Stat 642 Spring 2022 - Solutions for Assignment 10

Problem I. (24 points) This a RCBD with fields being the blocks. There is a 2x3 factorial treatment structure. This is a valid crossed treatment structure because the three levels associated with No Nitrogen Inhibitor are unique treatments because the nitrogen was applied at three different times. The time of applying nitrogen to the fields has 3 fixed levels and Inhibitor has two fixed levels, None or .5 lb/acre. The EU and MU are the plots in each field.

1. model: $y_{ijk} = \mu + b_k + \tau_i + \gamma_j + (\tau\gamma)_{ij} + e_{ijk}$, $i = 1, 2$; $j = 1, 2, 3$; $k = 1, 2, 3$; where

    y_{ijk} amount of Nitrogen uptake in Field k using Nitrogen Inhibitor (NI) i during Date (D) j

    b_k is the random field effect with $b_k \sim iid\ N(0, \sigma_B^2)$

    τ_i is the fixed effect due to ith level of nitrogen inhibitor (NI), with $\tau_2 = 0$

    γ_j is the fixed effect due to jth level of Date (D), with $\gamma_3 = 0$

    $(\tau\gamma)_{ij}$ is the fixed interaction effect of combining the ith level of NI with the jth level of D, with $(\tau\gamma)_{2j} = 0$ and $(\tau\gamma)_{i3} = 0$

    e_{ijk} is residual effect due to all other factors with $e_{ijk} \sim iid\ N(0, \sigma_e^2)$

    b_k and e_{ijk} are mutually independent

- From the residual plots on the next page, the normality and equal variance conditions appear to be satisfied.

    AOV Table ($a = 2, b = 3, r = 3$):

Source	df	SS	MS	F	$Pr > F$	Expected MS
Field	2	395.243	197.622			$6\sigma_F^2 + \sigma_e^2$
NI	1	548.909	548.909	14.77	.0032	$9Q_{NI} + \sigma_e^2$
D	2	1426.990	713.495	19.20	.0004	$9Q_D + \sigma_e^2$
NI*D	2	453.614	226.807	6.10	.0185	$3Q_{D*NI} + \sigma_e^2$
Residual	10	371.523	37.152			σ_e^2
Total	17	3196.280				

    From PROC MIXED output: $\hat{\sigma}_F^2 = 26.7449$ and $\hat{\sigma}_e^2 = 37.1523$

2. Marginal mean for NI: $Var(\hat{\mu}_{i.}) = Var(\bar{Y}_{i..}) = Var(\bar{b}_. + \bar{e}_{i..}) = \frac{\sigma_F^2}{3} + \frac{\sigma_e^2}{(3)(3)} = \frac{3\sigma_F^2 + \sigma_e^2}{9} = \frac{EMS_F + EMSE}{18} \Rightarrow$

    $\widehat{SE}(\hat{\mu}_{i.}) = \sqrt{\frac{\hat{\sigma}_F^2}{3} + \frac{\hat{\sigma}_e^2}{(3)(3)}} = \sqrt{\frac{26.7449}{3} + \frac{37.1523}{9}} = 3.6115$ or

    $\widehat{SE}(\hat{\mu}_{i.}) = \sqrt{\frac{MS_F + MSE}{18}} = \sqrt{\frac{197.622 + 37.152}{18}} = 3.6115$

    Marginal mean for D: $Var(\hat{\mu}_{.j}) = Var(\bar{Y}_{.j.}) = Var(\bar{b}_. + \bar{e}_{.j.}) = \frac{\sigma_F^2}{3} + \frac{\sigma_e^2}{(2)(3)} = \frac{2\sigma_F^2 + \sigma_e^2}{6} = \frac{EMS_F + 2EMSE}{18} \Rightarrow$

    $\widehat{SE}(\hat{\mu}_{i.}) = \sqrt{\frac{\hat{\sigma}_F^2}{3} + \frac{\hat{\sigma}_e^2}{(2)(3)}} = \sqrt{\frac{26.7449}{3} + \frac{37.1523}{6}} = 3.8868$ or

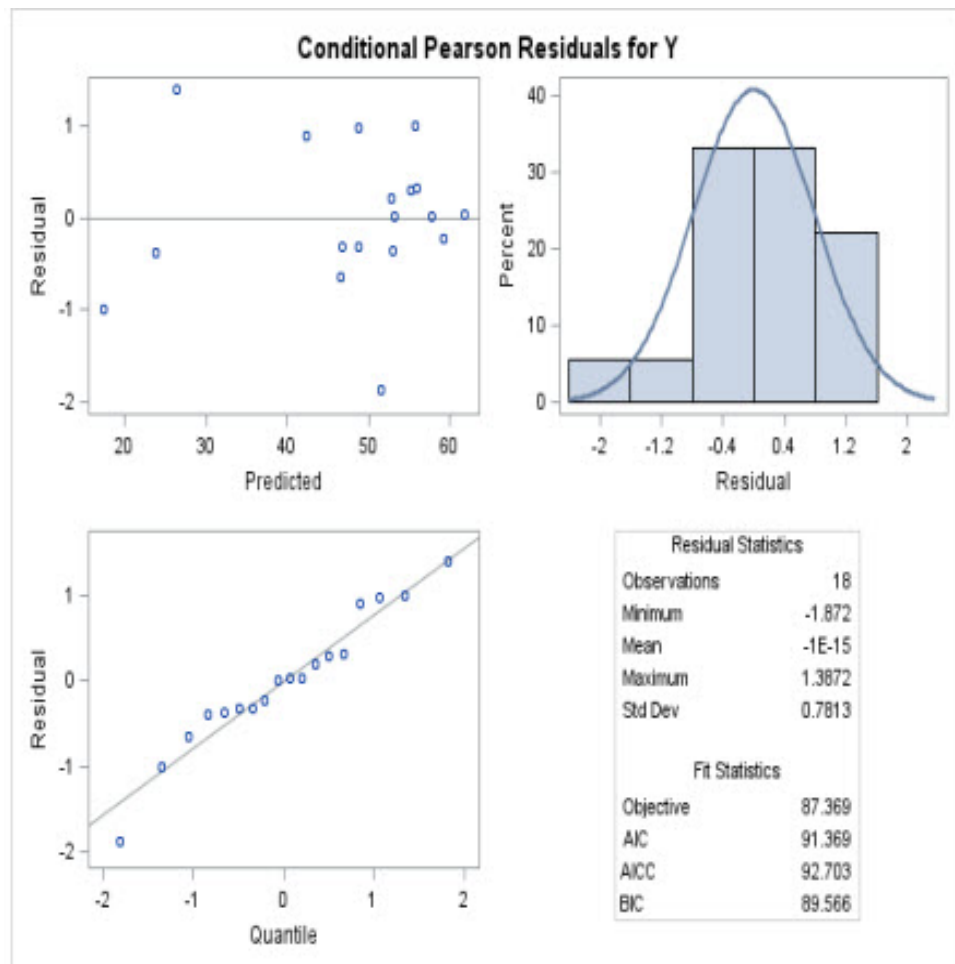 $\widehat{SE}(\hat{\mu}_{i.}) = \sqrt{\frac{MS_F + 2MSE}{18}} = \sqrt{\frac{197.622 + 2(37.152)}{18}} = 3.8868$

    Cell (Treatment) means : $Var(\hat{\mu}_{ij}) = Var(\bar{Y}_{ij.}) = Var(\bar{b}_. + \bar{e}_{ij.}) = \frac{\sigma_F^2}{3} + \frac{\sigma_e^2}{3} = \frac{\sigma_F^2 + \sigma_e^2}{3} = \frac{EMS_F + 5EMSE}{18} \Rightarrow$

    $\widehat{SE}(\hat{\mu}_{i.}) = \sqrt{\frac{\hat{\sigma}_F^2}{3} + \frac{\hat{\sigma}_e^2}{3}} = \sqrt{\frac{26.7449}{3} + \frac{37.1523}{3}} = 4.6151$ or

    $\widehat{SE}(\hat{\mu}_{i.}) = \sqrt{\frac{MS_F + 5MSE}{18}} = \sqrt{\frac{197.622 + 5(37.152)}{18}} = 4.6151$

3. From the AOV table, there is significant evidence of an interaction between NI and D (p-value = .0185) and significant evidence of nonzero main effects NI has p-value=.0032 and D has p-value=.0004. However, because there was significant evidence of an interaction, the main effects are not interpretable.

4. Relative efficiency Computation: $t = 6$, $r = 3$ and $K = \frac{r(t-1)}{rt-1} = 0.8824$, $H = \frac{MS_{Block}}{MSE} = 5.319$. Thus, $RE = K + (1 - K)H = 1.51$. We would need $\approx 51\%$ more EU's in a CRD to obtain same degree of precision as was obtained in RCBD. Need $r * RE = (3)(1.51) = 4.53 \approx 5$ reps/TRT in a CRD.

5. The following plots of the Conditional Studentized Residuals indicate that the residuals are approximately normally distributed with equal variances.

Conditional Pearson Residuals for Y

Residual Statistics	
Observations	18
Minimum	-1.872
Mean	-1E-15
Maximum	1.3872
Std Dev	0.7813

Fit Statistics	
Objective	87.369
AIC	91.369
AICC	92.703
BIC	89.566

- If the conditions of normality and/or equal variances were violated, then the Friedman rank based procedure could be applied to the data by creating 6 treatments, the cross of Nitrogen Inhibitor levels and Application Time.

Problem II. (12 points)

1. Latin Square Design: Blocks = Day: $S,\ M,\ W,\ F$; Blocks = Streams: $S1,\ S2,\ S3,\ S4$

    TRT = StreamLocation:$L1,\ L2,\ L3,\ L4$

	Stream			
Day	S1	S2	S3	S4
S	L1	L2	L3	L4
M	L2	L3	L4	L1
W	L3	L4	L1	L2
F	L4	L1	L2	L3

Source	df
Day	3
Stream	3
Location	3
Error	6
Total	15

2. This is a Latin Square Design with 2 subsamples on each observation.

   ANOVA table

Source	df
Day	3
Stream	3
Location	3
Error	6
Subsample	16
Total	31

Problem III (15 points)

1. Check the constraints with t=7, b=7, r=4, k=4:

   R1 $N = tr = 28 = bk$

   R2 $\lambda = \frac{r(k-1)}{(t-1)} = \frac{4(4-1)}{(7-1)} = 2$ is an integer and is the number of times each pair of TRT's appear together in the same block.

   R3 $b = 7 \geq 4 = 7$

   R4 $\lambda = 2 < 4 = r < 7 = b$

   All four restrictions hold thus the design is a BIBD.

2. From the SAS output, there is strong evidence that the residuals do not have a normal distribution and the seven treatment populations have unequal variances. A log transformation appears to yield normally distributed residuals with equal variances. The analysis will be conducted on the transformed data: log of breakdown times.

   From the SAS output, the ratio of MS_{VOLT} to MSE is $F = \frac{17.3282}{.015646} = 1107.52$ with

   p-value = $1 - pf(1107.52, 6, 6) = 7.3 \times 10^{-9}$. Thus, there is significant evidence of a difference in the mean time to breakdown across the seven voltages.

3. LS Means and Raw Means are given below:

   ```
        Voltage        LS Mean           Raw Mean
          V24           5.6182             5.6472
          V28           5.2389             5.2715
          V32           3.8639             3.8777
          V36           2.1783             2.1947
          V40           1.8711             1.8580
          V44           0.9347             0.8919
          V48          -0.4060            -0.4418
   ```

   Notice that the LS Means from PROC MIXED differ from the Raw Means in that the LS Means are adjusted for the block (Day) effects.

4. The test for a linear trend has a p-value less than .0001 so there is significant evidence that there exists a linear trend in the mean time to breakdown with increasing voltage. The sign of the estimated contrast is negative so we can conclude that there is a decreasing trend in the mean time to breakdown with increasing voltage.

Problem IV. (24 points) This is a RCBD with two Field as the blocks. There is a split plot treatment assignment with the Whole Plot treatment being Phosphorus level and EU being half of a field. The Split Plot treatment is the cross of Water with Nitrogen (3x3) and the EUs are the 9 regions within each half of a field.

1. $y_{ijk\ell} = \mu + \tau_i + d_j + (\tau d)_{ij} + \gamma_k + \beta_\ell + (\gamma\beta)_{k\ell} + (\tau\gamma)_{ik} + (\tau\beta)_{i\ell} + (\tau\gamma\beta)_{ik\ell} + e_{ijk\ell}$ $i,j = 1,2$ $k,\ell = 1,2,3$

- τ_i - fixed effect of Phosphorus with $\tau_2 = 0$; $\quad d_j$ - random effect of Field iid $N(0,\sigma_d^2)$;

- $(\tau d)_{ij}$ - random effect of Field and Phosphorus interaction (whole plot error term) iid $N(0,\sigma_{(\tau d)}^2)$;

- γ_k - fixed effect of Water with $\gamma_3 = 0$; $\quad \beta_\ell$ - fixed effect of Nitrogen with $\beta_3 = 0$;

- $(\gamma\beta)_{k\ell}$ interaction effect of Water and Nitrogen with $(\gamma\beta)_{3\ell} = 0$ and $(\gamma\beta)_{k3} = 0$;

- $(\tau\gamma)_{ik}$ interaction effect of Phosphorus and Water with $(\tau\gamma)_{2k} = 0$ and $(\tau\gamma)_{i3} = 0$;

- $(\tau\beta)_{i\ell}$ interaction effect of Phosphorus and Nitrogen with $(\tau\beta)_{2\ell} = 0$ and $(\tau\beta)_{i3} = 0$;

- $(\tau\gamma\beta)_{ik\ell}$ interaction effect of Phosphorus, Water and Nitrogen with $(\tau\gamma\beta)_{2k\ell} = 0$, $(\tau\gamma\beta)_{i3\ell} = 0$, and $(\tau\gamma\beta)_{ik3} = 0$;

- $e_{ijk\ell}$ - All unspecified effects iid $N(0,\sigma_e^2)$; and d_j, $(\tau d)_{ij}$ and $e_{ijk\ell}$ are mutually independent.

- From the residual plots in the SAS output, the normality and equal variance conditions appear to be satisfied.

  In the following calculations of variance, only the random terms in the model need to be considered. There are methods of finding the standard errors: Using the variance component estimates from PROC MIXED and using the MS's from PROC MIXED.

  a. Difference of Two Phosphorus Means:
  $$Var(\bar{y}_{i...} - \bar{y}_{h...}) = Var(\bar{b}_. - \bar{b}_.) + Var((\tau b)_{i.} - (\tau b)_{h.}) + Var(\bar{e}_{i...} - \bar{e}_{h...}) =$$
  $$= 0 + 2\frac{\sigma_{B*P}^2}{2} + 2\frac{\sigma_e^2}{18} = 2\frac{9*\sigma_{B*P}^2 + \sigma_E^2}{18} = 2\frac{EMS_{P*F}}{18} \Rightarrow$$
  - $\hat{SE}(\hat{\mu}_{i..} - \hat{\mu}_{h..}) = \sqrt{27.5625/9} = 1.75$ - using MOM-AOV estimators
  - $\hat{SE}(\hat{\mu}_{i..} - \hat{\mu}_{h..}) = \sqrt{.8654 + 6.3206/9} = 1.2521$ - using REML's

  b. Difference of Two Nitrogen Means:
  $$Var(\bar{y}_{...\ell} - \bar{y}_{...p}) = Var(\bar{b}_. - \bar{b}_.) + Var((\tau b)_{..} - (\tau b)_{..}) + Var(\bar{e}_{...\ell} - \bar{e}_{...p}) = 2\sigma_e^2/12 = \frac{EMS_E}{6} \Rightarrow$$
  - $\hat{SE}(\hat{\mu}_{..l} - \hat{\mu}_{..p}) = \sqrt{\frac{6.320347}{6}} = 1.0264$ - using MOM-AOV and REML estimators

  c. Difference of Two Water Means:
  $$Var(\bar{y}_{..k.} - \bar{y}_{..m.}) = Var(\bar{b}_. - \bar{b}_.) + Var((\tau b)_{..} - (\tau b)_{..}) + Var(\bar{e}_{..k.} - \bar{e}_{..m.}) = 2\sigma_e^2/12 = \frac{EMS_E}{6} \Rightarrow$$
  - $\hat{SE}(\hat{\mu}_{.k.} - \hat{\mu}_{.m.}) = \sqrt{\frac{1}{6}6.3206} = 1.0264$ for both MOM-AOV and REML estimators

  d. Difference of Two Water at Two Nitrogen Means:
  $$Var(\bar{y}_{..kl} - \bar{y}_{..mp}) = Var(\bar{b}_. - \bar{b}_.) + Var((\tau b)_{..} - (\tau b)_{..}) + Var(\bar{e}_{..kl} - \bar{e}_{..mp}) = 0 + 0 + 2\sigma_e^2/4 \Rightarrow$$
  $$\hat{SE}(\hat{\mu}_{.kl} - \hat{\mu}_{.mp}) = \sqrt{\frac{1}{2}6.3206} = 1.7777$ \text{ for both MOM-AOV and REML estimators}$$

2. The AOV table and tests are given below

Source	df	Sum of Squares	Mean Squares	F-STAT	P-Value
FIELD	1	0.666944	0.666944	*	*
PHOSPHORUS	1	1.246944	1.246944	0.09	0.8160
PHOSPHORUS*FIELD	1	27.562500	27.562500	*	*
WATER	2	751.842222	375.921111	59.48	$< .0001$
NITROGEN	2	2768.648889	1384.324444	219.03	$< .0001$
NITROGEN*WATER	4	242.079444	60.519861	9.58	0.0004
PHOSPORUS*WATER	2	0.808889	0.404444	0.06	0.9383
PHOSPORUS*NITROGEN	2	12.708889	6.354444	1.01	0.3879
PHOSPORUS*NITROGEN*WATER	4	13.872778	3.468194	0.55	0.7026
Error	16	101.125556	6.320347		

Effect	Num. DF	Denom. DF	F Value	$Pr > F$	Conclusion
PHOSPHORUS	1	1	0.09	0.8160	Main Effect of P is not Sign.
WATER	2	16	59.48	$< .0001$	Not Interpretable due to N*W
NITROGEN	2	16	219.02	$< .0001$	Not Interpretable due to N*W
N*W	4	16	9.57	0.0004	Sign. Evidence of a N*W Interaction
P*W	2	16	0.06	0.9383	P*W Interaction Not Sign.
P*N	2	16	1.01	0.3879	P*N Interaction Not Sign.
P*N*W	4	16	0.55	0.7027	3-Factor Interaction Not Sign.

Problem V. (15 points) This is a CRD with two reps of the treatments which are a cross of Density and Moisture (3x3). The factor Day is a Repeated Measures variable.

1. The profile depicting a Density by Moisture Interaction is given below:

5
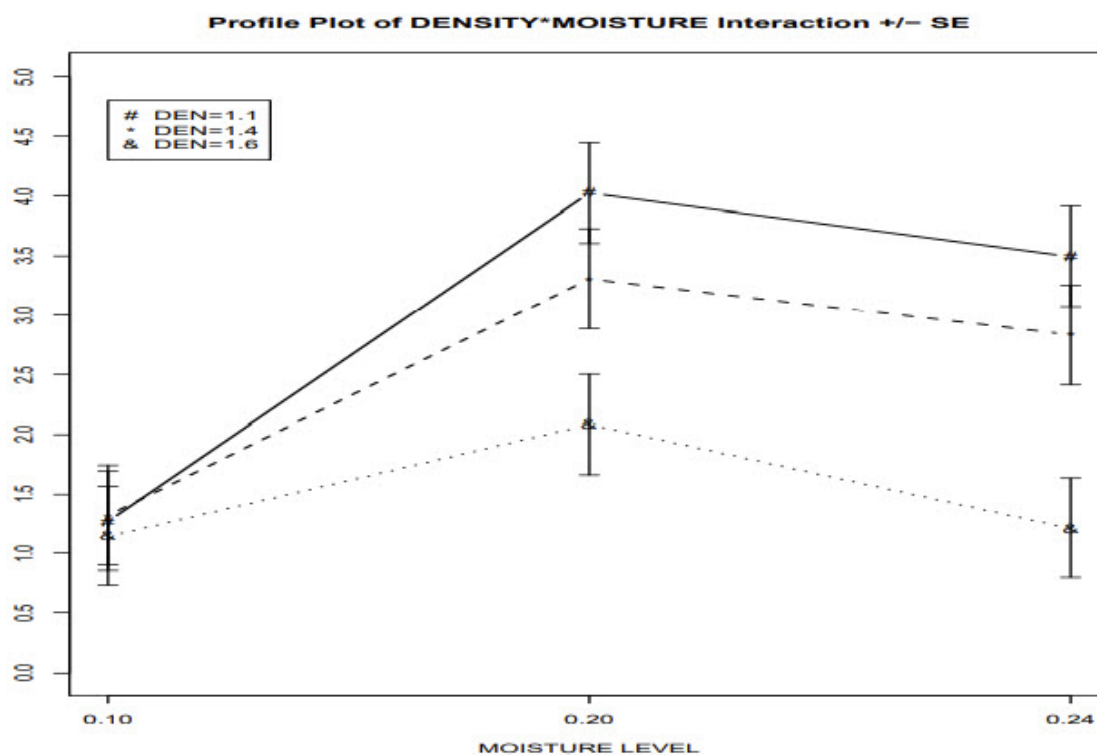
The AOV table from an AR(1) analysis:

```
          Type 3 Tests of Fixed Effects

              Num     Den
Effect         DF      DF    F Value   Pr > F
DEN             2       9      6.29    0.0196
MO              2       9     16.31    0.0010
DEN*MO          4       9      1.37    0.3179
DAY             2      18     28.30    <.0001
DEN*DAY         4      18      0.59    0.6722
MO*DAY          4      18     13.80    <.0001
DEN*MO*DAY      8      18      2.22    0.0761
```

From the AOV table, we conclude that there is not significant (p-value=.0761) evidence of a 3-way interaction between Density, Moisture, and Day. Amongst the two-way interactions, there is not significant evidence of a DEN*DAY (p-value=.6722) or DEN*MOISTURE (p-value=.3179) interaction. However, there is significant (p-value< .0001) evidence of an interaction between Moisture and Day. There is significant evidence of a MOISTURE main effect (p-value=.0010), DENSITY main effect (p-value=.0196) and a DAY main effect (p-value< .0001) but the interpretation of these effects is complicated by the two-way interaction between Moisture and Day. In order to evaluate the linear and quadratic trends across day, it is necessary to consider different levels of Moisture:

```
                   Contrasts
                  Num     Den
Label              DF      DF    F Value   Pr > F
LINEAR MO=.1        1      18     60.77    <.0001
QUAD MO=.1          1      18     19.58    0.0003
LINEAR MO=.2        1      18     20.39    0.0003
QUAD MO=.2          1      18      2.28    0.1488
LINEAR MO=.24       1      18      0.52    0.4814
QUAD MO=.24         1      18      8.25    0.0101
LINEAR DEN=1.1      1      18     30.94    <.0001
QUAD DEN=1.1        1      18      0.10    0.7580
LINEAR DEN=1.4      1      18     14.42    0.0013
QUAD DEN=1.4        1      18      0.02    0.9036
LINEAR DEN=1.6      1      18     13.47    0.0017
QUAD DEN=1.6        1      18      0.02    0.8861
```

From the above using $\alpha_{PC} = .05/12 = .0042$, it is observed that there are significant Linear Trends across Day at Moisture = .1, .2 but not at Moisture=.24. The linear trends were all significant at the three levels of Density. For the Quadratic Trends across Day, there was significant evidence of a quadratic trend across Day only at Moisture = .1.

Because of the two-way interaction between Moisture and Day, the grouping of Moisture Levels must be done separately for Day. If we use the Tukey-Kramer Adjusted p-values, our conclusions would have a very large probability for Type II error because we would be adjusting for $\binom{9}{2} = 36$ pairs of comparisons. In actuality, we are only making $3 \times \binom{3}{2} = 9$ pairs of comparisons. Therefore, we will use a Bonferroni adjusted comparison by using the unadjusted p-values from SAS and compare them to $\alpha_{pc} = .05/9 = .0056$

6

Comparison of Least Squares Means with UnAdjusted p-values

Day	p-values Moisture Comp.		
	.1 vs .2	.1 vs .24	.2 vs .24
1	.0041	.8081	.0024
2	< .0001	.0002	.0895
3	< .0001	.0004	.3033

Comparing the p-values in the above table to $\alpha_{pc} = .0056$ we obtain the following groupings:

Day	Moisture Groupings	
1	G1 = (.1,.24)	G2=(.20)
2	G1 = (.1)	G2 = (.20, .24)
3	G1 = (.1)	G2 = (.20, .24)

2. Using the output from MANOVA, we obtain the following results:

```
Contrast Variable: DAY_LINEAR

Source                      DF      Type III SS     Mean Square     F Value     Pr > F

Mean                         1      11.85654444     11.85654444      42.74      0.0001
DEN                          2       0.46888889      0.23444444       0.85      0.4610
MO                           2       5.25468889      2.62734444       9.47      0.0061
DEN*MO                       4       0.97727778      0.24431944       0.88      0.5125
Error                        9       2.49690000      0.27743333
```

```
Contrast Variable: DAY_QUADRATIC

Source                      DF      Type III SS     Mean Square     F Value     Pr > F

Mean                         1       0.00005926      0.00005926       0.00      0.9769
DEN                          2       0.01187407      0.00593704       0.09      0.9158
MO                           2       2.67871852      1.33935926      20.04      0.0005
DEN*MO                       4       1.16644815      0.29161204       4.36      0.0310
Error                        9       0.60160000      0.06684444
```

Summarizing the 3 days with a linear trend, we find that the interaction between Density and Moisture is not significant (p-value=.5125) and the main effect of Density is not significant (p-value=.4610) but there is a significant difference across the three levels of Moisture (p-value=.0061).

Summarizing the 3 days with a quadratic trend, we find that there is significant evidence of an interaction between Density and Moisture (p-value=.0310) and the main effect of Density is not significant (p-value=.9158) but there is a significant difference across the three levels of Moisture (p-value=.0005).

Problem VI. (10 points) This is a CRD with treatment being the three Alloys. There is a covariate in the study, the Diameter of the weld.

1. model: $Y_{ij} = \mu + \beta(X_{ij} - \bar{X}_{..}) + \tau_i + e_{ij}$, $i = 1, 2, 3$, $j = 1, \cdots, 6$, where τ_i is fixed effect of ith Alloy, X_{ij} is weld diameter, β is coefficient for linear regression of weld strength and e_{ij} is experimental error. We assume that 1) the slopes are the same for all alloys, 2) alloys do not affect the value of the weld diameter X_{ij}, 3) the weld diameters are measured without error and 4) $e_{ij} \sim iid\ N(0, \sigma_e^2)$.

   - From the residual plots in the SAS output, the normality and equal variance conditions appear to be satisfied.

   a. ANOVA tables:

   i. Fitting model I: $Y_i = \beta_0 + \beta_1 I_{1i} + \beta_2 I_{2i} + \beta_3 X_i + \beta_4 I_{1i} X_i + \beta_5 I_{2i} X_i + e_i$.

Source	df	SS	MS	F	$Pr > F$
A	2	5716.36	2858.18	74.90	$< .0001$
D	1	2838.92	2838.92	74.40	$< .0001$
D*A	2	258.27	129.13	3.38	0.0683
Error	12	457.899	38.158		

   ii. Fitting model II: $Y_i = \beta_0 + \beta_1 I_{1i} + \beta_2 I_{2i} + \beta_3 X_i + e_i$.

Source	df	SS	MS	F	$Pr > F$
A	2	2005.97	1002.99	19.61	$< .0001$
D	1	2838.90	2838.92	55.50	$< .0001$
Error	14	716.164	51.155		

   b. Tests:

   i. H_0 : Alloys have common Slopes vs H_1 : Alloys have different Slopes.
   We obtain $p - value = 0.0683 > 0.05$ and so fail to reject H_0. Thus, we conclude there is not a significant difference in the treatment slopes.

   ii. H_0 : No Alloy Difference vs H_1 : Main Effect due to Alloys.
   We obtain $p - value < 0.0001$ and so reject H_0. Thus, we conclude there is a significant difference in the adjusted treatment means. Next, do comparisons on the adjusted treatment means by constructing CIs on the difference between two adjusted means.

2. i. Adjusted treatment means using the model with equal slopes but different intercepts:

Alloy	Least Squares Line	$\hat{\mu}_i^{UNADJ}$	$\hat{\mu}_i^{ADJ}$	Average Diameter
1	$\hat{Y} = -36.751 + 5.505X$	50.417	61.733	15.833
2	$\hat{Y} = -20.439 + 5.505X$	88.750	78.045	19.833
3	$\hat{Y} = -47.596 + 5.505X$	51.500	50.888	18.000

   ii. Standard errors:

   $$\widehat{SE}(\hat{\mu}_i^{ADJ}) = \sqrt{MSE_2 \left(\frac{1}{r_i} + \frac{(\bar{X}_{i.} - \bar{X}_{..})^2}{\sum_{i=1}^{t} \sum_{j=i}^{r_i} (X_{ij} - \bar{X}_{i.})^2} \right)}$$

   $\Rightarrow \widehat{SE}(\hat{\mu}_1^{ADJ}) = 3.2914$, $\widehat{SE}(\hat{\mu}_2^{ADJ}) = 3.2543$, $\widehat{SE}(\hat{\mu}_3^{ADJ}) = 2.9210$.

   $$\widehat{SE}(\hat{\mu}_i^{ADJ} - \hat{\mu}_{i'}^{ADJ}) = \sqrt{MSE_2 \left(\frac{1}{r_i} + \frac{1}{r_{i'}} + \frac{(\bar{X}_{i.} - \bar{X}_{i'.})^2}{\sum_{i=1}^{t} \sum_{j=i}^{r_i} (X_{ij} - \bar{X}_{i.})^2} \right)}$$

   $\Rightarrow \widehat{SE}(\hat{\mu}_1^{ADJ} - \hat{\mu}_2^{ADJ}) = 5.0784$, $\widehat{SE}(\hat{\mu}_1^{ADJ} - \hat{\mu}_3^{ADJ}) = 4.4289$, $\widehat{SE}(\hat{\mu}_2^{ADJ} - \hat{\mu}_3^{ADJ}) = 4.3459$.

3. The regression lines for a model with common slopes, common slope relating Weld Strength to Diameter is 5.505, are given in part 3. The regression lines for a model allowing different slopes are given in part 5. From the two scatterplots, we observe that Alloy 2 appears to have a large slope relative to the slopes for Alloys 1 and 3. However, note that this mainly due to a single point (Dia=22.5, Strength = 119.5). If this point is removed, the slope for Alloy 2 is reduced and now nearly matches the slopes for the other two alloys.
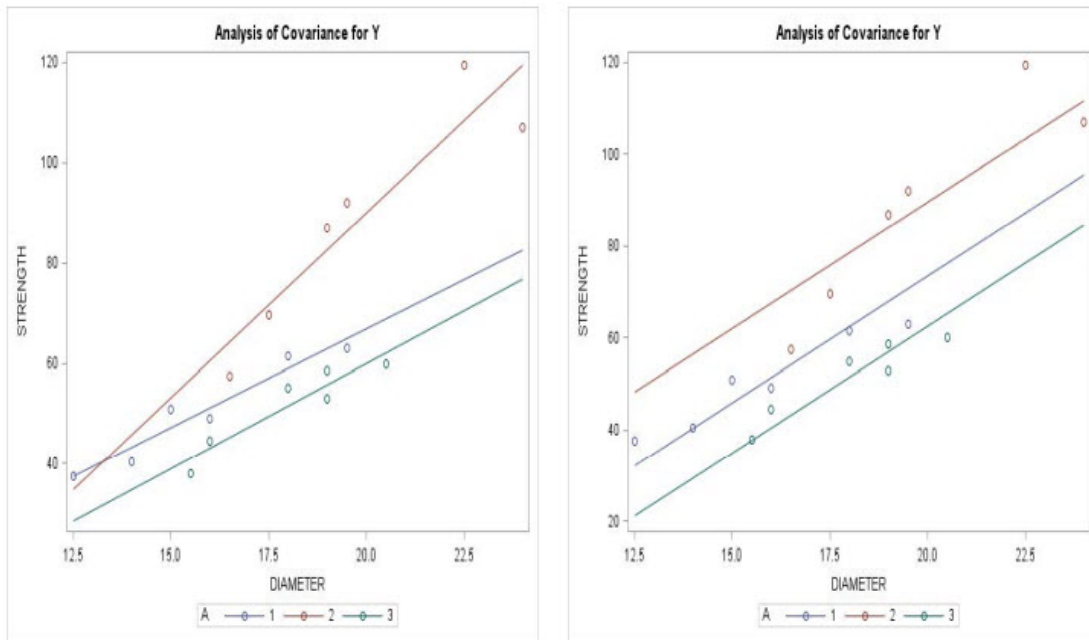
4. From PROC MIXED,

Alloy	Alloy	Estimate	Standard Error	DF	t Value	$Pr > |t|$	Adjustment	Adj P
1	2	-16.3120	5.0784	14	-3.21	0.0063	Tukey-Kramer	0.0162
1	3	10.8449	4.4289	14	2.45	0.0281	Tukey-Kramer	0.0681
2	3	27.1569	4.3459	14	6.25	< .0001	Tukey-Kramer	< .0001

We conclude that there is a significant Alloy effect. From Tukey, we conclude that Alloys 1,3 are not significantly different.

- In ANCOVA, when testing H_0 : Equal Slopes vs H_1 : Unequal Slopes, the consequences of a Type II error are more crucial than the consequences of a Type I error: If we make a Type I error, then we will use a model with unequal slopes when in fact the slopes are equal. Thus, we will need to make comparisons of the treatment differences at selected values of the covariate. The only real loss would be smaller d.f. of error. If we make a Type II error, then we will use a model having equal slopes when in fact slopes are unequal. Thus, we will evaluate treatment differences at covariate=0. However, the size of the differences in treatment means will vary depending on the value of the covariate. By testing H_0 vs H_1 with large α ($\alpha \approx 0.25$) we will be greatly reducing chance of Type II error.

Using the results from the model allowing different slopes we obtain the following results:

Using the GLM model statement: MODEL Y=A D*A /NOINT SOLUTION; with D not in the statement but NOINT included will yield the estimates of the slopes and intercepts along with their standard errors:

Alloy	Parameter	Estimate	Standard Error of Estimate
A1	Intercept	-11.65	17.1271
A2	Intercept	-57.04	19.1093
A3	Intercept	-23.91	25.9739
A1	Slope	3.92	1.0699
A2	Slope	7.35	0.9551
A3	Slope	4.19	1.4362

The above values can also be obtained from the statement: MODEL Y=A D D*A / SOLUTION;

$\hat{\beta}_1 = 4.189 - 0.269 = 3.92$, $\hat{\beta}_2 = 4.189 + 3.161 = 7.35$, $\hat{\beta}_3 = 4.189$.

$\widehat{SE}(\hat{\beta}_i) = \sqrt{\frac{MSE_1}{\sum_{j=1}^r (X_{ij} - \bar{X}_{i\cdot})^2}}$

$\sum_{j=1}^r (X_{1j} - \bar{X}_{1\cdot})^2 = 33.333$, $\sum_{j=1}^r (X_{2j} - \bar{X}_{2\cdot})^2 = 41.8333$,

$\sum_{j=1}^r (X_{3j} - \bar{X}_{3\cdot})^2 = 18.5$, $MSE_1 = 38.158$. Thus,

$\widehat{SE}(\hat{\beta}_1) = 1.0699$, $\widehat{SE}(\hat{\beta}_2) = 0.9551$, $\widehat{SE}(\hat{\beta}_3) = 1.4362$.

We would next recompute the adjusted treatment means using the model with different slopes and compare these values for differences. From the SAS output using LSMEANS, we obtain the adjusted treatment means using the model with different slopes and different intercepts:

Adjusted treatment means using the model with unequal slopes:

Alloy	Least Squares Line	$\hat{\mu}_i^{UNADJ}$	$\hat{\mu}_i^{ADJ(DiffSlopes)}$	$\hat{\mu}_i^{ADJ(SameSlope)}$
1	$\hat{Y} = -11.650 + 3.920X$	50.417	58.474	61.733
2	$\hat{Y} = -57.037 + 7.351X$	88.750	74.457	78.045
3	$\hat{Y} = -23.905 + 4.189X$	51.500	51.035	50.888

Also, from PROC MIXED,

Alloy	Alloy	Estimate	Standard Error	DF	t Value	$Pr > |t|$	Adjustment	Adj P
1	2	-15.9827	4.5831	12	-3.49	0.0045	Tukey-Kramer	0.0116
1	3	77.4399	4.1931	12	1.77	0.1014	Tukey-Kramer	0.2194
2	3	23.4226	4.0241	12	5.82	$< .0001$	Tukey-Kramer	.0002

The two sets of equations are considerably different relative to their slopes and intercepts and the adjusted treatment means have changed from the values obtained using the equations with a common slope. However, the conclusions concerning the differences in the Adjusted means are similar to the conclusions reached using the common slopes model.