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STAT 642
Assign 09
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(See H.O.10 pg 10)

Problem 1: For each the three designs given below identify the following characteristics.

C_1: The fraction of the full Design:

C_2: The # of required generators:

C_3: The # of implicit generators:

C_4: The # of aliases for each estimable effect:

C_5: The # of EU's (runs required)

(1) Design: 2^{6-2}

C_1: $2^{-2} = \frac{1}{4}$

C_2: 2

C_3: $2^2 - 2 - 1 = 1$

C_4: $2^{6-2} = 2^4 = 16$

C_5: $2^{6-2} = 16$

(2) Design: 2^{7-3}

C_1: $2^{-3} = \frac{1}{8}$

C_2: 3

C_3: $2^3 - 3 - 1 = 4$

C_4: $2^{7-3} = 2^4 = 16$

C_5: $2^{7-3} = 2^4 = 16$

(3) Design: 2^{7-4}

C_1: $2^{-4} = \frac{1}{16}$

C_2: 4

C_3: $2^4 - 4 - 1 = 11$

C_4: $2^{7-4} = 2^3 = 8$

C_5: $2^{7-4} = 2^3 = 8$

(see HO 10 pg 20) → **Problem 2**: Construct a 2^{n-3} fractional factorial design using ABCF, CDEF, ADFG as the design generators.

- (i): Defining Contrasts: $I = ABCF, CDEF, ADFG$
- (ii): Implicit Defining contrasts: $2^3 - 1 = 4$
  - $ABCF \cdot CDEF = ABDE$
  - $ABCF \cdot ADFG = BCDG$
  - $CDEF \cdot ADFG = ACEG$
  - $ABCF \cdot CDEF \cdot ADFG = BEFG$

[* where does the 4th implicit contrast come from? (multiply all defining contrasts together)]

(iii) Resolution = 4

(iv) Alias sets for all main effects & two factor interactions:

Alias Set								
1	I	ABCF	CDEF	ADFG	ABDE	BCDG	ACEG	BEFG
2	A	BCF	ACDEF	DFG	BDE	ABCDG	CEG	ABEFG
3	B	ACF	BCDEF	ABDFG	CDG	CDG	ABEG	EFG
4	C	ABF	DEF	ACDFG	ABCDE	BDG	AEG	BCEFG
5	D	ABCDF	CEF	AFG	ABE	BCG	ACDEG	BDEFG
6	E	ABCEF	CDF	ADEFG	ABD	BCDEG	ACG	BFG
7	F	ABC	CDE	ADG	ABDEF	BCDFG	ACEFG	BEG
8	G	ABCFG	CDEFG	ADF	ABDEG	BCD	ACE	BEF
9	AB	CF	ABCDEF	BDFG	DE	ACDG	BCEG	AEFG
10	AC	BF	ADEF	CDFG	BCDE	ABDG	EG	ABCEFG
11	AD	BCDF	ACEF	FG	BE	ABCG	CDEG	ABDEFG
12	AE	BCEF	ACDF	DEFG	BD	ABCDEG	CG	ADFG
13	AF	BC	ACDE	DG	BDEF	ABCDGF	CEFG	ABEG
14	AG	BCFG	ACDEFG	DF	BDEG	ABCD	CE	ADEF
15	BG	ACFG	BCDEFG	ABDF	ADEG	CD	ABCE	EF
16	ABG	CFG	ABCDEFG	BDF	DEG	ACD	BCE	AEF

(1) Treatments: A, B, C, D, E, F, G, AB, AC, AD, AE, AF, AG, BG, ABG

where A indicates the treatment consists of factor A at its high level & BCDEG at their low levels.

(2) Implicit generators (see above)

(3) Resolution = 4. (1) Main effects are not confounded w/ other main effects or two way interactions, some two way interactions are confounded w/ each other. (2) The smallest # of factors in our sequences is 4.

Problem III: A 2^{5-2} fractional factorial experiment is designed using generators ABC & BCDE. It is known that factors A, B & C do not interact w/ one another and factors C, D, E do not interact w/ one another.

1.) Which effects can be estimated ignoring three factor interactions or higher.

- Implicit contrast(I): ABC · BCDE = ADE => Resolution 3 => only main effects are not confounded w/ main effects.

Alias set

1	I	ABC	BCDE	ADE
2	A	BC	ABCDE	DE
3	B	AC	CDE	ABDE
4	C	AB	BDE	ACDE
5	D	ABCD	BCE	AE
6	E	ABCE	BCD	AD
7	AD	BCD	ABCE	E - confounded w/ E
8	AE	BCE	ABCD	D - confounded w/ D
9	BD	ACD	CE	ABE
10	BE	ACE	CD	ABD

- So we can estimate the main effects of all factors and the two way interactions for BD & BE.

(2) Is it possible to have an improved resolution 2^{5-2} design for this situation.

- No, we cannot have a higher resolution design.
- The resolution is equal to the number of factors in the smallest defining contrast, including the implicit ones.
- If one of our defining contrasts contains ≤ 3 elements, then our resolution will be ≤ 3. If both of our defining contrasts contain > 3 elements there are two scenarios:

(1) Both defining contrasts contain 4 elements.

- since we have 5 factors, these contrasts can differ by at most 1 element => our implicit contrast has 1 element => our resolution = 1.

(2) One defining contrast contains 4 elements & the other contains 5 elements.

- same as above, the two contrasts can differ by at most 1 element => resolution = 1.

(Sec H.010 pg 9) → **Problem IV**: In a resolution VIII 2^{n-p}

(1) Main effects are not confounded w/ which effects

- Main effects are not confounded w/ any 6-way or lower interactions and possibly some 7-way and higher interactions.

(2) Two-Way Interaction effects are not confounded w/ which effects?

- Two-way interaction effects " " 5-way or lower "
  " 6-way and higher interactions

(3) Three-way Interaction effects are not confounded w/ which effects?

- Three way interaction effects " " 4-way or lower "
  " 5-way or higher interactions.

(4) four-way Interaction effects are not confounded w/ which effects?

- Four way interaction effects " " 3-way or lower "
  " 4-way or higher interactions.

(KE 4010 Pg 21) **Problem 5:** Construct a Resolution 4 design that is a $\frac{1}{8}$ fraction of a 2^8 factorial. Make sure to provide your design generators and display a list of the treatments to be used in the experiment.

- Let our factors be A, B, C, D, E, F, G, H
  - Design Generators: ABCF, ABDG, BCDEH
  - Implicit Contrasts:
    - $ABCF \cdot ABDG = CDFG$
    - $ABCF \cdot BCDEH = ADEFH$
    - $ABDG \cdot BCDEH = ACEGH$
    - $ABCF \cdot ABDG \cdot BCDEH = BEFGH$
  - See figure 1. (Treatments are indicated by the first column, starting in the second row. The labels in the treatments indicates the factors which are at their high level in the treatment and the factors which are not included in the test label are those factors which are at their low level in the treatment.)

Problem VI: (See SAS program: ASSIGN9 - PROBVI-SP22)

(1) What is the resolution of the design?

Resolution 5: the minimum # of elements in one of our defining contrasts (we only have 1 defining contrast, no implicit contrasts) is 5. Also see table on pg 21.

(2) Show how the (+) & (-) signs for the levels of Factor E were determined for this design.

- We are given in the problem set up that $E = -ABCD$.
- for each row, if you multiply -1 times the product of the elements in the row for columns A, B, C, & D you get the value for E in each row.

(3) Show the alias structure for this design.

Alias set

1	I	-ABCDE
2	A	-BCDE
3	B	-ACDE
4	C	-ABDE
5	D	-ABCE
6	E	-ABCD
7	AB	-CDE
8	AC	-BDE
9	AD	-BCE
10	AE	-BCD
11	BC	-ADE
12	BD	-ACE
13	BE	-ACD
14	CD	-ABE
15	CE	-ABD
16	DE	-ABC

Problem V5 (cont'd)

(4) What assumptions must be made to estimate main effects and two factor interactions free of any other effects?

- In order to estimate main effects and two factor interactions we must first construct the experiment to be of at least Resolution V so that the main effects are not confounded w/ any other main effects or two-factor interactions and the two-factor interactions are not confounded w/ any other two factor interactions. Furthermore we must <u>assume</u> that any 3-factor or higher order interaction effects are either nonexistent or negligible.

(5) Estimate the main effects and two-factor interactions & their standard errors?

<u>see table 2</u>

- The reason we cannot estimate the standard errors is b/c the $df_E = 0$.