Statistics 630 - Assignment 9
(due Wednesday, 17 November 2021)

Important: When referring to the estimator of a parameter be sure to use distinctive notation (define, if necessary). For example, $\bar{X}$ as the estimator of a mean μ, $\hat{\theta}$ as an estimator of parameter θ. Estimators and parameters are not the same thing, so do not label them the same.

1. Chapter 6 Exercise 6.5.1. To do this right, let $\theta = \sigma^2$ and find the score, Fisher information, etc., as functions of θ. Add the following.
   (b) Recall from Exercise 6.2.12 (Assignment 8) that the MLE for θ in this case (with $\mu = \mu_0$ known) is $\widehat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^n (X_i - \mu_0)^2$. Use the theory of MLE's to determine the asymptotic distribution of $\widehat{\sigma}^2$. Does it agree with the CLT (applied to $Y_i = (X_i - \mu_0)^2$)?

2. Recall Exercise 6.2.7 (Assignment 8). Now do the following. It may help to identify the distribution of $-\log(X_i)$ (and hence its mean and variance).
   (a) Show that the MLE for α is consistent.
   (b) Also, find the Fisher information and determine the asymptotical normal distribution for the MLE of α.

3. Let $T_1, \ldots, T_n$ be iid Poisson(λ). Recall that the MLE for λ is $\hat{\lambda} = \bar{T}_n$.
   (a) Find the Fisher information and confirm that the asymptotic variance for $\hat{\lambda}$ is *exactly* $\mathsf{Var}(\hat{\lambda})$ (which is not generally true).
   (b) Now suppose, for whatever reason, you want to estimate $\theta = \frac{1}{\sqrt{\lambda}}$. What is the MLE for θ? Use the delta method to get an asymptotic distribution for this estimator. [Note: the estimator technically does not have a finite mean, let alone finite variance! Nevertheless, the asymptotic distribution is correct.]
   (c) Use the data provided in the data file poisson_sample.csv (in the R Files module in Canvas) to estimate λ and θ, and then estimate the asymptotic standard error based on the asymptotic variance found in part (b).

4. Chapter 6 Exercises 6.3.1, 6.3.2. Do not compute a P-value (just yet) but instead assess the hypothesis by first getting the confidence interval and then determining whether $\mu = 5$ is inside the confidence interval. (The confidence intervals for the two exercises will differ.)

5. Chapter 6 Exercise 6.3.8. Compute *both* the Wald and score intervals and assess the hypothesis with each by determining whether $\theta = 0.65$ is inside the confidence interval. (Do not compute a P-value.)

6. Chapter 6 Exercise 6.5.4. Construct a Wald interval. Use that interval to assess the hypothesis and omit the power calculation. Add the following.
   (b) Construct an approximate level $\gamma = 0.95$ confidence interval based on the *asymptotic* pivot $\frac{\hat{\lambda}-\lambda}{\sqrt{\lambda/n}}$ (which gives the score interval).

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(c) Carry out a simulation to determine which interval has better coverage properties. (Generate $N \geq 10,000$ Poisson($\lambda = 11$) samples of size $n = 20$ and compute both types of intervals for each sample. "coverage" = "interval contains the true value of λ".)

7. Chapter 6 Exercises 6.5.5, 6.5.6. You may choose a method to use, but clearly indicate which method it is.