

Introduction to Astrometry

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What is astrometry?

- Astrometry is the measure of the position of celestial bodies.
- Provides description of the motions and orbits of objects, which can be used to study their properties and the forces that govern them.

Domains for astrometry

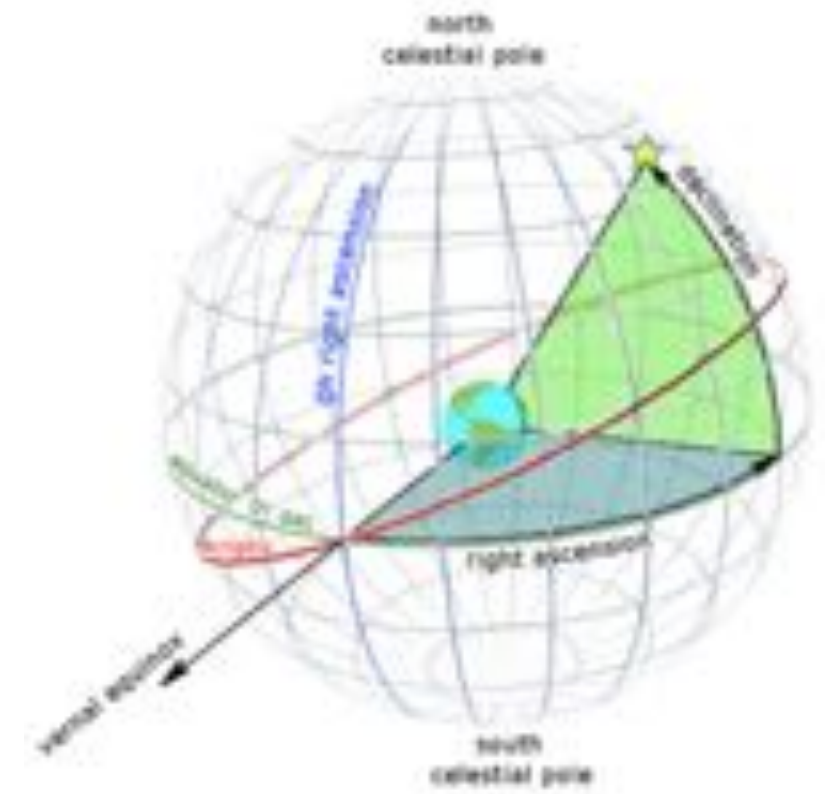
- **Extragalactic:** measuring distribution and motion of galaxies gives insight into the early Universe.
- **Galactic:** *Parallax*: determine distances to stars. Can be used to measure absolute luminosities, actual diameters of stars, actual dimensions of orbits, binary pulsars: general relativity, proper motions: unseen companions. Kinematics of stellar groups. References for photometry.
- **Solar system:** the Sun, major planets, planetary satellites, asteroids and comets.
- **Earth-Moon System:** monitoring artificial satellites from the ground: gravitational field of the Earth, rotation of the Earth, plate tectonics, synchronisation of clocks, trajectories of satellite missions.

Coordinates in the sky: terminology

- **Right Ascension** (RA) measured in hours, minutes, and seconds. By definition RA of Sun is 0 on the vernal equinox, March 21. There are 24h of RA around the celestial equator.
 - **Declination** (Dec) measured in degrees, arcminutes (′), and arcseconds (″). Range between -90° and $+90^\circ$ where 0° is at the celestial equator; directly overhead the equator of the Earth.
 - **Hour Angle** (HA) the angle between the observer's meridian (circle of constant longitude) and the meridian of RA which passes through the star. Runs from -12 to 12h.
 - **Local Sidereal time** (LST) hour angle of the Vernal Equinox.
- **Astrometry Data Exchange Standard (ADES):** new IAU-approved format for submitting and distributing astrometry of solar-system bodies. With ADES, RA and DEC are measured in decimal degrees.

Coordinates in the sky: Exercises

- Right Ascension (RA) measured in hours, minutes, and seconds. By definition RA of Sun is 0 on the vernal equinox, March 21. There are 24h of RA around the celestial equator.
 - Declination (Dec) measured in degrees, arcminutes ($'$), and arcseconds ($''$). Range between -90° and $+90^\circ$ where 0° is at the celestial equator; directly overhead the equator of the Earth.
 - Hour Angle (HA) the angle between the observer's meridian (circle of constant longitude) and the meridian of RA which passes through the star. Runs from -12 to 12h.
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- Ex 1.: You want to find a star to observe close to Zenith. What should the RA and Dec be?
A: $RA=LST$, $Dec=Latitude$
 - Ex 2.: You want to move your telescope towards the West. Do you increase or decrease the RA?
A: Decrease



Precession

- The Earth's spin axis is not fixed in space, but precesses slowly due to the gravitational pulls of the Sun and Moon with a period of 25,770 years. It means that the RA and Dec of stars are slowly changing in time.
- Star catalogues need to specify the equinox (reference date) when listing the RA and Dec of a star. Approximate expressions for the changes in the RA (α) and Dec (δ) since equinox 2000.0 are

$$\Delta\alpha = M + N \sin\alpha \tan\delta$$
$$\Delta\delta = N \cos\alpha$$

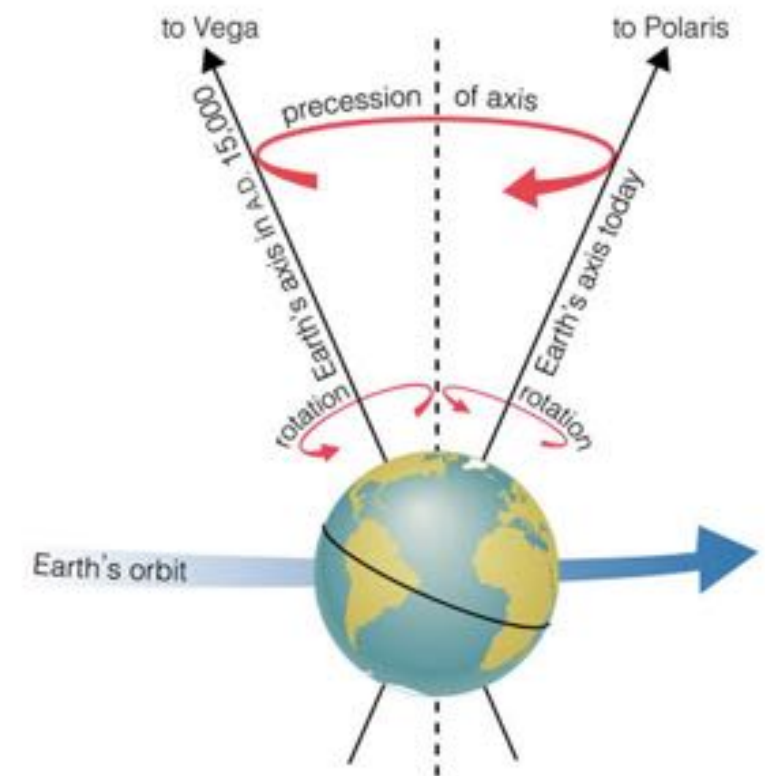
where M and N are given by

$$M = 1.2812323^\circ T + 0.0003879^\circ T^2 + 0.0000101^\circ T^3$$

$$N = 0.5567530^\circ T - 0.0001185^\circ T^2 + 0.0000116^\circ T^3$$

T is defined as $T = (t - 2000.0)/100$

and t is the current date in fractions of a year.



International Celestial Reference System

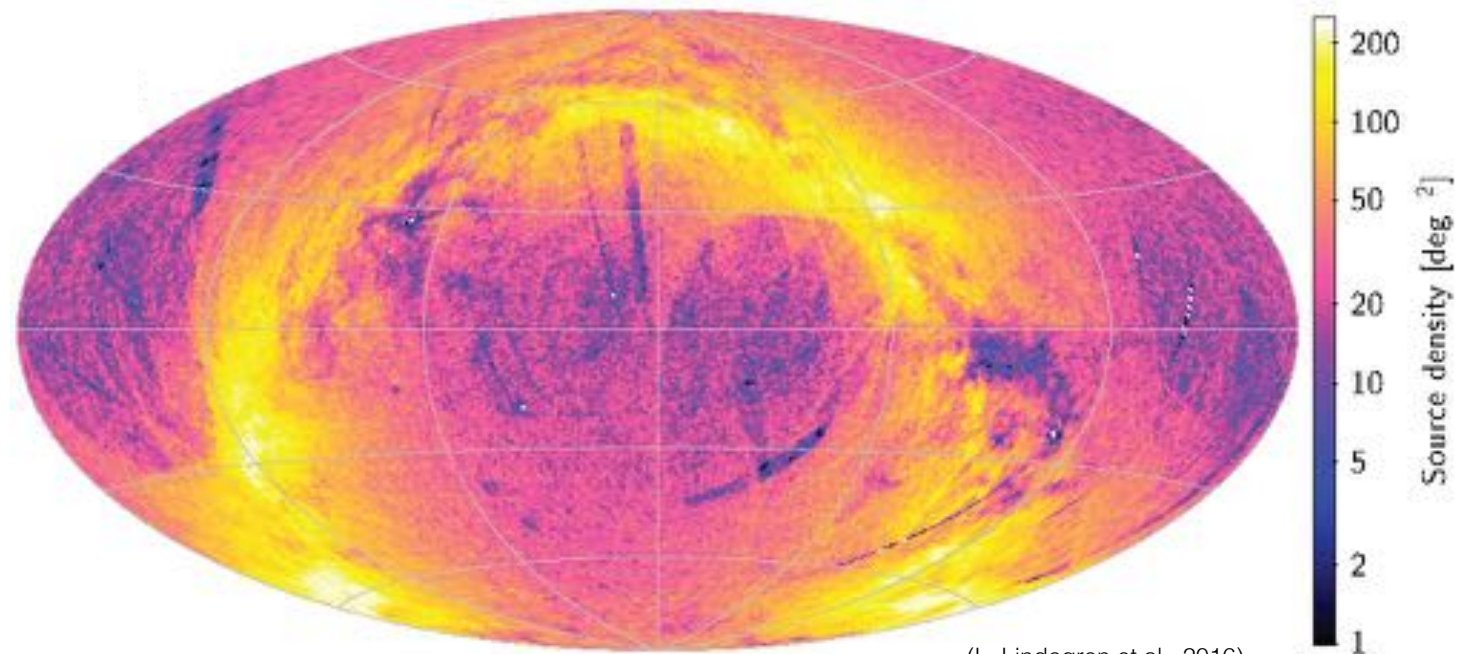
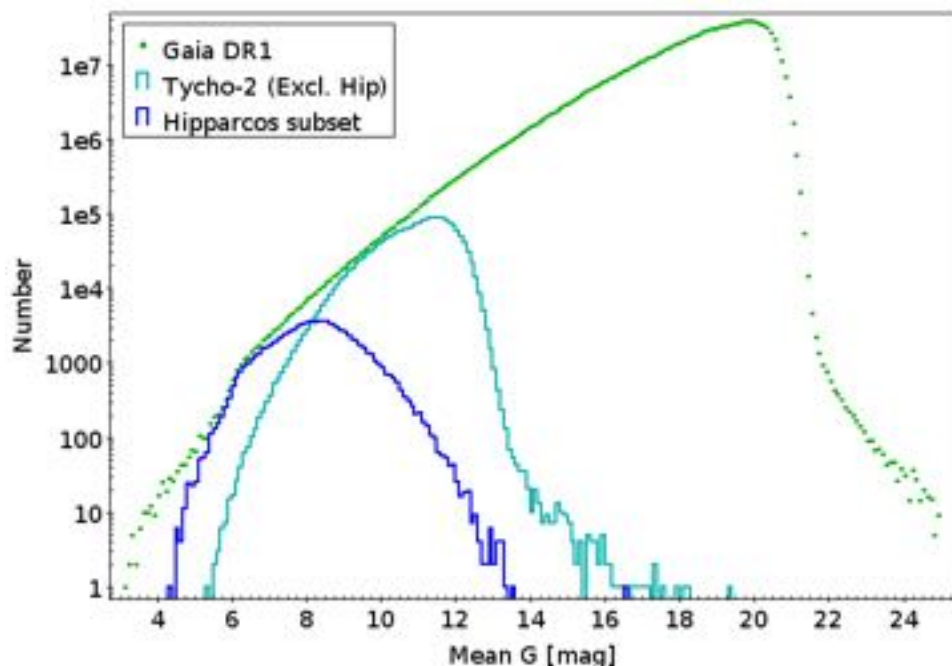
- Equatorial coordinates are defined by the rotation axis of the Earth: need to know where the North Celestial Pole actually is at any given time.
- International Celestial Reference System (ICRS), and is defined by the positions of a specific set of extragalactic objects, which are assumed to have no proper motions. Modern positions are usually quoted in the ICRS system, but the ICRS axes are consistent to better than 0.1" with the 2000.0 equinox.

Proper motions

- A star's motion through space is called its **proper motion**.
- For a star with significant proper motion we must specify, in addition to the RA, Dec and equinox, the star's proper motion, and when the position was measured (epoch). If no epoch is given, it is assumed to be the same as the equinox for an entry.

Astrometric catalogues

- Tycho-2: HIPPARCOS (ESA) since 1991 epoch errors in proper motions are accumulating.
- USNO-B1.0: Schmidt plates
- 2MASS: infra-red
- Large number of fainter stars: UCAC4, PPMX and PPMXL, SDSS.
- Gaia DR1: 2016. No proper motions until DR2; TGAS: Tycho-Gaia Astrometric Solution



(L. Lindegren et al., 2016)

- Catalogue limitations

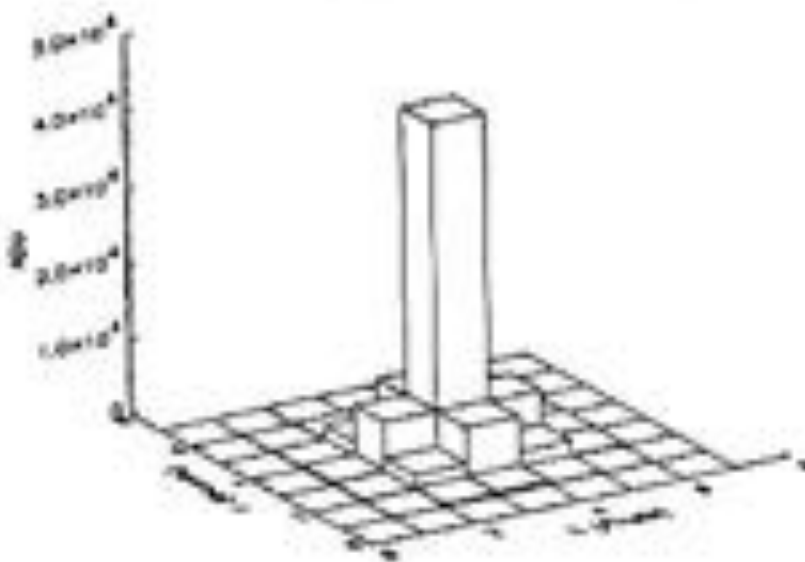
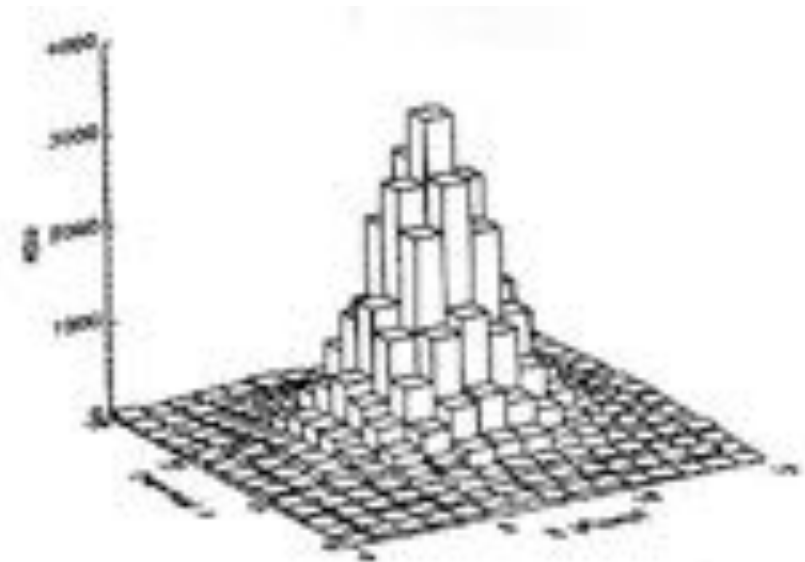
- Typically, every catalog contains systematic errors, that is, errors in position that are similar in direction and magnitude for objects that are in the same area of the sky, or are of the same magnitude or colour. Systematic errors mean that the reference frame is warped, or is effectively different for different classes of objects.
- Caused by: sky coverage, random errors, absence of proper motions etc.
- For older catalogs like USNO-B1.0 and PPMXL, random error of 0.15 arcsec is typical after outlier rejection, and systematic errors can be even larger than that.

Observation: Pixel Sampling

- Choosing the correct binning is an important consideration. Higher binning enables a faster readout time. *But*, need to ensure PSFs are well sampled.

sampling parameter, r : **$r = \text{FWHM} / \text{pixel size}$**

- Undersampled when $r < 1.5$. Undersampled PSFs are not well represented by a Gaussian function.
- Nyquist sampling: optimal sampling when FWHM is $\sim 2\times$ pixel size. More rigorously: for a Gaussian PSF, optimum $\text{FWHM} = 2.355$ pixels.
- Ex.: For a $0.0822''$ pixel size, and best seeing of $0.8''$, what is the optimum binning?
A: $0.8 / (2.355 * (0.0822)) \rightarrow 4 \times 4$
- Trade-off between object detection (put as much light into one pixel as possible) and astrometric accuracy (critical sampling). From experience, optimum pixel size = seeing / 1.6

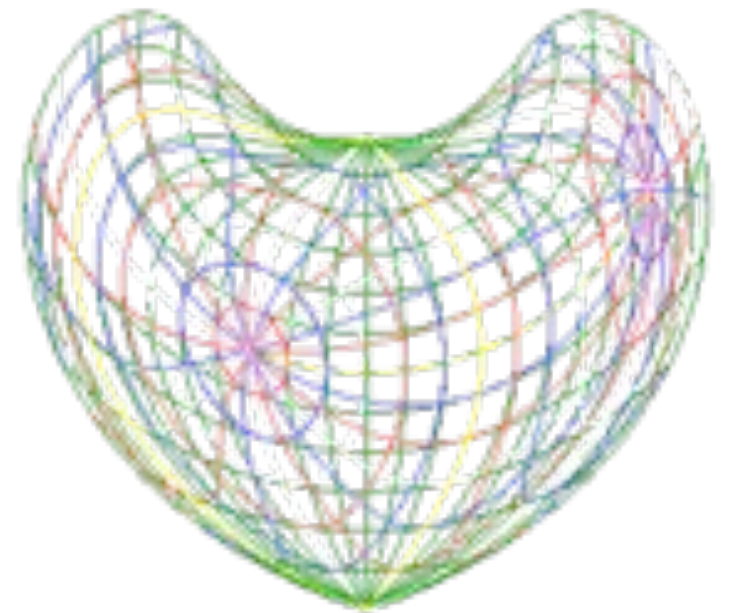


Flexible Image Transport System

- Flexible Image Transport System (FITS) is the most commonly used digital file format in astronomy. FITS is designed specifically for scientific data and includes many provisions for describing photometric and spatial calibration information, together with image origin metadata.
- FITS is also often used to store non-image data, such as spectra, photon lists, data cubes, or even structured data such as multi-table databases. A FITS file may contain several extensions, and each of these may contain a data object.
- FITS also supports tabular data with named columns and multidimensional rows.

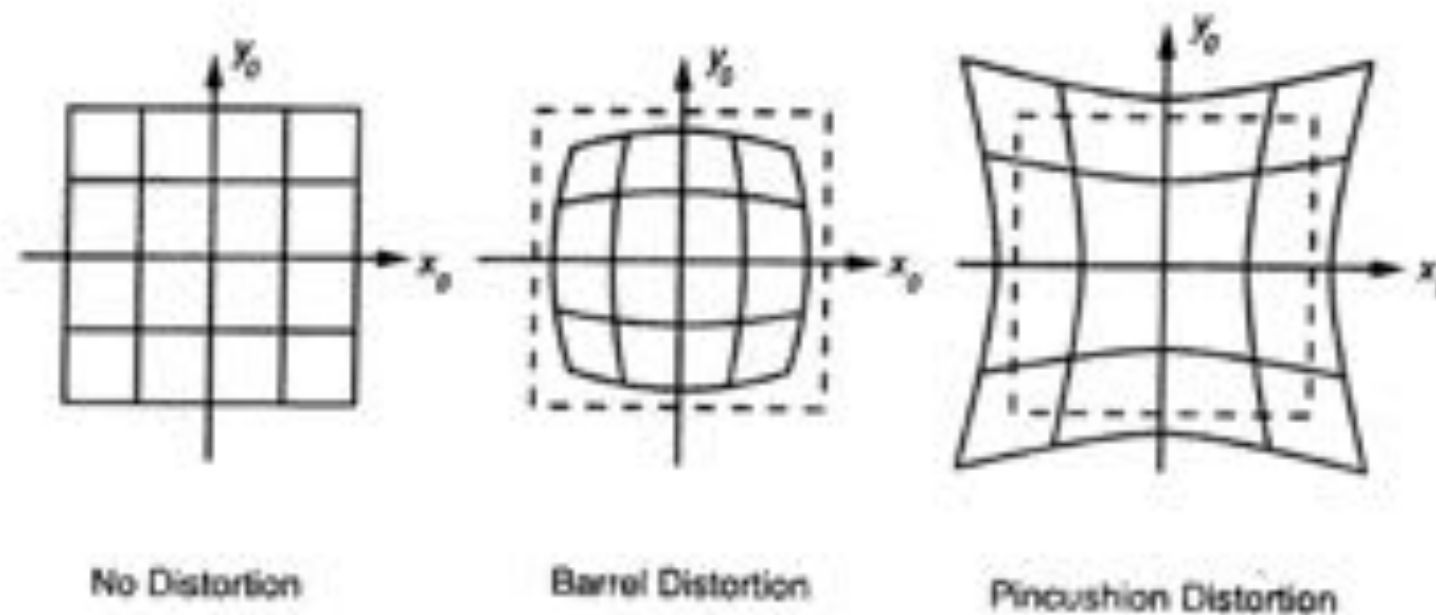
WCS

- World Coordinate Systems (WCS) are any coordinate systems that describe the physical coordinates associated with a data array, such as sky coordinates (RA and Dec, galactic latitude and longitude, etc.) for an astronomical image.
- The FITS "World Coordinate System" (WCS) standard defines keywords and usage that provide for the description of astronomical coordinate systems in a FITS image header.



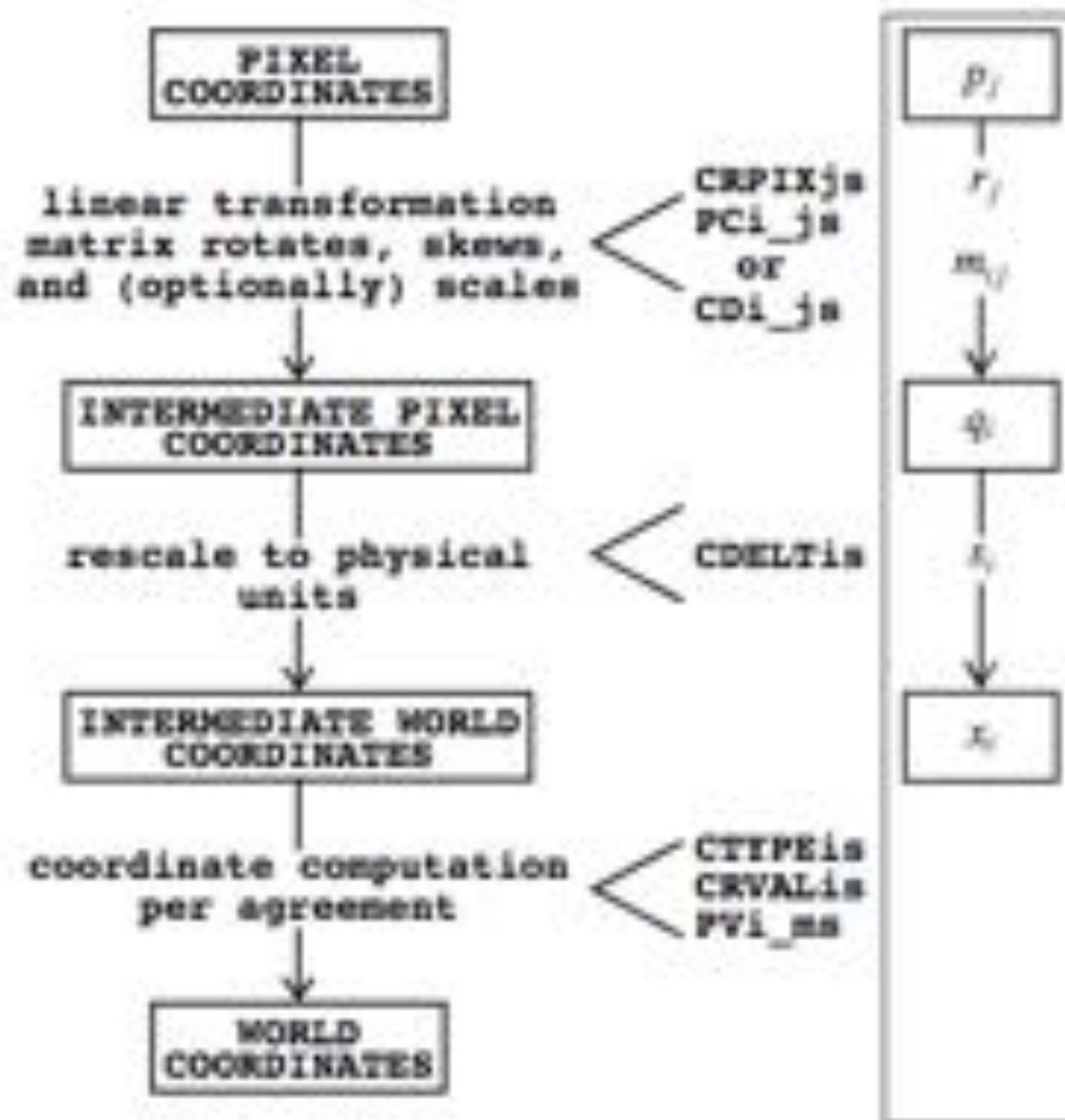
Distortion

- Distortions are caused by imperfect optics e.g. when the entrance pupil of the telescope has field curvature.
- Distortions can be described by high-order polynomials or spline functions with many empirically-derived coefficients.
- For any given instrument, these coefficients can be stored in the FITS headers to be applied by WCS interpreter.



Astrometric Calibration:

Converting pixels to sky coordinates

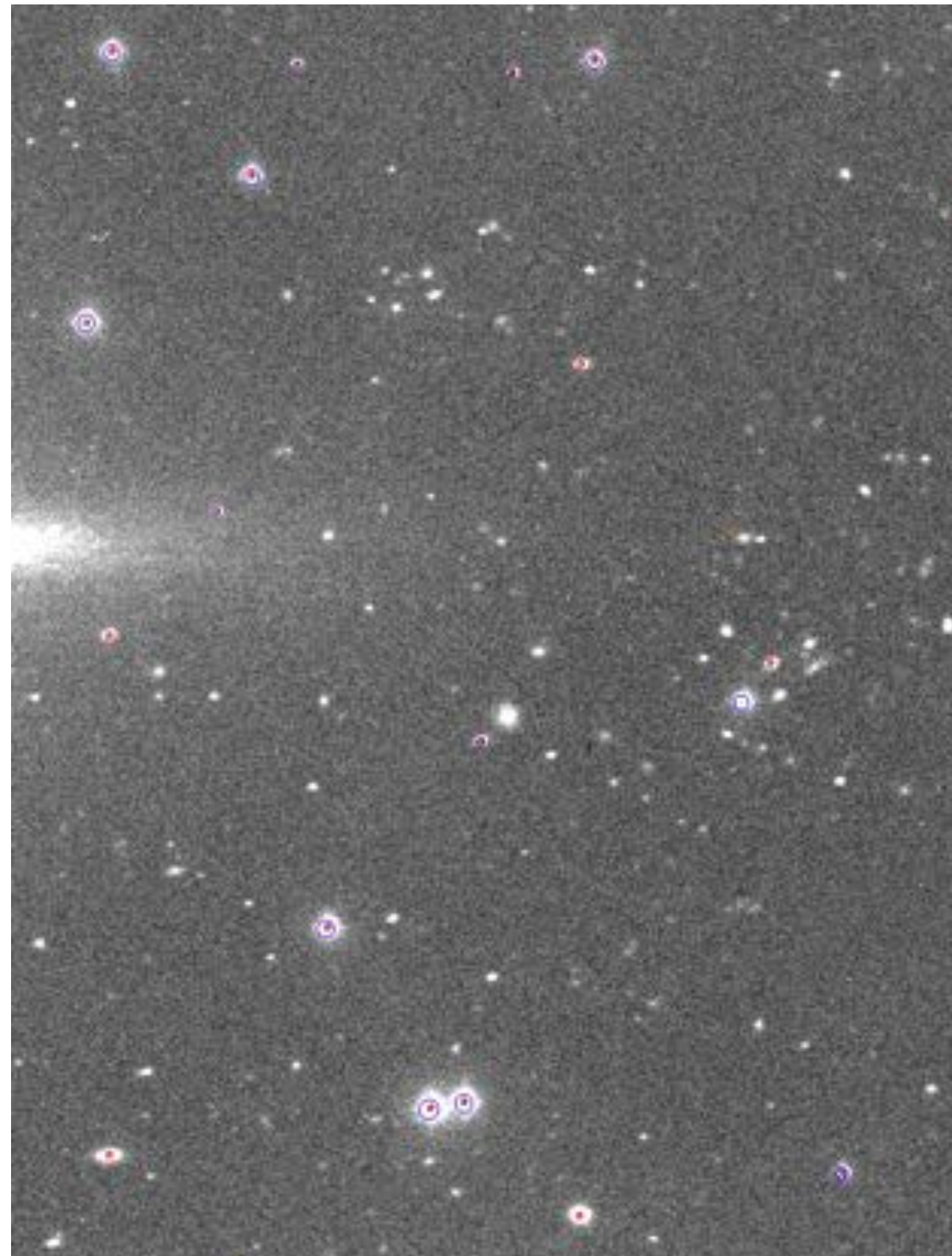


Astrometry Procedure

1. Bias, flat-field correction
2. Catalogue matching to convert pixels to WCS
3. Identify target pixels
4. Centroiding
5. Finding optimum aperture size & estimate uncertainties

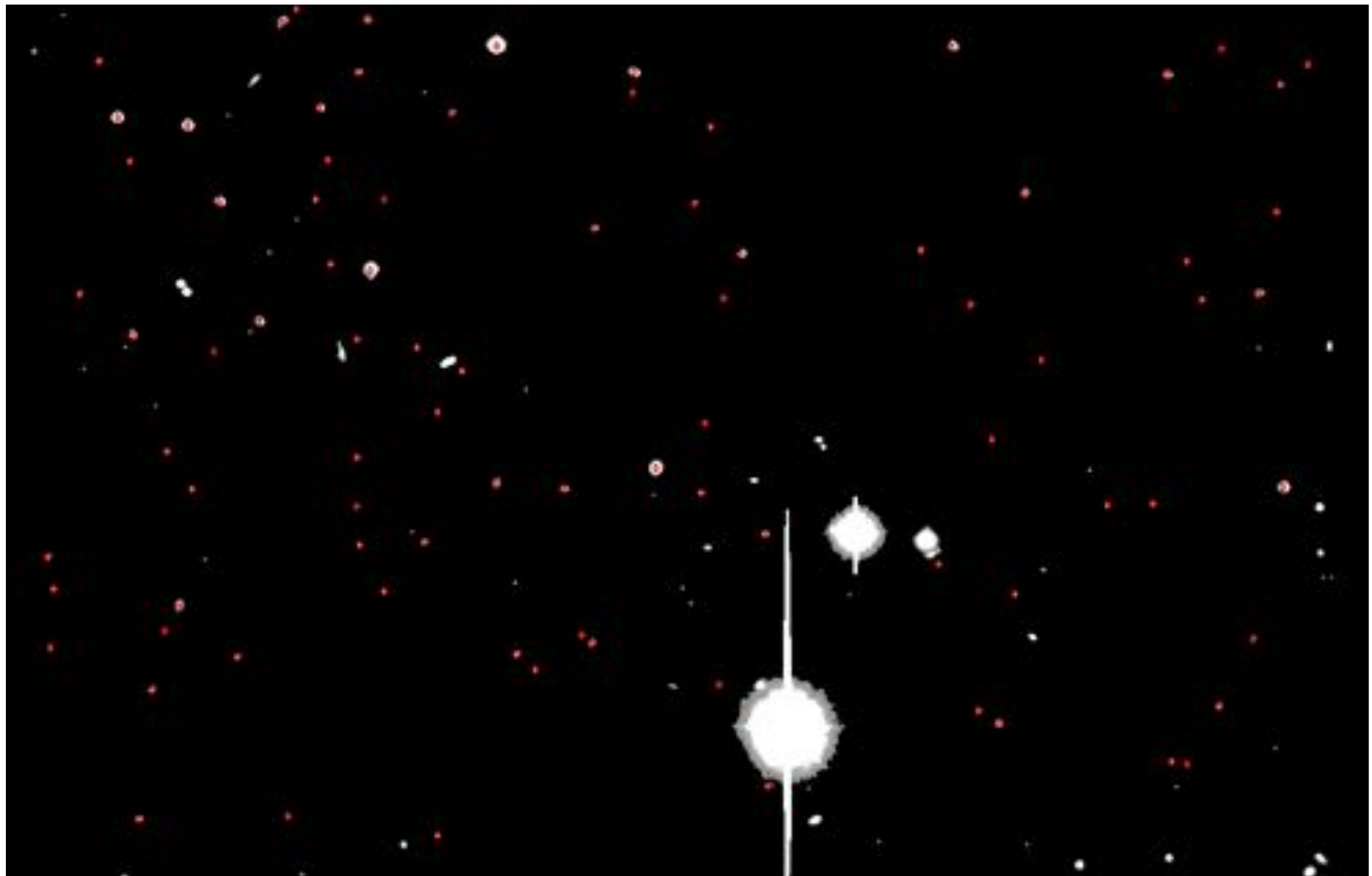
Catalogue Matching

- Use telescope pointing or WCS to determine starting region for matching
- Identify stars in image
- Match image centres with catalogue centres using correlations.
- Apply correction to target position.



Star Finding

- E.g. by locating maxima on rescaled image



Uncertainties:

Reduced Chi-squared fitting

- Fit 2D Gaussian model to PSF and calculate reduced Chi-square statistic.

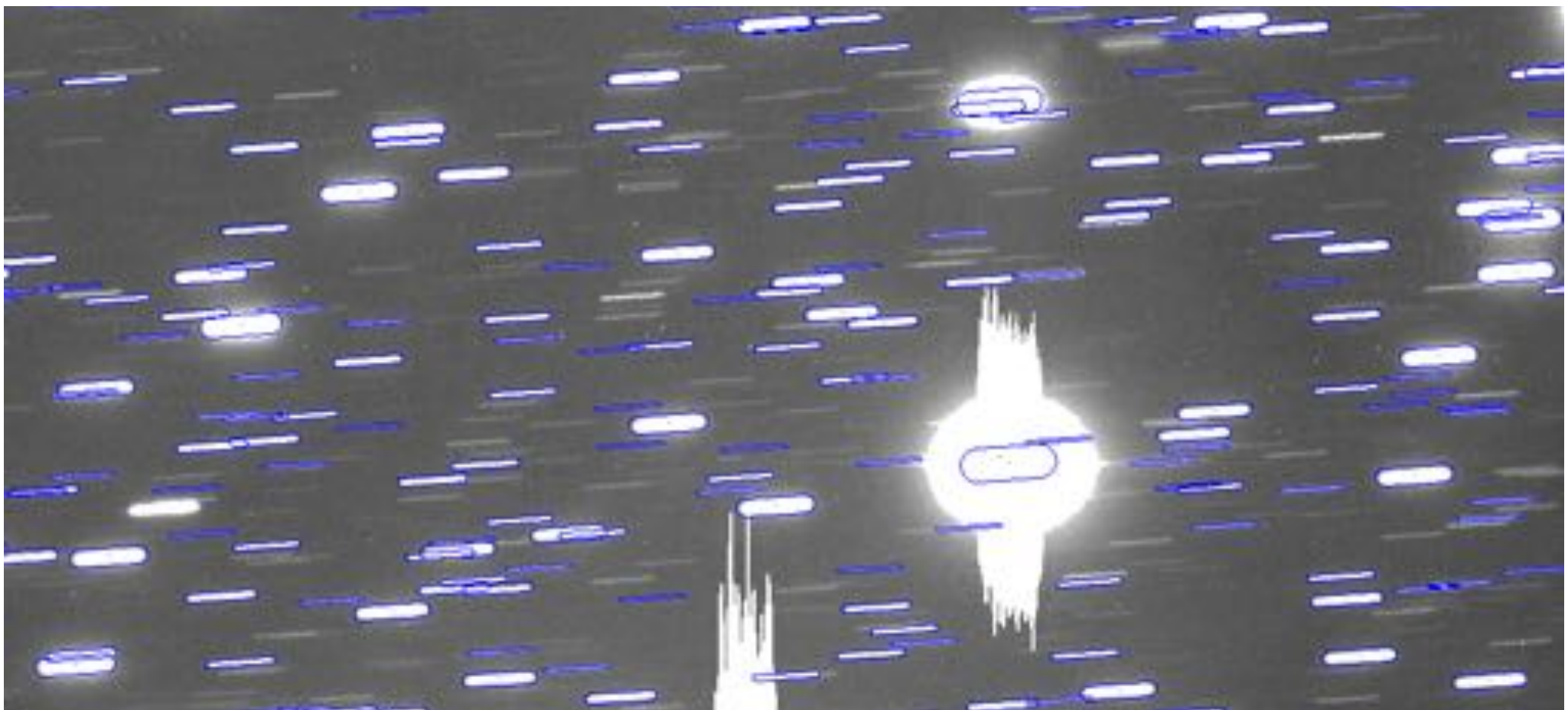
$$\chi^2 = \sum_{i=1}^n \left(\frac{z_i - f(x_i, y_i)}{\sigma_i} \right)^2$$

$$\chi_{\text{red}}^2 = \frac{\chi^2}{N_{\text{DoF}}}$$

- If χ_{red}^2 is larger than one, it is considered a “bad” fit, whereas if $\chi_{\text{red}}^2 < 1$, it is considered an overfit.
- Minimise χ_{red}^2 and rescale the errors such that the value of χ_{red}^2 is exactly equal to one.

Aside: Trailed Images

- Trailed images can consist of a Gaussian PSF convolved with a boxcar function.
- A trailed catalog must be constructed by shifting the catalog positions.
- Matching is done via fitting to the trailed PSF model.

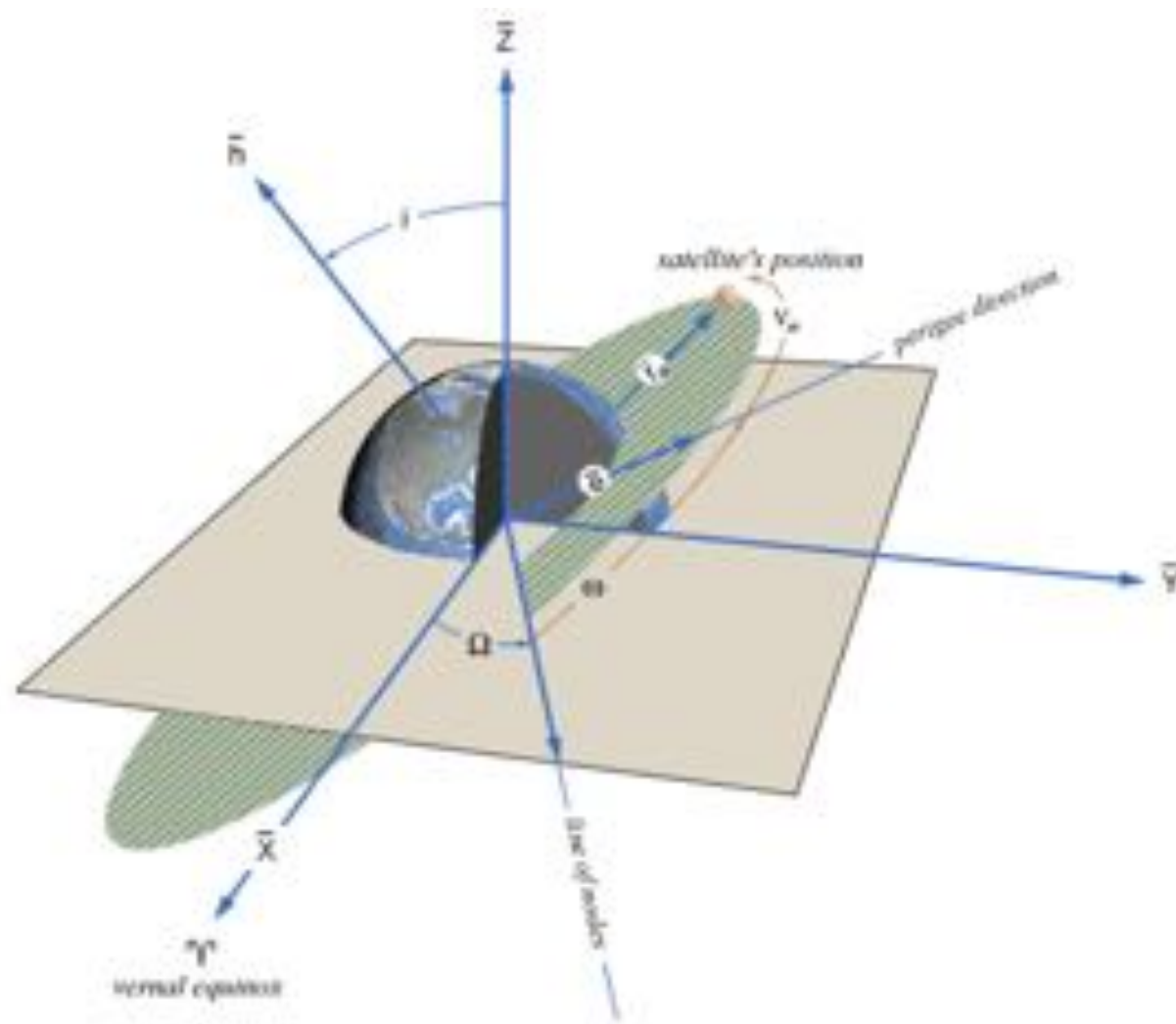


Timing

- Timing error is usually systematic, which is a problem, because systematic error does not go down as root N. Lots of observations made with a bad clock can alter an orbit solution.
- Testing clock accuracy: equip telescope with GPS receiver, observe GPS satellites.



Orbits



- a - defines the size of the orbit
- e - defines the shape of the orbit
- i - defines the orientation of the orbit with respect to the Earth's equator
- Ω - defines where the low point, perigee, of the orbit is with respect to the Earth's surface
- ω - defines the location of the ascending and descending orbit locations with respect to the Earth's equatorial plane
- V - defines where the satellite is within the orbit with respect to perigee.

- To describe (3-D) motion in 3-D space, need 6 parameters: e.g. position (x, y, z) and velocity (x', y', z').
- For an orbit - more useful: 2 describing ellipse (a, e), 3 describing its orientation (i, Ω, ω), 1 describing position (v) along ellipse.
- 1 observation: (i, j) position + time (t). Need at least 3 positions to fit a unique ellipse.