Stat 642 Spring 2022 - Solutions for Assignment 7

Problem 1. (30 points) This is a 4×3 CRD with two crossed factors, Type of Crop and Amount of Nitrogen, both having fixed levels. There are four reps/treatment with EU=MU=Growth Chamber.

a. Cell Mean Model: $Y_{ijk} = \mu_{ij} + e_{ijk};\ i = 1,2,3;\ j = 1,2,3,4;\ k = 1,2,3,4$, where Y_{ijk} is acetylene reduction from the kth growth chamber receiving ith Nitrogen level with the jth Crop, μ_{ij} is the mean response of ith Nitrogen level, jth Crop, and $e_{ijk} \sim iid\ N(0, \sigma_e^2)$.
Effects Model: $Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + e_{ijk}$, where τ_i is the fixed effect of ith Nitrogen level, β_j is the fixed effect of jth Crop, $(\tau\beta)_{ij}$ are interaction effects between Nitrogen and Crop, with $\tau_3 = 0,\ \beta_4 = 0,\ (\tau\beta)_{3j} = (\tau\beta)_{i4} = 0$ for $i = 1,2,3; j = 1,2,3,4$.

b. Based on the B-F-L test which has p-value = 0.0389 and the plot of the residuals, there is an indication that the assumption of equality of variance is invalid.

The Shapiro-Wilk test has p-value< .0001, the Stem-Leaf plot appears heavy tailed with numerous outliers, and the normal probability plot has the residuals both above and below a straight line; therefore, the normal condition appears not to be valid.

There is not an index of time or space relative to the measurements or experimental units so a valid measure of correlation in the residuals is not available.

c. Using the Box-Cox procedure, a transformation X = log(Y) was suggested. An evaluation of the AOV conditions using log(Y) yielded:

• Based on the B-F-L test which has p-value = 0.1940 and the plot of the residuals, the evidence indicates that the assumption of equality of variance is valid.

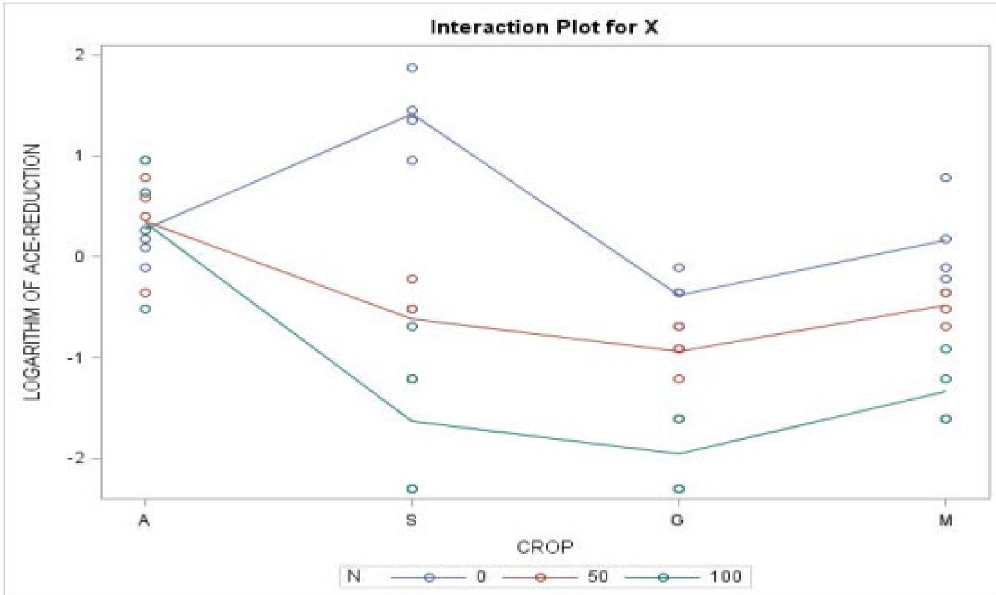
The Shapiro-Wilk test has p-value= .9289, the Stem-Leaf plot appears symmetric with no outliers, and the normal probability plot has the residuals very close to a straight line; therefore, the normal condition appears to be valid for the transformed data.

The ANOVA table from SAS is given below:

```
Dependent Variable: X =  log(ACE-CONC)
                                        Sum of
Source                    DF           Squares    Mean Square   F Value   Pr > F
Model                     11       41.23719964     3.74883633     18.41   <.0001
Error                     36        7.32907897     0.20358553
Corrected Total           47       48.56627862

Source                    DF       Type III SS    Mean Square   F Value    Pr > F
C                          3       12.46379492     4.15459831     20.41    <.0001
N                          2       18.34306101     9.17153051     45.05    <.0001
C*N                        6       10.43034372     1.73839062      8.54    <.0001
```

(d) Test for Interaction: $H_0 : \mu_{ij} - \mu_{ij'} = \mu_{i'j} - \mu_{i'j'}$ for all (i, i', j, j') vs $H_1 : \mu_{ij} - \mu_{ij'} \neq \mu_{i'j} - \mu_{i'j'}$ for some choice (i, i', j, j'):
Test statistic is $F = \frac{MS_{C\times N}}{MSE} = \frac{1.73839}{0.20358553} = 8.54$ with $F = 8.54 > 2.36 = F_{0.05,6,36}$ and
$p - value = 1 - pf(8.54, 6, 36) = 8.494528e - 06 < 0.05$, we reject H_0. Thus, we conclude that there is significant evidence of an interaction between Nitrogen level and Crop. The ANOVA table shows both main effect have very small p-values and hence there is strong evidence of a main effect of both Nitrogen and Crop however, neither of these results have meaningful interpretation.

(e) The profile plot is given here and the graph confirms the conclusion from the test of hypotheses. There is very little change in the mean level of log-acetylene with increasing levels of nitrogen for Alfalfa but there is a strong decrease in log-acetylene with increasing nitrogen for the other three crops.



(f) Because of the significant interaction, the four types of Crops will be grouped separately for each of the three levels of Nitrogen using the unadjusted p-values with $\alpha_{PC} = .05/18 = .0028$ because there are $(3)\binom{4}{2} = 18$ pairs of treatment means being compared.

Similarly, you can group the levels of Nitrogen at each of the four level of Crops using the unadjusted p-values with $\alpha_{PC} = .05/12 = .0042$ because there are $(4)\binom{3}{2} = 12$ pairs of treatment means being compared.

```
 Least Squares Means
UnAdjusted for Multiple Comparisons

C   N       X LSMEAN    Standard Error  LSMEAN Number
A   0      0.28194567    0.22560226          1
A   50     0.35625855    0.22560226          2
A   100    0.33722599    0.22560226          3
S   0      1.41172630    0.22560226          4
S   50    -0.61219190    0.22560226          5
S   100   -1.62557254    0.22560226          6
G   0     -0.37796440    0.22560226          7
G   50    -0.93242536    0.22560226          8
G   100   -1.95601150    0.22560226          9
M   0      0.16056871    0.22560226          10
M   50    -0.47933067    0.22560226          11
M   100   -1.33478484    0.22560226          12
```

Dependent Variable: X

i/j	1	2	3	4	5	6
1		0.8171	0.8634	0.0011	0.0081	<.0001
2	0.8171		0.9528	0.0021	0.0044	<.0001
3	0.8634	0.9528		0.0018	0.0052	<.0001
4	0.0011	0.0021	0.0018		<.0001	<.0001
5	0.0081	0.0044	0.0052	<.0001		0.0031
6	<.0001	<.0001	<.0001	<.0001	0.0031	
7	0.0458	0.0273	0.0312	<.0001	0.4676	0.0004
8	0.0005	0.0003	0.0003	<.0001	0.3222	0.0365
9	<.0001	<.0001	<.0001	<.0001	0.0002	0.3073
10	0.7059	0.5435	0.5832	0.0004	0.0206	<.0001
11	0.0224	0.0128	0.0148	<.0001	0.6796	0.0010
12	<.0001	<.0001	<.0001	<.0001	0.0296	0.3681

i/j	7	8	9	10	11	12
1	0.0458	0.0005	<.0001	0.7059	0.0224	<.0001
2	0.0273	0.0003	<.0001	0.5435	0.0128	<.0001
3	0.0312	0.0003	<.0001	0.5832	0.0148	<.0001
4	<.0001	<.0001	<.0001	0.0004	<.0001	<.0001
5	0.4676	0.3222	0.0002	0.0206	0.6796	0.0296
6	0.0004	0.0365	0.3073	<.0001	0.0010	0.3681
7		0.0908	<.0001	0.1001	0.7525	0.0049
8	0.0908		0.0028	0.0015	0.1642	0.2154
9	<.0001	0.0028		<.0001	<.0001	0.0594
10	0.1001	0.0015	<.0001		0.0525	<.0001
11	0.7525	0.1642	<.0001	0.0525		0.0110
12	0.0049	0.2154	0.0594	<.0001	0.0110	

Groupings of the levels of Crops for each level of Nitrogen:

Nitrogen = 0: G1 = {A, G, M }; G2 = {S }
Nitrogen = 50: G1 = {A, S, M }; G2 = {S, G, M }
Nitrogen = 100: G1 = {A }; G2 = {S, G, M }

Groupings of the levels of Nitrogen for each level of Crop:

Crop = Alfalfa: G1 = {0, 50, 100 }
Crop = Soybean: G1 = {0 }; G2 = {50}; G3 = {100 }
Crop = Guar: G1 = {0, 50 }; G2 = {100 }
Crop = Mungbean: G1 = {0, 50 }; G2 = {50, 100 }

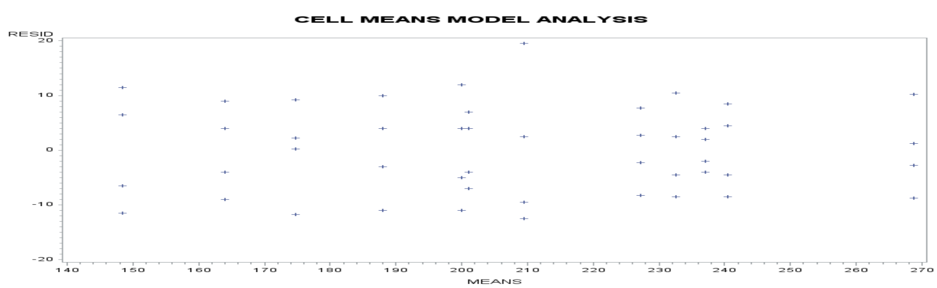
- The both sets of groupings are generally confirmed after viewing the profile plot. However, for the Nitrogen 50 level, the mean for Alfalfa would appear to be considerably larger than the means for Soybean and Mungbean. This seeming contradiction reflects that after taking into account the size of the the standard errors of the estimated means, there is not significant evidence of a difference in the true treatment means.

Problem 2. (40 points) The design is a CRD with a 2x2x3 factorial treatment structure with 4 reps/treatment. The three factors have fixed levels because the levels of the factors were directly (not randomly) selected by the researchers. The fabric specimen is both the EU and the MU in this experiment.

Many of the questions will be answered using SAS output

a. Cell Mean Model: $Y_{ijkl} = \mu_{ijk} + e_{ijkl}$; $i = 1,2$; $j = 1,2$; $k = 1,2,3; l = 1,2,3,4$, where Y_{ijkl} is observation on the lth EU receiving ith level of Surface with the jth level of Filler with the kth level of Proportion of Filler, μ_{ijk} is the mean response of ith level of Surface, jth level of Filler, and kth level of Proportion of Filler, and $e_{ijkl} \sim iid\ N(0, \sigma_e^2)$.
Effects Model: $Y_{ijkl} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \gamma_k + (\tau * \gamma)_{ik} + (\beta * \gamma)_{jk} + (\tau * \beta * \gamma)_{ijk} + e_{ijkl}$, with $\tau_2 = 0$, $\beta_2 = 0$, $(\tau\beta)_{2j} = 0$, $(\tau\beta)_{i2} = 0$, $\gamma_3 = 0$, $(\tau * \gamma)_{2k} = 0$, $(\tau * \gamma)_{i3} = 0$, $(\beta * \gamma)_{2k} = 0$, $(\beta * \gamma)_{j3} = 0$, $(\tau * \beta * \gamma)_{ij3} = 0$, $(\tau * \beta * \gamma)_{i2k} = 0$, $(\tau * \beta * \gamma)_{2jk} = 0$ for $i = 1,2; j = 1,2; k = 1,2,3$, and $e_{ijkl} \sim iid\ N(0, \sigma_e^2)$.

b. The B-F test has a p-value of 0.5086 which would indicate that there is not much evidence of inequalities in the 12 treatment variances. Also, an examination of the plot of the residuals versus the estimated treatment means does not yield any apparent non-constancy in the treatment variances.



- The Shapiro-Wilk test has a p-value = .1327, the box plot of the residuals is only slightly right skewed with no outliers, and the normal reference plot has the residuals very close to a straight line.
- There is no conceivable index for the residuals (no time ordering or spatial ordering) so a test of correlation is not possible. A discussion with the manufacturer's engineers who ran the experiment provided assurance that the data was not correlated due to the manner in which the experiment was conducted.

c. The ANOVA table from SAS is given below:

Source	df	SS	MS	F	$Pr > F$
TRT	11	54616.73	4965.16	62.27	$< .0001$
S	1	27408.52	27408.52	345.94	$< .0001$
F	1	2146.69	2146.69	27.09	$< .0001$
S*F	1	336.02	336.02	4.24	.0467
P	2	22795.29	11397.65	143.86	0.0019
S*P	2	315.04	157.52	1.99	0.1517
F*P	2	1297.63	648.81	8.19	0.0012
S*F*P	2	317.54	158.77	2.00	0.1496
Error	36	2852.25	79.23		
Total	47	57468.98			

d. Marginal and Cell means:

$\widehat{SE}(\hat{\mu}_{i\cdot\cdot}) = \widehat{SE}(\bar{Y}_{i\cdot\cdot\cdot}) = \sqrt{\frac{MSE}{rbc}}$, $\widehat{SE}(\hat{\mu}_{\cdot j\cdot}) = \widehat{SE}(\bar{Y}_{\cdot j\cdot\cdot}) = \sqrt{\frac{MSE}{rac}}$, $\widehat{SE}(\hat{\mu}_{\cdot\cdot k}) = \widehat{SE}(\bar{Y}_{\cdot\cdot k\cdot}) = \sqrt{\frac{MSE}{rab}}$

$\widehat{SE}(\hat{\mu}_{ij\cdot}) = \widehat{SE}(\bar{Y}_{ij\cdot\cdot}) = \sqrt{\frac{MSE}{rc}}$, $\widehat{SE}(\hat{\mu}_{i\cdot k}) = \widehat{SE}(\bar{Y}_{i\cdot k\cdot}) = \sqrt{\frac{MSE}{rb}}$, $\widehat{SE}(\hat{\mu}_{\cdot jk}) = \widehat{SE}(\bar{Y}_{\cdot jk\cdot}) = \sqrt{\frac{MSE}{ra}}$

$\widehat{SE}(\hat{\mu}_{ijk}) = \widehat{SE}(\bar{Y}_{ijk\cdot}) = \sqrt{\frac{MSE}{r}}$

Surface	$\bar{Y}_{i\cdots}$	$\widehat{SE}(\bar{Y}_{i\cdots})$
1	231.54	1.8169
2	183.75	1.8169

Filler	$\bar{Y}_{\cdot j\cdot\cdot}$	$\widehat{SE}(\bar{Y}_{\cdot j\cdot\cdot})$
1	214.33	1.8169
2	200.96	1.8169

Proportion	$\bar{Y}_{\cdot\cdot k\cdot}$	$\widehat{SE}(\bar{Y}_{\cdot\cdot k\cdot})$
25	180.75	2.2253
50	208.06	2.2253
75	234.13	2.2253

Filler	Surface	Proportion	$\bar{Y}_{ijk\cdot}$	$\widehat{SE}(\bar{Y}_{ijk\cdot})$
1	1	25	201.0	4.4505
1	1	50	237.0	4.4505
1	1	75	268.75	4.4505
1	2	25	164.0	4.4505
1	2	50	188.0	4.4505
1	2	75	227.25	4.4505
2	1	25	209.5	4.4505
2	1	50	232.5	4.4505
2	1	75	240.5	4.4505
2	2	25	148.5	4.4505
2	2	50	174.75	4.4505
2	2	75	200.0	4.4505

Surface	Proportion	$\bar{Y}_{i\cdot k}$	$\widehat{SE}(\bar{Y}_{ijk\cdot})$
1	25	205.25	3.1470
1	50	234.75	3.1470
1	75	254.63	3.1470
2	25	156.25	3.1470
2	50	181.38	3.1470
2	75	213.63	3.1470

Filler	Proportion	$\bar{Y}_{\cdot jk}$	$\widehat{SE}(\bar{Y}_{ijk\cdot})$
1	25	182.5	3.1470
1	50	212.5	3.1470
1	75	248.0	3.1470
2	25	179.0	3.1470
2	50	203.63	3.1470
2	75	220.25	3.1470

Filler	Surface	$\bar{Y}_{ij\cdot}$	$\widehat{SE}(\bar{Y}_{ijk\cdot})$
1	1	235.58	2.5695
1	2	193.08	2.5695
2	1	227.50	2.5695
2	2	174.42	2.5695

e.
- Test of three-factor Interaction:
  $H_0 : [(\mu_{ijk} - \mu_{ij'k}) - (\mu_{i'jk} - \mu_{i'j'k})] = [(\mu_{ijk'} - \mu_{ij'k'}) - (\mu_{i'jk'} - \mu_{i'j'k'})]$ for all (i, i', j, j', k, k')
  H_1 : Equality does NOT hold for some choice of (i, i', j, j', k, k').
  Test statistic is $F = \frac{MS_{S*F*PF}}{MSE} = \frac{158.77083}{79.22917} = 2.00$. $p - value = 0.1496 > 0.05$ and so fail to reject H_0. Thus, we conclude that there is NOT significant evidence of an interaction among Surface, Filler and Proportion of Filler. Because the three-way interaction was NOT significant, conduct tests of the three sets of 2-way interactions.
- Test of two-factor Interaction:

i. $H_0 : \mu_{ij\cdot} - \mu_{ij'\cdot} = \mu_{i'j\cdot} - \mu_{i'j'\cdot}$ for all (i, i', j, j'): Test statistic is $F = \frac{MS_{S*F}}{MSE} = \frac{336.02083}{79.22917} = 4.24$. $p - value = 0.0467 < 0.05$ and so reject H_0. We conclude that there is significant evidence of an interaction between Surface and Filler.

ii. $H_0 : \mu_{i\cdot k} - \mu_{i\cdot k'} = \mu_{i'\cdot k} - \mu_{i'\cdot k'}$ for all (i, i', k, k'): Test statistic is $F = \frac{MS_{S*P}}{MSE} = \frac{157.52083}{79.22917} = 1.99$. $p - value = 0.1517 > 0.05$ and so fail to reject H_0. We conclude that there is NOT significant evidence of an interaction between Surface and Proportion of Filler.

iii. $H_0 : \mu_{\cdot jk} - \mu_{\cdot jk'} = \mu_{\cdot j'k} - \mu_{\cdot j'k'}$ for all (j, j', k, k'): Test statistic is $F = \frac{MS_{F*PF}}{MSE} = \frac{648.81250}{79.22917} = 8.19$. $p - value = 0.0012 < 0.05$ and so reject H_0. We conclude that there is significant evidence of an interaction between Filler and Proportion of Filler.

- Next, compare the two Filler means:

i. Because of the significant F*P and F*S interactions, it is necessary to compare the levels of Filler at fixed levels of Proportion and Surface. Thus, there are 5 separate comparisons of the two Types of Fillers:

From the following SAS output using the unadjusted p-values with $\alpha_{PC} = \frac{.05}{5} = 0.01$

Filler	Surface	LSMEAN	LSMEAN Number
1	1	235.58	1
1	2	193.08	2
2	1	227.50	3
2	2	174.42	4

i/j	1	2	3	4
1		$< .0001$	0.0325	$< .0001$
2	$< .0001$		$< .0001$	$< .0001$
3	0.0325	$< .0001$		$< .0001$
4	$< .0001$	$< .0001$	$< .0001$	

Pairs of Means are declared to be different if the $p - value < 0.01$

Thus, our Groups are as follows:

Surface	Groupings of Levels of Filler
1	G_1={F_1, F_2}
2	G_1={F_1}; G_2={F_2}

ii. From the following SAS output using the unadjusted p-values with $\alpha_{PC} = \frac{.05}{5} = 0.01$,

Filler	Proportion	LSMEAN	LSMEAN Number
1	25	182.50	1
1	50	212.50	2
1	75	248.00	3
2	25	179.00	4
2	50	203.63	5
2	75	220.25	6

i/j	1	2	3	4	5	6
1		$< .0001$	$< .0001$	0.4368	< 0.0001	$< .0001$
2	$< .0001$		$< .0001$	$< .0001$	0.0538	0.0902
3	$< .0001$	$< .0001$		$< .0001$	$< .0001$	$< .0001$
4	0.4368	$< .0001$	$< .0001$		$< .0001$	$< .0001$
5	$< .0001$	0.0538	$< .0001$	$< .0001$		0.0006
6	$< .0001$	0.0902	$< .0001$	$< .0001$	0.0006	

Pairs of Means are declared to be different if the $p - value < 0.01$

Thus, our Groups are as follows:

Proportion	Groupings of Levels of Filler
25	G_1={F_1, F_2}
50	G_1={F_1, F_2}
75	G_1={F_1}; G_2={F_2}

f. Because of the significant F*P interaction, compare the levels of Proportion at fixed levels of Filler:

From the following SAS output using the unadjusted p-values with $\alpha_{PC} = \frac{.05}{6} = 0.0083$,

Filler	Proportion	LSMEAN	LSMEAN Number
1	25	182.5	1
1	50	212.25	2
1	75	248.0	3
2	25	179.0	4
2	50	203.63	5
2	75	220.25	6

i/j	1	2	3	4	5	6
1		$< .0001$	$< .0001$	0.4368	< 0.0001	$< .0001$
2	$< .0001$		$< .0001$	$< .0001$	0.0538	0.0902
3	$< .0001$	$< .0001$		$< .0001$	$< .0001$	$< .0001$
4	0.4368	$< .0001$	$< .0001$		$< .0001$	$< .0001$
5	$< .0001$	0.0538	$< .0001$	$< .0001$		0.0006
6	$< .0001$	0.0902	$< .0001$	$< .0001$	0.0006	

Pairs of Means are declared to be different if the $p - value < 0.0083$

Thus, we would have the following groupings:

Filler	Groupings of Levels of Proportion
1	G_1={P=25%}, G_2={P=50%}; G_3={ P=75%}
2	G_1={P=25%}, G_2={P=50%}; G_3={ P=75%}

g. Because of the significant interaction between type of Filler and Proportion of Filler ($F * P$), the test of Linear and Quadratic trends in Proportion must be conducted separately for each of the two types of Filler:

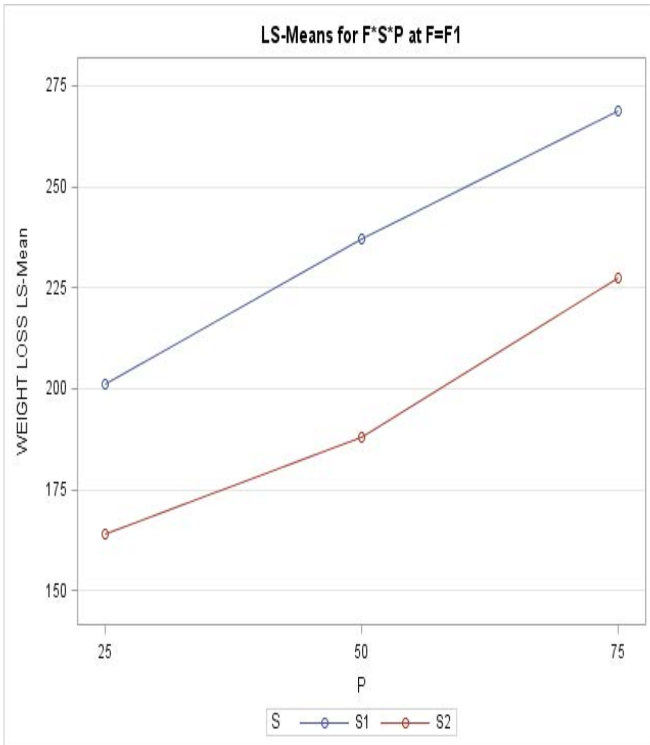
- Test for linear and quadratic trends in the mean weight loss across the levels of proportion of filler.

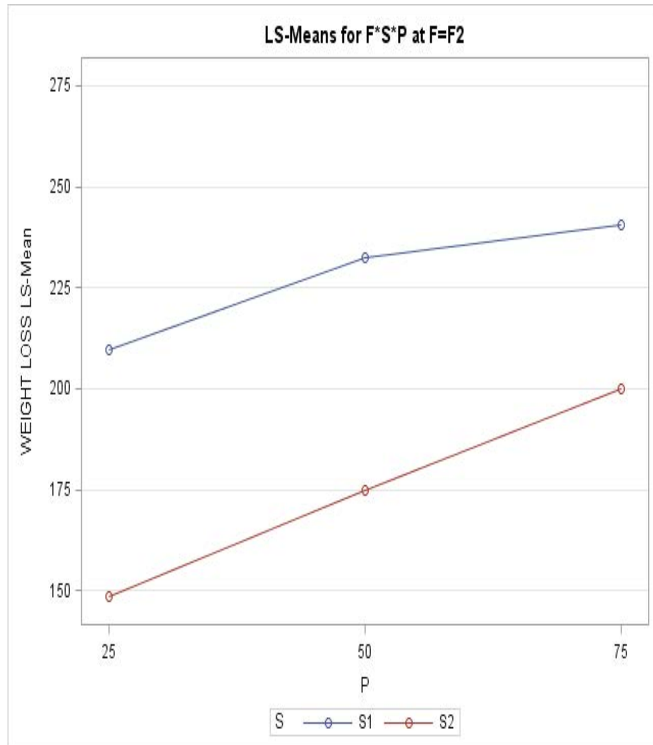
```
Contrast                     DF    Contrast SS    Mean Square   F Value   Pr > F

PropLinear-Filler1            1       17161.00       17161.00    216.60   <.0001
PropQuadratic-Filler1         1          40.33          40.33      0.51   0.4801
PropLinear-Filler2            1        6806.25        6806.25     85.91   <.0001
PropQuadratic-Filler2         1          85.33          85.33      1.08   0.3063
```

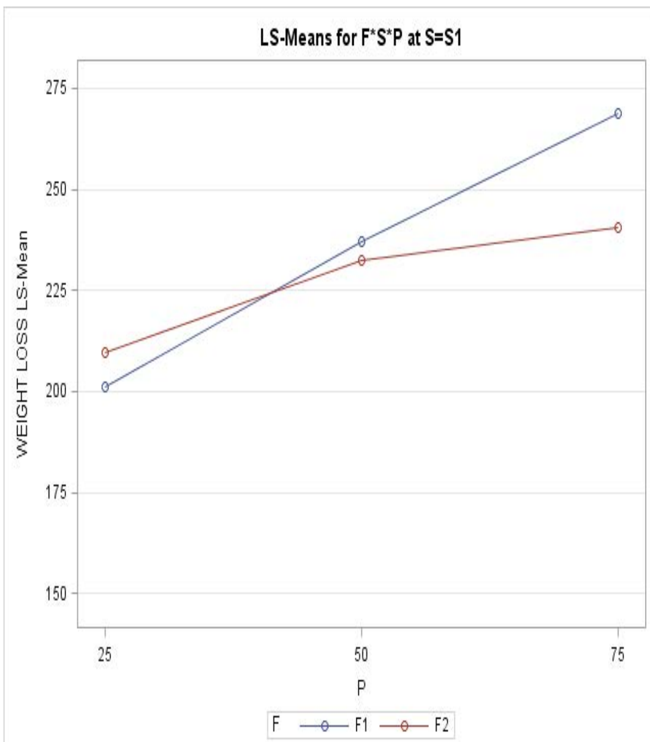
The interaction between Filler and Proportion of Filler is significant and Proportion of Filler is quantitative. An examination of a trend in the mean weight loss with increasing values of Proportion must be done separately at each level of Filler. Compare p–values to $\alpha = .05/4 = .0125$ for each level of Filler. Thus conclude that there is significant linear trend in the mean weight loss at level 1 and level 2 of Filler and there is not a quadratic trend in the mean weight loss at level 1 nor at level 2 of Filler. These trends can be observed in the F*P profile plots displayed on the next page.

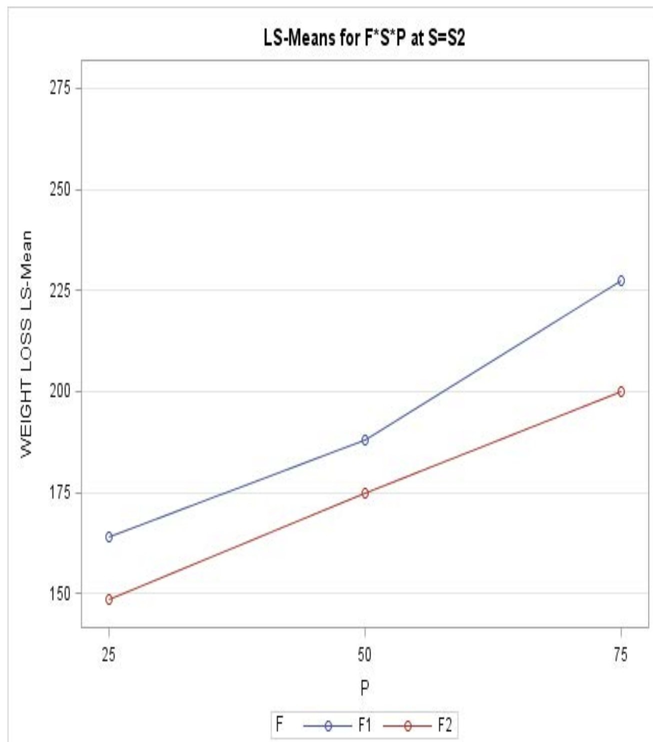
h. Profile plots:

The plots demonstrate explicitly the results displayed in the AOV Table. The F-test for a 3-way interaction indicates that there is not significant evidence of a 3-way interaction (p-value=.1426). An examination of the S*F*P profile plot reveals that the trends in the weight loss means for increasing proportion of Filler are very similar across the four combinations of Surface and Filler. A similar comment could be made for the profile plot for the S*P interaction (p=value=.0467). The interaction F*S (p-value = .0467) was marginally significant which is reflected in the profile plot where there is only a slightly larger difference between the mean weight losses of Surface 1 and Surface 2 across the two levels of Filler (52.5 at Filler 1 and 53.08 at Filler 2). However, the F*P interaction was highly significant (p-value = .0012). An examination of the F*P profile plot reveals that Filler type 1 had a distinctly greater slope in the linear trend in the weight loss means across the Proportions compare to the slope for Filler type 2.


LS-Means for F*S*P at F=F1
WEIGHT LOSS LS-Mean
275
250
225
200
175
150
25
50
75
P
S
S1
S2


LS-Means for F*S*P at F=F2
WEIGHT LOSS LS-Mean
275
250
225
200
175
150
25
50
75
P
S
S1
S2


LS-Means for F*S*P at S=S1
WEIGHT LOSS LS-Mean
275
250
225
200
175
150
25
50
75
P
F
F1
F2


LS-Means for F*S*P at S=S2
WEIGHT LOSS LS-Mean
275
250
225
200
175
150
25
50
75
P
F
F1
F2

Problem 3. (10 points)

a. R or J - Because of the significant interaction, the comparison should not be done on the marginal means

b. B - Need to perform trend analyzes across the levels of F_1 separately at each level of F_2 because of the significant interaction

Problem 4. (12 points)

a. C or K : Because $F_1 * F_3$ was significant but $F_1 * F_2$ and $F_1 * F_2 * F_3$ were not significant, we should examine trends in levels of F_1 separately over the levels of F_3, but averaged over the levels of F_2. That is, trends in $\hat{\mu}_{1 \cdot k}, \ \ldots, \ \hat{\mu}_{a \cdot k}$ separately for each value of $k = 1, \ldots, c$ or you could do a Tukey comparison of the means of F_1 separately over the levels of F_3, but averaged over the levels of F_2.

b. D: Because $F_1 * F_2 * F_3$ was significant, we should evaluate trends in the levels of F_1 separately over the levels of F_2 and F_3. That is, evaluate trend contrasts in the levels of $\hat{\mu}_{1jk}, \ \ldots, \ \hat{\mu}_{ajk}$ separately for each choice of (j, k) for $j = 1, \ldots, b; \ k = 1, \ldots, c$

c. G and H: Because $F_1 * F_2$ and $F_1 * F_3$ were significant but $F_1 * F_2 * F_3$ was not significant, we should compare the levels of F_1 to the control separately over the levels of F_2 but averaged over the levels of F_3 and separately over the levels of F_3 but averaged over the levels of F_2 . That is, compare $\hat{\mu}_{i \cdot k}$ to the Control mean and $\hat{\mu}_{ij \cdot}$ to the Control mean for $i = 1, \ldots, a; \quad j = 1, \ldots, b; \quad k = 1, \ldots, c$

d. P: Because $F_1 * F_2 * F_3$ was significant, we should perform Hsu's procedure over the levels of F_1 separately over all combinations of factors F_2 and F_3. That is, perform Hsu's procedure on the $\hat{\mu}_{ijk}$'s for fixed values of (jk).

Problem 5. (8 points)

1. The researcher's analysis was correct because there are 12 distinct treatments with the four treatments consisting of the 0 level of nitrogen combined with each of the four crops being distinctly different treatments due to there being nitrogen naturally occurring in the soil and the crops having different nitrogen fixation rates.

2. Take $\alpha = .05, \gamma_o = .80, t = 5, b = 2, \nu_1 = t - 1 = 4, \nu_2 = (5)(2)(r-1) = 10(r-1), D = 6, \hat{\sigma}_e^2 = 6.5$ and compute $\lambda = \frac{rbD^2}{2\sigma_e^2} = \frac{r(2)(36)}{(2)(6.5)} = 5.538r$ and $\phi = \sqrt{\lambda/t} = \sqrt{5.538r/5)} = 1.05247\sqrt{r}$. From Table IX, $\nu_1 = 4$, we find that $r = 3$ has power approximately .84 . Thus, r=3 yields the desired specification.

   Using R, power $= 1 - pf(qf(.95, 4, 20), 4, 20, 16.614) = .841$

3. **D** The linear trend across the levels of Factor F_1 averaged over the levels of Factor F_2 is

   C $= -3\mu_{1.} - \mu_{2.} + \mu_{3.} + 3\mu_{4.} = -3\mu_{11} - \mu_{21} + \mu_{31} + 3\mu_{41} + (-3\mu_{12} - \mu_{22} + \mu_{32} + 3\mu_{42})$

4. **B** The quadratic trend across the levels of factor B would use the contrast coefficients 1, -2, 1 (Table XI in textbook) and $L = \mu_{11} - 2\mu_{12} + \mu_{13} - \mu_{21} + 2\mu_{22} - \mu_{23} = (\mu_{11} - 2\mu_{12} + \mu_{13}) - (\mu_{21} - 2\mu_{22} + \mu_{23})$