STAT 608, Spring 2022 - Assignment 1

1. Question 1, page 38 in our textbook. Notice that to answer these questions correctly, you should be thinking like a statistician and talking about population parameters, not only sample statistics. That is, do some inference in every part, using as much everyday layperson's terminology as possible. For example, part b should not just say "The intercept is (or is not) 10,000." What does an intercept mean in context to someone who is selling movie tickets? Use the discussion board as needed to get all your details correct.

2. Show that $\text{Var}(Y_i|X_i = x_i) = \text{Var}(\epsilon_i)$ in the simple linear regression model. Don't overthink this; the answer is simple. What did you assume in answering this?

3. Define using only words what the least squares criterion is.

4. Question 4, page 40 in our textbook, except do:

   (a) Setup:

      i. Write down your design matrix $\mathbf{X}$.

      ii. Show, using matrix notation and starting with the principle of least squares, that the least squares estimate of β is

$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$$

   (b) As in text.

5. Show that the least-squares criterion applied to the "intercept-only" model, i.e.,

$$y_i = \beta_0 + \epsilon_i, \ i = 1, 2, \dots, n$$

   results in the least squares estimator of β_0: $\hat{\beta}_0 = \bar{y}$ by following these steps:

   (a) Write down your design matrix $\mathbf{X}$. (It won't be the same as any we've used in class.) Double check: does $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ give the set of equations listed above? Notice this model has no predictor variable.

   (b) Use the previously derived formula $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y}$ to get the least squares estimator.

6. Prove that $\text{Cov}(aX, bY) = ab\text{Cov}(X, Y)$.

7. Question 7, page 42 in our textbook.

8. Using $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y}$, finish our algebra from class and show that $\hat{\beta}_0 = \bar{y} - \frac{SXY}{SXX}\bar{x}$ for the simple linear regression case.

9. Show that for the usual regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, where the usual regression assumptions from question 4 apply, $\text{Var}(\mathbf{a}'\hat{\boldsymbol{\beta}}|\mathbf{X}) = \sigma^2\mathbf{a}' (\mathbf{X}'\mathbf{X})^{-1} \mathbf{a}$, where $\mathbf{a}$ is a constant vector.