STAT 608, Spring 2022 - Assignment 0 (Review - not for a grade)

1. Matrix Algebra Review

   Define matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 2 & 0 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & -1 \\ 3 & 0 \\ 2 & 1 \\ 0 & -2 \end{bmatrix}, \text{ and } \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

   (a) Calculate $\mathbf{A}'$, the transpose of $\mathbf{A}$.

   (b) Calculate $\mathbf{A}' + \mathbf{B}$.

   (c) Calculate $\mathbf{AB}$, the matrix product of $\mathbf{A}$ and $\mathbf{B}$.

   (d) Calculate $\mathbf{BA}$. Is $\mathbf{AB} = \mathbf{BA}$?

   (e) Is the matrix $\mathbf{AB}$ singular? Why or why not? (Invertible means nonsingular; see the Wikipedia page for invertible matrices for a review.)

   (f) Calculate the trace of $\mathbf{AB}$.

   (g) Write $(\mathbf{AB})'$ in another form algebraically: remove the parentheses.

   (h) Calculate $(\mathbf{AB})^{-1}$.

   (i) Write $\mathbf{I}_2$, the $2 \times 2$ identity matrix.

   (j) What is $\mathbf{I}_2\mathbf{A}$? Why?

   (k) Describe geometrically the space spanned by $\mathbf{C}$. That is, the space spanned by the two column vectors in the matrix $\mathbf{C}$. Assume we're working in three-dimensional space defined by axes $xyz$.

   (l) Calculate the projection matrix for $\mathbf{C}$. That is, what is the matrix that projects a vector in $x, y, z$ space onto the $x, y$ plane?

   (m) Project the vector $\mathbf{d} = [2\ 2\ 2]'$ onto the space spanned by $\mathbf{C}$.

   (n) Describe geometrically what you did in the previous step.

   (o) Are the vectors $\mathbf{d}$ and $\mathbf{f} = [1\ 0\ 0]'$ orthogonal? Why or why not? (Talk about a dot product in your answer.)

   (p) Calculate the dot product $\mathbf{1} \cdot \mathbf{1}$, where the vector $\mathbf{1} = [1\ 1\ \ldots 1]'$ is of length $n$.

   (q) Calculate the dot product $\mathbf{1} \cdot \mathbf{x}$, where $\mathbf{1}$ is defined as above and $\mathbf{x} = [x_1\ x_2\ \ldots x_n]'$.

   (r) Calculate the dot product $\mathbf{x} \cdot \mathbf{x}$, where $\mathbf{x}$ is defined as above.

   (s) Describe geometrically what the first eigenvector (sorted in order from highest eigenvalue to lowest) would tell you about the vector space.

2. Calculus Review

   Define

$$f(x, y) = 3x^2 + 2xy^2 - y$$

   (a) Calculate $\frac{\partial}{\partial x} f(x, y)$.

(b) Calculate $\frac{\partial}{\partial y} f(x, y)$.

3. Log Review

   (a) Calculate $\log(e)$. (Note that statisticians usually write "log" instead of "ln" when they mean log base $e$.)

   (b) Rewrite $\log\left(\frac{x}{y}\right)$ in terms of a difference.

   (c) Rewrite $\log\left(x^n\right)$ in terms of a product.

   (d) Solve $\log(x) = y$ for $x$.

4. Statistics and Linear Regression Review

   After regressing eight patients' weights (in kg) on their height (in cm), a doctor found the following output.

| Coefficient | Estimate | Std. Error | t-value | $Pr(>\|t\|)$ |
|---|---|---|---|---|
| Intercept | -129.1667 | 24.3610 | -5.302 | 0.001826 |
| Height | 1.1667 | 0.1521 | ???? | 0.000257 |

   (a) Write down the least squares regression line using $\hat{y}$ = predicted weight and $x$ = height.

   (b) What weight does the model predict for someone who is 160 cm tall?

   (c) Interpret the slope of the line in the context of the model.

   (d) Interpret the standard error of the slope in the context of the model.

   (e) Calculate the t-statistic for testing whether the slope is statistically significant.

   (f) Are height and weight linearly associated? Explain. (Assume assumptions are met.)

   (g) A journal might report that height is a *significant* predictor of weight. Explain what this means in context, as if to someone with no statistical background.

   (h) Calculate a 95% confidence interval for the slope.

   (i) Interpret your interval above in context.