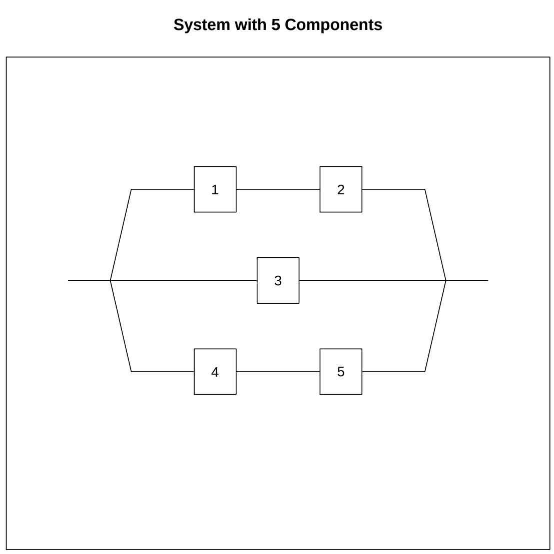
Statistics 630 - Assignment 2
(due Friday, 17 September 2021)

Instructions: (same as those given in the first assignment)
The material covered by this assignment is primarily in Lectures 04–07 and Chapters 1–2 of the textbook.

1. (a) Prove Bonferroni's inequality: $\mathsf{P}(A \cap B) \geq \mathsf{P}(A) + \mathsf{P}(B) - 1$.
   (b) In statistics we often talk about the event that our statistical procedure will lead to a correct (true) conclusion. Suppose A and B are such events (for two different procedures, but in the same experiment) and each has probability 95%. According to Bonferroni's inequality, what can we say about the chance that both procedures lead to correct conclusions?
   (c) Extrapolate to 3 events. Specifically, suppose 3 statistical procedures each have probability $1 - \alpha/3$ of resulting in a correct conclusion and show that the probability that all 3 are correct is at least $1 - \alpha$.
2. Chapter 1 Exercises 1.5.9, 1.5.14, 1.5.18(a,b,c).
3. The system shown below has five components which act independently. Each component fails with probability p. Find the probability that the system fails.



4. If a parent has genotype Aa, he transmits either A or a to an offspring, each with probability 1/2. The gene he transmits to one offspring is independent of the gene he transmits to all other offspring. Consider a parent with three children (labeled 1,2,3) and the following events: $B = \{1 \text{ and } 2 \text{ have the same gene}\}$, $C = \{2 \text{ and } 3 \text{ have the same gene}\}$, $D = \{1 \text{ and } 3 \text{ have the same gene}\}$. Show that all these events are pairwise independent, but *not mutually independent*.
5. Chapter 2 Exercise 2.1.5. Add the following.
   (b) Show that $I_{A \cup B} = \max(I_A, I_B)$.
6. Chapter 2 Exercise 2.1.8. In fact, compute $W(s)$ and $Z(s)$ for all $s \in S$.

(continued next page)

7. Chapter 2 Exercise 2.2.4.

8. Chapter 2 Exercises 2.3.9, 2.3.10, 2.3.13, 2.3.14.

9. Chapter 2 Exercise 2.3.15. For (b) use the interpretation that the first basket is obtained on the 10th attempt, and for (c) that the second basket is obtained on the 10th attempt.

10. Use R to check the accuracy of the binomial approximation to the hypergeometric distribution. You can use the `dbinom` and `dhyper` functions to compute probabilities for the two distributions. Compare the distributions for $n = 10$, $M/N = 0.6$ and $N = 50, 100, 1000$. The syntax in R for computing the hypergeometric probability mass function is `dhyper(x,M,N-M,n)`. Note that `x` can be a vector in either function. So, for example, `dbinom(0:3,10,.4)` returns a vector with binomial(10,.4) probabilities for $x = 0, 1, 2, 3$.