STATISTICS 641 - ASSIGNMENT 7

DUE DATE: NOON, MONDAY, NOVEMBER 22, 2021

Name ____

Email Address ____

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- Read Handout 12
- Supplemental Reading from Devore book: Chapters 8

P1. (10 points) A researcher presents you with the following data Y_i, $i = 1, 2, \ldots 29$, which is independent and identically normally distributed with a mean μ and variance σ^2.

1. Set up the rejection region for testing the hypotheses $\mathrm{H}_0 : \mu = 20$ vs $\mathrm{H}_1 : \mu \neq 20$, based on $Y_1, \ldots, Y_{29}$ and $\alpha = 0.05$. Assume σ is unknown.
2. Calculate the power of your test for the following values of the true parameter μ:

   19.9, 19.95, 19.99, 20, 20.05, 20.1, 20.15, 20.2, 20.25, 20.3, 20.4, 20.5.

   The researcher provides the estimate, $\hat{\sigma} = .43$. This value of σ should just be used in the power calculation and not in the actual testing procedure. Use your results to sketch a power curve for your test. Be sure to label your axes clearly.
3. Determine the necessary sample size so that the power at $\mu = 20.15$ is at least 0.80 for a level $\alpha = .05$ test of $\mathrm{H}_0 : \mu \leq 20$ vs $\mathrm{H}_1 : \mu > 20$.

P2. (10 points) A new additive has been formulated to reduce the reaction time in a chemical process. With the previously used additive, the average reaction time was 10 minutes. In order to evaluate the effectiveness of the new additive, 15 batches of the material are formulated and the new additive is placed in the batches. From previous studies, reaction times appear to have a normal distribution.

1. The mean reaction time from the 15 batches was 8.7 minutes with a standard deviation of 2 minutes. Is there significant evidence using an $\alpha = .01$ test that the average reaction time has been reduced? Include the p-value with your decision.
2. The process engineer had claimed that the new additive will reduce the average reaction time by at least 1.5 minutes. What is the probability that the experiment will be able to detect a reduction of the average reaction to 8.5 minutes or smaller using $\alpha = .01$?
3. A new study is to be designed. What sample size is needed for an $\alpha = .05$ test to have at least an 80% chance to detect that the average reaction time is 9 minutes or less?

P3. (10 points) A new device has been developed which allows patients to evaluate their blood sugar levels. The most widely device currently on the market yields widely variable results. The new device is evaluated by 25 patients having nearly the same distribution of blood sugar levels yielding the following data:

125	123	117	123	115	112	128	118	124
111	116	109	125	120	113	123	112	118
121	118	122	115	105	118	131		

1. Is there significant evidence ($\alpha = .10$) that the standard deviation in the readings from the new device is less than 10?
2. Compute the probability of a Type II error in using your test from part 1. for the following values of σ: 5, 6, 7, 8, 9, 10
3. Construct an upper 90% confidence bound on the standard deviation of the new device. Is this bound consistent with your answer to the question in part 1.?

P4. (10 points) Refer to the blood sugar device data in Problem 3.

1. Is there significant ($\alpha = .05$) evidence that median blood sugar readings was less than 120 in the population from which the 25 patients were selected? Use the sign test and report the p-value.
2. Is there significant ($\alpha = .05$) evidence that median blood sugar readings was less than 120 in the population from which the 25 patients were selected? Use the Wilcoxon signed rank test and report the p-value.
3. Place a 90% upper bound on the median blood sugar reading.

P5. (10 points) The current method of identifying patients at risk of sudden cardiac death can be identified with 80% accuracy. A change in the method has hopefully improved the accuracy. To evaluate the new method, 100 people are tested and the new method produced the result on 92 of the 100 people.

1. Place a 95% confidence interval on the accuracy of the device.
2. Is there substantial evidence ($\alpha = .05$) that the improved method has increased the accuracy over the current method?
3. Compute the power of the test in part 2. to detect that the accuracy of the improved method is 75%, 80%, 85%, 90%, 95%.
4. How many patients would need to be included in a new study in order to have a power value of 80% if the new method had an accuracy of 90%.

P6. **Multiple Choice (50 points) SELECT ONE** of the following letters **(A, B, C, D, or E)** corresponding to the **BEST** answer. Show details for partial credit.

(MC1.) Suppose that $X_1, \cdots, X_n$ are to be used to construct a 95% prediction interval for a normal population. The researcher notes that the data was collected by an automatic sampler which may result in $X_1, \cdots, X_n$ having a high positive correlation. If the prediction interval was computed using the formula: $\bar{X} \pm t_{.025,n-1} S\sqrt{1+\frac{1}{n}}$, the resulting interval

A. will have a level of confidence less than 95%.

B. will have a level of confidence greater than 95%.

C. will have a level of confidence equal to 95%.

D. will be too wide.

E. none of the above

(MC2.) In a level $\alpha = .05$ test of $H_o : \mu \leq 17$ versus $H_1 : \mu > 17$, where μ is the mean of a normally distributed population, the sample size needed to have a Type II error rate of at most 0.10 whenever $\mu > 17 + .5 * \sigma$ is

A. 13

B. 22

C. 35

D. 70

E. need the non-central t cdf in order to determine sample size

(MC3.) The power of a test of the hypothesis: $H_1 : \mu < \mu_o$

A. is one minus the probability of a Type II error at μ_o

B. is the probability of a Type II error

C. varies depending on the value of μ

D. is not a function of the value of α

E. none of the above

(MC4.) In testing the hypotheses $H_o : \sigma \leq 23.8$ versus $H_1 : \sigma > 23.8$, where σ is the standard deviation of a normally distributed population, an $\alpha = .05$ test was run using a independent random sample of size $n = 10$. The probability of a Type II error when $\sigma = 47.9$ is

A. .05

B. .10

C. .90

D. .95

E. need noncentral Chi-squared tables to compute power

(MC5.) A researcher wants to determine if there is an increase in the likelihood that people will purchase a product after a redesign of the product. The current market share is 20%. Initially, the researcher was planning on using a random sample of n=20 persons with an $\alpha = .05$ test to evaluate the product. He wants you to calculate the chance that the study will fail to detect that preference for the product has been increased if in fact the preference for the new product is 40%. This chance is

A. .316

B. .416

C. .596

D. .950

E. cannot be determined with the given information

(MC6.) A random sample of n=15 from a normally distributed population is used to construct a level $\alpha = .01$ test of $H_o : \mu \leq 20$ versus $H_1 : \mu > 20$, where μ is the mean of the population. The probability of a Type II error for $\mu > 20 + .8\sigma$ is at most

A. .05

B. .22

C. .32

D. .55

E. cannot be determined from the given information

(MC7.) A psychologist is investigating the IQ level of young children who have been in a head start program. She wants to determine if the variation in IQ scores for the population of head start students is smaller than the variation in the general population of children under the age of 6 which has a variation of $\sigma = 10.2$. She also informs you that the distribution of IQ scores is highly right skewed. Suppose she uses the test: reject H_o is $\frac{(n-1)S^2}{(10.2)^2} < \chi^2_{.95,n-1}$, where S is the standard deviation from a random sample of n head start students, to test whether σ is less than 10.2 with an α value of 0.05.

A. the actual level of significance will be greater than 0.05.

B. the actual level of significance will be less than 0.05.

C. the actual level of significance will be very close to 0.05.

D. the actual level of significance will be exactly 0.05.

E. it is impossible to determine the effect of skewness on the actual level of significance.

Use the following information for Problems MC8-MC9.

A process engineer samples a continuous flow of the company's product 200 times per day and obtains the following pH levels in the product : $X_1, \cdots, X_{200}$. He determines that the daily pH levels are related by $X_t = \theta + \rho X_{t-1} + e_t$, where the e_ts have independent $N(0, \sigma_e^2)$ distributions and $\rho \approx .92$.

(MC8.) The engineer constructs a nominal 95% confidence interval for the average daily pH level, μ, using the formula $\bar{X} \pm t_{.025,199}(s/\sqrt{200})$, where $\bar{X}$ and s are the sample mean and standard deviation for a given days pH levels. The true coverage probability of this confidence interval

A. is 0.95.

B. is much less than 0.95.

C. is very close to 0.95.

D. is much greater than 0.95.

E. may be greater than 0.95 or less than 0.95 depending on the distribution of the X_t's.

(MC9.) Refer to Problem MC8. The nominal pH level of the product is 5.3. The process engineer wants to test if the pH on a given day is different from 5.3. He uses $t = \frac{\bar{X}-5.3}{s/\sqrt{200}}$ as his test statistic. Next, he uses the t-distribution with df=199 to compute the p-value of the observed data. The computed p-value will be

A. correct because the sample size is large.

B. much smaller than the correct p-value.

C. much larger than the correct p-value.

D. very close to the correct p-value because the sample size is large.

E. may be greater or less than the correct value depending on the size of σ.