SOLUTIONS STAT 641 - EXAM II

I. (40 points) CIRCLE (A, B, C, D, or E) corresponding to the **BEST** answer. Only one letter is allowed per question. Partial credit will be given on problems if you show your calculations.

(1.) **C.** the metallurgist wanted a lower bound with 99% of population values greater than the bound

(2.) **B.** $F(\cdot)$ has a shifted t-distribution with df=3 because the plot reveals a symmetric distribution with tails heavier than a normal distribution.

(3.) **A.** This is the definition of a C.I.

(4.) **C.** If the estimator is biased with a small variance then most of its values will be near its expected value

(5.) **B.** Because of the positive correlation, $S/\sqrt{n}$ will under estimate the true standard error of $\hat{L}$ and hence the confidence interval will be too narrow to be a 90% C.I.

(6.) **E.**

A. If $\hat{\theta} = Y_{(n)}$ then the asymptotic distribution is an extreme value distribution

B. The accuracy of the bootstrap procedure also depends on how well the edf, $\hat{F}$, matches the population cdf F

C. Recall the examples in HO 10.

D. The Box-Cox procedure works well only with data from skewed or heavy-tailed distributions but not from mutli-modal distributions

E. Thus, none of A-D are true

(7.) **E.** None of the above statements are true because $\bar{Y}$ is an unbiased estimator of μ for all sample sizes.

(8.) **D.**

(9.) **D.**

(10.) **B.** $n = \frac{Z^2_{.025}\hat{p}(1-\hat{p})}{(.05)^2} \leq \frac{(1.96)^2 .\hat{2}(1-.2)}{(.05)^2} = 245.9 \ \Rightarrow$ use n = 246.

Part II. (60 points)

1. The square root transformation provides an excellent fit to the data because:

   a. The plotted points $(X_{(i)},\ Q_Z(u_i)), i = 1, \ldots, 60$ are very close to a straight line

   b. The SW test yields W=.974 which has an associated .10 <p-value < 0.50 for the X = $\sqrt{Y}$ data values

(2.) The $P = .90,\ \gamma = .95$ tolerance interval would be obtained by first noting that the $X = \sqrt{Y}$ has a normal distribution and computing a (P=.9, γ= .95) tolerance interval for the distribution of X:

$\bar{X} \pm K_{.90,.95} S_X = 15.31 \pm (1.96)(8.80) = 15.31 \pm 17.248 = (-1.938,\ 32.558) = (0,\ 32.558)$, X is a nonnegative random variable.

Next, invert the endpoints of the tolerance interval on the distribution of X to obtain the tolerance interval for the distribution of lifetimes, $Y = X^2$:

$(0,\ 32.558^2) = (0,\ 1060.023)$ which implies $(0,\ 1060023)$ is a (.9, .95) tolerance interval for the distribution of the lifetimes.

A less efficient distribution-free Tolerance Interval considering n=60 would be obtained by using $m = 2$ from the table yielding $(Y_{(1)},\ Y_{(60)}) = (.1,\ 1129.2)$. Thus, $(100,\ 1129200)$ would be a less efficient tolerance interval for the distribution of lifetimes.

(3.) A 95% C.I. on the proportion of cords receiving the prescribed stress that would have a lifetime greater than 750,000 hours is given by

Let Y be the number of cords out of the 60 having lifelength greater than 750,000. From the data B=8. Because, $n = 60 > 40$ and $\min(n\hat{p},\ n(1-\hat{p}) = 8 > 5$, the Agresti-Coull C.I. would be appropriate.

$\tilde{Y} = B + .5(1.96)^2 = 9.9208 \quad \tilde{n} = n + (1.96)^2 = 63.8416 \ \Rightarrow\ \tilde{p} = 9.9208/63.8416 = .1554$

The A-C 95% C.I. on p is $.1554 \pm 1.96\sqrt{.1554(1-.1554)/63.8416} = .1554 \pm .0889 = (.067,\ .244)$

(4.) Find n such that we are 99% confident that $\bar{Y}$ is within 5000 hours of μ_Y.

From the data, an estimate of σ_Y would be $S = 291.0$. Also, tentatively using the Central Limit Theorem to approximate the sampling distribution of $\frac{|\bar{Y}-\mu|}{\sigma_Y/\sqrt{n}}$ we obtain the following:

$$.99 = P\left[|\bar{Y}-\mu| < 5000/1000\right] = P\left[\frac{|\bar{Y}-\mu|}{\sigma_Y/\sqrt{n}} < \frac{5}{\sigma_Y/\sqrt{n}}\right] \approx P\left[|Z| < \frac{5}{\sigma_Y/\sqrt{n}}\right] \text{ also, } P\left[|Z| < 2.576\right] = .99 \ \Rightarrow$$

$$\frac{5}{\sigma_Y/\sqrt{n}} = 2.576 \ \Rightarrow\ n \approx \frac{(2.576)^2(\hat{\sigma}_Y)^2}{(5)^2} = \frac{(2.576)^2(291.0)^2}{(5)^2} = 22476.97 \ \Rightarrow\ \text{would need a sample size of } n = 22477$$

Exam 2 Scores for STAT 641

Min = 52, Q(.25) = 77, Q(.5) = 83, Mean = 81.9 , Q(.75) = 90, Max = 100