Statistics 630 - Assignment 11

(due Wednesday, 8 December 2021)

1. Suppose $U_1, \ldots, U_n$ is a random sample from Uniform(0,θ) where θ is unknown. Recall that $M_n = \max(U_1, \ldots, U_n)$ is the MLE for θ.
   (a) Consider the test for $H_0 : \theta = 1$ versus $H_a : \theta = \theta_1$, with $\theta_1 > 1$, that rejects H_0 when $M_n > c$ for some $c < 1$. Identify c so that the test has size α. (Recall $M_n \leq c$ iff every $U_i \leq c$.)
   (b) Determine the power of the test, as a function of θ_1.
   (c) Show that this is the uniformly most powerful (UMP) test of size α for $H_0 : \theta = 1$ versus $H_a : \theta > 1$.
2. Exercises 6.3.26, 6.3.27. Also, explain why the test described here (namely, "reject H_0 if p-value $\leq \alpha$") is the same as the test that rejects H_0 when $Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \geq z_{1-\alpha}$.
3. Chapter 8 Exercise 8.2.3. First identify the hypotheses and explain why.
4. Recall Exercises 6.3.1 and 6.3.2 (Assignment 9). Now compute the p-value for each. *Do not do a power calculation.*
5. Recall Exercise 6.3.8 (Assignment 9). Now compute p-values using both the Wald and score statistics.
6. Suppose X is a statistic with binomial$(100, \theta)$ distribution. Consider the test that rejects $H_0 : \theta = 0.5$ in favor of $H_a : \theta \neq 0.5$ when $|X - 50| > 10$. Use the normal approximation to answer the following:
   (a) What is α?
   (b) Derive the (approximate) power function and graph it as a function of θ.
7. Chapter 8 Exercise 8.2.4.
8. Chapter 8 Exercise 8.2.6. No computation is necessary. Just think about what "having not ruled the null hypothesis out" signifies here.
9. Recall Exercise 6.2.19 from Assignment 8 (Hardy-Weinberg model), for which you found the likelihood and score functions and the MLE. Now determine the Wald and score tests for a two-sided size α test of $H_0 : \theta = 0.5$ (that is, that the two alleles are equally likely A or a and are independent).
10. Chapter 8 Exercise 8.2.20. Express the test in terms of a sufficient statistic (and its sampling distribution). (While it gives a way to express the power function as an integral, you can answer both parts (a) and (b) without the hint, by using the Neyman-Pearson lemma and what we have said about what makes a UMP test.) Add
    (c) Derive the generalized likelihood ratio for testing $H_0 : \lambda = \lambda_0$ versus $H_a : \lambda \neq \lambda_0$, and describe how to use it to conduct the test.
    (d) Use the data of Problem 3 in Assignment 9 to test the hypotheses $H_0 : \lambda = 2$ versus $H_a : \lambda \neq 2$ with $\alpha = 0.01$.