Cost Functions

EC 311 - Intermediate Microeconomics

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Outline

Chapter 07

- Topics
  - Cost Curves (7.3)
  - Average and Marginal Costs (7.4)

Cost Curves

What The Hell Is A Cost Function?

Our goal for the second half of the class is to minimize costs.

We will do this by deriving a **minimzed cost function** but what even is it?

Let's begin by introducing some useful notation:

- The **minimized cost function** will be $C^*(Q)$
  - This function tells us that for any given quantity (Q), $C^*(Q)$ represents the cheapest way possible to produce Q

We will leverage this information:

> **Knowing $C^*(Q)$ means that we do not have to solve the cost minimization problem to figure out how much it will cost you to produce a target quantity**

Cost Functions

The general formula for a **cost function** is:

$$C^*(Q) = wL^*(Q) + rK^*(Q)$$

To find this we will

- Find the MRST and set it equal to the price ratio
- Plug into our Q constraint and find a L^* and K^* in terms of Q
- Find the cost of the optimal L^* and K^*

Last lecture we found levels of L^* and K^* when we knew Q. Now we will keep Q as a variable so we can find costs for any possible quantity.

Cost Function Example - Step 1

Let's say we are faced with the following problem:

$$\min 10L + 10K \quad s.t. \quad \bar{Q} = f(L,K) = L^{1/4}K^{1/4}$$

We begin by finding the MRTS and set it equal to the Price Ratio

MRTS

$$\begin{aligned} \text{MRTS} &= \frac{MU_L}{MU_K} \\ &= \frac{1/4 \cdot L^{-3/4}K^{1/4}}{1/4 \cdot L^{1/4}K^{-3/4}} \\ &= \frac{K^{1/4}K^{3/4}}{L^{1/4}L^{3/4}} = \frac{K}{L} \end{aligned}$$

Price Ratio

$$\frac{w}{r} = \frac{10}{10} = 1$$

Set Them Equal

$$\frac{K}{L} = 1 \rightarrow K = L$$

Cost Function Example - Step 2

$$K = L \quad \text{recall: } \bar{Q} = f(L, K) = L^{1/4}K^{1/4}$$

We found our Optimality Condition for K and L

Plug into Q-Constraint to find $L^*(Q)$ and $K^*(Q)$

Q-Constraint

$$Q = F(L, K) = L^{1/4}K^{1/4}$$
$$Q = L^{1/4}L^{1/4} = L^{1/2}$$
$$Q = L^{1/2}$$

Solve for $L^*(Q)$ and $K^*(Q)$

$$L^{1/2} = Q$$
$$(L^{1/2})^2 = Q^2$$
$$L^* = Q^2$$
$$K^* = Q^2$$

Cost Function Example - Step 3

$$L^* = Q^2 \quad \& \quad K^* = Q^2 \qquad \text{recall: } C(Q) = 10L + 10K$$

We found our optimal **Labor** and **Capital** choices are in terms of Q

Find $C^*(Q)$ using L^* and K^*

$$C^*(Q) = 10 \cdot L^*(Q) + 10 \cdot K^*(Q)$$
$$C^*(Q) = 10 \cdot Q^2 + 10 \cdot Q^2$$
$$C^*(Q) = 20 \cdot Q^2$$

Why Bother With Cost Functions?

To begin, they are very useful in industry

Let's think back to our Ducks jersey example. Nike wants you to make 20,000 jerseys. All you do is turn around and say it will cost

$$\begin{aligned} C^*(Q) &= 20 \cdot Q^2 \\ C^*(20,000) &= 20 \cdot 20,000^2 \\ C^*(20,000) &= 20 \cdot 400,000,000 = 8,000,000,000 \end{aligned}$$

But then with Bo Nix being gone, they only need 1,000 jerseys

So you say: That sucks, I thought we were better than just Bo, but okay. It'll cost:

$$C^*(1,000) = 20 \cdot 1,000^2 = 20,000,000$$

Why Bother With Cost Functions?

More generally, as the factory manager, you would just share your cost function with Nike and they can:

- Use it to figure out the optimal number of jerseys tor produce, taking costs of production into account
- We will introduce profits later
- This decision is where **Supply** will come from

But before we can get into **Profit Maximization**, we will dive deeper into understanding cost functions

What Goes Into Cost Functions?

Beyond wages and rental rates, e**con**omists think about costs in a different way:

We consider ALL foregone alternatives that we could have used our resources in

We call this **opportunity cost**:

- It is the value of inputs when used for their best alternative use

Opportunity Cost

Let’s consider of a non-economics example **Kyler Murray**

NFL Quarterback for the AZ Cardinals

He has a 5yr/ $230,500,000 contract

But he also played baseball in college and nearly went pro

Understanding Cost Functions

Knowing what is considered as costs in economics helps us with the theory part of production

- It will play a larger role when we deal with profit maximization

For now, we can figure out how we expect cost functions to behave:

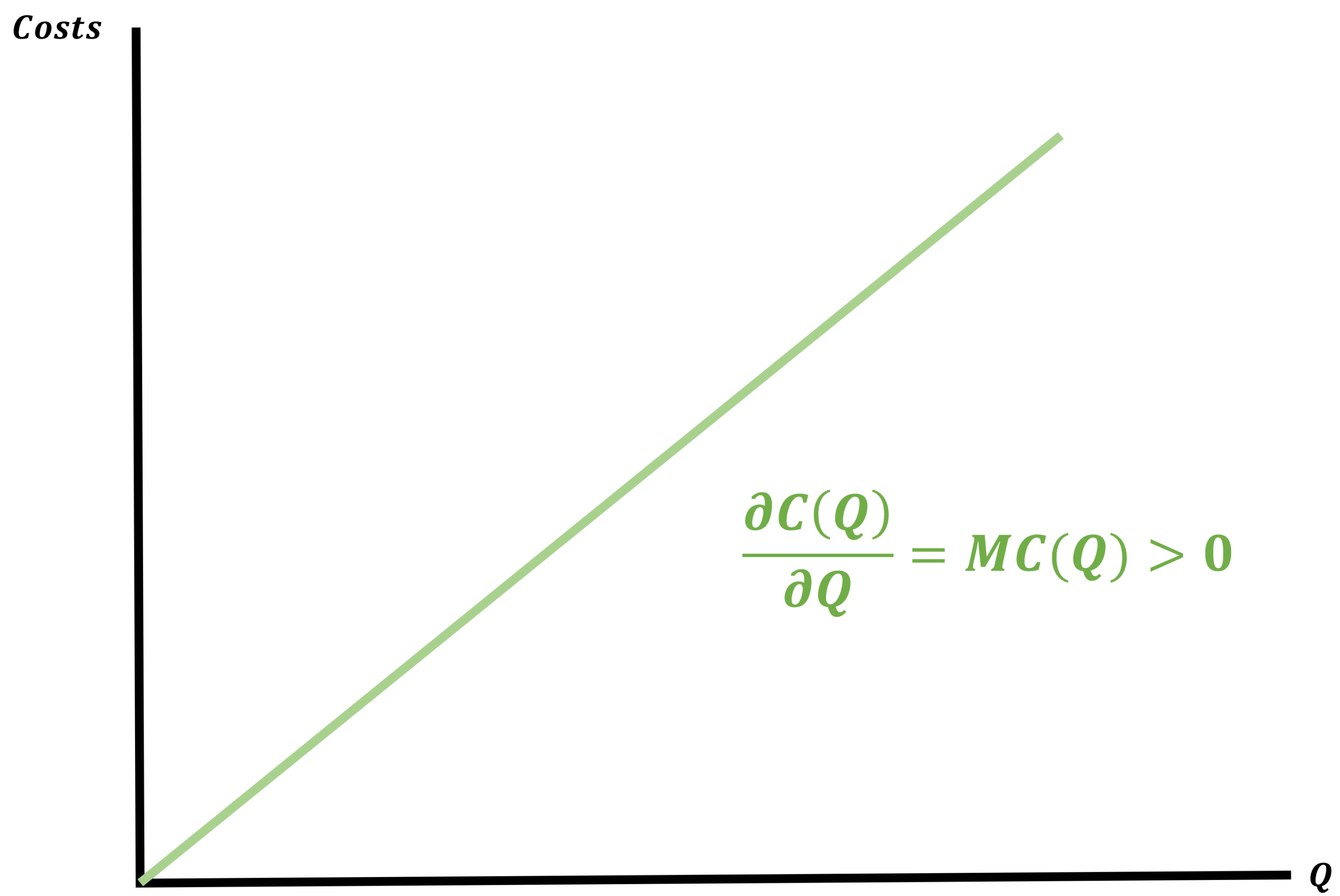
Assume a generic cost function such that $C = F(Q)$

- When Q **increases** what do we expect to happen to C?
  - They should **INCREASE**
- It usually costs more to make more goods
- This means that the first derivative $(C'(Q))$ should be?
  - Positive $\rightarrow C'(Q) > 0$

Derivative of $C(Q)$

The derivative of $C(Q)$ is **very important**

So much so that we give it a name: **Marginal Cost (MC)**



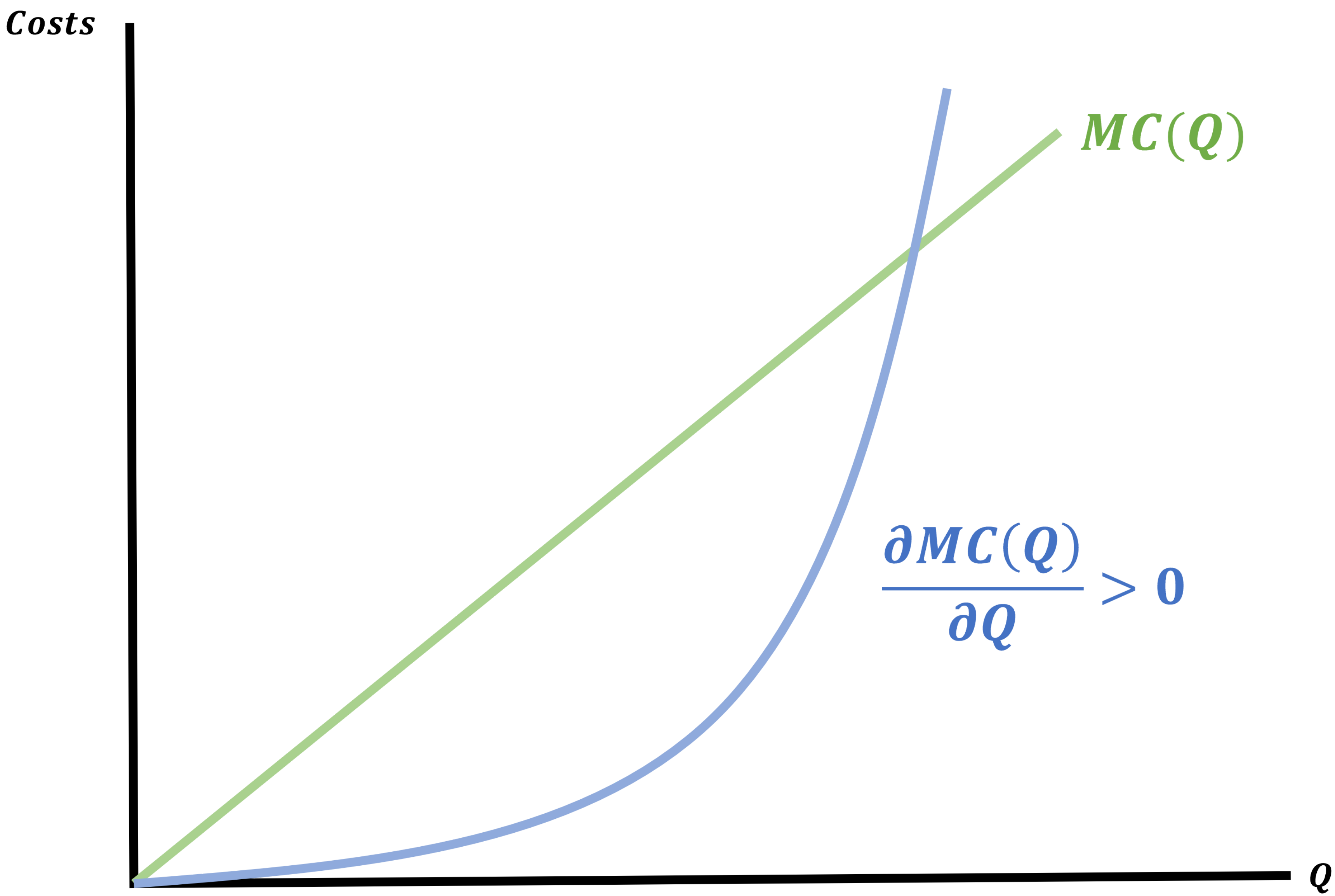
Marginal Costs

We will add an additional assumption to make our lives easier

Assume that firms production functions exhibit decreasing returns

- This has a direct implication on our **Marginal Cost curve**
  - $MC(Q)$ will increase as Q increases
  - It becomes more expensive to make an additional unit as you make more and more
- So **Decreasing Returns to Scale (DRS)** = **Increasing Marginal Costs**

Marginal Cost Curves

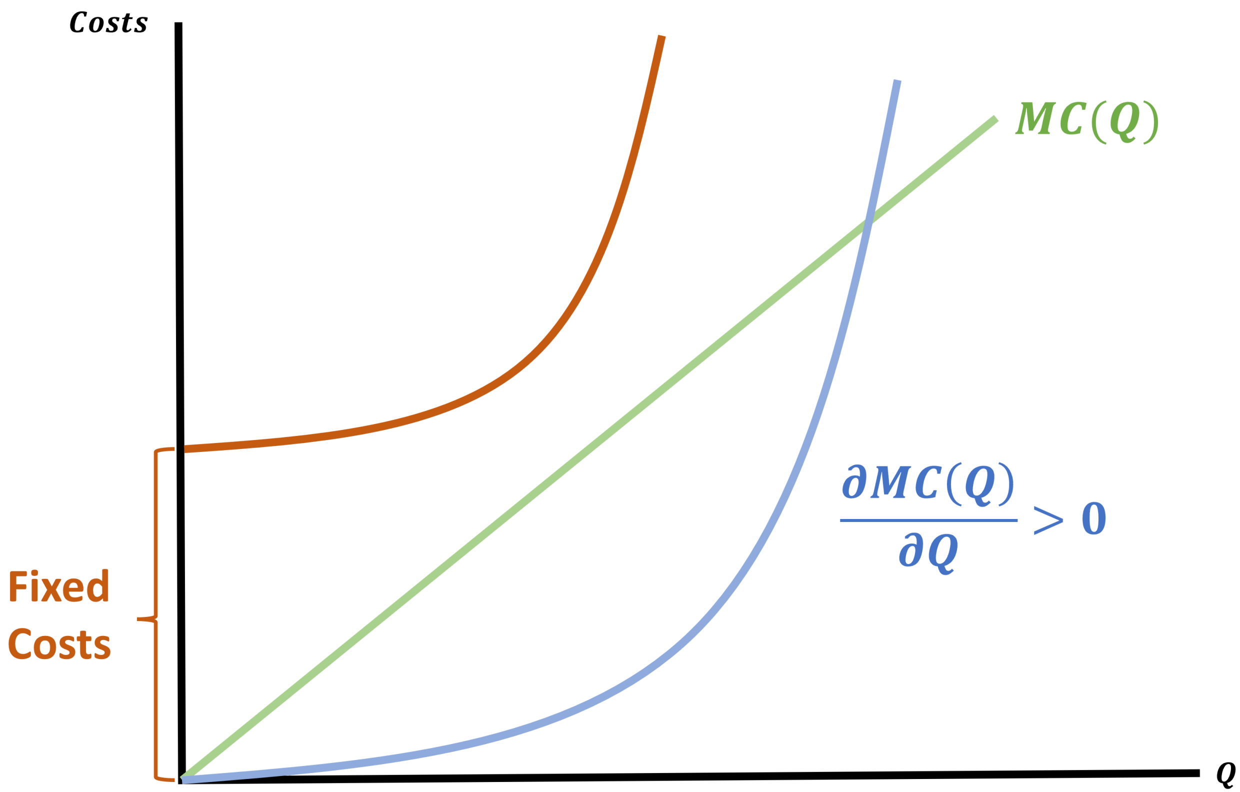


Cost of Producing Nothing?

Does it actually cost nothing to make nothing?

We will assume that there exists some form of overhead or **fixed costs** associated with producing goods

- A business pays rent on their warehouse no matter how empty or full it is
- A restaurant has to purchase a license in order to serve food/alcohol



How Realistic are Cost Functions?

A good thinker is initially skeptical. So let's cast some doubt on our **Increasing Marginal Cost assumption**

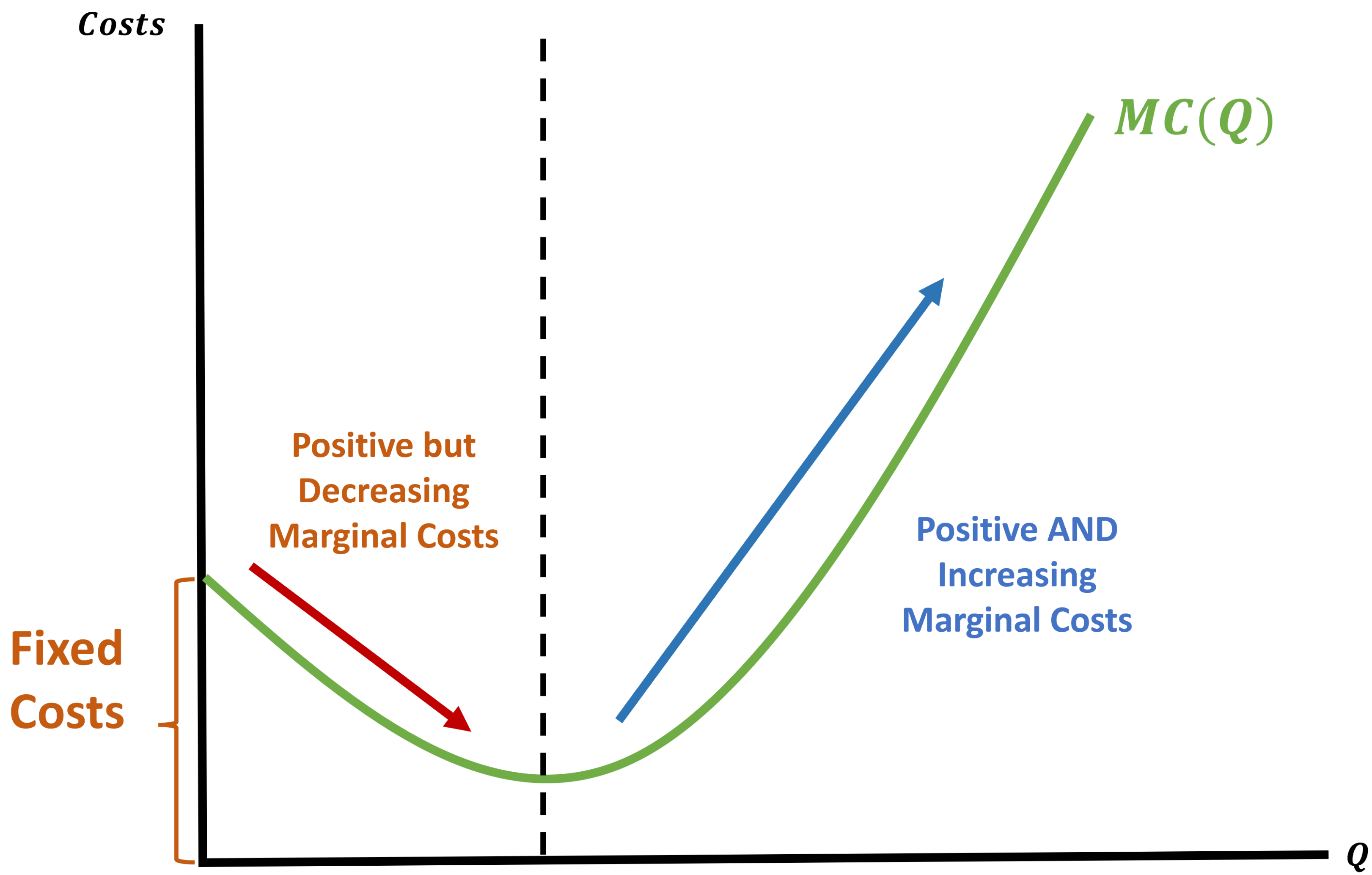
Isn't producing in bulk sometimes much easier than producing small quantities?

- The existence of Costco tells us yes

So this is true, but only up to a point

- We are perfectly comforable modeling firms that **initially experience decreasing marginal costs** but at a high-enough quantity produced, they **must experience increasing marginal costs**

Initial Decreasing Marginal Costs with Increasing Marginal Costs At High Quantity



Types of Cost Functions - Quadratic

We will be dealing with two types of cost functions: **Quadratic and Cubic**:

Quadratic

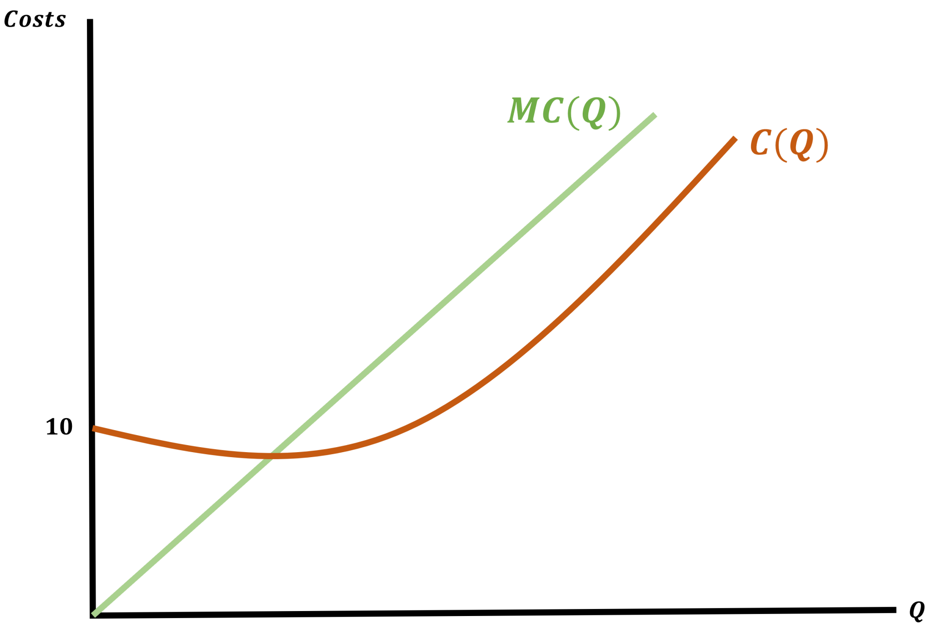
MC is always increasing

$$C(Q) = Q^2 + 10$$

$$\frac{\partial C(Q)}{\partial Q} = MC = 2Q$$

$$\frac{\partial MC}{\partial Q} = 2 > 0$$

How Does the Graph Look?



Types of Cost Functions - Cubic

Cubic

MC is initially decreasing, and eventually increasing

$$C(Q) = \frac{1}{3}Q^3 - Q^2 + 2Q + 12$$

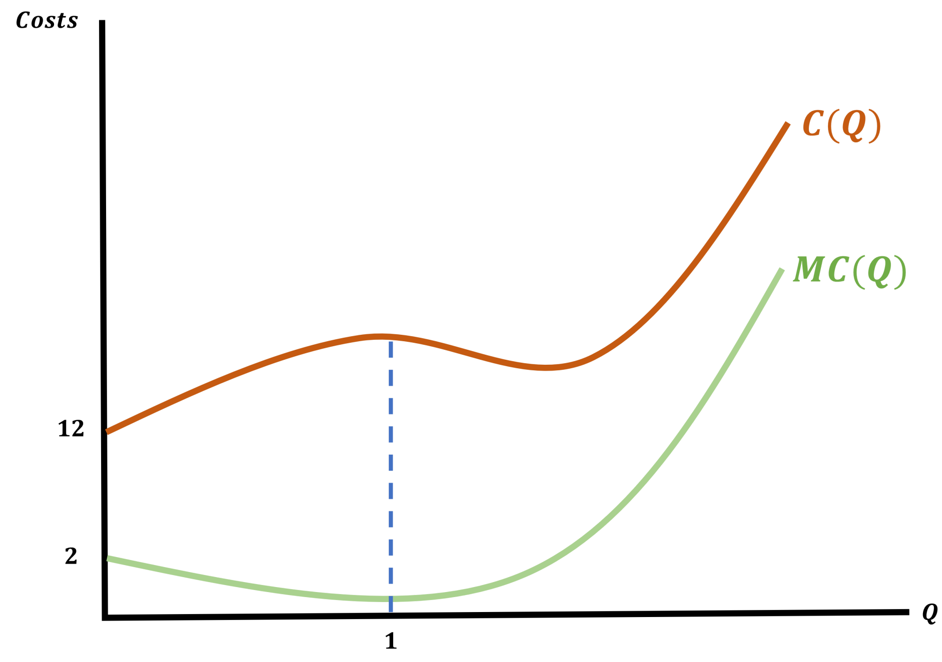
$$\frac{\partial C(Q)}{\partial Q} = MC = Q^2 - 2Q + 2$$

$$\frac{\partial MC}{\partial Q} = 2Q - 2$$

$$2Q - 2 = 0$$
$$2Q = 2$$
$$Q = 1$$

When does it switch from (-) to (+)?

$$Q < 1 \rightarrow \text{Decreasing}$$
$$Q > 1 \rightarrow \text{Increasing}$$



Total Cost

Up to now we have expressed firm's costs as

$$TC(Q) = w \cdot L^*(Q) + r \cdot K^*(Q)$$

Which let's us know the cheapest way to produce a given target Q

We can also express costs as a function of quantity

$$TC(Q) = f(Q) + F$$

Where F is a non-negative constant

We will split these costs up by type (i.e. More Cost Functions!)

Decomposing Costs

Costs will fall into one of two categories:

Fixed Costs

Costs that are paid even if the firm produces nothing $\rightarrow C(0)$

Variable Costs

Costs that are increasing in the quantity produced (e.g. materials and labor used to produce each unit)

In its simplest form, total cost can be written as

$$\text{Total Cost} = \text{Variable Costs} + \text{Fixed Costs}$$

Fixed Costs (FC)

These are costs the firm has to pay **even if it produces 0 units of output**

- **Also known as the intercept of the Cost Function**

$$FC = TC(0) = f(0) + F = F$$

We can find these by setting $Q = 0$

Find the Fixed Costs for the following Cost Function

$$C(Q) = Q^2 + 10$$

$$C(0) = 0^2 + 10 = 10$$

Variable Costs (VC)

These are the increasing costs that the firm pays **for every unit of quantity produced**

- **These are costs of producing Q, when we ignore all Fixed Costs**

$$\begin{aligned} TC(Q) &= VC + FC \\ VC &= TC(Q) - FC \end{aligned}$$

Find the Variable Costs for the following Cost Function

$$C(Q) = Q^2 + 10$$

$$VC = Q^2$$

Decomposing Costs

It is always the case that

$$C(Q) = VC(Q) + FC$$

Costs will always be the sum of the variable costs and the fixed cost

Let's practice:

Decompose the following Cost Function into FC, VC, and MC

$$C(Q) = \frac{1}{3}Q^3 - Q^2 + 2Q - 12$$

$$\begin{aligned} FC &= 12 \\ VC &= \frac{1}{3}Q^3 - Q^2 + 2Q \\ MC &= Q^2 - 2Q + 2 \end{aligned}$$

Average & Marginal Costs

Decomposing Costs: Average Costs

Although Total, Variable, and Fixed Costs are important to Cost Functions we do not really care for them on their own

What we care about are the **Average of these costs**

Now we will introduce **Average Costs** which are quite literally just the average of the previous costs

Average Total Costs

$$ATC(Q) = \frac{C(Q)}{Q}$$

Average Fixed Costs

$$AFC(Q) = \frac{FC}{Q}$$

Average Variable Costs

$$AVC(Q) = \frac{VC(Q)}{Q}$$

Decomposing Costs: Average Costs

For the following Cost Function

$$C(Q) = Q^2 + 10$$

Find ATC, AFC, and AVC

$$ATC = Q + \frac{10}{Q} \qquad AFC = \frac{10}{Q} \qquad AVC = Q$$

Graphing Cost Functions

Graphing cost functions is very helpful to understanding what is going on and how optimal choices are made

For the Cost Function

$$C(Q) = \frac{1}{3}Q^3 - Q^2 + 2Q + 12$$

We will graph the **MC, AFC, AVC, and ATC**

But first we will derive them

Finding and Graphing Marginal Costs

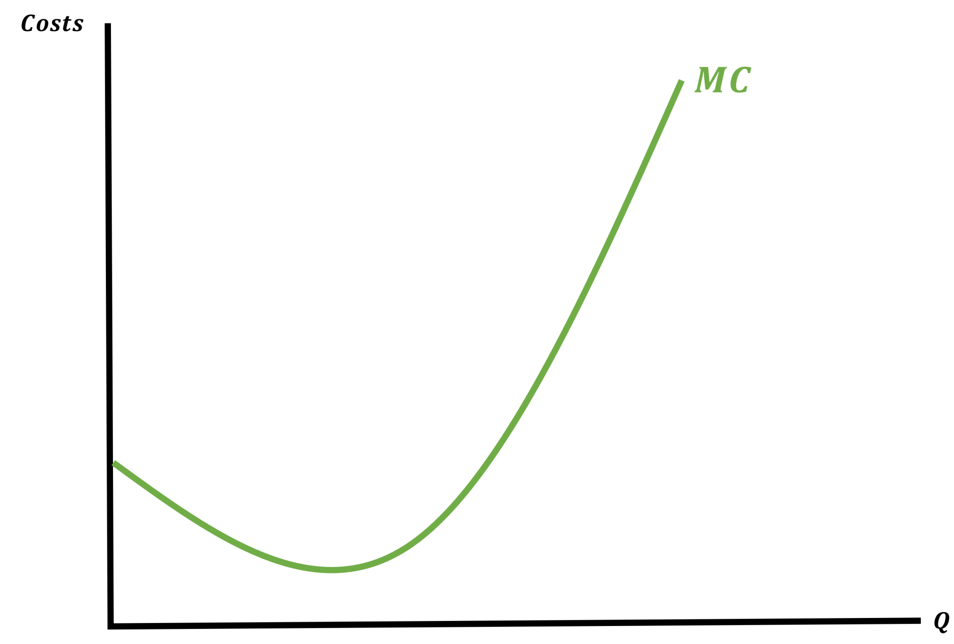
$$C(Q) = \frac{1}{3}Q^3 - Q^2 + 2Q + 12$$

Find the MC

The Marginal Cost

$$MC = \frac{\partial C(Q)}{\partial Q} = Q^2 - 2Q + 2$$

Graphing MC



Finding and Graphing Average Fixed Costs

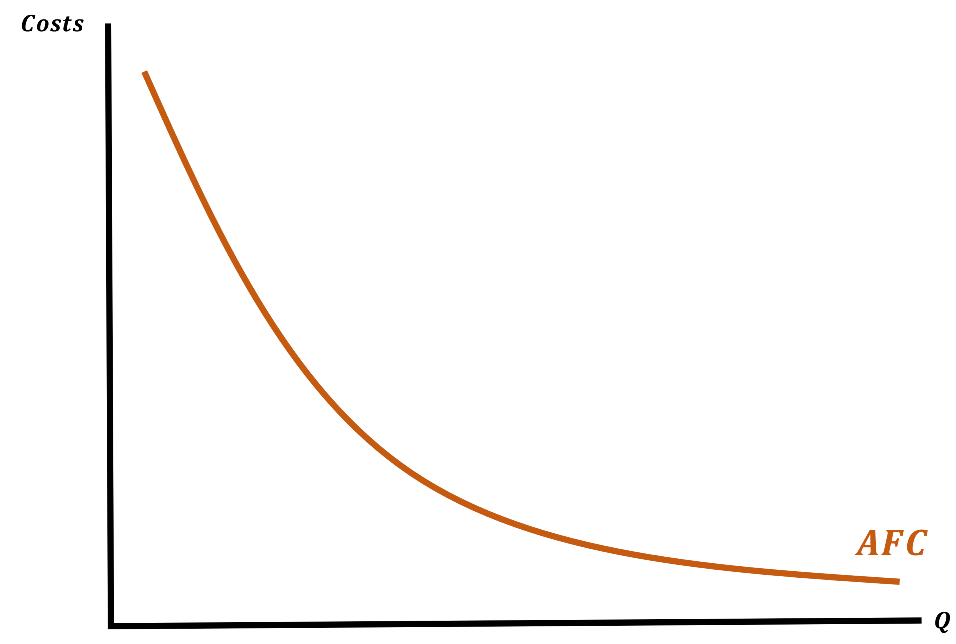
$$C(Q) = \frac{1}{3}Q^3 - Q^2 + 2Q + 12$$

Find the AFC

The Average Fixed Cost

$$AFC = \frac{FC}{\partial Q} = \frac{12}{Q}$$

Graphing AFC



Finding and Graphing Average Variable Costs

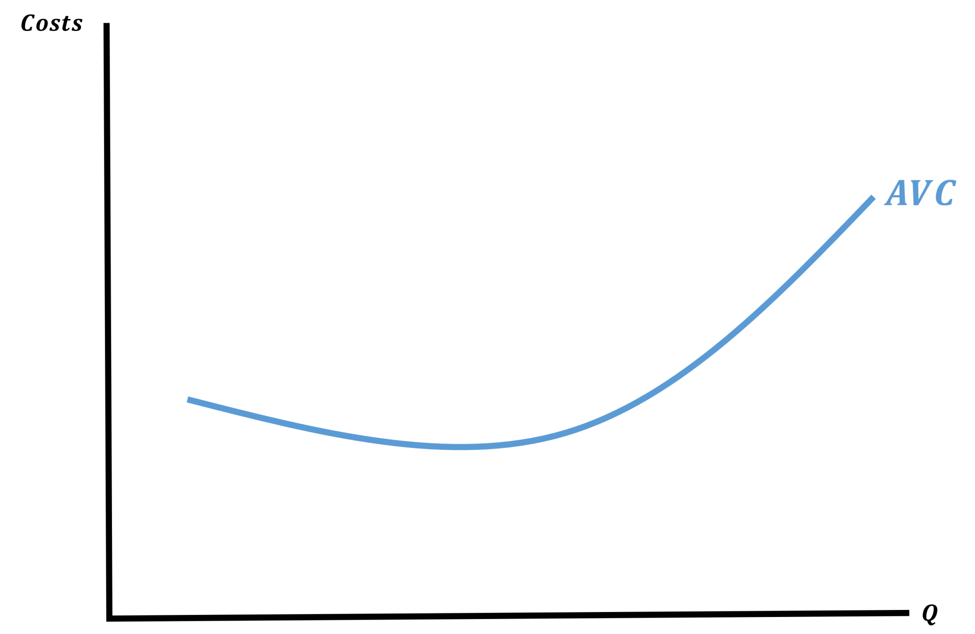
$$C(Q) = \frac{1}{3}Q^3 - Q^2 + 2Q + 12$$

Find the AVC

The Average Variable Cost

$$AVC = \frac{VC}{\partial Q} = \frac{1}{3}Q^2 - Q + 2$$

Graphing AVC



Finding and Graphing Average Total Costs

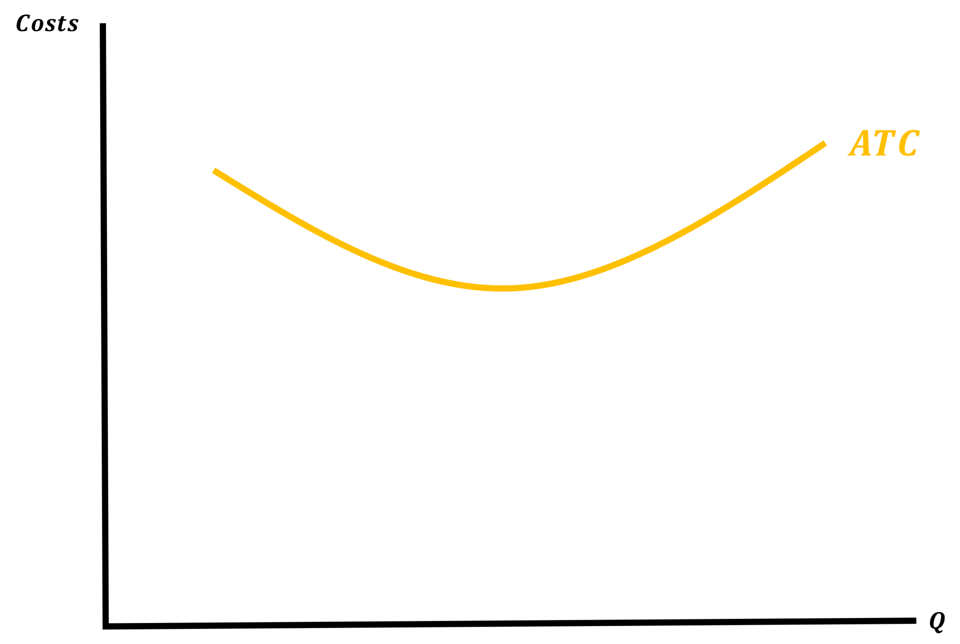
$$C(Q) = \frac{1}{3}Q^3 - Q^2 + 2Q + 12$$

Find the ATC

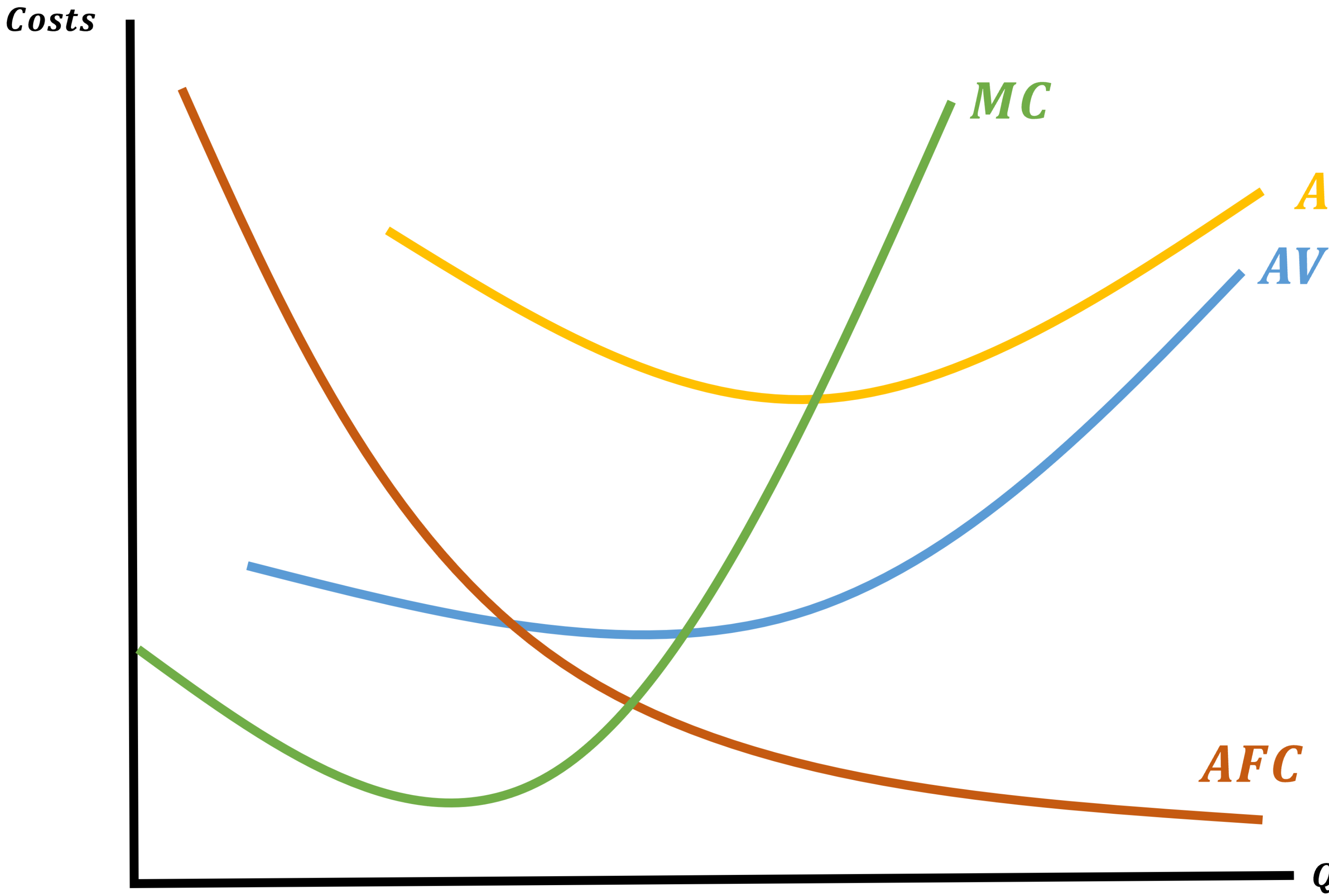
The Average Total Cost

$$ATC = \frac{C(Q)}{Q} = \frac{1}{3}Q^2 - Q + 2$$

Graphing ATC



Graphings Costs: All Together



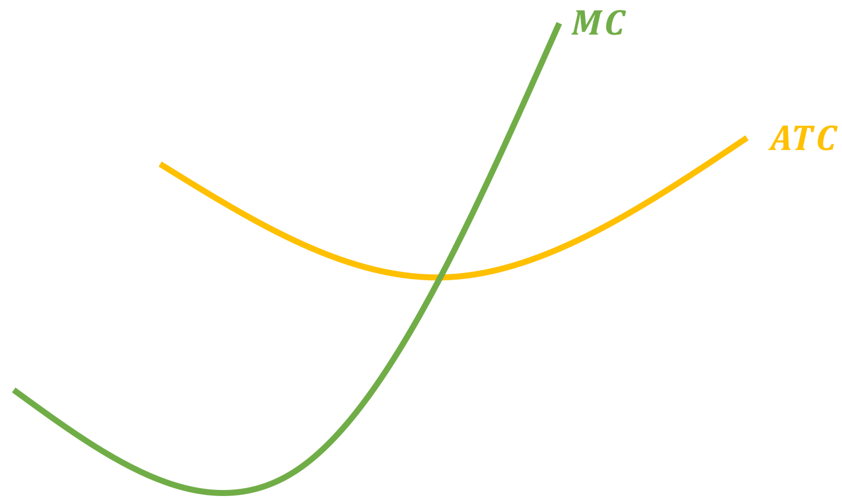
Key Things About Graphing Costs

As always, the scale is not very important

But be sure to get the **most important facts down**:

- They are all convex (open upwards)
- **AVC** is always below the **ATC**
- Where **the curves cross is crucial**
  - The **MC** crosses the **AVC** and **ATC** at their **minimum values**
- As always, label your curves and axis!

Importance of Average Total & Marginal Cost Relationship



What is so special about the point where **MC** and **ATC** cross?

- It is where the **ATC** is at it's minimum value
- Mathematically, this is where the derivative of **ATC** $= 0$
  - This implies that the **MC = ATC**

Let's look at an example that will show us what I mean intuitively

Average Total & Marginal Cost Relationship

Imagine we measure the height of everyone in class and find the average height

Then someone new joins the class

What happens to the **average height** if the new person is **shorter than the average**?

- It **decreases**
- When the **Marginal Person** is **shorter than the average**, the **average decreases**
- When the **Marginal Person** is **taller than the average**, the **average increases**

Average Total & Marginal Cost Relationship

At our sweet spot where **MC = ATC** at the minimum of the **ATC** we can say:

- When the **Marginal Cost** is less than the **Average Total Cost**, the **ATC** must be **decreasing**
- When the **Marginal Cost** is larger than the **Average Total Cost**, the **ATC** must be **increasing**

Therefore, when **MC = ATC**, the **average total cost** is switching from **decreasing** to **increasing**

- That is the definition of a minimum

All of this will be useful for our next topic: **Profit Maximization**