Jack Radoni
STAT 608
Assignment 04
3/9/22

1.) ϕ That for the model $y_i = \alpha + e_i$ the errors are independent w/ mean 0. Also, suppose that the measurements are taken using one device for the first n_1 measurements and then a more precise device was used for the next n_2 measurements. Thus $var(e_i) = \sigma^2$ $i = 1, \dots, n_1$ & $var(e_i) = \frac{\sigma^2}{2}$ $i = n_1+1, \dots, n$

(a) ignore the fact that the errors have different variances, derive the LSE of $\hat{\alpha}$ using matrix notation & $\hat{\alpha} = (x'x)^{-1}x'y$

$$X = \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{n} \qquad x'x = [1\ 1 \cdots 1]\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = n \qquad \Rightarrow \quad (x'x)^{-1}x'y = \frac{1}{n}\sum y_i = \bar{y} = \hat{\alpha}$$

$$x'y = [1\ 1 \cdots 1]\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \sum_{i=1}^{n} y_i$$

$$\boxed{\hat{\alpha} = [\bar{y}]}$$

(b) Derive the weighted LSE for α, $\hat{\alpha}_{WLS}$

$$\text{let } w_i = \begin{cases} 1 \text{ if } i = 1, \dots, n_1 \\ 2 \text{ if } i = n_1+1, \dots, n \end{cases}$$

- We know from H.O. 4, slide 8: $\hat{\beta}_{WLS} = (X'WX)^{-1}X'Wy$ where W is: $W = \begin{bmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_n \end{bmatrix}$. In our case then:

$$W = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & 2 & 0 \\ 0 & \cdots & 0 & 2 \end{bmatrix} \quad \text{w/ } w_{ii} = 1 \text{ for } i = 1, \dots, n_1 \text{ \& } w_{ii} = 2 \text{ for } i = n_1+1, \dots, n$$

$$\underset{1\times n}{x'W} = [1\ 1 \cdots 1]\underset{n\times n}{\begin{bmatrix} 1 & & 0 \\ & \ddots\ 2 \ddots & \\ 0 & & 2 \end{bmatrix}} = [\underbrace{1\ 1 \cdots 1}_{n_1}\ \underbrace{2 \cdots 2}_{n-n_1 = n_2}]$$

$$x'Wx = [1 \cdots 1\ 2 \cdots 2]\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = n_1 + 2(n-n_1) \Rightarrow (x'Wx)^{-1} = \frac{1}{n_1+2(n-n_1)} = \frac{1}{n_1+2n_2}$$

$$x'Wy = [1 \cdots 1\ 2 \cdots 2]\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \sum_{i=1}^{n_1} y_i + 2\sum_{i=n_1+1}^{n} y_i = n_1\bar{y}_1 + 2n_2\bar{y}_2$$

$$(x'Wx)^{-1}x'Wy = \boxed{\hat{\alpha}_{WLS} = \frac{n_1\bar{y}_1 + 2n_2\bar{y}_2}{n_1+2n_2}}$$

1.) (cont'd)

(c) If $n_1 = n_2$ ~ $n_1 + n_2 = n$. Compute the expected values and variances of the two estimators above. Which is a better estimator & why? (Use $MSE(\hat{\alpha}) = Bias(\hat{\alpha})^2 + var(\hat{\alpha})$ as your definition of better)

if $n_1 = n_2 \Rightarrow \hat{\alpha}_{WLS} = \dfrac{n_1\bar{y}_1 + 2n_2\bar{y}_2}{n_1 + 2n_2} = \dfrac{\frac{n}{2}\bar{y}_1 + n\bar{y}_2}{\frac{3}{2}n} = \frac{1}{3}\bar{y}_1 + \frac{2}{3}\bar{y}_2$

$\hat{\alpha}_{OLS}$:

- $E[\hat{\alpha}|X] = E[(X'X)^{-1}X'y|X] = (X'X)^{-1}X' E[y|X]$
$= (X'X)^{-1}X' E[X\alpha|X] = (X'X)^{-1}X'X\alpha = \alpha$

- $Bias(\hat{\alpha}) = E[\hat{\alpha}] - \alpha = 0$

ignore covariances b/c e_i are iid.

- $var(\hat{\alpha}) = var\left(\frac{1}{n}\sum y_i\right) = \frac{1}{n^2} var\left(\sum y_i\right) = \frac{1}{n^2}[var(y_1) + \cdots + var(y_n)]$
$= \frac{1}{n^2}[var(y_1) + \cdots + var(y_{n_1}) + var(y_{n_1+1}) + \cdots + var(y_n)]$
$= \frac{1}{n^2}[n_1\sigma^2 + n_2\sigma^2/2] = \frac{1}{n^2}\left[\frac{2n\sigma^2}{4} + \frac{n\sigma^2}{4}\right] = \frac{3\sigma^2}{4n}$ $(n = n_1 + n_2)$

$\boxed{MSE[\hat{\alpha}] = \frac{3\sigma^2}{4n}}$

$\hat{\alpha}_{WLS}$:

- $E[\hat{\alpha}|X,W] = E[(X'WX)^{-1}X'Wy|XW] = (X'WX)^{-1}X'W\,E[y|XW]$
$= (X'WX)^{-1}X'W\,E[X\alpha|X,W] = (X'WX)^{-1}X'WX\alpha = \alpha$

- $Bias(\hat{\alpha}_{WLS}) = E[\hat{\alpha}_{WLS}] - \alpha = 0$.

- $var(\hat{\alpha}_{WLS}) = var\left(\frac{1}{3n}\sum_{i=1}^{n_1} y_i + \frac{2}{3n}\sum_{i=n_1+1}^{n} y_i\right) = \frac{1}{9n^2} var\left(\sum_{i=1}^{n_1} y_i\right) + \frac{4}{9n^2} var\left(\sum_{i=n_1+1}^{n} y_i\right)$
$= \frac{1}{9n_1^2}[var(y_1) + \cdots + var(y_{n_1})] + \frac{4}{9n_2^2}[var(y_{n_1+1}) + \cdots + var(y_n)]$
$= \frac{n_1}{9n_1^2}\sigma^2 + \frac{4n_2}{9n_2^2}\frac{\sigma^2}{2} = \frac{\sigma^2}{9n_1} + \frac{2\sigma^2}{9n_2}$ $(n_1 = n_2 = \frac{1}{2}n \Rightarrow 9n_1 = 9n_2 = \frac{9n}{2})$

$= \frac{2\sigma^2 + 4\sigma^2}{9n} = \frac{6\sigma^2}{9n} = \frac{2\sigma^2}{3}$

$\boxed{MSE[\hat{\alpha}_{WLS}] = \frac{2\sigma^2}{3}}$

$\boxed{\hat{\alpha}_{WLS} \text{ is a better estimator b/c it is unbiased (as is } \hat{\alpha}_{OLS}\text{) and has a smaller variance.}}$

2.) (Question 2, Chp 4)

- Consider regression through the origin (i.e. straight line regression w/ population intercept known to be 0) w/ $var(e_i | x_i) = x_i^2 \sigma_i^2$. The corresponding regression model is

$$y_i = \beta x_i + e_i \quad (i = 1, \dots, n)$$

- Find an explicit expression for the variance of the weighted least squares estimate of β.

- Let $w_i = \frac{1}{x_i^2}$ $\Rightarrow$ $W = \begin{bmatrix} 1/x_1^2 & & 0 \\ & \ddots & \\ 0 & & 1/x_n^2 \end{bmatrix}$; $X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$; $y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$

- $\hat{\beta} = (X'WX)^{-1} X'Wy$

  - $X'W = \underset{1 \times n}{[x_1 \cdots x_n]} \underset{n \times n}{\begin{bmatrix} 1/x_1^2 & & 0 \\ & \ddots & \\ 0 & & 1/x_n^2 \end{bmatrix}} = \left[\frac{1}{x_1} \cdots \frac{1}{x_n}\right]$

  - $X'WX = \left[\frac{1}{x_1} \cdots \frac{1}{x_n}\right] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = n$

  - $X'Wy = \left[\frac{1}{x_1} \cdots \frac{1}{x_n}\right] \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \sum_{i=1}^{n} \frac{y_i}{x_i}$

- $\hat{\beta} = \frac{1}{n} \sum_{i=1}^{n} y_i / x_i$

3.) (Question 3, Chp 4) * (see graph outputs in book)

- The Sunday April 15, 2007 issue of the Houston Chronicle included a section devoted to real estate prices in Houston. In particular, data are presented on the 2006 median price per square ft for 1922 subdivisions. The data (HoustonRealEstate.txt) can be found on the book website.

Interest centers on developing a regression model to predict

y_i = 2006 median price per sq ft from

x_{1i} = % New Homes (% houses in '06 built in '05 or '06)

x_{2i} = % Foreclosures (% of those sold in 2006 that were classified as foreclosures)

for $i = 1, \dots, 1922$ subdivisions.

- The first model considered was: $Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + e$. fit using weight $w_i = n_i$

og attempt (talk to prof. wendy)

?

our Y_i are really $\tilde{y}_i$ if $y_i \sim N$

Then $\tilde{y}_i = \bar{y}_i$

$var(\bar{y}_i) = \frac{\sigma_{y_i}^2}{n}$

(a) Explain why it is necessary to use WLS and why $w_i = n_i$ is appropriate.

- From slide 2 in the Chp 4 notes: we use weighted least squares when y_i is the average or median of n_i observations. In our case y_i is the 2006 median price per sq ft. Thus, the variance of the responses at each pt $var(Y_i) \propto \frac{1}{n_i}$. Thus, the weight $w_i = n_i$ is appropriate.

3.) (Contd)

(b) Explain why the model is not valid.

① Looking at the plot of std residuals vs fitted values we see that the standardized residuals do not randomly fluctuate around 0, which implies the model isn't valid.

② looking at a plot of $\sqrt{|\text{std residuals}|}$ vs fitted values we can see we do not have constant error variance

③ The two predictor variables x_1 & x_2 seem to be highly correlated which means we might have issues w/ multicollinearity in the model.

(C) Describe what steps you would take to obtain a valid regression model.

- I would try power transformations on y, x_1 and/or x_2.

4.) Return to question 4 from homework 2. Now, p the variance in y is proportional to the # of coins being put on the scale. I recommend double-checking using both linear-algebra and (if you're working in R) the linear model function lm(y~x, weight = w) where w is a vector of weights, and you invent your own y.

(a) Design an appropriate matrix of weights.

- $y_i = \beta_1 \text{coin}_1 + \beta_2 \text{coin}_2 + \varepsilon_i$ where coin_1 & coin_2 are indicator variables indicating if coin 1 & coin 2 were on the scale respectively.
- We can split up ε_i into $e_{1i} + e_{2i}$ where e_1 is the random error due to coin 1 & e_2 is the random error due to coin 2.
- $\text{var}(y_i) = \text{Var}(e_{1i} + e_{2i}) = \text{var}(e_{1i}) + \text{var}(e_{2i})$
- assuming the variances are equal: $\text{var}(y_i) = 2\sigma_e$

$\Rightarrow$ $w_i = \frac{1}{n_i}$ where n_i is the # of coins on the scale.

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

4.) (cont'd)

(b) Calculate the new least-squares estimates of the weights of the coins using weighted least squares

$$\hat{\beta}_{WLS} = (X'WX)^{-1}X'W\underset{\sim}{y} \quad ; \quad X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_4 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

• $$X'W = \underset{2\times4}{\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}} \underset{4\times4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1/2 & 1/2 \end{bmatrix}$$

• $$(X'WX) = \begin{bmatrix} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow (X'WX)^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

• $$X'W\underset{\sim}{y} = \begin{bmatrix} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_4 \end{bmatrix} = \begin{bmatrix} y_1 + 1/2(y_3 + y_4) \\ y_2 + 1/2(y_3 + y_4) \end{bmatrix}$$

• $$\hat{\beta}_{WLS} = (X'WX)^{-1}X'W\underset{\sim}{y} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} y_1 + 1/2(y_3+y_4) \\ y_2 + 1/2(y_3+y_4) \end{bmatrix} = \begin{bmatrix} 2y_1 + y_3 + y_4 - y_2 - 1/2y_3 - 1/2y_4 \\ 2y_2 + y_3 + y_4 - y_1 - 1/2y_3 - 1/2y_4 \end{bmatrix}$$

$$\boxed{\hat{\beta}_{WLS} = \begin{bmatrix} 2y_1 - y_2 + 1/2(y_3+y_4) \\ 2y_2 - y_1 + 1/2(y_3+y_4) \end{bmatrix}}$$

5.) For the model $y_i = \beta_0 + \beta_1 x_i + e_i$ the errors are iid w/ mean 0. The four observed values of x_i are $x' = [1\ 2\ 3\ 4]$. The estimator of β_1 is $\tilde{\beta}_1 = [y_4 + 2y_3 - 2y_2 - y_1]/5$.
For this model, do the following:

(a) Is $\tilde{\beta}$ unbiased? why or why not?

$$E[\tilde{\beta}] = E\left[\frac{y_4+2y_3-2y_2-y_1}{5}\right] = \frac{1}{5} E[y_4+2y_3-2y_2-y_1]$$

$$= \frac{1}{5}(E[y_4] + 2E[y_3] - 2E[y_2] - E[y_1])$$

$$= \frac{1}{5}(E[\beta_0+\beta_1(4)] + 2E[\beta_0+\beta_1(3)] - 2E[\beta_0+\beta_1(2)] - E[\beta_0+\beta_1(1)])$$

$$= 1/5(\cancel{\beta_0} + \cancel{2\beta_0} - \cancel{2\beta_0} - \cancel{\beta_0} + 4\beta_1 + 2(3)\beta_1 - 2(2)\beta_1 - \beta_1)$$

$$E[\tilde{\beta}] = 1/5(5\beta_1) = \beta_1 \Rightarrow \boxed{\tilde{\beta} \text{ is unbiased}}$$

(b) What is the sampling variance of $\tilde{\beta}_1$?

$$var(\tilde{\beta}_1) = var((y_4+2y_3-2y_2-y_1)/5)$$

$$= \frac{1}{25} var(\beta_0+\beta_1(4)+e_1 + 2(\beta_0+\beta_1(3)+e_3) - 2(\beta_0+\beta_1(2)+e_2) - (\beta_0+\beta_1(1)+e_1))$$

$$= \frac{1}{25} var(e_1 + 2e_3 - 2e_2 - e_1)$$

$$= \frac{1}{25}(var(e_1) + 4var(e_3) + 4var(e_2) + var(e_1)) \text{ (+ covariance terms = 0 b/c iid.)}^*$$

$$= \frac{1}{25}(\sigma_e^2 + 4\sigma_e^2 + 4\sigma_e^2 + \sigma_e^2) = \frac{10\sigma_e^2}{25} = \boxed{\frac{2}{5}\sigma_e^2 = var(\tilde{\beta})}$$

(c) Get the usual LSE of β_1 & calculate its variance.

$$\hat{\beta} = (X'X)^{-1}X'y \quad ; \quad X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ 1 & x_4 \end{bmatrix} \quad ; \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$\cdot\ X'X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \end{bmatrix}\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ 1 & x_4 \end{bmatrix} = \begin{bmatrix} 4 & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \quad ; \quad \sum x_i = 1+2+3+4 = 10, \quad \sum x_i^2 = 1+4+9+16 = 30$$

$$= \begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix} \Rightarrow (X'X)^{-1} = \frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix}$$

$$\cdot\ X'y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

$$\hat{\beta} = (X'X)^{-1}X'y = \frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix}\begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 30\sum y_i - 10\sum y_i x_i \\ 4\sum x_i y_i - 10\sum y_i \end{bmatrix}$$

don't need all this.

$$\cdot\ Var(\hat{\beta}) = \sigma^2(X'X)^{-1} \Rightarrow var(\hat{\beta}_1) = \frac{4\sigma^2}{20} = \frac{\sigma^2}{5}. \text{ (see chp 2 notes slide 48)}$$

(d) Compare the sampling variance of $\hat{\beta}_1$ w/ $\tilde{\beta}_1$

- $\hat{\beta}_1$ is a better estimator of β_1 as they are both unbiased, but $var(\hat{\beta}_1) < var(\tilde{\beta}_1)$

6.) A food manufacturing company is interested in modeling whether people prefer X_1 = Type A or Type B hot dog buns w/ their hot dogs. They also want to control for X_2 = different amounts of sodium in the hot dogs themselves and are testing hot dog buns at a variety of sodium contents, giving each taster both a hot dog & a bun w/ no condiments. The response variable is y = perceived taste of bun on a scale of 1:10.

(a) In order to find out whether Type A or Type B is preferred, is it necessary to have an interaction term?

- No. If we wanted to find out if the relationship btwn the perceived taste of the bun and the bun type was different across sodium levels in the hot dogs

(b) $y_i = \beta_0 + \beta_1 I(A)_i + \beta_2 \text{Sodium}_i$ $\quad i=1,\dots,n$ where $I(A) = \begin{cases} 1 \text{ if } x_{1i} = A \\ 0 \text{ if } x_{1i} = B \end{cases}$ * ask about this part!!!*

β_0: The mean perceived taste of bun Type B when sodium levels are 0

β_1: The difference btwn the mean perceived taste of bun Type A and mean perceived taste of bun type B, holding sodium content constant ?

β_2: The mean change in perceived taste of the bun for a 1 unit change in sodium content, holding bun type constant.

$H_0: \beta_0 = \beta_1 = \beta_0 = 0$ $\quad H_A$: At least one $\beta_i \neq 0$. (F test).

- (If we reject null above:) $H_0: \beta_0 = 0,\ H_a: \beta_0 \neq 0$; $H_0: \beta_1 = 0,\ H_1: \beta_1 \neq 0$; $H_0: \beta_2 = 0,\ H_1: \beta_2 \neq 0$

(c) Whether or not you added an interaction term above, assume now that it was added and it is statistically significant. How should we interpret this interaction in context?

$y_i = \beta_0 + \beta_1 I(A)_i + \beta_2 \text{Sodium}_i + \beta_3 (I(A)_i * \text{Sodium})$

- β_3: The difference between the mean change in perceived taste of bun type A for a 1 unit increase in sodium content and the mean change in perceived taste of bun type B for a 1 unit increase in sodium content.

7.) In a one-way ANOVA model w/ $k=3$ groups & 4 obs per group:

(a) Use the F-statistic in Model Reduction Method 2 to derive a statistic for testing whether the average of the means of the first two groups is the same as the mean of the third group. That is, create the F-statistic for testing $H_0: \frac{1}{2}(\mu_1+\mu_2)=\mu_3$

(Hint: Don't fit a model w/ any intercept)

• $H_0: \frac{1}{2}(\mu_1+\mu_2) - \mu_3 = 0$ $H_a: \frac{1}{2}(\mu_1+\mu_2) - \mu_3 \neq 0$.

$A = [0.5\ 0.5\ -1]$; $h = 0$

$\hat{e}' = [\hat{e}_1\ \hat{e}_2\ \cdots\ \hat{e}_{24}]$

$$X' = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}_{(p+1)\times r} \Rightarrow p=2, r=1$$

$$F = \frac{(A\hat{\beta}-h)'(A(X'X)^{-1}A')^{-1}(A\hat{\beta}-h)/r}{SSE/(n-p-1)}$$

• $(X'X)^{-1} = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$ • $(A(X'X)^{-1}A')^{-1} = 0.\overline{666}$

• $(A\hat{\beta}-h)' = \hat{\mu}_1+\hat{\mu}_2-2\hat{\mu}_3$

$$F = \frac{\frac{8}{3}\left(\frac{1}{2}(\hat{\mu}_1+\hat{\mu}_2)-\hat{\mu}_3\right)^2/1}{SSE/9} = \frac{24\left(\frac{1}{2}(\hat{\mu}_1+\hat{\mu}_2)-\hat{\mu}_3\right)^2}{SSE}$$

(b) $\hat{\mu}_1 = 5.6$, $\hat{\mu}_2 = 7.9$, $\hat{\mu}_3 = 6.1$ & $SSE = 12.5$. Test Hypo at $\alpha = 0.05$.

$F = 0.792 < F_{1,9} = 5.12$. Fail to reject H_0, we don't have significant evidence at the $\alpha = 0.05$ level to conclude that $\frac{1}{2}(\mu_1+\mu_2)+\mu_3 \neq 0$.