Statistics 630 - Assignment 1
(partial solutions)

Note: I highly recommend that you do not just read the solutions but rather that you *fix your mistakes* by carefully redoing the problems you missed. This will help you avoid similar mistakes in the future. If you still have questions, please post them on the discussion board or come see me or the grader.

These solutions are not meant to be complete. In some cases only the final answer is shown, with no derivation and not calculated entirely. In others, the main ideas are provided but they may not include all the necessary details or the full explanation and justification that would be expected of you. Some problems or parts are not shown (including some that have answers in the back of the textbook). [Comments in square brackets are not part of the solution.]

These solutions might also suggest shortcuts or notational devices that could improve your presentation for later work.

1. Exer. 1.2.2.

   (c) Any four outcomes out of the eight will give probability $\frac{1}{2}$, hence there are $\binom{8}{4} = 70$ such events.

2. (c) $A \cap C = \{(5,4),(6,4)\}$,
   $B \cup C = \{(2,1),(3,2),(4,3),(5,4),(6,5),(3,1),(4,2),(5,3),(6,4),(4,1),(5,2),(6,3),(5,1),$
   $(6,2),(6,1),(1,4),(2,4),(3,4),(4,4)\}$,
   $A \cap (B \cup C) = \{(5,4),(6,3),(6,4),(6,5)\}$.

   (e) $\mathsf{P}(A \cap C) = \frac{1}{18}$, $\mathsf{P}(B \cup C) = \frac{19}{36}$, $\mathsf{P}(A \cap (B \cup C)) = \frac{1}{9}$.

   (f) No, $\frac{1}{18} \neq \frac{5}{18} \times \frac{1}{6}$.

   (g) Same as $\mathsf{P}(\text{sum equals } 7) = \frac{1}{6}$.

3. Exer. 1.3.2. $\frac{1}{3}$ and $\frac{1}{6}$, resp. Hint: make a table.

	6:00	not 6:00	
11:00	1/3	1/6	1/2
not 11:00	1/3	1/6	1/2
	2/3	1/3	1

   Exer. 1.3.4. 15% and 25%, resp. By the properties of probability,

$$\max(\mathsf{P}(A), \mathsf{P}(B)) \le \mathsf{P}(A \cup B) \le \min(\mathsf{P}(A) + \mathsf{P}(B), 1).$$

   Exer. 1.3.10(a). This is done by repeatedly using the basic two-event property. First,

$$\begin{aligned}\mathsf{P}(A \cup B \cup C) &= \mathsf{P}(A) + \mathsf{P}(B \cup C) - \mathsf{P}(A \cap (B \cup C)) \\ &= \mathsf{P}(A) + \mathsf{P}(B) + \mathsf{P}(C) - \mathsf{P}(B \cap C) - \mathsf{P}((A \cap B) \cup (A \cap C)). \qquad (1)\end{aligned}$$

   Looking at the last term in (1),

$$\begin{aligned}\mathsf{P}((A \cap B) \cup (A \cap C)) &= \mathsf{P}(A \cap B) + \mathsf{P}(A \cap C) - \mathsf{P}((A \cap B)) \cap (A \cap C)) \\ &= \mathsf{P}(A \cap B) + \mathsf{P}(A \cap C) - \mathsf{P}(A \cap B \cap C). \qquad (2)\end{aligned}$$

   Plugging (2) back into (1) then gives the desired equality.

(continued next page)

4. Exer. 1.4.6. The sum is *less than* 4 if and only if one card is an ace and the other is either an ace or a 2.

$$1 - \mathsf{P}(\text{sum is less than } 4) = 1 - \frac{4(3) + 2(4)(4)}{52(51)} = 0.983409.$$

   Exer. 1.4.12. $\frac{1}{6} \times \mathsf{P}(\text{at least one Head}) = \frac{7}{48}$. Alternatively, the sample space is $\mathcal{S} = \{(i, j) : i \in \{1, 2 \ldots, 6\}, j \in \{0, 1, 2, 3\}\}$ and find $\mathsf{P}(\{(1, 1), (2, 2), (3, 3)\})$.

5. (a) Everyone should get a different value. If you tried the simulation more than once, you will get different values. My estimate was 0.38957. [If you recall confidence intervals for a proportion then you might note that this is accurate to plus/minus 0.003 with 95% confidence.]
   (b) 0.391434.
   (c) 119, by trial and error. You can test multiple values in your R code, for example with (101:150) to get probabilities for samples of sizes 101–150.

6. Exer. 1.5.8. Use the definition of conditional probability (and the multiplication rule).

$$\mathsf{P}(\text{snow} \mid \text{accident}) = \frac{\mathsf{P}(\text{accident} \mid \text{snow})\mathsf{P}(\text{snow})}{\mathsf{P}(\text{accident})} = \cdots = 0.80.$$

   Exer. 1.5.10. Using values from Exer. 1.4.11 and the fact that $\{\text{all red}\} \subset \{\text{all same}\}$,

$$\mathsf{P}(\text{all red} \mid \text{all same}) = \frac{\mathsf{P}(\text{all red})}{\mathsf{P}(\text{all same})} = \frac{\frac{\binom{5}{3}\binom{6}{3}}{\binom{12}{3}\binom{18}{3}}}{\frac{\binom{5}{3}\binom{6}{3}}{\binom{12}{3}\binom{18}{3}} + \frac{\binom{7}{3}\binom{12}{3}}{\binom{12}{3}\binom{18}{3}}} = \cdots = 0.025316.$$

7. Exer. 1.5.7. Let F = "fastball", C = "curveball" and H = "home run".
   (a) $\mathsf{P}(H) = 0.08 \times 0.80 + 0.05 \times 0.20 = 0.074$.
   (b) $\mathsf{P}(C \mid H) = \frac{0.05 \times 0.20}{.074} = 0.135135$.
   (c) $\mathsf{P}(C \mid H^c) = \frac{0.95 \times 0.20}{1 - .074} = 0.205184$.