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STAT 642
Final Exam
5/5/22

Problem I:

- Fish (4 levels - Fixed) RV (Carcinogen level in bladder section)
- Assessment Technique (3 levels - Fixed)
- Mice (Litter) (4 levels - Random)

EU: Mice (Litter) - Trt (Fish)

Bladder Section (Mouse, Litter) - Technique

(1) D
(2) D
(3) B, C
(4) A, E
(5) B
(6) (a) Response Variable - Carcinogen level in Bladder Section
(b) Covariates - None
(c) Subsampling - None
(d) Repeated Measures - None

(7) A
(8) B
(9) B
(10) D

$$y_{ijkl} = a_i + b_j + \tau_k + (ab)_{ij} + (a\tau)_{ik} + (b\tau)_{jk} + (ab\tau)_{ijk} + e_{ijkl}$$

Problem II ; F - (3 levels - fixed) L - (2 levels - fixed) O(L) - (4 levels - Random)

$a=3,\ b=2,\ c=4,\ r=2$

SV	levels	Q_F	Q_L	$\sigma^2_{O(L)}$	Q_{F*L}	$\sigma^2_{F*O(L)}$	σ^2_e
F	3	16	0	0	0	2	1
L	2	0	24	16	0	2	1
O(L)	8	0	0	6	0	2	1
F*L	6	0	0	0	8	2	1
F*O(L)	24	0	0	0	0	2	1
Error	48	0	0	0	0	0	1

SV	DF	MS	EMS
F	2	41.40	$\sigma^2_e + 2\sigma^2_{F*O(L)} + 16Q_F$
L	1	4.08	$\sigma^2_e + 2\sigma^2_{F*O(L)} + 6\sigma^2_{O(L)} + 24Q_L$
O(L)	6	11.99	$\sigma^2_e + 2\sigma^2_{F*O(L)} + 6\sigma^2_{O(L)}$
F*L	2	9.52	$\sigma^2_e + 2\sigma^2_{F*O(L)} + 8Q_{F*L}$
F*O(L)	12	5.49	$\sigma^2_e + 2\sigma^2_{F*O(L)}$
Error	24	2.33	σ^2_e
Total	47		

(2) For Interaction

- $H_0: Q_{F*L}=0 \quad H_a: Q_{F*L}\neq 0$
- Test Statistic : $F^* = MS_{F*L}/MS_{F*O(L)} = \frac{9.52}{5.49} = 1.73406$
- Decision : Fail to reject H_0 at the $\alpha=0.05$ level
  $F_{0.05,2,12} = 3.885 > 1.73406 = F^*$

For Fixture :

- $H_0: Q_F=0$; $H_a: Q_F\neq 0$
- Test statistic : $F^* = MS_F/MS_{F*O(L)} = 41.40/5.49 = 7.541$
- Decision : Reject H_0 and conclude we have significant evidence of differences in the trt means due to fixture at the $\alpha=0.05$ level.
  - $F_{0.05,2,12} = 3.885 < 7.541 = F^*$

Problem II (continued):

For Assembly Layout:

- $H_0: Q_L = 0 \qquad H_a: Q_L \neq 0$
- Test statistic $F^* = MS_L / MS_{O(L)} = \frac{4.08}{11.99} = 0.34028$
- Decision: Fail to reject H_0 at the $\alpha = 0.05$ level.

$F_{0.05,1,6} = 5.987 > 0.34028 = F^*$

(3) Compute a 95% CI for the mean time to assemble a circuit board using Layout 2. Justify your answer clearly with the estimated mean, SE of the mean & also critical value

$y_{ijkl} = \mu + \tau_i + \gamma_j + b_{k(j)} + (\tau\gamma)_{ij} + d_{ik(j)} + e_{ijkl}$

$$\boxed{\hat{\bar{y}}_{\cdot 2 \cdot\cdot} = 26.38}$$

$$var(\bar{y}_{\cdot j \cdot\cdot}) = var(\mu + \bar{\tau}_{\cdot} + \gamma_j + \bar{b}_{\cdot(j)} + \overline{(\tau\gamma)}_{\cdot j} + \bar{d}_{\cdot\cdot(j)} + \bar{e}_{\cdot j \cdot\cdot})$$
$$= var(\bar{b}_{\cdot(j)} + \bar{d}_{\cdot\cdot(j)} + \bar{e}_{\cdot j \cdot\cdot})$$
$$= var(\bar{b}_{\cdot(j)}) + var(\bar{d}_{\cdot\cdot(j)}) + var(\bar{e}_{\cdot j \cdot\cdot})$$
$$\frac{\sigma^2_{O(L)}}{4} + \frac{\sigma^2_{\tau \cdot O(L)}}{(3)(4)} + \frac{\sigma^2_e}{(6)(3)(4)}$$

- $\hat{\sigma}^2_e = 2.33$;
- $\sigma^2_{\tau O(L)} = \frac{5.49 - 2.33}{2} = 1.58$
- $\sigma^2_{O(L)} = \frac{11.99 - 2(1.58)}{6} = 1.471667$

$$var(\bar{y}_{\cdot j \cdot\cdot}) = \frac{1.471667}{4} + \frac{1.58}{12} + \frac{2.33}{72}$$
$$= 0.3679 + 0.131667 + 0.0323611 = 0.532$$

$$\boxed{\hat{SE}(\hat{\mu}_{\cdot 2 \cdot}) = 0.72938}$$

95% CI: $\bar{y}_{\cdot 2 \cdot\cdot} \pm q_{tukey}(\frac{0.05}{2}, v)$ where v is the Satterwaithe df for the $SE(\hat{\mu}_{\cdot j \cdot})$

Problem III: 7 Factors A, B, C, D, E, F, G each having 2 levels
- Conduct 2^{7-3} FF design.
- Generators: ABDG = -1, ACDF = +1 BCDE = -1

(1) How many trts would be observed in this experiment?

$2^{7-3} = 2^4 = \underline{16 \text{ trts}}$

(2) Would the trt (A, B, C, D, E, F, G) = (+, -, +, +, -, +, +) be used in the experiment.

$\boxed{\text{NO: b/c BCDE} = -1 \text{ but for the given treatment BCDE} = (-1\cdot1\cdot1\cdot-1) = 1 \neq -1.}$

(3) What is the resolution of this design:

Implicit contrasts: $2^3 - 3 - 1 = 4$.
- ABDG · ACDF = BCFG
- ABDG · BCDE = ACEG
- ACDF · BCDE = ABEF
- ABDG · ACDF · BCDE = BCFG · BCDE = DEFG
- Resolution IV (4) → We know this design is Resolution IV b/c the # of factors in the shortest generator is 4.

	ABDG	ACDF	BCDE	BCFG	ACEG	ABEF	DEFG
A	BDG	CDF	ABCDE	ABCFG	CEG	BEF	ADEFG

- The interactions shown above would need to be negligible in order to be able to obtain an estimate of the main effect of Factor A.

Problem IV:

(1) E

(2) C

(3) E

(4) D

(5) D

(6) C

(7) A - if savings could be made using Tukey's HSD on the adjusted trt means at specified values of the covariate

(8) D

(9) C

$MS_{Area} = 36$, $MSE = 12$

$$RE = \frac{SS_{B} + r(t-1)MSE}{(rt-1)MSE}$$

$MS_{Area} = 36$, $r = 10 \Rightarrow SS_{Area} = 36(10-1) = 324$

$$RE = \frac{324 + 10(3)12}{39(12)} = 1.4615$$

$\Rightarrow$ it would take $\approx$ 50% more obs in a CRD to achieve the same precision