Statistics 630 - Assignment 6
(due Wednesday, 20 October 2021)

View lectures 16–20.

1. (a) Chapter 3 Exercise 3.3.25. You can do this directly from the joint pmf, but here is a simpler alternative approach. First observe that $X_i \sim \text{binomial}(n, \theta_i)$ (recall Example 2.8.5 in the text) and $X_i + X_j \sim \text{binomial}(n, \theta_i + \theta_j)$ if $i \neq j$ (why? – think about combining categories). Then use those facts and properties of variance and covariance to get two expressions for $\mathsf{Var}(X_i + X_j)$ which you can use to solve for the desired covariance.
   (b) Use the above to find $\mathsf{Corr}(X_i, X_j)$. How does the correlation change with n?
2. (a) Show that the variance for the beta(a, b) distribution is $\frac{ab}{(a+b)^2(a+b+1)}$. (Recall Exer. 3.2.22.)
   (b) Suppose $(X_1, X_2) \sim \text{Dirichlet}(a_1, a_2, a_3)$ (recall Exer. 2.7.17). It can be shown (and you may assume) that $X_1 + X_2 \sim \text{beta}(a_1 + a_2, a_3)$. Use an argument similar to part (a) of Problem 1 to obtain $\mathsf{Cov}(X_1, X_2)$.
3. Chapter 3 Exercises 3.4.5, 3.4.12, 3.4.16.
4. Chapter 3 Exercises 3.4.20, 3.4.23. (Why is it necessary that $t < \lambda$?)
5. Chapter 3 Exercises 3.5.4, 3.5.11 (note errors in textbook solution!), 3.5.16.
6. Let T have an exponential(λ) distribution, and conditional on T, let U be uniform on $[0, T]$. Find the unconditional mean and variance of U.
7. Chapter 3 Exercise 3.6.10. Add
   (c) Compare the bound in part (b) to the exact probability.
8. Chapter 4 Exercises 4.2.10, 4.2.11.