STATISTICS 642 - EXAM I - March 2, 2022 - SOLUTIONS

Problem I. (30 points) CIRCLE (A, B, C, D, or E) corresponding to the **BEST** answer.

1. **C**. Note that $\lambda = \frac{4(2)}{8} = 1$ where $t = 9, r = 4, k = 3$
2. **D**
3. **B**
4. **E**

5i. **B**

5ii. **B**

6. **A**. Notice the standard constraint
7. **A**
8. **B**
9. **A**

Problem II. (40 points)

1. Using the results in the AOV table from the SAS output,

   $F = 24.3$ with p-value$< .0001$ in testing $H_o : \mu_{1500} = \mu_{1600} = \mu_{1700} = \mu_{1800} = \mu_{1900}$ versus H_1 : differences in the μ_i.

   From these values, we can conclude:

   Yes, there is significant evidence of a difference in the average strength of the steel for different heat-treat temperatures.

   Using the Tukey Adjusted p-values from the SAS output to make the comparisons, there is no significant evidence that the following group means have differences:(1500, 1600), (1600,1900), (1900,1800), (1800, 1700) with their pvalues, 0.1203, 0.6842, 0.2941, 0.0508.

   There is significant evidence that the average strength of heat-treat of temp=1500 differs from temp=1700, 1800, 1900. There is significant evidence that the average strength of heat-treat of temp=1600 differs from temp=1700, 1800.

2. There are 5 equally spaced levels for the treatment factor. Thus, we can test for 4 trends using the mutually orthogonal contrasts from Table XI.

   The correct value of α_{PC} for conducting the 4 tests is $\alpha_{PC} = 1 - (1 - .05)^{1/4} = .01274$.

   The significant trend contrasts are those contrasts having p-values less than .01274:

   We have the following contrast:
   • Linear(p-value$< .0001$), trend Quadratic(p-value$< .0001$) trend, Cubic(p-value $= .0588$) trend, and Quartic(p-value $= .0039$) trend. The Linear, Quadratic, Quartic trends have the p-values less than .01274.

   Thus, there appears to be a quartic relationship between average strength and the temperature.

3. The 95% confidence interval for μ_{1900} :
   $\hat{\mu}_{1900} \pm t_{.025,n-t}\widehat{SE}\hat{\mu}_{1900} = 6.4583 \pm (2.004)\frac{1.087985}{\sqrt{12}}$.
   Then 95% confidence interval for this mean is (5.8289, 7.0877)

4. With $\alpha = .05, t = 5, \sigma_e = 1.1, r = 20, D = 1.25;$ $\Rightarrow$ $\lambda = \frac{rD^2}{2\sigma_e^2} = \frac{20\times 1.25^2}{2\times(1.1)^2} = 12.9132;$
   $\Phi = \sqrt{\lambda/t} = \sqrt{12.9132/5} = 1.6071$

- From Table IX - Tables for STAT 642 with $\nu_1 = t - 1 = 4, \nu_2 = t(r-1) = 95, \alpha = .05, \Phi = 1.6$, power is approximately between 0.77 and 0.85 which is less than .90. Thus, r=20 reps is too small to achieve the power specification.

- From R, $\gamma(12.91) = P[F \geq F_{.05,4,95} | \lambda = 12.91] = 1 - pf(2.4675, 4, 95, 12.91) = .8127$

Problem III. (36 points) Provide the details for each of the following items:

1. Type of Randomization:

   Completely Randomized Design with a Split Plot Treatment Assignment

   There are two separate randomizations. The anxiety-tension combinations first randomly assign to each subject. Eventhough there are four responses from each subject, the randomization is restricted so that all four of those responses will be at the same anxietytension combination. Anxiety and tension are thus whole-plot treatment factors. Each subject does four memory trials. The trial type is randomized to the four trials for a given subject. Thus the four trials for a subject are the split plots, and the trial type is the split-plot treatment. At the whole-plot level, the anxiety-tension combinations are assigned according to a CRD, so there is no blocking.

2. Type of Treatment Structure:

   $2 \times \ 2 \times 4$ crossed factorial structure with anxiety, tension and type crossed

3. Identify each of the factors as being Fixed or Random:

- anxiety - 2 fixed levels; tension - 2 fixed levels; type - 4 fixed levels; subjects - 12 random levels

4. Describe the experimental units:

   EU is a subject

5. Describe the measurement units:

   MU is subject's grip

6. Identify any covariates:

   subject's age

EXAM 1 SCORES n = 59

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Min = 29,   Q(.25) =  70,   Q(.5) = 76,   Mean = 73.7288,  Q(.75) =  84,   Max =  94
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