Statistics 630 - Assignment 1
(due Friday, 10 September 2021)

Instructions:

- The textbook exercises are in the book by Michael Evans and Jeffrey Rosenthal. This assignment covers material from Lectures 01–04.
- Assignments are *due by the end of class time.*
- Whether you write out the solutions by hand or in a text document, be sure that they are *neat, legible and in order* (even if you choose to solve them in a different order).
- Section 600: Please turn your assignments in at the front of class on the due date.
- Section 700: Please use the link provided in Canvas.

1. Chapter 1 Exercises 1.2.2, 1.2.9.

2. Two 6-sided dice are thrown sequentially and the values they show are recorded.

   (a) List the sample space. (There are 36 outcomes.)
   (b) List the outcomes that make up the following events: $A =$ "the sum of the two values is at least 9", $B =$ "the value of the first die is higher than the value of the second die" and $C =$ "the second die has value 4".
   (c) List the elements of the following events: $A \cap C$, $B \cup C$, $A \cap (B \cup C)$.
   (d) Illustrate the events of part (c) in Venn diagrams with regions depicting A, B and C.
   (e) Assume the outcomes are equally likely (they each have the same probability) and find the probabilities of the events in part (c).
   (f) Can $\mathsf{P}(A \cap C)$ be computed by multiplying the probabilities of A and C?
   (g) Imagine this experiment being repeated many times. What would be the longterm proportion of all the experiments for which the sum of the two dice is 7?

3. Chapter 1 Exercises 1.3.2, 1.3.4, 1.3.8, 1.3.10(a).

4. Chapter 1 Exercises 1.4.1, 1.4.6, 1.4.11, 1.4.12.

5. Long ago we posted grades identified by the last 4 digits of a student's social security number. (It is illegal now!) Assume each of the 10,000 configurations 0000 to 9999 are equally likely.

   (a) Use R to estimate the probability that at least two students in a class of 100 share the same 4 digits. Do this by simulating 100,000 samples of size 100, with replacement, from the population $\{0, 1, 2 \ldots, 9999\}$ and then determining the proportion of times that a sample has at least one duplicate. (*Note:* if x is the sample then `(length(unique(x))!=100)` returns value TRUE, which computes to 1, when there is at least one duplicate. This is a bit simpler than the method shown in the birthday problem example.)
   (b) Find the actual probability and compare it to your estimate. (Recall the birthday problem discussed in class.)
   (c) What is the smallest class enrollment for which the probability that at least two students have the same 4 digits is at least 0.50?

(continued next page)

6. Chapter 1 Exercises 1.5.1, 1.5.8, 1.5.10.

7. Chapter 1 Exercise 1.5.7. Add

   (b) Suppose we are given (or know or assume) that the batter hits a home run. What is the (conditional) probability that he was thrown a curve ball?

   (c) As in (b), suppose the batter does *not* hit a home run. What is the probability that he was thrown a curve ball?