

STAT 608, Spring 2022 - Assignment 7

1. For an AR(1) process:

$$e_t = \rho e_{t-1} + \nu_t$$

for $t = 2, 3, \dots, n$ and where $\nu_1, \nu_2, \dots, \nu_n$ are iid $N(0, \sigma_\nu^2)$, show that

$$\text{Corr}(e_t, e_{t-2}) = \rho^2$$

2. Consider the regression model:

$$y_t = \beta_0 + \beta_1 x_t + e_t$$

where the e_t follow an AR(1) process: $e_t = \rho e_{t-1} + \nu_t$, where the ν_t are iid $N(0, \sigma_\nu^2)$. Conduct a simulation to examine the coverage probabilities of nominal 95% confidence intervals for mean response when $\rho = 0.1, 0.2, \dots, 0.9$ and when usual least squares is used to fit the model (i.e., assuming no serial correlation). Provide either a table or plot showing the actual coverage probabilities as a function of ρ . In your simulation, use $x_1, x_2, \dots, x_n \stackrel{iid}{\sim} U(0, 1)$, $\beta_0 = 0.5$, $\beta_1 = 1.5$, $n = 50$, and $\sigma_\nu^2 = 0.5$. In each simulation, compute a CI for mean response when $x = 0.5$. Use $M = 1000$ simulations for each value of ρ .

3. Exercise 9.2 from the textbook.