Partial Solutions to STATISTICS 642 - EXAM II April 6, 2022

Problem I (44 points)

1. (16 pts) Do the necessary conditions for testing hypotheses and constructing confidence intervals appear to be satisfied? Justify your answers.

   Using the SAS output:

   C_1 Normality: Shapiro-Wilks test has p-value= .9536 and the 36 plotted points in the normal probability plot are very close to straight line which confirms an excellent fit of the normal distribution to the residuals

   C_2 Equal Variance: Because there are only 2 reps/treatment, the Brown-Forsythe's test of homogeneity of variances is not valid. The plot of the residuals vs means indicates a decreasing spread in the residuals as a function of the means. Thus, the condition of equal variances may be violated.

2. (10 pts) Group the three Humidity Levels on the basis of the mean growth of the fungi. Use $\alpha_F = .05$.

   The T*H*M (p-value =.2205) and T*H (p-value =.1572) interactions are not significant but M*H (p-value =.0002) is significant. This implies that when making inferences about the levels of Humidity, it is necessary to make the inferences separately at each level of Growth Media averaged over the Time levels. Using the p-values displayed in the M*H unadjusted p-value matrix on page 5 of SAS output for the M*H LS Means, we have by comparing the p-values to $\alpha_{pc} = .05/6 = .00833$. The Tukey adjusted p-values are adjusting for $\binom{6}{2} = 15$ comparisons and the problem is requesting only 6 of the 15. The Tukey adjusted p-values would be too conservative which may lead to inflated probabilities of Type II errors.

- For $M_1 : G_1 = \{45\%\}, G_2 = \{60\%\}\ and\ G_3 = \{85\%\}$

- For $M_2 : G_1 = \{45\%, 60\%\}\ and\ G_2 = \{85\%\}$

3. (10 pts) The T*H*M (p-value =.2205) interaction is not significant but both M*H (p-value =.0002) and M*T (p-value =.0003) are significant. This implies that when making inferences about differences in the two Media, it is necessary to make the inferences separately at each level of Humidity averaged over the levels of Time and separately at each level of Time averaged over the levels of Humidity. Thus it is necessary to use the unadjusted p-values for T*M and H*M means (6= $3\binom{2}{2} + 3\binom{2}{2}$ pairs) with $\alpha_{pc} = .05/6 = .00833$. The Tukey adjusted p-values would adjust for $2\binom{6}{2} = 30$ pairs whereas the problem is requesting only 6 of the 30 pairwise comparisons. The Tukey adjusted p-values would be too conservative which may lead to inflated probabilities of Type II errors.

   For all 6 comparisons of the two Media,

- $H = 45\% : G_1 = \{M_1\}, G_2 = \{M_2\}$
- $H = 60\% : G_1 = \{M_1\}, G_2 = \{M_2\}$
- $H = 85\% : G_1 = \{M_1\}, G_2 = \{M_2\}$
- $T = 25 : G_1 = \{M_1\}, G_2 = \{M_2\}$
- $T = 50 : G_1 = \{M_1\}, G_2 = \{M_2\}$
- $T = 75 : G_1 = \{M_1\}, G_2 = \{M_2\}$

   we obtain $p-value < .00833$ which would imply that there is significant evidence of a difference in the mean growth of the fungus for the two Media for the 3 levels of Time and for the three levels of Humidity.

4. (8 pts) A 95% confidence interval for the mean growth of fungus over a 50 hour period in Growth Media M_1 with a humidity of 60%.

   $\hat{\mu}_{50,60\%,M_1} = 6.45$ with $\hat{SE} = \sqrt{MSE/2} = \sqrt{4.63556/2} = 1.522$, with df=18. Therefore, a 95% C.I. for the mean is

$$\hat{\mu}_{50,60\%,M_1} \pm t_{.025,18}\hat{SE} = 6.45 \pm (2.101)(1.522) = 6.45 \pm 3.2 = (3.25,\ 9.65)$$

Problem II (56 points) CIRCLE (A, B, C, D, or E) corresponding to the BEST answer. Only ONE LETTER should be CIRCLED for each of the 14 questions.

(1.) C

(2.) A

(3.) B. All received partial credit if answered A.

(4.) D. positive correlation results in wider interval. All of you received the credit even if you marked B because majority of the students answered as B

(5.) D

(6.) D

(7.) (i) C, (ii) B

(i) If the interaction between Type and Thickness is significant:

From Table XI, the coefficients for a linear trend in t=4 levels of a factor are (-3,-1,1,3). Because there is an interaction between Type of Capsule and Wall Thickness, the two contrasts in Wall Thickness, one for each Type of Capsule, are as follows:

- T_1: $C_1 = -3\mu_{11} - \mu_{12} + \mu_{13} + 3\mu_{14}$
- T_2: $C_2 = -3\mu_{21} - \mu_{22} + \mu_{23} + 3\mu_{24}$

(ii) If the interaction between Type and Thickness is not significant:

Because there is not an interaction between Type of Capsule and Wall Thickness, the two contrasts in Wall Thickness, one for each Type of Capsule, will be combined:

- $C = 3\mu_{.1} - \mu_{.2} + \mu_{.3} + 3\mu_{.4} = -3\mu_{11} - \mu_{12} + \mu_{13} + 3\mu_{14} - 3\mu_{21} - \mu_{22} + \mu_{23} + 3\mu_{24}$

(8.) D

(9.) E

(10.) E. A researcher is designing an experiment having 2 factors: Factor F_1 with 2 fixed levels and Factor F_2 with 4 fixed levels. The researcher decides to use $r = 9$ replications in the experiment and $\alpha = .01$ test. What is the power of the test to detect a difference of at least 18 units in two or more of the treatment means using the estimate $\hat{\sigma}_e = 9$:

- $t = 2 \cdot 4 = 8, \quad \sigma_e = 9, \quad \alpha = .01 \quad D = 18 \Rightarrow \lambda = \frac{(r)(D)^2}{2(\sigma_e)^2} = \frac{(9)(18)^2}{2(9)^2} = 18 \quad \Rightarrow \quad \phi = \sqrt{\lambda/t} = \sqrt{\frac{18}{8}} = 1.5$
- $\nu_1 = t - 1 = 7 \Rightarrow$ Use Table IX on page 610 in STAT 642 Tables with $\nu_2 = (r-1)t = (9-1)(8) = 64$ and $\phi = 1.5, \nu_2 = 64, \alpha = .01$, we read from the graph Power $= .65 < 0.80$. Thus, 9 reps is not sufficient to achieve the desired power.

(11.) D

(12.) A

(13.) C

<u>EXAM 2 SCORES:</u> n = 58

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Min =  37, Q(.25) =  72.75, Q(.5) = 84.5 , Mean = 80.1, Q(.75) = 90, Max =  99
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