5. There are only two firms in an industry. Firm A has cost function $C_a(a) = 20a$ and firm B has cost function $C_b(b) = b^2$. Demand in the industry is given by $P = 120 - Q$ where $Q = a + b$.

   (a) Assuming they engage in Cournot competition, write down the profit function for each firm and simplify.

$P = 120 - Q = 120 - a - b$

Firm A:

$\Pi_a = P \cdot a - C_a(a)$

$= (120 - a - b)a - 20a$

$= 120a - a^2 - ab - 20a$

$\Pi_A = 100a - ab - a^2$ //

Firm B:

$\Pi_B = (120 - a - b)b - b^2$

$= 120b - ab - b^2 - b^2$

$\Pi_B = 120b - ab - 2b^2$ //

   (b) What is firm A's best response function?

$\frac{\partial \Pi_a}{\partial a} = 0 \rightarrow 100 - b - 2a = 0$

$\hookrightarrow 2a = 100 - b$

$BR_a = a^* = 50 - \frac{1}{2}b$

   (c) What is firm B's best response function?

$\frac{\partial \Pi_B}{\partial b} = 0 \rightarrow 120 - a - 4b = 0$

$\hookrightarrow 4b = 120 - a$

$BR_b = b^* = 30 - \frac{1}{4}a$

(d) Find the Nash Equilibrium

Plug in BR_b into BR_a:

$a^* = 50 - \frac{1}{2}b$

$a^* = 50 - \frac{1}{2}(30 - \frac{1}{4}a^*)$

$a^* = 50 - 15 + \frac{1}{8}a^*$

$\frac{7}{8}a^* = 35$

$\longrightarrow a^* = 40$

$b^* = 30 - \frac{1}{4}a^*$

$b^* = 30 - \frac{1}{4}(40)$

$b^* = 30 - 10 = 20$

THE NE IS $a^* = 40$, $b^* = 20$.

(e) How much profit does each firm earn in the Nash Equilibrium?

$P = 120 - Q = 120 - a^* - b^* = 120 - 40 - 20 = 60$

$\Pi_a = P \cdot a - C_a(a) = 60 \cdot 40 - 20(40) = 2400 - 800 = 1600$

$\Pi_b = P \cdot b - C_b(b) = 60 \cdot 20 - (20)^2 = 1200 - 400 = 800$

	Cournot	Stackelberg
A	1600	>1600
B	800	<800

with A leading

6. Imagine two firms are competing in Cournot competition. Firm A has the cost function $C(Q_A) = 6Q_A$. The market demand curve is $P = 54 - 2Q$. Write out Firm A's profit function. Find Firm A's best response function.

$\pi_a = P \cdot Q_A - C(Q_A)$; $P = 54 - 2Q = 54 - 2(Q_A + Q_B) = 54 - 2Q_A - 2Q_B$

$\pi_a = (54 - 2Q_A - 2Q_B)Q_A - 6Q_A$

$= 54Q_A - 2Q_A^2 - 2Q_BQ_A - 6Q_A$

$\pi_a = 48Q_A - 2Q_A^2 - 2Q_BQ_A$

$BR_A \Rightarrow \frac{\partial \pi_a}{\partial Q_A} = 0$

$\rightarrow 48 - 4Q_A - 2Q_B = 0 \rightarrow 4Q_A = 48 - 2Q_B$

$Q_A^* = 12 - \frac{1}{2}Q_B$

7. Imagine two firms are competing in Cournot competition. Firm A's reaction function is $Q_A = 30 - \frac{Q_B}{2}$ and firm B's reaction function is $Q_B = 35 - Q_A$. What are the Cournot equilibrium quantities for each firm?

Plug Q_B into Q_A:

$Q_A = 30 - Q_B/2$

$Q_A = 30 - \left(\frac{35 - Q_A}{2}\right)$

$2Q_A = 60 - 35 + Q_A$

$Q_A^* = 25$

$Q_B^* = 35 - 25 = 10$

Stack: 1. Find followers BR

2. Plug BR_F into π_L

8. Imagine that Duck Mountain is the only place to ski in the area. Their cost function is $C(Q) = 50Q$, where Q represents the number of week passes they sell. The demand curve for week passes is $P = 1050 - Q_D$.

   (a) Write down Duck Mountain's profit function, using the fact that they are a monopoly. What is their marginal revenue function? What is their marginal cost function?

$P = 1050 - Q$

$$\pi = P \cdot Q - C(Q) = \underbrace{(1050 - Q)Q}_{\text{Revenue}} - \underbrace{50Q}_{\text{Costs}}$$

$$MR = \frac{\partial Rev}{\partial Q} = 1050 - 2Q$$

$$MC = \frac{\partial Costs}{\partial Q} = 50$$

   (b) What is the profit-maximizing price and quantity that duck Mountain charges?

Quantity is given by $MR = MC$

$MR = MC$

$1050 - 2Q = 50$

$2Q = 1000$

$Q_m^* = 500$

Price comes from demand curve

$P = 1050 - Q$

$P_m^* = 1050 - 500 = 550$

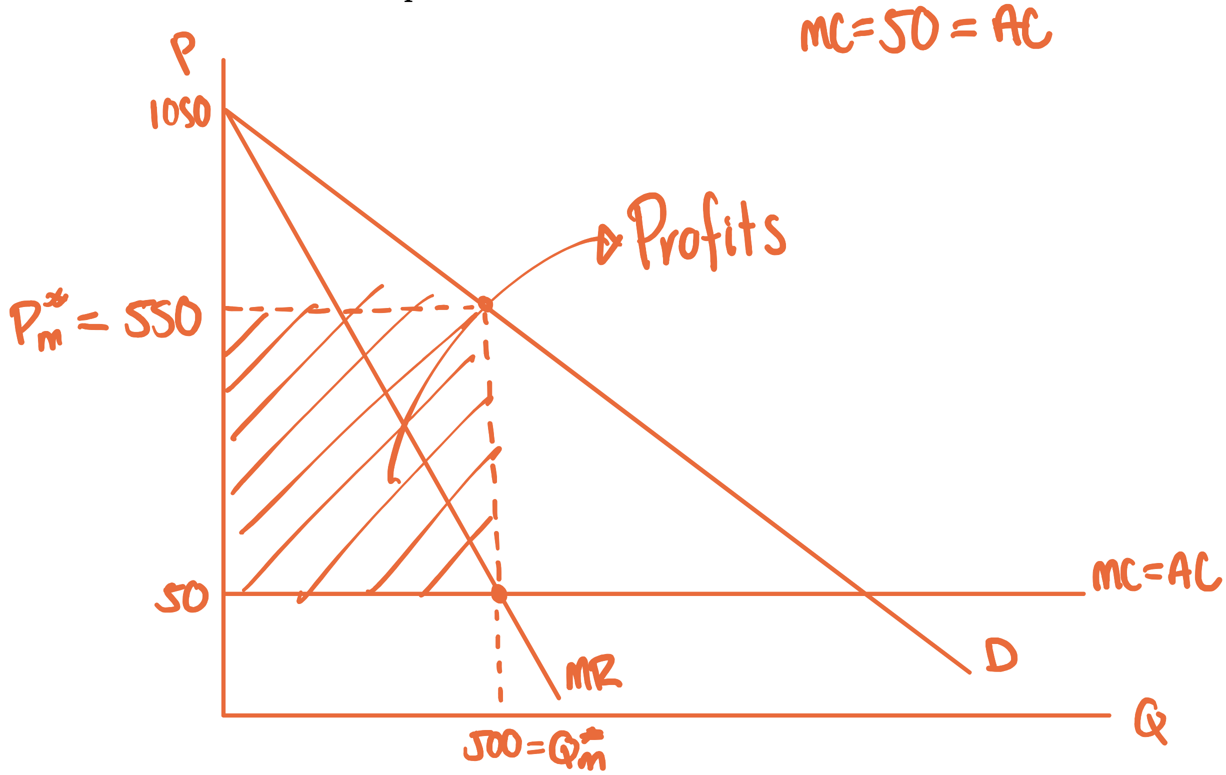
(c) On a graph with (Q, P) axis, graph the market demand curve, and Duck Mountain's marginal cost, average cost and marginal revenue curves.

$C(Q) = 50Q \rightarrow AC = 50$

Hint: Two for these curves will be the same.

Add the profit-maximizing Q and P for Duck Mountain. Shade in the area associated with Duck Mountain's profits.

$MC = 50 = AC$



(d) A new company is planning to buy land near Duck Mountain and open another ski resort called Beaver Mountain. They have the same cost function $C(Q) = 50Q$. Write down Beaver Mountain's profit function, assuming they will engage in imperfect competition with Duck Mountain. Find Beaver Mountain's best response function.

$$\pi_B = P \cdot Q_B - C(Q_B) = (1050 - Q_B - Q_D)Q_B - 50Q_B$$

$$BR_B \Rightarrow \frac{\partial \pi_B}{\partial Q_B} = 0 \longrightarrow 1050 - 2Q_B - Q_D - 50 = 0$$

$$\Rightarrow 2Q_B = 1050 - 50 - Q_D$$

$$BR_B = Q_B^* = 500 - \frac{1}{2}Q_D$$

(e) Since Duck Mountain is already established, they will get to be the leader in Stackelberg competition with Beaver Mountain. Write down Duck Mountain's profit fucntion, incorporating their anticipation of what Beaver Mountain will do. What impact will the opening of Beaver Mountain have on market quantity and price (just the direction of the impact, no math required)?

Duck → Leader

Beaver → Follower

$$\pi_D = P \cdot Q_D - C(Q_D) = (1050 - Q_B - Q_D)Q_D - 50Q_D$$

$$Q_B = BR_B = 500 - \frac{Q_D}{2}$$

$$\pi_D = \left(1050 - Q_D - \left(500 - \frac{Q_D}{2}\right)\right)Q_D - 50Q_D$$

$$\pi_D = 1050Q_D - Q_D^2 - 500 + \frac{Q_D^2}{2} - 50Q_D$$

↳ Monopoly → Stackelberg

Quantity ↑ & Price ↓ because there is more competition is Stack.

(f) Imagine that Duck Mountain and Beaver Mountain switch from competing on the quantity of passes they offer to engage in a price war according to Bertrand Competition. What is the market equilibrium price and quantity? What are each firm's profits?

Set $P = MC \longrightarrow MC = 50$

$P^* = 50$

To find Q^* use Demand Curve: $P = 1050 - Q$

$50 = 1050 - Q \longrightarrow Q^* = 1000$

Because $P = MC$, firms are in Zero profit condition:

↳ $\pi_D = \pi_B = 0$

9. There are only two firms in an industry. Firm A has cost function $C(Q_A) = 20Q_A$ and firm B has cost function $C(Q_B) = Q_B^2$. Demand in the industry is given by the demand curve $P = 120 - Q$.

   (a) Write down the profit function for both firms, assuming they are in Cournot competition (do not simplify)

   $P = 120 - Q = 120 - Q_A - Q_B$

   $\pi_A = P \cdot Q_A - C(Q_A) = (120 - Q_A - Q_B)Q_A - 20Q_A$

   $\pi_B = (120 - Q_A - Q_B)Q_B - Q_B^2$

   (b) Find firm A's reaction function.

   $\frac{\partial \pi_A}{\partial Q_A} = 0$

   $\rightarrow 120 - 2Q_A - Q_B - 20 = 0$

   $\rightarrow 2Q_A = 120 - Q_B - 20 \rightarrow Q_A^* = 50 - \frac{Q_B}{2}$

   (c) Find firm B's reaction function

   $\frac{\partial \pi_B}{\partial Q_B} = 0$

   $\rightarrow 120 - Q_A - 2Q_B - 2Q_B = 0$

   $4Q_B = 120 - Q_A$

   $Q_B^* = 30 - \frac{1}{4}Q_A$

(d) What are the Cournot equilibrium quantities for each firm?

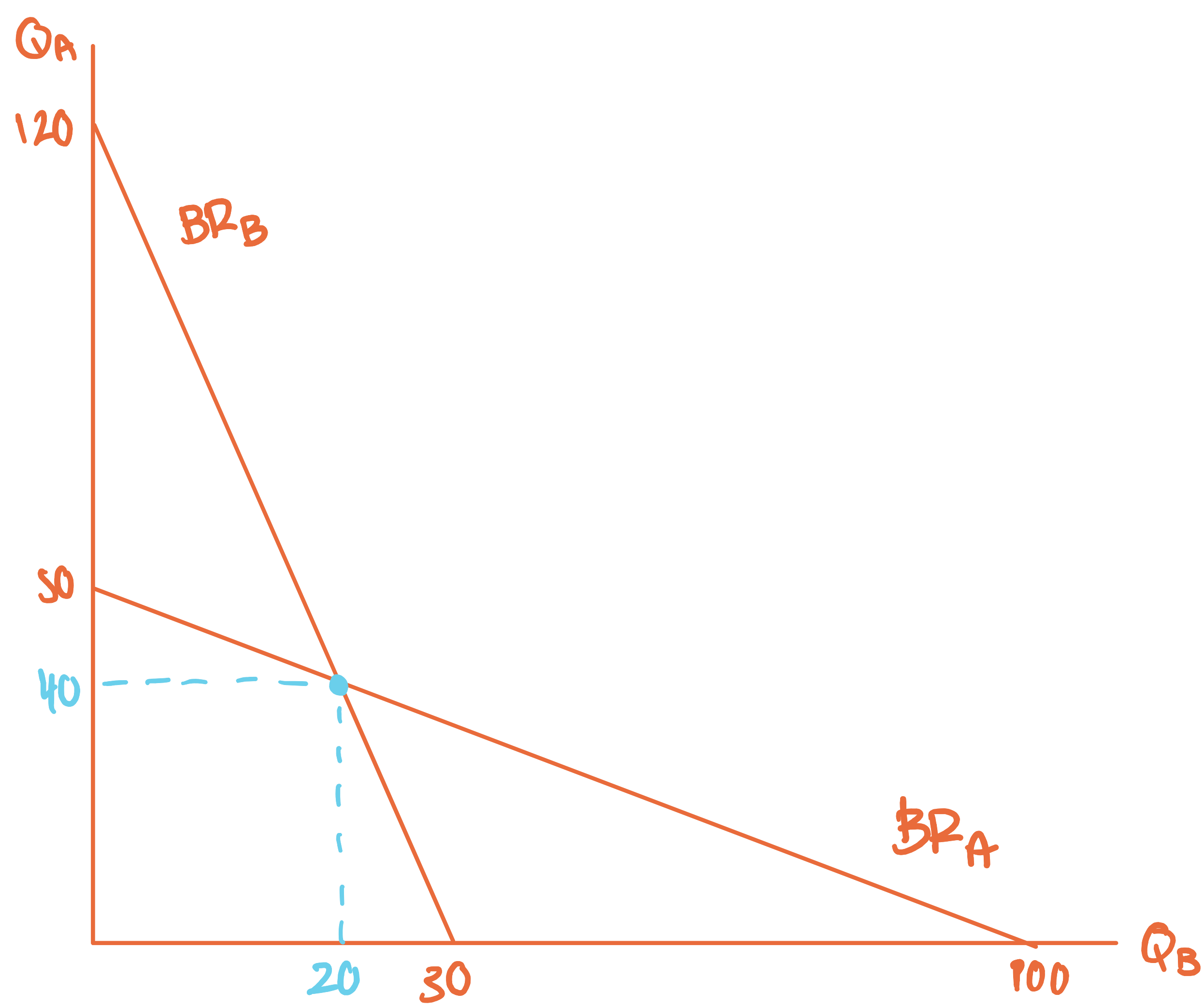
Plug in BR functions into each other

$$Q_A = 50 - \left(\frac{30 - Q_A/4}{2}\right) = 50 - 15 + \frac{Q_A}{8}$$

$$\frac{7}{8} Q_A = 35 \longrightarrow Q_A^* = 40$$

$$Q_B^* = 30 - \frac{1}{4}(40) = 30 - 10 = 20$$

(e) In the (Q_B, Q_A) plane, graph the reaction function of both firms and label the Cournot equilibrium.



(f) Imagine the nature of competition switches: the firms are now able to compete by naming prices (Bertrand competition). In Bertrand competition, firm A will be able to price firm B out of the market. If firm A would like to supply 40 units by themselves, what price should they set (or what should they set their price just barely less than)? What is their profit at that point?

$P = 120 - Q$; @ $Q = 40 \rightarrow P = 80$

$MC(Q_B) = 2Q_B \rightarrow 2(40) = 80.$

Firm B is willing to supply 40 units at $P = 80$ since $MC = P$.

↳ Firm A can charge 79.99.

Firm B is unwilling to supply 40 units since $P < MC$

$\Pi_A = P \cdot Q_A - C(Q_A) = (79.99)(40) - 20 \cdot 40$

$\Pi_A \approx 2400$

(f) Imagine the nature of competition switches: the firms are now able to compete by naming prices (Bertrand competition). In Bertrand competition, firm A will be able to price firm B out of the market. If firm A would like to supply 40 units by themselves, what price should they set (or what should they set their price just barely less than)? What is their profit at that point?

$P = 80 \rightarrow \pi_B = 0$

$P = 79.99 \rightarrow \pi_B < 0$

$MC_B = 80$ @ $Q = 40$

$MC_A = 20$ @ $Q = 40$