STAT 608, Spring 2022 - Assignment 6

1. For logistic regression with one predictor, we use the model

$$\log\left(\frac{\theta(x)}{1-\theta(x)}\right) = \beta_0 + \beta_1 x$$

   (a) Show that solving for the probability of success for a given value of the predictor, $\theta(x)$, gives

$$\theta(x) = \frac{\exp(\beta_0 + \beta_1 x)}{1 + \exp(\beta_0 + \beta_1 x)}$$

   (b) and

$$\theta(x) = \frac{1}{1 + \exp(-\{\beta_0 + \beta_1 x\})}$$

2. On page 285 of the text, it says "When X is a dummy variable, it can be shown that the log odds are also a linear function of x." Suppose that X is a dummy variable, taking the value 1 with probability π_j, $j = 0, 1$, conditional on $Y = 0, 1$.

   (a) Show that the log odds are a linear function of x.

   (b) Define the slope and intercept for the linear function.

3. On page 284 of the text, the author quotes Cook and Weisberg: "When conducting a binary regression with a skewed predictor, it is often easiest to assess the need for x and $\log(x)$ by including them both in the model so that their relative contributions can be assessed directly." Show that indeed the log odds are a function of x and $\log(x)$ for the gamma distribution.

4. Chapter 8, Question 4