^2 \quad (3.16)$$

$$\mu_t = \mu_{t-1} + \sigma \omega_t \quad (3.17)$$

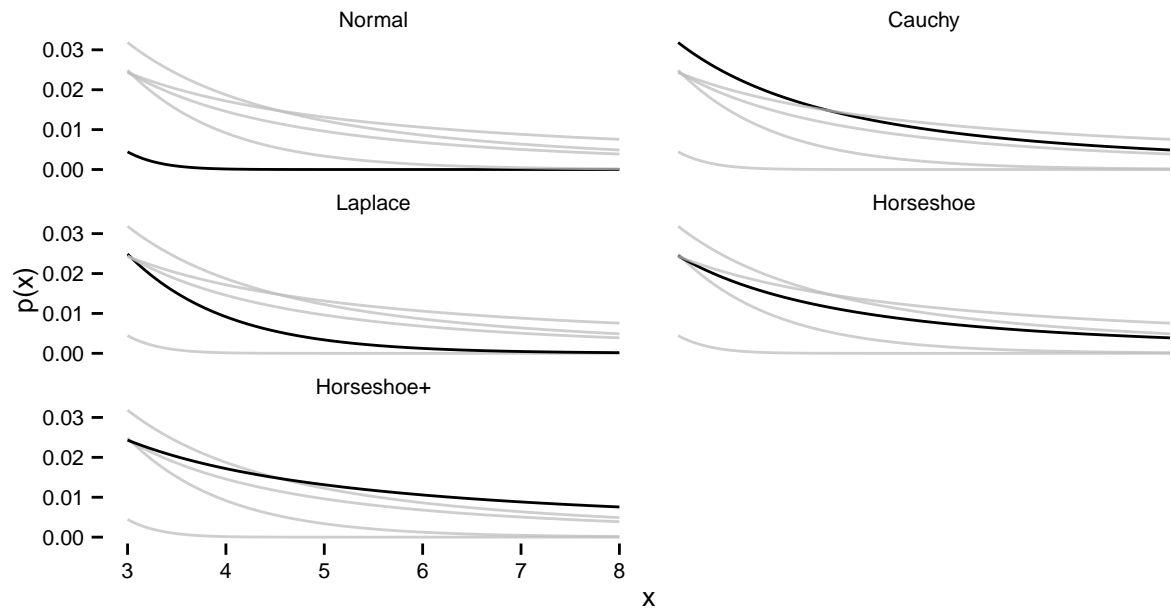
where  $\omega_t$  is given a shrinkage prior distribution that induces sparsity. This work will propose horseshoe and horseshoe+ distributions as those shrinkage priors, and compare them other shrinkage priors such as the Student's  $t$  and double exponential distributions.

The method proposed here is able to estimate the values of  $\mu$  even when it is subject to large jumps or follows a step-function. However, unlike methods using discrete state space models, it does not directly provide probabilities that any given location is a change-points. The researcher can identify change points from the posterior distribution of  $\omega$ . This is similar to the auxiliary residual test of de Jong and Penzer (1998). But for many practical purposes, visual inspection by the researcher of a plot of the posterior estimates of  $\mu$  should be sufficient. As noted earlier, for many data generating processes, the hypothesis that  $\omega_t = 0$  is implausible, so testing for change points is non-sensical. Rather, change point models are used to ease interpretation, so an informal method of visualizing is enough. If the change in  $\mu$  is not distinguishable in a plot, then it is unlikely to be of much substantive importance.

While this one-group approach does not provide clear posterior probabilities that locations are change points, it is a reasonable model of change point processes for sev-



(a) Near zero.



(b) Tail region.

Figure 3.1: Comparison of the density functions of normal, Cauchy, Laplace, horseshoe, and horseshoe+ distributions. All functions have location and scale parameters of 0. The Cauchy distribution is a special case of the Student's  $t$  distribution with degrees of freedom equal to 1.



eral reasons.<sup>4</sup> First, for many time-varying parameter processes typically modeled with a change-point model, the data-generating process is probably more similar to a situation in which the parameter is changing in all periods, but most of these changes are small relative to that of a few periods. In other words, the system errors  $\omega$  are never zero; it is just that in most periods they are relatively small. This seems plausible for many processes modeled by political scientists in which there is no physical reason to think there are discrete states, apart from perhaps institutional changes. It may be useful for the researcher to model that parameter with change points for parsimony and interpretation, but the data-generating process is likely closer to one in which things are usually changing a little, but in some periods they change a lot. Most of the data generating processes considered by political science papers using change points fall into this category: example Supreme court dissent and consensus (Calderia and Zorn 1998), wage growth in OECD states (Western and Kleykamp 2004), casualties in the Iraq War (Spirling 2007b), presidential use of force (Park 2010), and campaign contributions (Blackwell 2012). Second, even in change point models with discrete locations of change points, after marginalizing over the posterior distribution of the change points, the posterior distribution of the change in  $\mu$  will never be exactly zero. The shrinkage prior effectively estimates this distribution of  $\mu$ .

### 3.3 Estimation and Implementation in Stan

A helpful feature of the model as laid out in the previous section is that it can be expressed as Gaussian dynamic linear model, which has properties useful for its computation and generalization. A Gaussian dynamic linear model (GDLM), or linear Gaussian state space model, is a model represented by the system of equations (Durbin and Koop-

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<sup>4</sup>These points are analogous to those made by Polson and Scott (2012, pp. 2-3) with regard to the use of shrinkage priors in variable selection.

man 2012; West and Harrison 1997; Petris, Petrone, and Campagnoli 2009; Shumway and Stoffer 2010, Ch 6),

$$y_t \sim \mathcal{N}(b_t + F_t \theta_t, V_t) \quad (3.18)$$

$$\theta_t \sim \mathcal{N}(g_t + G_t \theta_{t-1}, W_t) \quad (3.19)$$

In these equations, the observed data,  $y_t$ , is a linear function of some latent states,  $\theta_t$ , which are a function of their previous values  $\theta_{t-1}$ . Equation (3.18) is the *observation equation*, where  $y_t$  (observation vector) is a  $r \times 1$  vector of observed data,  $\theta_t$  (state equation) is a  $p \times 1$  vector of unobserved states,  $b_t$  is an  $r \times 1$  vector,  $F_t$  is a  $r \times p$  matrix, and  $V_t$  (observation variance) is an  $r \times r$  covariance matrix. Equation (3.19) is the *state equation* equation, which relates the current state to its previous value;  $g_t$  is an  $p \times 1$  vector,  $G_t$  is a  $p \times p$  matrix, and  $W_t$  (state variance) is an  $p \times p$  covariance matrix. The vectors and matrices,  $\Phi = \{b_t, g_t, F_t, G_t, V_t, W_t\}_{t \in 1:n}$  are *system matrices*, which in applications will often be functions of parameters. GDLMs are a general class of models which includes many common time series models, including SARIMA models, structural time series (Harvey 1990), dynamic factor models, seemingly unrelated regression, and linear regression with time varying coefficients, among others (Durbin and Koopman 2012, Ch. 3).

The model in Section 3.2, Equations (3.2) and (3.4), is a GDLM. In this case,  $\beta_t = g_t = 0$ ,  $F_t = G_t = 1$ , and  $\epsilon_t$  in Equation (3.2) are distributed  $\epsilon_t \sim \mathcal{N}(0, V_t)$ , and  $\omega_t \sim \mathcal{N}(0, V_t)$ . Even though the proposed distributions for  $\omega_t$  are not normal, since they are all representable as scale-mixtures of normal distributions, they are normal conditional on the values of  $\lambda_t$ . These models are similar to what is often called the *local level model* (Durbin and Koopman 2012, Ch 2.), except that they have time-varying state variances,  $W_t$ . In other words, the change point model discussed here is simply a local level model with a sparsity-inducing shrinkage prior on the state variance.

That these change point models can be represented as linear Gaussian state space models is useful for two reasons. First, it suggests how these models can be extended beyond simple changes in means. Any sort of GDLM, which includes many common models, can be adjusted to account for change points in any of its states, by using a shrinkage prior distribution for its system variance,  $W$ .<sup>5</sup>

Second, since both the observation and system equations are multivariate normal, there are analytical solutions that allow for efficiently computing its likelihood and sampling the latent states from their posterior distribution. The Kalman filter calculates the values of  $p(\theta_t | y_{1:(t-1)})$  and  $(\theta_t | y_{1:t})$ , and can be used to calculate the likelihood of  $p(y | \Phi)$ . Importantly, the Kalman filter calculate the likelihood of  $y$  without needing the values of the latent states  $\theta$ . The derivations of the Kalman filter can be found in most time-series texts, including Durbin and Koopman (2012, Ch. 5–7) WestHarrison1997. Given the results of the Kalman filter there are several methods to sample  $\theta$  from  $p(\theta | y, \Phi)$ , a process called Forward-Filtering Backwards-Smoothing (FFBS) or simulation smoothing (Carter and Kohn 1994; Frühwirth-Schnatter 1994; De Jong and Shephard 1995; Durbin and Koopman 2002; Durbin and Koopman 2012, Ch 4.9). I make use of these methods to efficiently sample both the latent states,  $\theta$ , and other parameters of the model in Stan.

Stan is a probabilistic programming language, with its own BUGS-like language, and interfaces to several programming languages, including R (Stan Development Team 2015; Carpenter et al. 2015). GDLMs could be estimated in Stan by directly translating the models described Equations (3.18) and (3.19) into a Stan model.<sup>6</sup> Although the horseshoe and horseshoe+ distributions are not directly implemented in Stan since they have not closed form, but they are easily implemented in Stan through their hierarchical representation.

However, GDLMs can be estimated more efficiently in Stan by marginalizing over the

<sup>5</sup>Though the shrinkage prior distribution will need to be a scale-mixture of normals.

<sup>6</sup>Since these models are hierarchical, it will be more efficient to implement them using the computational tricks in Section ... of X.

latent states. The sampling methods implemented in Stan, the primary of which is HMC-NUTS (Hoffman and Gelman 2014), only require the calculation of a likelihood from the user.<sup>7</sup> The Kalman filter can be used to calculate the likelihood  $p(y|\dots)$  by marginalizing over the latent states,  $\theta_t$ . Marginalizing over parameters is required when estimating models with discrete parameters in Stan, e.g. mixture models (Stan Development Team 2015, p. 104). Although it is not necessary to sample from GDLMs, it helps the efficiency of sampling by reducing the correlation between the latent states and the other parameters of the model. If needed, the latent states can be sampled using a simulation smoothing method discussed earlier.<sup>8</sup> This two-step process is an example of partially collapsed Gibbs-sampling (van Dyk and Park 2008).

To summarize, an efficient method to sample GDLMs in Stan is:

1. Sample  $\vartheta$  from  $p(\vartheta|y)$  using HMC in Stan. This requires integrating out the latent states,  $\theta$ , and calculating  $p(y|\vartheta)$ , which is done using a Kalman filter.
2. Sample  $p(\theta|y, \vartheta)$  using a simulation smoother for a GDLM as in (Carter and Kohn 1994; Frühwirth-Schnatter 1994; De Jong and Shephard 1995; Durbin and Koopman 2002; Durbin and Koopman 2012, Ch 4.9).

I use this method to sample all the models in this work. This required implementing the Kalman filter and simulation smoothing methods in Stan. I provide a full set of user-defined Stan functions that implements Kalman filters, smoothers, and simulation sampling available at Arnold (2015c). Section 3.7 provides the code for one of the Stan modules used in this paper.<sup>9</sup>

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<sup>7</sup>Technically, it also requires derivatives of the likelihood, but these are generated automatically by Stan using its automatic differentiation engine.

<sup>8</sup>In Stan, the calculation of  $p(y|\cdot)$  is done in the `transformed parameter` or `model` blocks, while the sampling of the latent states is done in the `generated quantities` block.

<sup>9</sup>The full code for all models run in this work are available at <https://github.com/jrnold/dlm-shrinkage>.

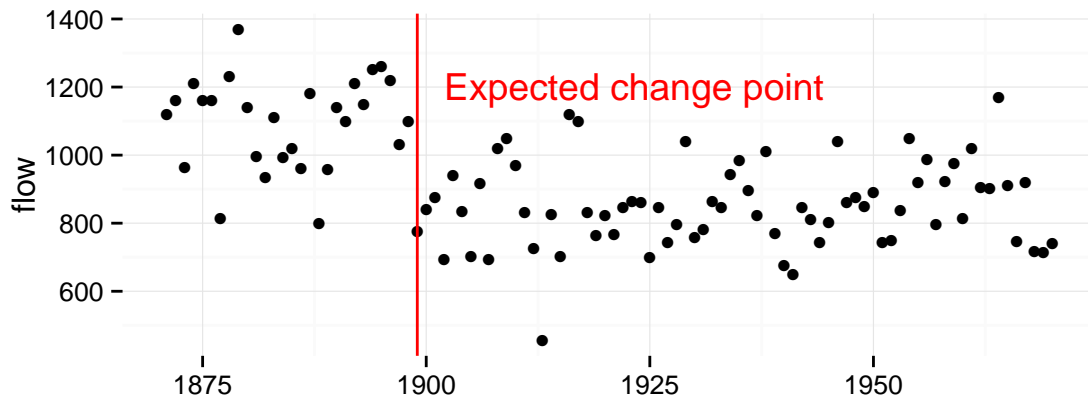


Figure 3.2: Annual flow of the Nile River, 1871–1970. Previous work has found a break point in this series near 1899.

### 3.4 Example: Annual Nile River Flows

A classic dataset that has been analyzed in many works and texts on times series and structural breaks is the annual flow volume of the Nile River between 1871 and 1970 (Cobb 1978; Balke 1993; de Jong and Penzer 1998; Durbin and Koopman 2012; Commandeur, Koopman, and Ooms 2011).<sup>10</sup> Figure 3.2 plots the annual Nile river flow data. This series seems to show a single large shift in the average level of the annual flow around 1899. This level shift is attributed to construction of a dam at Aswan that started operation in 1902 or climate changes that reduced rainfall in the area (Cobb 1978, p. 278). This example is used to see if the methods discussed earlier are able to recover this change point.

I compare several models of this data. In all models,  $y_t$  is the annual flow of the Nile River,<sup>11</sup> and consists of  $n = 100$  annual observations from 1871 to 1970. In all models the

<sup>10</sup>The dataset is included with R as `Nile` in the included package `datasets`.

<sup>11</sup>Discharge at Aswan in  $10^8 m^3$ .

observations are normally distributed given the mean,  $\mu_t$ .<sup>12</sup>

$$y_t \sim \mathcal{N}(\mu_t, \sigma^2) \quad (3.20)$$

The models differ in how they model the possibly time-varying mean,  $\mu_t$ . I estimate the following models:

**Constant** In this model, the mean is constant.

$$\mu_t = \mu_0 \quad \text{for all } t \quad (3.21)$$

**Intervention** This model includes an indicator variable for all years including and after 1899. This models a situation in which the researcher knows the change points in a series, and is manually accounting for them.

$$\mu_t = \begin{cases} \mu_0 & t < 1899 \\ \mu_0 + \omega & t \geq 1899 \end{cases} \quad (3.22)$$

**Normal** In this model, the system errors are distributed normal. This corresponds to the typical local level model (Durbin and Koopman 2012, Ch. 2; West and Harrison 1997, Ch. 2).<sup>13</sup>

$$\mu_t = \mu_{t-1} + \omega_t \quad \omega_t \sim N(0, \tau^2) \quad (3.23)$$

**StudentT** In this model, the system errors are distributed Student  $t$ .

$$\mu_t = \mu_{t-1} + \omega_t \quad \omega_t \sim \mathcal{T}_v(0, \tau) \quad (3.24)$$

---

<sup>12</sup> This may not be appropriate since that data contains possible outliers at 1879, 1913, and 1964. However, modeling outliers is not the purpose of this analysis, so I ignore that.

<sup>13</sup>For example, as implemented in the R function `StructTS`.

The prior distribution of the degrees of freedom parameter  $\nu$  is that suggested by Juárez and Steel (2010), a Gamma distribution with shape parameter 2 and rate parameter 0.1 (mode 10, mean 20, and variance 200), which places most of mass in the relevant region of the space.

**Laplace** In this model, the system errors are distributed Laplace (double exponential):

$$\mu_t = \mu_{t-1} + \omega_t \quad \omega_t \sim \mathcal{L}(0, \tau) \quad (3.25)$$

**Horseshoe** In this model, the system errors are distributed horseshoe.

$$\begin{aligned} \mu_t &= \mu_{t-1} + \omega_t \quad \omega_t \sim \mathcal{N}(0, \tau^2 \lambda_t^2) \\ \lambda_t &\sim \mathcal{C}^+(0, 1) \end{aligned} \quad (3.26)$$

**Horseshoe+** In this model, the system errors are distributed horseshoe.

$$\begin{aligned} \mu_t &= \mu_{t-1} + \omega_t \quad \omega_t \sim \mathcal{N}(0, \tau^2 \lambda_t^2) \\ \lambda_t &\sim \mathcal{C}^+(0, \eta_t) \\ \eta_t &\sim \mathcal{C}^+(0, 1) \end{aligned} \quad (3.27)$$

For the global scale parameter,  $\tau$ , in **Laplace**, **StudentT**, **Horseshoe**, and **Horseshoe+**, I use the half-Cauchy prior  $\tau \sim \mathcal{C}^+(0, \frac{1}{n})$ . In **Normal** and **Intervention**,  $\tau$  is given a semi-informative half-Cauchy prior with a scale equal to a scalar multiple of the standard deviation of the data.

Figure 3.3 plots the posterior distribution of  $\mu$  for each model. The **Normal** model does not show a clean break at 1899, instead it estimates a change occurring over several years. The **Laplace** model looks similar to the **Normal** model. While the Laplace distribution

achieves sparsity in maximum likelihood estimates because it places the posterior mode of parameters at zero, when the posterior mean is used as an estimate it does not. Since it does not concentrate much mass near zero, it is not surprising that it does not perform much differently than the normal distribution (Park and Casella 2008). Both the Horseshoe and Horseshoe+ models produce a posterior distribution of  $\mu$  that appears similar to the step function in the Intervention model. Additionally, the StudentT also produces estimates similar to the Intervention, but with slightly wider posterior distributions than the horseshoe models. Figure 3.3 plots the posterior distribution of the system errors,  $\omega$ , for each model. The Horseshoe, Horseshoe+, and StudentT models all estimate  $E(\omega_i|y)$  near zero for all years but 1899.

Table 3.1 includes several model comparison statistics. First, I compare the models on their fit to the observed data using the root mean squared error (RMSE) ( $RMSE(y)$ ), and the expected log predictive density calculated using the Widely Applicable Information Criterion ( $WAIC$ ) ( $elpd_{WAIC}$ ) and importance sampled leave-one-out (LOO) cross-validation ( $elpd_{loo}$ ). The RMSE is defined as  $\sqrt{\frac{1}{n} \sum_i (y_i - E(\mu_i|y))^2}$ . The expected log probability density statistics are measures of the model's fit to a new out-of-sample observation. The log probability density of a new observation is the expected value of the posterior density of a future observation,  $\log p(\tilde{y}|\theta, y)$ , where  $\tilde{y}$  is the value of a future observation, and  $\theta$  includes all parameters in the model. Since the value of the future observation,  $\tilde{y}$ , is unknown, the expected log probability density averages over the predictive distribution of  $\tilde{y}$ ,  $elpd = E_f(\log p(\tilde{y}|\theta, y_i))$ . However, the distribution of future values is in general also unknown, which is where the two approximations come in.  $elpd_{WAIC}$  approximates  $elpd$  using an information criteria similar to AIC, BIC or DIC, taking the in-sample log-likelihood and penalizing it for model complexity.  $elpd_{loo}$  approximates  $elpd$  using leave-one-out cross validation. See Gelman, Carlin, et al. (2013), Gelman and Vehtari (2014), or Gelman, Hwang, and Vehtari (2014) for more thorough discussion of  $elpd$  and predictive



measures for Bayesian model comparison.<sup>14</sup> In the RMSE, a lower value indicates a better fit, for elpd, a higher value indicates is a better fit. Second, I compare the fits of the model to the “true” values of  $\mu_t$ . But, since this is real data, I do not know the true values of  $\mu_t$ . Instead, I will compare the other models to posterior mean estimate of Intervention model. The column  $\text{RMSE}(\mu)$  of Table 3.1 is the root mean squared error of the models compared to the posterior mean of  $\mu$  as estimated by the Intervention model, defined as  $\sqrt{\frac{1}{n} \sum (\mathbb{E}(\mu_t|y) - \bar{\mu}_t)^2}$ , where  $\bar{\mu}_t$  is the posterior mean of  $\mu_t$  in the Intervention model. As with comparisons of fit to the observed data, the Horseshoe and Horseshoe+ models have the lowest RMSE, although the StudentT model is close.

model	RMSE(y)	elpd <sub>WAIC</sub>	elpd <sub>loo</sub>	RMSE( $\mu$ )
Constant	168.38	-656.49	-656.49	111.24
Intervention	126.39	-629.09	-629.10	
Normal	143.82	-641.87	-641.88	37.52
StudentT	137.46	-637.61	-637.66	13.50
Laplace	140.89	-640.43	-640.49	25.75
Horseshoe	136.48	-635.89	-635.98	9.44
Horseshoe+	136.42	-637.47	-638.26	12.59

Table 3.1: Model comparison statistics for models of the Nile Rive annual flow data.

### 3.5 Change Points in Levels and Trends

The use of sparsity inducing priors can be extended to model change points in trends in addition to the level.<sup>15</sup> The local level model considered in section 3.2 can be extended to

<sup>14</sup> The way these are calculated does not fully account for the time-series nature of this data. The measures presented here should be seen as approximating the fit of the model to a previously missing value within the time-series. To calculate  $\text{elpd}_{WAIC}$  and  $\text{elpd}_{loo}$ , I use the `loo` R package, which implements the methods described in Gelman and Vehtari (2014).

<sup>15</sup> Kim et al. (2009) and Tibshirani (2014) consider similar problems in a maximum likelihood framework with  $\ell_1$  regularization.

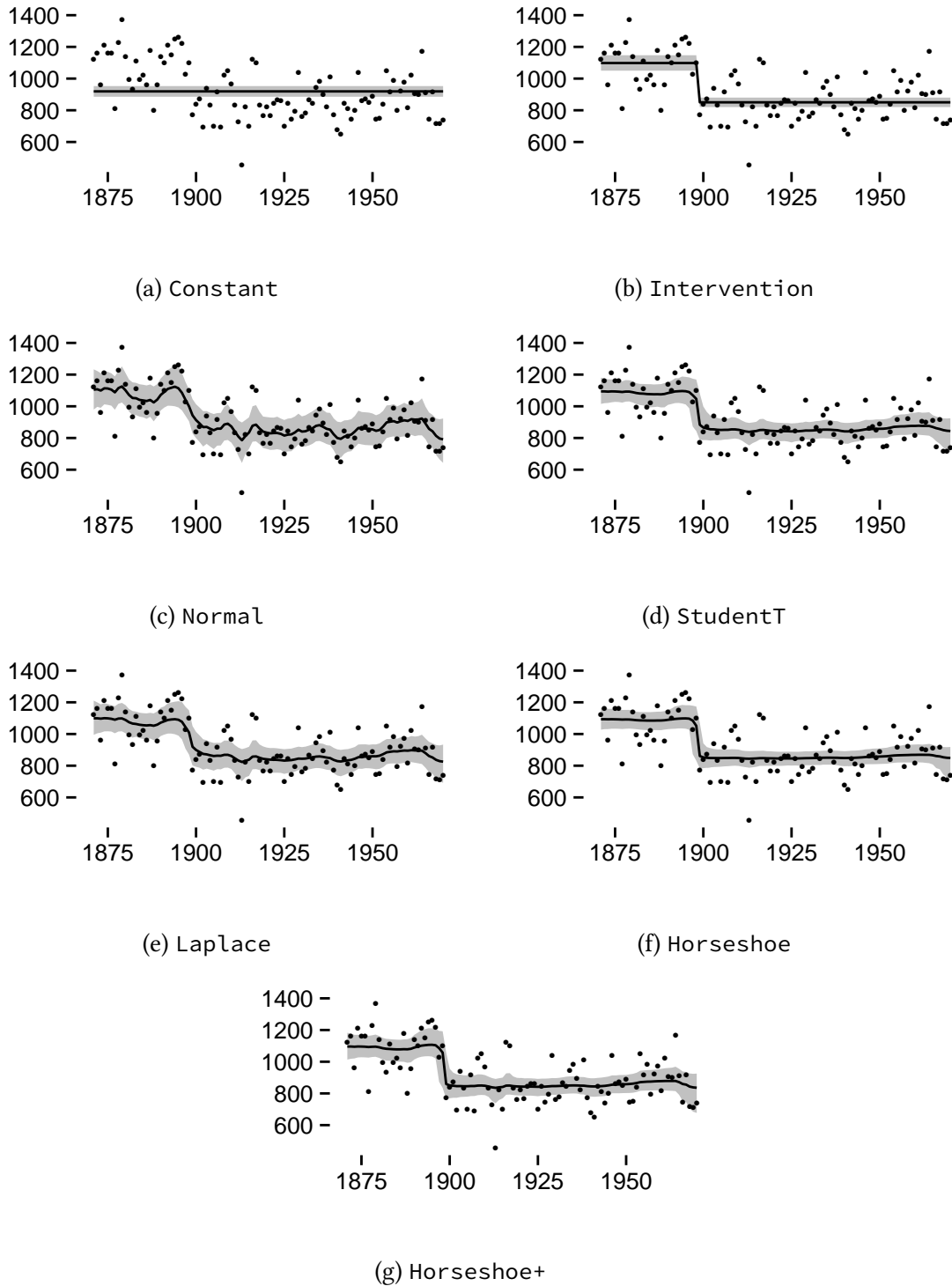


Figure 3.3: Posterior distributions of  $\mu_t$  for models of the Nile River annual flow data. The line is the posterior mean; the range of the ribbon the 2.5–97.5% percentiles of the posterior distribution.

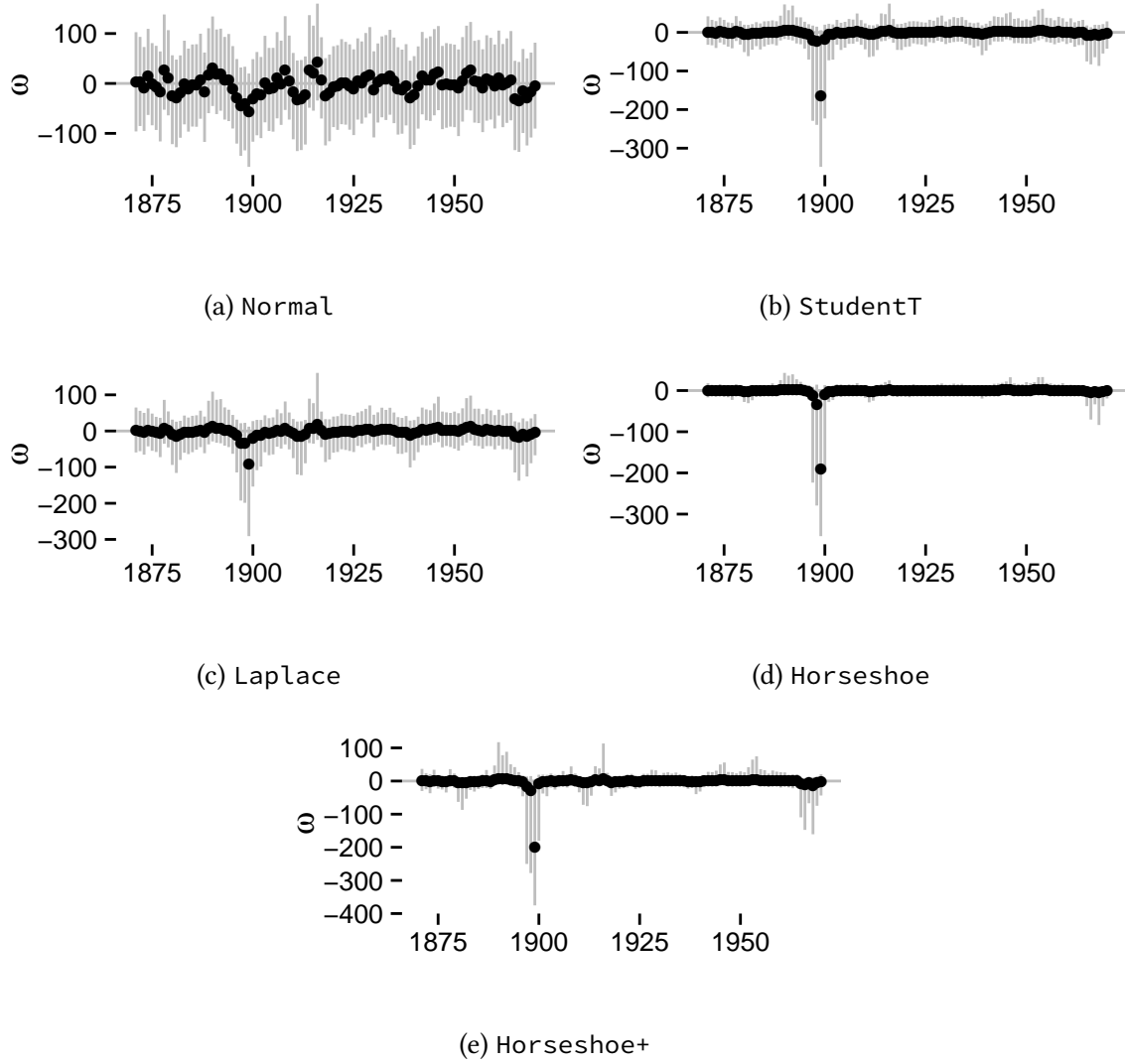


Figure 3.4: Posterior distributions of  $\omega_t$  for models of the Nile River annual flow data. The point is the posterior mean; the range of the line is the 2.5–97.5% percentiles of the posterior distribution.

a local trend model (Durbin and Koopman 2012, Ch 3.2; West and Harrison 1997, Ch 7),

$$\begin{aligned} y_t &= \mu_t + \epsilon_t & \epsilon_t &\sim \mathcal{N}(0, \sigma^2) \\ \mu_t &= \mu_{t-1} + \alpha_{t-1} + \omega_{1,t} \\ \alpha_t &= \alpha_{t-1} + \omega_{2,t} \end{aligned} \quad (3.28)$$

In Equation (3.28), there are two states:  $\mu_t$  is the current level, and  $\alpha_t$  is the current trend (change in the level). The level is changing over time both due to the current value of the trend,  $\alpha_t$ , and errors,  $\omega_{1,t}$ . While the trend is changing over time due to error,  $\omega_{2,t}$ . This model will allow for change points in both the level and trend if sparsity inducing shrinkage priors are used for  $\omega_1$  and  $\omega_2$ . The system errors in a local trend model could be modeled with an arbitrary covariance structure, but in this work, I follow the suggestion of (West and Harrison 1997, Ch 7.),

$$\begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix}' \sim \mathcal{N}(0, L \text{diag}(W_1^2, W_2^2)L'); \quad L = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad (3.29)$$

Since the sparsity inducing shrinkage priors discussed can be represented as scale-mixtures of normals,  $\omega$ , is distributed as:

$$\begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} \tau_1^2 \lambda_{1,t}^2 + \tau_2 \lambda_{2,t}^2 & \tau_2 \lambda_{2,t}^2 \\ \tau_2 \lambda_{2,t}^2 & \tau_2 \lambda_{2,t}^2 \end{bmatrix}\right) \quad (3.30)$$

This means that this model is a GDLM as in Section 3.3, and can be sampled with the method described in that section.

### 3.6 Example: George W. Bush Approval Ratings

Ratkovic and Eng (2010) use the approval ratings for George W. Bush, displayed in Figure 3.5. George W. Bush's approval ratings are difficult to fit with a typical smoothing methods because it was subject to two large jumps in his approval ratings: September 11th, 2001 and at the start of the Iraq War on March 20, 2003. The data used in this example consist of 270 polls between February 01, 2001 and February 01, 2001 from the Roper Center Public Opinion <sup>16</sup>

$$\begin{aligned}
 y_t &= \mu_t + \epsilon_t & \epsilon_t &\sim \mathcal{N}(0, \sigma^2) & (3.31) \\
 \mu_t &= \alpha_t + \mu_{t-1} + \partial\mu_{t-1} + \omega_{1,t} \\
 \partial\mu_t &= +\partial\mu_{t-1} + \omega_{2,t} \\
 \begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix} &\sim \mathcal{N}\left(0, \begin{bmatrix} \tau_1^2 \lambda_{1,t}^2 + \tau_2 \lambda_{2,t}^2 & \tau_2 \lambda_{2,t} \\ \tau_2^2 \lambda_{2,t} & \tau_2^2 \lambda_{2,t} \end{bmatrix}\right)
 \end{aligned}$$

**Normal** System errors are distributed normal.  $\lambda_{i,t} = 1$  for all  $i$  for all  $t$ ,  $\alpha_t = 0$  for all  $t$ .

**Intervention** System errors are distributed normal,  $\lambda_{i,t} = 1$  for all  $i$  for all  $t$ . There are manual interventions after 9/11 and the the Iraq War. The values of  $\alpha$  for those two dates are non-zero and estimated, all other  $\alpha_t$  are zero. This corresponds to a manual intervention for known change points.

**Horseshoe** The system errors,  $\omega_{i,t}$ , are distributed horseshoe, with  $\alpha_t = 0$  for all  $t$ .

**Horseshoe+** The system errors,  $\omega_{i,t}$ , are distributed horseshoe+, with  $\alpha_t = 0$  for all  $t$ .

Figures 3.6 and 3.7 plot the posterior distribution of  $\mu_t$  for the models. The Normal model shows Bush's approval rating rising before 9/11 and shows a lot of small varia-

<sup>16</sup>From [http://webapps.ropercenter.uconn.edu/CFIDE/roper/presidential/webroot/presidential\\_rating\\_detail.cfm?allRate=True&presidentName=Bush#.UbeB8HUbyv8](http://webapps.ropercenter.uconn.edu/CFIDE/roper/presidential/webroot/presidential_rating_detail.cfm?allRate=True&presidentName=Bush#.UbeB8HUbyv8).

tion. The Horseshoe and Horseshoe+ models more closely resemble the Intervention model: Bush's approval rating are loping downward or steady until 9/11, and the approval rating is mostly smooth apart from those sharp jumps. Table 3.2 shows the RMSE and expected log predictive densities of these models. The horseshoe models fit the data better than a normal distribution, but less well than the Intervention model.

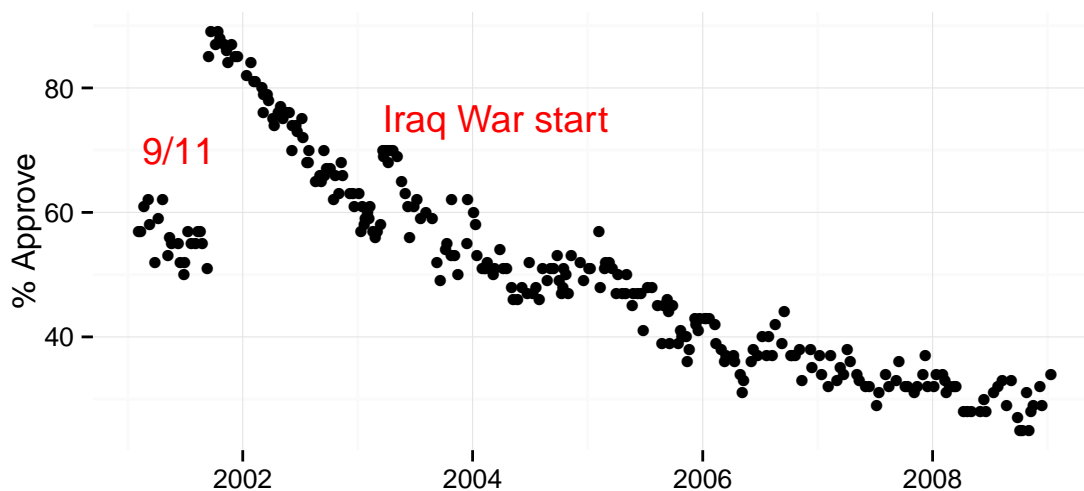
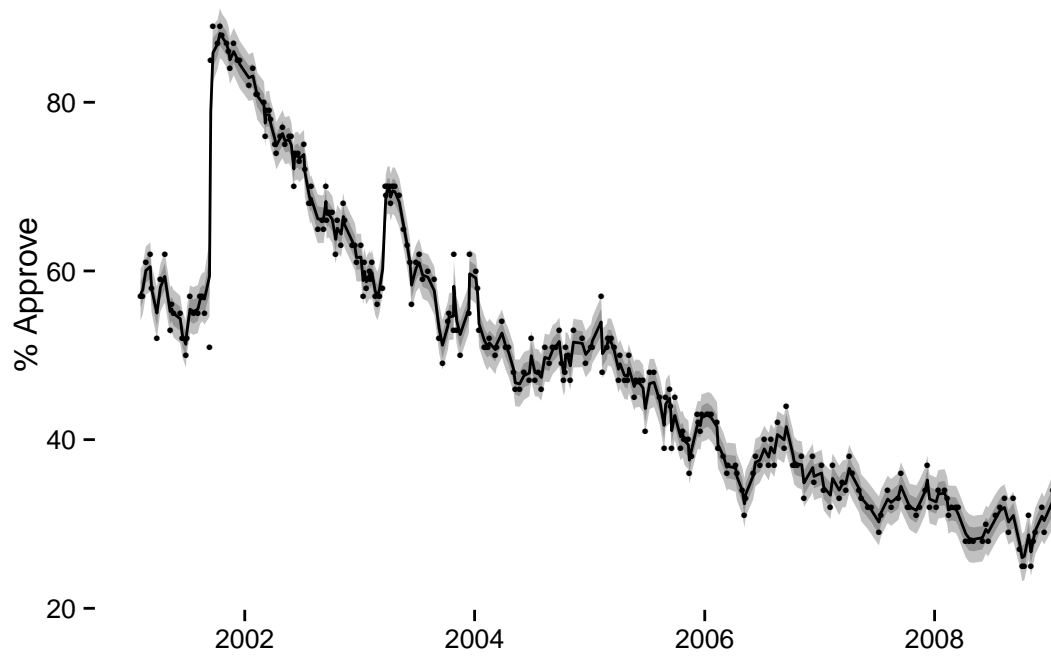


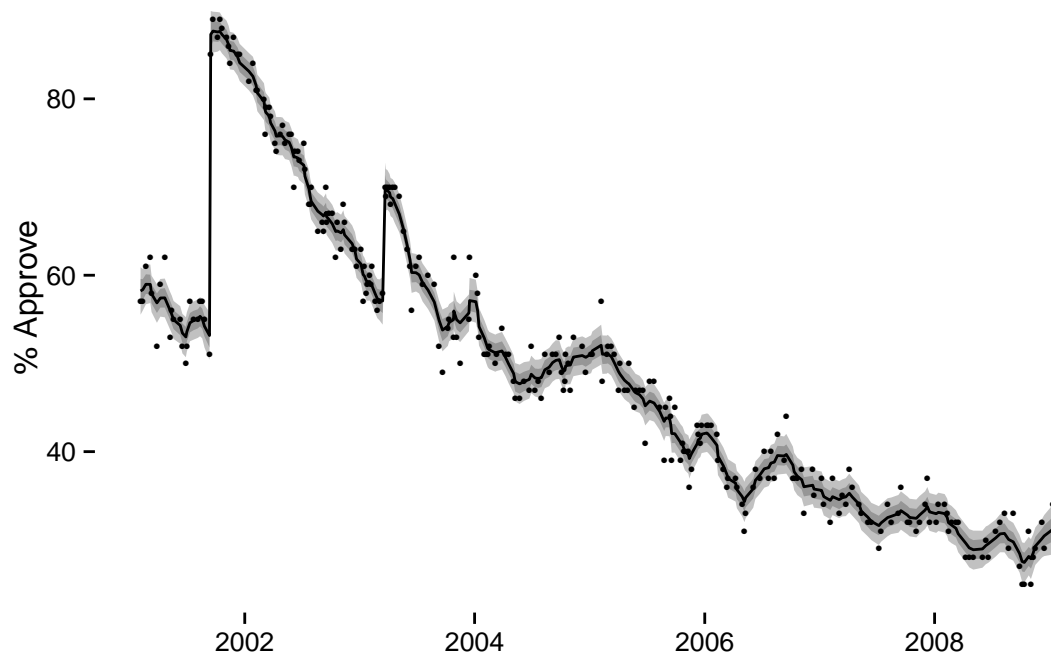
Figure 3.5: Approval ratings of President George W. Bush

model	RMSE	$\text{elpd}_{WAIC}$	$\text{elpd}_{loo}$
Normal	3.73	-748.51	-749.43
Intervention	2.80	-666.24	-666.84
Horseshoe	3.44	-680.65	-681.15
Horseshoe+	3.43	-682.56	-683.61

Table 3.2: Model comparison statistics for models of President George W. Bush's approval rating.

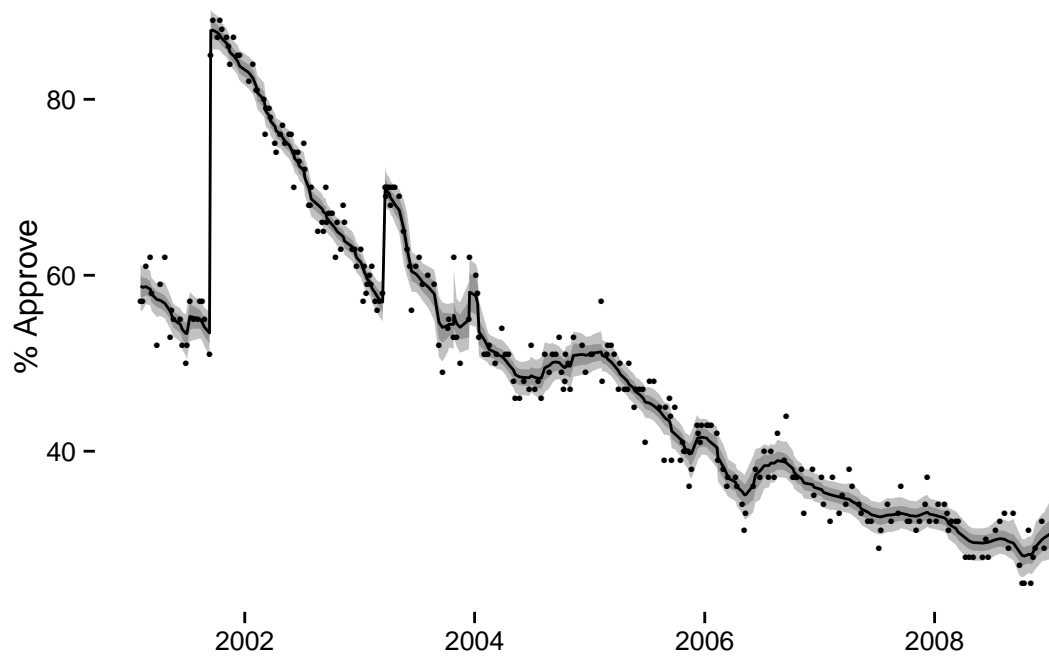


(a) Normal

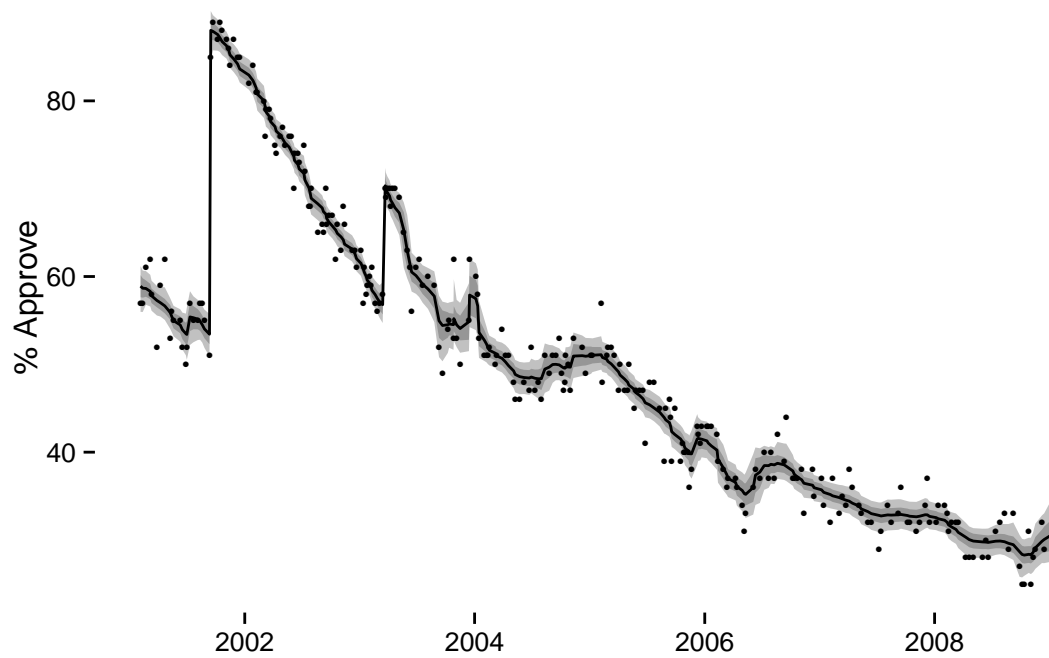


(b) Intervention

Figure 3.6: Posterior distribution of  $\mu$  for Normal and Intervention models.



(a) Horseshoe



(b) Horseshoe+

Figure 3.7: Posterior distribution of  $\mu$  for Horseshoe and Horseshoe+ models.



### 3.7 Conclusion

This work proposes modeling change points using sparse shrinkage priors, such as the horseshoe, over the change in a parameter. This approach has several useful features. It does not require choosing a specific number of change points, and the sparsity of the changes can be estimated from the data. Although it does not directly estimate the probability of change points, it is closer to the DGP of many political processes in which there is never no change, but are characterized by many periods of small changes, and a few periods of large changes. Many models are represented as Gaussian dynamic linear models, and thus can be sampled using efficient algorithms for that class of model. This work provides a partially collapsed Gibbs sampler method to estimate GDLMs in Stan, as well Stan code that implements Kalman filtering and smoothing in Stan.

The most promising feature of this approach is that it is flexible and can be applied to a variety of models. For example, to model a linear regression with independent change points, and  $K$  variables,

$$\begin{aligned} y &\sim \mathcal{N}(\alpha_t + \beta X, \sigma^2) \\ \alpha_t &\sim \mathcal{N}(\alpha_{t-1}, \sigma^2 \tau_\alpha^2 \lambda_{\alpha,t}^2) \\ \beta_{t,k} &\sim \mathcal{N}(\beta_{t-1,k}, \sigma^2 \tau_{\beta,k}^2 \lambda_{\beta,t}^2) \quad \text{for } k \in 1, \dots, K \end{aligned} \tag{3.32}$$

where the local variance components,  $\lambda_\alpha$  and  $\lambda_\beta$  are given prior distributions corresponding to a sparse shrinkage distribution such as the horseshoe or horseshoe+. This model corresponds to a GDLM with latent states  $\alpha$  and  $\beta$ , and thus the partially collapsed Gibbs sampling method in Section 3.3 can be used to efficiently estimate it. As written, this would correspond to independent change points in the parameters,  $\alpha$  and  $\beta$ . If the researcher wanted to impose a restriction that large changes occurred at the same time for

all distributions, they could set  $\tau_\alpha = \tau_{\beta,1} = \dots = \tau_{\beta,K}$  and  $\lambda_\alpha = \alpha_{\beta,1} = \dots = \alpha_{\beta,K}$ .

## Example Stan Program

An example of a change point model. See the replication data for this work to see the code for all the Stan models estimated. The DLM related user-defined functions in the functions block are excluded.

```
data {
  int<lower = 1> n;
  vector[n] y;
  int miss[n];
  real m0;
  real<lower = 0.0> C0;
  real<lower = 0.0> s;
  real<lower = 0.0> w;
}

parameters {
  real<lower = 0.0> sigma;
  real<lower = 0.0> tau;
  vector<lower = 0.0>[n] lambda;
}

transformed parameters {
  vector[n] log_lik;
  vector[6] dlm[n + 1];
  vector[n] W;
```

---

```

for (i in 1:n) {
  W[i] <- pow(sigma * tau * lambda[i], 2);
}

{
  vector[n] V;
  V <- rep_vector(pow(sigma, 2), n);
  dlm <- dlm_local_level_filter(n, y, miss, V, W, m0, C0);
  log_lik <- dlm_local_level_filter_loglik(n, dlm, miss);
}

}

model {
  real ll;

  sigma ~ cauchy(0.0, s);
  tau ~ cauchy(0.0, w);
  lambda ~ cauchy(0.0, 1);
  increment_log_prob(sum(log_lik));
}

generated quantities {
  vector[1] mu[n + 1];
  vector[1] omega[n];
  vector[1] kalman[n];

  {
    matrix[1, 1] G_tv[n];

```

```
G_tv <- rep_array(rep_matrix(1.0, 1, 1), n);
mu <- dlm_filter_bsample_rng(n, 1, 1, G_tv, dlm);
}
for (i in 1:n) {
  omega[i] <- mu[i + 1] - mu[i];
}
for (i in 1:n) {
  kalman[i] <- dlm_get_C(i, 1, 1, dlm) * dlm_get_Q_inv(i, 1, 1, dlm);
}
}
```

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