The following table provides C.I.s for a variety of situations and parameters.

Parameter	Population Conditions	Endpoints of Confidence Intervals
Q(p)	$X_1, \cdots, X_n$ iid cont. cdf	$(X_{(r)}, X_{(s)})$, where r,s selected using Binomial(n,p) tables
μ	$X_1, \cdots, X_n$ iid $N(\mu, \sigma^2)$ σ unknown	$\bar{X} \pm t_{(\alpha/2,df)} \frac{S}{\sqrt{n}}$ where t has d.f.= n-1
$\mu_1 - \mu_2$	$X_1, \cdots, X_{n1}$ iid $N(\mu_1, \sigma_1^2)$ $Y_1, \cdots, Y_{n2}$ iid $N(\mu_2, \sigma_2^2)$ X's,Y's ind., $\sigma_1 = \sigma_2$	$(\bar{X} - \bar{Y}) \pm t_{(\alpha/2,df)} S_P \sqrt{\frac{1}{n1} + \frac{1}{n2}}$ where t has d.f.= $n1 + n2 - 2$
$\mu_1 - \mu_2$	$X_1, \cdots, X_{n1}$ iid $N(\mu_1, \sigma_1^2)$ $Y_1, \cdots, Y_{n2}$ iid $N(\mu_2, \sigma_2^2)$ X's,Y's ind., $\sigma_1 \neq \sigma_2$	$(\bar{X} - \bar{Y}) \pm t_{(\alpha/2,df)} \sqrt{\frac{S_1^2}{n1} + \frac{S_2^2}{n2}}$ where t has d.f.= $\frac{(C+1)^2(n1-1)(n2-1)}{C^2(n2-1)+(n1-1)}$, and $C = \frac{S_1^2/n_1}{S_2^2/n_2}$
$\mu_1 - \mu_2$	$(X_1, Y_1), \cdots, (X_n, Y_n)$ iid with $D_i = X_i - Y_i$ $N(\mu_D, \sigma_D^2)$	$\bar{D} \pm t_{(\alpha/2,df)} S_D/\sqrt{n}$ where t has d.f. = n-1
p	Y is Bin(n,p) $min(n\hat{p}, n(1-\hat{p}) \geq 5$ and $n \leq 40$	$\tilde{p} \pm \frac{Z_{(\alpha/2)}\sqrt{n}\sqrt{\hat{p}(1-\hat{p}) + \frac{1}{4n}Z^2_{(\alpha/2)}}}{n + Z^2_{\alpha/2}}$ $Z_{(\alpha/2)}$ upper N(0,1) percentile $\tilde{Y} = Y + Z^2_{\alpha/2}/2$, $\tilde{n} = n + Z^2_{\alpha/2}$, $\tilde{p} = \frac{\tilde{Y}}{\tilde{n}}$
p	Y is Bin(n,p) $min(n\hat{p}, n(1-\hat{p}) \geq 5$ and $n > 40$	$\tilde{p} \pm Z_{(\alpha/2)} \sqrt{\tilde{p}(1-\tilde{p})/\tilde{n}}$ $Z_{(\alpha/2)}$ upper N(0,1) percentile $\tilde{Y} = Y + Z^2_{\alpha/2}/2$, $\tilde{n} = n + Z^2_{\alpha/2}$, $\tilde{p} = \frac{\tilde{Y}}{\tilde{n}}$
p	Y is Bin(n,p) $min(n\hat{p}, n(1-\hat{p}) < 5$	Use Binomial Tables
$p_1 - p_2$	Count Data $min(n\hat{p}_i, n(1-\hat{p}_i) \geq 5$	$\hat{p}_1 - \hat{p}_2 \pm Z_{(\alpha/2)} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ $Z_{(\alpha/2)}$ upper N(0,1) percentile
σ	Normal Data	$\left(\frac{\sqrt{n-1}\;S}{\sqrt{\chi^2_{(\alpha/2,n-1)}}}, \frac{\sqrt{n-1}\;S}{\sqrt{\chi^2_{(1-\alpha/2,n-1)}}} \right)$, $\chi^2_{(\alpha/2,n-1)}$ and $\chi^2_{(1-\alpha/2,n-1)}$ upper percentiles- Chi-square tables
$\frac{\sigma_1}{\sigma_2}$	Normal Data	$\left(\frac{S_1}{S_2} \sqrt{\frac{1}{F_{(\alpha/2,n_1-1,n_2-1)}}}, \frac{S_1}{S_2} \sqrt{F_{(\alpha/2,n_2-1,n_1-1)}} \right)$, $F_{(\alpha/2,n_1-1,n_2-1)}$ and $F_{(\alpha/2,n_2-1,n_1-1)}$ upper percentiles- F-tables
β	Exponential Data	$\left(\frac{2n\bar{Y}}{\chi^2_{(\alpha/2,2n)}}, \frac{2n\bar{Y}}{\chi^2_{(1-\alpha/2,2n)}} \right)$, $\chi^2_{(\alpha/2,2n)}$ and $\chi^2_{(1-\alpha/2,2n)}$ upper percentiles- Chi-square tables
θ	Parameter in pdf	$\left(\hat{\theta} - Z_{\alpha/2}\widehat{SE}(\hat{\theta}),\; \hat{\theta} + Z_{\alpha/2}\widehat{SE}(\hat{\theta}) \right)$, $\hat{\theta}$ is MLE of θ

TABLES FROM HANDOUTS 9 & 11:
GOODNESS OF FIT - INTERVAL ESTIMATORS TABLES

Table 1: Percentiles for GOF Measures (Completely Specified Distributions)

		Upper Percentiles							
Statistic	Modified Statistic	.25	.15	.10	.05	.025	.01	.005	.001
D_n	$D_n(\sqrt{n} + .12 + .11/\sqrt{n})$	1.019	1.138	1.224	1.358	1.480	1.628	1.731	1.950
W_n^2	$(W_n^2 - \frac{.4}{n} + \frac{.6}{n^2})(1 + \frac{1}{n})$	0.209	0.284	0.347	0.461	0.581	0.743	0.869	1.167
A_n^2	For all $n \geq 5$	1.248	1.610	1.933	2.492	3.070	3.857	4.500	6.000

Table 2: CDF for Anderson-Darling (Completely Specified Distributions)

z	G(z)	z	G(z)	z	G(z)	z	G(z)	z	G(z)	z	G(z)
0.05	0.0000	0.75	0.4815	1.45	0.8111	2.15	0.9239	2.85	0.9674	3.80	0.9891
0.10	0.0000	0.80	0.5190	1.50	0.8235	2.20	0.9285	2.90	0.9692	3.90	0.9902
0.15	0.0000	0.85	0.5537	1.55	0.8350	2.25	0.9328	2.95	0.9710	4.00	0.9913
0.20	0.0096	0.90	0.5858	1.60	0.8457	2.30	0.9368	3.00	0.9726	4.25	0.9934
0.25	0.0296	0.95	0.6154	1.65	0.8556	2.35	0.9405	3.25	0.9795	4.50	0.9950
0.30	0.0618	1.00	0.6427	1.70	0.8648	2.40	0.9441	3.30	0.9807	4.60	0.9955
0.35	0.1036	1.05	0.6680	1.75	0.8734	2.45	0.9474	3.35	0.9818	4.70	0.9960
0.40	0.1513	1.10	0.6912	1.80	0.8814	2.50	0.9504	3.40	0.9828	4.80	0.9964
0.45	0.2019	1.15	0.7127	1.85	0.8888	2.55	0.9534	3.45	0.9837	4.90	0.9968
0.50	0.2532	1.20	0.7324	1.90	0.8957	2.60	0.9561	3.50	0.9846	5.00	0.9971
0.55	0.3036	1.25	0.7503	1.95	0.9021	2.65	0.9586	3.55	0.9855	5.50	0.9983
0.60	0.3520	1.30	0.7677	2.00	0.9082	2.70	0.9610	3.60	0.9863	6.00	0.9990
0.65	0.3930	1.35	0.7833	2.05	0.9138	2.75	0.9633	3.65	0.9870	7.00	0.9997
0.70	0.4412	1.40	0.7973	2.10	0.9190	2.80	0.9654	3.70	0.9878	8.00	0.9999

Table A29 Critical Values for the Shapiro–Wilk Test for Normality

n	Critical Value $\alpha = 1\%$	2%	5%	10%	50%
3	0.753	0.756	0.767	0.789	0.959
4	0.687	0.707	0.748	0.792	0.935
5	0.686	0.715	0.762	0.806	0.927
6	0.713	0.743	0.788	0.826	0.927
7	0.730	0.760	0.803	0.838	0.928
8	0.749	0.778	0.818	0.851	0.932
9	0.764	0.791	0.829	0.859	0.935
10	0.781	0.806	0.842	0.869	0.938
11	0.792	0.817	0.850	0.876	0.940
12	0.805	0.828	0.859	0.883	0.943
13	0.814	0.837	0.866	0.889	0.945
14	0.825	0.846	0.874	0.895	0.947
15	0.835	0.855	0.881	0.901	0.950
16	0.844	0.863	0.887	0.906	0.952
17	0.851	0.869	0.892	0.910	0.954
18	0.858	0.874	0.897	0.914	0.956
19	0.863	0.879	0.901	0.917	0.957
20	0.868	0.884	0.905	0.920	0.959
21	0.873	0.888	0.908	0.923	0.960
22	0.878	0.892	0.911	0.926	0.961
23	0.881	0.895	0.914	0.928	0.962
24	0.884	0.898	0.916	0.930	0.963
25	0.888	0.901	0.918	0.931	0.964
26	0.891	0.904	0.920	0.933	0.965
27	0.894	0.906	0.923	0.935	0.965
28	0.896	0.908	0.924	0.936	0.966
29	0.898	0.910	0.926	0.937	0.966
30	0.900	0.912	0.927	0.939	0.967
31	0.902	0.914	0.929	0.940	0.967
32	0.904	0.915	0.930	0.941	0.968
33	0.906	0.917	0.931	0.942	0.968
34	0.908	0.919	0.933	0.943	0.969
35	0.910	0.920	0.934	0.944	0.969
36	0.912	0.922	0.935	0.945	0.970
37	0.914	0.924	0.936	0.946	0.970
38	0.916	0.925	0.938	0.947	0.971
39	0.917	0.927	0.939	0.948	0.971
40	0.919	0.928	0.940	0.949	0.972
41	0.920	0.929	0.941	0.950	0.972
42	0.922	0.930	0.942	0.951	0.972
43	0.923	0.932	0.943	0.951	0.973
44	0.924	0.933	0.944	0.952	0.973
45	0.926	0.934	0.945	0.953	0.973
46	0.927	0.935	0.945	0.953	0.974
47	0.928	0.928	0.946	0.954	0.974
48	0.929	0.937	0.947	0.954	0.974
49	0.929	0.937	0.947	0.955	0.974
50	0.930	0.938	0.947	0.955	0.974

Source: Adapted from Shapiro, S. S. and Wilk, M. B. (1965), "An Analysis of Variance Test for Normality (Complete Samples)," *Biometrika*, 52, 591–611.

Table 3: Modifications and Percentiles for GOF Measures for Normal Distributions with μ and σ Unknown

		Upper Percentiles							
Statistic	Modified Statistic	.50	.25	.15	.10	.05	.025	.01	.005
D_n	$D_n(\sqrt{n} - .01 + .85/\sqrt{n})$	-	-	0.775	0.819	0.895	0.995	1.035	-
W_n^2	$W_n^2(1 + \frac{.5}{n})$	0.051	0.074	0.091	0.104	0.126	0.148	0.179	0.201
A_n^2	$A_n^2(1 + \frac{.75}{n} + \frac{2.25}{n^2})$	0.341	0.470	0.561	0.631	0.752	0.873	1.035	1.159

Table 4: Modifications and Percentiles for GOF Measures for Exponential Distribution with β Unknown

		Upper Percentiles							
Statistic	Modified Statistic	.25	.20	.15	.10	.05	.025	.01	.005
D_n	$(D_n - \frac{0.2}{n})(\sqrt{n} + 0.26 + \frac{0.5}{\sqrt{n}})$	-	-	0.926	0.995	1.094	1.184	-	-
W_n^2	$W_n^2(1.0 + \frac{0.16}{n})$	0.116	0.130	0.148	0.175	0.222	0.271	0.338	0.390
A_n^2	$A_n^2(1.0 + \frac{0.6}{n})$	0.736	0.816	0.916	1.062	1.321	1.591	1.959	2.244

Table 5: Modifications and Percentiles for A-D Measure for Extreme Value Distribution with Unspecified Parameters

		Upper Percentiles				
Statistic	Modified Statistic	.25	.10	.05	.025	.01
A_n^2	$A_n^2(1.0 + \frac{0.2}{\sqrt{n}})$	0.474	0.637	0.757	0.877	1.038

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VII.3 CONFIDENCE INTERVALS FOR MEDIANS

If the observations $x_1, x_2, \ldots, x_n$ are arranged in ascending order $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$, a $100(1-\alpha)\%$ confidence interval on the median of the population can be found. This table gives values of k and α such that one can be $100(1-\alpha)\%$ confident that the population median is between $x_{(k)}$ and $x_{(n-k+1)}$.

CONFIDENCE INTERVALS FOR THE MEDIAN

n	Largest k	Actual $\alpha \leq 0.05$	Largest k	Actual $\alpha \leq 0.01$	N	Largest k	Actual $\alpha \leq 0.05$	Largest k	Actual $\alpha \leq 0.01$
6	1	0.031			36	12	0.029	10	0.004
7	1	0.016			37	13	0.047	11	0.008
8	1	0.008	1	0.008	38	13	0.034	11	0.005
9	2	0.039	1	0.004	39	13	0.024	12	0.009
10	2	0.021	1	0.002	40	14	0.038	12	0.006
11	2	0.012	1	0.001	41	14	0.028	12	0.004
12	3	0.039	2	0.006	42	15	0.044	13	0.008
13	3	0.022	2	0.003	43	15	0.032	13	0.005
14	3	0.013	2	0.002	44	16	0.049	14	0.010
15	4	0.035	3	0.007	45	16	0.036	14	0.007
16	4	0.021	3	0.004	46	16	0.026	14	0.005
17	5	0.049	3	0.002	47	17	0.040	15	0.008
18	5	0.031	4	0.008	48	17	0.029	15	0.006
19	5	0.019	4	0.004	49	18	0.044	16	0.009
20	6	0.041	4	0.003	50	18	0.033	16	0.007
21	6	0.027	5	0.007	51	19	0.049	16	0.005
22	6	0.017	5	0.004	52	19	0.036	17	0.008
23	7	0.035	5	0.003	53	19	0.027	17	0.005
24	7	0.023	6	0.007	54	20	0.040	18	0.009
25	8	0.043	6	0.004	55	20	0.030	18	0.006
26	8	0.029	7	0.009	56	21	0.044	18	0.005
27	8	0.019	7	0.006	57	21	0.033	19	0.008
28	9	0.036	7	0.004	58	22	0.048	19	0.005
29	9	0.024	8	0.008	59	22	0.036	20	0.009
30	10	0.043	8	0.005	60	22	0.027	20	0.006
31	10	0.029	8	0.003	61	23	0.040	21	0.010
32	10	0.020	9	0.007	62	23	0.030	21	0.007
33	11	0.035	9	0.005	63	24	0.043	21	0.005
34	11	0.024	10	0.009	64	24	0.033	22	0.008
35	12	0.041	10	0.006	65	25	0.046	22	0.006

Factors for Determining Two-sided Tolerance Limits

	$\gamma = 0.90$			$\gamma = 0.95$			$\gamma = 0.99$		
		P			P			P	
n	0.900	0.950	0.990	0.900	0.950	0.990	0.900	0.950	0.990
2	15.512	18.221	23.423	31.092	36.519	46.944	155.569	182.720	234.877
3	5.788	6.823	8.819	8.306	9.789	12.647	18.782	22.131	28.586
4	4.157	4.913	6.372	5.368	6.341	8.221	9.416	11.118	14.405
5	3.499	4.142	5.387	4.291	5.077	6.598	6.655	7.870	10.220
10	2.546	3.026	3.958	2.856	3.393	4.437	3.617	4.294	5.610
15	2.285	2.720	3.565	2.492	2.965	3.885	2.967	3.529	4.621
20	2.158	2.570	3.372	2.319	2.760	3.621	2.675	3.184	4.175
25	2.081	2.479	3.254	2.215	2.638	3.462	2.506	2.984	3.915
30	2.029	2.417	3.173	2.145	2.555	3.355	2.394	2.851	3.742
35	1.991	2.371	3.114	2.094	2.495	3.276	2.314	2.756	3.618
40	1.961	2.336	3.069	2.055	2.448	3.216	2.253	2.684	3.524
45	1.938	2.308	3.032	2.024	2.412	3.168	2.205	2.627	3.450
50	1.918	2.285	3.003	1.999	2.382	3.129	2.166	2.580	3.390
60	1.888	2.250	2.956	1.960	2.335	3.068	2.106	2.509	3.297
70	1.866	2.224	2.922	1.931	2.300	3.023	2.062	2.457	3.228
80	1.849	2.203	2.895	1.908	2.274	2.988	2.028	2.416	3.175
90	1.835	2.186	2.873	1.890	2.252	2.959	2.001	2.384	3.133
100	1.823	2.172	2.855	1.875	2.234	2.936	1.978	2.357	3.098
150	1.786	2.128	2.796	1.826	2.176	2.859	1.905	2.271	2.985
200	1.764	2.102	2.763	1.798	2.143	2.816	1.866	2.223	2.921
250	1.750	2.085	2.741	1.780	2.121	2.788	1.839	2.191	2.880
300	1.740	2.073	2.725	1.767	2.106	2.767	1.820	2.169	2.850
350	1.732	2.064	2.713	1.757	2.094	2.752	1.806	2.152	2.828
400	1.726	2.057	2.703	1.749	2.084	2.739	1.794	2.138	2.810
450	1.721	2.051	2.695	1.743	2.077	2.729	1.785	2.127	2.795
500	1.717	2.046	2.689	1.737	2.070	2.721	1.777	2.117	2.783
550	1.713	2.041	2.683	1.733	2.065	2.713	1.770	2.109	2.772
600	1.710	2.038	2.678	1.729	2.060	2.707	1.765	2.103	2.763
650	1.707	2.034	2.674	1.725	2.056	2.702	1.759	2.097	2.755
700	1.705	2.032	2.670	1.722	2.052	2.697	1.755	2.091	2.748
750	1.703	2.029	2.667	1.719	2.049	2.692	1.751	2.086	2.742
800	1.701	2.027	2.664	1.717	2.046	2.688	1.747	2.082	2.736
850	1.699	2.025	2.661	1.715	2.043	2.685	1.744	2.078	2.731
900	1.697	2.023	2.658	1.712	2.040	2.682	1.741	2.075	2.727
950	1.696	2.021	2.656	1.711	2.038	2.679	1.738	2.071	2.722
1000	1.695	2.019	2.654	1.709	2.036	2.676	1.736	2.068	2.718
∞	1.645	1.960	2.576	1.645	1.960	2.576	1.645	1.960	2.576

Factors for Determining One-sided Tolerance Limits

	$\gamma = 0.90$			$\gamma = 0.95$			$\gamma = 0.99$		
		P			P			P	
n	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
2	10.253	13.090	18.500	20.581	26.260	37.094	103.029	131.426	185.617
3	4.258	5.311	7.340	6.155	7.656	10.553	13.995	17.370	23.896
4	3.188	3.957	5.438	4.162	5.144	7.042	7.380	9.083	12.387
5	2.742	3.400	4.666	3.407	4.203	5.741	5.362	6.578	8.939
10	2.066	2.568	3.532	2.355	2.911	3.981	3.048	3.738	5.074
15	1.867	2.329	3.212	2.068	2.566	3.520	2.521	3.102	4.222
20	1.765	2.208	3.052	1.926	2.396	3.295	2.276	2.808	3.832
25	1.702	2.132	2.952	1.838	2.292	3.158	2.129	2.633	3.601
30	1.657	2.080	2.884	1.777	2.220	3.064	2.030	2.515	3.447
35	1.624	2.041	2.833	1.732	2.167	2.995	1.957	2.430	3.334
40	1.598	2.010	2.793	1.697	2.125	2.941	1.902	2.364	3.249
45	1.577	1.986	2.761	1.669	2.092	2.898	1.857	2.312	3.180
50	1.559	1.965	2.735	1.646	2.065	2.862	1.821	2.269	3.125
60	1.532	1.933	2.694	1.609	2.022	2.807	1.764	2.202	3.038
70	1.511	1.909	2.662	1.581	1.990	2.765	1.722	2.153	2.974
80	1.495	1.890	2.638	1.559	1.964	2.733	1.688	2.114	2.924
90	1.481	1.874	2.618	1.542	1.944	2.706	1.661	2.082	2.883
100	1.470	1.861	2.601	1.527	1.927	2.684	1.639	2.056	2.850
150	1.433	1.818	2.546	1.478	1.870	2.611	1.566	1.971	2.740
200	1.411	1.793	2.514	1.450	1.837	2.570	1.524	1.923	2.679
250	1.397	1.777	2.493	1.431	1.815	2.542	1.496	1.891	2.638
300	1.386	1.765	2.477	1.417	1.800	2.522	1.475	1.868	2.608
350	1.378	1.755	2.466	1.406	1.787	2.506	1.461	1.850	2.585
400	1.372	1.748	2.456	1.398	1.778	2.494	1.448	1.836	2.567
450	1.366	1.742	2.448	1.391	1.770	2.484	1.438	1.824	2.553
500	1.362	1.736	2.442	1.385	1.763	2.475	1.430	1.814	2.540
550	1.358	1.732	2.436	1.380	1.757	2.468	1.422	1.806	2.530
600	1.355	1.728	2.431	1.376	1.752	2.462	1.416	1.799	2.520
650	1.352	1.725	2.427	1.372	1.748	2.456	1.411	1.792	2.512
700	1.349	1.722	2.423	1.368	1.744	2.451	1.406	1.787	2.505
750	1.347	1.719	2.420	1.365	1.741	2.447	1.401	1.782	2.499
800	1.344	1.717	2.417	1.363	1.737	2.443	1.397	1.777	2.493
850	1.343	1.714	2.414	1.360	1.734	2.439	1.394	1.773	2.488
900	1.341	1.712	2.411	1.358	1.732	2.436	1.390	1.769	2.483
950	1.339	1.711	2.409	1.356	1.729	2.433	1.387	1.766	2.479
1000	1.338	1.709	2.407	1.354	1.727	2.430	1.385	1.762	2.475
∞	1.282	1.645	2.326	1.282	1.645	2.326	1.282	1.645	2.326

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Tolerance Intervals (P, γ): $[X_{(r)}, X_{(n-s+1)}]$

Values of $m = r + s$ such that we may assert with confidence at least γ that 100 P percent of a population lies between the rth smallest and the sth largest of a random sample of n from that population (continuous distribution function assumed)

n	$\gamma = 0.50$					$\gamma = 0.75$					$\gamma = 0.90$					$\gamma = 0.95$					$\gamma = 0.99$				
P	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99
50	25	12	5	2	0	22	10	3	1	—	20	9	2	1	—	19	8	2	—	—	16	6	1	—	—
55	28	14	5	3	0	25	12	4	2	—	23	10	3	1	—	21	9	2	—	—	19	7	1	—	—
60	30	15	6	3	0	27	13	4	2	—	25	11	3	1	—	24	10	2	1	—	21	8	1	—	—
65	33	16	6	3	0	30	14	5	2	—	27	12	4	1	—	26	11	3	1	—	23	9	2	—	—
70	35	17	7	3	1	32	15	5	2	—	30	13	4	1	—	28	12	3	1	—	25	10	2	—	—
75	38	19	7	4	1	35	16	6	2	—	32	14	4	1	—	30	13	3	1	—	27	16	2	—	—
80	40	20	8	4	1	37	17	6	3	—	34	15	5	2	—	33	14	4	1	—	30	11	2	—	—
85	43	21	8	4	1	39	19	7	3	—	37	16	5	2	—	35	15	4	1	—	32	12	3	—	—
90	45	22	9	4	1	42	20	7	3	—	39	17	5	2	—	37	16	5	1	—	34	13	3	1	—
95	48	24	9	5	1	44	21	7	3	—	41	18	6	2	—	39	17	5	2	—	36	14	3	1	—
100	50	25	10	5	1	47	22	8	3	—	44	20	6	2	—	42	18	5	2	—	38	15	4	1	—
110	55	27	11	5	1	51	24	9	4	—	48	22	7	3	—	46	20	6	2	—	43	17	4	1	—
120	60	30	12	6	1	56	27	10	4	—	53	24	8	3	—	51	22	7	2	—	47	19	5	1	—
130	65	32	13	6	1	61	29	11	5	—	58	26	9	3	—	56	25	8	3	—	52	21	6	2	—
140	70	35	14	7	1	66	31	12	5	1	62	28	10	4	—	60	27	8	3	—	56	23	6	2	—
150	75	37	15	7	1	71	34	12	6	1	67	31	10	4	—	65	29	9	3	—	61	26	7	2	—
170	85	42	17	8	2	81	39	14	7	1	77	35	12	5	—	74	33	11	4	—	70	30	9	3	—
200	100	50	20	10	2	95	46	17	8	1	91	42	15	6	—	88	40	13	5	—	84	36	11	4	—
300	150	75	30	15	3	144	70	26	12	2	139	65	23	10	1	136	63	22	9	1	130	58	19	7	—
400	200	100	40	20	4	193	94	36	17	3	187	89	32	15	2	184	86	30	13	1	177	80	27	11	—
500	250	125	50	25	5	242	118	45	22	3	236	113	41	19	2	232	109	39	17	2	224	103	35	14	1
600	300	150	60	30	6	292	143	55	26	4	284	136	51	23	3	280	133	48	21	2	272	126	44	18	1
700	350	175	70	35	7	341	167	65	31	5	333	160	60	28	4	328	156	57	26	3	319	149	52	22	2
800	400	200	80	40	8	390	192	74	36	6	382	184	69	32	5	377	180	66	30	4	367	172	61	26	2
900	450	225	90	45	9	440	216	84	41	7	431	208	79	37	5	425	204	75	35	4	415	195	70	30	3
1000	500	250	100	50	10	489	241	94	45	8	480	233	88	41	6	474	228	85	39	5	463	219	79	35	3

Tolerance Interval (P, γ): $[X_{(1)}, X_{(n)}]$

Confidence γ with which we may assert that 100 P percent of the population lies between the largest and smallest of a random sample of n from that population (continuous distribution assumed)

n	$P = .50$	$P = .75$	$P = .90$	$P = .95$	$P = .99$	n	$P = .75$	$P = .90$	$P = .95$	$P = .99$
3	.50	.16	.03	.01	.00	17	.95	.52	.21	.01
4	.69	.26	.05	.01	.00	18	.96	.55	.22	.01
5	.81	.37	.08	.02	.00	19	.97	.58	.25	.02
6	.89	.47	.11	.03	.00	20	.98	.61	.26	.02
7	.94	.56	.15	.04	.00	25	.99	.73	.36	.03
8	.96	.63	.19	.06	.00	30	1.00−	.82	.45	.04
9	.98	.70	.23	.07	.00	40		.92	.60	.06
10	.99	.76	.26	.09	.00	50		.97	.72	.09
11	.99	.80	.30	.10	.01	60		.99	.81	.12
12	1.00−	.84	.34	.12	.01	70		.99	.87	.16
13		.87	.38	.14	.01	80		1.00−	.91	.19
14		.90	.42	.15	.01	90			.94	.23
15		.92	.45	.17	.01	100			.96	.26
		.94	.49	.19	.01					

Table of Common Distributions

Discrete Distributions

Bernoulli(p)

pmf	$P(X=x\|p)=p^x(1-p)^{1-x};\quad x=0,1;\quad 0\le p\le 1$
mean and variance	$\mathrm{E}X=p,\quad \mathrm{Var}\,X=p(1-p)$
mgf	$M_X(t)=(1-p)+pe^t$

Binomial(n,p)

pmf	$P(X=x\|n,p)=\binom{n}{x}p^x(1-p)^{n-x};\quad x=0,1,2,\ldots,n;\quad 0\le p\le 1$
mean and variance	$\mathrm{E}X=np,\quad \mathrm{Var}\,X=np(1-p)$
mgf	$M_X(t)=[pe^t+(1-p)]^n$
notes	Related to Binomial Theorem (Theorem 3.2.2). The *multinomial* distribution (Definition 4.6.2) is a multivariate version of the binomial distribution.

Discrete uniform

pmf	$P(X=x\|N)=\frac{1}{N};\quad x=1,2,\ldots,N;\quad N=1,2,\ldots$
mean and variance	$\mathrm{E}X=\frac{N+1}{2},\quad \mathrm{Var}\,X=\frac{(N+1)(N-1)}{12}$
mgf	$M_X(t)=\frac{1}{N}\sum_{i=1}^{N}e^{it}$

Geometric(p)

pmf	$P(X=x\|p)=p(1-p)^{x-1};\quad x=1,2,\ldots;\quad 0\le p\le 1$
mean and variance	$\mathrm{E}X=\frac{1}{p},\quad \mathrm{Var}\,X=\frac{1-p}{p^2}$

mgf	$M_X(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -\log(1-p)$
notes	$Y = X - 1$ is negative binomial$(1, p)$. The distribution is *memoryless*: $P(X > s \mid X > t) = P(X > s - t)$.

Hypergeometric

pmf	$P(X = x \mid N, M, K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}; \quad x = 0, 1, 2, \ldots, K;$ $M - (N - K) \leq x \leq M; \quad N, M, K \geq 0$
mean and variance	$\mathrm{E}X = \frac{KM}{N}, \quad \mathrm{Var}\, X = \frac{KM}{N}\frac{(N-M)(N-K)}{N(N-1)}$
notes	If $K \ll M$ and N, the range $x = 0, 1, 2, \ldots, K$ will be appropriate.

Negative binomial(r, p)

pmf	$P(X = x \mid r, p) = \binom{r+x-1}{x} p^r (1-p)^x; \quad x = 0, 1, \ldots; \quad 0 \leq p \leq 1$
mean and variance	$\mathrm{E}X = \frac{r(1-p)}{p}, \quad \mathrm{Var}\, X = \frac{r(1-p)}{p^2}$
mgf	$M_X(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r, \quad t < -\log(1-p)$
notes	An alternate form of the pmf is given by $P(Y = y \mid r, p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$, $y = r, r+1, \ldots$. The random variable $Y = X + r$. The negative binomial can be derived as a gamma mixture of Poissons. (See Exercise 4.32.)

Poisson(λ)

pmf	$P(X = x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; \quad x = 0, 1, \ldots; \quad 0 \leq \lambda < \infty$
mean and variance	$\mathrm{E}X = \lambda, \quad \mathrm{Var}\, X = \lambda$
mgf	$M_X(t) = e^{\lambda(e^t - 1)}$

Beta(α, β)

pdf	$f(x\|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \le x \le 1, \quad \alpha > 0, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \frac{\alpha}{\alpha+\beta}, \quad \mathrm{Var}\, X = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
mgf	$M_X(t) = 1 + \sum_{k=1}^{\infty} \left(\prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r} \right) \frac{t^k}{k!}$
notes	The constant in the beta pdf can be defined in terms of gamma functions, $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. Equation (3.2.18) gives a general expression for the moments.

Cauchy(θ, σ)

pdf	$f(x\|\theta,\sigma) = \frac{1}{\pi\sigma} \frac{1}{1+\left(\frac{x-\theta}{\sigma}\right)^2}, \quad -\infty < x < \infty; \quad -\infty < \theta < \infty, \quad \sigma > 0$
mean and variance	do not exist
mgf	does not exist
notes	Special case of Student's t, when degrees of freedom $= 1$. Also, if X and Y are independent $\mathrm{n}(0,1)$, X/Y is Cauchy.

Chi squared(p)

pdf	$f(x\|p) = \frac{1}{\Gamma(p/2)2^{p/2}} x^{(p/2)-1} e^{-x/2}; \quad 0 \le x < \infty; \quad p = 1, 2, \ldots$
mean and variance	$\mathrm{E}X = p, \quad \mathrm{Var}\, X = 2p$
mgf	$M_X(t) = \left(\frac{1}{1-2t}\right)^{p/2}, \quad t < \frac{1}{2}$
notes	Special case of the gamma distribution.

Double exponential(μ, σ)

pdf	$f(x\|\mu,\sigma) = \frac{1}{2\sigma} e^{-\|x-\mu\|/\sigma}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$
mean and variance	$\mathrm{E}X = \mu, \quad \mathrm{Var}\, X = 2\sigma^2$
mgf	$M_X(t) = \frac{e^{\mu t}}{1-(\sigma t)^2}, \quad \|t\| < \frac{1}{\sigma}$
notes	Also known as the *Laplace* distribution.

Exponential(β)

pdf	$f(x\|\beta) = \frac{1}{\beta} e^{-x/\beta}, \quad 0 \le x < \infty, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \beta, \quad \mathrm{Var}\, X = \beta^2$
mgf	$M_X(t) = \frac{1}{1-\beta t}, \quad t < \frac{1}{\beta}$
notes	Special case of the gamma distribution. Has the *memoryless* property. Has many special cases: $Y = X^{1/\gamma}$ is *Weibull*, $Y = \sqrt{2X/\beta}$ is *Rayleigh*, $Y = \alpha - \gamma \log(X/\beta)$ is *Gumbel*.

F

pdf	$f(x\|\nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{x^{(\nu_1-2)/2}}{\left(1+\left(\frac{\nu_1}{\nu_2}\right)x\right)^{(\nu_1+\nu_2)/2}};$ $0 \le x < \infty; \quad \nu_1, \nu_2 = 1, \ldots$
mean and variance	$\mathrm{E}X = \frac{\nu_2}{\nu_2-2}, \quad \nu_2 > 2,$ $\mathrm{Var}\, X = 2\left(\frac{\nu_2}{\nu_2-2}\right)^2 \frac{(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}, \quad \nu_2 > 4$
moments (mgf does not exist)	$\mathrm{E}X^n = \frac{\Gamma\left(\frac{\nu_1+2n}{2}\right)\Gamma\left(\frac{\nu_2-2n}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\frac{\nu_2}{\nu_1}\right)^n, \quad n < \frac{\nu_2}{2}$
notes	Related to chi squared ($F_{\nu_1,\nu_2} = \left(\frac{\chi^2_{\nu_1}}{\nu_1}\right) / \left(\frac{\chi^2_{\nu_2}}{\nu_2}\right)$, where the χ^2s are independent) and t ($F_{1,\nu} = t_\nu^2$).

Gamma(α, β)

pdf	$f(x\|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad 0 \le x < \infty, \quad \alpha, \beta > 0$
mean and variance	$\mathrm{E}X = \alpha\beta, \quad \mathrm{Var}\, X = \alpha\beta^2$
mgf	$M_X(t) = \left(\frac{1}{1-\beta t}\right)^\alpha, \quad t < \frac{1}{\beta}$
notes	Some special cases are exponential ($\alpha = 1$) and chi squared ($\alpha = p/2$, $\beta = 2$). If $\alpha = \frac{3}{2}$, $Y = \sqrt{X/\beta}$ is *Maxwell*. $Y = 1/X$ has the *inverted gamma distribution*. Can also be related to the Poisson (Example 3.2.1).

Logistic(μ, β)

pdf	$f(x\|\mu, \beta) = \frac{1}{\beta} \frac{e^{-(x-\mu)/\beta}}{[1+e^{-(x-\mu)/\beta}]^2}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \mu, \quad \mathrm{Var}\, X = \frac{\pi^2\beta^2}{3}$

mgf	$M_X(t) = e^{\mu t}\Gamma(1-\beta t)\Gamma(1+\beta t), \quad \lvert t\rvert < \frac{1}{\beta}$
notes	The cdf is given by $F(x\lvert\mu,\beta) = \frac{1}{1+e^{-(x-\mu)/\beta}}$.

Lognormal(μ, σ^2)

pdf	$f(x\lvert\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}, \quad 0 \le x < \infty, \quad -\infty < \mu < \infty,$ $\sigma > 0$
mean and variance	$\mathrm{E}X = e^{\mu+(\sigma^2/2)}, \quad \mathrm{Var}\,X = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$
moments (mgf does not exist)	$\mathrm{E}X^n = e^{n\mu+n^2\sigma^2/2}$
notes	Example 2.3.5 gives another distribution with the same moments.

Normal(μ, σ^2)

pdf	$f(x\lvert\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty,$ $\sigma > 0$
mean and variance	$\mathrm{E}X = \mu, \quad \mathrm{Var}\,X = \sigma^2$
mgf	$M_X(t) = e^{\mu t+\sigma^2t^2/2}$
notes	Sometimes called the *Gaussian* distribution.

Pareto(α, β)

pdf	$f(x\lvert\alpha,\beta) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad a < x < \infty, \quad \alpha > 0, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \frac{\beta\alpha}{\beta-1}, \quad \beta > 1, \quad \mathrm{Var}\,X = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}, \quad \beta > 2$
mgf	does not exist

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t

pdf	$f(x\lvert\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\left(\frac{x^2}{\nu}\right)\right)^{(\nu+1)/2}}, \quad -\infty < x < \infty, \quad \nu = 1, \ldots$
mean and variance	$\mathrm{E}X = 0, \quad \nu > 1, \quad \mathrm{Var}\,X = \frac{\nu}{\nu-2}, \quad \nu > 2$
moments (mgf does not exist)	$\mathrm{E}X^n = \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{\nu-n}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\nu^{n/2}$ if $n < \nu$ and even, $\mathrm{E}X^n = 0$ if $n < \nu$ and odd.
notes	Related to F $(F_{1,\nu} = t_\nu^2)$.

Uniform(a, b)

pdf	$f(x\|a,b) = \frac{1}{b-a}, \quad a \le x \le b$
mean and variance	$\mathrm{E}X = \frac{b+a}{2}, \quad \mathrm{Var}\,X = \frac{(b-a)^2}{12}$
mgf	$M_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$
notes	If $a = 0$ and $b = 1$, this is a special case of the beta ($\alpha = \beta = 1$).

Weibull(γ, β)

pdf	$f(x\|\gamma,\beta) = \frac{\gamma}{\beta} x^{\gamma-1} e^{-x^\gamma/\beta}, \quad 0 \le x < \infty, \quad \gamma > 0, \quad \beta > 0$
mean and variance	$\mathrm{E}X = \beta^{1/\gamma}\Gamma\left(1 + \frac{1}{\gamma}\right), \quad \mathrm{Var}\,X = \beta^{2/\gamma}\left[\Gamma\left(1 + \frac{2}{\gamma}\right) - \Gamma^2\left(1 + \frac{1}{\gamma}\right)\right]$
moments	$\mathrm{E}X^n = \beta^{n/\gamma}\Gamma\left(1 + \frac{n}{\gamma}\right)$
notes	The mgf exists only for $\gamma \ge 1$. Its form is not very useful. A special case is exponential ($\gamma = 1$).

Table A.1 Cumulative Binomial Probabilities (*cont.*)

c. $n = 15$

		p														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	0.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
	2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
	4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
	5	1.000	1.000	.998	.939	.852	.722	.403	.151	.034	.004	.001	.000	.000	.000	.000
	6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
x	7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
	8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
	9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
	10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
	11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.920	.833	.451	.171	.010
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.987	.965	.794	.537	.140

d. $n = 20$

		p														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	.818	.358	.122	.012	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.983	.736	.392	.069	.024	.008	.001	.000	.000	.000	.000	.000	.000	.000	.000
	2	.999	.925	.677	.206	.091	.035	.004	.000	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.984	.867	.411	.225	.107	.016	.001	.000	.000	.000	.000	.000	.000	.000
	4	1.000	.998	.957	.630	.415	.238	.051	.006	.000	.000	.000	.000	.000	.000	.000
	5	1.000	1.000	.989	.804	.617	.416	.126	.021	.002	.000	.000	.000	.000	.000	.000
	6	1.000	1.000	.998	.913	.786	.608	.250	.058	.006	.000	.000	.000	.000	.000	.000
	7	1.000	1.000	1.000	.968	.898	.772	.416	.132	.021	.001	.000	.000	.000	.000	.000
	8	1.000	1.000	1.000	.990	.959	.887	.596	.252	.057	.005	.001	.000	.000	.000	.000
	9	1.000	1.000	1.000	.997	.986	.952	.755	.412	.128	.017	.004	.001	.000	.000	.000
	10	1.000	1.000	1.000	.999	.996	.983	.872	.588	.245	.048	.014	.003	.000	.000	.000
x	11	1.000	1.000	1.000	1.000	.999	.995	.943	.748	.404	.113	.041	.010	.000	.000	.000
	12	1.000	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.102	.032	.000	.000	.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.214	.087	.002	.000	.000
	14	1.000	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.383	.196	.011	.000	.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.585	.370	.043	.003	.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.775	.589	.133	.016	.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.909	.794	.323	.075	.001
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.976	.931	.608	.264	.017
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.997	.988	.878	.642	.182

Table A.1 Cumulative Binomial Probabilities (*cont.*)

e. $n = 25$

		p														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	.778	.277	.072	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.974	.642	.271	.027	.007	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.998	.873	.537	.098	.032	.009	.000	.000	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.966	.764	.234	.096	.033	.000	.000	.000	.000	.000	.000	.000	.000	.000
	4	1.000	.993	.902	.421	.214	.090	.009	.000	.000	.000	.000	.000	.000	.000	.000
	5	1.000	.999	.967	.617	.378	.193	.029	.002	.000	.000	.000	.000	.000	.000	.000
	6	1.000	1.000	.991	.780	.561	.341	.074	.007	.000	.000	.000	.000	.000	.000	.000
	7	1.000	1.000	.998	.891	.727	.512	.154	.022	.001	.000	.000	.000	.000	.000	.000
	8	1.000	1.000	1.000	.953	.851	.677	.274	.054	.004	.000	.000	.000	.000	.000	.000
	9	1.000	1.000	1.000	.983	.929	.811	.425	.115	.013	.000	.000	.000	.000	.000	.000
	10	1.000	1.000	1.000	.994	.970	.902	.586	.212	.034	.002	.000	.000	.000	.000	.000
	11	1.000	1.000	1.000	.998	.980	.956	.732	.345	.078	.006	.001	.000	.000	.000	.000
x	12	1.000	1.000	1.000	1.000	.997	.983	.846	.500	.154	.017	.003	.000	.000	.000	.000
	13	1.000	1.000	1.000	1.000	.999	.994	.922	.655	.268	.044	.020	.002	.000	.000	.000
	14	1.000	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.030	.006	.000	.000	.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.071	.017	.000	.000	.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.149	.047	.000	.000	.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.273	.109	.002	.000	.000
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.439	.220	.009	.000	.000
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.622	.383	.033	.001	.000
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.786	.579	.098	.007	.000
	21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.904	.766	.236	.034	.000
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.968	.902	.463	.127	.002
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.993	.973	.729	.358	.026
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.996	.928	.723	.222

SOURCE: Adapted from L. L. Chao (1980), *Statistics for Management*, Wadsworth, Inc.

Table A.2 Cumulative Poisson Probabilities

		λ									
		.1	.2	.3	.4	.5	.6	.7	.8	0.9	1.0
	0	.905	.819	.741	.670	.607	.549	.497	.449	.407	.368
	1	.995	.982	.963	.938	.910	.878	.844	.809	.772	.736
	2	1.000	.999	.996	.992	.986	.977	.966	.953	.937	.920
x	3		1.000	1.000	.999	.998	.997	.994	.991	.987	.981
	4				1.000	1.000	1.000	.999	.999	.998	.996
	5							1.000	1.000	1.000	.999
	6										1.000

Table A.2 Cumulative Poisson Probabilities *(cont.)*

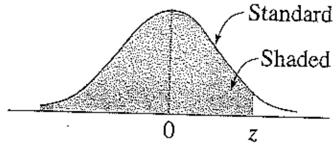
	λ										
	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	15.0	20.0
0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
11			.999	.995	.980	.947	.888	.803	.697	.185	.021
12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
13				.999	.996	.987	.966	.926	.864	.363	.066
14				1.000	.999	.994	.983	.959	.917	.466	.105
15					.999	.998	.992	.978	.951	.568	.157
16					1.000	.999	.996	.989	.973	.664	.221
17						.999	.998	.995	.986	.749	.297
18						1.000	.999	.998	.993	.819	.381
19							1.000	.998	.997	.875	.470
20								1.000	.998	.917	.559
21									.999	.947	.644
22									1.000	.967	.721
23										.981	.787
24										.989	.843
25										.994	.888
26										.997	.922
27										.998	.948
28										.999	.966
29										1.000	.978
30											.987
31											.992
32											.995
33											.997
34											.999
35											.999
36											1.000

SOURCE: L. L. Chao (1974), *Statistics: Methods and Analysis*, 2nd ed. New York: McGraw-Hill.

Table

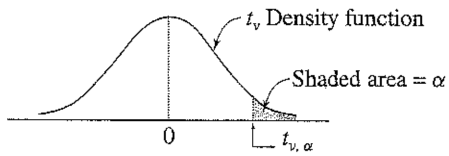
z
−3.4
−3.3
−3.2
−3.1
−3.0
−2.9
−2.8
−2.7
−2.6
−2.5
−2.4
−2.3
−2.2
−2.1
−2.0
−1.9
−1.8
−1.7
−1.6
−1.5
−1.4
−1.3
−1.2
−1.1
−1.0
−0.9
−0.8
−0.7
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1
−0.0

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$ (*cont.*)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0,4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1,7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

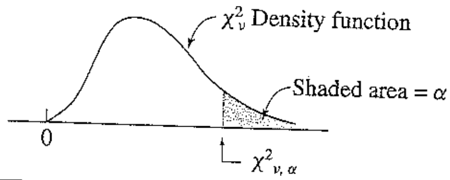
Table A.4 Critical Values $t_{v,\alpha}$ for the t-Distribution



v	α						
	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	1.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

SOURCE: This table is produced with the kind permission of the Trustees of Biometrika from E. S. Pearson and H. O. Hartly (eds.) *The Biometrika Tables for Statisticians*, vol. 1, 3rd ed. (1966) Biometrika.

Table A.5 Critical Values $\chi^2_{v,\alpha}$ for the Chi-square Distribution

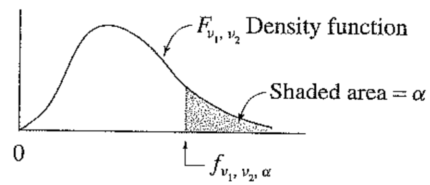


v	α = .995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.420	65.473
40*	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

* For $v > 40$, $x^2_{v,\alpha} \simeq \left(1 - \frac{2}{9v} + z_\alpha \sqrt{\frac{2}{9v}}\right)^3$.

SOURCE: This table is produced with the kind permission of the Trustees of Biometrika from E. S. Pearson and H. O. Hartly (eds.) *The Biometrika Tables for Statisticians*, vol. 1, 3rd ed. (1966) Biometrika.

Table A.6 Critical Values $f_{v_1,v_2,\alpha}$ for the F-Distribution ($\alpha = .05$) (*cont.*)



Degrees of freedom for the denominator (v_2)	Degrees of freedom for the numerator (v_1) 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.49	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.69	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.81	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.91	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.09	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.59	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.81	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00