STATISTICS 641 - EXAM I - SOLUTIONS

Problem I. (5 points EACH)

(1.) D. Strata are the five States, Counties are clusters of Residents, randomly select 20 counties from each state, then randomly select 150 residents from each of the 100 counties.

(2.) D. Simulate value from a Poisson distribution with $\lambda = (5)(.4) = 2$. Using the Poisson table in the attached tables, $P[F \leq 2] = .677 < .682 < .857 = P[F \leq 3]$. Therefore, the simulated value for F is 3.

(3.) A. Number of successes in a sequence of 30 iid Bernoulii trials

(4.) A. $.05 = P[D > \tau] \Rightarrow P[D \leq \tau] = .95 \Rightarrow \tau = Q(.95) \Rightarrow .95 = .75 + .25\tau \Rightarrow \tau = .8$

(5.) E. The value of pdf is not a probability, in HO 3 displayed a pdf where $f(t) > 1$. By def, $f(25)$ is the rate of change in the cdf,$F(t)$, at $t = 25$

(6.) B. $(\theta_1,\ \theta_2,\ \theta_3)$ = (scale, shape, location) parameters

(7.) C. Use the standard member of the family, Z, to compute $P[|Y - \theta_1| \leq k\theta_2] = P[|Z| \leq k]$

(8.) A. See discussion on page 33 in HO 4 and the multiple plots of estimated pdf in Handout 4.

(9.) B. See discussion on page 9 in Handout 6.

(10.) B. See discussion on page 27 in Handout 5. Also, recall that the Cauchy distribution is symmetric but both μ and σ are undefined.

(11.) D. Recall the examples in HO 5, "elephant vs house" and the plots on page 15 in HO 5 where a bimodal distribution has skewness=0 and kurtosis = 3

(12.) E. The recorded amount, Y_i, for the censored rats has $Y_i \leq T_i$, where T_i is the amount of insecticide that would have been in the rat's bladder after 30 days if the rat would have survived to 30 days.

Problem II (40 Points)

(A.) $\hat{\alpha} = 47.7866,\ \hat{\gamma} = 9.3863,\ \Rightarrow\ \hat{P}[V_F > 50] = 1 - \hat{F}(50) = e^{-(50/47.7866)^{9.3863}} = .2166$

(B.) $V_{CR} = Q(.28) \Rightarrow$ for distribution-free: Use the Kaplan-Meier Estimator

$\hat{V}_{CR} = inf\{t : \hat{S}(t) \leq 1 - .28\} = 43.3$ Using linear interpolation, $\hat{V}_{CR} = 42.46$

For MLE, $u = F(y_u) = 1 - e^{-(y/\alpha)^\gamma} \Rightarrow Q(u) = y_u = \alpha[-log(1-u)]^{1/\gamma} \Rightarrow$

$\hat{V}_{CR} = \hat{Q}(.28) = \hat{\alpha}[-log(1 - .28)]^{1/\hat{\gamma}} = 47.7866[-log(1 - .28)]^{1/9.3863} = 42.44$

EXAM I SCORES: n = 77

Min = 22, Q(.25) = 64.25, Q(.5) = 79.5, Mean = 76.12, Q(.75) = 89, Max = 100