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Assignment 08
STAT 642
4/14/22

1.)

F_1 \ F_2	1	2	3	4
1	$\bar{y}_{11}$	*	*	$\bar{y}_{14}$
2	$\bar{y}_{21}$	$\bar{y}_{22}$	$\bar{y}_{23}$	*
3	$\bar{y}_{31}$	*	$\bar{y}_{33}$	$\bar{y}_{34}$

(a) $C_1 = \mu_{11} - \mu_{14} - \mu_{31} + \mu_{34} \iff C_1 = (\mu_{11} + \mu_{34}) - (\mu_{14} + \mu_{31})$

(see H.O. 8 pg 30) → (i) Interaction

(ii) Estimable

(b) $C_2 = \mu_{11} + \mu_{21} + \mu_{31} - \mu_{13} - \mu_{23} - \mu_{33}$

(i) Main Effect - F_2

(ii) No

(c) $C_5 = \mu_{11} - \mu_{14} + \mu_{21} - \mu_{24} + \mu_{31} - \mu_{34} = (\mu_{11} + \mu_{21} + \mu_{31}) - (\mu_{14} + \mu_{24} + \mu_{34})$

(i) Main Effect - F_2

(ii) No

2.) Two factors: F_1 (3 levels), F_2 (4 levels)

F_1 \ F_2	1	2	3	4
1	$\bar{y}_{11}$	*	$\bar{y}_{13}$	$\bar{y}_{14}$
2	*	$\bar{y}_{22}$	$\bar{y}_{23}$	$\bar{y}_{24}$
3	$\bar{y}_{31}$	*	$\bar{y}_{33}$	$\bar{y}_{34}$

(a) Write two contrasts for the main effects of F_1. Select contrasts which involve maximum number of treatment means. Are your contrasts orthogonal?

- $C_1 = (\mu_{11} + \mu_{13} + \mu_{14}) - (\mu_{31} + \mu_{33} + \mu_{34})$ $[1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ -1\ 0\ -1\ -1]$
- $C_2 = (\mu_{13} + \mu_{14}) - (\mu_{23} + \mu_{24})$ → $[0\ 0\ 1\ 1\ 0\ 0\ -1\ -1\ 0\ 0\ 0\ 0]$
- NO, they aren't orthogonal.

(b) Write two contrasts which would evaluate interaction effects for F_1 & F_2. Are they orthogonal.

- $C_1 = (\mu_{11} - \mu_{13}) - (\mu_{22} - \mu_{23}) - (\mu_{31} - \mu_{32}) = 0$
- $C_1 = (\mu_{21} - \mu_{22}) - (\mu_{31} - \mu_{32}) = 0$
- NO, they aren't orthogonal.

3.) F_1: Traffic Signal Type (3 levels - Fixed) $(S - \tau)$

F_2: Method to measure traffic delays (2 levels - Fixed) $(M - \beta)$

F_3: Intersection (Signal) (2 levels - Random) $(I(S) - a)$

F_4: Traffic type (2 levels - fixed) $(T - \gamma)$

(a) Write a model for this study:

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + a_{l(i)} + (a\gamma)_{l(i)k} + (a\beta)_{l(i)j} + e_{ijkl}$$

$i = 1,2,3;\ j = 1,2;\ k = 1,2;\ l = 1,2;\ \tau_3 = \beta_2 = \gamma_2 = 0$

• $(\tau\beta)_{3j} = (\tau\beta)_{i2} = (\tau\gamma)_{3k} = (\tau\gamma)_{i2} = (\tau\beta\gamma)_{3jk} = (\tau\beta\gamma)_{i2k} = (\tau\beta\gamma)_{ij2} = 0.$

(b) Provide an Anova Table for this study.

° See Figure 1

(c) Provide the Expected Mean Squares for all sources.

• See figure 1

(d) What can you conclude at the $\alpha = 0.05$ levels about the effect of Type of Traffic Signal, Measuring Method, & Level of Traffic on the average Traffic delay?

For Type of traffic signal, Measurement Method and level of trafic we doesn't have significant evidence to conclude at the $\alpha = 0.05$ level to conclude that any of them have an effect on the average traffic delay.

3) (c) Provide estimates of the variance components & the proportions of the total variance in traffic delay measurements

$$\sigma_y^2 = \sigma^2_{I(S)} + \sigma^2_{I(S)*T} + \sigma^2_{I(S)*M} + \sigma^2_e = 156.10 + 13.9775 + 13.9183 + 4.1683 = 188.1641$$

- Proportion due to $I(S)$: $\sigma^2_{I(S)}/\sigma^2_y = 0.829595$
- Proportion due to $I(S)*T$: $\sigma^2_{I(S)*T}/\sigma^2_y = 0.07428356418$
- Proportion due to $I(S)*M$: $\sigma^2_{I(S)*M}/\sigma^2_y = 0.07396894519$
- Proportion due to error: $\sigma^2_e/\sigma^2_y = 0.02215247223$

a = 4
b = 6
c = 3
r = 6

4.)

R: σ_A^2 F: Q_B R: $\sigma^2_{A\times B}$ R: $\sigma^2_{C(B)}$ R: $\sigma^2_{A\times C(B)}$ σ^2_e

Source	DF	MS	
A	3	24.5	$(5)(3)(6)\sigma_A^2 + (3)(6)\sigma^2_{A\times B} + 6\sigma^2_{A\times C(B)} + \sigma^2_e$
B	4	19.7	$(4)(3)(6)Q_B + (3)(6)\sigma^2_{A\times B} + (4)(6)\sigma^2_{C(B)} + 6\sigma^2_{A\times C(B)} + \sigma^2_e$
A×B	12	8.9	$(3)(6)\sigma^2_{A\times B} + \sigma^2_e$
C(B)	10	7.5	$(4)(6)\sigma^2_{C(B)} + 6\sigma^2_{A\times C(B)} + \sigma^2_e$
A×C(B)	30	6.8	$6\sigma^2_{A\times C(B)} + \sigma^2_e$
Error	300	5.8	σ^2_e

(b) Test for a significant AB interaction ($\alpha = 0.05$) while the ANOVA table is providing the MS not SS for each source of variation.

- Since the 3 way interaction is not included in the model, use MSE as our denominator in the test stat.

$$F = MS_{A\times B}/MS_{A\times C(B)} = 8.9/6.8 = 1.309 < 2.09 = F_{0.05,12,30}$$

- Fail to reject $H_0: \sigma^2_{AB} = 0$ at the $\alpha = 0.05$ level.

(c) Test for a significant B main effect ($\alpha = 0.05$)

$H_0: Q_B = 0$ $H_a: Q_B \neq 0$.

$$M = MS_{AB} + MS_{C(B)} - MS_{A*C(B)} = 9.6$$

- Satterthwaite DF:

$$df = \frac{M^2}{\frac{(MS_{AB})^2}{12} + \frac{MS_{C(B)}^2}{10} + \frac{MS_{A*C(B)}^2}{30}} = 6.6942$$

$$F = \frac{MS_B}{M} = \frac{19.7}{9.6} = 2.052 \text{ w/ p-value} = pf(2.052, 4, 6.6942) = 0.1952 > 0.05$$

- Fail to reject H_0 at the $\alpha = 0.05$ level.

4.) (d) Compute the variance of the difference in the estimated treatment means for levels 1 & 2 of Factor B: $\bar{y}_{\cdot 1 \cdot\cdot} - \bar{y}_{\cdot 2 \cdot\cdot}$. Provide an estimate of this var and the df of the estimate.

$$var[\bar{y}_{\cdot 1 \cdot\cdot} - \bar{y}_{\cdot 2 \cdot\cdot}] = \frac{2[MS_{AB} + MS_{C(B)} - MS_{A \times C(B)}]}{(4)(3)(6)} = \boxed{0.2667}$$

$\boxed{df = 6.6942}$

(e) Compute the value of Tukey-Kramer HSD w/ $\alpha = 0.05$, that would be used to determine which pairs of means across the levels of factor B are different

$$HSD = q_{0.05, 5, v}\sqrt{\frac{M}{(4)(3)(6)}} = qtukey(0.95, 5, 6.69420)\sqrt{\frac{9.6}{72}} = 5.1257\sqrt{\frac{9.6}{72}} = \boxed{1.87116}$$

5.) (a) F_1 - Run (4 levels - Random); F_2 - Patient (5 levels random)

model: $y_{ijk} = \mu + a_i + b_j + (ab)_{ij} + e_{ijk}$; $i = 1, \ldots, 4$; $j = 1, \ldots, 5$; $r = 2$.

Source	DF	EMS
F_1	3	$\sigma_e^2 + 2\sigma^2_{F_1 * F_2} + 10\sigma^2_{F_1}$
F_2	4	$\sigma_e^2 + 2\sigma^2_{F_1 * F_2} + 8\sigma^2_{F_2}$
$F_1 * F_2$	12	$\sigma_e^2 + 2\sigma^2_{F_1 * F_2}$
error		σ_e^2

(b) A (4 levels - Random), B(A) (3 levels, Random), C(A,B) (2 levels, random)
D(A,B,C) (3 levels, Random)

Source	df	EMS
A	3	$\sigma^2_d + 3\sigma^2_{C(AB)} + 6\sigma^2_{B(A)} + 18\sigma^2_A$
B(A)	6	$\sigma^2_d + 3\sigma^2_{C(AB)} + 6\sigma^2_{B(A)}$
C(A,B)	6	$\sigma^2_d + 3\sigma^2_{C(AB)}$
D(AB,C)	12	σ^2_d

5.) (contd)

(c) A, B fixed ; C (A, B) (Random 6-levels). D (fixed - 5 levels) CRD w/ 6 replications.

Source	DF	EMS
A	2	$\sigma^2_e + 6\sigma^2_{B*C(AB)} + 30\sigma^2_{C(A,B)} + 360 Q_A$
B	1	$\sigma^2_e + 6\sigma^2_{D*C(AB)} + 30\sigma^2_{C(A,B)} + 540 Q_B$
AB	2	$\sigma^2_e + 6\sigma^2_{D*C(A,B)} + 30\sigma^2_{C(A,B)} + 180 Q_{A*B}$
C(A,B)		$\sigma^2_e + 6\sigma^2_{D*C(A,B)} + 30\sigma^2_{C(A,B)}$
D	4	$\sigma^2_e + 6\sigma^2_{D*C(A,B)} + 216 Q_D$.
AD	8	$\sigma^2_e + 6\sigma^2_{D*C(A,B)} + 72 Q_{A*D}$
BD	4	$\sigma^2_e + 6\sigma^2_{D*C(A,B)} + 108 Q_{B*D}$
ABD	8	$\sigma^2_e + 6\sigma^2_{D*C(A,B)} + 36 Q_{A*B*D}$
C(A,B)*D		$\sigma^2_e + 6\sigma^2_{D*C(A,B)}$
Error		σ^2_e