Statistics 630 - Assignment 7
(due Friday, 29 October 2021)

Use R for simulation, data computation, graphing, etc. You do not need to report your routines (R commands) – just show the results. But I recommend that you save your routines for later reference.

1. Chapter 4 Exercise 4.2.12. Note: $M_n = \bar{X}_n = \frac{1}{n}(X_1 + \cdots + X_n)$, the mean of a sample of n data. Use R to generate the random exponential variables with `rexp`. The second argument for `rexp` (a value that you provide) is the parameter λ in the book's notation. The mean of a vector `x` is given by `mean(x)`.

2. Chapter 4 Exercise 4.4.4.

3. Chapter 4 Exercise 4.4.12. Add
(d–f) Determine the exact distribution of the average time to service for the first n customers, when $n = 16$, 36, 100. (Hint: use the mgf.) Then use the `pgamma` function in R to find the exact probability and compare it to the normal approximation.

4. Chapter 4 Exercise 4.4.16.

5. Chapter 4 Exercises 4.6.1, 4.6.2, 4.6.7.

6. Chapter 4 Exercises 4.6.10. The book says "compute the distribution of" but all you need is to identify the distribution using the results in Section 4.6.

7. Suppose $X_1, \ldots, X_n$ are iid random variables from the exponential(λ) distribution. Write down the joint pdf for the random vector $(X_1, \ldots, X_n)$ and show that it can be expressed in terms of n, λ and $x_1 + \cdots + x_n$.

8. Suppose $T_1, \ldots, T_n$ are iid random variables from the binomial$(4, \theta)$ distribution. Write down the joint pmf for the random vector $(T_1, \ldots, T_n)$ and show that it can be factored as $a(\theta)g(t_1, \ldots, t_n)h(t_1 + \cdots + t_n, \theta)$.

9. Recall the Laplace pdf $f(x) = \frac{1}{2}\mathrm{e}^{-|x|}$ (Exer. 2.4.22 and Exer. 3.4.16). This can be generalized to a *location-scale* family with parameters (μ, β) by $f(x) = \frac{1}{2\beta}\mathrm{e}^{-|x-\mu|/\beta}$. Let $X_1, \ldots, X_n$ be iid random variables from this distribution, for some (μ, β), and write down the joint pdf for the random vector $(X_1, \ldots, X_n)$. Simplify as possible.

10. Use R to simulate $N = 10^4$ random samples $(Z_1, \ldots, Z_n)$ from the normal(0,1) distribution and compute $T = \max(Z_1, \ldots, Z_n)$ for each sample. Use $n = 20$ (but you can try larger n for comparison, if you have time and would like to.) `max(x)` gets the maximum value in a vector `x`. Use `hist` and `boxplot` to obtain a histogram and box-plot of your N values of T. Comment on the histogram shape and symmetry.