Statistics 630 - Assignment 8
(due Friday, 5 November 2021)

Important: When referring to the estimator of a parameter be sure to use distinctive notation (define, if necessary). For example, $\bar{X}$ as the estimator of a mean μ, $\hat{\theta}$ as an estimator of parameter θ. Estimators and parameters are not the same thing, so *do not label them the same.*

1. Chapter 6 Exercise 6.2.4. Add
   (b) Show that $\sum_{i=1}^{n} X_i$ is sufficient for θ.
   (c) Evaluate the bias, variance, and mean squared error of the maximum likelihood estimator for θ.
   (d) What is the MLE for θ^2? Is it unbiased? If not, what is its bias?
   (e) What is the MLE for $\mathsf{P}(X_i = 0)$?

2. Chapter 6 Exercise 6.2.7. Ignore the hint, as you can express the pdf in a form that does not have gamma functions. Add
   (b) Provide a statistic (reduced from the sample itself) that is sufficient for α.
   (c) What is the MLE for $\mathsf{Var}(X_i)$?
   (d) Compute $\mathsf{E}(X_i)$ and use this to obtain a method of moments estimator for α.

3. $W_1, \ldots, W_n$ are iid from the distribution with pdf $f(w) = \frac{3w^2}{\beta^3} I_{[0,\beta]}(w)$.
   (a) Write down the likelihood function and use it to find the MLE for β. *Careful – note the support of f*; it may help to first sketch what the likelihood function looks like.
   (b) Find a method of moments estimator for β. Is it unbiased?

4. Chapter 6 Exercise 6.2.12. Add
   (b) Evaluate the bias, variance, and mean squared error of this estimator.
   (c) Compare its mean squared error to those of S^2 and $\hat{\sigma}^2$ from Example 45 on slides 42–44 of the Chapter 6 lecture notes. Which of the *unbiased* estimators has smallest MSE? Why is this reasonable? (Hint: as a general rule of thumb, the more you can assume the better your estimation can be.)

5. Chapter 6 Exercise 6.2.8. Note: this equation would need to be solved numerically; do not try to do it yourself.

6. Chapter 6 Exercise 6.2.19. Hint: review the multinomial model (Example 6.1.5) and note that the parameter space is reduced to one-dimension in this exercise (the Hardy-Weinberg model).

7. Suppose $X_1, ..., X_n$ is a random sample from an exponential(λ) distribution. Find the estimator for λ of the form $L_a = \frac{a}{\sum_{i=1}^{n} X_i}$ with the smallest mean squared error. That is, find a to minimize $\mathsf{E}((L_a - \lambda)^2)$, and give the minimum value. Hint: you will first need to take note of the distribution of $T = \sum_{i=1}^{n} X_i$ and use that to find $\mathsf{E}(1/T)$ and $\mathsf{E}(1/T^2)$.