Statistics 630 – Exam I
Partial Solutions

These solutions are relatively complete but (except for the multiple choice questions) they may not include all the steps or details I would expect. **Please rework your solutions** to the best of your ability, paying close attention to notation, until you are satisfied you can do them on your own. You can post questions on the discussion board or see me during my office hours if you still have difficulty or doubts. [Comments in square brackets are not part of the solution.]

1. (b). Assuming $a > 0$, $\mathsf{P}(aX + b \le ax_p + b) = \mathsf{P}(X \le x_p) = p$. [If $a < 0$ the p-quantile for $aX + b$ would actually be $ax_{1-p} + b$.]

2. (c). The expression given shows $f(w, z) = f_Z(z) \times f_{W|Z}(w|z)$, from which we identify the marginal distribution for Z and the conditional distribution for W, given $Z = z$.

3. (d). Sampling without replacement from each school, and then the product rule.

4. (a). $f_S(s)$ must be a function of s only, and (c) is not a pdf.

5. Let T be the observed total.

   (a) There are 3 ways out of 36 for two dice to total 4 and 3 ways out of $6^3 = 216$ for three dice to total 4. Thus,

   $$\mathsf{P}(T = 4) = \mathsf{P}(\text{black})\mathsf{P}(T = 4|\text{black}) + \mathsf{P}(\text{white})\mathsf{P}(T = 4|\text{white}) = \frac{2}{9}\frac{3}{36} + \frac{7}{9}\frac{3}{216} = \frac{12 + 7}{3 \times 216}.$$

   [Use proper notation for (conditional) probabilities.]

   (b) The first term in the middle expression above is $\mathsf{P}(\text{black}, T = 4)$. Hence,

   $$\mathsf{P}(\text{black}|T = 4) = \frac{\mathsf{P}(\text{black}, T = 4)}{\mathsf{P}(T = 4)} = \frac{12}{12 + 7}.$$

6. For $t \le 0$, $F_X(t) = F_Y(t) = 0$. For $t > 0$,

   $$F_X(t) = \int_0^t e^{-x}\,dx = (-e^{-x})\,\Big|_0^t = 1 - e^{-t},$$

   which we have seen multiple times before. Integrating by parts,

   $$F_Y(t) = \int_0^t xe^{-x}\,dx = x(-e^{-x})\,\Big|_0^t - \int_0^t (-e^{-x})\,dx = 1 - (1 + t)e^{-t} < F_X(t).$$

   [**Check that you have proper cdfs.** Also, $\int_0^t e^{-t}\,dt$ and $\int_0^t te^{-t}\,dt$ are *not* meaningful!]

7. Let M be the number of my ant hills and N the number of my neighbor's. By independence,

   $$\mathsf{P}(M \ge 4, N < 4) = (1 - \mathsf{P}(M \le 3))\mathsf{P}(N \le 3) = \Big(1 - \Big(1 + 7 + \frac{7^2}{2} + \frac{7^3}{6}\Big)e^{-7}\Big)\Big(1 + 3 + \frac{3^2}{2} + \frac{3^3}{6}\Big)e^{-3}.$$

8. $\mathsf{P}(X \le x) = \mathsf{P}(R^{2/3} \le x) = \mathsf{P}(R \le x^{3/2}) = 1 - e^{-x^3/4}$. Taking a derivative, $f_X(x) = \frac{3x^2}{4}e^{-x^3/4}$, for $x > 0$. [$\int_0^x \frac{3x^2}{4}e^{-x^3/4}\,dx$ is also *not meaningful*.]