Statistics 630 - Assignment 2
(partial solutions)

Note: I highly recommend that you do not just read the solutions but rather that you fix your mistakes by carefully redoing the problems you missed. This will help you avoid similar mistakes in the future. If you still have questions, please post them on the discussion board or come see me or the grader.

These solutions are not meant to be complete. In some cases only the final answer is shown, with no derivation and not calculated entirely. In others, the main ideas are provided but they may not include all the necessary details or the full explanation and justification that would be expected of you. Some problems or parts are not shown (including some that have answers in the back of the textbook). [Comments in brackets are not part of the solution.]

These solutions might also suggest shortcuts or notational devices that could improve your presentation for later work.

1. (a) Use the two-event inclusion-exclusion formula (Thm. 1.3.3 in the book) and the fact $\mathsf{P}(A \cup B) \leq 1$.

   (c) Either apply part (a) twice or apply Thm. 1.3.4 in the book to the *complements* of the events.

2. Exer. 1.5.14. $\mathsf{P}(A^c \cap B) = \mathsf{P}(B) - \mathsf{P}(A \cap B) = (1 - \mathsf{P}(A))\mathsf{P}(B)$ iff $\mathsf{P}(A \cap B) = \mathsf{P}(A)\mathsf{P}(B)$.

   Exer. 1.5.18. Note that the unconditional probability that the host opens door B is $\frac{1}{2} \times \mathsf{P}(A) + 1 \times \mathsf{P}(C) = \frac{1}{2}$. Then

$$\mathsf{P}(C \mid \text{host opens B}) = \frac{\mathsf{P}(C)\mathsf{P}(\text{host opens B} \mid C)}{\mathsf{P}(\text{host opens B})} = \frac{\frac{1}{3} \times 1}{\frac{1}{2}} = \frac{2}{3}.$$

   So you are twice as likely to win if you switch to door C, given the host opens B.

3. The top channel does not fail if nether component 1 nor 2 fails, i.e., with probability $(1-p)^2$, by independence. So it fails with probability $1 - (1-p)^2$. Similarly for the bottom. Then, again by independence, all three channels fail (which is when the system fails) with probability $p(1-(1-p)^2)^2$, which simplifies to $p^3(2-p)^2$.

4. Any two siblings have the same gene with probability $\frac{1}{2}$. They all have the same gene with probability $\frac{1}{4}$ (2 out of 8 equally likely outcomes).

6. Exer. 2.1.8. $W(s) = 1, 0, 1, 1, 1$ for $s = 1, 2, 3, 4, 5$, respectively. You can also express this as $1 - I_{\{2\}}$. Note that $X - Y = I_{\{1\}} \geq 0$. So $W = X - Y + Z \geq Z$.

7. Exer. 2.2.4.

   (a) $\mathsf{P}(W = w) = \frac{1}{6}$ for $w = 5, 12, 31, 68, 129, 220$, and $= 0$ otherwise.

   (c) $\mathsf{P}(ZW = x) = \frac{1}{6}$ for $x = 5, 24, 93, 272, 645, 1320$, and $= 0$ otherwise.

   (e) $\mathsf{P}(V + W = r) = \frac{1}{6}$ for $r = 6, 12 + \sqrt{2}, 31 + \sqrt{3}, 70, 129 + \sqrt{5}, 220 + \sqrt{6}$, and $= 0$ otherwise.

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Exer. 2.3.10. $\mathsf{P}(X^2 \leq 15) = 1 - \mathsf{P}(X \geq 4) = 1 - (\frac{4}{5})^4$. (Thinking about a sequence of trials: the first four trials are not all failures.)

Exer. 2.3.14. Let Y be the number of times the event occurs in 20 rolls. Then Y binomial$(20, \frac{2}{3})$ and $\mathsf{P}(Y = k) = \binom{20}{k}(\frac{2}{3})^k(\frac{1}{3})^{20-k}$ for $k = 0, 1, 2 \ldots, 20$. In particular, $\mathsf{P}(Y = 5) = 1.423 \times 10^{-4}$.

8. With $N = 50$,

	hypergeometric	binomial	difference
0	1.798588e-05	0.0001048576	-8.687172e-05
1	4.905241e-04	0.0015728640	-1.082340e-03
2	5.334450e-03	0.0106168320	-5.282382e-03
3	3.063889e-02	0.0424673280	-1.182844e-02
4	1.034063e-01	0.1114767360	-8.070483e-03
5	2.150850e-01	0.2006581248	1.442688e-02
6	2.800586e-01	0.2508226560	2.923595e-02
7	2.259296e-01	0.2149908480	1.093878e-02
8	1.082579e-01	0.1209323520	-1.267440e-02
9	2.785585e-02	0.0403107840	-1.245494e-02
10	2.924864e-03	0.0060466176	-3.121754e-03

With $N = 1000$,

	hypergeometric	binomial	difference
0	9.793903e-05	0.0001048576	-6.918575e-06
1	1.502901e-03	0.0015728640	-6.996336e-05
2	1.033436e-02	0.0106168320	-2.824731e-04
3	4.193348e-02	0.0424673280	-5.338480e-04
4	1.111929e-01	0.1114767360	-2.838344e-04
5	2.013295e-01	0.2006581248	6.714024e-04
6	2.520856e-01	0.2508226560	1.262926e-03
7	2.155291e-01	0.2149908480	5.382428e-04
8	1.204228e-01	0.1209323520	-5.095340e-04
9	3.970499e-02	0.0403107840	-6.057947e-04
10	5.866412e-03	0.0060466176	-1.802054e-04