TMA4265 Stochastic Modeling
Project 2

Background information

- The deadline for the project is Monday 23 Oct, 14:00.
- The projects (three in all) will count 20% of the final mark.
- The project work is required to be admitted to the exam.
- In order to pass this project a reasonable attempt must be made to solve all problems.
- The project should be done in groups of two or three persons. You can sign up as a group in Blackboard.
- Submit the project by uploading the project report as a pdf file to your group folder in Blackboard. Attach the code as separate files.
- Your project report should include both equations with calculations, plots and interpretation as text.
- Computer-code should be written in `Matlab`, `R` or `Python`. Please try to make your code readable and add comments to describe what you do.
- Lectures on 20 and 23 Oct will be set aside to work on the project. You will get assistance in the lecture room on Friday 8:15-10:00.

Problem 1 : Insurance claims

Let $N(t)$ denote the number of claims received by an insurance company from time 0 to time t. Assume that $N(t)$ is a Poisson process. Here, the continuous time index $t \geq 0$ denotes days from January 1st, 0:00.00.

a) Set the intensity to a constant $\lambda(t) = 3$.
What is the probability that there are more than 175 claims before March 1st (59 days)?
Simulate multiple Poisson processes with intensity $\lambda(t) = 3$, and use the simulations to verify your answers.
Plot the processes $N^b(t)$, for different realizations $b = 1, \ldots, 100$, as a function of time.

b) Set the intensity to $\lambda(t) = 2 + \cos(t\pi/182.5)$.
What is the probability that there are more than 175 claims before March 1st (59 days)?

Simulate multiple Poisson processes with this non-constant intensity, and use the simulations to verify your answer.
Plot the processes $N^b(t)$, for different realizations $b = 1, \ldots, 100$ as a function of time, and compare with a).

Hint: You can use thinning to simulate such an inhomogeneous Poisson process. This is based on generating a homogeneous Poisson process with rate $\lambda^\star = \max[\lambda(t)]$, *giving* N_{59} *events at times* $T_1, \ldots, T_{N_{59}}$, *and next keeping points with probability* $\lambda(T_i)/\lambda^\star$, *for all* i. *Otherwise an event is removed. Example code is on the lecture notes on the Blackboard homepage for the course.*

Assume that the monetary claims are independent. These claim amounts are also independent of the claim arrival times. Every claim amount (in mill. kr.) has a log-Gaussian distribution with parameters $\mu = -2$ and $\sigma^2 = 1^2$. I.e. claim amount $C_i = \exp(Y_i)$, where $Y_i \sim N(\mu, \sigma^2)$, $i = 1, 2, \ldots$. The total claim amount at time t is then defined by $Z = \sum_{i=1}^{N(t)} C_i$.

c) Use double expectation and double variance to compute the expected total claim amount(s) before March 1st, and the variance in this total claim amount when $\lambda(t) = 3$.
Simulate claim arrival times according to a Poisson distribution with constant intensity $\lambda(t) = 3$, and with $\lambda(t) = 2 + \cos(t\pi/182.5)$. Simulate claim amounts using the log-Gaussian distribution specified above.
Compare and discuss results for the two different intensities.

d) The insurance company must ensure that they can cover the total claim amount for the entire year. This must be done at the beginning of the year, so each claim is discounted to January 1st using a discount rate $\alpha = 0.001$. This means that a multiplication factor of $\exp(-\alpha t)$ is used for a claim at time t to reflect this amount on January 1st. The total discounted amount of claims is then $Z_{\text{disc}} = \sum_{i=1}^{N(365)} \exp(-\alpha T_i) C_i$, where T_i is the time of the ith claim.

Use simulations to answer the following, for both $\lambda(t) = 3$ and for $\lambda(t) = 2 + \cos(t\pi/182.5)$:

- Approximate the expected total discounted amount of claims.
- How much money should the insurance company hold to be 95 percent sure it can cover the total (sum of) claims during the next year?

Problem 2 : Server jobs

Assume that a computer server can handle 32 jobs in parallel, no matter their size. When all 32 units are on their tasks, an arriving job is forwarded to another server, no matter the size of that incoming job. Arrivals to the server follow a homogeneous Poisson process with rate $\lambda = 25$ per hour, and each job takes an exponential time, with rate parameter $\mu = 1$ per hour. This is then a birth-death process with finite bounds, and the number of jobs running on the server is $N(t) \in \{0, 1, \dots, 32\}$.

a) Use long-term equality of rates in and out of states to derive the equilibrium probabilities $P[N(t) = n]$, $n = 0, 1 \dots, 32$, of the system. Display the probabilities and interpret the result.

b) Assume $N(0) = 0$. Simulate realizations $N^b(t)$, $b = 1, \dots, 100$, of the process on the computer, each for $7 \cdot 24$ hours, and answer the following:

- Study the transient part of the process - How long time does it approximately take to reach the equilibrium distribution?
- Verify the long-term probabilities of a).
- How many jobs (per hour) are on average forwarded to the other server?