Three Essays on Conflict and Financial Markets, and Political Methodology

by

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Biographical Sketch

The author was born in Pasadena, California. He attended Dartmouth College, and graduated with a Bachelor of Arts degree in Economics and Government in 2004. He worked as a research assistant and economist in the Money and Payments Studies group in the Economic Research division at Federal Reserve Bank of New York between 2004 and 2007. He began doctoral studies in Political Science at the University of Rochester in 2007. He was awarded a Sproull Fellowship in 2004 and received the Master of Arts degree in 2011 from the University of Rochester. He pursued his research in Political Science under the direction of Randall Stone.

Abstract

This dissertation consists of three distinct papers. In the first paper, I estimate the effects of battles in the American Civil War on the bond yields of U.S. government bonds. Since the yields of U.S. government bonds were primarily a function of the expected outcome of the war, they are a proxy for the expected cost of war to the United States. While Confederate victories increased the expected cost of the war to the United States, Union victories had little effect on war cost expectations. The effects of the battles are hard to explain in a private information theory of war, since the largest effects occur on expected war costs occur late in the war, in the summer of 1864. Surprisingly, major battles explain little of the variation in expectations of war costs. In the second paper, I estimate the market assessed *ex ante* probability of the onset of the American Civil War using U.S. government and state bonds. Surprisingly, financial markets were surprised by the Battle of Fort Sumter and the start of the war. Prior to Abraham Lincoln's election in November 1860, the market assigned almost no probability to a war. Even after secession of several states and the week before the Battle of Fort Sumter, the market assigned a negligible probability to war onset, approximately 5 percent. In the third paper, I provide a general method to estimate change points using Bayesian sparse shrinkage priors, such as the horseshoe distribution (Carvalho, Polson, and Scott 2010). This method is flexible and can estimate change points in a variety of models without having to assume a particular number of change points. Since many of these models can be represented as Gaussian dynamic linear

models, I provide a method to efficiently sample these models using a partially collapsed Gibbs sampler in the Bayesian programming language Stan. Additionally, I provide Stan functions to perform Kalman filtering, smoothing, and sampling.

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Chapter 1

Bonds and Battles: Financial Market Reactions to Battlefield Events in the American Civil War

1.1 Introduction

Understanding what leads to the the start of war is inextricably linked to what leads to the end of war (Blainey 1988, p. x). Understanding how wars ends requires understanding how military combat leads to a resolution of the conflict between the belligerents which was not reached without war, for military combat is the the "means" that war uniquely adds to the political bargaining process. This is what Gartner (1998) calls "opening up the black box of war." But opening the "black box of war" has been difficult for quantitative analysis because the relevant intra-war data is rare (Reiter 2003; Reiter 2009). Even data on battles is not readily available for large numbers of wars. Due to the lack of multi-war intra-war data, the preferred approach to the empirical study of the bargaining theory of war with intra-war data has been qualitative case studies (Reiter 2003; Reiter 2009, Chapter 9). There are some implications of the bargaining model of war that can be tested with within-case data, such as those regarding the offers made throughout the war. But the use of within-case data prohibits the use of several key dependent variables of interest in these models—the cost, duration, and outcome of a war—since there is no variation in these variables within a single war.

This work proposes a quantitative case study method to understand the effects of battles on war outcomes using the prices or yields of financial assets. Some financial assets, in particular many sovereign bonds, can be used as a measure of expectations about war termination when the payoffs of these assets are largely contingent on the war outcome. This allows for quantitative analysis of battles or other war events on war outcomes, by using prices as a proxy for the time-varying expected or predicted war outcome in place of the observed war outcome. This provides an alternative method with which to analyze the intra-war events using the small number of cases for which high quality data is available. Additionally, since prices incorporate expectations about the future, they provide a particularly good measure of surprising events. Surprising events are crucial in private information theories of war, but it is difficult to know what events were surprising to contemporaries. Since prices of financial assets only respond to new information, they provide a natural measure of surprising events, which are particularly important in informational theories of war (Shirkey 2009). This can also used to identify which events were seen as most important in the war, with the price itself already controlling for all previous events in the war because they are incorporated into the prior expectation of that event.

In this work, I focus on the case of the American Civil War due to the quality of its battle data, and the appropriateness of its financial assets for this purpose. I estimate the effects of militarily significant battles on U.S. government bond yields. The yields of U.S. government bonds were almost certainly primarily driven by expectations of the cost of the war in this case. Thus, the bond yields of U.S. government can act as a proxy for expectations of the war cost, given the available information at the time. Modeling the effects of battles on these yields estimates how Confederate and Union victory affected expectations about the cost and duration of the war. I find the following. First, the effect of battles on bond yields, and, by extension, expectations of the war cost, was asymmetric. On average, Confederate victories increased yields by approximately 5%, while, on average, Union victories had little to no effect. This is consistent with the asymmetric strategic environment of the American Civil War. The United States had to win battles to recapture the seceding states, while the Confederacy had only to keep the Union from capturing its territory. It also suggests that U.S. victories were expected, at least by New York investors, while Confederate victories were surprising. This may have meant that there was little belief that the United States would lose the war, so any Confederate victory only served to extend the expected duration of the war. Second, the battles with the largest estimated effects on bond yields were Confederate victories in May–June, 1864. These battles were part of the Overland and Richmond-Petersburg campaigns in which U.S. forces advanced Richmond, the Confederate capital, and were met with heavy resistance. Third, although including battles in model of the bond yields improves the model fit, the improvement is surprisingly small. This is consistent with many events within a war, not just major battles, having an influence on war expectations.

This work adds to an economic history literature on the effects of war events in the American Civil War on currencies and other financial assets (Mitchell 1903; Mitchell 1908; Schwab 1901; Roll 1972; Calomiris 1988; Davis and Pecquet 1990; Willard, Guinnane, and Rosen 1996; McCandless 1996; Smith and Smith 1997; Brown and Burdekin 2000; Weidenmier 2000; Weidenmier 2002; Haber et al. 2014). This work contributes to that literature by using the yields of U.S. government bonds, which unlike Greenbacks (U.S. currency), provide a time series that spans the entire war. This work estimates the effects of both Confederate and Union victories, as well as that of individual major battles. It also connects that literature to political science theories of war, and shows how its insights can be used not just to understand how markets work, but also to better understand how war works. This work builds on a large and growing literature which measures impact or identifies important political events using financial (North and Weingast 1989; North and Weingast 2000; Frey and Kucher 2000a; Sussman and Yafeh 2000; Wells and Wills 2000; Herron 2000; Eldor and Melnick 2004; Chen and Siems 2004; Greenstone 2007) or prediction markets (Wolfers and Zitzewitz 2004; Arrow et al. 2008; Wolfers and Zitzewitz 2009).

1.2 Financial Markets and the Study of War

Financial markets relate to the study of war in several ways. First, they provide a measure of the the costs of war (Schneider and Troeger 2006; Guidolin and La Ferrara 2010). Second, financial markets are themselves a key actor or mechanism in several theories of war. This includes the "Capitalist Peace" (Gartzke 2007; Dafoe and Kelsey 2014), in which markets can provides signals of the costs of war to actors, and Slantchev (2012), in which debt financing creates a commitment problem. Third, in some cases, the prices of financial assets act similarly to a prediction market for the expected onset or outcome of war. The implied expectations of war outcomes derived from the prices of these financial assets can be used to assess theories of war initiation and termination.[1] In this work, I focus on the third case, and use the yields of U.S. government bonds as a measure of expectations of the outcome of the American Civil War.[2]

[1]See Chapter 2 for an analysis of how financial markets assessed the probability of the onset of the American Civil War.

[2] I use the term "outcome" to mean the state of the world at the end of the war, including the duration, cost, and victor of the war. I use the term "result" to refer to the specific outcome of who "won" or "lost" the war, which can be context specific.

1.2.1 How the Prices of Financial Assets Relate to War Outcomes

A financial asset is a claim on a stream of future, possibly uncertain, cash flows.[3] For example, a coupon bond pays a set number of coupons at specified times and its face value on maturity. A stock pays dividends at potentially uncertain times. Both because these cash flows are in the future and because their payment may be uncertain, these cash flows are discounted. The current price of a financial asset, P_t, is the value of those discounted cash flows,

$$P_t = \sum_{j=1}^{H} \frac{\mathrm{E}_{t+j}\left(C_{t+j} | \mathscr{I}_t\right)}{1 + \mathrm{E}_t\left(\delta_{t+j} | \mathscr{I}_t\right)} \tag{1.1}$$

where H is the number of cash flows, C_{t+j} is the cash flow at time $t + j$, and δ_{t+j} is the discount rate for time $t + j$, and $\mathscr{I}_t$ is the information available to the market at the current time t. The discount rate often consists of a risk free interest rate and an asset-specific premium, both of which may be time-varying. The riskiness of the asset can also be incorporated in $\mathrm{E}(C_t | \mathscr{I}_t)$. For example, if there is a positive probability of default, $\mathrm{E}(C_t)$ would be a function of the probability of default and the amounts received in default and non-default states. The key feature of Equation (1.1) is that all of the price inputs are expectations conditioned on current information. Thus, events change the price through changing the market's information set.

Since the prices of financial assets are a function of expectations about future cash flows and risk premia, assets in which those cash flows or risk premia are largely contingent on some outcome of a war are themselves a proxy for expectations about that war outcome. For this purpose, the ideal asset would be a binary option, which would pay out a non-zero amount if the war outcome of interest occurs and zero otherwise. Then, with only some assumptions about the risk-preferences of the market, the probability of the

[3] This discussion largely follows the discussion in Guidolin and La Ferrara (2010, p. 673); refer to them for more detail. See also Haber et al. (2015). See any introductory finance textbook or course notes for more information. See Chan-Lau (2006) for an overview of market-based measures of sovereign risk.

war outcome is easily calculated from the price of the asset. Binary options are often used in "prediction markets" for events. There are several examples of prediction markets for political events. The Iowa Electronic Market is a prediction market for U.S. presidential and Congressional elections.[4] Intrade, until its closure in 2013, had prediction markets for multiple political and foreign policy events. These prediction markets included U.S. presidential and other elections, whether Saddam Hussein would be removed as the president of Iraq, when Osama Bin Laden would be either killed or captured, and when there would be a U.S. or Israeli airstrike against Iran.[5]

Unfortunately for the researcher of conflict, these prediction markets in political events are not widespread at present, and not available historically for wars. However, financial assets in which the cash flows or risk premia are contingent on a war outcome are effectively prediction markets for that war outcome. In particular, the sovereign bonds issued by the belligerents in a war are likely to be highly sensitive to war outcomes. These bonds respond to two features of the war outcome. First, the riskiness of a belligerent's sovereign bond is a function of the expected cost of the war. A more costly war almost directly implies either or both more debt issued or higher taxes, that cannot be used to service existing debt. Both of these imply a higher probability of default for the bond, and a lower price of that bond. Wars are also costly to the belligerents in more ways than just direct government expenditures. The destruction of human capital, military and civilian casualties, and physical capital affects on the expected economic growth of that country, and the future resources with which to pay off the debt.[6] Note that these expectations of

[4] See Wolfers and Zitzewitz (2004) for an overview of prediction markets. Rhode and Strumpf (2004) discusses historical betting markets on U.S. presidential elections in the late nineteenth-early twentieth century, which unfortunately for me were not formalized until 1884. See http://tippie.uiowa.edu/iem/ for the Iowa Electronic Market.

[5] The Saddam contract issued by Tradesports is used in Leigh, Wolfers, and Zitzewitz (2003). See http://intrade-archive.appspot.com/event.jsp?event=4272 and http://intrade-archive.appspot.com/event.jsp?event=37985 for the Intrade contracts on Osama bin Laden and an airstrike against Iran, respectively.

[6] See Goldin and Lewis (1975) for a calculation of the costs of the American Civil War.

the cost of war incorporate both expectations about both the intensity of the war and its duration. Second, the riskiness of a belligerent's sovereign bond is a function of whether the belligerent is expected to win or lose the war. Victory in war can gain the belligerent more territory and greater resources to pay off debt; a loss can mean the converse. In some cases, a defeat can lead to a loss of sovereignty and a default on their debt. This is relatively rare in inter-state wars, but more common in civil wars in which the rebel side issued debt (Haber et al. 2015). A loss of sovereignty followed by a repudiation was the case for holders of the debt and currency of the Confederate States.[7] More generally, losing states may tend to default on their debt with a higher probability (Slantchev 2012). The price of a bond is generally some weighted function of all of these expectations about what the outcome of the war will be and how those outcomes will affect the ability of the belligerents to repay their debt. For some bonds, it may easy to back out the probability of a specific war outcome. Haber et al. (2015) focus on cases in which rebels issued bonds in civil wars (American Civil War, Chinese Civil War, and Spanish Civil War) and estimate the probability that the rebel side is victorious. But it may not be possible to disentangle expectations about the cost and outcomes of the war for the belligerents. However, there is some advantage to the way in which financial assets weight these war outcomes. By putting all war outcomes on the same scale, namely, their fiscal effect on that belligerent, the prices of sovereign bonds are able to provide a single, if incomplete, measure of the war outcome for each belligerent that incorporates both the costs and benefits of the war to that belligerent. Additionally, because interest rates are expectations and are generally available at a high frequency, they provide a real-time measure war expectations. Sovereign bonds are not the only financial asset which could be used as a proxy for war outcomes, stocks Chen and Siems (2004), Schneider and Troeger (2006), and Wolfers

[7]Section 4 of the Fourteenth Amendment, passed after the American Civil War, explicitly repudiates Confederate debt.

and Zitzewitz (2009), exchange rates Hall (2004), and commodities Wolfers and Zitzewitz (2009), may respond to conflict in some cases and may be able to be used as proxies for expectations about war outcomes. However, while there are commonalities, the specifics of how a financial asset relates to war outcomes are not universal, and in each case the researcher should carefully consider how the financial asset relates to the war outcome of interest. When using a financial asset price fas a proxy for an expected war outcome, it is also necessary that its price is primarily influenced by that war outcome and not other factors. For example, when studying the Iraq War, it would be implausible to use the interest rates of U.S. Treasury bonds or the S&P 500 index since the effect of the war one way or another is likely a small influence relative to other economic factors.[8] It would be plausible to use bonds issued by Iraq, if they had issued in 2003, because the outcome or initiation of a war with the U.S. would plausibly be the most important factor influencing them. This is not out of concern of confounding, but a need for the the signal (war outcome) to noise (other factors that cannot be controlled for) ratio of the price needs to be large for it to be plausible to use the price as a proxy for a war outcome. Using sovereign bonds, or most financial assets, to proxy for war expectations only works for war outcomes so long as those outcomes has some influence on either cash flows or risk premia. Some policy objectives of a war, *e.g.* national pride or the abolition of slavery in the American Civil War, may be important to the belligerents and of interest to the researcher, but if they do not affect future cash flows or risk premia, then they will have little effect on the prices of financial assets.

In this work, instead of prices, I use the yields to maturity (yields) of coupon bonds. The yield to maturity of a coupon bond can be found by taking a known price and cash

[8] It is still possible to ask questions such as how much influence did events related to the Iraq War have on the stock market, as in Wolfers and Zitzewitz (2009). It would just be difficult to ask the inverse question which may be of more interest to political science research; given that the stock market is proxying expectations of the Iraq War, what events had the largest influence on it.

flow schedule in Equation (1.1) and solving for a discount rate. For example, assuming continuous compounding for simplicity, the yield to maturity, y, of a bond is the solution to

$$P_t = \sum_{j=1}^{H} C_{t+j} e^{-yj}. \tag{1.2}$$

Equation (1.2) shows that the interest rate moves inversely to the price: a higher yield corresponds to a lower price, and vice-versa. An increase in the risk of a bond lowers its price and raises its yield.

1.2.2 How Financial Markets Can Improve the Study of War

Since Fearon (1995), the dominant theory of war has been the bargaining theory of war.[9] While there is a large theoretical literature that has formalized much of the theory (Filson and Werner 2002; Slantchev 2003; Smith and Stam 2004; Powell 2004; Leventoğlu and Slantchev 2007; Langlois and Langlois 2009; Wolford, Reiter, and Carrubba 2011), direct empirical evidence of the theory is still limited (Reiter 2009). Financial asset prices can improve the discipline's understanding of war in two ways: expanding the number of cases which can be used for analysis by allowing for intra-case quantitative analysis, and providing a measure of surprising events.

The bargaining theory of war is essentially a formalization of Clausewitz's famous dictum: "War is merely the continuation of policy by other means (Clausewitz 1989, p. 87)." The bargaining theory of war sees war as part of a bargaining process. The central puzzle of war is, given that war is costly, why did the sides not reach an agreement without incurring the costs of war. In particular, at the end of a war, some agreement is reached, even if that involves the complete capitulation of one of the sides. So why was that agreement not reached prior to the war and without incurring the costs of the war. Fearon

[9] See Reiter (2003), Powell (2006), and Reiter (2009) for discussions.

(1995) and Powell (2006) propose two frictions that would prevent an *ex ante agreement*: private information and commitment problems.[10] But this leads to a second puzzle: given that these frictions can lead to conflict instead of a peaceful bargain, how does the actual fighting in the conflict resolve that friction so that the sides can come to an agreement and return to peace. Leventoğlu and Slantchev (2007, p. 757) describe theories which answer this question as *complete* and *coherent* theories of war: "[A theory of war] must be complete, which means it must account for war's outbreak and termination; and it must be coherent, which means that its account of termination must explain how fighting has resolved the cause." While some evidence for the bargaining theory of war can come from studying war onset, testing the "core of the theory" requires intra-war data: estimates of capabilities, estimates of resolve, exchange of offers, and combat data (Reiter 2003). Since these data are difficult to acquire, there have been few empirical studies that attempt this (Reiter 2003; Ramsay 2008; Reiter 2009; Weisiger 2015).

Financial asset price data is useful to this research agenda in two ways: it provides a method for quantitative analysis of a single war, and it provides a measure of surprising events. First, financial asset price data allows for within-case quantitative analysis of war termination. Instead of using the cross-sectional variation in war outcomes from multiple wars, financial asset prices provide measures of expected war outcomes that are time-varying within each war. Being able to use intra-case variation is useful to researchers, because, apart from controlling for case specific attributes, the data demands of the bargaining theory of war means that few cases have the requisite data. While data on the sides' estimates of resolve and capabilities and (often secret) bargaining offers are unsurprisingly scarce, even data on battles is also of surprisingly poor quality and quantity.

[10] War can also occur due to the sides the being risk loving or due to domestic politics. Fearon (1995) dismisses risk lovingness as implausible. Many of the domestic politics explanations are explanations that generate a private information or commitment problem that allows for conflict as a solution to the bargaining problem.

While there exists a Correlates of War for wars, there does not exist a similar dataset for battles, and the available data on battles is limited in both quantity and quality. The most commonly used and comprehensive battle dataset for inter-state wars is the CDB90 or HERO dataset (HERO et al. 1984; CAA 1991), but that data is both dated and of questionable quality (Reiter 2003, p. 32; Biddle and Long 2004). This lack of available data was one motivation for the use of the American Civil War in this work, since it is a war for which data on battles is available, and of relatively high quality and comprehensiveness.[11] Even if battle data were more plentiful, the ability to compare features of battles across wars may be fundamentally limited due to the heterogeneity of combat across time and space (Reiter 2009). As such, Reiter (2003) and Reiter (2009) suggest that the best methodology for the empirical analysis of the bargaining theory of war is qualitative analysis on limited cases. But in within-case analysis, some important dependent variables—war outcome, *e.g.* duration—cannot be used in the analysis, since only one war outcome is observed for each war. But, as discussed in Section 1.2.1, the prices of some financial assets may act as time-varying expected values of the war outcome. This allows researchers to conduct within-case quantitative analysis using cases with the requisite battle data, and use the prices of carefully-chosen financial assets as an outcome variable that proxies for war outcomes, such as the duration, cost or result.

Second, financial asset price changes naturally provide a measure of surprising events and new information. Surprising events are particularly important in private information theories of war, as only new information should affect beliefs. However, it is difficult for the researcher to know what was surprising to contemporaries; see Shirkey (2009) for an attempt to code surprising events. Prices provide a natural measure of surprising events, since as discussed in Section 1.2.1, prices are expectations of future events given

[11] More recently work by Weisiger (2015) and Cochran and Long (2014) have offered multi-war data. See CAA (1991) and Helmbold (1995) for a discussion of the difficulties in compiling battle data.

the current information available to the market, and changes in prices correspond to new information available to the market,

The prices of financial assets have several other characteristics that make them a useful addition to the conflict researcher's toolbox for analyzing war.[12] First, they incorporate only *ex ante* information. This distinguishes price data from sources written after the fact, including participant memoirs fact or secondary sources (Willard, Guinnane, and Rosen 1996, p. 1001; Frey and Kucher 2000b, p. 188). Those sources can suffer from hindsight bias because the author is aware of future events. Second, investors have strong incentives to be truthful in their assessments because the payoffs from their investments are contingent on the accuracy of their assessments. This distinguishes prices from other sources, such as surveys or diaries of decision makers, which, like prices, have only the information available at the time, but in which the author may have strategic incentives to deceive (Reiter 2009, p. 57).[13] Finally, price data is often available at high frequencies, often daily.

[12]See Willard, Guinnane, and Rosen (1996), North and Weingast (2000), and Frey and Kucher (2000a) for other discussions on the use of price data to analyze historical events.

[13] Although the investors in the American Civil War had their own preferences on the war, their investments seemed motivated by their preference for profits. Cornwallis (1879, pp. 5,7) writes,

> Sectional feeling often entered largely into the bull and bear contests in the Gold Room, and Union men and rebel sympathizers fought their battles sometimes, as much to gratify this as to make money.
>
> The heaviest speculative orders were sent from Washington and Baltimore, and next to these, from Louisville, Kentucky, owing to these cities being in close communication to the seat of war and the rebel lines; and the operators there, almost to a man, were "bulls" in feeling, and strong Secessionists. But though seldom or never found selling "short" they were quick to sell out their "long" gold — that is the gold they were carrying — whenever the Confederate arms met with a reverse, and as quick to buy it back again when the market seemed to "touch bottom".

Mitchell (1903, p. 210) writes,

> Viewed in a broad way, it is therefore a serious mistake to look on the gold market as a place where a few gamblers were tossing the premium about to suit their selfish schemes; a much saner view is that it was the place where the community's estimate of the government's credit was visibly recorded. Here, as in other markets, those operators succeeded who forecast the future correctly, and men who tried to advance the price of gold when public confidence was increasing, or to depress it when confidence was on the wane, learned to their cost that they were not masters of the situation.

This is in contrast with documentary evidence, which is often either produced at lower frequency or missing for large periods of time (Reiter 2009, p. 57). The high frequency of price data provides more data points to the researcher, and expands the set of questions that can be answered.

This is not to claim that prices of financial assets dominate other data sources. They simply provide another tool for conflict researchers. Financial asset prices have several shortcomings when studying conflict, and are not appropriate in all cases. First, these prices do not directly measure the beliefs of the belligerents. Prices of financial data generally incorporate public information, and do not incorporate the private information available to decision makers. However, the causal mechanism in private information theories of war is the convergence of the beliefs of decision makers. Yet, in those theories the means by which beliefs converge is that conflict reveals private information, making that private information public. It may be possible that many private information models of war could restated so that they have implications for changes in the expected outcome of the war conditional on public information, for which financial asset prices can server as a measure. Second, an appropriate financial asset may not be available in many conflicts. In many areas in which researchers are most interested in understanding conflict, *e.g.* civil wars in developing countries, a financial asset that is closely tied to the war outcome may not exist. Even now, not all developing countries issue sovereign bonds. Additionally, those countries prone to war may not have liquid markets, so the prices of available assets may not be accurate assessments of market beliefs. Even states with liquid and well-developed markets in peace-time often intervene in their financial markets during war. This distorts the price signals from those assets (Haber et al. 2015, p. 12). Finally, in any case in which prices are used, they cannot be used without domain specific knowledge. The researcher will need to apply their substantive knowledge of the assets, market, and conflict in order to find a financial asset that is related to expected war outcomes, and to understand how that asset relates to the war outcomes which are of interest to the researcher.

1.2.3 The Means and Mechanisms of War

While I motivate the use of financial asset prices to study war from the perspective of the bargaining theory of war, I do not attempt to directly test the private information and commitment theories of war in this work.[14] Instead, I focus on the effects of military combat — victories and defeats in major battles — on the expected cost of the war. I do not attempt to distinguish whether these victories and defeats influence the expected war termination through the revelation of private information or through resolving commitment problems. I do this because distinguishing between private information and commitment theories of war using only battle data is harder than previously appreciated. Observing only battle outcomes, such as victories or casualties, is insufficient to identify the causal mechanism of the war without relying on a well-specified model of the war process. These models do not exist at the moment. Models of that kind would require a deeper study of military processes. By estimating the effects of battles on war termination, and identifying which battles seemed to have the largest effect, this work may in small part contribute. This section explains the reasoning behind this claim.

In the bargaining theory of war, war combines a political bargaining process with military combat process.[15] These processes are distinct yet interconnected. For example, in costly-process formal models of war, war is modeled as alternating rounds of "bargaining" and "fighting", *e.g.* Slantchev (2003). The bargaining and combat are distinct, but the outcome of the game depends on both. As discussed in the previous section, a complete and coherent theory of war needs to explain how fighting resolves the friction—private

[14] See Ramsay (2008), Weisiger (2015), and Reiter (2009) for works which attempt to do so.

[15] "Combat is a violent clash between at least two politically distinct groups organized to wield force. War consists of sustained and substantial episodes of combat." (Reiter 2003)

information or a commitment problem—that prevented an agreement without war. The means of war is military combat, the immediate goals of which are the destruction of military forces, the destruction of civilian assets, or the control of territory (Reiter 2003, p. 30).[16] These means of war must lead to war termination by resolving the private information or commitment problems. But there may exist multiple immediate means of combat and multiple channels by which each of those means influence war termination. Figure 1.1 illustrates the relationship between means, mechanisms ans war outcomes. In one view, resolution of private information or the commitment problem are causes of war termination. In another view, military combat is a cause of war termination and resolution of private information and commitment problems are mediators. And in general, it is difficult to identify the causal effects of mediators (Keele 2015).

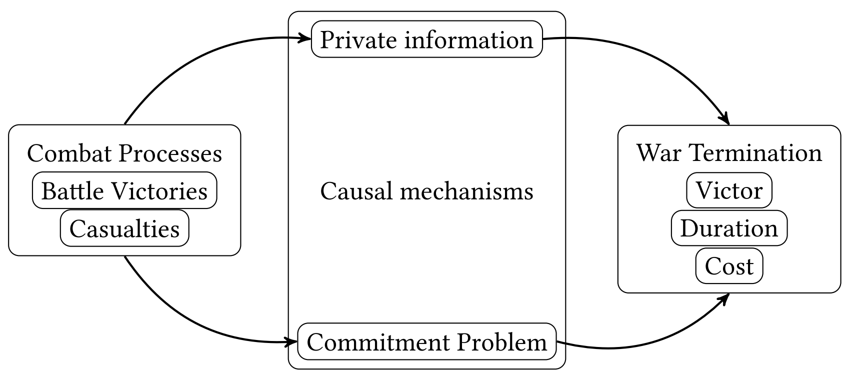

Figure 1.1: Causal Diagram of combat and war termination

With regard to war, identifying the channels through which combat influences the war termination is empirically quite difficult. Almost any observable feature of military combat alters the actual state of the war, and potentially resolves a commitment problem, and also alters the beliefs of the participants about the combat process in which they

[16] While formal models of war can be categorized by whether they are models of the private information mechanism, commitment problem, or both, they can also be categorized by how they model military combat within the war. In some models battles are important primarily because they influence the probability of war victory (Powell 2004; Wagner 2000; Leventoğlu and Slantchev 2007; Slantchev 2003; Smith and Stam 2004). In other models, battles are important because they impose costs on the sides (Filson and Werner 2002; Powell 2004; Leventoğlu and Slantchev 2007).

are involved, and potentially resolves a private information problem. There are a couple ways in which this effects might be identified. First, if the beliefs of the participants were observed, then the effects of war events on the beliefs of the participants could be measured. A problem with this approach is that it requires data on the beliefs of the participants, which is difficult to acquire. Even if first-hand accounts are available, it is not clear whether these are accurate indicators of those beliefs. Another problem is that it is insufficient to simply observe changes in the beliefs of one participant over time, as in Reiter (2009). The causal mechanism in informational theories of war is the convergence of beliefs of the belligerents to a common distribution, which requires knowing the beliefs of both sides. Second, it could be identified if commitment problem that started the war were known and changes it were observable. Third, it could be identified through a structural model of how military combat influences war outcomes. Then it may be possible to back out how belligerents would update their beliefs and how combat could result in military victory or resolve commitment problems. However, this would require more work which focus on generating models of military combat, even if these models are not related tied to political variables. To understand how fighting resolves the frictions in bargaining, either changes in those frictions must be observed, or it is necessary to understand the process that underlies the fighting. One difficulty in the later is that military combat may be extremely heterogeneous, both across time and space (Reiter 2003). But that is not a reason to ignore the study of military combat, but a call to describe and explain those patterns. A better understanding of the the military combat process in war may have different implications for these models, and improve the ability to distinguish between these causal mechanisms.[17]

[17] For example, Walter (2009) conjectures that insurgency is slower to reveal information than other forms of war. Rigorously evaluating theories in this manner would require better theories of the actual military rationale between insurgency versus conventional war, the characteristics of each, and when belligerents choose to use each.

1.3 Why The American Civil War?

The American Civil War has two features that make it particularly suited for this analysis. It is a war with plentiful, and high quality battle-level data, and in which there are financial assets which proxy expected war outcomes well. In terms of its battle data, the American Civil War is one of the best documented wars, and the battle data for it is discussed in Section 1.5. [18] In Section 1.4, I discuss why financial assets issued by the belligerents of the American Civil War may be good proxies for its war outcome.

Like any case, its generalizability is limited by its similarity to other observations. But despite its age, the American Civil War has several similarities to contemporary civil wars. First, the American Civil War is, if not the first modern war, a that marked the transition to modern warfare. This war introduced several of the technologies which came to prominence in World War I and later wars: rifled artillery, telegraph, machine guns, barbed wire, ironclad warships, and submarines (89 Fuller 1956; Weiss 1966, p. 760). Second, the American Civil War is primarily a conventional war. This places it in the plurality of post-Cold War civil wars, which are more often conventional wars than insurgencies (Kalyvas and Balcells 2010, p. 423). Finally, by the Civil War, the United States was wealthy enough that the power parity adjusted GDP per capita of the United States and Confederate States in 1860 would classify them them as lower-middle income countries today.[19]

The American Civil War is also an interesting case for study in its own right. Its impor-

[18]For example, *The Official Records of the Union and Confederate Armies* (U.S. War Dept. 1880-1901), published between 1880 and 1901, consists of 128 books in 70 volumes totaling 139 thousand pages.

[19]The GDP per capita of the US in 1860 was $2,241 in 1990 GK international dollars. In 2010, Cambodia had a GDP per capita of $2,450, Pakistan $2,494, and Ghana $1,922 (Bolt and Zanden 2013). The southern states were poorer than the northern states, with a per capita consumption of about 70 percent of the overall U.S. level (Goldin and Lewis 1975, p. 324). The GDP per capita of the pre-war Confederacy is similar to that of Angola, Iraq and Senegal in 2010.The estimated GDP per capita of the southern states in 1860 at 70% that of the northern states was $1,568. In 2010, the following countries had approximately the same real GDP per capita: Angola $1,600, Iraq $1,610, and Senegal $1,507 (Bolt and Zanden 2013).

tance to American history is obvious, and need not be stated.[20] But, given the voluminous literature on the subject, it is surprisingly understudied in at least two regards. First, the international relations literature has largely ignored this conflict. While the inter-state war literature considers it a civil war, and the civil war literature focuses on the post-1945 era (Reiter 2009, pp. 140-141; Poast 2012, p. 2). Exceptions to this are Reiter (2009) and Poast (2012). Second, almost none of the existing work on the American Civil War conflict, including the few examples in international relations, use quantitative methodologies.[21] In applying international relations theory and quantitative methods to the study of this war, this work contributes a droplet to the ocean that is the literature on the American Civil War.

1.4 U.S. Government Bond Yield Data

1.4.1 The Fives of 1874

I use the yields of long-run U.S. government bonds as a measure of the expected outcome of the American Civil War. While the yields U.S. government bonds were almost certainly primarily driven by war expectations, they were plausibly influenced by both expectations of U.S. victory and expectations of war costs. However, it is likely that expectations of war costs is the more prominent of these two factors. In this work, I use the the yields of the Fives of 1874, a U.S. government coupon bond. It is the only U.S. government bond that was both issued prior to the war and quoted throughout the war, meaning that it is the only time series of a single asset that spans the war.

The Fives of 1874 were a coupon bond authorized by the Act of June 14, 1858 to pay

[20] See McPherson (2003) among too many others to list.
[21] Weiss (1966) for an example from operations research.

for a budget deficit due to the Panic of 1857, a financial panic and recession.[22] As their name suggests, the Fives of 1874 were a coupon bond with a coupon rate of 5 percent per-annum paid semi-annually, and redeemable in 1874. Thus, during the period considered here, 1861–1865, the Fives had a maturity of 13–9 years. Like most U.S. government bonds of this period, the Fives of 1874 paid interest and principal in specie (gold dollars). They continued to pay interest in specie even after the U.S. Treasury ceased the redemption of Greenbacks (currency) in gold.[23]

Data on the prices of the Fives of 1874 come from tables in the two leading monthly financial magazines of the era, the *Bankers' Magazine and Statistical Register* and the *The Merchants' Magazine and Commercial Chronicle* (Mitchell 1903, p. 186). In these publications the prices of bonds, not their yields, were quoted. The prices were quoted in currency, and not in gold dollars, even though they paid interest in gold dollars. I converted the prices to gold dollars using the current exchange rate of Greenbacks to gold dollars in New York, using the Mitchell (1908) data.[24] The yields of the bonds were calculated by converting prices to gold dollars accounting for the current exchange rate and the expected depreciation of the gold dollar.[25] Calculating the price and yield in gold dollars

[22] See Bayley (1882, p. 76), De Knight (1900, pp. 78–79), U.S. Treasury (1863, pp. 42-43), and Homer and Sylla (2005, pp. 300-301, 305).

[23] Greenback refers to several non-interest bearing notes issued by the U.S. as the currency during the war. Greenbacks were originally redeemable in gold dollars at par on demand. However, the Treasury ceased redemption of these notes for gold on January 1st, 1862. U.S. currency floated against the gold dollar from January 1st, 1862 until the resumption of convertibility on January 1st, 1879 (Mitchell 1908; Dewey 1918; Willard, Guinnane, and Rosen 1996).

[24] Available in digital form at http://eh.net/wp-content/uploads/2013/11/greenback.txt from Willard, Guinnane, and Rosen (1996). I made some corrections to it where there were incorrect transcriptions from the original source.

[25] Since the price of bonds was quoted in currency but it paid interest and principal in currency, calculating the yields is difficult since they also incorporated beliefs about the future prices of gold dollars (Macaulay 1938, Appendix A; Roll 1972; Calomiris 1988; Homer and Sylla 2005, pp. 302-303). Most bonds, including the Fives of 1874, were priced in U.S. currency but paid specie and often principal in gold dollars. This work accounts for this using a simple method to calculate the future price path of the dollar; it assumes that currency with appreciate at the risk free interest rate of 5 percent until it reaches parity with a gold dollar. This follows from an assumption that the risk free rate is constant over the period, and the gold dollar is priced as a risk-free zero-coupon bond with an unknown redemption date McCandless 1996. Roll (1972) and Calomiris (1988) estimate the expected depreciation of gold dollars in Greenbacks using differentials in

rather than currency is a meaningful decision, because while the price and yield in gold dollars fluctuated throughout the war, the price and yield in currency were stable.[26] This suggests that investors were less worried about a complete default than an implicit default in which government would redeem interest or principal payments in depreciated currency rather than specie. Both sources quote the bonds at irregular, but approximately weekly frequencies. [27] For the analysis, data from these two sources are combined to form a single series.

I collected and provide the financial data used in this work, as well as additional financial and economic from the American Civil War period in the American Civil War Era Financial Data Collection (ACWFD) (Arnold 2015b).[28] This data collection contains bond prices and exchange rates published in the *Bankers' Magazine* and *Merchants' Magazine*, bid-level auction data for U.S. government bond auctions, and miscellaneous financial and economic data for this period from a variety of primary and secondary sources.

Figures 1.2a and 1.2b plot the price in gold dollars and the yield to maturity of the Fives of 1874. Prior to mid-1860, the yield was low with little variation, fluctuating between 4.5 and 5 percent.[29] After Lincoln's election in November 1860, the interest rate rose to 6 percent. After the initiation of the war with the Battle of Sumter (April 12–14, 1861), the yield rose to 8.2 percent. During the war, between April 1861 and April 1865, the yield fluctuated between 5.2 and 14.7 percent, reaching its peak on July 30, 1864. After the end of conflict in April 1865, the interest rate returned to stability at 6.5 percent, a rate higher than its pre-war level.

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bonds, however those methods cannot be applied to the period considered here.

[26]See Roll (1972) for an analysis of this phenomena.

[27]*Bankers' Magazine*: 3–15 days, with a median of 10 days; *Merchants' Magazine*: 4–23 days, with a median of 7 days.

[28]Also available at The dataset is at http://github.com/jrnold/civil_war_era_findata.

[29] This is actually below was generally considered risk-free rate in the U.S. for that period, 5 percent, which Elder (1863, p. 29) calls the "natural rate of interest on [government] Loans". During this period British 3 percent consols had a yield of between of 3.10 and 3.28 (Homer and Sylla 2005, p. 193).

The Fives of 1874 were one of several bonds issued by the United States government during this period, but it has several advantages over other bonds for the purpose of this analysis. The Fives of 1874 were the only coupon bond regularly quoted for the entire war, April 1861–May 1865, in the sources considered here. The reason it is the only such bond, is that while the United States issued much debt during the Civil War, it had issued little debt in the antebellum period, and thus there were few outstanding bond issues at the start of the war.[30] The Sixes of 1848–1868 was the only other bond widely traded at the start of the war, but *Bankers' Magazine* stops quoting these shortly after the start of the war, in September 1861. Another bond, the Sixes of 1861–1881 were issued under acts in February and July 1861, just prior to and just after the start of the war. They are not regularly quoted in the *Bankers' Magazine* until September 1861, five months after the start of the war. The United States issued several other bonds of various maturities and rates throughout the war, but they provide even shorter time series. In many cases these loans also are have exotic features relative to government bonds issued today. They often contained options to exchange the bond for other bond issues or were callable by the government. Both of these features make it difficult to calculate yields for these bonds.[31] Using the Fives of 1874 instead of these other bonds does not make a substantive difference for the analysis, since as Figure 1.3b shows, the patterns of the yields of the Fives of 1874 is similar to the yields of all the other U.S. government bonds. Willard, Guinnane, and Rosen (1996), McCandless (1996), and Smith and Smith (1997) use the exchange rate of Greenbacks to gold dollars in their analyses. Since the United States did not issue Greenbacks until after the start of

[30] After paying off the debt issued during the War of 1812, the U.S. government had no outstanding debt between 1835 and 1842 (Homer and Sylla 2005, p. 297). The debt as a percentage of GDP rose from 1.9 percent in 1860, before the war, to 31.4 percent in 1866, its peak just after the war (CBO 2012a; CBO 2012b).

[31] The 5-20's and 10-40's were bonds callable by the government after 5 (10) years and redeemable in 20 (40) years. The 7-30's were bonds that had coupon rates of 7.30 percent (2 cents per day), were redeemable in 3 years, but included the option to be exchanged for either the Sixes of 1881 or 5-20's. See Bayley (1882), De Knight (1900), and Homer and Sylla (2005, pp. 297–309) for overviews of U.S. debt issues during this period.

the war, and did not suspend the redemption of Greenbacks for gold until January 1, 1862, any analysis using only Greenback prices cannot cover the first eight months of the war, April–December 1861. This is the period which should have the highest informational content in an informational theory of war, and includes battles such as the First Battle of Bull Run, and the Battle of Wilson's Creek.[32] Apart from the loss of observations, using Greenbacks instead of the Fives of 1874 would not make a substantive difference, since as Figure 1.3a shows, the price of Fives of 1874 in gold dollars is similar in both level and trend as the the price of Greenbacks in gold dollars.

The American Civil War is an ideal case for which to use prices to infer expectations of war termination for three reasons. First, The war was of such severity that the nearly the entire budget of the U.S. government was war-related military spending, and thus fiscal expectations corresponded to military spending expectations. The War and Navy Departments accounted for 76 percent (in 1862) to 61 percent (in 1865) of the expenditures (U.S. Treasury 1861b; U.S. Treasury 1861a; U.S. Treasury 1862; U.S. Treasury 1863; U.S. Treasury 1864; U.S. Treasury 1865). The fraction of the budget spent directly on the military by the Union fell over the course the war because the fraction spent on paying the principal and interest on its debt increased from 20 (1862) to 36 (1865) percent of the budget. The overwhelming the majority of that debt had been issued to fund the war, so debt service payments can be considered delayed military spending. As McCandless (1996, p. 668) states, "[d]uring the war, the single most important indicator of future government expenditures is the process of the war itself."

Second, there is extensive economic history literature establishing that war events are the single best determinant of bond and currency prices during the American Civil War: (Mitchell 1903; Mitchell 1908; Calomiris 1988; Willard, Guinnane, and Rosen 1996;

[32]Poast (2012) argues that the First Battle of Bull Run was pivotal in avoiding British recognition of the Confederate States.

McCandless 1996; Smith and Smith 1997), graybacks (Schwab 1901; Weidenmier 2002), southern prices (Burdekin and Langdana 1993), Confederate bonds (Davis and Pecquet 1990; Brown and Burdekin 2000; Oosterlinck and Weidenmier 2007), and Union bonds (Roll 1972).[33]

Third, statements by and the actions of contemporary investors indicated the importance that war events had on the prices. Members of the government, military, and reporters all engaged in investment speculation based on their knowledge of war events (Cornwallis 1879, pp. 5-7; Mitchell 1903; Willard, Guinnane, and Rosen 1996, p. 1004). Investment firms even had their own correspondents stationed in cities close to the war fronts to relay the latest news (Cornwallis 1879, pp. 5-7). Finally, magazines and newspapers often cited war events as the cause of recent price movements. For example, the issue of *Bankers' Magazine* after Gettysburg and Vicksburg (Jul 23, 1863), cites these battles as the cause of the recent changes in market prices, =

> The month of July has been a very active one with numerous fluctuations, almost daily, in the market values of gold, with unusual changes in the current values of stocks. The advices as to the war movements are of a most satisfactory kind, leading to a fall in gold from 146 1/2 at the close of June, to 123 1/2, the 20th inst., at which dates the public had learned the capitulation of Vicksburgh and of Port Hudson, the defeat of Lee in Maryland and Pennsylvania, and of successful results at other points for the Union forces. (*The Bankers Magazine* 1864, p. 159)

[33]One notable exception is Burdekin and Weidenmier (2001), which shows that the prices of graybacks in Richmond and Houston diverged due to differential application of monetary reform in late 1864.

1.4.2 U.S. Government Bond Yields are Primarily a Function of the Expected Cost of the War

Since the cash flows of U.S. government bonds are not directly related to a war outcome, in order to use yields as proxy for the expectation of a war outcome, war outcomes need to influence future cash flows, and war expectations need to be the primary factor influencing the yield. U.S. government bond yields are plausibly related to both expectations about whether the United States would win the war, and as the cost of the war to the U.S. government. These two expectations have possibly cross-cutting implications on bond yields. An increasing probability of U.S. victory would make bonds less risky (higher price; lower yield), while a increasing expected war cost would make bonds more risky (lower price; higher yield). Unfortunately, it is not possible to separate these effects using only U.S. government bond yield data. However, it is likely that U.S. government bond bond yields mostly responded to and can be considered a proxy for the expected war costs to the United States.

The risk on U.S. government bonds depended on the war cost for several reasons. The war was funded primarily through debt, and thus, expectations of a longer or more vigorously fought war directly implied an increase in government debt. The proportions of the U.S. government revenues for each source of funding was as follows: 72–89% from loans, 11–25% from taxes, and 0–4% from miscellaneous sources (tariffs, land sales). In its analysis of the future fiscal situation, the 1863 *Annual Report of the Treasury* notes that taxes and other revenue sources covered non-war expenses and debt payments in its budget, but debt issued were needed to pay for the military expenditures needed to wage the war (U.S. Treasury 1863, pp. 10-13). [34] The budgetary effect of the war is clear from comparing forecasts of the U.S. budget for the 1866 fiscal year made before and after the war had ended. In *The Annual Report of the Treasury* issued on December 6, 1864, the

[34] See also Godfrey (1976, p. 14).

forecast deficit for FY 1866, assuming that the war continued, was $470 million, with $1,168 million in expenditures (U.S. Treasury 1864, p. 13). In the following year's *Annual Report*, issued in December, 1865, after the conclusion of the war, the Treasury estimated a surplus of $112 million, with expenditures of only $396 million dollars (U.S. Treasury 1865). This forecast was close to what would be the actual surplus, $132 million (U.S. Treasury 1866, p. 2). Thus, both expectations of the government military expenditures and the length of the war should have directly affected investors' expectations of the eventual debt level of the U.S. government. This increased debt would increase the possibility of default or the temptation of the government to repay its debt in currency rather than specie.

The future resources of the United States to pay off its debt also depended on the result of the war, namely, whether the seceding states would be reincorporated into the United States and increase the available resources to pay the costs incurred in the war. The size and wealth of the seceding Southern states was much smaller than that of the Northern and Western states. In 1860, the states that would make up the Confederacy accounted for only 29 percent of the overall U.S. population,[35] and 25 percent of the total wealth.[36]

The manner in which U.S. government could have defaulted on its debt obligations takes several plausible forms in addition to simply completely defaulting. The government could have simply stopped paying interest, as the Southern states did at the start of the war (*The Bankers Magazine* 1862, p. 159)[810,947]BankersMagazine1860, for the remainder of the war. After the United States started issuing Greenbacks which were not redeemable on demand in gold, it could have paid either the interest or principal of bonds in currency

[35] Southern states included VA, TN, GA, NC, AL, MS, LA, SC, TX, AR, FL Eicher (2001, p. 5). Wealth is used instead of GDP, since GDP was not invented until later. Thus the total wealth of individuals, as reported in the Census, is a better measure of what contemporaries knew, since that is how they measured it.

[36] Elder (1865, p. 12). Wealth is defined as the value personal property excluding slaves, which was asked on the 1860 Census. It may be a better measure of how contemporaries would have viewed the value of the South than GDP because GDP had not yet been invented. The closest contemporaries had to GDP was an approximation that the value of yearly production was about 25-27 percent of total wealth (Elder 1865, p. 7; U.S. Treasury 1865, p. 24).

instead of specie. Since the Greenback traded at a discount to a gold dollar, this would This as a plausible concern, as there was a controversy among investors regarding a different bond issue, the 5-20's, which were explicit on repaying the interest in specie but were ambiguous as to whether the principal would be repaid in specie or currency (Roll 1972). Another example is that the Act of March 3, 1863 (12 Stat 710) declared that the interest on certificates of indebtedness issued under the Act of March 1, 1862 (12 Stat 352) was payable in currency (Bayley 1882, p. 81). Since these were issued after the suspension of convertibility of Greenbacks to gold this may have not been a surprise to investors, but it suggests that repayment in currency was a possibility. That the yields on U.S. government bonds when calculated in gold rose during the war, while those calculated in currency remained low and steady near 5 percent (Homer and Sylla 2005, p. 305), implies that to the extent that investors viewed U.S. government bonds as risky, it was that they would be repaid in depreciated currency rather than specie.If investors feared a complete default, then yields calculated in currency should have also risen during the war. This is why it is important to use yields and prices calculated in gold, which did respond to war risk, rather than yield and prices in in currency as in Homer and Sylla (2005), which are much less sensitive to that risk, since they ignore the risk due to repayment in deprciated currency.

Some statements by government officials and market participants suggest that they were more concerned with the cost and duration of the war than with the the reincorporation of the seceding states. William Elder, a leading economist of the era, in an 1863 analysis of the ability of the United States to repay its debt, stated "The Rebellion leaves our capital in real and personal property just where it was before the secession. … So far now this is only a loss of that which we have not had, and at best or worst, a very small one in any time of need (Elder 1863, p. 19)."[37] The 1863 *Annual Report of the Treasury De-*

[37] However, Elder (1863) does emphasize the importance of the resources of the Western territories, which perhaps strengthens the argument of Weingast (1998) that the United States was concerned that the secession of Southern states could lead to the secession of Western states.

partment described this relationship between war costs and the interest rate it paid on its bonds:

> It will not escape observation that the average [interest] rate is now increasing, and it is obvious that it must continue to increase with the increase of the proportion of the interest-bearing to the non-interest-bearing debt. And as the amount of the latter, consisting of United States notes and fractional currency, cannot be materially augmented without evil consequences of the most serious character, the rate of interest must increase with the debt, and approach continually the highest average. That must be greater or less in proportion to the duration and cost of the war. (U.S. Treasury 1863, p. 13)

The striking difference in the trend of the prices of U.S. bonds and that of the Graybacks (Confederate currency) in Figure 1.3 is also evidence that U.S. bond yields were less driven by expectations of U.S. victory than by the war cost. That figure shows that the price of Graybacks in gold dollars decreases over the entire course of the war. Although the price of Graybacks is partially determined by the money supply (Burdekin and Weidenmier 2001), the price of Confederate bonds issued in Amsterdam and payable in gold followed a similar pattern, decreasing from mid-1863 onward (Haber et al. 2015). The prices of Confederate assets were likely primarily determined by whether the United States would win the war and repudiate their debt (Haber et al. 2015). Thus, if the U.S. bonds primarily reflected the probability of U.S. victory, then the expected trend of its prices (yields) would increase (decrease) immediately after the start of the war, and almost monotonically increase (decrease) throughout the war, or at least from mid-1863 onward. However, Figure 1.2b shows that while the yields of U.S. government bonds fluctuate throughout the war, they do not decrease over the entire course of the war. Instead, they spike in mid-1864 at a point at which U.S. victory should have been highly probable.

This suggests that U.S. government bond yields do not primarily reflect expectations of U.S. victory, and instead reflect some other factor, which is likely war costs, by elimination.

Finally, the financial benefit of victory to the United States and war costs are inversely related. The longer and more destructive the war, the less the ability of the Southern states to assist in the repayment of the debt issued by the United States to pay for the war. Immediately after the war, the U.S. government attempted to collect direct taxes from the Southern states, but it soon suspended that collection after it became clear that the Southern states could not pay them. Treasury Secretary Hugh McCullough justified this suspension by noting that the inhabitants of the Southern states "had been subject to heavy taxation by the government which was attempted to be established in opposition to that of the United States, and had been greatly exhausted by the ravages of war." (U.S. Treasury 1865, p. 29). In analyses of the repayment of debt after the war by the Treasury Department and others (Elder 1865; U.S. Treasury 1865; Walker 1865), the wealth of the Confederate states was assumed to remain constant between 1860 and 1870 due to the destruction of the war, and it was assumed that they would not be able to contribute to the repayment of the debt until at least after 1870.

While the precise relationship between these bond yields and outcomes of the American Civil War is unclear, it is clear that these bond yields almost completely reflect war expectations. As noted earlier in this section, military expenses accounted for the overwhelming majority of U.S. government expenditures between 1861 and 1865, so there are no other budgetary concerns that could plausibly have been affecting investor's expectations. The importance of the war is also clear from comparing the level and variance of U.S. government bond yields during the war with the preceding period. During 1856–1860, yields on U.S. government bonds were stable at below 5 percent. At the height of the the Panic of 1857, the yield of U.S. government bonds (Sixes of 1868) only reached

5.1 percent.[38] Since a major financial crisis had little effect on bond yields prior to the war, there are certainly no non-war omitted factors affecting the yields. This is not to say that the battles in the war are the only factor affecting the bond yields. Government policies, elections, and other events could and almost certainly did have some affect on bond yields. But these events were affecting government bond yields through their effects on investors' expectations of the war, and not through other channels.

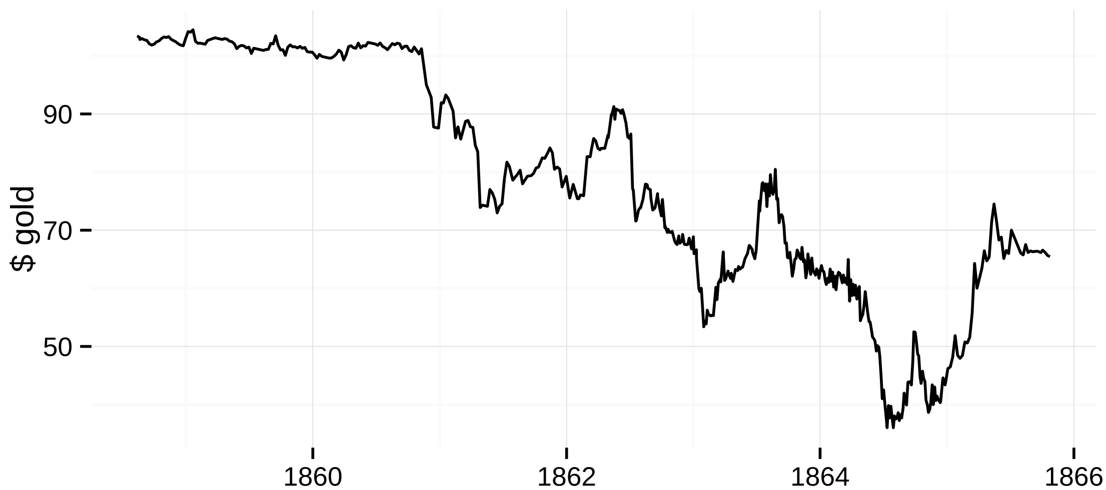
In summary, the U.S. government bond yields reflected some combination of expectations of the cost and the victor of the war, weighted by how each of these would affect the ability of the U.S. government to repay its debts. However, it is more likely that these yields are largely reflecting expectations about the cost and duration of the war.

1.5 Battle Data

A theoretical and practical issue in the empirical study of war is how to operationalize military combat within wars.[39] There are two major issues in the operationalization of military combat. The first is "what the unit of observation?" Some work uses events, *e.g.* battles or campaigns, as the unit of observation (Reiter and Stam 1998; Reiter and Stam 2002; Ramsay 2008; Reiter 2009, pp. 59-60; Pilster and Böhmelt 2011; Grauer and Horowitz 2012), while other work uses time-period aggregated values, *e.g.* monthly casualties (Weisiger 2015). The second is "how to measure military success?" Some work uses victory or defeat in battle (Reiter and Stam 1998; Reiter and Stam 2002; Grauer and Horowitz 2012), while other work uses casualty-based measures, such as loss-exchange ratios (Biddle 2004; Biddle and Long 2004; Pilster and Böhmelt 2011). In this work, I quantify military combat in the American Civil War by focusing my analysis on a set of important

[38] Data from the *Bankers' Magazine* and my calculations. Also see Homer and Sylla (2005).

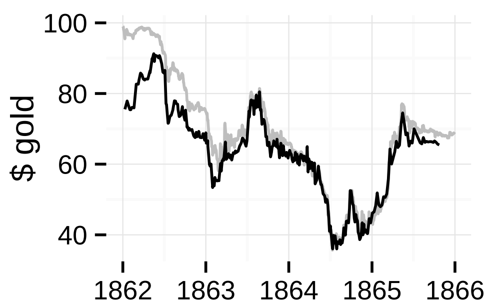
[39] See the discussions in Reiter and Stam (1998), Biddle and Long (2004), Reiter (2003), Grauer and Horowitz (2012), and Weisiger (2015).



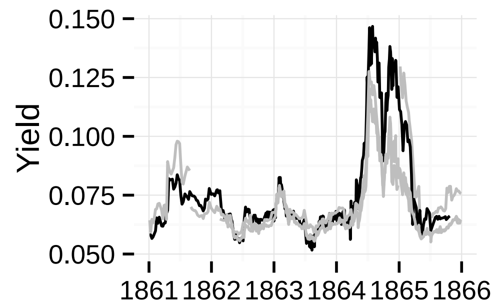
(a) Price in gold dollars. Par value is $100.

(b) Yield to maturity.

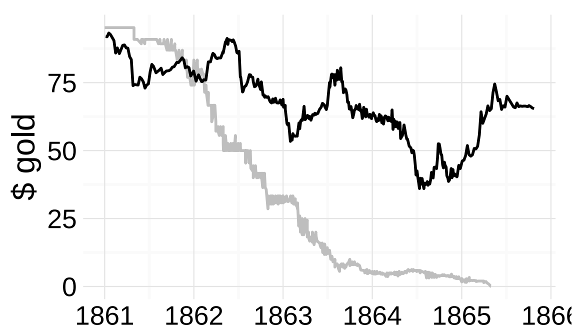
Figure 1.2: Price and yield of the Fives of 1874 from its issue in 1858 until the end of the Civil War in 1865. The yield is steady at near 5 percent until the election of Lincoln and the secession of the Southern States in late 1860-early 1861. During the American Civil War, the yields are variable, spiking in the middle of 1864.



(a) Price in gold dollars for the Fives of 1874 compared to the price of Greenbacks, 1862-1865.



(b) Yield to maturity of the Fives of 1874 compared to other U.S. bonds.



(c) Price in gold dollars of the Fives of 1874 compared to $100 in Graybacks (Confederate currency).

Figure 1.3: The Fives of 1874 compared with other government and state issued bonds and currency.

battles, and define success in combat by the battle result: Union or Confederate victory. An advantage of this method is that it allows for the estimation of individual battle effects with fewer assumptions on which features of those battles relate to war outcomes.

The battles used in this analysis are listed in Table 1.1. This set of battles consists of those which the National Park Service's Civil War Sites Advisory Commission (CWSAC) classified as the most important of the war (CWSAC 1993a). These data come from the 1993 CWSAC report, *Civil War Sites Advisory Commission Report on the Nation's Civil War Battlefields* (CWSAC 1993a; CWSAC 1993b). The CWSAC was a commission established in accordance with a 1990 law to preserve Civil War battlefields and was "asked to identify the nation's historically significant Civil War sites; determine their relative importance; determine their condition; assess threats to their integrity; and recommend alternatives for preserving and interpreting them (CWSAC 1993b)." As part of that process they identified 383 battles of the over 10,500 military engagements listed in the *Official Records of the American Civil War*, as the "principal battles" of the war based on their "military significance." As to the comprehensiveness of the list, CWSAC (1993a) states,

> These [battle] sites encompass virtually all of the principal land battles that were of special strategic, tactical, or thematic importance to local operations, campaigns, theaters, or to the war as a whole … The more than 10,500 conflict sites excluded from our inventory were relatively unimportant as individual military actions. These conflicts were the venues and actions that implemented the war between and beyond the dramatic major engagements.

These principal battles were further classified into four categories of military significance: "A", "having a decisive influence on a campaign and a direct impact on the course of the war", to "D", "having a limited influence on the outcome of their campaign or operation but achieving or affecting important local objectives". This work uses 43 battles with an "A" military significance classification.[40] While this is a subset of the battles in the American Civil War, it is still a relatively large number of battles by most war standards and similar in number and composition as the list of battles provided by other sources.[41] Livermore (1900) includes 63 battles, Bodart (1908) includes 50, and CAA (1991) includes 49. That this set of battles is a subset of battles in the American Civil War is largely due to the completeness of the records and richness of the data on the American Civil War.

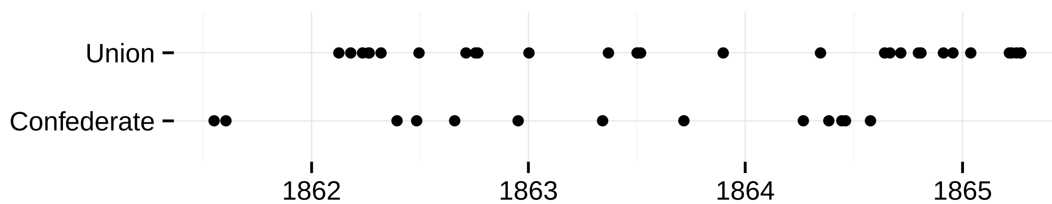


Figure 1.4: Dates of major battles of the American Civil War and their outcomes

The CWSAC (1993b) assigns a result to each battle, which is one of "Union victory", "Confederate victory", or "Inconclusive". While CWSAC (1993a) does not explicitly describe the methodology used to classify the battle outcomes, the classifications largely agree with other sources (Fox 1898; Livermore 1900; Bodart 1908; CAA 1991), several of which are themselves cited as sources of the report. Two of these sources, Fox (1898) and Livermore (1900), provide more detail on how they assigning the winner of a battle. Generally, the victorious side of a battle is considered to be the side that controls the battlefield

[40] Three CWSAC class "A" battles are omitted from the analysis. The Battle of Fort Sumter (SC, April 12–14, 1861) is omitted because it marks the start of the war and the analysis starts after this battle. Given the small size of Fort Sumter, no casualties, the effects of the battle can be attributed to marking the start of conflict. The Battle of Fort Blakely (AL, April 2–9, 1865) is omitted because news of battle does not reach New York City until after news of the surrender of Gen. Robert E. Lee's forces at Appomattox Court House on Apr 10, 1865. The Battle of Five Forks (VA, April 1, 1865) is omitted because I could not find reports of it in the *New York Times*; news of it was overshadowed by the Third Battle of Petersburg on April 2nd, which ended the Siege of Petersburg and started the Appomattox Campaign, which ended in the surrender of Robert E. Lee.

[41] In CAA (1991) the only wars with more battles than this are World War I, World War II, and the Napoleonic Wars.

at the end of battle. The data used in this work includes 30 Union victories, 13 Confederate victories, and no "Inconclusive" battles.[42] Figure 1.4 plots the date and outcomes of these battles over time. In the analysis, I will not explicitly include casualty or force size data. By selecting on the most militarily significant battles, I have already effectively selected those battles with the highest casualties and force sizes.

In order to estimate the effects of these battles on bond yields, I need the days on which the New York market received news about these battles. These are not the same as the dates of the battle, because news reaches the market with a lag. In general, the New York market received news about battles relatively quickly:

> Members of both Houses [of Congress], and of all political creeds, resident bankers, the lobby agents, clerks, and secretaries, haunted the War Department for the latest news from the seat of war. The daily registry of the Gold Room was a quicker messenger of successes or defeats than the tardier telegrams of the Associated Press. A private secretary of a high official, with no capital at all save his position, which gave him authentic information of every shaping of the chess game of war a full twenty hours in advance of the public, simply flashed the words "sell, buy" across the wires, and trusted to the honor of his broker for the rest. (Medbery 1870, p. 245)[43]

However, there was still a lag between the occurrence of a battle, and when news of it reached the market. More importantly for the analysis, this lag varied with the distance of the battle from New York and Washington, as well as idiosyncratic reasons. For example, the market knew nothing of Sherman's army after he moved north from Savannah until he

[42] In the original CWSAC data, there were two "Inconclusive" battles: The Wilderness, and Spotsylvania Court House. In the analysis, these were recoded as a Union victory and a Confederate victory, respectively, since with only two battles, I cannot estimate an average effect of "Inconclusive" battles. The Wilderness was reclassified as a Union victory, and Spotsylvania Court House as a Confederate victory based on how the results were reported in the *New York Times*.

[43] Originally quoted in Willard, Guinnane, and Rosen (1996).

	State	Theater	Dates	Result
First Bull Run	VA	Eastern	Jul 21, 1861	Confed
Wilson's Creek	MO	Trans-Mississippi	Aug 10, 1861	Confed
Fort Donelson	TN	Western	Feb 11–16, 1862	Union
Pea Ridge	AR	Trans-Mississippi	Mar 6–8, 1862	Union
Glorieta Pass	NM	Trans-Mississippi	Mar 26–28, 1862	Union
Shiloh	TN	Western	Apr 6–7, 1862	Union
Island Number Ten	MO	Western	Feb 28–Apr 8, 1862	Union
Forts Jackson & St. Philip	LA	Lower Seaboard	Apr 16–28, 1862	Union
First Winchester	VA	Eastern	May 25, 1862	Confed
Gaines's Mill	VA	Eastern	Jun 27, 1862	Confed
Malvern Hill	VA	Eastern	Jul 1, 1862	Union
Second Bull Run	VA	Eastern	Aug 28–30, 1862	Confed
Antietam	MD	Eastern	Sep 16–18, 1862	Union
Second Corinth	MS	Western	Oct 3–4, 1862	Union
Perryville	KY	Western	Oct 8, 1862	Union
Fredericksburg	VA	Eastern	Dec 11–15, 1862	Confed
Stones River	TN	Western	Dec 12, 1–Jan 12, 1	Union
Chancellorsville	VA	Eastern	Apr 30–May 6, 1863	Confed
Champion Hill	MS	Western	May 16, 1863	Union
Gettysburg	PA	Eastern	Jul 1–3, 1863	Union
Vicksburg	MS	Western	May 18–Jul 4, 1863	Union
Port Hudson	LA	Lower Seaboard	May 22–Jul 9, 1863	Union
Chickamauga	GA	Western	Sep 18–20, 1863	Confed
Missionary Ridge	TN	Western	Nov 23–25, 1863	Union
Mansfield	LA	Trans-Mississippi	Apr 8, 1864	Confed
the Wilderness	VA	Eastern	May 5–7, 1864	Union
Spotsylvania Court House	VA	Eastern	May 8–21, 1864	Confed
Cold Harbor	VA	Eastern	May 31–Jun 12, 1864	Confed
Second Petersburg	VA	Eastern	Jun 15–18, 1864	Confed
the Crater	VA	Eastern	Jul 30, 1864	Confed
Mobile Bay	AL	Western	Aug 2–23, 1864	Union
Jonesborough	GA	Western	Aug 31–Sep 1, 1864	Union
Opequon	VA	Eastern	Sep 19, 1864	Union
Cedar Creek	VA	Eastern	Oct 19, 1864	Union
Westport	MO	Trans-Mississippi	Oct 23, 1864	Union
Franklin (1864)	TN	Western	Nov 30, 1864	Union
Nashville	TN	Western	Dec 15–16, 1864	Union
Second Fort Fisher	NC	Eastern	Jan 13–15, 1865	Union
Bentonville	NC	Western	Mar 19–21, 1865	Union
Fort Stedman	VA	Eastern	Mar 25, 1865	Union
Third Petersburg	VA	Eastern	Apr 2, 1865	Union
Fort Blakely	AL	Western	Apr 2–9, 1865	Union
Appomattox Court House	VA	Eastern	Apr 9, 1865	Union

Table 1.1: List of the 43 militarily significant battles of the American Civil War included in this analysis.

arrived in Mitchell (1903, p. 204). As a measure of when information of the battle reaches the New York market, I coded when news about a battle was first reported in the *New York Times*. While battles in the Eastern theater were often reported the next day (Antietam, Gettysburg), news on battles in other theaters may not reach the market for days. The battle with the longest lag in news was the Battle of Glorietta Pass in New Mexico; it ended on March 28, 1862, but news of the battle did not reach New York city until April 15. For battles which lasted several days of approximately equal intensity, *e.g.* Chattanooga, I coded news as reaching the market on several days. For sieges which could last many days, *e.g.* Vicksburg, Port Hudson, I only coded when the news of the end of the siege was reported. A data driven approach to identifying the timing of this news is important since in an event-study analysis like this, the timing of the events is of key importance in identifying their effects. [44]

In using a small set of battles based on a *ex post* expert assessment of their military importance, it is important to note that this is not selecting on the dependent variable. In this case, that would be including explanatory variables by selecting periods of time that experienced large changes in yields. Instead, I am selecting a set of covariates, *i.e.* battles, which are expected to have large effects on the response variable based on prior information. This is no different than any other regression model, in which the researcher selects a relatively small set of covariates of all possible variables in the universe, because those covariates are *a priori* expected to plausibly have an effect on the response variable.

For this project, I created a public data collection of American Civil War battles, the American Civil War Battle Database (ACWBD) (Arnold 2015a).[45] The ACWBD combines, cleans, cross-references battle data from multiple primary and secondary sources, in-

[44] Measurement error due to how these dates are coded will be mitigated somewhat since prices are not observed daily, so usually the change in prices reflects the sum of the effects of several days.

[45] The data and source code of the data collection are available from https://github.com/jrnold/acw_battle_data.

cluding Phisterer (1883), Livermore (1900), Bodart (1908), Dyer (1908), Kennedy (1998), CWSAC (1993a), and NPS (2012). It is the most complete, machine-readable data on American Civil War battles of which I am aware.

1.6 Statistical Models of the Effects of Battles on Bond Yields

In this section, I present several models of the effects of battles on U.S. government bond yields. In all these models, the response variable is the yields of the Fives of 1875, as discussed in Section 1.4.1. The Fives data consistent of 314 observations between 1861-04-27, the first observation after the Battle of Fort Sumter and the official start of the war, and 1865-05-04. These occur at irregular frequencies, with gaps of 1–1 days between observations. In these models, I use the logarithm of the yield since Figure 1.2 suggests that the variance of yields is increasing in its level.

All of these models are of the form,

$$\log y_t = y_t^* + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}\left(0, \tau^2\right) \qquad \text{if } y_t \text{ not missing} \tag{1.3}$$

$$y_t^* = y_{t-1}^* + \delta_t + \eta_t \qquad \eta_t \sim \mathcal{T}_5(0, \sigma) \tag{1.4}$$

for days $t = 1, \dots, T$. Equation (1.3) states that the observed log-yields consist of a latent log-yield (y_t^*) and measurement error (ϵ_t). Equation (1.4) states that the first difference in the latent log-yields, $y_t^* - y_{t-1}^*$, is the sum of δ_t and a mean-zero innovation, η_t. The models differ in the definition of δ_t and this is where battle effects are included. The innovation η_t is distributed Student's t to allow for large events that are unmodeled, *i.e.* any non-battle events. If $\tau^2 = 0$, then Equations (1.3) and (1.4) would be equivalent to simply estimating a model with the first difference of $\log y_t$ as the dependent variable. With a non-zero τ, and supposing η was distributed normal, this model would be equivalent to an ARIMA(0, 1, 1) model in which the MA parameter is constrained to be negative (Petris, Petrone, and Campagnoli 2009, p. 91).

In this section, I estimate four models which differ in how δ_t is defined:

- Model $\mathscr{M}_1$ is benchmark model that does not include any battle effects.
- Model $\mathscr{M}_2$ includes variables for Confederate and Union victories, but no individual battle effects.
- Model $\mathscr{M}_3$ is like $\mathscr{M}_2$, but also includes random effects for each battle.
- Model $\mathscr{M}_4$ is like $\mathscr{M}_2$, but the effects of Confederate and Union victories are smoothly varying over time.

Model $\mathscr{M}_1$ includes no battle effects, $\delta_t = 0$ for all t. In the other models, δ_t is non-zero and time-varying, and is a function of battle outcomes, but these models differ how battles are modeled. In model $\mathscr{M}_2$, δ_t includes the effects of Confederate and Union victories.

$$\delta_t = \alpha + X_t \begin{bmatrix} \beta_C & \beta_U \end{bmatrix}' \tag{1.5}$$

$$X_{t,1} = \sum_{b \in B} W_{t,b} \qquad \text{if } b \text{ is a Confederate victory} \tag{1.6}$$

$$X_{t,2} = \sum_{b \in B} W_{t,b} \qquad \text{if } b \text{ is a Union victory} \tag{1.7}$$

$$W_{b,t} = \begin{cases} \frac{1}{n_b} & \text{If news of battle } b \text{ reached the market on day } t. \\ 0 & \text{otherwise} \end{cases} \tag{1.8}$$

where B is the set of all battles in the data, see Table 1.1. In this model, all Confederate victories have the same effect on the yield, and all Union victories have the same effect on the yield. In other words, given their result, battles are exchangeable. The parameters β_C and β_U are the average effects of Confederate and Union victories. The matrix W is is a $T \times |B|$ matrix, in which each column is a variable indicating if there was news about a battle on that day. News of battles is assumed to reach the market on the days the first news of the battle is reported in the *New York Times* and the following 5 days. For a given battle, all days with news are weighted equally, and the weights for each battle sum to one. n_b is the total number of days for which news of battle b reached the market. For example, the end of Gettysburg was reported in the news on July 2nd–4th, 1863, so July 2, 1863 through July 9, 1863 (July 2 plus 5 days) are equal to $\frac{1}{8}$ in $W_{.,\text{Gettysburg}}$, and all other days in $W_{.,\text{Gettsyburg}}$ are equal to 0. X_t is an $T \times 2$ matrix containing the sum of the news weights of Confederate and Union victories occurring on day t. Since each column of W_b sums to one, β_C is interpreted as the average effect of a single Confederate victory, and β_U as the average effect of a single Union victory. Finally, α is a parameter representing a constant trend in yields not due to the battles.

In model $\mathscr{M}_3$, δ also includes random effects for each battle,

$$\delta_t = \alpha + X_t \begin{bmatrix} \beta_C & \beta_U \end{bmatrix}' + W_t \gamma \tag{1.9}$$

$$\gamma_b \sim \mathscr{T}_7\left(0, \zeta^2\right) \tag{1.10}$$

where α and β have the same meaning as in $\mathscr{M}_2$, and γ is a $|B| \times 1$ vector with random effects for each battle. The random effects, γ, are given a Student's t distribution since it is expected that a few battles will have particularly large effects. The effect of an individual battle is the sum of its result (Confederate or Union victory) and random effect,

$$\omega_b = \begin{cases} \beta_C + \gamma_b & \text{if } b \text{ is a Confederate victory} \\ \beta_U + \gamma_b & \text{if } b \text{ is a Union victory} \end{cases} \tag{1.11}$$

In model $\mathscr{M}_4$, the effect of each battle, ω_b, is shrunk towards both the average effect of its result and and the effect of the previous previous battle of the same result. This is equivalent to a model in which the effects of Confederate and Union victories are smoothly time-varying with an $AR(1)$ process.[46]

$$\delta_t = \alpha + W_{C,t}\omega_C + W_{U,t}\omega_U \tag{1.12}$$

$$\omega_{C,b} \sim \mathcal{N}\left((1-\rho)\beta_C + \rho\omega_{C,b-1}, \rho\zeta^2\right) \qquad \text{for } b \in \{2, \dots, C\} \tag{1.13}$$

$$\omega_{C,1} \sim \mathcal{N}\left(\beta_C, \zeta^2\right) \tag{1.14}$$

$$\omega_{U,b} \sim \mathcal{N}\left((1-\rho)\beta_U + \rho\omega_{U,b-1}, \rho\zeta^2\right) \qquad \text{for } b \in \{2, \dots, U\} \tag{1.15}$$

$$\omega_{U,1} \sim \mathcal{N}\left(\beta_U, \zeta^2\right) \tag{1.16}$$

Where C is the number of Confederate victories, U is the number of Union victories. The Confederate and Union victories are ordered by their dates. For Confederate victories, the first battle was First Bull Run (July 21, 1861), and the last was the Crater (July 30, 1864). For Union victories, the first battle was Fort Donelson (February 11–16, 1862), and the last was Appomattox Court House (April 9, 1865). As an example, in this model the effect of the Battle of Gettysburg, ω_{13}, is shrunk towards both the average effect of Union victories, β_U, and the effect of the previous battle, ω_{12} (Champion Hill on May 16, 1863).

In all these models, since y^* represents log-yields, the parameters β, γ and ω can be interpreted as approximately the percent change in the yield associated with the result of

[46] $\omega_{C,t} \sim N(\beta_{t,C}, \zeta^2)N(\omega_{C,t-1}, \xi^2) = N((1-\rho)\beta_C + \rho\omega_{C,t-1}, \rho\zeta^2)$, where $\rho = \frac{\xi^2}{\xi^2+\zeta^2}$.

a battle.

All models are estimated using MCMC methods to sample from the posterior distribution. Samples are drawn using the NUTS-HMC algorithm (Hoffman and Gelman 2014) as implemented in the probabilistic programming language, Stan (Stan Development Team 2015). I found that the sampling efficiency of state space models, such as those estimated here, could be improved with a two step method. In the first step, I marginalize over the latent states of the state space model (y^* in this case) and use HMC-NUTS to sample the other parameters. In the second step, given the values of the parameters, I use the forward-filter backwards-sample algorithm (Frühwirth-Schnatter 1994; Carter and Kohn 1994) to sample the latent states. Together, these steps constitute a partially collapsed Gibbs sampler (van Dyk and Park 2008). I describe this method and its implementation in Stan in more detail in Chapter 3 of this dissertation. For all models, four chains were run, and convergence assessed using multiple-chain R hat statistics and the number of effective samples, as recommended in (Stan Development Team 2015) and Gelman, Carlin, et al. (2013).

In these models, I do not include indicators of other events in the war. While changes in policy, leadership, foreign policy events, negotiations and other events certainly influence war expectations, I do not have a coherent method for choosing a set of events. McCandless (1996) and Smith and Smith (1997) both include indicator variables for a few military, fiscal, and political events in the war. However, neither of these have a clear rule as to what constitutes major non-battle events in the war, and what does not. Smith and Smith (1997) includes indicator variables selected on the dependent variable, using the events mentioned in Mitchell (1903) as corresponding to large daily movements in the gold market.In Section 1.7, I propose a more coherent method for identifying war and non-war events.

These models do not include measures of inflation or the risk-free interest rate. Inflation is not included for several reasons. First, while inflation measured in U.S. currency (Greenbacks) rose during this period, inflation calculated in gold dollars did not (Mitchell 1903; Mitchell 1908). The price of gold closely tracked the price of goods during this period, so there was little inflation when the prices of goods were measured in gold dollars. Since the bonds considered here paid interest in gold dollars, and the bond yields are calculated with the cash flows converted to gold dollars, the inflation of currency is not explicitly a problem. Second, the influence of the war was such that inflation is plausibly a post-treatment variable that, like bond yields, responded to war expectations. Third, data on inflation is only available at frequencies much longer than used in the analysis; the Warren-Pearson Wholesale Price Index is only available at the monthly level (Warren and Pearson 1933). The risk free interest rate during this period is not included because there is not a good measure of the the risk free interest rate during this period. Before and after the American Civil War, the yields of U.S. government bonds could be considered approximately risk-free assets (Homer and Sylla 2005). But this work uses the yields of U.S. government bonds precisely because those bonds are risky during the American Civil War. Macaulay (1938) and Homer and Sylla (2005) consider indexes of railroad bonds and New England municipal and state bonds as risk free assets during the late nineteenth century. But railroad bonds were not yet safe assets at the time of the American Civil War; before the war they had much higher yields than U.S. government bonds. New England municipal and state bonds are the other candidate for risk-free assets, but when their yields are calculated in gold dollars, they are similar to those of U.S. government bonds for this period. Moreover, most New England states did not issue debt prior to the Civil War, and issued bonds during the Civil War to pay for their war expenditures (Martin 1871, pp. 86–87). Thus, yields of these bonds were also plausibly linked to war expectations. On a practical level, the currently available comprehensive source of these interest rates, Macaulay (1938), only provides them at monthly (railroad) or quarterly (New England

Model	Result	Mean	2.5%	50%	97.5%
$\mathscr{M}_2$	Confederate	0.050	0.013	0.050	0.085
$\mathscr{M}_3$	Confederate	0.052	0.015	0.053	0.088
$\mathscr{M}_4$	Confederate	0.057	0.022	0.057	0.092
$\mathscr{M}_2$	Union	-0.015	-0.035	-0.015	0.005
$\mathscr{M}_3$	Union	-0.015	-0.036	-0.015	0.004
$\mathscr{M}_4$	Union	-0.018	-0.037	-0.018	0.002

Table 1.2: Summary statistics of the posterior distribution of the average effects of Confederate and Union victories on log-yields.

state and municipal bonds) frequencies. Other work in 19th century economic history uses the interest rate of British consols as a risk free rate (Bordo and Rockoff 1996). The rate of consols is inappropriate for this analysis because that risk free interest rate corresponded to the London market, and this analysis uses the New York market.

1.6.1 Average Effects of Union and Confederate Victories

For these models, the first question to ask is "what is the average effect of a Confederate or Union victory on the bond yield?" The average effects of Confederate and Union victories are defined slightly differently for the models. For model $\mathscr{M}_2$, the average effect of a Confederate victory is the parameter β_C, and the average effect of a Union victory is the parameter β_U.[47] For models $\mathscr{M}_3$ and $\mathscr{M}_4$, the average effect is the average of the battle effects. For these models, the average effect of a Confederate victory is the average of the effects of the individual Confederate victories, $\frac{1}{C}\sum_{b \in C} \omega_b$, and, similarly, the average effect of a Union victory is $\frac{1}{U}\sum_{b \in U} \omega_b$.

Table 1.2 shows the summary statistics of the posterior distribution of the average effects of Confederate and Union victories for the models. The estimated average effect of a Confederate victory on yields is positive, meaning that Confederate victories were associated with an increase in the expected cost of the war. A posterior mean of the average

[47]Model $\mathscr{M}_1$ does not include battles.

effect of a Confederate victory is estimated to have increased the yield approximately 5–5.7 percent, with the 95% central credible intervals including the range of 1.3% to 9.2% The posterior mean of the average effect of a Union victory is negative, meaning that Union victories were associated with an decrease in the expected cost of the war. However, the posterior mean is small in magnitude, -1.5% to -1.8%, and the central 95% percent credible intervals of all models contain zero. There are two explanations for this asymmetry in effects of victories on the expected cost of the war. First, individual Union victories had a smaller effect on the overall cost and duration of the war than Confederate victories. This is because the strategic situation of the war was itself asymmetric. As military historian, J.F.C. Fuller described it, "to reestablish the Union, the North must conquer the South, and to maintain the Confederacy and all it stood for, the South must resist invasion (Fuller 1942, p. 177)." Likewise, McPherson (2003, p. 336) wrote, "The South could 'win' the war by not losing; the North could win only by winning." Since the United States had to win victories and take terroritory while the Confederacy had to only to delay and resist implies a differential value of Union and Confederate victories. Second, the market may have been expecting Union victories, so they were less surprising on average than Confederate victories.

1.6.2 Individual Battle Effects

While model $\mathscr{M}_2$ assumes that all Confederate and Union victories have the same effect, Models $\mathscr{M}_3$ and $\mathscr{M}_4$ estimate effects for individual battles in addition to the average effects of Confederate victories. The second set of questions to ask is about the variation in the effects of individual battles. How much variation was there in the effects of battles? How did the effects of Union and Confederate victories change over time? Which battles had the largest effects?

Figure 1.5 plots the battle effects, ω_b, for each battle, for each model. Figure 1.6 plots the battle effects for model $\mathscr{M}_3$, for each battle. Since the battle effects in model $\mathscr{M}_4$ can be considered time-varying values of effect of Confederate and Union victories, Figure 1.7 plots the battle effects for Confederate and Union victories over time. One thing to note is that the estimates of battle-level effects are highly uncertain; in all models most battles have 95 percent central credible intervals which include zero.

The battles with the largest effects are the Confederate victories in the summer of 1864 in the Overland Campaign (Spotsylvania Courthouse, Cold Harbor) and the start of the Richmond-Petersburg Campaign (Second Petersburg). In the battles of Spotsylvania Courthouse and Cold Harbor, U.S. forces commanded by Ulysses S. Grant suffered large casualties while advancing towards the Confederate capital, Richmond. In the of Battle of Second Petersburg, the United States failed break the Confederate lines and decisively capture Richmond. This defeat began the Siege of Petersburg which would last from June 1864 until the United States broke through the Confederate lines at the Third Battle of Petersburg on April 2, 1865. Reiter (2009, pp. 151-152) cites this period as a turning point in the war, during which the United States "neared exhaustion".

The next largest effect is the Confederate victory at the Battle of Gaine's Mill. This was the major Confederate victory in the Seven Days Battles and Peninsula Campaign in June 1862. In the Peninsula Campaign, the U.S. Army of the Potomac, commanded by General George McClellan, moved to almost within artillery distance of Richmond. However, in the Seven Days battles, Confederate forces commanded by Robert E. Lee fought a series of battles, largely Confederate victories, which pushed U.S. forces back from Richmond. This was cited as a turning point of the war by Fuller (1942): "[The Peninsula Campaign] campaign was certainly one of [the Civil War's] most important [campaigns], because, in spite of numerous blunders, the Army of Northern Virginia, under the guidance of Lee, saved Richmond. Had Richmond fallen into Federal hands, the whole course of the war would have been changed (Fuller 1942, p. 206)." The Seven Day's battles are cited by Reiter (2009, p. 145) as one of the turning points in the war, when "it was becoming increasingly apparent that the war would not end quickly." In this campaign, both sides had opportunities to bring the war to an end, the United States by capturing Richmond, and the Confederacy by destroying the Army of the Potomac, but neither was able to (Fuller 1942, Chapter6).

The estimates of the effects of Union battles are smaller in magnitude and 95 percent central credible intervals of all Union victories contain zero. However, the results are plausible in that the battles of Gettysburg (July 1–3, 1863) and Vicksburg (July 4, 1863) are estimated as having among the largest battle effects. This is despite these two battles occurring almost simultaneously, which makes it hard to disentangle their effects. The Union victory with the largest estimated effect is the Second Battle of Fort Fisher which blockaded the Wilmington, N.C. , the last unblockaded Confederate seaport. Reiter (2009, p. 154) cites the capture of Fort Fisher as final turning point of the war.

1.6.3 Model Comparisons

The third question is "how well do battles explain the variation in bond yields during the period considered?" If the yield movements are taken as a proxy of war expectations, this is equivalent to asking "How well do major battle results (as operationalized here) explain the expected cost and duration of the war?"

To answer this, I compare the models using the root mean squared error (RMSE) and the WAIC, which approximates the expected log predictive density or leave-one-out cross-validation. Table 1.3 shows the RMSE, WAIC, and expected log predictive density (elpd) of all the models. The Widely Applicable Information Criterion (WAIC) (Watanabe 2010) is an information criteria similar to AIC or the Deviance Information Criterion (DIC). Unlike

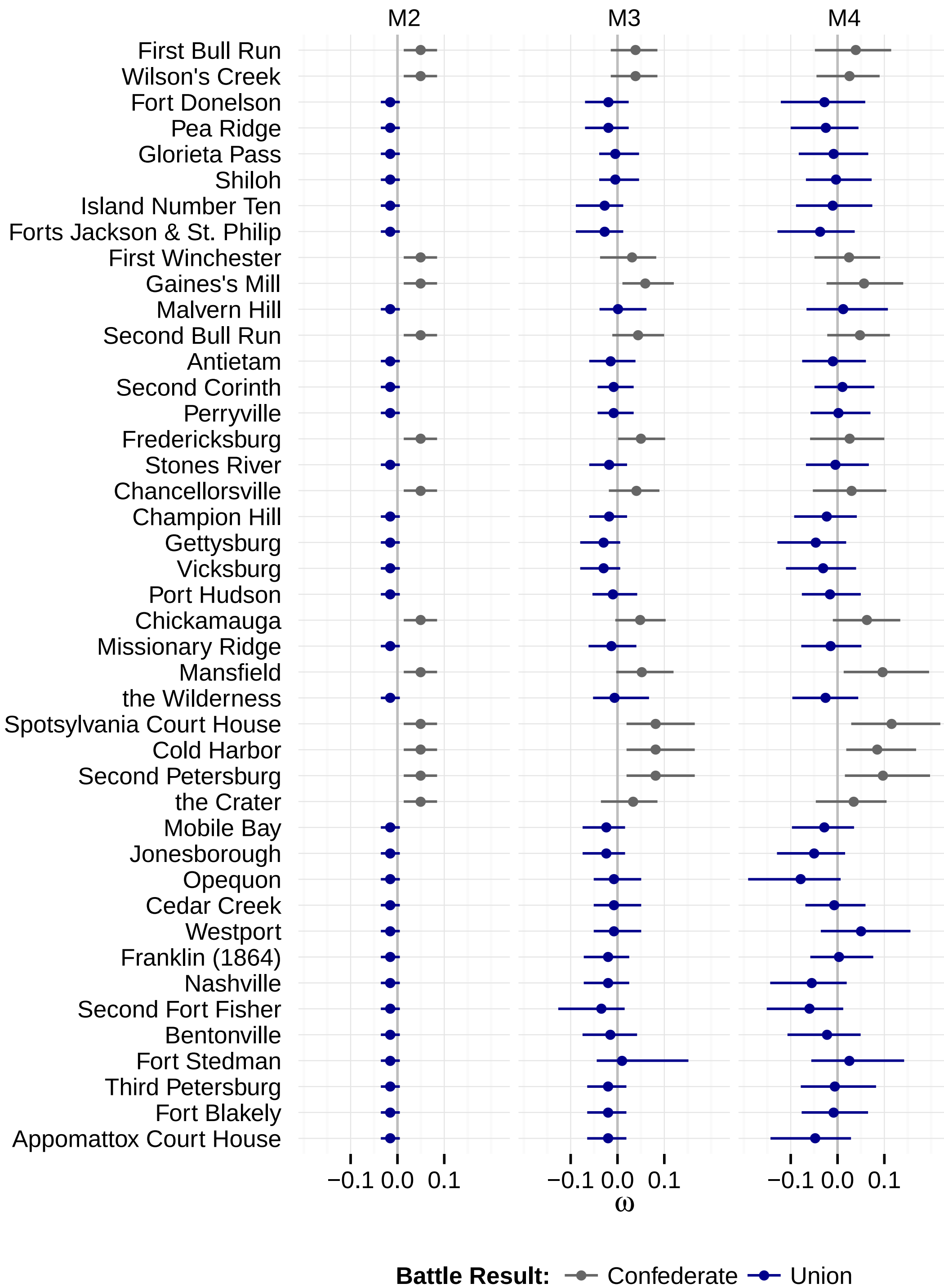


Figure 1.5: Battle effects, ω_b, for all battles for all models. These are ordered by end date of the battle. Union victories are in dark blue. Confederate victories are in gray.

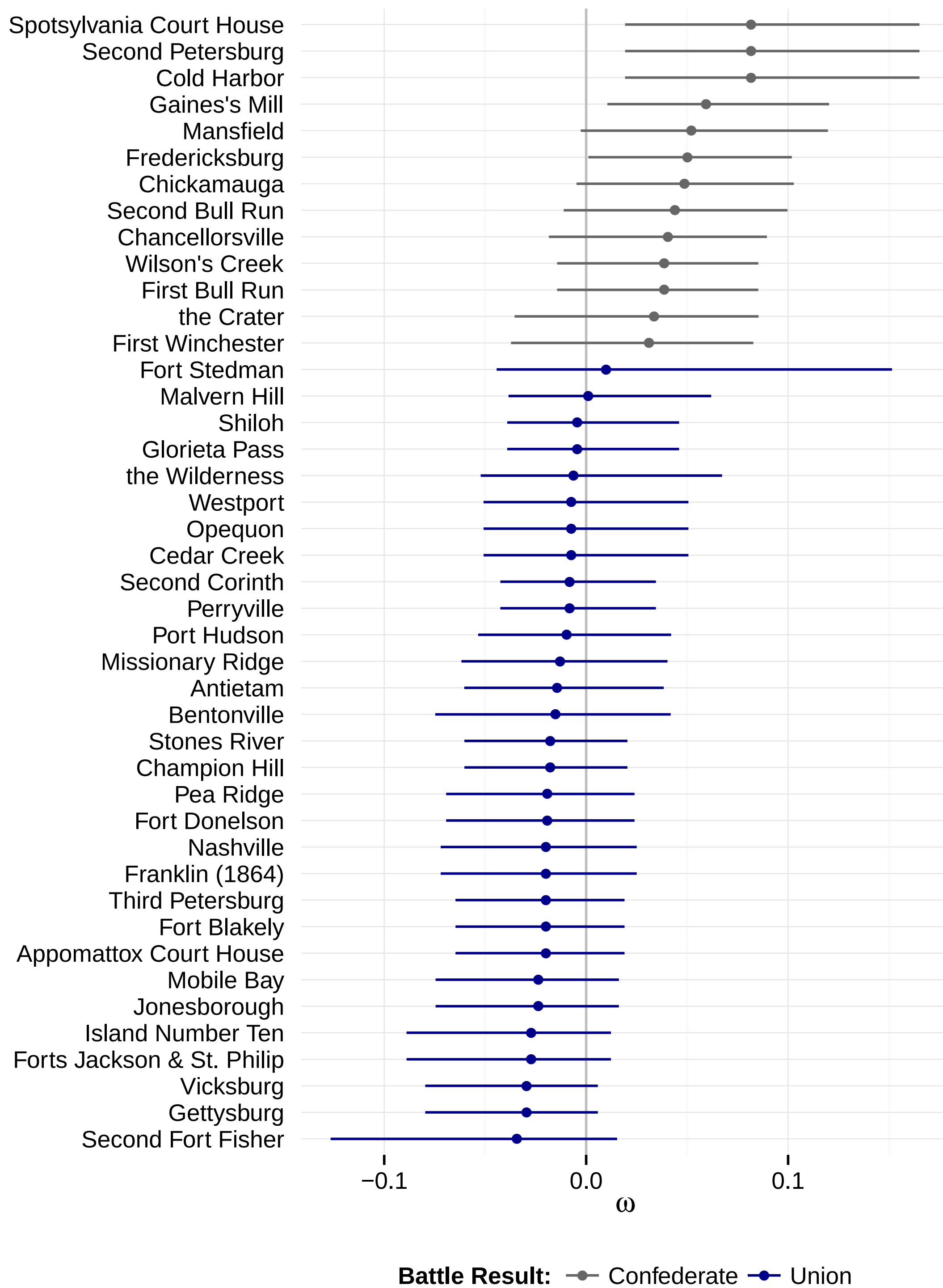


Figure 1.6: Battle effects, ω_b, for model $\mathscr{M}_3$. These are ordered from largest to smallest. Union victories are in dark blue. Confederate victories are in gray.

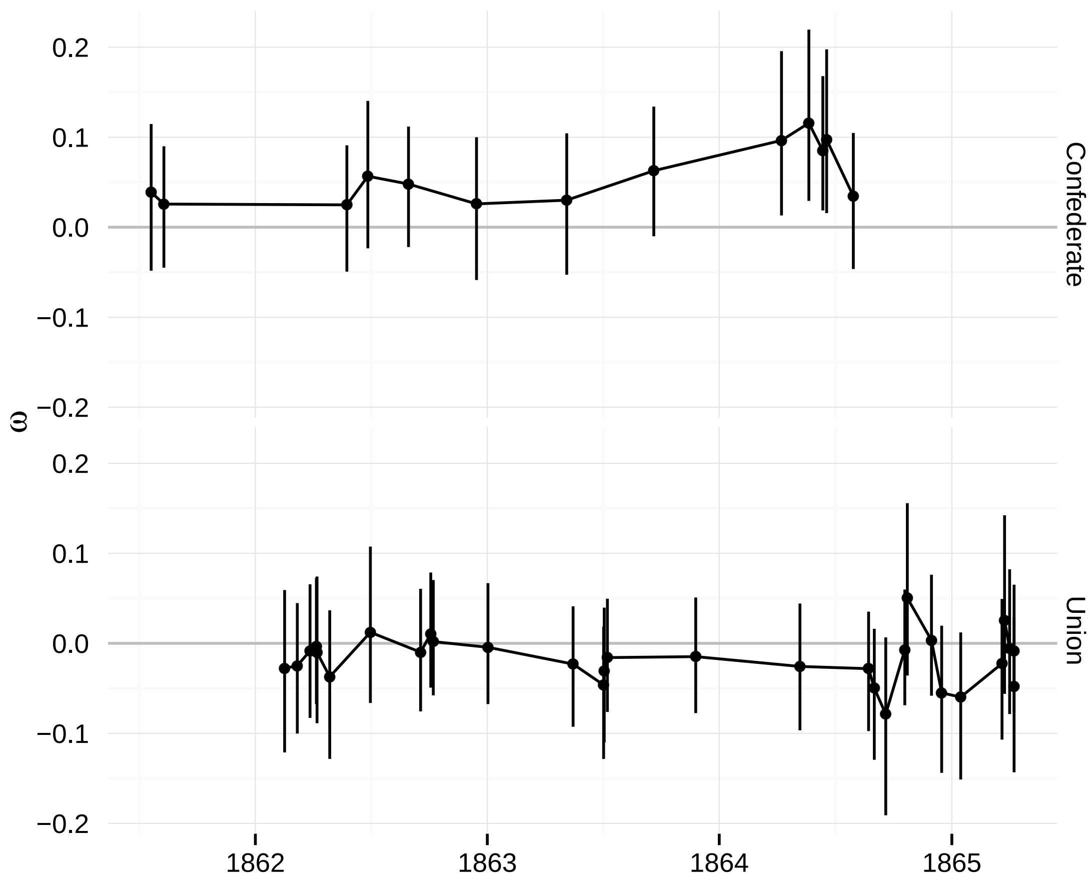


Figure 1.7: Battle effects, ω_b, for all battles for model $\mathscr{M}_4$, by battle result.

either AIC or DIC is it fully Bayesian, and is calculated by integrating over the posterior distribution rather than using point estimates. The expected log predictive density (elpd) is the expected log probability of a new (out-of-sample) data point. WAIC and other information criteria approximate the elpd by adjusting the log-likelihood with a penalty for the complexity of the model. See Gelman, Carlin, et al. (2013, Ch. 7), Gelman, Hwang, and Vehtari (2014), and Gelman and Vehtari (2014) for more on Bayesian information criteria for model comparison.

The RMSE of the models that include battle effects is lower than that of the benchmark model $\mathscr{M}_1$. However, the decrease in RMSE is relatively small.[48] The WAIC and elpd also show that the models with battles improve the predictive ability of the models, but that the improvement is small. Although models $\mathscr{M}_2$–$\mathscr{M}_3$ all improve the WAIC (lower values) and elpd (higher values), the improvement is less than the standard errors of these in model $\mathscr{M}_1$.[49] As operationalized in this work, battles appear to improve the predictive and explanatory power of the model of bond yields, but that improvement is small.

The relatively small magnitude of the effects of individual battles is seen in Table 1.4. The average effect of a single Union victory is less than 1 standard deviation of the standard deviation of the system error η; Confederate victories are larger than than 1 sd of η, but less than 2 standard deviations. Considering that there are only 43 battles, but 1469 values of η_t, neither of those magnitudes seems particularly large.

There are several interpretations for the limited explanatory power of battles in this analysis. First, major battles may be the key factor in war, but information diffuses slowly over time, while my model assumes that all effects of a battle occur within a short time of its conclusion. Second, smaller battles and non-battle events may not be as influential as large battles, but cumulatively they may have a large influence on the war outcome.

[48]The R^2 of model $\mathscr{M}_4$ using $\mathscr{M}_2$ as the null model is only about 0.17.

[49]See Gelman and Vehtari (2014) for method to calculate standard errors for WAIC, and the R package **loo** for an implementation.

Finally, perhaps improvements in the models employed in this analysis would produce different results. Perhaps I could better model the time-series processes, lag structure of battles, or operationalize combat differently. While I could make the models in this analysis more complicated, a better approach to distinguish between these interpretations is not better models, but better data. I conclude this work by discussing one such approach in Section 1.7.

Model	RMSE	$\text{elpd}_{\text{WAIC}}$	$\text{se}(\text{elpd}_{\text{WAIC}})$	WAIC	se(WAIC)
$\mathscr{M}_1$	0.052	785.829	16.479	-1571.658	32.959
$\mathscr{M}_2$	0.051	788.187	15.814	-1576.374	31.628
$\mathscr{M}_3$	0.050	788.828	15.961	-1577.657	31.922
$\mathscr{M}_4$	0.048	791.867	15.640	-1583.733	31.280

Table 1.3: Comparison of models using WAIC and RMSE. RMSE is the root mean squared error of each model. $\text{elpd}_{\text{WAIC}}$ is the expected log predictive density implied by the Widely Applicable Information Criterion (WAIC). The standard errors for WAIC and $\text{elpd}_{\text{WAIC}}$ are calculated as in Gelman and Vehtari (2014).

Model	$\text{sd}(\eta)$	$\text{E}(\tilde{\beta}_C)$	$\text{E}(\tilde{\beta}_U)$
$\mathscr{M}_2$	0.040	0.050	-0.015
$\mathscr{M}_3$	0.039	0.052	-0.015
$\mathscr{M}_4$	0.037	0.057	-0.018

Table 1.4: The posterior mean of the average effect of Confederate and Union victories relative to the standard deviation of the innovations, η.

1.7 Discussion

I conclude by discussing a few surpising results, how those results relate to the bargaining theory of war, and some thoughts on future directions for this research and research combining war data and financial market data. First, the largest estimated effects of battles occur over three years into the war, in May and June of 1864, when U.S. forces were advancing towards Richmond but suffering extremely heavy casualties. That the largest effects would occur so late in the war seems to fit better with commitment theories of war than informational theories of war. This pattern not fit well with the naive learning models of war, in which early battles should be the most informative. Nor does it seem to fit well with the case in which heavier fighting revealed the resolve of the United States to continue fighting and the Confederates to fight to end if need be. If that were the case, then each side would have updated its expectations, an agreement would be reached and the expected the cost (duration) of the war would decline. This pattern could fit with a commitment theory of war in which this marked the point at which expectations moved from there being some expectation of a settlement to one in which to it being nearly certain that the war would have to extend to total victory. What this is not consistent with, is the idea that because the Union was incurring heavy costs, it was near exhaustion, and would quit the war. Instead, as the United States incurred heavier costs, the market expected the United States to incur even heavier costs in the future. If the market it expected the Union to quit due to exhaustion, it would have expected lower future costs, and the yield on U.S. bonds would have decreased. Second, the major battles explain little of the variation in bond yields. To the extent that these yields were driven by war cost expectations, this means that many features of the war influence expectations of the war cost apart from major battles. This is similar to the results of bargaining models by Powell (2004) and Slantchev (2003) that what happens off the battlefield can be as or more important than what happens on the battlefield.

On its own, the results of this work are primarily limited to questions of the American Civil War since it is but a single case. However, this method may allow for analyses of more wars, which together may uncover patterns and stylized facts useful in extending and building better theories of the relationship between military combat and war outcomes.

There are several methodological issues that this analysis had to confront. They are "What information about battles did the market receive?", "When did the market receive that information?", and "What other non-battle information did the market receive?" In this work, I handled the the first by assuming that only a small set of battles were important enough to have any effect, and that the market only cared about the results, Union or Confederate victory, of those battles. The assumption was that contemporaries would have also seen that same set of battles as the most important, and that they would classify the results of battles similarly to how they were coded in the data. As a measure of when the market received information about battles, I used the day on which the battle was first reported in newspapers. Since I had no data on the non-battle information that would be important to the market, I only modeled it with a fat-tailed distribution in the error term. A means to improve on this methodology is suggested by the way in which I handled the problem of when news reached the market; I consulted contemporary newspapers to determine when information about battles was publicly known. The way in which I used newspaper data to identify when the market received information about the battles could be extended to address all three questions. A rough sketch of the analysis would be to use computational natural language processing tools, *e.g.* topic models, to extract events or topics from the text of contemporary newspapers and regress those topics on market movements. The information that the market received about battles would correspond to battle-related topics inferred from the newspaper articles. When the market received that information would simply be the days on which those topics appeared in newspapers. The same method would also provide measures of non-battle events. These would be the topics that do not correspond to This approach would provide a set of control variables for non-battle events, without the researcher needing to make arbitrary decisions about which events were important. This sort of analysis, which combines natural language processing to generate events with market movements as measures of war expectations seems a promising approach to understanding the effects of combat on war outcomes where the events are not always either clearly identified or discrete.

Chapter 2

Financial Markets and the Onset of the American Civil War

```
## Warning in readChar(con, 5L, useBytes = TRUE): cannot open compressed
file './acw_onset_and_markets/doc/analysis.RData', probable reason 'No
such file or directory'
## Error in readChar(con, 5L, useBytes = TRUE): cannot open the con-
nection
```

Chapter 3

Bayesian Change Points and Linear Filtering in Dynamic Linear Models using Shrinkage Priors

3.1 Introduction

Political and social processes are rarely, if ever, constant over time, and, as such, social scientists have a need to model that change (Büthe 2002; Lieberman 2002). Moreover, these political process are often marked by both periods of stability and periods of large changes (Pierson 2004). If the researcher has a strong prior about or wishes to test a specific location of a change, they may include indicator variables, an example in international relations is the ubiquitous Cold War dummy variable. Other approaches estimate locations of these change using change point or structural break models (Calderia and Zorn 1998; Western and Kleykamp 2004; Spirling 2007b; Spirling 2007a; Park 2010; Park 2011; Blackwell 2012).

This offers a different Bayesian approach to modeling change points. I combine a continuous latent state space approach, e.g. dynamic linear models, with recent advances in Bayesian shrinkage priors (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2010; Polson and Scott 2010). These sparse shrinkage priors are the Bayesian analog to regularization and penalized likelihood approaches such as the LASSO (Tibshirani 1996) in maximum likelihood. An example of such an approach, is a model of change points in the mean of normally distributed observations,

$$
\begin{aligned}
y_t &= \mu_t + \epsilon_t \qquad \epsilon \text{ iid}, \mathrm{E}(\epsilon) = 0 \\
\mu_t &= \mu_{t-1} + \omega_t
\end{aligned}
\tag{3.1}
$$

Since this is a change point model, the change in the mean, ω_t, should be sparse, with most values at or near zero, and a few which can be large. To achieve this estimate, I model ω with a shrinkage prior distribution that has most of its mass concentrated near zero to shrink values of ω_t to zero, but has wide tails which will not shrink the non-zero ω_t at the change points. The estimated posterior distribution of the μ will resemble a step function, with the steps being the estimated change points. The particular shrinkage priors that will be used in this work are the horseshoe (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2010) and horseshoe+ distributions (Bhadra et al. 2015).

Modeling change points using sparse shrinkage prior distributions has several advantageous features. First, it does not require specifying the number of change points *ex ante*. The sparsity of the parameter changes can be estimated from the data. Second, although this method will not directly provide a posterior distribution of the locations of the change points, it is likely more representative of the data generating processes commonly observed in the social science domain. Traditional change point models have a data generating process in which there are periods of exactly no change, with a few periods of any change. But many social science processes are more akin parameter that is always always changing, but which changes by relatively small amounts in most periods, but changes by very large amounts in a few periods. Third, this method is flexible and extensible in that it can be adapted to a variety of models. This work considers the cases of change points in the level and both the level and trend of a single parameter. But, it can also be applied to linear regressions with time varying parameters, changes in seasonality, variance, and a variety of other models.

Fourth, this method is computationally efficient. Most shrinkage distributions, and all those used in this work, are representable as scale-mixtures of normal distributions. This allows the model to be expressed as a Gaussian dynamic linear models (GDLMs) (West and Harrison 1997). GDLMs are a class of models than incorporates many common time series models, including ARIMA, structural time series, and linear regression with time-varying parameters. and use specialized methods for sampling from GDLMs, *e.g.* the forward-filter backwards-sampling (FFBS) algorithm (Carter and Kohn 1994)Fruehwirth-Schnatter1994.

Fifth, this method is implementable and efficiently implemented in a popular Bayesian probabilistic programming language, Stan. Since Stan does not directly sample discrete variables, standard Bayesian change point methods based on a discrete state space such as (Chib 1998), are either difficult, requiring marginalizing over the discrete states, or impossible. Since in this approach all parameters are continuous, it can be directly implemented in Stan. However, since in many cases, the change point problem can be represented more efficient methods specific to GDLMs can be used. A complementary contribution of this work is that it provides a method to efficiently estimate Gaussian dynamic linear models (GDLMs) in Stan. These had previously been difficult to sample in general purpose Bayesian programming languages, such as JAGS (Jackman 2009, p. 477). This work provides a complete set of functions to perform Kalman filtering, smoothing, and backward sampling within Stan. This allows for efficiently estimating GDLMs in Stan using a partially collapsed Gibbs sampler in which Stan's standard algorithms are used to estimate parameters after marginalizing over the latent states, and the latent states of the GDLM are sampled using FFBS.

This work presents two examples of this approach to change points. The first calculates change points in the level of a time series, using the example of the annual flow of the Nile River, 1870-1970. The second calculates change points in both the level and trend of a time series, using the example of approval ratings for President George W. Bush.

3.2 Change points as a Variable Selection and Shrinkage Problem

For simplicity, I start with a model of change points in the level of a time-series, and later generalize to other cases. In this case, there are n ordered observations, $y_1, \dots, y_n$, with a time-varying mean, μ_t:

$$y_t = \mu_t + \epsilon_t \quad \epsilon_t \text{ are iid with } \mathrm{E}(\epsilon) = 0. \tag{3.2}$$

Suppose that there are M change points, with ordered change-point locations, $\tau_1, \dots, \tau_M$, and the convention that $\tau_0 = 0$ and $\tau_{M+1} = n$. This splits the mean into M segments, with values of the mean $\mu_1^*, \dots, \mu_M^*$, such that

$$\mu_t = \mu_m^* \quad \text{if } \tau_m \leq t \leq \tau_{m+1} \tag{3.3}$$

There are a variety of approaches to the change point problem in both classical (frequentist) (Page 1954; Hinkley 1970; Bai and Perron 2003; Olshen et al. 2004; Bai and Perron 1998; Killick, Fearnhead, and Eckley 2012) and Bayesian statistics (Yao 1984; Barry and Hartigan 1993; Chib 1998; Fearnhead 2006; Fearnhead and Liu 2007).

An alternative approach to the change point problem is to rewrite the problem in Equation (3.3) to focus on the changes in the mean (system errors), $\omega_t = \mu_t - \mu_{t-1}$, rather than the locations of the change points. In this case, replace Equation (3.3) with

$$\mu_t = \mu_{t-1} + \omega_t \tag{3.4}$$

In a change point model, the system errors, ω_t, are sparse, meaning that most values of ω_t are zero, and only a few are non-zero. In this formulation, the times of the change points are not directly estimated. Instead, the change points are those times at which ω_t is non-zero. This formulation turns the change-point problem from one of segmentation, to one of variable selection, in which the goal is to find the non-zero values of ω_t and estimate their values. The natural Bayesian approach to this variable selection problem is to explicitly model the values of ω as a discrete mixture between a point mass at zero for the non-change points, and an alternative distribution (Mitchell and Beauchamp 1988; Efron 2008):

$$\omega_t = \rho g(\omega) + (1 - \rho)\delta_0 \tag{3.5}$$

where g is a distribution of the non-zero ω, and δ_0 is a Dirac delta distribution (point mass) at 0. Equation (3.5) is a so-called "spike and slab" prior (Mitchell and Beauchamp 1988). Since Equation (3.5) explicitly models the two groups of zero and non-zero parameters, Efron (2008) calls this the two-group answer to the two-group problem. Spike and slab priors are convenient because they directly provide a posterior probability that a parameter is non-zero, or in this case that a time is a change point. Giordani and Kohn (2008) propose using the representation in Equation (3.4) with the a discrete mixture distribution for ω, as in Equation (3.5), to estimate change points.

Recent work in Bayesian computation has focused on one-group solutions to the variable selection problem. These combine shrinkage and variable selection through the use of continuous distributions with large spike at zero to shrink parameters towards zero and wide tails to avoid shrinking the non-zero parameters (Polson and Scott 2010). Numerous sparse sparse shrinkage priors have been proposed, but this paper will consider the Student's t (Tipping 2001), the Laplace or double exponential (Park and Casella 2008)Hans2009, the horseshoe (Carvalho, Polson, and Scott 2010), and horseshoe+ distributions (Bhadra et al. 2015).

Student's t: The Student's t distribution for $x \in \mathbb{R}$ with scale $\tau \in \mathbb{R}^+$ and degrees of freedom $\nu \in \mathbb{R}^+$,

$$p(\omega_t|\tau,\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{\omega_t^2}{\nu}\right) \tag{3.6}$$

The Student's t distribution can also be expressed as a scale mixture of normals,[1]

$$\begin{aligned} \omega_t|\tau,\nu,\lambda_t &\sim \mathcal{N}\left(0,\tau^2\lambda_t^2\right) \\ \lambda_t^2|\nu &\sim \mathcal{IG}\left(\frac{\nu}{2},\frac{\nu}{2}\right) \end{aligned} \tag{3.8}$$

(Tipping 2001) uses the Student's t for sparse shrinkage by letting the degrees of freedom $\nu \to 0$.

Laplace: The Laplace (double exponential) distribution with scale τ,

$$p(\omega_t|\tau) = \frac{1}{2\tau}\exp\left(-\frac{|\omega_t|}{\tau}\right) \tag{3.9}$$

The Laplace distribution can also be expressed as a scale-mixture of normal distributions,[2]

$$\begin{aligned} \omega_t|\tau,\lambda_t &\sim \mathcal{N}\left(0,\tau^2\lambda_t^2\right) \\ \lambda_t^2 &\sim \mathcal{E}\left(\frac{1}{2}\right) \end{aligned} \tag{3.11}$$

The Laplace distribution is the distribution that corresponds to the ℓ_1 penalty used

[1] $\mathcal{IG}(x|\alpha,\beta)$ is an inverse-gamma distribution for $x \in \mathbb{R}^+$ with shape $\alpha \in \mathbb{R}^+$ and inverse-scale $\beta \in \mathbb{R}^+$,

$$\mathcal{IG}(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{-(\alpha-1)}\exp\left(\beta\frac{1}{x}\right) \tag{3.7}$$

[2] $\mathcal{E}(x|\beta)$ is the exponential distribution for $x \in \mathbb{R}^+$ with inverse-scale (rate) parameter $\beta \in \mathbb{R}^+$,

$$\mathcal{E}(x|\beta) = \beta\exp(-\beta x) \tag{3.10}$$

in the LASSO estimator (Park and Casella 2008; Hans 2009). However, although an ℓ_1 penalty is able to produce sparse estimates in a maximum likelihood framework because, it does not produce sparse posterior means in Bayesian estimation (Park and Casella 2008). Another problem with using the Laplace distribution as a shrinkage prior is that its tails are narrow, so it tends to excessively shrink large signals (Carvalho, Polson, and Scott 2010).

Horseshoe: The horseshoe distribution (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2010) does not have an analytical form, but is defined hierarchically as a scale-mixture of normals

$$\begin{aligned} \omega_t | \lambda_t, \tau &\sim \mathcal{N}\left(0, \tau^2 \lambda_t^2\right) \\ \lambda_t &\sim \mathcal{C}^+(0, 1) \end{aligned} \tag{3.12}$$

where $\mathcal{C}^+(x|0, s)$ denotes half-Cauchy distribution with a scale parameter s, and density

$$p(x|s) = \frac{2}{\pi x \left(1 + \left(\frac{x}{s}\right)^2\right)} \tag{3.13}$$

The Horseshoe distribution has some theoretically attractive properties for shrinkage and variable selection (Carvalho, Polson, and Scott 2009; Carvalho, Polson, and Scott 2010; Datta and Ghosh 2012; Pas, Kleijn, and Vaart 2014).

Horseshoe+ The horseshoe+ distribution Bhadra et al. (2015) is similar to the Horseshoe distribution, but with an additional hyper-prior on λ_t,

$$\begin{aligned} \omega_t | \lambda_t, \eta_t, \tau &\sim \mathcal{N}\left(0, \tau^2 \lambda_t^2\right) \\ \lambda_t &\sim \mathcal{C}^+\left(0, \eta_t\right) \\ \eta_t &\sim \mathcal{C}^+(0, 1) \end{aligned} \tag{3.14}$$

The shrinkage distributions considered here are global-local scale mixtures of normal distributions. These distributions contain a global variance component, τ, and local variance components, λ_t (Polson and Scott 2010). The global variance component, τ, concentrates the prior distribution around zero, while the local variance components, λ_t, allow individual parameters to be large without shrinking them towards zero. The choice of the prior distribution for the global variance component, τ, is particularly important in these shrinkage distributions as it effectively controls the sparsity of the estimates, which in this application is the number of change points. As per the suggestion in Bhadra et al. (2015) and Pas, Kleijn, and Vaart (2014), I use the prior distribution

$$\tau \sim \mathscr{C}^{+}\left(0, \frac{1}{n}\right) \tag{3.15}$$

where n is the number of observations. [3] That these are distributions can all be expressed as normal distributions, conditional the values of λ_t and τ, is useful computationally. This property means that many change point problems can be expressed as Gaussian dynamic linear models, and can make use of computationally efficient methods as discussed in Section 3.3.

These Bayesian sparse shrinkage priors are analogous to the sparse regularization and penalized likelihood approaches in maximum likelihood, of which the LASSO estimator (Tibshirani 1996) and its the numerous variations are the most prominent and popular examples. Several papers have proposed using LASSO-like penalties and maximum likelihood to estimate change-points (Tibshirani et al. 2005; Harchaoui and Lévy-Leduc 2010; Chan, Yau, and Zhang 2014). This is a Bayesian extension of that approach.

[3] Other suggestions include $\tau \sim \mathscr{C}^{+}(0,1)$, $\tau \sim \mathscr{U}(0,1)$ where $\mathscr{U}()$ is the uniform distribution, and a plug-in value of p/n, where p is the expected number of non-zero parameters. See Polson and Scott (2012), Pas, Kleijn, and Vaart (2014), and Bhadra et al. (2015).

To summarize, the proposed model for change points in the level of a time-series is,

$$y_t \sim \epsilon_t \qquad \epsilon_t \text{ iid}, \mathrm{E}(\epsilon) = 0, \mathrm{Var}(\epsilon) = \sigma^2 \tag{3.16}$$

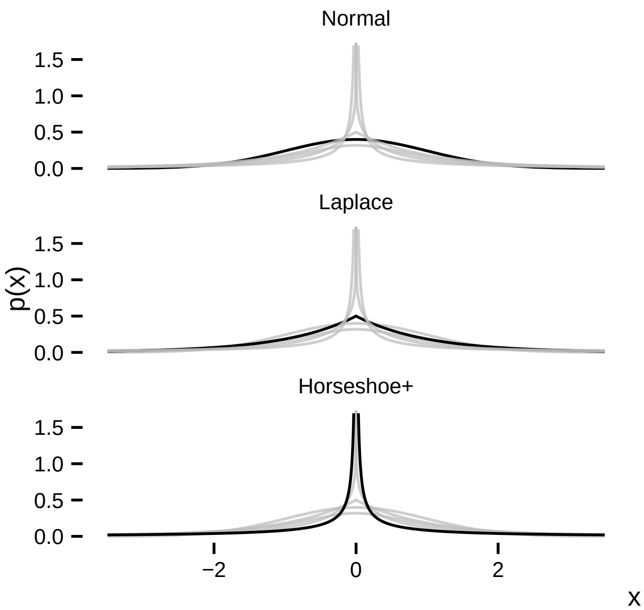
$$\mu_t = \mu_{t-1} + \sigma\omega_t \tag{3.17}$$

where ω_t is given a shrinkage prior distribution that induces sparsity. The values of ω_t is multiplied by the observation variance in order to avoid multi-modal posterior distributions (Polson and Scott 2010, p. 8). I propose using the horseshoe and horseshoe+ distributions as those shrinkage priors, and compare them the Student's t and Laplace distributions.

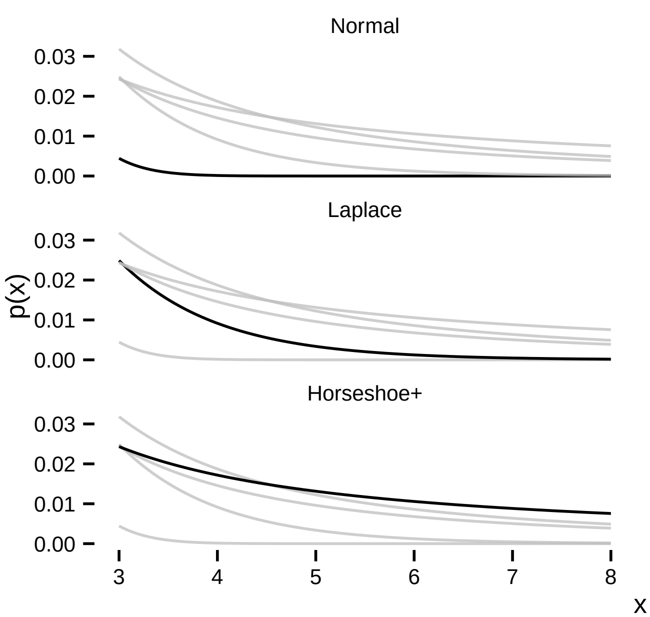
The method proposed here is able to estimate the values of μ even when it is subject to large jumps or follows a step-function. However, unlike methods using discrete state space models, it does not directly a provide probability that a given location is a change point. Instead, the researcher can identify "change points" from the magnitudes of the posterior distribution of ω. When the posterior distribution of ω_t is far from zero, it is change point, and when it includes or is close to zero, it is not. This is similar to the auxiliary residual test of de Jong and Penzer (1998). For many practical purposes, visual inspection by the researcher of a plot of the posterior estimates of μ should be sufficient. As noted earlier, for many data generating processes, the hypothesis that $\omega_t = 0$ is implausible, so testing it is nonsensical. Rather, change point models are used to ease interpretation, so an informal visual method should suffice for interpretation. If the change in μ is not distinguishable in a plot, then it is unlikely to be of much substantive importance.

While this one-group approach does not provide clear posterior probabilities of the locations of change points, it is a reasonable model of change point processes for several reasons.[4] First, for many time-varying parameter processes typically modeled with a

[4]These points are similar to those made by Polson and Scott (2012, pp. 2-3) with regard to the use of



(a) Near zero.



(b) Tail region.

Figure 3.1: Comparison of the density functions of normal, Cauchy, Laplace, horseshoe, and horseshoe+ distributions. All functions have location and scale parameters of 0. The Cauchy distribution is a special case of the Student's t distribution with degrees of freedom equal to 1.

change-point model in the social sciences, the data generating process is probably more similar to one in which the parameter is changing in all periods, but most of those periods the changes are small relative to the magnitudes of changes in a few periods. In other words, the system errors, ω, are never zero, but they are relatively small in most periods. This seems plausible for many processes modeled by political scientists in which there is no physical reason to think there are actually discrete states. Thus the researcher is modeling the parameter to take on a few values for parsimony and interpretation, and not primarily because it matches the data generating process. Most of the data generating processes considered by political science papers using change points fall into this category: Supreme court dissent and consensus (Calderia and Zorn 1998), wage growth in OECD states (Western and Kleykamp 2004), casualties in the Iraq War (Spirling 2007b), presidential use of force (Park 2010), and campaign contributions (Blackwell 2012). Second, even in change point models where the model had discrete states, after marginalizing over the posterior distribution of the discrete states, the posterior distribution of the change in μ will never be exactly zero. The shrinkage prior effectively is approximating that posterior distribution of each μ_t after marginalizing over those states.

3.3 Estimation and Implementation in Stan

The model in the previous section is an example of a Gaussian dynamic linear model (GDLM), also known as linear Gaussian state space models.[5] This this model can be expressed as a GDLM is useful because there are efficient algorithms to calculate the likelihood and sample from these models, and since GDLMs are a particularly flexible class of models, it provides ways to generalize it. A GDLM is represented system of equations (Durbin and Koopman 2012; West and Harrison 1997; Petris, Petrone, and Campagnoli

shrinkage priors in variable selection.

[5]See Beck (1989) and Martin and Quinn (2002) for examples of GDLMs in political science.

2009; Shumway and Stoffer 2010, Ch 6),

$$y_t \sim \mathcal{N}\left(b_t + F_t\theta_t, V_t\right) \tag{3.18}$$

$$\theta_t \sim \mathcal{N}\left(g_t + G_t\theta_{t-1}, W_t\right) \tag{3.19}$$

In these equations, the observed data, y_t, is a linear function of the latent states, θ_t, which are a function of their previous values, θ_{t-1}. Equation (3.18) is the *observation equation*, where y_t (observation vector) is an $r \times 1$ vector of observed data, θ_t (state equation) is a $p \times 1$ vector of the latent states, b_t is an $r \times 1$ vector, F_t is a $r \times p$ matrix, and V_t (observation variance) is an $r \times r$ covariance matrix. Equation (3.19) is the *state equation* equation, which relates the current latent states to their previous values; g_t is a $p \times 1$ vector, G_t is a $p \times p$ matrix, and W_t (state variance) is an $p \times p$ covariance matrix. The vectors and matrices, $\Phi = \{b_t, g_t, F_t, G_t, V_t, W_t\}_{t \in 1:n}$, are *system matrices.* In applications, the system matrices, will often be functions of parameters. GDLMs are a general class of models which includes many common time series models, including SARIMA, structural time series (Harvey 1990), dynamic factors, seemingly unrelated regression, and linear regression with time varying coefficients, among others (Durbin and Koopman 2012, Ch. 3).

The model in Equations (3.2) and (3.4) is a GDLM if $\epsilon_t \sim \mathcal{N}(0, V)$ and $\omega_t \sim \mathcal{N}\left(0, W_t\right)$. In this case, $\beta_t = g_t = 0$, $F_t = G_t = 1$. Even though the proposed distributions for ω_t are not normal, since they are all scale-mixtures of normal distributions, they are normal conditional on the values of λ_t.These models are similar to the *local level model* (Durbin and Koopman 2012, Ch 2.), except that they have time-varying state variances, $W_t = \tau^2\lambda_t^2$. In other words, the change point model discussed here is simply a local level model with a sparse shrinkage prior on the state variance.

That these change point models can be represented as GDLMs is useful for two reasons.

First, it suggests how these models can be extended beyond the simple change in level model in Section 3.2. Any sort of GDLM, which, as noted, includes many common models, can be adjusted to account for change points in in any of its states, by using a shrinkage prior distribution for its system variance, as long as the prior distribution is a scale-mixture of normals.[6]

Second, since in a GDLM both the observation and system equations are multivariate normal, there are analytical solutions that allow for efficiently computing its likelihood and sampling the latent states from their posterior distributions. The Kalman filter calculates the values of $p(\theta_t | y_{1:(t-1)})$ and $p(\theta_t | y_{1:t})$, and can be used to calculate the likelihood of $p(y|\Phi)$. Importantly, the Kalman filter can calculate the likelihood without needing the values of the latent states, θ. The derivations of the Kalman filter can be found in most time-series texts, including Durbin and Koopman (2012, Ch. 5–7) and West and Harrison (1997), and thus are not presented here. Given the results of the Kalman filter there are several methods to sample θ from $p(\theta | y, \Phi)$, a process called Forward-Filtering Backwards-Smoothing (FFBS) or simulation smoothing (Carter and Kohn 1994; Frühwirth-Schnatter 1994; De Jong and Shephard 1995; Durbin and Koopman 2002; Durbin and Koopman 2012, Ch 4.9). I make use of these methods to efficiently sample both the latent states, θ, and other parameters of the model in `Stan`.

`Stan` is a probabilistic programming language, with a BUGS-like modeling language, and interfaces to several programming languages, including `R` (Stan Development Team 2015; Carpenter et al. 2015). GDLMs can be directly estimated in `Stan` by translating the model described by Equations (3.18) and (3.19) into a `Stan` model. However, GDLMs can be estimated more efficiently in `Stan` by marginalizing over the latent states, θ. The sampling methods implemented in `Stan`, of which the default is HMC-NUTS (Hoffman and Gelman

[6] That the shrinkage prior be a scale-mixture of normals is not required, but then the model would no longer be a GDLM, and the efficient methods discussed next cannot be used in estimation.

2014), only require the calculation of a likelihood from the user.[7] The Kalman filter can be used to calculate the likelihood $p(y|\Phi)$, marginalizing out the latent states, θ_t. Marginalizing out parameters is required when estimating models with discrete parameters, such as mixture models, in Stan (Stan Development Team 2015, p. 104). Although it is not necessary to marginalize over parameters to sample from GDLMs in Stan, it helps the efficiency of sampling by reducing the correlation between the latent states and the other parameters of the model.The latent states can then be sampled using FFBS. [8] To summarize, an efficient method to sample GDLMs in Stan is:

1. Sample ϑ from $p(\vartheta|y)$ using HMC in Stan. This requires integrating out the latent states, θ, and calculating $p(y|\vartheta)$, which is done using a Kalman filter.

2. Sample $p(\theta|y, \vartheta)$ using a simulation smoother for a GDLM as in (Carter and Kohn 1994; Frühwirth-Schnatter 1994; De Jong and Shephard 1995; Durbin and Koopman 2002; Durbin and Koopman 2012, Ch 4.9).

This two-step process is an example of a partially collapsed Gibbs-sampler (van Dyk and Park 2008). I use this method to sample all the models in this work.

This required implementing Kalman filter and simulation smoothing methods in Stan. Along with this work, I provide a full set of user-defined Stan functions that implement the Kalman filter, smoother, and simulation sampling Arnold (2015c). Section 3.A provides the code for one of the Stan models used in this paper which uses these functions.[9]

[7] Technically, it also requires derivatives of the likelihood, but these are generated automatically by Stan using its automatic differentiation engine.

[8] In Stan, the calculation of $p(y|.)$ is done in the `transformed parameter` or `model` blocks, while the sampling of the latent states is done in the `generated quantities` block.

[9] The full code for all models run in this work are available at https://github.com/jrnold/dlm-shrinkage.

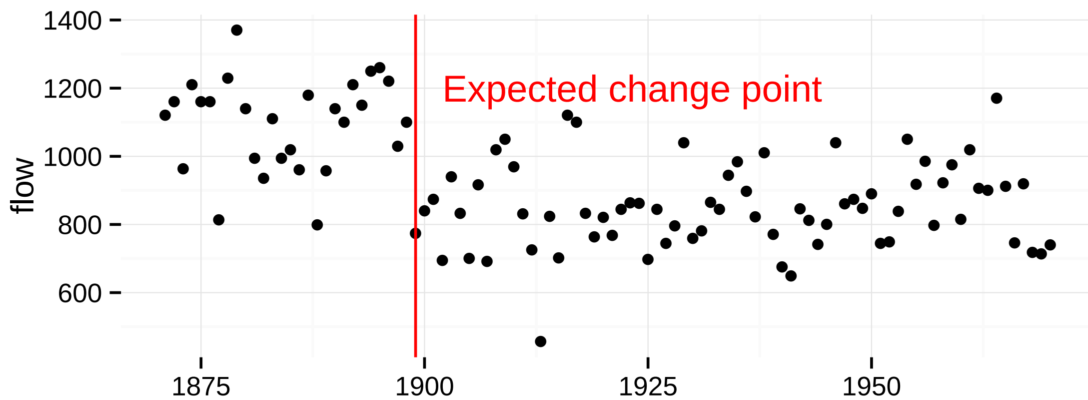


Figure 3.2: Annual flow of the Nile River, 1871–1970. Previous work has found a break point in this series near 1899.

3.4 Example: Annual Nile River Flows

A classic dataset that has been analyzed in many works and texts on time series and structural breaks is the annual flow volume of the Nile River between 1871 and 1970 (Cobb 1978; Balke 1993; de Jong and Penzer 1998; Durbin and Koopman 2012; Commandeur, Koopman, and Ooms 2011).[10] Figure 3.2 plots this data. This series seems to show a single large shift in the average level of the annual flow around 1899. This level shift is attributed to construction of a dam at Aswan that started operation in 1902 or to climate changes that reduced rainfall in the area (Cobb 1978, p. 278).

I compare several models of this data. In all models, y_t is the annual flow of the Nile River,[11] and consists of $n = 100$ annual observations from 1871 to 1970. In all models, each observation y_t is distributed normal with mean μ_t and a common variance, σ^2.[12]

$$y_t \sim \mathcal{N}\left(\mu_t, \sigma^2\right) \tag{3.20}$$

[10] The dataset is included with `R` as `Nile` in the included package **datasets**.

[11] Discharge at Aswan in $10^8 m^3$.

[12] This may not be appropriate since that data contains possible outliers at 1879, 1913, and 1964. However, modeling outliers is not the purpose of this analysis, so I ignore that.

The models differ in how they model the possibly time-varying mean, μ_t. I estimate the following models:

`Constant` In this model, the mean is a constant.

$$\mu_t = \mu_0 \quad \text{for all } t \tag{3.21}$$

`Intervention` This model includes an indicator variable for all years including and after 1899. This models a situation in which the researcher knows, or suspects they know, the change points, and is manually accounting for them.

$$\mu_t = \begin{cases} \mu_0 & t < 1899 \\ \mu_0 + \omega & t \geq 1899 \end{cases} \tag{3.22}$$

`Normal` In this model, the system errors are distributed normal. This corresponds to the a local level model (Durbin and Koopman 2012, Ch. 2; West and Harrison 1997, Ch. 2).[13]

$$\mu_t = \mu_{t-1} + \omega_t \quad \omega_t \sim N(0, \tau^2) \tag{3.23}$$

`StudentT` In this model, the system errors are distributed Student t with degrees of freedom ν,

$$\mu_t = \mu_{t-1} + \omega_t \quad \omega_t \sim \mathcal{T}_\nu(0, \tau) \tag{3.24}$$

The prior distribution of the degrees of freedom parameter ν is that suggested by Juárez and Steel (2010), a Gamma distribution with shape parameter 2 and rate parameter 0.1 (mode 10, mean 20, and variance 200). This places most of mass in the relevant region of the space—away from 0 but less than 30, after which the distri-

[13]For example, the local level model implemented in the `R` function `StructTS`.

bution is effectively indistinguishable from a normal distribution.

`Laplace` In this model, the system errors are distributed Laplace (double exponential):

$$\mu_t = \mu_{t-1} + \omega_t \quad \omega_t \sim \mathscr{L}(0, \tau) \tag{3.25}$$

`Horseshoe` In this model, the system errors are distributed horseshoe.

$$\begin{aligned} \mu_t &= \mu_{t-1} + \omega_t \quad \omega_t \sim \mathcal{N}\left(0, \tau^2 \lambda_t^2\right) \\ \lambda_t &\sim \mathscr{C}^+(0, 1) \end{aligned} \tag{3.26}$$

`Horseshoe+` In this model, the system errors are distributed horseshoe+.

$$\begin{aligned} \mu_t &= \mu_{t-1} + \omega_t \quad \omega_t \sim \mathcal{N}\left(0, \tau^2 \lambda_t^2\right) \\ \lambda_t &\sim \mathscr{C}^+\left(0, \eta_t\right) \\ \eta_t &\sim \mathscr{C}^+(0, 1) \end{aligned} \tag{3.27}$$

For the global scale parameter, τ, in the `Laplace`, `StudentT`, `Horseshoe`, and `Horseshoe+` models, I use the half-Cauchy prior $\tau \sim \mathscr{C}^+\left(0, \frac{1}{n}\right)$. In the `Normal` and `Intervention` models, τ is given a semi-informative half-Cauchy prior with a scale equal to a multiple of the standard deviation of the data.

Figure 3.3 plots the posterior distribution of μ for each model. The `Normal` model does not show a clean break at 1899, instead it estimates a change occurring over several years. The `Laplace` model looks similar to the `Normal` model. While the Laplace distribution achieves sparsity in maximum likelihood estimates because because they use the mode as the estimate, it does produce sparse posterior mean estimates. Since the distribution does not concentrate much mass near zero, it is not surprising that it does not perform much differently than the normal distribution (Park and Casella 2008). Both the `Horseshoe` and `Horseshoe+` models produce a posterior distribution of μ that appears similar to the step function in the `Intervention` model. Additionally, the `StudentT` also produces estimates similar to the `Intervention`, but with slightly wider posterior distributions than the horseshoe models. Figure 3.3 plots the posterior distribution of the system errors, ω, for each model. The `Horseshoe`, `Horseshoe+`, and `StudentT` models all estimate $\mathrm{E}(\omega_t|y)$ near zero for all years but 1899.

Table 3.1 compares the models using several statistics. First, I compare the models on their fit to the in-sample data using the root mean squared error (RMSE), $\mathrm{RMSE}(y)$, The RMSE is defined as $\sqrt{\frac{1}{n}\sum_i (y_i - \mathrm{E}(\mu_i|y))^2}$, where $\mathrm{E}(\mu_i|y)$ is the posterior mean of μ_i. I also compare the models based on their expected fit to out-of-sample data with the expected log predictive density calculated using two methods: the Widely Applicable Information Criterion (WAIC), elpd_{WAIC}, and leave-one-out (LOO) cross-validation, elpd_{loo}. The log probability density of a new observation is the expected value of the posterior density of a future observation, $\log \mathrm{E}(p(\tilde{y}|\theta))$. Since the value of the future observation, $\tilde{y}$, is unknown, the expected log probability density averages over the predictive distribution of $\tilde{y}$, $\mathrm{elpd} = \mathrm{E}_f(\log p(\tilde{y}|\theta, y_i))$, where f is the distribution of $\tilde{y}$. However, the distribution of future values is in general also unknown, which why two approximations are used. elpd_{WAIC} approximates the elpd using an information criteria similar to AIC, BIC or DIC, taking the in-sample log-likelihood and penalizing it for model complexity. elpd_{loo} approximates the elpd using leave-one-out cross validation. See Gelman, Carlin, et al. (2013), Gelman and Vehtari (2014), or Gelman, Hwang, and Vehtari (2014) for more thorough discussion of elpd and predictive measures for Bayesian model comparison.[14] In the RMSE, a lower

[14] The way these are calculated does not fully account for the time-series nature of this data. The measures presented here should be seen as approximating the fit of the model to a previously missing value within the time-series. To calculate elpd_{WAIC} and elpd_{loo}, I use the **loo** `R` package (Vehtari, Gelman, and Gabry 2015), which implements the methods described in Gelman and Vehtari (2014).

value indicates a better fit, for elpd, a higher value indicates is a better fit. The `Horseshoe` and `Horseshoe+` models both have the best fit in terms of RMSE and elpd values of the shrinkage priors, though neither fits either the in-sample or out-of-sample data as well as the `Intervention` data. Surprisingly, the `StudentT` model is close in performance to the horseshoe models.

Second, I compare the fits of the model to the "true" values of μ_t. But, since this is real data, I do not know the true values of μ_t. Instead, I will will compare the other models to posterior mean estimate of `Intervention` model. The column RMSE(μ) of Table 3.1 is the root mean squared error of the models compared to the posterior mean of μ as estimated by the `Intervention` model, defined as $\sqrt{\frac{1}{n}\sum(\mathrm{E}(\mu_t|y) - \bar{\mu}_t)^2}$, where $\bar{\mu}_t$ is the posterior mean of μ_t in the `Intervention` model. As with comparisons of fit to the observed data, the `Horseshoe` and `Horseshoe+` models have the lowest RMSE, although the `StudentT` model is close.

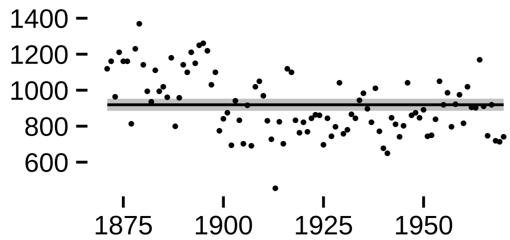
model	RMSE(y)	elpd$_{WAIC}$	elpd$_{loo}$	RMSE(μ)
`Constant`	168.38	-656.49	-656.49	111.24
`Intervention`	126.39	-629.09	-629.10	
`Normal`	143.82	-641.87	-641.88	37.52
`StudentT`	137.46	-637.61	-637.66	13.50
`Laplace`	140.89	-640.43	-640.49	25.75
`Horseshoe`	136.48	-635.89	-635.98	9.44
`Horseshoe+`	136.42	-637.47	-638.26	12.59

Table 3.1: Model comparison statistics for models of the Nile Rive annual flow data.

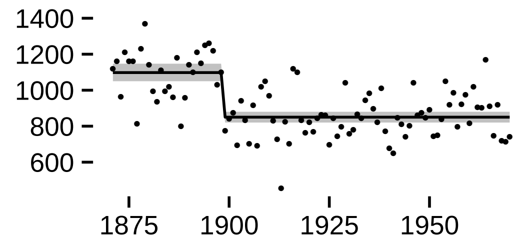
3.5 Change Points in Levels and Trends

The use of sparsity inducing priors can be extended to model change points in trends in addition to the level.[15] The local level model considered in section 3.2 can be extended to

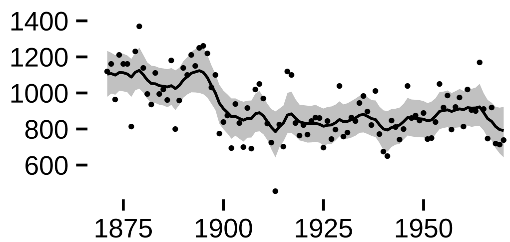
[15] Kim et al. (2009) and Tibshirani (2014) consider similar problems in a maximum likelihood framework with ℓ_1 regularization.



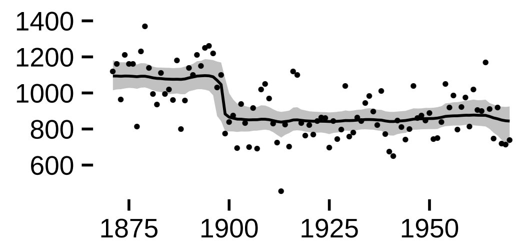
(a) `Constant`



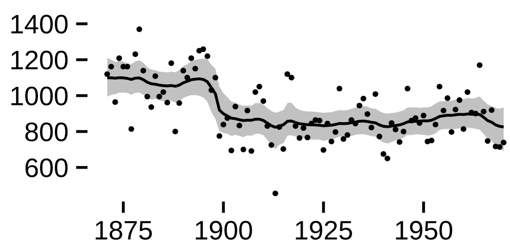
(b) `Intervention`



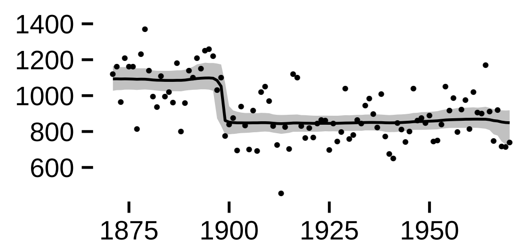
(c) `Normal`



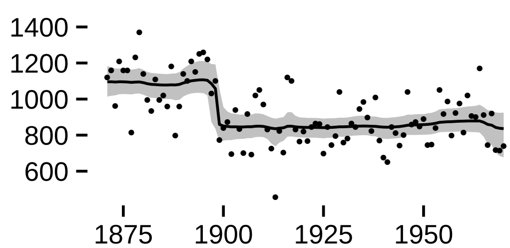
(d) `StudentT`



(e) `Laplace`

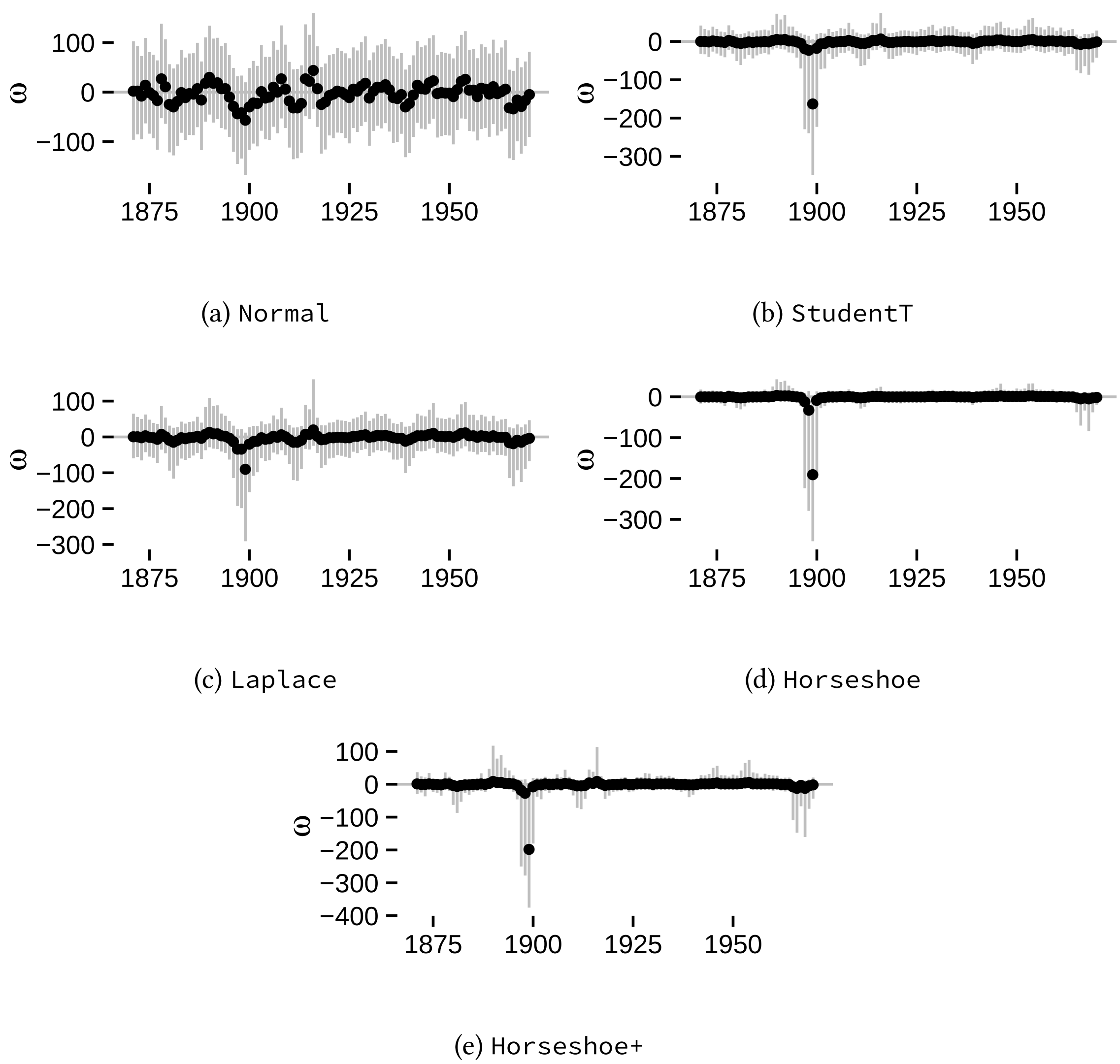


(f) `Horseshoe`



(g) `Horseshoe+`

Figure 3.3: Posterior distributions of μ_t for models of the Nile River annual flow data. The line is the posterior mean; the range of the ribbon the 2.5–97.5% percentiles of the posterior distribution.



(a) `Normal`

(b) `StudentT`

(c) `Laplace`

(d) `Horseshoe`

(e) `Horseshoe+`

Figure 3.4: Posterior distributions of ω_t for models of the Nile River annual flow data. The point is the posterior mean; the range of the line is the 2.5–97.5% percentiles of the posterior distribution.

a local trend model (Durbin and Koopman 2012, Ch 3.2; West and Harrison 1997, Ch 7),

$$
\begin{aligned}
y_t &= \mu_t + \epsilon_t & \epsilon_t &\sim \mathcal{N}\left(0, \sigma^2\right) \\
\mu_t &= \mu_{t-1} + \alpha_{t-1} + \omega_{1,t} \\
\alpha_t &= \alpha_{t-1} + \omega_{2,t}
\end{aligned} \tag{3.28}
$$

In Equation (3.28), there are two states: μ_t is the current level, and α_t is the current trend (change in the level). The level is changing over time both due to the current value of the trend, α_t, and the system errors, $\omega_{1,t}$. While the trend is changing over time only due to the system error, $\omega_{2,t}$. This model will allow for change points in both the level and trend if sparsity inducing shrinkage priors are used for ω_1 and ω_2. The system errors in a local trend model could be modeled with an arbitrary covariance structure, but in this work, I follow the suggestion of West and Harrison (1997, Ch 7.),

$$
\begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix}' \sim \mathcal{N}\left(0, L \operatorname{diag}(W_1^2, W_2^2) L'\right); \quad L = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{3.29}
$$

Since the sparsity inducing shrinkage priors discussed can be represented as scale-mixtures of normals, the previous equation can be expressed as:

$$
\begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} \tau_1^2 \lambda_{1,t}^2 + \tau_2 \lambda_{2,t}^2 & \tau_2 \lambda_{2,t} \\ \tau_2^2 \lambda_{2,t} & \tau_2^2 \lambda_{2,t} \end{bmatrix}\right) \tag{3.30}
$$

where τ_1 and $\lambda_{1,t}$ are the global and local variance components for the level, and τ_2 and $\lambda_{2,t}$ are the global and local variance components for the trend. Since this model is a GDLM, it can be efficiently sampled using the methods in 3.3.

3.6 Example: George W. Bush Approval Ratings

As an example of a time series that is a smooth curve with jumps Ratkovic and Eng (2010) use the approval ratings for George W. Bush, displayed in Figure 3.5. George W. Bush's approval ratings are difficult to fit with typical smoothing methods because it was subject to two large jumps, September 11th, 2001, and at the start of the Iraq War on March 20, 2003. The data used in this example consists of 270 polls between February 04, 2001 and January 11, 2009 from the Roper Center Public Opinion [16]

$$
\begin{aligned}
y_t &= \mu_t + \epsilon_t & \epsilon_t &\sim \mathcal{N}\left(0, \sigma^2\right) \qquad (3.31) \\
\mu_t &= \alpha_t + \mu_{t-1} + \partial\mu_{t-1} + \omega_{1,t} \\
\partial\mu_t &= +\partial\mu_{t-1} + \omega_{2,t} \\
\begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix} &\sim \mathcal{N}\left(0, \begin{bmatrix} \tau_1^2 \lambda_{1,t}^2 + \tau_2 \lambda_{2,t}^2 & \tau_2 \lambda_{2,t} \\ \tau_2^2 \lambda_{2,t} & \tau_2^2 \lambda_{2,t} \end{bmatrix}\right)
\end{aligned}
$$

`Normal` System errors are distributed normal. $\lambda_{i,t} = 1$ for all i for all t, $\alpha_t = 0$ for all t.

`Intervention` System errors are distributed normal, $\lambda_{i,t} = 1$ for all i for all t. There are manual interventions after 9/11 and the the Iraq War. The values of α_t for those two dates are non-zero and estimated, all other α_t are set to zero. This corresponds to a manual intervention for known change points.

`Horseshoe` The system errors, $\omega_{i,t}$, are distributed horseshoe, with $\alpha_t = 0$ for all t.

`Horseshoe+` The system errors, $\omega_{i,t}$, are distributed horseshoe+, with $\alpha_t = 0$ for all t.

Figures 3.6 and 3.7 plot the posterior distribution of μ_t for the models. The `Normal` model shows Bush's approval rating rising before 9/11 and is rough, with much small

[16] From http://webapps.ropercenter.uconn.edu/CFIDE/roper/presidential/webroot/presidential_rating_detail.cfm?allRate=True&presidentName=Bush#.UbeB8HUbyv8.

variation between the jumps. The `Horseshoe` and `Horseshoe+` models more closely resemble the `Intervention` model: Bush's approval rating are loping downward or steady until 9/11, and otherwise the approval ranting is mostly smooth. Table 3.2 shows the RMSE and expected log predictive densities of the these models. The horseshoe models fit the the data better than a normal distribution, but less well than the `Intervention` model.

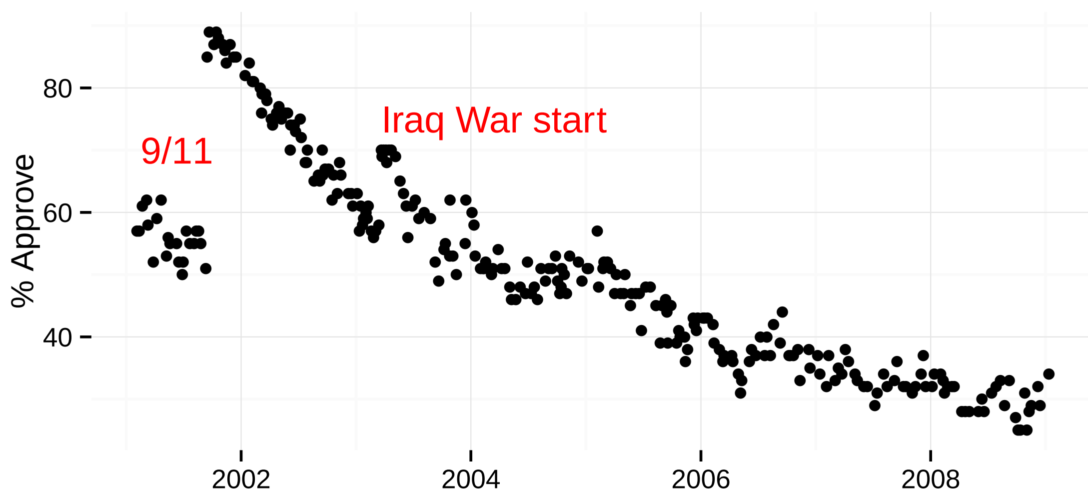
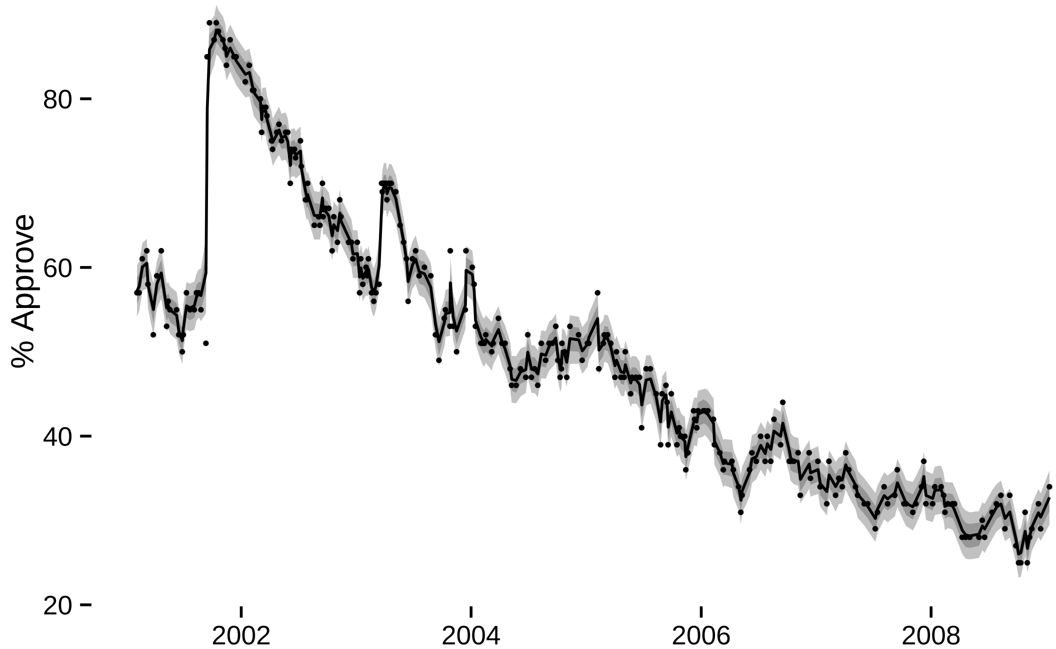


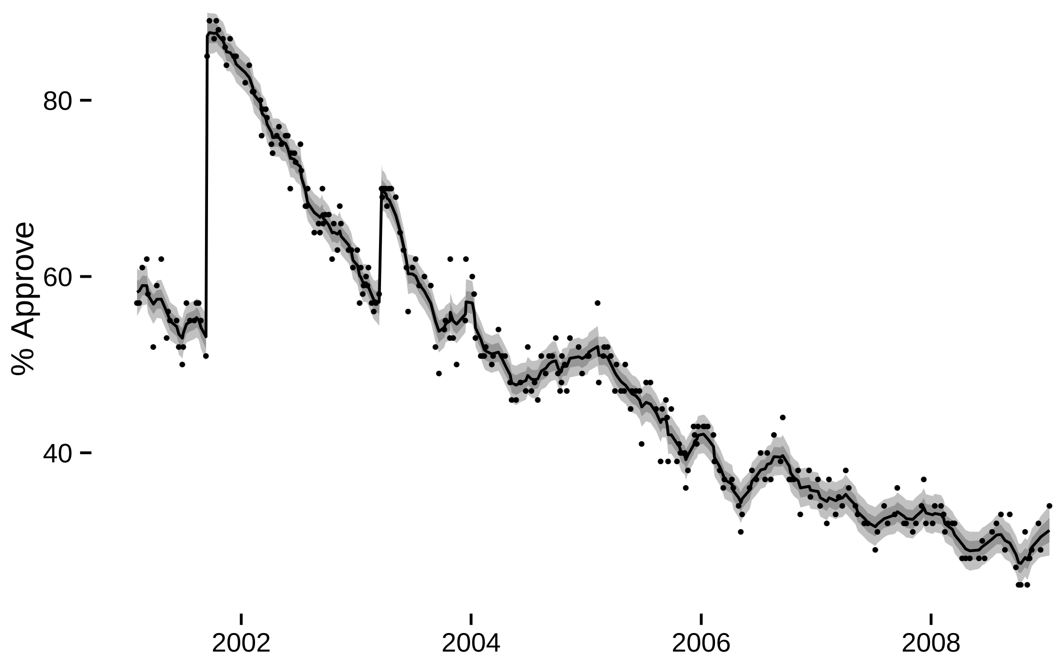
Figure 3.5: Approval ratings of President George W. Bush

model	RMSE	elpd_{WAIC}	elpd_{loo}
`Normal`	3.73	-748.51	-749.43
`Intervention`	2.80	-666.24	-666.84
`Horseshoe`	3.44	-680.65	-681.15
`Horseshoe+`	3.43	-682.56	-683.61

Table 3.2: Model comparison statistics for models of President George W. Bush's approval rating.

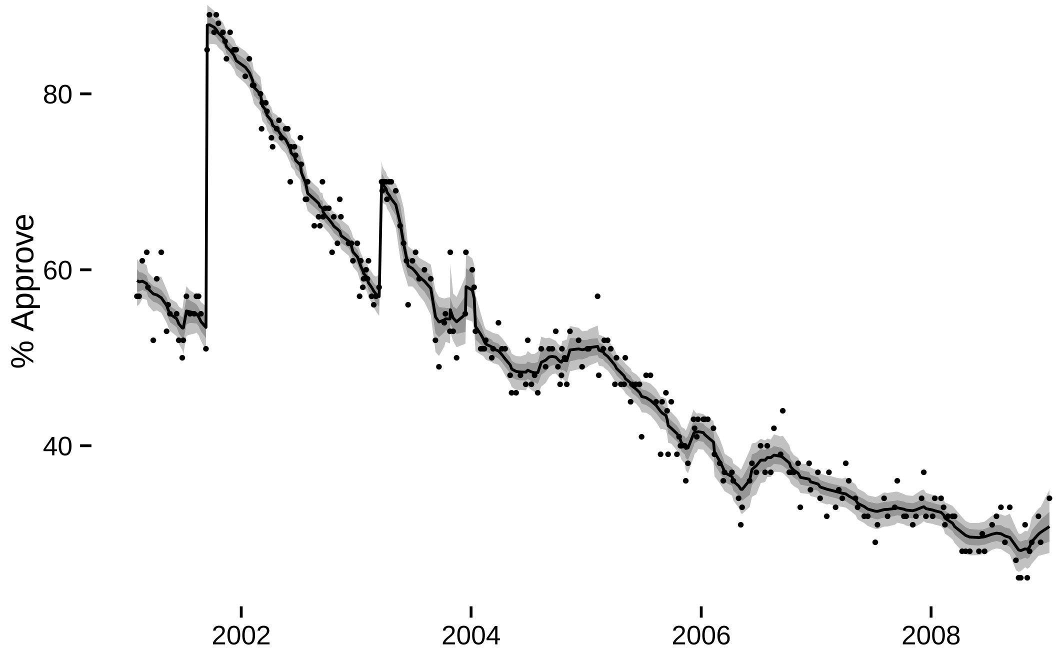


(a) `Normal`

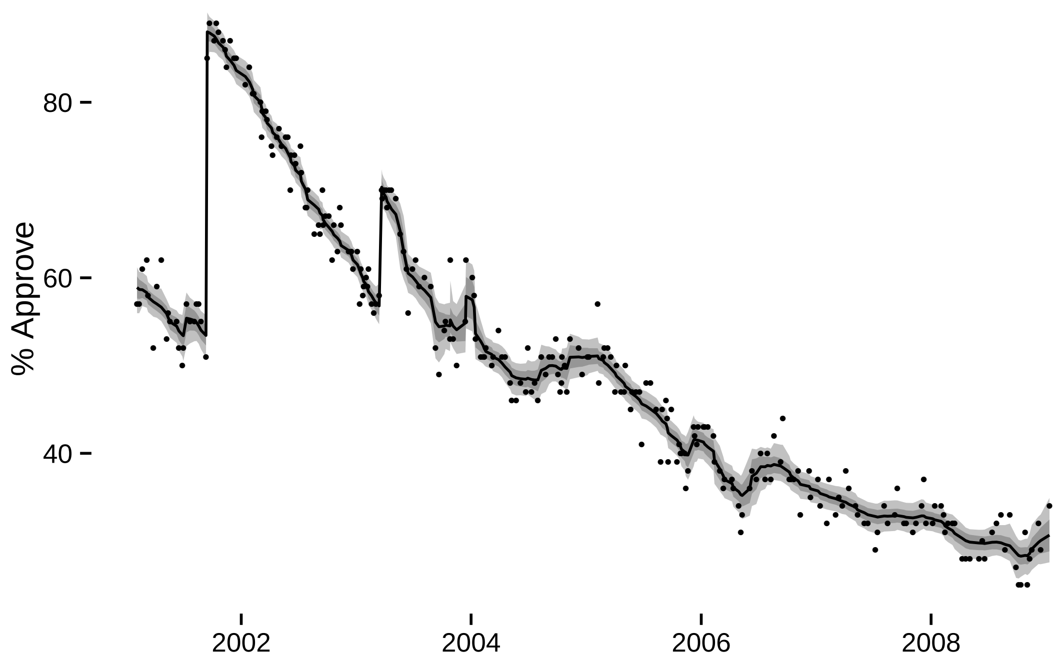


(b) `Intervention`

Figure 3.6: Posterior distribution of μ for `Normal` and `Intervention` models.



(a) `Horseshoe`



(b) `Horseshoe+`

Figure 3.7: Posterior distribution of μ for `Horseshoe` and `Horseshoe+` models.

3.7 Conclusion

This work proposes modeling change points through the use sparse shrinkage priors, such as the horseshoe, for the change in a parameter. This approach has several useful features. It does not require choosing a specific number of change points, and the sparsity of the changes can be estimated from the data. Although it does not directly estimate the probability of change points, it is closer to the data generating process of many political processes in which there is always change, but which are characterized by many periods of small changes, and only a few periods of large changes. Since these shrinkage priors are scale-mixtures of normal distributions, these models fall into the class of Gaussian dynamic linear models, and, thus, can be sampled using efficient algorithms specific to that class of models. This work provides a partially collapsed Gibbs sampler method to estimate GDLMs in Stan, as well Stan code that implements Kalman filtering and smoothing in Stan.

The most promising feature of this approach is that it is flexible and can be applied to a variety of models. For example, the following model is a linear regression with independent change points, and K variables,

$$
\begin{aligned}
y &\sim \mathcal{N}\left(\alpha_t + \beta X, \sigma^2\right) \\
\alpha_t &\sim \mathcal{N}\left(\alpha_{t-1}, \sigma^2 \tau^2_{\alpha} \lambda^2_{\alpha,t}\right) \\
\beta_{t,k} &\sim \mathcal{N}\left(\beta_{t-1,k}, \sigma^2 \tau^2_{\beta,k} \lambda^2_{\beta,t}\right) \quad \text{for } k \in 1, \dots, K
\end{aligned}
\tag{3.32}
$$

where the local variance components, λ_α and λ_β are given prior distributions corresponding to a sparse shrinkage distribution such as the horseshoe or horseshoe+. This model corresponds to a GDLM with latent states α and β, and thus the partially collapsed Gibbs sampling method in Section 3.3 can be used to efficiently estimate it. As written, this would correspond to independent change points in the parameters, α and β. If the researcher wanted to impose a restriction that large changes occurred at the same time for all distributions, they could set $\tau_{\alpha} = \tau_{\beta,1} = \cdots = \tau_{\beta,K}$ and $\lambda_{\alpha} = \alpha_{\beta,1} = \cdots = \alpha_{\beta,K}$. This is one example, but these sparse shrinkage parameters can be applied to any model with a time-varying parameter with support $\mathbb{R}$. If that model is of the class of GDLMs, then there are efficient methods to sample it, if not, the model can still be estimated in Stan using its usual algorithms.

3.A Example Stan Program

An example of a change point model implemnted in Stan. See the replication data for this work to see the code for all the Stan models estimated. The DLM related user-defined functions in the `functions` block are excluded. The code for them can be found in Arnold (2015c) or https://raw.githubusercontent.com/jrnold/dlm-shrinkage/master/stan/includes/dlm.stan.

```
data {
  int<lower = 1> n;
  vector[n] y;
  int miss[n];
  real m0;
  real<lower = 0.0> C0;
  real<lower = 0.0> s;
  real<lower = 0.0> w;
}
parameters {
  real<lower = 0.0> sigma;
```

```
  real<lower = 0.0> tau;
  vector<lower = 0.0>[n] lambda;
}
transformed parameters {
  vector[n] log_lik;
  vector[6] dlm[n + 1];
  vector[n] W;

  for (i in 1:n) {
    W[i] <- pow(sigma * tau * lambda[i], 2);
  }
  {
    vector[n] V;
    V <- rep_vector(pow(sigma, 2), n);
    dlm <- dlm_local_level_filter(n, y, miss, V, W, m0, C0);
    log_lik <- dlm_local_level_filter_loglik(n, dlm, miss);
  }

}
model {
  real ll;

  sigma ~ cauchy(0.0, s);
  tau ~ cauchy(0.0, w);
  lambda ~ cauchy(0.0, 1);
  increment_log_prob(sum(log_lik));
```

```
}
generated quantities {
  vector[1] mu[n + 1];
  vector[1] omega[n];
  vector[1] kalman[n];

  {
    matrix[1, 1] G_tv[n];

    G_tv <- rep_array(rep_matrix(1.0, 1, 1), n);
    mu <- dlm_filter_bsample_rng(n, 1, 1, G_tv, dlm);
  }
  for (i in 1:n) {
    omega[i] <- mu[i + 1] - mu[i];
  }
  for (i in 1:n) {
    kalman[i] <- dlm_get_C(i, 1, 1, dlm) * dlm_get_Q_inv(i, 1, 1, dlm);
  }

}
```

let3.A3.1

Bibliography

Arnold, Jeffrey (2015a). “American Civil War Battle Database”. URL: http://dx.doi.org/10.6084/m9.figshare.1515995 (cit. on p. 36).

— (2015b). “American Civil War Era Financial Data”. URL: http://dx.doi.org/10.6084/m9.figshare.1513853 (cit. on p. 20).

— (2015c). “Dynamic Linear Model Functions in Stan”. URL: http://dx.doi.org/10.6084/m9.figshare.1553209 (cit. on pp. 68, 82).

Arrow, Kenneth J. et al. (2008). “The Promise of Prediction Markets”. *Science* 320.5878, pp. 877–878. DOI: 10.1126/science.1157679. eprint: http://www.sciencemag.org/content/320/5878/877.full.pdf. URL: http://www.sciencemag.org/content/320/5878/877.short (cit. on p. 4).

Bai, Jushan and Pierre Perron (1998). “Estimating and Testing Linear Models with Multiple Structural Changes”. *Econometrica* 66.1, pp. 47–78. ISSN: 00129682. URL: http://www.jstor.org/stable/2998540 (cit. on p. 58).

— (2003). “Computation and analysis of multiple structural change models”. *Journal of Applied Econometrics* 18.1, pp. 1–22. ISSN: 1099-1255. DOI: 10.1002/jae.659. URL: http://dx.doi.org/10.1002/jae.659 (cit. on p. 58).

Balke, Nathan S. (1993). “Detecting Level Shifts in Time Series”. *Journal of Business & Economic Statistics* 11.1, pp. 81–92. ISSN: 07350015. URL: http://www.jstor.org/stable/1391308 (cit. on p. 69).

Barry, Daniel and J. A. Hartigan (1993). “A Bayesian Analysis for Change Point Problems”. *Journal of the American Statistical Association* 88.421, pp. 309–319. ISSN: 01621459. URL: http://www.jstor.org/stable/2290726 (cit. on p. 58).

Beck, Nathaniel (1989). “Estimating Dynamic Models Using Kalman Filtering”. *Political Analysis* 1.1, pp. 121–156. DOI: 10.1093/pan/1.1.121. eprint: http://pan.oxfordjournals.org/content/1/1/121.full.pdf+html. URL: http://pan.oxfordjournals.org/content/1/1/121.abstract (cit. on p. 65).

Bhadra, Anindya, Jyotishka Datta, Nicholas G. Polson, and Brandon Willard (2015). “The Horseshoe+ Estimator of Ultra-Sparse Signals”. eprint: 1502.00560 (cit. on pp. 56, 60–62).

Biddle, Stephen (2004). *Military Power: Explaining Victory and Defeat*. Pinceton University Press. ISBN: 9780691116457. URL: http://books.google.com/books?id=WwLClEbUIT4C (cit. on p. 29).

Biddle, Stephen and Stephen Long (2004). “Democracy and Military Effectiveness: A Deeper Look”. *Journal Of Conflict Resolution* 48, p. 525. DOI: 10.1177/0022002704266118. URL: http://jcr.sagepub.com/cgi/content/abstract/48/4/525 (cit. on pp. 11, 29).

Blackwell, Matthew (2012). “Game-changers: Detecting shifts in the Flow of campaign contributions”. URL: http://www.mattblackwell.org/files/papers/gamechangers.pdf (cit. on pp. 55, 65).

Blainey, Geoffrey (1988). *The Causes of War*. 3, reprint. Simon and Schuster. ISBN: 9780029035917. URL: http://books.google.com/books?id=Mcknp3tt0LMC (cit. on p. 1).

Bodart, Gaston (1908). *Militär-historisches kreigs-lexikon, (1618-1905)*. Stern. URL: http://books.google.com/books?id=Eo4DAAAAYAAJ (cit. on pp. 33, 37).

Bolt, Jutta and Jan Luiten van Zanden (2013). *The First Update of the Maddison Project; Re-estimating Growth Before 1820*. Working Paper 4. The Maddison Project (cit. on p. 17).

Bordo, Michael D. and Hugh Rockoff (1996). “The Gold Standard as a “Good Housekeeping Seal of Apova”. *The Journal of Economic History* 56 (02), pp. 389–428. ISSN: 1471-6372. DOI: 10.1017/S0022050700016491. URL: http://journals.cambridge.org/article_S0022050700016491 (cit. on p. 43).

Brown William O., Jr. and Richard C. K. Burdekin (2000). “Turning Points in the U.S. Civil War: A British Perspective”. *The Journal of Economic History* 60.1, pp. 216–231. ISSN: 00220507. URL: http://www.jstor.org/stable/2566803 (cit. on pp. 3, 23).

Burdekin, Richard C. K. and Farrokh K. Langdana (1993). “War Finance in the Southern Confederacy, 1861-1865”. *Explorations in Economic History* 30.3, pp. 352–376. ISSN: 0014-4983. DOI: 10.1006/exeh.1993.1015. URL: http://www.sciencedirect.com/science/article/B6WFJ-45PMM8X-D/2/0332a9f88be4f3e45940cc43f8263a42 (cit. on p. 23).

Burdekin, Richard C. K. and Marc D. Weidenmier (2001). “Inflation Is Always and Everywhere a Monetary Phenomenon: Richmond vs. Houston in 1864”. *The American Economic Review* 91.5, pp. 1621–1630. ISSN: 00028282. URL: http://www.jstor.org/stable/2677945 (cit. on pp. 23, 27).

Büthe, Tim (2002). “Taking Temporality Seriously: Modeling History and the Use of Narratives as Evidence”. *American Political Science Review* null (03), pp. 481–493. ISSN: 1537-5943. DOI: 10.1017/S0003055402000278. URL: http://journals.cambridge.org/article_S0003055402000278 (cit. on p. 55).

Calderia, Gregory A. and Christopher J. W. Zorn (1998). “Of Time and Consensual Norms in the Supreme Court”. *American Journal of Political Science* 42.3, pp. 874–902. ISSN: 00925853. URL: http://www.jstor.org/stable/2991733 (cit. on pp. 55, 65).

Calomiris, Charles W. (1988). “Price and Exchange Rate Determination during the Greenback Suspension”. *Oxford Economic Papers*. New Series 40.4, pp. 719–750. ISSN: 00307653. URL: http://www.jstor.org/stable/2663038 (cit. on pp. 3, 19, 22).

Carpenter, Bob, Andrew Gelman, Matt Hoffman, Daniel Lee, Ben Goodrich, Micahel Betancourt, Marcus A. Brubaker, Jiquiang Guo, Peter Li, and Allend Riddell (2015). “Stan: A Probabilistic Programming Language”. *Journal of Statistical Software* ? (?). URL: http://www.stat.columbia.edu/~gelman/research/published/stan-paper-revision-feb2015.pdf (cit. on p. 67).

Carter, C. K. and R. Kohn (1994). "On Gibbs sampling for state space models". *Biometrika* 81.3, pp. 541–553. DOI: 10.1093/biomet/81.3.541. eprint: http://biomet.oxfordjournals.org/content/81/3/541.full.pdf+html. URL: http://biomet.oxfordjournals.org/content/81/3/541.abstract (cit. on pp. 41, 57, 67, 68).

Carvalho, Carlos M., Nicholas G. Polson, and James G. Scott (2009). "Handling Sparsity via the Horseshoe". *Journal of Machine Learning and Research: Workshop and Conference Proceedings* 5, pp. 73–80 (cit. on pp. 55, 56, 61).

— (2010). "The horseshoe estimator for sparse signals". *Biometrika* 97.2, pp. 465–480. DOI: 10.1093/biomet/asq017. eprint: http://biomet.oxfordjournals.org/content/97/2/465.full.pdf+html. URL: http://biomet.oxfordjournals.org/content/97/2/465.abstract (cit. on pp. iii, 55, 56, 59, 61).

Chan, Ngai Hang, Chun Yip Yau, and Rong-Mao Zhang (2014). "Group LASSO for Structural Break Time Series". *Journal of the American Statistical Association* 109.506, pp. 590–599. DOI: 10.1080/01621459.2013.866566. eprint: http://dx.doi.org/10.1080/01621459.2013.866566. URL: http://dx.doi.org/10.1080/01621459.2013.866566 (cit. on p. 62).

Chan-Lau, Jorge A. (2006). "Market-Based Estimation of Default Probabilities and Its Application to Financial Market Surveillance". WP/06/06. URL: http://www.imf.org/external/pubs/ft/wp/2006/wp06104.pdf (cit. on p. 5).

Chen, Andrew H. and Thomas F. Siems (2004). "The effects of terrorism on global capital markets". *European Journal of Political Economy* 20.2, pp. 349–366. ISSN: 0176-2680. DOI: 10.1016/j.ejpoleco.2003.12.005. URL: http://www.sciencedirect.com/science/article/pii/S0176268003001022 (cit. on pp. 4, 7).

Chib, Siddhartha (1998). "Estimation and comparison of multiple change-point models". *Journal of Econometrics* 86.2, pp. 221–241. ISSN: 0304-4076. DOI: DOI:10.1016/S0304-4076(97)00115-2. URL: http://www.sciencedirect.com/science/article/B6VC0-3VM1XM5-2/2/469ee3cba827365611dee3677f0babc6 (cit. on pp. 57, 58).

Clausewitz, Carl von (1989). *On war*. Ed. and trans. by Michael Howard and Peter Paret. Indexed edition. Princeton University Press. ISBN: 9780691018546. URL: http://books.google.com/books?id=ygPEQgAACAAJ (cit. on p. 9).

Cobb, George W. (1978). "The problem of the Nile: Conditional solution to a changepoint problem". *Biometrika* 65.2, pp. 243–251. DOI: 10.1093/biomet/65.2.243. eprint: http://biomet.oxfordjournals.org/content/65/2/243.full.pdf+html. URL: http://biomet.oxfordjournals.org/content/65/2/243.abstract (cit. on p. 69).

Cochran, Kathryn and Stephen B. Long (2014). "Measuring Military Effectiveness: Calculating Casualty Loss-Exchange Ratios for Multilateral Interstate Wars, 1816-1978". working paper. URL: https://facultystaff.richmond.edu/~slong/CochranLong2015.pdf (cit. on p. 11).

Commandeur, Jacques J. F., Siem Jan Koopman, and Marius Ooms (2011). "Statistical Software for State Space Methods". *Journal of Statistical Software* 41.1, pp. 1–18. ISSN: 1548-7660. URL: http://www.jstatsoft.org/v41/i01 (cit. on p. 69).

Congressional Budget Office (2012a). *The 2012 Long-Term Budget Outlook*. URL: http://www.cbo.gov/publication/43288 (cit. on p. 21).

Congressional Budget Office (2012b). *The 2012 Long-Term Budget Outlook: Supplemental Data*. URL: http://www.cbo.gov/sites/default/files/cbofiles/attachments/43288-LTBOSuppTables_0.xls (visited on 01/25/2013) (cit. on p. 21).

Cornwallis, K. (1879). *The Gold Room and the New York Stock Exchange and Clearing House*. Atlas series. A. S. Barnes. URL: http://books.google.com/books?id=55tAAAAAIAAJ (cit. on pp. 12, 23).

Dafoe, Allan and Nina Kelsey (2014). "Observing the capitalist peace: Examining market-mediated signaling and other mechanisms". *Journal of Peace Research* 51.5, pp. 619–633. DOI: 10.1177/0022343314536423. eprint: http://jpr.sagepub.com/content/51/5/619.full.pdf+html. URL: http://jpr.sagepub.com/content/51/5/619.abstract (cit. on p. 4).

Datta, Jyotishka and Jayanta. K. Ghosh (2012). "Asymptotic Properties of Bayes Risk for the Horseshoe Prior". *Bayesian Analysis* 7 (4), pp. 771–792. DOI: 10.1214/13-BA805. URL: http://projecteuclid.org/euclid.ba/1362406654 (cit. on p. 61).

Davis, George K. and Gary M. Pecquet (1990). "Interest Rates in the Civil War South". *The Journal of Economic History* 50.1, pp. 133–148. ISSN: 00220507. URL: http://www.jstor.org/stable/2123441 (cit. on pp. 3, 23).

de Jong, Piet and Jeremy Penzer (1998). "Diagnosing Shocks in Time Series". *Journal of the American Statistical Association* 93.442, pp. 796–806. ISSN: 01621459. URL: http://www.jstor.org/stable/2670129 (cit. on pp. 63, 69).

De Jong, Piet and Neil Shephard (1995). "The simulation smoother for time series models". *Biometrika* 82.2, pp. 339–350. DOI: 10.1093/biomet/82.2.339. eprint: http://biomet.oxfordjournals.org/content/82/2/339.full.pdf+html. URL: http://biomet.oxfordjournals.org/content/82/2/339.abstract (cit. on pp. 67, 68).

Dewey, Davis Rich (1918). *Financial History of the United States*. American citizen series. Longmans, Green. URL: http://books.google.com/books?id=l3oaAAAAMAAJ (cit. on p. 19).

Durbin, J. and S. J. Koopman (2002). "A Simple and Efficient Simulation Smoother for State Space Time Series Analysis". *Biometrika* 89.3, pp. 603–615. ISSN: 00063444. URL: http://www.jstor.org/stable/4140605 (cit. on pp. 67, 68).

Durbin, J. and S.J. Koopman (2012). *Time Series Analysis by State Space Methods: Second Edition*. Oxford Statistical Science Series. OUP Oxford. ISBN: 9780199641178. URL: http://books.google.com/books?id=fOq39Zh0olQC (cit. on pp. 65–70, 76).

Dyer, F.H. (1908). *A compendium of the War of the Rebellion*. A Compendium of the War of the Rebellion. The Dyer publishing company. URL: http://books.google.com/books?id=OBkNAQAAMAAJ (cit. on p. 37).

Efron, Bradley (2008). "Microarrays, Empirical Bayes and the Two-Groups Model". *Statist. Sci.* 23.1, pp. 1–22. DOI: 10.1214/07-STS236. URL: http://dx.doi.org/10.1214/07-STS236 (cit. on p. 59).

Eicher, David J. (2001). *The Longest Night: A Military History of the Civil War*. Ed. by James M. McPherson. Simon & Schuster. ISBN: 9780743218467. URL: http://books.google.com/books?id=1p94XzYASDAC (cit. on p. 25).

Elder, W. (1863). *Debt and resources of the United States: and the effect of secession upon the Trade and Industry of the United States*. Ringwalt & Brown, printers. URL: http://books.google.com/books?id=Yw7TAAAAMAAJ (cit. on pp. 20, 26).

— (1865). *How Our National Debt Can be Paid: The Wealth, Resources, and Power of the People of the United States, Issued by Jay Cooke*. How Our National Debt Can be Paid: The Wealth, Resources, and Power of the People of the United States, Issued by Jay Cooke v. 1. Sherman. URL: http://books.google.com/books?id=gRgsAAAAYAAJ (cit. on pp. 25, 28).

Eldor, Rafi and Rafi Melnick (2004). "Financial Markets and Terrorism". working papers. URL: www.idc.ac.il/publications/files/111.pdf (cit. on p. 4).

Fearnhead, Paul (2006). "Exact and efficient Bayesian inference for multiple changepoint problems". *Statistics and Computing* 16.2, pp. 203–213. ISSN: 0960-3174. DOI: 10.1007/s11222-006-8450-8. URL: http://dx.doi.org/10.1007/s11222-006-8450-8 (cit. on p. 58).

Fearnhead, Paul and Zhen Liu (2007). "On-line inference for multiple changepoint problems". *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 69.4, pp. 589–605. ISSN: 1467-9868. DOI: 10.1111/j.1467-9868.2007.00601.x. URL: http://dx.doi.org/10.1111/j.1467-9868.2007.00601.x (cit. on p. 58).

Fearon, James D. (1995). "Rationalist Explanations for War". *International Organization* 49.3, pp. 379–414. DOI: 10.1017/S0020818300033324. URL: http://dx.doi.org/10.1017/S0020818300033324 (cit. on pp. 9, 10).

Filson, Darren and Suzanne Werner (2002). "A Bargaining Model of War and Peace: Anticipating the Onset, Duration, and Outcome of War". *American Journal of Political Science* 46.4, pp. 819–837. ISSN: 00925853. URL: http://www.jstor.org/stable/3088436 (cit. on pp. 9, 15).

Fox, William Freeman (1898). *Regimental losses in the American Civil War, 1861-1865. A treatise on the extent and nature of the mortuary losses in the Union regiments, with full and exhaustive statistics compiled from the official records on file in the state military bureaus and at Washington*. Albany Pub. Co. URL: http://books.google.com/books?id=R5YukgAACAAJ (cit. on p. 33).

Frey, Bruno S. and Marcel Kucher (2000a). "History as Reflected in Capital Markets: The Case of World War II". *The Journal of Economic History* 60.02, pp. 468–496. DOI: 10.1017/S002205070000022X. eprint: http://journals.cambridge.org/article_S002205070000022X. URL: http://journals.cambridge.org/action/displayAbstract?fromPage=online&aid=351410&fulltextType=RA&fileId=S002205070000022X (cit. on pp. 4, 12).

— (2000b). "World War II as reflected on capital markets". *Economics Letters* 69, pp. 187–191 (cit. on p. 12).

Frühwirth-Schnatter, Sylvia (1994). "Data Augmentation And Dynamic Linear Models". *Journal of Time Series Analysis* 15.2, pp. 183–202. ISSN: 1467-9892. DOI: 10.1111/j.1467-9892.1994.tb00184.x. URL: http://dx.doi.org/10.1111/j.1467-9892.1994.tb00184.x (cit. on pp. 41, 67, 68).

Fuller, J. F. C. (1942). *Decisive Battles of the U.S.A.* New York: Thomas Yoseloff (cit. on pp. 44–46).

Fuller, J. F. C. (1956). *The Decisive Battles of the Western World, and Their Influence Upon History: From the American Civil War to the end of the Second World War*. Vol. 3. The Decisive Battles of the Western World, and Their Influence Upon History. Eyre & Spottiswoode. URL: http://books.google.com/books?id=nizTAAAAMAAJ (cit. on p. 17).

Gartner, Scott Sigmund (1998). "Opening up the Black Box of War". *The Journal of Conflict Resolution* 42.3, pp. 252–258. ISSN: 00220027. URL: http://www.jstor.org/stable/174513 (cit. on p. 1).

Gartzke, Erik (2007). "The Capitalist Peace". *American Journal of Political Science* 51.1, pp. 166–191. URL: http://dx.doi.org/10.1111/j.1540-5907.2007.00244.x (cit. on p. 4).

Gelman, A., J.B. Carlin, H.S. Stern, D.B. Dunson, A. Vehtari, and D.B. Rubin (2013). *Bayesian Data Analysis*. 3rd. Chapman & Hall/CRC Texts in Statistical Science. Taylor & Francis. ISBN: 9781439840955. URL: https://books.google.com/books?id=ZXL6AQAAQBAJ (cit. on pp. 41, 50, 72).

Gelman, Andrew, Jessica Hwang, and Aki Vehtari (2014). "Understanding predictive information criteria for Bayesian models". *Statistics and Computing* 24.6, pp. 997–1016. ISSN: 0960-3174. DOI: 10.1007/s11222-013-9416-2. URL: http://dx.doi.org/10.1007/s11222-013-9416-2 (cit. on pp. 50, 72).

Gelman, Andrew and Andrew Vehtari (2014). "WAIC and Cross-validation in STAN". URL: http://www.stat.columbia.edu/~gelman/research/unpublished/waic_stan.pdf (cit. on pp. 50, 51, 72).

Giordani, Paolo and Robert Kohn (2008). "Efficient Bayesian Inference for Multiple Change-Point and Mixture Innovation Models". *Journal of Business & Economic Statistics* 26.1, pp. 66–77. DOI: 10.1198/073500107000000241. eprint: http://amstat.tandfonline.com/doi/pdf/10.1198/073500107000000241. URL: http://amstat.tandfonline.com/doi/abs/10.1198/073500107000000241 (cit. on p. 59).

Godfrey, J.M. (1976). *Monetary expansion in the Confederacy*. Dissertations in American economic history. Arno Press. ISBN: 9780405110344. URL: http://books.google.com/books?id=xIRGwgW3OYUC (cit. on p. 24).

Goldin, Claudia and Frank D. Lewis (1975). "Economic Cost of the Civil War: Estimates and Implications". *Journal of Economic History* 35.2, pp. 299–326. URL: http://www.jstor.org/stable/2119410 (cit. on pp. 6, 17).

Grauer, Ryan and Michael C. Horowitz (2012). "What Determines Military Victory? Testing the Modern System". *Security Studies* 21.1, pp. 83–112. DOI: 10.1080/09636412.2012.650594. eprint: http://dx.doi.org/10.1080/09636412.2012.650594. URL: http://dx.doi.org/10.1080/09636412.2012.650594 (cit. on p. 29).

Greenstone, Michael (2007). *Is The 'Surge' Working? Some New Facts*. Department of Economics Working Paper 07-24. MIT. URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1014427 (cit. on p. 4).

Guidolin, Massimo and Eliana La Ferrara (2010). "The economic effects of violent conflict: Evidence from asset market reactions". *Journal of Peace Research* 47.6, pp. 671–684. DOI: 10.1177/0022343310381853. eprint: http://jpr.sagepub.com/content/47/6/671.full.pdf+html. URL: http://jpr.sagepub.com/content/47/6/671.abstract (cit. on pp. 4, 5).

Haber, Stephen, Kris James Mitchener, Kim Oosterlinck, and Marc C. Weidenmier (2014). *Predicting Winners in Civil Wars*. CEPR Discussion Paper DP10109. CEPR. URL: http://ssrn.com/abstract=2513468 (cit. on p. 3).

Haber, Stephen, Kris James Mitchener, Kim Oosterlinck, and Marc D. Weidenmier (2015). "Victory or Repudiation? Predicting Winners in Civil Wars using International Financial Markets". *Journal of Banking & Finance*. ISSN: 0378-4266. DOI: http://dx.doi.org/10.1016/j.jbankfin.2015.07.009. URL: http://www.sciencedirect.com/science/article/pii/S0378426615002034 (cit. on pp. 5, 7, 13, 27).

Hall, George J. (2004). "Exchange rates and casualties during the first world war". *Journal of Monetary Economics* 51.8, pp. 1711–1742. ISSN: 0304-3932. DOI: DOI:10.1016/j.jmoneco.2004.02.001. URL: http://www.sciencedirect.com/science/article/B6VBW-4DSB10S-2/2/3bc7658152bcc006a7df02095246d23a (cit. on p. 8).

Hans, Chris (2009). "Bayesian lasso regression". *Biometrika* 96.4, pp. 835–845. DOI: 10.1093/biomet/asp047. eprint: http://biomet.oxfordjournals.org/content/96/4/835.full.pdf+html. URL: http://biomet.oxfordjournals.org/content/96/4/835.abstract (cit. on p. 61).

Harchaoui, Z. and C. Lévy-Leduc (2010). "Multiple Change-Point Estimation With a Total Variation Penalty". *Journal of the American Statistical Association* 105.492, pp. 1480–1493. DOI: 10.1198/jasa.2010.tm09181. eprint: http://dx.doi.org/10.1198/jasa.2010.tm09181. URL: http://dx.doi.org/10.1198/jasa.2010.tm09181 (cit. on p. 62).

Harvey, Andrew C. (1990). *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press. ISBN: 9780521405737. URL: http://books.google.com/books?id=Kc6tnRHBwLcC (cit. on p. 66).

Helmbold, Robert L. (1995). *Personnel Attrition Rates in Historical Land Combat Operations. Some Empirical Relations among Force Sizes, Battle Durations*. Research Paper CAA-RP-1995-1. U.S. Army Concepts Analysis Agency. URL: http://handle.dtic.mil/100.2/ADA298124 (cit. on p. 11).

Herron, Michael C. (2000). "Estimating the Economic Impact of Political Party Competition in the 1992 British Election". *American Journal of Political Science* 44.2, pp. 326–337. ISSN: 00925853. URL: http://www.jstor.org/stable/2669314 (cit. on p. 4).

Hinkley, David V. (1970). "Inference about the change-point in a sequence of random variables". *Biometrika* 57.1, pp. 1–17. DOI: 10.1093/biomet/57.1.1. eprint: http://biomet.oxfordjournals.org/content/57/1/1.full.pdf+html. URL: http://biomet.oxfordjournals.org/content/57/1/1.abstract (cit. on p. 58).

Historical Evaulation and Research Organization, T. N. Dupuy, G. P. Hayes, C. C. Johnson, C. R. Smith, and B. Bader (1984). *Analysis of Factors That Have Influenced Outcomes of Battles and Wars: A Data Base of Battles and Engagements. Volume 1. Main Report. Selected Battles 1600 - 1973*. Tech. rep. U. S. Army Concepts. URL: http://handle.dtic.mil/100.2/ADB086797 (cit. on p. 11).

Hoffman, Matthew D. and Andrew Gelman (2014). "The No-U-turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo". *Journal of Machine Learning Research* 15.1, pp. 1593–1623. ISSN: 1532-4435. URL: http://dl.acm.org/citation.cfm?id=2627435.2638586 (cit. on pp. 41, 67).

Homer, Sidney and Richar E. Sylla (2005). *A history of interest rates*. Wiley finance series. Wiley. ISBN: 9780471732839. URL: http://books.google.com/books?id=OOQKf4asZ9EC (cit. on pp. 19–21, 26, 29, 42).

Jackman, S. (2009). *Bayesian Analysis for the Social Sciences*. Wiley Series in Probability and Statistics. Wiley. ISBN: 9780470686638. URL: http://books.google.com/books?id=QFqyrNL8yEkC (cit. on p. 57).

Juárez, Miguel A. and Mark F. J. Steel (2010). "Non-Gaussian dynamic Bayesian modelling for panel data". *Journal of Applied Econometrics* 25.7, pp. 1128–1154. ISSN: 1099-1255. URL: http://dx.doi.org/10.1002/jae.1113 (cit. on p. 70).

Kalyvas, Stathis N. and Laia Balcells (2010). "International System and Technologies of Rebellion: How the End of the Cold War Shaped Internal Conflict". *American Political Science Review* 104.03, pp. 415–429. DOI: 10.1017/S0003055410000286. eprint: http://journals.cambridge.org/article_S0003055410000286. URL: http://dx.doi.org/10.1017/S0003055410000286 (cit. on p. 17).

Keele, Luke (2015). "The Statistics of Causal Inference: A View from Political Methodology". *Political Analysis* 23.3, pp. 313–335. DOI: 10.1093/pan/mpv007. eprint: http://pan.oxfordjournals.org/content/23/3/313.full.pdf+html. URL: http://pan.oxfordjournals.org/content/23/3/313.abstract (cit. on p. 15).

Kennedy, Frances H. and Conservation Fund (Arlington, Va.) (1998). *The Civil War battlefield guide*. Houghton Mifflin. ISBN: 9780395740125. URL: http://books.google.com/books?id=qHObJArDHZMC (cit. on p. 37).

Killick, R., P. Fearnhead, and I. A. Eckley (2012). "Optimal Detection of Changepoints With a Linear Computational Cost". *Journal of the American Statistical Association* 107.500, pp. 1590–1598. DOI: 10.1080/01621459.2012.737745. eprint: http://dx.doi.org/10.1080/01621459.2012.737745. URL: http://dx.doi.org/10.1080/01621459.2012.737745 (cit. on p. 58).

Kim, S., K. Koh, S. Boyd, and D. Gorinevsky (2009). "ℓ_1 Trend Filtering". *SIAM Review* 51.2, pp. 339–360. DOI: 10.1137/070690274. eprint: http://dx.doi.org/10.1137/070690274. URL: http://dx.doi.org/10.1137/070690274 (cit. on p. 73).

Langlois, Catherine C. and Jean-Pierre P. Langlois (2009). "Does Attrition Behavior Help Explain the Duration of Interstate Wars? A Game Theoretic and Empirical Analysis". *International Studies Quarterly* 53.4, pp. 1051–1073. ISSN: 1468-2478. DOI: 10.1111/j.1468-2478.2009.00568.x. URL: http://dx.doi.org/10.1111/j.1468-2478.2009.00568.x (cit. on p. 9).

Leigh, Andrew, Justin Wolfers, and Eric Zitzewitz (2003). *What do Financial Markets Think of War in Iraq?* Working Paper 9587. NBER. URL: http://www.nber.org/papers/w9587 (cit. on p. 6).

Leventoğlu, Bahar and Branislav L. Slantchev (2007). "The Armed Peace: A Punctuated Equilibrium Theory of War". *American Journal of Political Science* 51.4, pp. 755–771. ISSN: 1540-5907. URL: http://dx.doi.org/10.1111/j.1540-5907.2007.00279.x (cit. on pp. 9, 10, 15).

Lieberman, Robert C. (2002). "Ideas, Institutions, and Political Order: Explaining Political Change". *American Political Science Review* null (04), pp. 697–712. ISSN: 1537-5943. DOI:

10.1017/S0003055402000394. URL: http://journals.cambridge.org/article_S0003055402000394 (cit. on p. 55).

Livermore, Thomas Leonard (1900). *Number and Losses in the Civil War in America, 1861-65*. Houghton, Mifflin and Company. URL: http://books.google.com/books?id=Qw8pAAAAYAAJ (cit. on pp. 33, 37).

Macaulay, F.R. (1938). *Some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in the United States since 1859*. Publications of the National Bureau of Economic Research. National Bureau of Economic Research. URL: http://papers.nber.org/books/maca38-1 (cit. on pp. 19, 42).

Martin, Andrew D. and Kevin M. Quinn (2002). "Dynamic Ideal Point Estimation via Markov Chain Monte Carlo for the U.S. Supreme Court, 1953-1999". *Political Analysis* 10.2, pp. 134–153. DOI: 10.1093/pan/10.2.134. eprint: http://pan.oxfordjournals.org/cgi/reprint/10/2/134.pdf. URL: http://pan.oxfordjournals.org/cgi/content/abstract/10/2/134 (cit. on p. 65).

Martin, Joseph G. (1871). *Seventy-three Years' History of the Boston Stock Market, from Januay 1, 1798, to January 1, 1871*. Martin G. Joseph. URL: http://books.google.com/books?id=g5jyU-B-nikC (cit. on p. 42).

McCandless George T., Jr. (1996). "Money, Expectations, and the U.S. Civil War". *The American Economic Review* 86.3, pp. 661–671. ISSN: 00028282. URL: http://www.jstor.org/stable/2118219 (cit. on pp. 3, 19, 21–23, 41).

McPherson, James M. (2003). *Battle cry of freedom: the Civil War era*. Oxford History of the United States. Oxford University Press. ISBN: 9780195168952. URL: http://books.google.com/books?id=GXfGuNAvm7AC (cit. on pp. 18, 44).

Medbery, James K. (1870). *Men and Mysteries of Wall Street*. URL: http://books.google.com/books?id=mKtAHoe3UfwC (cit. on p. 34).

Mitchell, T. J. and J. J. Beauchamp (1988). "Bayesian Variable Selection in Linear Regression". *Journal of the American Statistical Association* 83.404, pp. 1023–1032. DOI: 10.1080/01621459.1988.10478694. eprint: http://www.tandfonline.com/doi/pdf/10.1080/01621459.1988.10478694. URL: http://www.tandfonline.com/doi/abs/10.1080/01621459.1988.10478694 (cit. on p. 59).

Mitchell, Wesley Clair (1903). *A History of the Greenbacks*. Vol. 9. Decennial Publications of the University of Chicago. Chicago University Press. URL: http://books.google.com/books?id=EnUXAQAAIAAJ (cit. on pp. 3, 12, 19, 22, 23, 36, 41, 42).

— (1908). *Gold, Prices, and Wages Under the Greenback Standard*. The University Press. URL: http://books.google.com/books?id=d3crAAAAYAAJ (cit. on pp. 3, 19, 22, 42).

North, Douglass C. and Barry R. Weingast (1989). "Constitutions and Commitment: The Evolution of Institutions Governing Public Choice in Seventeenth-Century England". *The Journal of Economic History* 49 (04), pp. 803–832. ISSN: 1471-6372. DOI: 10.1017/S0022050700009451. eprint: http://journals.cambridge.org/article_S0022050700009451. URL: http://dx.doi.org/10.1017/S0022050700009451 (cit. on p. 3).

— (2000). "Introduction: Institutional Analysis and Economic History". *The Journal of Economic History* 60.02, pp. 414–417. DOI: 10.1017/S002205070000019X. eprint: http:

//journals.cambridge.org/article_S002205070000019X. URL: http://dx.doi.org/10.1017/S002205070000019X (cit. on pp. 3, 12).

Olshen, Adam B., E. S. Venkatraman, Robert Lucito, and Michael Wigler (2004). "Circular binary segmentation for the analysis of array-based DNA copy number data". *Biostatistics* 5.4, pp. 557–572. DOI: 10.1093/biostatistics/kxh008. eprint: http://biostatistics.oxfordjournals.org/content/5/4/557.full.pdf+html. URL: http://biostatistics.oxfordjournals.org/content/5/4/557.abstract (cit. on p. 58).

Oosterlinck, Kim and Marc D. Weidenmier (2007). *Victory or Repudiation? The Probability of the Southern Confederacy Winning the Civil War*. Working Paper 13567. NBER. URL: http://www.nber.org/papers/w13567.pdf (cit. on p. 23).

Page, E. S. (1954). "Continuous Inspection Schemes". *Biometrika* 41.1/2, pp. 100–115. ISSN: 00063444. URL: http://www.jstor.org/stable/2333009 (cit. on p. 58).

Park, Jong Hee (2010). "Structural Change in U.S. Presidents' Use of Force". *American Journal of Political Science* 54.3, pp. 766–782. ISSN: 1540-5907. DOI: 10.1111/j.1540-5907.2010.00459.x. URL: http://dx.doi.org/10.1111/j.1540-5907.2010.00459.x (cit. on pp. 55, 65).

— (2011). "Changepoint Analysis of Binary and Ordinal Probit Models: An Application to Bank Rate Policy Under the Interwar Gold Standard". *Political Analysis*. DOI: 10.1093/pan/mpr007. eprint: http://pan.oxfordjournals.org/content/early/2011/03/22/pan.mpr007.full.pdf+html. URL: http://pan.oxfordjournals.org/content/early/2011/03/22/pan.mpr007.abstract (cit. on p. 55).

Park, Trevor and George Casella (2008). "The Bayesian Lasso". *Journal of the American Statistical Association* 103.482, pp. 681–686. DOI: 10.1198/016214508000000337. eprint: http://amstat.tandfonline.com/doi/pdf/10.1198/016214508000000337. URL: http://amstat.tandfonline.com/doi/abs/10.1198/016214508000000337 (cit. on pp. 59, 61, 72).

Pas, S. L. van der, B. J. K. Kleijn, and A. W. van der Vaart (2014). "The Horseshoe Estimator: Posterior Concentration around Nearly Black Vectors". *Electronic Journal of Statistics* 8, Number2, 2585–2618. DOI: doi:10.1214/14-EJS962. eprint: 1404.0202. URL: http://projecteuclid.org/euclid.ejs/1418134265 (cit. on pp. 61, 62).

Petris, G., S. Petrone, and P. Campagnoli (2009). *Dynamic Linear Models with R*. Use R! Springer. ISBN: 9780387772370. URL: http://books.google.com/books?id=VCt3zVq8TO8C (cit. on pp. 38, 65).

Phisterer, Frederick (1883). *Statistical Records of the Armies of the United States*. C. Scribner's Sons. URL: http://books.google.com/books?id=cVNHr_nnLlYC (cit. on p. 37).

Pierson, Paul (2004). *Politics in Time: History, Institutions, and Social Analysis*. Princeton University Press. ISBN: 9780691117157. URL: http://books.google.com/books?id=nVtptUoWuO4C (cit. on p. 55).

Pilster, Ulrich and Tobias Böhmelt (2011). "Coup-Proofing and Military Effectiveness in Interstate Wars, 1967–99". *Conflict Management and Peace Science* 28.4, pp. 331–350. DOI: 10.1177/0738894211413062. eprint: http://cmp.sagepub.com/content/28/4/331.full.pdf+html. URL: http://cmp.sagepub.com/content/28/4/331.abstract (cit. on p. 29).

Poast, Paul (2012). “Lincoln’s Gamble: How the Southern Secession Crisis Became the American Civil War” (cit. on pp. 18, 22).

Polson, Nicholas G. and James G. Scott (2010). “Shrink Globally, Act Locally: Sparse Bayesian Regularization and Prediction”. *Bayesian Statistics* (cit. on pp. 55, 59, 62, 63).

— (2012). “On the Half-Cauchy Prior for a Global Scale Parameter”. *Bayesian Analysis* 7 (4), pp. 887–902. DOI: 0.1214/12-BA730 (cit. on pp. 62, 63).

Powell, Robert (2004). “Bargaining and Learning While Fighting”. *American Journal of Political Science* 48.2, pp. 344–361. URL: http://dx.doi.org/10.1111/j.0092-5853.2004.00074.x (cit. on pp. 9, 15, 52).

— (2006). “War as a Commitment Problem”. *International Organization* 60.01, pp. 169–203. DOI: 10.1017/S0020818306060061. eprint: http://journals.cambridge.org/article_S0020818306060061. URL: http://journals.cambridge.org/action/displayAbstract?fromPage=online&aid=365095&fulltextType=RA&fileId=S0020818306060061 (cit. on pp. 9, 10).

Ramsay, Kristopher W. (2008). “Settling It on the Field: Battlefield Events and War Termination”. *Journal of Conflict Resolution* 52.6, pp. 850–879. DOI: 10.1177/0022002708324593. eprint: http://jcr.sagepub.com/content/52/6/850.full.pdf+html. URL: http://jcr.sagepub.com/content/52/6/850.abstract (cit. on pp. 10, 14, 29).

Ratkovic, Marc T. and Kevin H. Eng (2010). “Finding Jumps in Otherwise Smooth Curves: Identifying Critical Events in Political Processes”. *Political Analysis* 18.1, pp. 57–77. DOI: 10.1093/pan/mpp032. eprint: http://pan.oxfordjournals.org/content/18/1/57.full.pdf+html. URL: http://pan.oxfordjournals.org/content/18/1/57.abstract (cit. on p. 77).

Reiter, Dan (2003). “Exploring the Bargaining Model of War”. *Perspectives on Politics* 1.01, pp. 27–43. DOI: 10.1017.S1537592703000033. eprint: http://journals.cambridge.org/article_S1537592703000033. URL: http://journals.cambridge.org/action/displayAbstract?fromPage=online&aid=145049&fulltextType=RA&fileId=S1537592703000033 (cit. on pp. 1, 9–11, 14–16, 29).

— (2009). *How wars end*. Princeton University Press. ISBN: 9780691140605. URL: http://books.google.com/books?id=-_Avp1TNYjMC (cit. on pp. 1, 9–14, 16, 18, 29, 45, 46).

Reiter, Dan and Alan C. Stam III (1998). “Democracy, War Initiation, and Victory”. *The American Political Science Review* 92.2, pp. 377–389. ISSN: 00030554. URL: http://www.jstor.org/stable/2585670 (cit. on p. 29).

— (2002). *Democracies at War*. Pinceton University Press. URL: http://books.google.com/books?id=8yzs3V8EXXwC (cit. on p. 29).

Rhode, Paul W. and Koleman S. Strumpf (2004). “Historical Presidential Betting Markets”. *The Journal of Economic Perspectives* 18.2, pp. 127–141. ISSN: 08953309. URL: http://www.jstor.org/stable/3216894 (cit. on p. 6).

Roll, Richard (1972). “Interest Rates and Price Expectations During the Civil War”. *The Journal of Economic History* 32.2, pp. 476–498. ISSN: 00220507. URL: http://www.jstor.org/stable/2116826 (cit. on pp. 3, 19, 20, 23, 26).

Schneider, Gerald and Vera E. Troeger (2006). “War and the World Economy: Stock Market Reactions to International Conflicts”. *Journal of Conflict Resolution* 50.5, pp. 623–645.

DOI: 10.1177/0022002706290430. eprint: http://jcr.sagepub.com/content/50/5/623.full.pdf+html. URL: http://jcr.sagepub.com/content/50/5/623.abstract (cit. on pp. 4, 7).

Schwab, John Christopher (1901). *The Confederate States of America, 1861-1865: a financial and industrial history of the South during the Civil War*. Yale bicentennial publications. C. Scribner's Sons. URL: http://books.google.com/books?id=4TEOAAAAIAAJ (cit. on pp. 3, 23).

Service, National Park (2012). *Civil War Sites Advistory Commission Report Update & Resurvey*. URL: http://www.nps.gov/hps/abpp/CWSII/CWSII.htm (cit. on p. 37).

Shirkey, Zachary C. (2009). *Is This a Private Fight or Can Anybody Join?* Farnham, Surrey, GBR: Ashgate Publishing Group. URL: http://site.ebrary.com/lib/uwash/docDetail.action?docID=10325898 (cit. on pp. 2, 11).

Shumway, R.H. and D.S. Stoffer (2010). *Time Series Analysis and Its Applications*. Springer Texts in Statistics. Springer. ISBN: 9781441978653. URL: http://books.google.com/books?id=NIhXa6UeF2cC (cit. on p. 66).

Slantchev, Branislav L. (2003). "The Principle of Convergence in Wartime Negotiations". *American Political Science Review* 97.04, pp. 621–632. DOI: http://dx.doi.org/10.1017/S0003055403000911. URL: http://dx.doi.org/10.1017/S0003055403000911 (cit. on pp. 9, 14, 15, 52).

— (2012). "Borrowed Power: Debt Finance and the Resort to Arms". *American Political Science Review* 106 (04), pp. 787–809. ISSN: 1537-5943. DOI: 10.1017/S0003055412000378. URL: http://journals.cambridge.org/article_S0003055412000378 (cit. on pp. 4, 7).

Smith, Alastair and Allan C. Stam (2004). "Bargaining and the Nature of War". *The Journal of Conflict Resolution* 48.6, pp. 783–813. ISSN: 00220027. URL: http://www.jstor.org/stable/4149796 (cit. on pp. 9, 15).

Smith, Gregor W. and R. Todd Smith (1997). "Greenback-Gold Returns and Expectations of Resumption, 1862–1879". *The Journal of Economic History* 57.03, pp. 697–717. DOI: 10.1017/S0022050700019100. eprint: http://journals.cambridge.org/article_S0022050700019100. URL: http://journals.cambridge.org/action/displayAbstract?fromPage=online&aid=4125280&fulltextType=RA&fileId=S0022050700019100 (cit. on pp. 3, 21, 23, 41).

Spirling, Arthur (2007a). "Bayesian Approaches for Limited Dependent Variable Change Point Problems". *Political Analysis* 15.4, pp. 387–405. DOI: 10.1093/pan/mpm022. eprint: http://pan.oxfordjournals.org/content/15/4/387.full.pdf+html. URL: http://pan.oxfordjournals.org/content/15/4/387.abstract (cit. on p. 55).

— (2007b). """Turning Points" in the Iraq Conflict". *The American Statistician* 61.4, pp. 315–320. DOI: 10.1198/000313007X247076. eprint: http://pubs.amstat.org/doi/pdf/10.1198/000313007X247076. URL: http://pubs.amstat.org/doi/abs/10.1198/000313007X247076 (cit. on pp. 55, 65).

Staff of the Civil War Sites Advisory Commission (1993a). *Civil War Sites Advisory Commission Report on the Nation's Civil War Battlefields*. National Parks Services. URL: http://www.nps.gov/hps/abpp/cwsac/cws0-1.html (cit. on pp. 32, 33, 37).

— (1993b). *Civil War Sites Advisory Commission Report on the Nation's Civil War Battlefields: Technical Volume II: Battle Summaries.* National Parks Services. URL: http://www.nps.gov/hps/abpp/battles/tvii.htm (cit. on pp. 32, 33).

Stan Development Team (2015). *Stan Modeling Language Users Guide and Reference Manual, Version 2.7.0.* URL: http://mc-stan.org/ (cit. on pp. 41, 67, 68).

Sussman, Nathan and Yishay Yafeh (2000). "Institutions, Reforms, and Country Risk: Lessons from Japanese Government Debt in the Meiji Era". *The Journal of Economic History* 60.02, pp. 442–467. DOI: 10.1017/S0022050700000218. eprint: http://journals.cambridge.org/article_S0022050700000218. URL: http://dx.doi.org/10.1017/S0022050700000218 (cit. on p. 4).

The Bankers Magazine (1862). Vol. 16. Warren, Gorham & Lamont, Incorporated. URL: https://books.google.com/books?id=B10mAQAAIAAJ (cit. on p. 25).

The Bankers Magazine (1864). Vol. 18. Warren, Gorham & Lamont, Incorporated. URL: http://books.google.com/books?id=D14mAQAAIAAJ (cit. on p. 23).

Tibshirani, Robert (1996). "Regression Shrinkage and Selection via the Lasso". *Journal of the Royal Statistical Society. Series B (Methodological)* 58.1, pp. 267–288. ISSN: 00359246. URL: http://www.jstor.org/stable/2346178 (cit. on pp. 55, 62).

Tibshirani, Robert, Michael Saunders, Saharon Rosset, Ji Zhu, and Keith Knight (2005). "Sparsity and Smoothness via the Fused Lasso". *Journal of the Royal Statistical Society. Series B (Statistical Methodology)* 67.1, pp. 91–108. ISSN: 13697412. URL: http://www.jstor.org/stable/3647602 (cit. on p. 62).

Tibshirani, Ryan J. (2014). "Adaptive piecewise polynomial estimation via trend filtering". *The Annals of Statistics* 42.1. arXiv: 1304.2986, pp. 285–323. ISSN: 0090-5364. DOI: 10.1214/13-AOS1189. URL: http://arxiv.org/abs/1304.2986 (visited on 07/07/2014) (cit. on p. 73).

Tipping, Michael E. (2001). "Sparse bayesian learning and the relevance vector machine". *J. Mach. Learn. Res.* 1, pp. 211–244. ISSN: 1532-4435. DOI: 10.1162/15324430152748236. URL: http://dx.doi.org/10.1162/15324430152748236 (cit. on pp. 59, 60).

United States. Dept. of the Treasury (1861a). *Annual Report of the Secretary of the Treasury on the State of the Finances, December 9, 1861.* URL: http://fraser.stlouisfed.org/publication/?pid=194 (cit. on p. 22).

— (1861b). *Annual Report of the Secretary of the Treasury on the State of the Finances, July 4, 1861.* URL: http://fraser.stlouisfed.org/publication/?pid=194 (cit. on p. 22).

— (1862). *Annual Report of the Secretary of the Treasury on the State of the Finances, Fiscal Year Ending June 30, 1862.* URL: http://fraser.stlouisfed.org/publication/?pid=194 (cit. on p. 22).

— (1863). *Annual Report of the Secretary of the Treasury on the State of the Finances, Fiscal Year Ending June 30, 1863.* URL: http://fraser.stlouisfed.org/publication/?pid=194 (cit. on pp. 19, 22, 24, 27).

— (1864). *Annual Report of the Secretary of the Treasury on the State of the Finances, Fiscal Year Ending June 30, 1864.* URL: http://fraser.stlouisfed.org/publication/?pid=194 (cit. on pp. 22, 25).

United States. Dept. of the Treasury (1865). *Annual Report of the Secretary of the Treasury on the State of the Finances, Fiscal Year Ending June 30, 1865*. URL: http://fraser.stlouisfed.org/publication/?pid=194 (cit. on pp. 22, 25, 28).

— (1866). *Annual Report of the Secretary of the Treasury on the State of the Finances, Fiscal Year Ending June 30, 1866*. URL: http://fraser.stlouisfed.org/publication/?pid=194 (cit. on p. 25).

United States. Dept. of the Treasury and Rafael A. Bayley (1882). *The national loans of the United States, from July 4, 1776, to June 30, 1880*. Govt. Print. Off. URL: http://books.google.com/books?id=Ce4JAAAAIAAJ (cit. on pp. 19, 21, 26).

United States, Register of the Treasury and William F. De Knight (1900). *History of the currency of the country and of the loans of the United States from the earliest period to June 30, 1896*. Document (United States. Dept. of the Treasury) 1943. Government Printing Office. URL: http://books.google.com/books?id=0cQmAQAAMAAJ (cit. on pp. 19, 21).

United States War Department (1880-1901). *The War of the Rebellion: a Compilation of the Official Records of the Union and Confederate Armies*. Washington, D.C.: Government Printing Office. URL: http://ebooks.library.cornell.edu/m/moawar/waro.html (cit. on p. 17).

U.S. Army Concepts Analysis Agency (1991). *Database of Battles-Version 1990 (Computer Diskette)*. URL: http://www.dtic.mil/docs/citations/ADM000121 (cit. on pp. 11, 33).

van Dyk, David A and Taeyoung Park (2008). "Partially Collapsed Gibbs Samplers". *Journal of the American Statistical Association* 103.482, pp. 790–796. DOI: 10.1198/016214508000000409. eprint: http://dx.doi.org/10.1198/016214508000000409. URL: http://dx.doi.org/10.1198/016214508000000409 (cit. on pp. 41, 68).

Vehtari, Aki, Andrew Gelman, and Jonah Gabry (2015). *loo: Efficient leave-one-out cross-validation and WAIC for Bayesian models*. R package version 0.1. URL: https://github.com/jgabry/loo (cit. on p. 72).

Wagner, R. Harrison (2000). "Bargaining and War". *American Journal of Political Science* 44.3, pp. 469–484. ISSN: 00925853. URL: http://www.jstor.org/stable/2669259 (cit. on p. 15).

Walker, George (1865). *The Wealth, Resources, and Public Debt of the United States*. E. Couchman. URL: https://books.google.com/books?id=rAgZAAAAYAAJ (cit. on p. 28).

Walter, Barbara F. (2009). "Bargaining Failures and Civil War". *Annual Review of Political Science* 12.1, pp. 243–261. DOI: 10.1146/annurev.polisci.10.101405.135301. eprint: http://www.annualreviews.org/doi/pdf/10.1146/annurev.polisci.10.101405.135301. URL: http://www.annualreviews.org/doi/abs/10.1146/annurev.polisci.10.101405.135301 (cit. on p. 16).

Warren, George F. and Frank A. Pearson (1933). *Prices*. New York: John Wiley & Sons (cit. on p. 42).

Watanabe, Sumio (2010). "Asymptotic Equivalence of Bayes Cross Validation and Widely Applicable Information Criterion in Singular Learning Theory". *Journal of Machine

Learning Research 11, pp. 3571–3594. URL: http://www.jmlr.org/papers/v11/watanabe10a.html (cit. on p. 46).

Weidenmier, Marc D. (2000). "The Market for Confederate Cotton Bonds". *Explorations in Economic History* 37.1, pp. 76–97. ISSN: 0014-4983. DOI: DOI:10.1006/exeh.2000.0732. URL: http://www.sciencedirect.com/science/article/B6WFJ-45F4HNG-7/2/485cb493c030ac09196cfdf96b51d150 (cit. on p. 3).

— (2002). "Turning Points in the U. S. Civil War: Views from the Grayback Market". *Southern Economic Journal* 68.4, pp. 875–890. ISSN: 00384038. URL: http://www.jstor.org/stable/1061497 (cit. on pp. 3, 23).

Weingast, Barry R. (1998). "Political Stability and Civil War: Instiutions, Commitment, and American Democracy". In: ed. by Robert H. Bates, Avner Greif, Margaret Levi, Jean-Laurent Rosenthal, and Barry R. Weingast. Princeton University Press. Chap. 4 (cit. on p. 26).

Weisiger, Alex (2015). "Learning from the Battlefield: Information, Domestic Politics, and Interstate War Duration". URL: http://www.polisci.upenn.edu/~weisiger/learning_battlefield_012015.pdf (cit. on pp. 10, 11, 14, 29).

Weiss, Herbert K. (1966). "Combat Models and Historical Data: The U.S. Civil War". *Operations Research* 14.5, pp. 759–790. ISSN: 0030364X. URL: http://www.jstor.org/stable/168777 (cit. on pp. 17, 18).

Wells, John and Douglas Wills (2000). "Revolution, Restoration, and Debt Repudiation: The Jacobite Threat to England's Institutions and Economic Growth". *The Journal of Economic History* 60.02, pp. 418–441. DOI: 10.1017/S0022050700000206. eprint: http://journals.cambridge.org/article_S0022050700000206. URL: http://dx.doi.org/10.1017/S0022050700000206 (cit. on p. 4).

West, M. and J. Harrison (1997). *Bayesian forecasting and dynamic models.* Springer series in statistics. Springer. ISBN: 9780387947259. URL: http://books.google.com/books?id=jcl8lD75fkYC (cit. on pp. 57, 65, 67, 70, 76).

Western, Bruce and Meredith Kleykamp (2004). "A Bayesian Change Point Model for Historical Time Series Analysis". *Political Analysis* 12.4, pp. 354–374. DOI: 10.1093/pan/mph023. eprint: http://pan.oxfordjournals.org/content/12/4/354.full.pdf+html. URL: http://pan.oxfordjournals.org/content/12/4/354.abstract (cit. on pp. 55, 65).

Willard, Kristen L., Timothy W. Guinnane, and Harvey S. Rosen (1996). "Turning Points in the Civil War: Views from the Greenback Market". *The American Economic Review* 86.4, pp. 1001–1018. ISSN: 00028282. URL: http://www.jstor.org/stable/2118316 (cit. on pp. 3, 12, 19, 21–23, 34).

Wolfers, Justin and Eric Zitzewitz (2004). "Prediction Markets". *Journal of Economic Perspectives* 18.2, pp. 107–126. DOI: 10.1257/0895330041371321. URL: http://www.aeaweb.org/articles_detail.php?doi=10.1257/0895330041371321 (cit. on pp. 4, 6).

— (2009). "Using Markets to Inform Policy: The Case of the Iraq War". *Economica* 76.302, pp. 225–250. DOI: 10.1111/j.1468-0335.2008.00750.x. URL: http://dx.doi.org/10.1111/j.1468-0335.2008.00750.x (cit. on pp. 4, 7, 8).

Wolford, Scott, Dan Reiter, and Clifford J. Carrubba (2011). “Information, Commitment, and War”. *Journal of Conflict Resolution* 55.4, pp. 556–579. DOI: 10.1177/0022002710393921. eprint: http://jcr.sagepub.com/content/55/4/556.full.pdf+html. URL: http://jcr.sagepub.com/content/55/4/556.abstract (cit. on p. 9).

Yao, Yi-Ching (1984). “Estimation of a Noisy Discrete-Time Step Function: Bayes and Empirical Bayes Approaches”. *The Annals of Statistics* 12.4, pp. 1434–1447. ISSN: 00905364. URL: http://www.jstor.org/stable/2241012 (cit. on p. 58).