

STATISTICS 642 - ASSIGNMENT 4

DUE DATE: 8 am Central, THURSDAY, February 24, 2022

Name (**Typed**) _____

Email Address (**Typed**) _____

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STATISTICS 642 - ASSIGNMENT #4

- Read Handout 4
- Supplemental Reading: Chapter 4 in the Design & ANOVA book
- Hand in the following problems by 8 am Central, THURSDAY, February 24, 2022:

The following study will be used in answering questions 1 and 2:

An experiment was conducted to investigate heat loss for a coating placed on commercial thermal panes. There were five thicknesses of the coating applied to the panes: 0 (Control), 20, 40, 60, and 80 units. Ten panes were randomly assigned to each of the thicknesses. The interior temperature during the test period was set at 70°F and the average heat loss through the pane was recorded over a range of exterior temperatures. The heat loss data is given in the following table:

Thickness	Heat Loss Data									
0	10.2	10.8	10.1	10.9	11.1	11.8	11.3	11.9	9.3	9.9
20	9.2	9.8	9.1	9.9	10.1	10.8	10.3	10.9	9.3	9.8
40	9.0	9.9	9.2	9.8	10.0	10.8	10.2	10.7	9.9	9.0
60	8.1	8.1	8.0	8.9	8.2	8.9	8.1	8.8	9.2	9.9
80	7.2	7.8	7.1	7.9	8.1	8.8	8.3	8.9	9.3	9.8

1. (24 points) - Use the data from the heat loss study to answer the following questions. Contrasts of interest to the researchers were
 - C1: Control vs Average of the means of the four Thickness levels
 - C2: Linear trend across the 5 Thickness Levels
 - C3: Quadratic trend across the 5 Thickness Levels
 - C4: Cubic trend across the 5 Thickness Levels
 - a. Are the four contrasts mutually orthogonal? Justify your answer. If not, select three contrasts from the four contrasts which are mutually orthogonal.
 - b. Provide an estimate of each contrast along with the standard error of the estimator.
 - c. Use the Scheffè test at the $\alpha = .05$ level of significance to test the significance of the four contrasts.
 - d. Use a Bonferroni test with $\alpha_E = .05$ level of significance to test the significance of the four contrasts.
 - e. Test the three trend contrasts simultaneously using the matrix approach from HO 4.
 - f. Is there a trend in the mean heat losses as a function of the coating thicknesses? Justify your answer.
2. (12 points) - Use the data from the heat loss study to answer the following questions:
 - a. Which thickness(es) have the smallest mean heat loss with a probability of correct selection of 0.95.
 - b. Do any of the coatings have a mean heat loss less than the mean heat loss for the panes with no coating? Use $\alpha = .05$ in your answer.
 - c. Which pairs of the 5 treatment means are different using the Tukey procedure at the .05 significance level.

3. (4 points) - In a DOE textbook, the author states, “Suppose you are testing M hypotheses. Equality of the relationship $\alpha_F \leq M\alpha_{PC}$, holds when the M tests are independent. Is this statement true? If yes, provide a proof. If no, provide a condition under which equality does hold.
4. (6 points) - A completely randomized design with $t = 4$ fixed effects treatments and $n_1 = 5$, $n_2 = 4$, $n_3 = 5$, $n_4 = 6$, reps/treatment was run. The experimenter constructed two contrasts in the treatment means:

$$C_1 = 3\mu_1 + \mu_2 - \mu_3 - 3\mu_4 \quad C_2 = -\mu_1 + \mu_2 + \mu_3 - \mu_4$$

- Are the two contrasts orthogonal? Justify your answer.
 - Are the sample estimators of the two contrasts independently distributed? Justify your Answer.
 - Construct a contrast other than C_1 which is orthogonal to contrast C_2 .
5. (24 points) - A study was conducted to separately analyze the effects of 10 SNPs comparing people with Type I diabetes versus controls. The p-values from the 10 separate analyses are given in the following table.

SNP	p-value	SNP	p-value
1	.0001	6	.0911
2	.0058	7	.2012
3	.0132	8	.5718
4	.0289	9	.8912
5	.0498	10	.9011

- With no adjustment for multiple testing which SNPs have significant effects? Use a per comparison rate of 0.05.
 - Using a Bonferroni adjustment for multiple testing which SNPs have significant effects? Use a FWER of 0.05.
 - Using the FDR method to adjust for multiple testing which SNPs have significant effects? Use a FDR of 0.05.
6. (30 points) - Answer True or False to each of the following statements and include a short justification for your answer:
- If 5 contrasts C_1, C_2, C_3, C_4, C_5 are mutually orthogonal, then their related 5 sum of squares $SS_{C_1}, SS_{C_2}, SS_{C_3}, SS_{C_4}, SS_{C_5}$, are independent.
 - For $C_1 = \sum_{i=1}^t k_i \mu_i$ and $C_2 = \sum_{i=1}^t d_i \mu_i$, if $\sum_{i=1}^t k_i d_i = 0$ then C_1 and C_2 are orthogonal.
 - If 5 contrasts C_1, C_2, C_3, C_4, C_5 are mutually orthogonal, then $SS_{TRT} = SS_{C_1} + SS_{C_2} + SS_{C_3} + SS_{C_4} + SS_{C_5}$
 - A multiple comparison procedure which controls the False Discovery Rate (FDR) at level .01 will find fewer pairs of treatment means to be different than a multiple comparison procedure which controls the Family Wide Error Rate (FWER) at level .01.
 - The Tukey HSD procedure has an exact FWER.
 - The Fisher’s protected LSD procedure has an exact FWER.
 - The SNK procedure has an exact FWER.
 - The Dunnett procedure has an exact FWER.
 - The Hsu procedure has an exact FWER.
 - An experiment was conducted in which the treatments in a CRD are $t=5$ dose levels of a drug: 0, 5, 15, 30, 50 ml/kg of weight of the EU. There were $r=10$ EU’s per treatment. The contrasts specified in Table XI in the textbook can be used to conduct a test for Linear, Quadratic, Cubic, and Quartic trends in the dose level means.