Demand

EC 311 - Intermediate Microeconomics

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Outline

Chapter 5

- Topics
  - Income Changes (5.1)
  - Price Changes (5.2)
  - Substitution and Income Effect (5.3)
  - Changes in Another Good's Price (5.4)
  - Individual Demand $\Rightarrow$ Market Demand (5.5)

Applications of Micro Theory: Demand Functions

There's An Actual Purpose to This

All this abstract math stuff has a purpose:

- At the start, I mentioned that firms, non-profits, and government agencies all have an incentive to model demand across every major industry
- They can ask: "If we raise the price of our product by 2%, how will our sales numbers react?"

Applications

The models used to estimate demand responses are derived from utility maximization

For example, Dutch Bros sets up a utility that they think represents preferences for their coffee and related goods

- They can derive and estimate the associated demand function from their sales data

Up to now, we have only solved utility maximization problems with the goal of finding the **level of demand** (aka we found **numbers**)

Now we will solve these problems without specifying either price or income

This makes it more general, which has its advantages:

- It allows us to analyze how demand responds when prices or income change
- The math stays the same, just that we have more unknowns now

How Do We Find Demand Functions?

Let's begin with the workhorse of this course: Cobb-Douglas

$$\max U(x, y) = x^{\alpha} y^{\beta} \;\; \text{s.t.} \;\; P_x \cdot x + P_y \cdot y = M$$

Recall how we solve this problem:

1. Find the MRS and set it equal to the Price Ratio
2. Solve this equality for one of the goods $\longrightarrow$ Optimality Condition
3. Plug the Optimality Condition into the Budget Constraint
4. Use the found demand for one good and plug it into the Optimality Condition or Budget Constraint to find demand for the other good

Cobb-Douglas General Solution - Step 1

$$\max U(x,y) = x^\alpha y^\beta \;\; \text{s.t.} \;\; P_x \cdot x + P_y \cdot y = M$$

Find the MRS and the Price Ratio

$$MRS = \frac{MU_x}{MU_y} = \frac{\alpha x^{\alpha-1} y^\beta}{\beta x^\alpha y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{x^{\alpha-1-\alpha}}{y^{\beta-1-\beta}} = \frac{\alpha}{\beta} \cdot \frac{x^{-1}}{y^{-1}} = \frac{\alpha}{\beta} \cdot \frac{y}{x}$$

$$\text{Price Ratio} = \frac{P_x}{P_y}$$

Set them equal to each other

$$\text{MRS} = \text{Price Ratio}$$
$$\frac{\alpha}{\beta} \cdot \frac{y}{x} = \frac{P_x}{P_y}$$

Cobb-Douglas General Solution - Step 2

$$\frac{\alpha}{\beta} \cdot \frac{y}{x} = \frac{P_x}{P_y}$$

> **Solve for y**

$$\begin{aligned} \frac{\alpha}{\beta} \cdot \frac{y}{x} &= \frac{P_x}{P_y} \\ \frac{y}{x} &= \frac{P_x}{P_y} \cdot \frac{\beta}{\alpha} \\ y^* &= \frac{\beta}{\alpha} \cdot \frac{P_x}{P_y} \cdot x \end{aligned}$$

Cobb-Douglas General Solution - Step 3

$$y^* = \frac{\beta}{\alpha} \cdot \frac{P_x}{P_y} \cdot x \;\; \& \;\; \text{BC: } P_x \cdot x + P_y \cdot y = M$$

> **Solve for the Demand of good x**

$$\begin{aligned}
M &= P_x \cdot x + P_y \cdot y \\
M &= P_x \cdot x + P_y \cdot \left(\frac{\beta}{\alpha} \cdot \frac{P_x}{P_y} \cdot x \right) \\
M &= P_x \cdot x + \frac{\beta}{\alpha} \cdot P_x \cdot x \\
M &= P_x \cdot x \left(1 + \frac{\beta}{\alpha} \right) \\
&\vdots \\
x^* &= \frac{M}{P_x} \cdot \frac{\alpha}{\alpha + \beta}
\end{aligned}$$

Cobb-Douglas General Solution - Step 4

$$x^* = \frac{M}{P_x} \cdot \frac{\alpha}{\alpha + \beta}$$

> **Use found demand of one good to find demand for the other good**

$$y^* = \frac{\beta}{\alpha} \cdot \frac{P_x}{P_y} \cdot x^*$$

$$y^* = \frac{\beta}{\alpha} \cdot \frac{P_x}{P_y} \cdot \frac{M}{P_x} \cdot \frac{\alpha}{\alpha + \beta}$$

$$y^* = \frac{\beta}{\alpha + \beta} \cdot \frac{M}{P_y}$$

Cobb-Douglas General Solution - Trick

When x^* and y^* are functions, we call them **Demand Functions**

$$x^* = \frac{\alpha}{\alpha+\beta} \cdot \frac{M}{P_x} \qquad \& \qquad y^* = \frac{\beta}{\alpha+\beta} \cdot \frac{M}{P_y}$$

> **Knowing a C-D utility looks like $U(x, y) = x^\alpha y^\beta$ what do you notice about these functions?**

Let me show you a trick for Cobb-Douglas utility functions

- They are a constant share of how much I can afford of each individual good
- This will always be true so you can use this to check your math in later problems

Demand Functions

In general, the demand for x and y are functions of three variables:

- P_x, P_y, and M
- This allows us to write demand functions as:
  - $x^* = f(P_x, P_y, M)$
  - $y^* = f(P_x, P_y, M)$
- We are interested in how demand responds to changes to all three arguments

Techniques to Find Demand Functions

We will learn how to find Demand Functions

Graphically

- Expansion Paths and Engel Curves

Mathematically

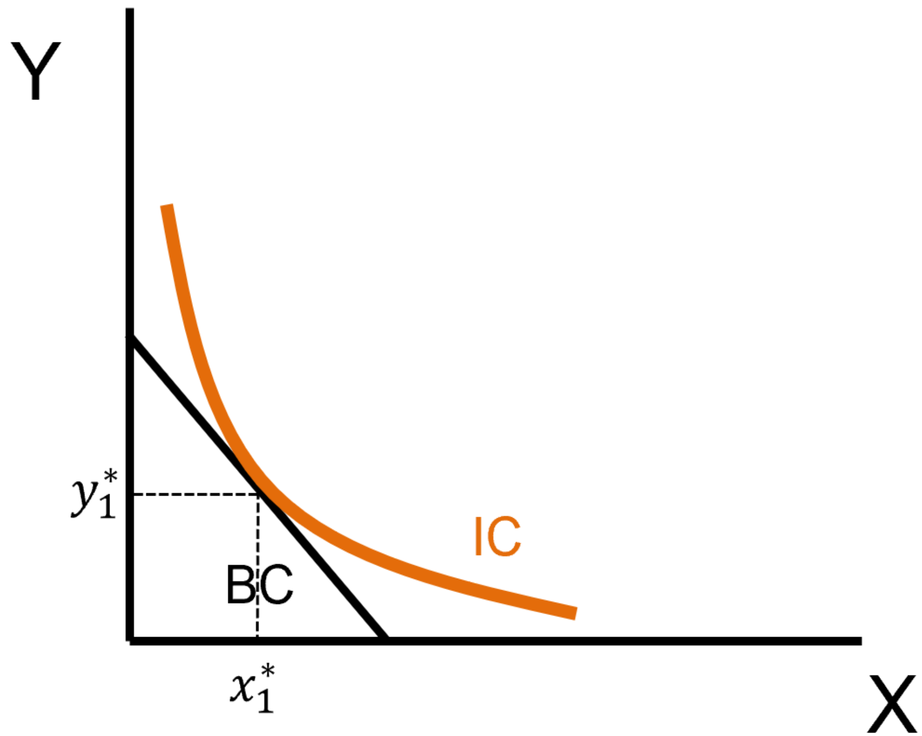
- Derivatives and Elasticities

- We will figure out how a change in any of the function parameters affects demand for both goods
- Let's begin with **Changes to Income**

Income Changes

Visualizing the Change

Let's start with a our standard optimized utility graph

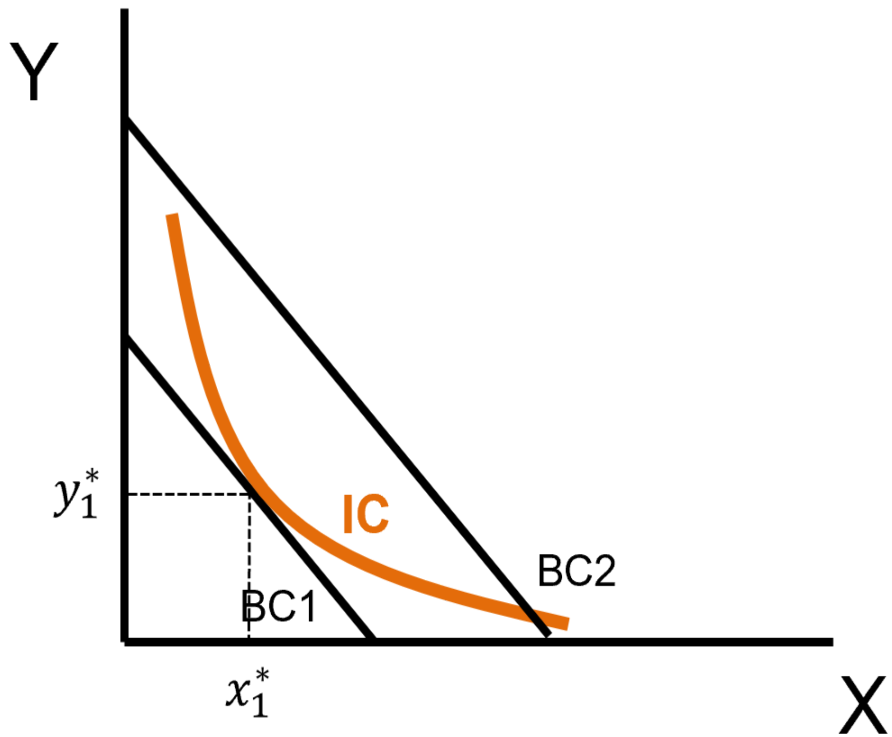


What would happen if we **Increase** our income?

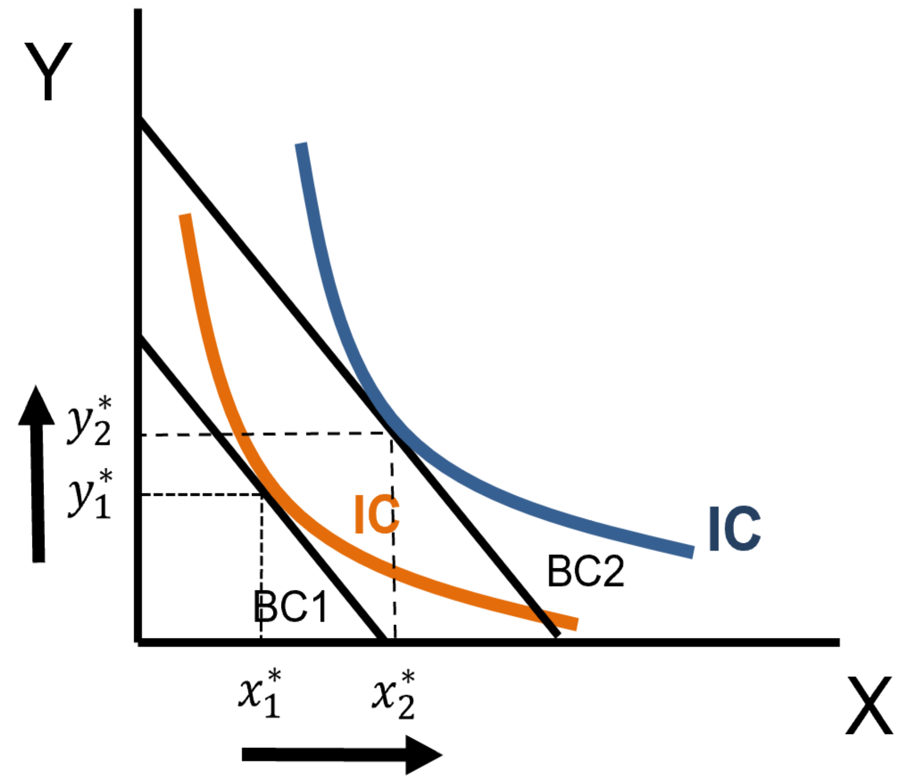
Visualizing the Change

With an increase in income, our budget constraint will:

Shift Outward



We find the new Maximizing Point



More Goods

The previous graphs tell us

- When income (M) goes up, you can now reach a higher IC and be better off (relatively)
- This graph also makes an important assumption

- Both goods ,x and y, are desirable
  - We call these **Normal Goods**

Normal Goods are desirable, which simply means that if we have more income we will consume more of them

Other Types of Goods?

Does it always have to be the case that consumption of both x and y has to increase in response to an increase in income?

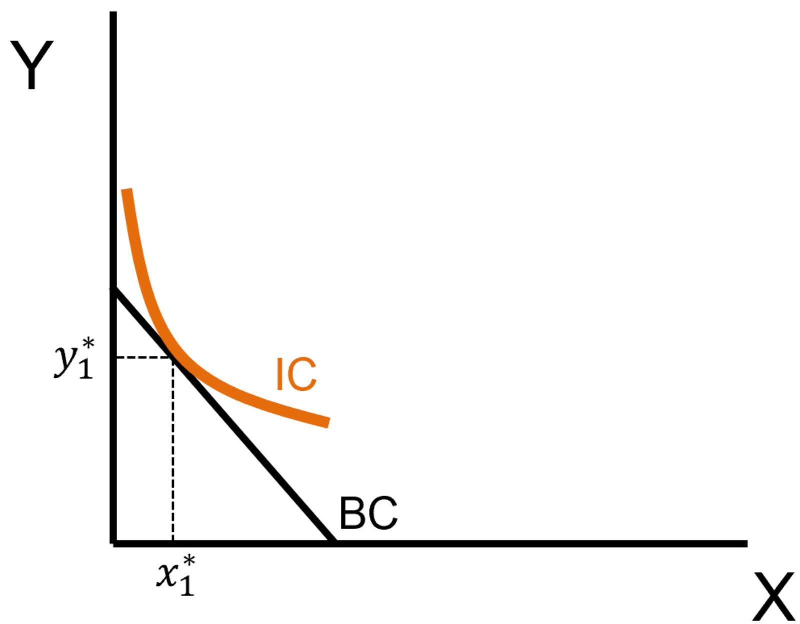
No! There are goods we call **Inferior Goods**

- These goods are consumed less when income increases
- A classic example of this is Maruchan ramen noodles

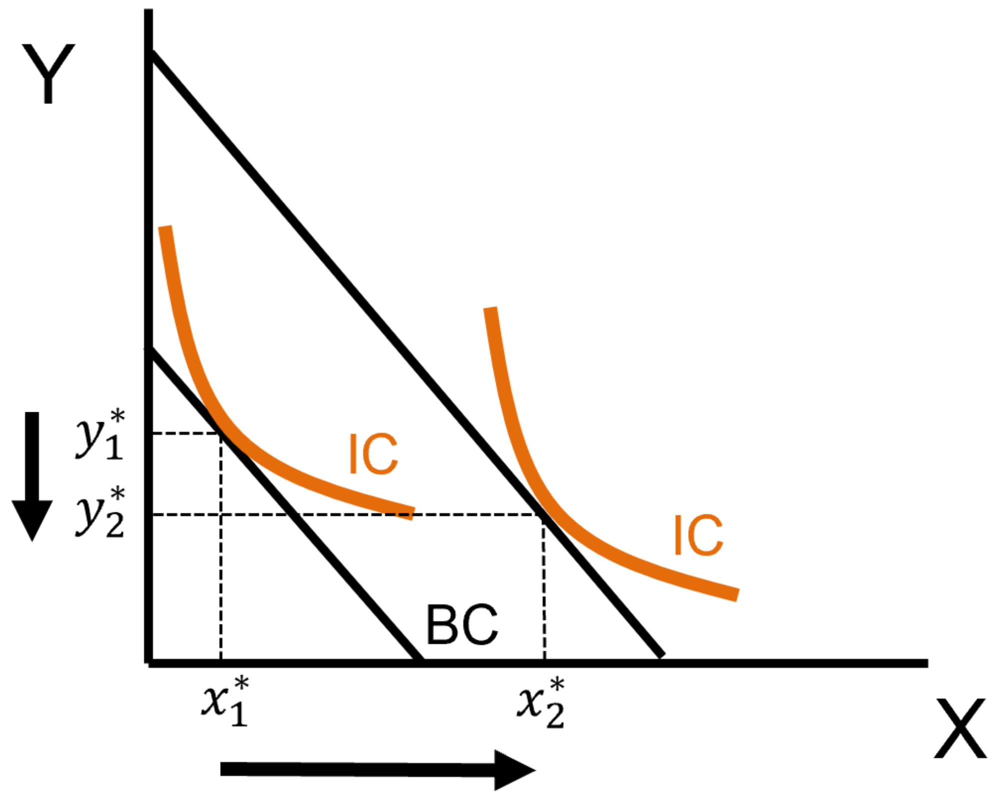
Inferior Goods - Graph

Let y be an inferior good, the optimized utility graph would look something like

Maximized Utility

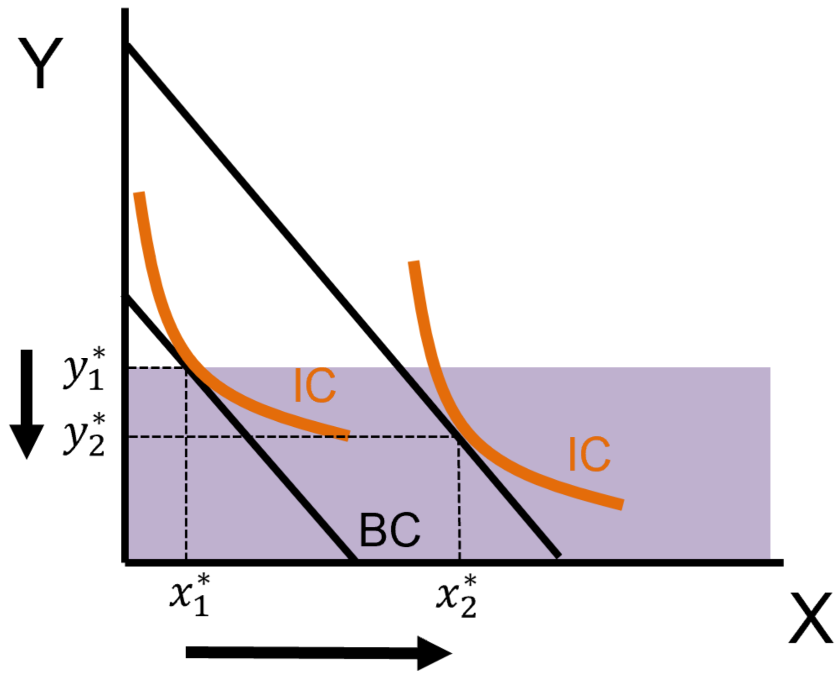


Increase in Income & Inferior Good y



Inferior Goods - Graph Intuition

With an inferior good (let's keep saying y) there is a predictable shift when income increases



The new bundle must be in **this region of the budget**

Can Both Goods Be Inferior?

As your income increases, can you buy less of both goods?

- No! **Why Not?**
- The Budget Constraint does not allow it
- Recall that the BC looks like: $P_x \cdot x + P_y \cdot y = M$

If your income increases, and you decrease the amount of x you consume (x is an inferior good), then there is a lot of income leftover that is required to be spent

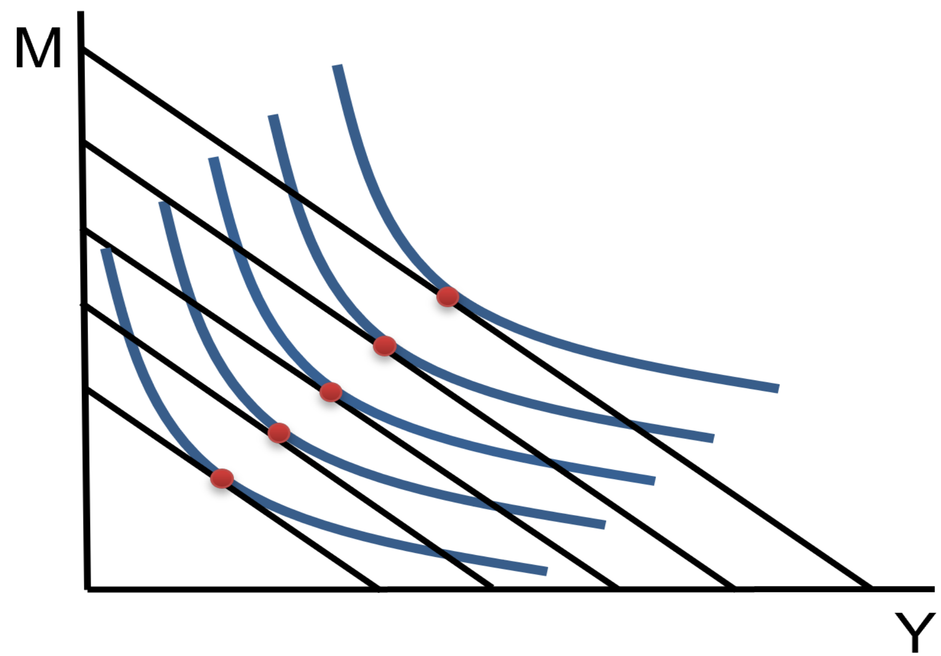
The only other option is to increase your consumption of y

> **In fact, if increasing your income has no effect on how much x you consume, you would still have to increase your consumption of y**

Engel Curves

This curve describes the relationship between M^* and y^*, with M on the vertical axis and y^* on the horizontal axis

Let's build this by parts, starting with a bunch of income levels and the associated y^* consumption levels

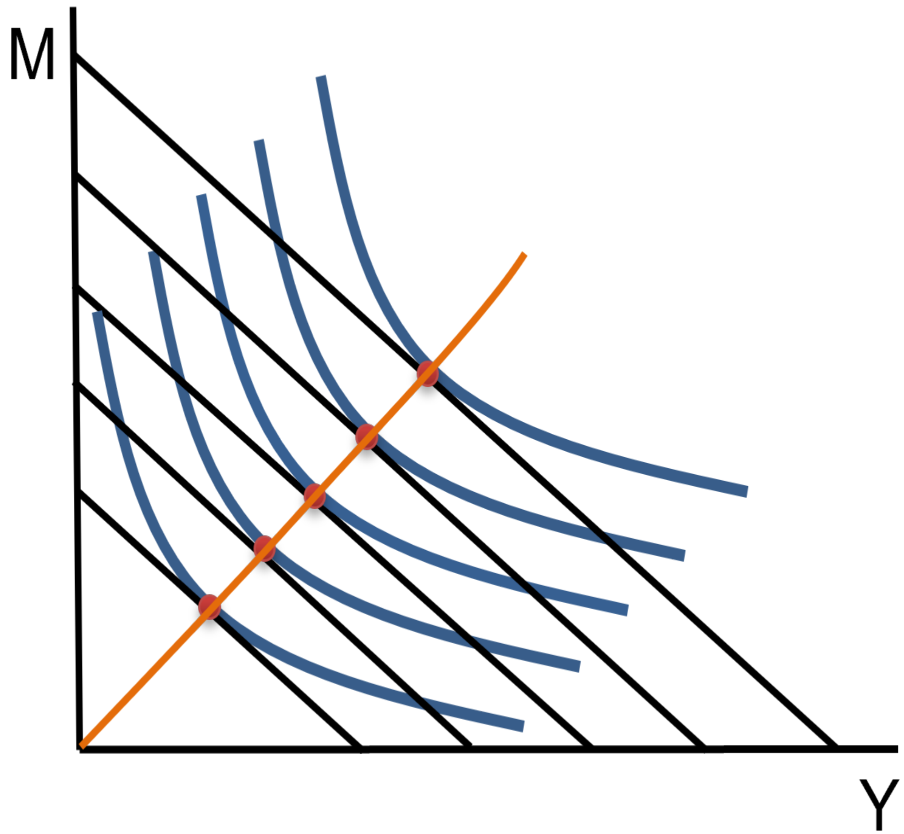


Notice the axis!

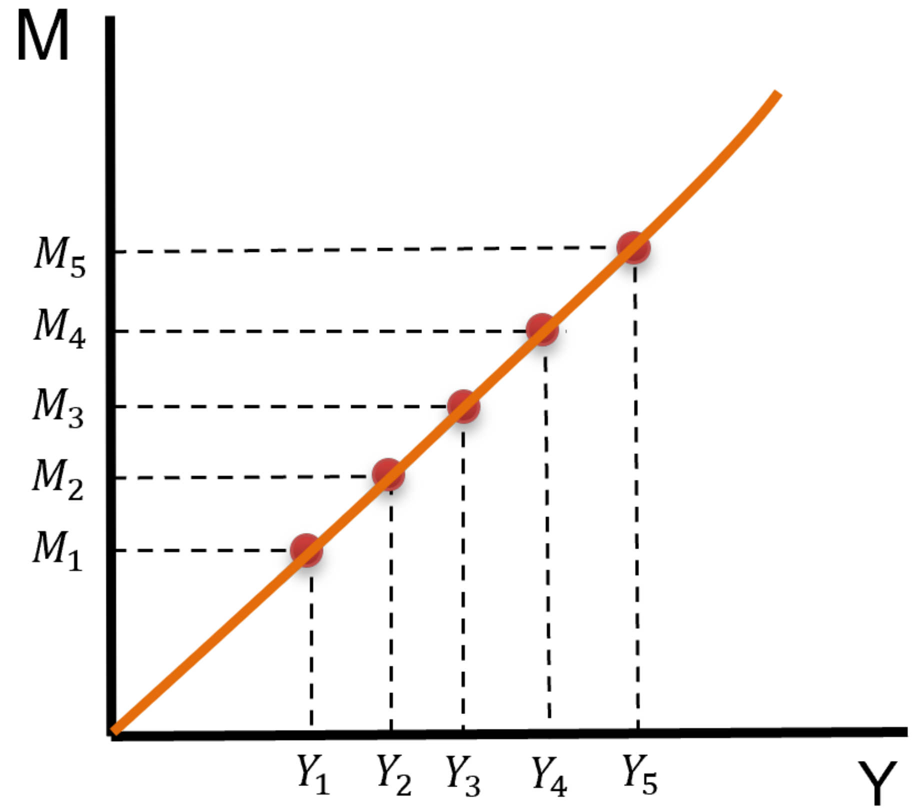
Engel Curves

If we connect all our optimal levels of y^* we find our **Engel Curve**

Connect the dots



Our Engel Curve



Engel Curves

The important thing to consider about Engel Curves is the **sign of the slope** $\rightarrow$ **It tells us if a good is normal or inferior**

- A **Positive Slope** means that when Income goes up, y^* increases and we say y is a **normal good**
- A **Negative Slope** means that when Income goes up, y^* decreases and we say y is an **inferior good**

We've seen that curves slopes can change, so we can ask ourselves:

> **Can a good be an inferior good over all income levels?**

Engel Curves - Inferior Goods Always?

Can in be the case that no matter what my current income is, an increase in income will decrease my consumption of a good?

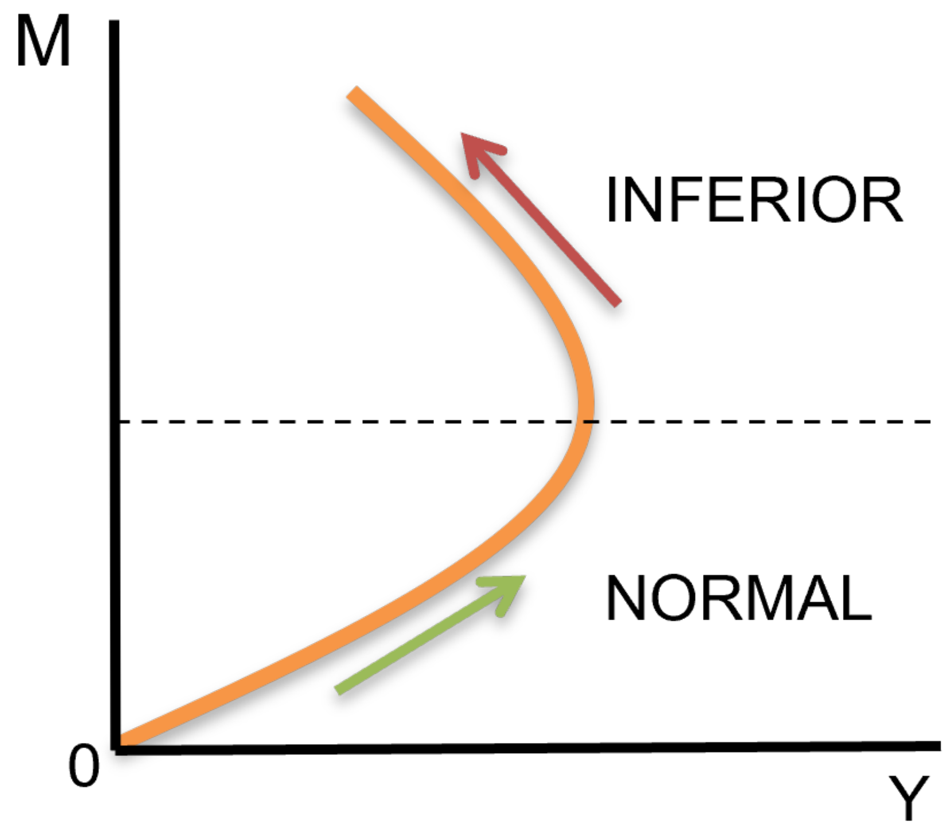
It is impossible!

- Even a good that is usually inferior, or better said, **inferior over common income levels must be a normal good over low enough levels of income**
  - I’ll show you an example of an Engel Curve with a negative slope and we’ll see why

Engel Curves - Inferior Goods Always?

Think about our Instant Ramen example. When in college, the more money you have, the more you buy Instant Ramen (so it is a normal good).

But once you graduate and get that promised pay increase from graduating, you buy Instant Ramen less and less the higher your Income becomes (inferior good)



Engel Curves - Mathematically

We can formalize Engel Curves as:

1. Engel Curves are graphs of the Demand Function
    - $x^* = f(P_x, P_y, M)$
2. Where we hold prices fixed and flip the axis

Engel Curves - Mathematically

Take our Cobb-Douglas example

$$U(x, y) = x^\alpha y^\beta \quad \text{where} \quad \alpha = \beta = P_x = 1$$

Also recall that

$$x^* = \frac{\alpha}{\alpha + \beta} \cdot \frac{M}{P_x}$$

Solving for M yields:

$$x^* = \frac{\alpha}{\alpha + \beta} \cdot \frac{M}{P_x}$$
$$x^* = \frac{1}{2} \cdot M = \frac{M}{2}$$
$$x^* = \frac{M}{2} \to M = 2x^*$$

What's the slope of this demand tell us?

$$\frac{\partial x}{\partial M} = \frac{1}{2} > 0$$

It's positive!

Engel Curves - Using Derivatives

We can show whether a good is normal or not over all income levels by taking the derivative with respect to M

$$\text{if } \frac{\partial x^*}{\partial M} > 0 \Rightarrow \text{Normal Good}$$

$$\text{if } \frac{\partial x^*}{\partial M} < 0 \Rightarrow \text{Inferior Good}$$

Back to Income Changes

We just saw that we can find whether a good is a normal or inferior by taking the partial derivative of the demand function w.r.t. income

- By knowing what category the good falls into, we can quickly estimate how demand will react to changes in income
- But this only tells us up or down, not magnitudes
- To find that, we will look at **elasticities**

Elasticities

They tell us how responsive demand is to income changes (later we will see price changes)

They follow the formula:

$$E_{x^*,M} = \frac{\partial x^*}{\partial M} \cdot \frac{M}{x^*}$$

There are 3 steps to finding an elasticity

1. Take the partial derivative of the good w.r.t. M
2. Multiply the partial derivative by the ratio of the **input variable** to the **response variable**
   - **What is changing** to **what we are estimating to chagne**
3. Substitute the original demand equation and simplify

Elasticities - Example

Let the Demand for x^* be:

$$x^* = \frac{M}{P_x}$$

> **1 - Take the partial derivative w.r.t. Income**

$$\frac{\partial x}{\partial M} = \frac{1}{P_x}$$

Elasticities - Example

The partial derivative is:

$$\frac{\partial x}{\partial M} = \frac{1}{P_x}$$

> **2 - Multiply the partial derivative by the ratio of input variable to the response variable** $\left(\frac{M}{x^*}\right)$

$$E_{x^*,M} = \frac{\partial x^*}{\partial M} \cdot \frac{M}{x^*} \rightarrow \frac{1}{P_x} \cdot \frac{M}{x^*}$$

Elasticities - Example

$$E_{x^*,M} = \frac{\partial x^*}{\partial M} \cdot \frac{M}{x^*} \rightarrow \frac{1}{P_x} \cdot \frac{M}{x^*}$$

> **3 - Substitute the original demand equation in and simplify**

$$E_{x^*,M} = \frac{1}{P_x} \cdot \frac{M}{x} = \frac{1}{P_x} \cdot \frac{M}{\frac{M}{P_x}} = \frac{1}{P_x} \cdot P_x = \frac{P_x}{P_x} = 1$$

So we would say that good x has an Income Elasticity of 1

Elasticities: What do They Mean?

They tell us how to translate a proportional change in x into a proportional change in y

So for the elasticity

$$E_{x,y} = \eta$$

If y goes up by 10%, x increases by $\eta \cdot 10\%$

Equivalently, if y goes down by 20%, x decreases by $\eta \cdot 20\%$

Elasticities can be

- Positive, negative, zero, or infinite

When considering Income Elasticities we know that

- Normal goods have positive elasticities
- Inferior goods have negative elastiticies

Elasticities: Responsiveness

We can describe how responsive goods are in terms of elasiticity

- If the **absolute value** of an elasticity is **greater than 1** (> 1) we say that there is an **elastic** response
  - $E_{x,M} = 2 \rightarrow M \uparrow 10\%,\ x \uparrow 20\%$
- If the **absolute value** of an elasticity is **less than 1** (< 1) we say that there is an **inelastic** response
  - $E_{x,M} = 0.5 \rightarrow M \uparrow 10\%,\ x \uparrow 5\%$
- If the **absolute value** of an elasticity is **equal to 1** $(= 1)$ we say that the response is **unit elastic**
- If the elastiscity is **exactly equal to 0** we call this **perfectly inelastic**
- If the elasticity is **infinite** (∞) we call this **perfectly elastic**

Own Price Changes

Price Changes are Important

Changing the price of a good and seeing what changes that causes is one of the most important types of changes we can consider

- This is what people in the real world most want to know about

An important thing to have in mind throughout this section is:

- If you hear/read “Demand Curve” or “Elasticity” without specifying if it’s about price or income, **the default is Own-Price Elasticity**

Price Changes

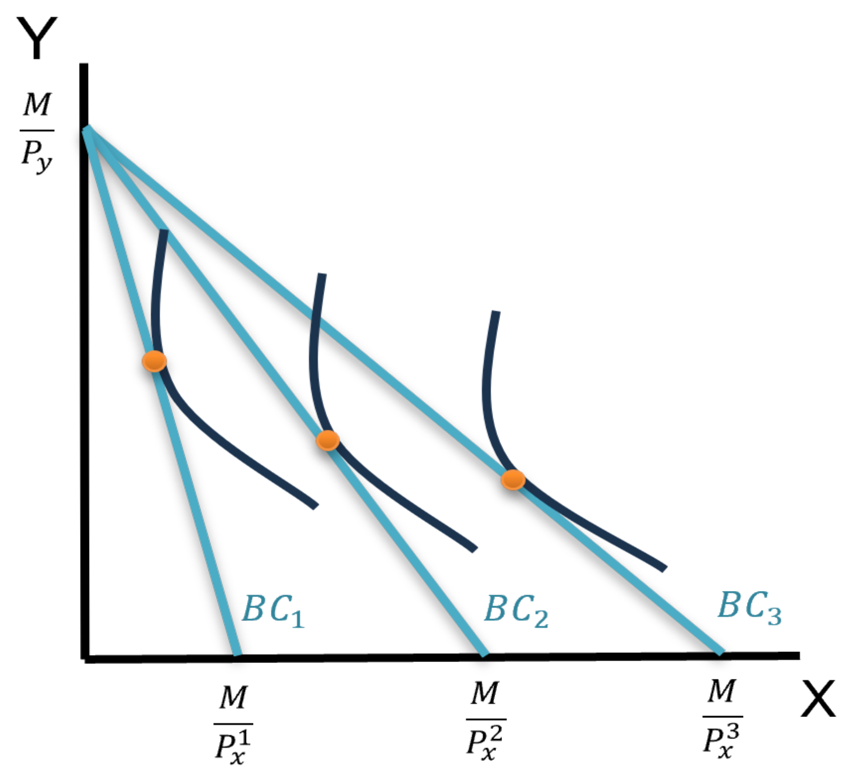
Let's recall an important effect of price changes on the Budget Constraint

> **What happens to the Budget Constraint when then price of a normal good x falls?**

What happens to the slope?

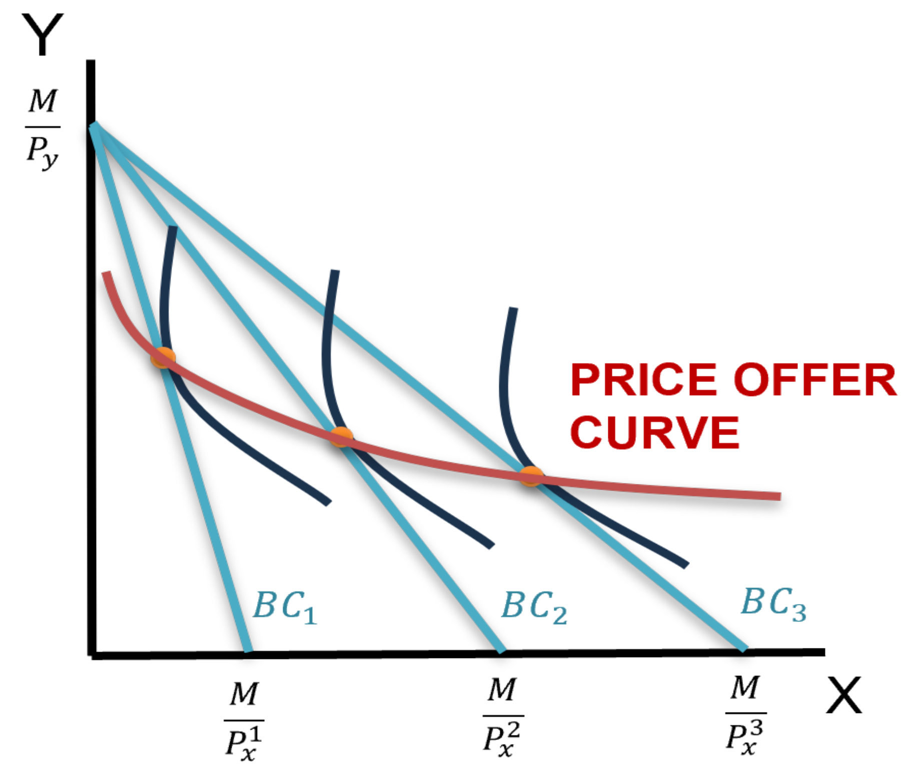
$$\frac{-P_x}{P_y} \rightarrow \downarrow P_x \rightarrow ???$$

Graph



Individual Demand

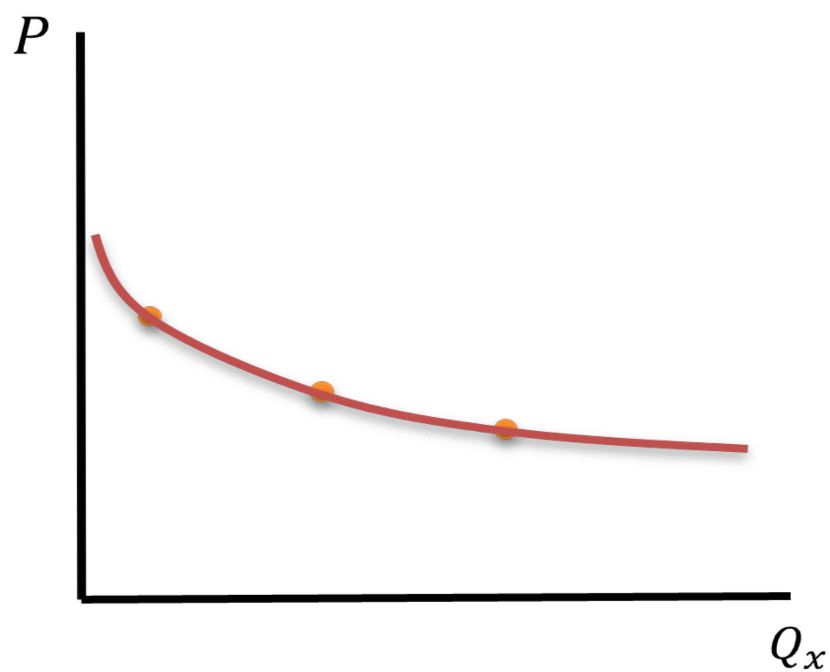
Similar to the Engel Curve, we can connect these optimal bundles to get an important curve



This is an **Individual Demand Curve**

Individual Demand Curve

If we clean it up a bit, we get our typical **downward sloping Demand Curve**

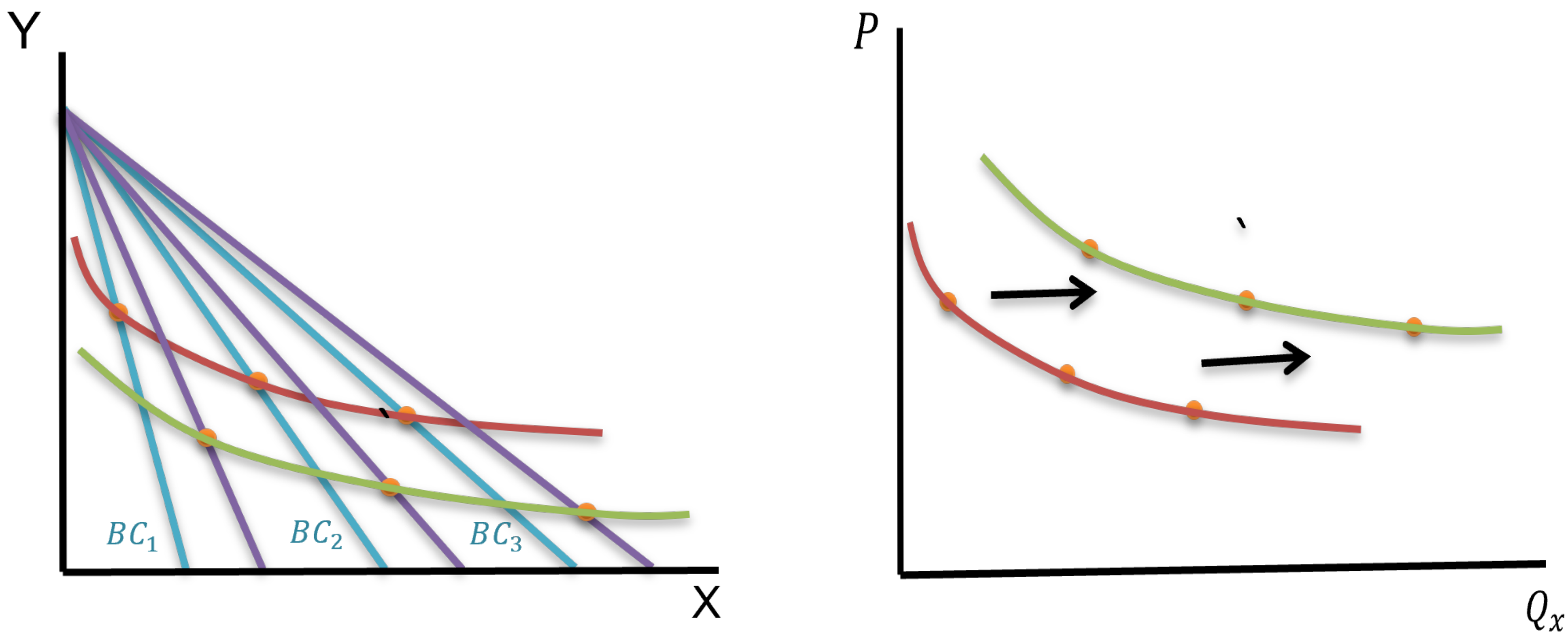


Notice I changed the axis slightly: I put Price on the vertical axis and Quantity of x demanded on the horizontal axis

Income Changes (Again)

If you recall from EC 201 we learned about demand shifters when income increases.

If we repeat this exercise with an increased income we can observe the **Demand Shift**



Individual Demand (Mathematically)

As we have done before, we also show the **Demand Curve Properties** through derivatives!

Let's show that the Demand Curve is downward sloping

- First, recall that the optimal consumption for a normal good y from a Cobb-Douglas Utility Function is:

$$y^* = \frac{\beta}{\alpha+\beta} \cdot \frac{M}{P_y} = \frac{\beta M}{\alpha+\beta} \cdot P_y^{-1}$$

- Second, we take the **derivative w.r.t. Price**

$$\frac{\partial y}{\partial P_y} = \frac{\beta M}{\alpha+\beta} \cdot (-1) P_y^{-2} = \frac{-\beta M}{\alpha+\beta} \cdot P_y^{-2} = \frac{-\beta M}{\alpha+\beta} \cdot \frac{1}{P_y^2} < 0$$

Price Elasticity

This concept is identical to income elasticity

I will refer to the **Elasticity of Demand** as the **elasticity of x with respect to P_x**

The formula is

$$E_{x^*,P_x} = \frac{\partial x^*}{\partial P_x} \cdot \frac{P_x}{x^*} \quad \text{where} \quad x^* = \frac{M}{2P_x}$$

Elasticity of Demand - Example

Let

$$E_{x^*,P_x} = \frac{\partial x^*}{\partial P_x} \cdot \frac{P_x}{x^*} \quad \text{where} \quad x^* = \frac{M}{2P_x}$$

Find $\frac{\partial x^*}{\partial P_x}$

$$\frac{\partial x^*}{\partial P_x} = \frac{-M}{2P_x^2}$$

Plug in our known values to the elasticity formula

$$E_{x^*,P_x} = \frac{-M}{2P_x^2} \cdot \frac{P_x}{x} = \frac{-M}{2P_x^2} \cdot \frac{P_x}{\frac{-M}{2P_x^2}} = \frac{-P_x}{P_x} = -1$$

Elasticity of Demand - Example 2

$$y^* = \frac{M}{P_y} - 10$$

> **Find** $\dfrac{\partial x^*}{\partial P_x}$

$$\frac{\partial y^*}{\partial P_y} = \frac{-M}{P_y^2}$$

> **Plug in our known values to the elasticity formula**

$$E_{y^*,P_y} = \frac{-M}{M - 10P_y}$$

Changes in Quantity Demanded

There are two major driving factors in changes in quantity when price changes:

1. Change that results from a change in the relative price of the two goods $\longrightarrow$ **Substitution Effect**
2. Change that results from a change in the purchasing power of the consumer's income $\longrightarrow$ **Income Effect**

However, we **cannot** observe both of these in the real world

We observed the **combined** effect of these two

So we have:

$$\text{Total Effect} = \text{Substitution Effect} + \text{Income Effect}$$

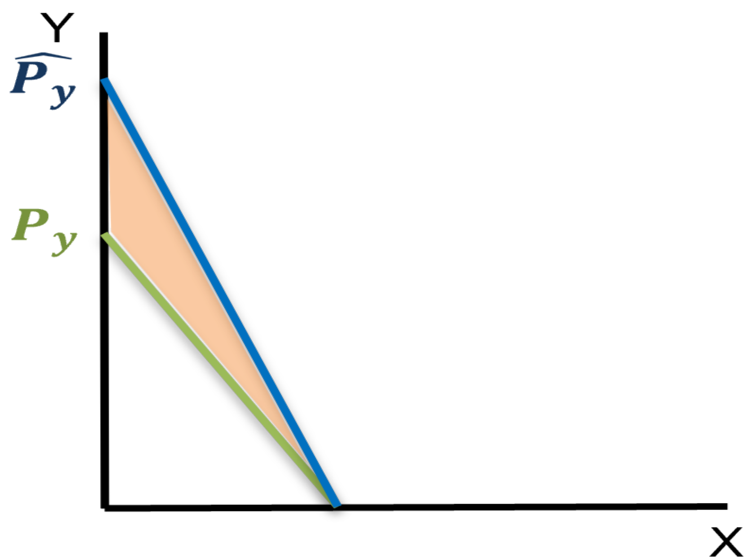
Substitution and Income Effect

Substitution effect

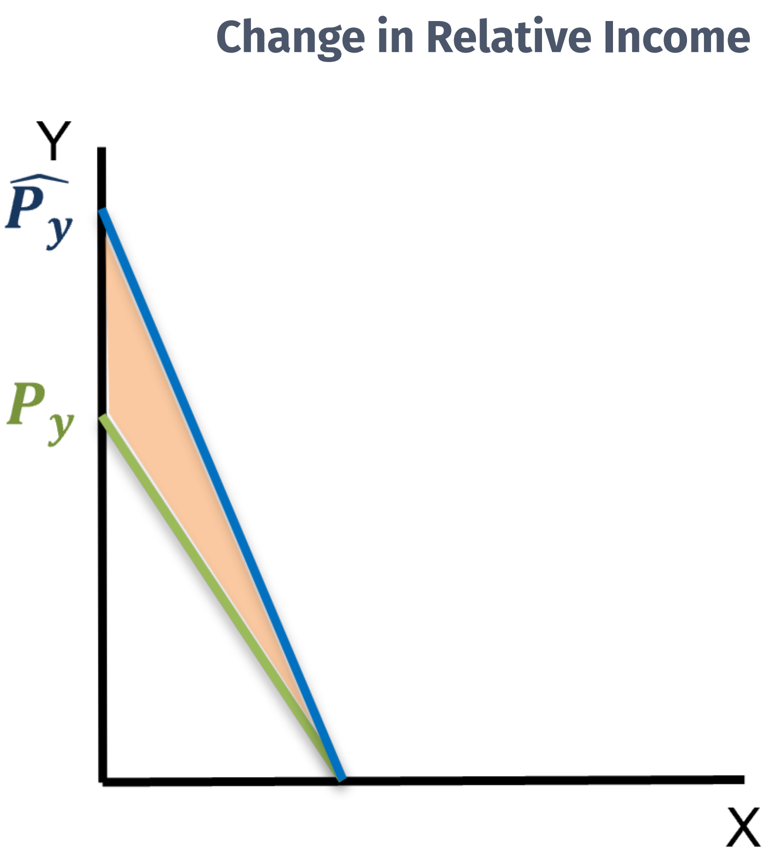
It is the change in bundle resulting from a change in the relative price of two goods

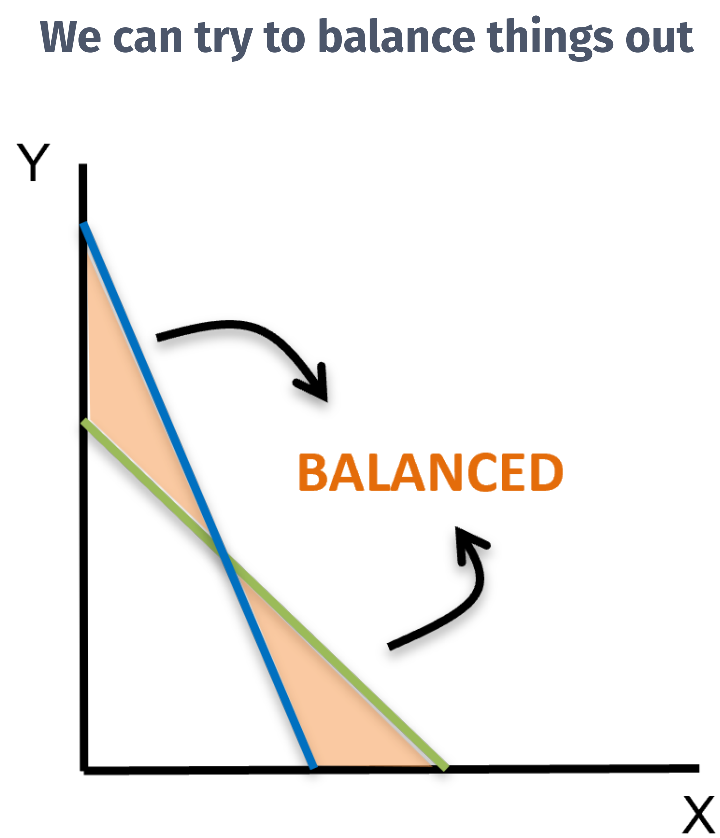
The relative price of both goods is simply the slope of the Budget Constraint

- Suppose that P_y decreases by some ϵ so that $\hat{P}_y = P_y - \epsilon$
  - How would this change the Budget Constraint?
  - $\text{Slope: } \dfrac{-P_x}{P_y} \quad \rightarrow \downarrow P_y \quad \rightarrow \quad \text{Slope gets Steeper}$



Forcing No Change in Relative Income





> The goal is to find where the consumer **does not** feel wealthier or poorer given the change in relative prices

Substitution Effect

> **We want to find where the consumer feels indifferent after price changes?**

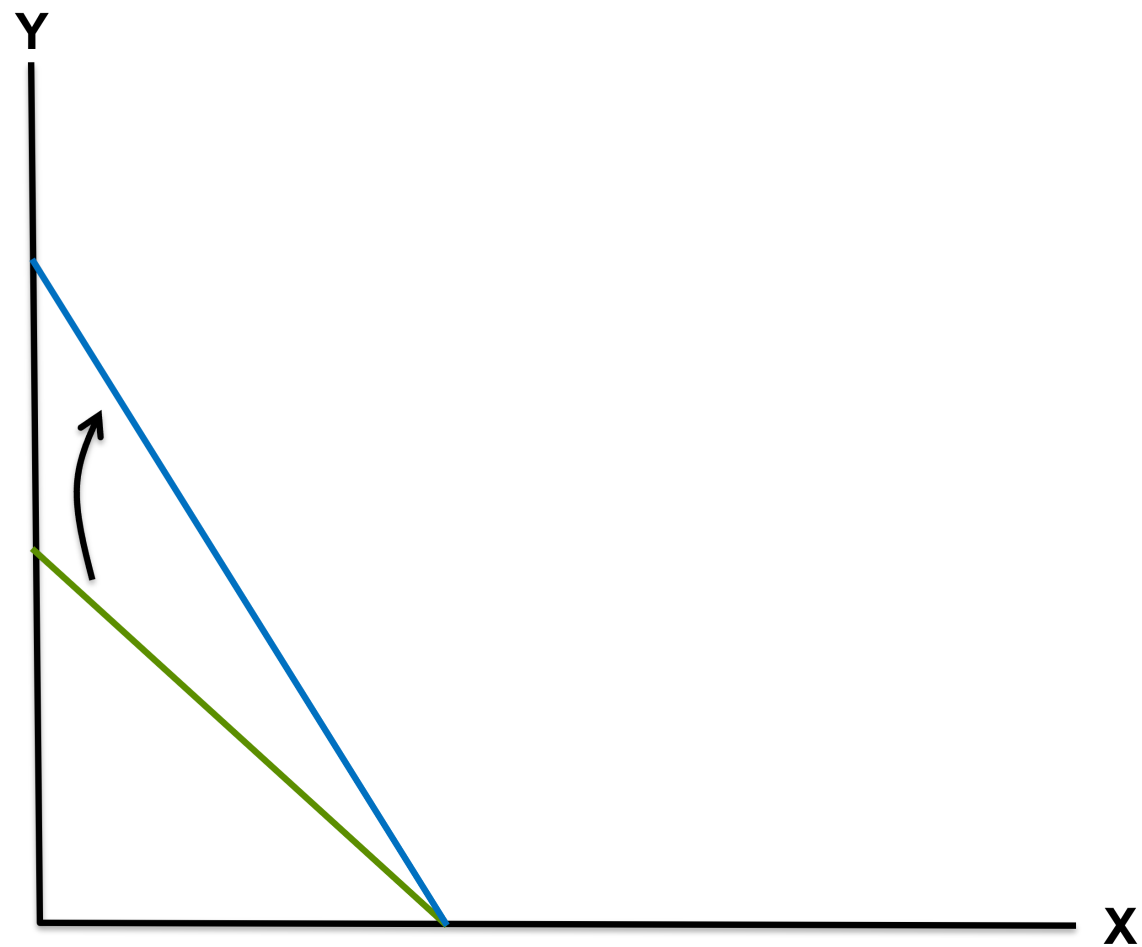
- We need to find a bundle on the same **Indifference Curve** but using a new Budget Constraint

Finding the Substitution Effect

1. Find the **New Budget Constraint**
2. Find the **Hypothetical Budget Constraint**
3. Find the **Optimal Bundle under Original Utility and Hypothetical BC**
4. Find the **Difference between Hypothetical and Original Bundles**

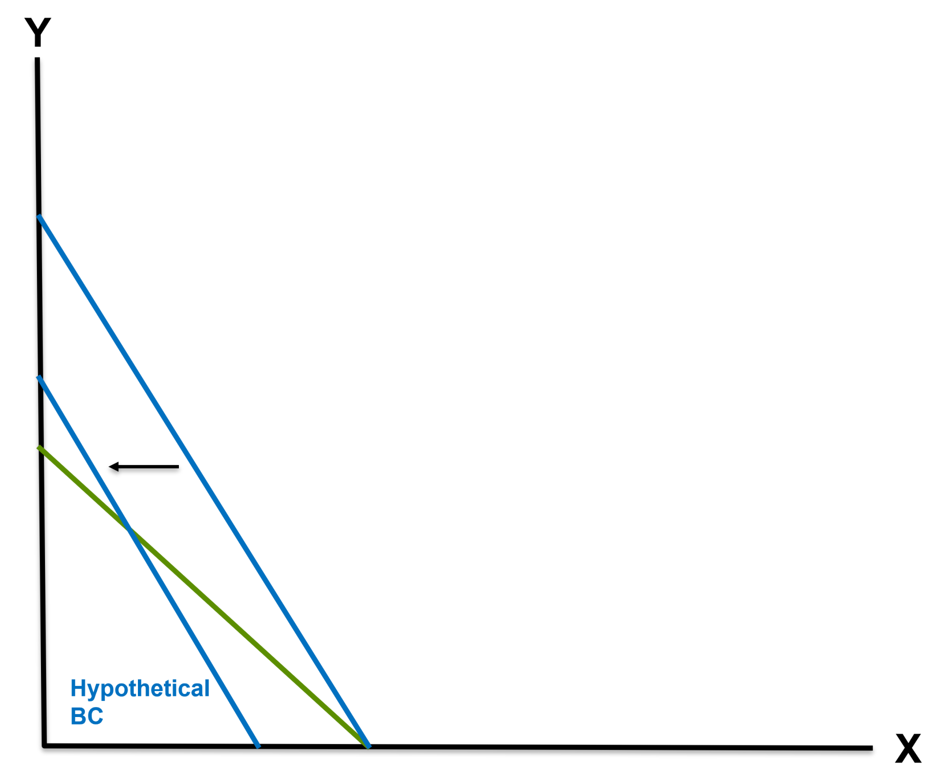
Finding the Substitution Effect - Step 1

Let's say P_y decreased. Find the New Budget Constraint



Finding the Substitution Effect - Step 2

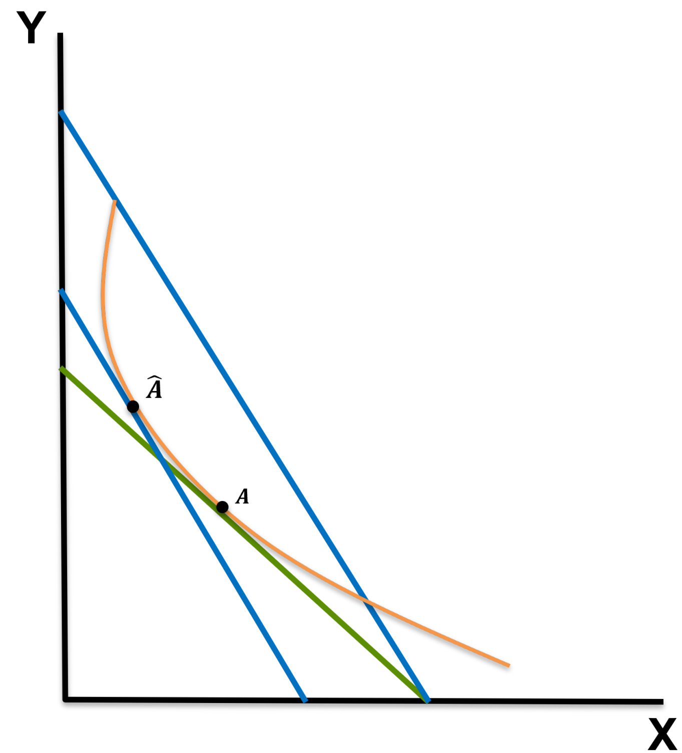
Find the Hypothetical Budget Constraint



We figuratively grab the new BC and shift it over

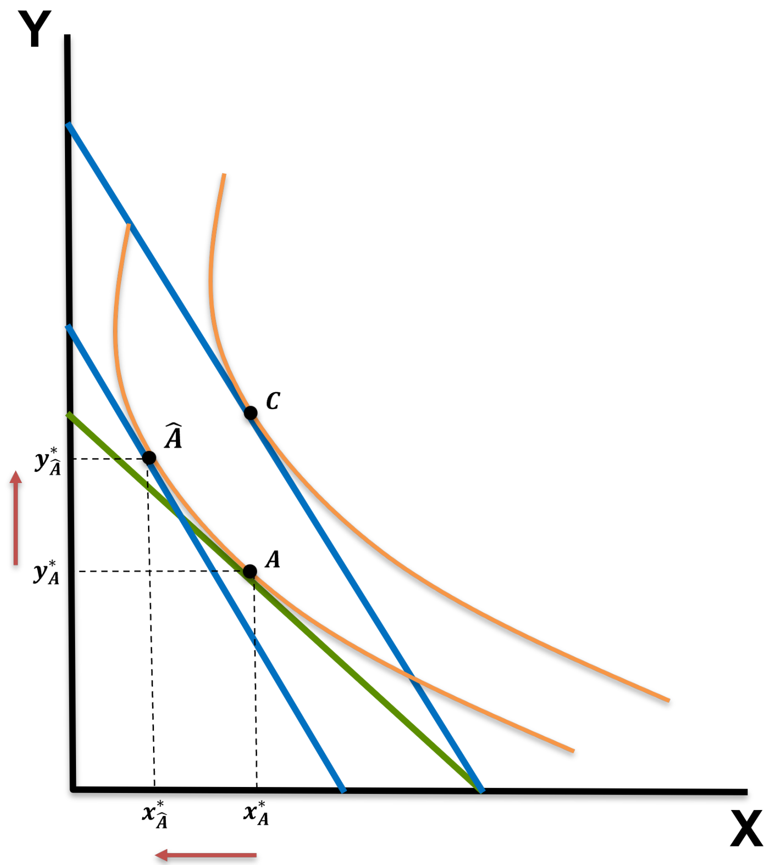
Finding the Substitution Effect - Step 3

Find the Optimal Bundle under Original Utility and Hypothetical



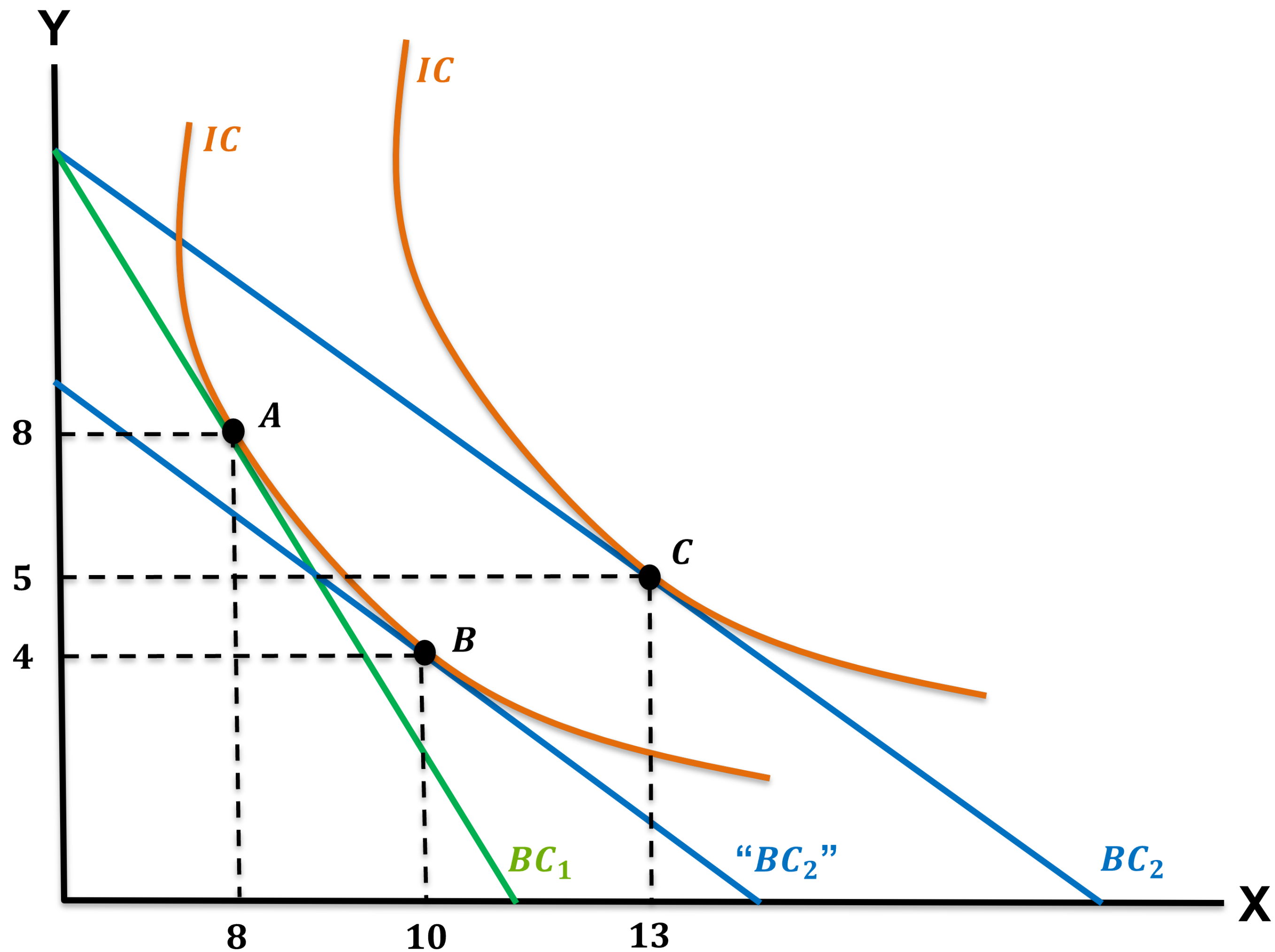
Finding the Substitution Effect - Step 4

Find the Difference between Hypothetical and Original Bundles



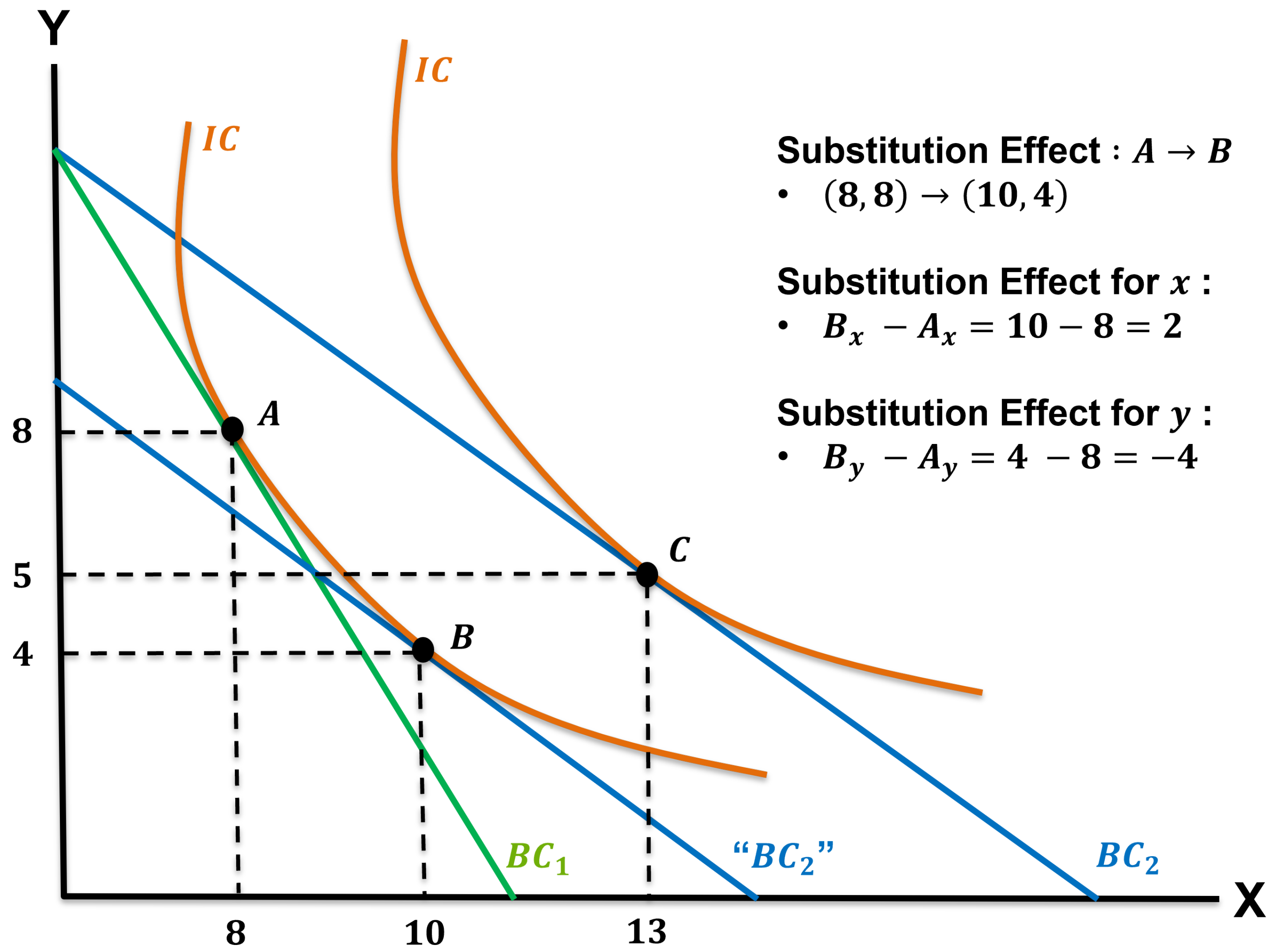
Substitution Effect - Example

Find the Substitution Effect of the following graph



Substitution Effect - Example Solution

Find the Substitution Effect of the following graph



Substitution Effect : $A \rightarrow B$

- $(8, 8) \rightarrow (10, 4)$

Substitution Effect for x :

- $B_x - A_x = 10 - 8 = 2$

Substitution Effect for y :

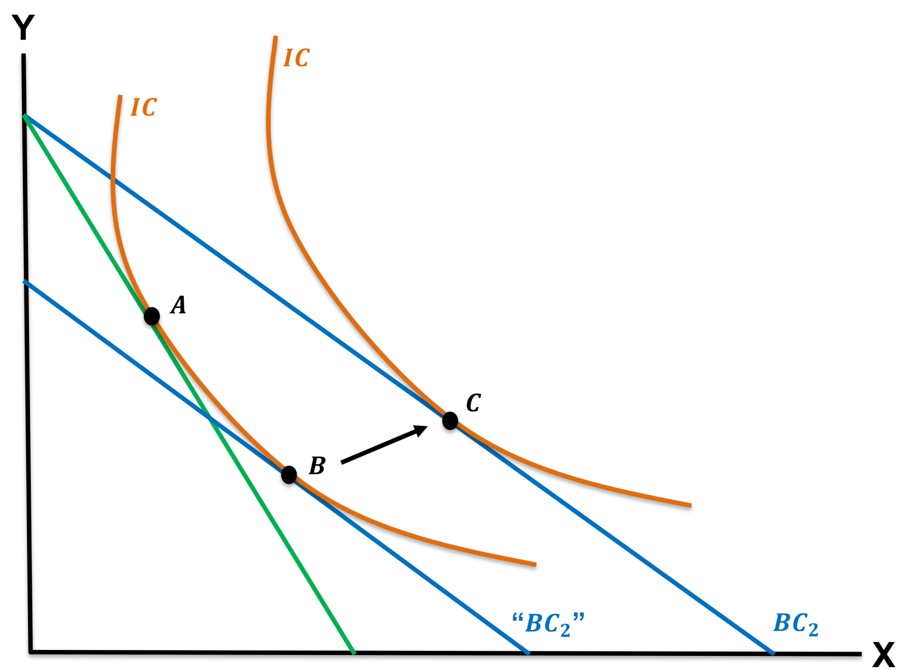
- $B_y - A_y = 4 - 8 = -4$

Income Effect

It is the change in bundle resulting from a change in the purchasing power of the consumer's income

- The relative income (purchasing power) can change even when income does not

> **We want to find the change in consumption from an increase in purchasing power if the relative prices were always at the new ratio**



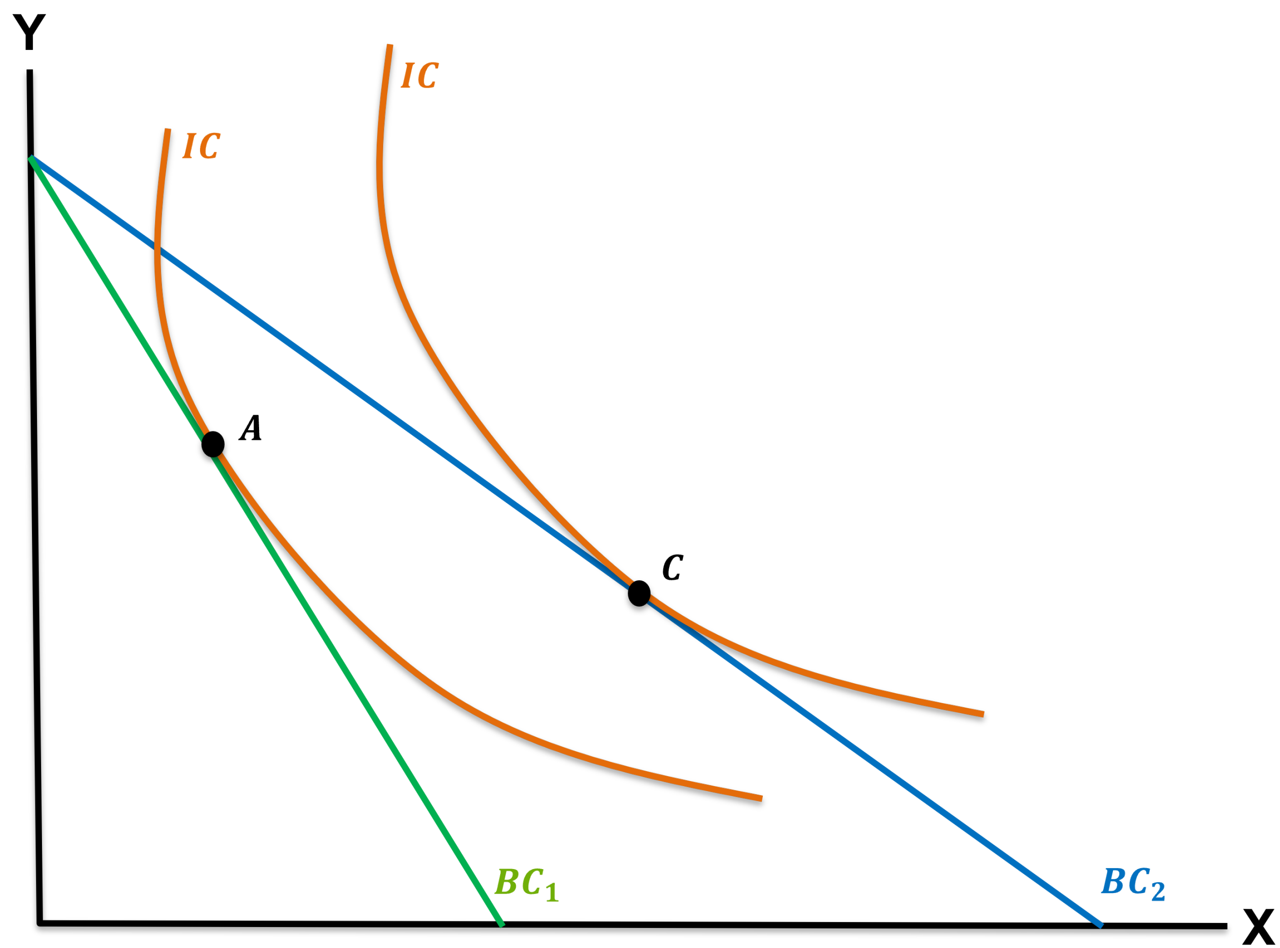
Finding the Income Effect

1. Find the **New Budget Constraint & the Optimal Bundle**
2. Find the **Hypothetical Budget Constraint**
3. Find the **Optimal Bundle under Original Utility and Hypothetical Budget Constraint**
4. Find the **Difference between Hypothetical and New Bundle**

> **Luckily it is a lot of the same steps so it is not a lot of extra work**

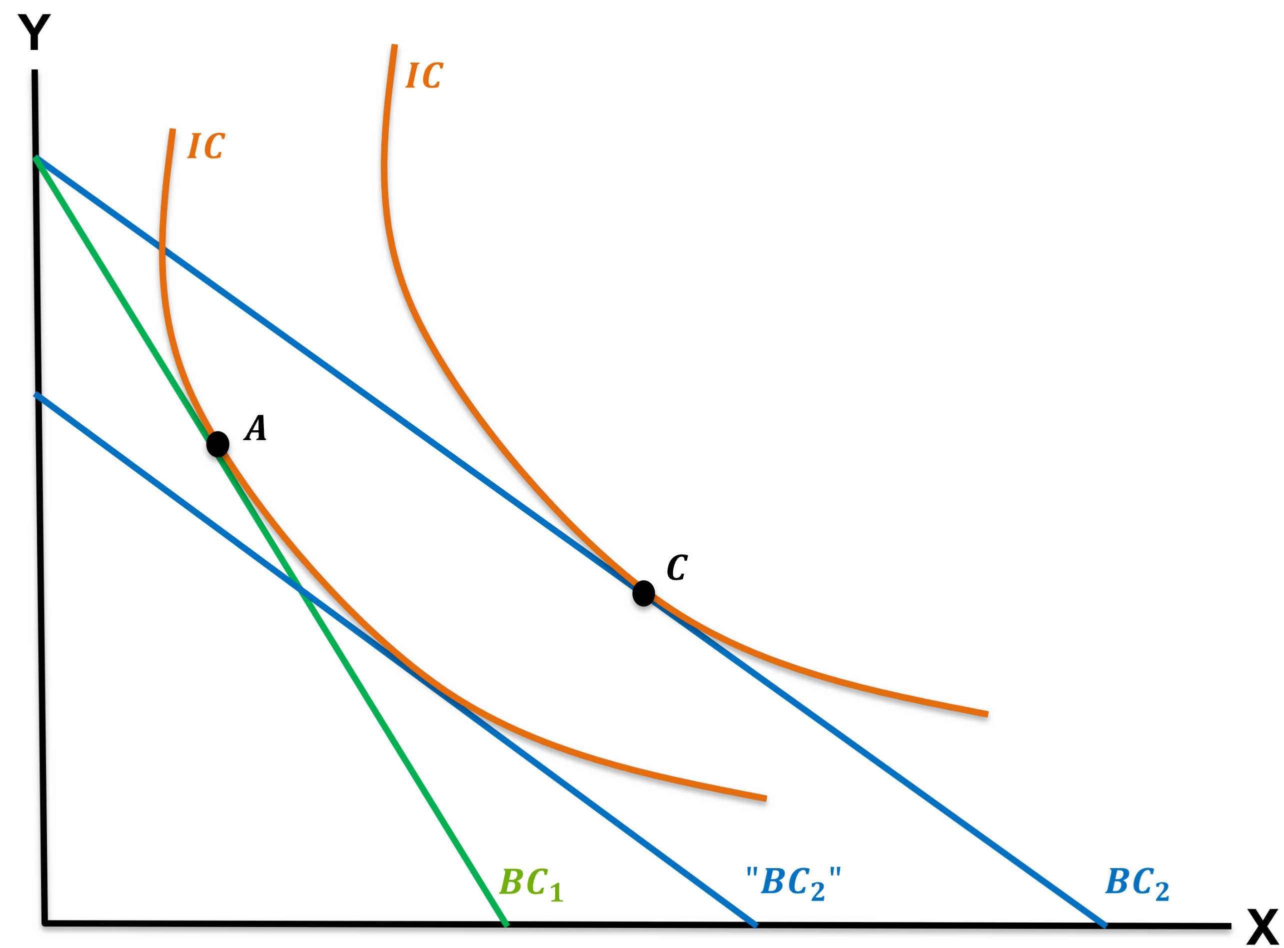
Finding the Income Effect - Step 1

Find the New Budget Constraint & the Optimal Bundle



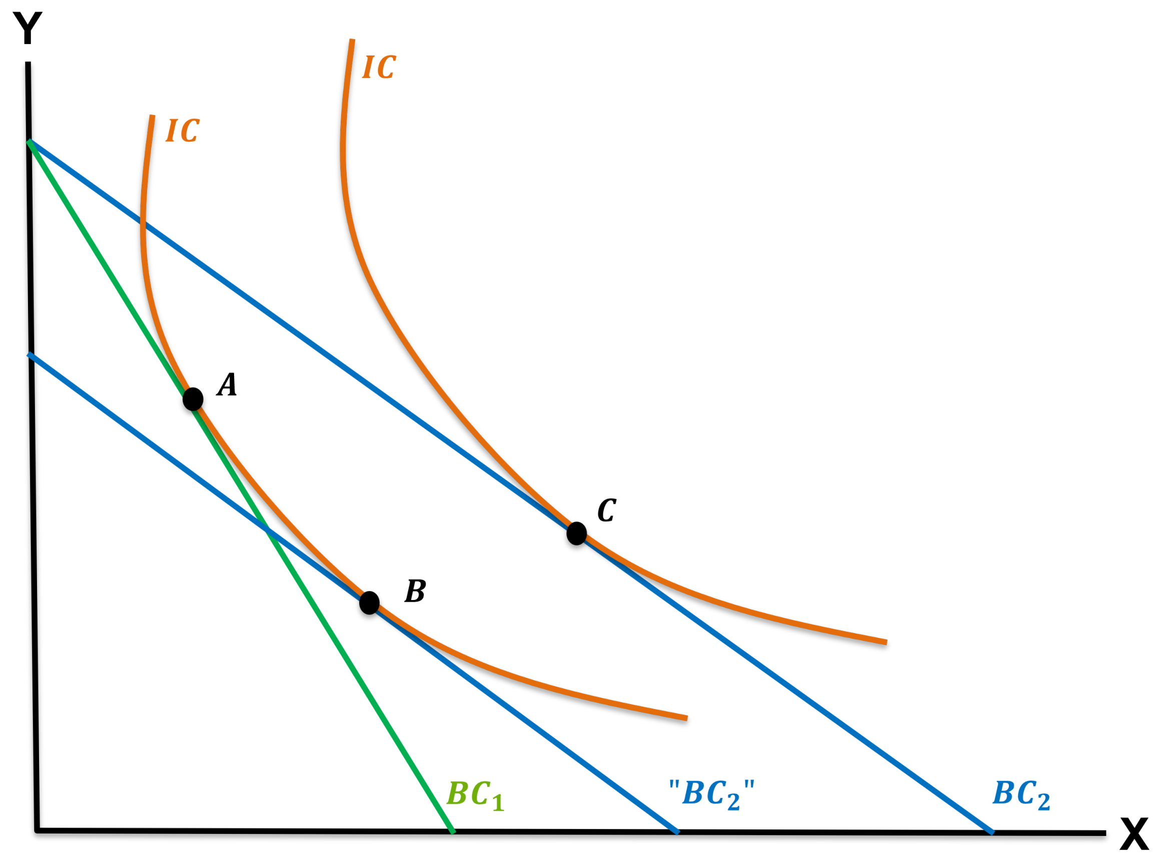
Finding the Income Effect - Step 2

Find the Hypothethical Budget Constraint



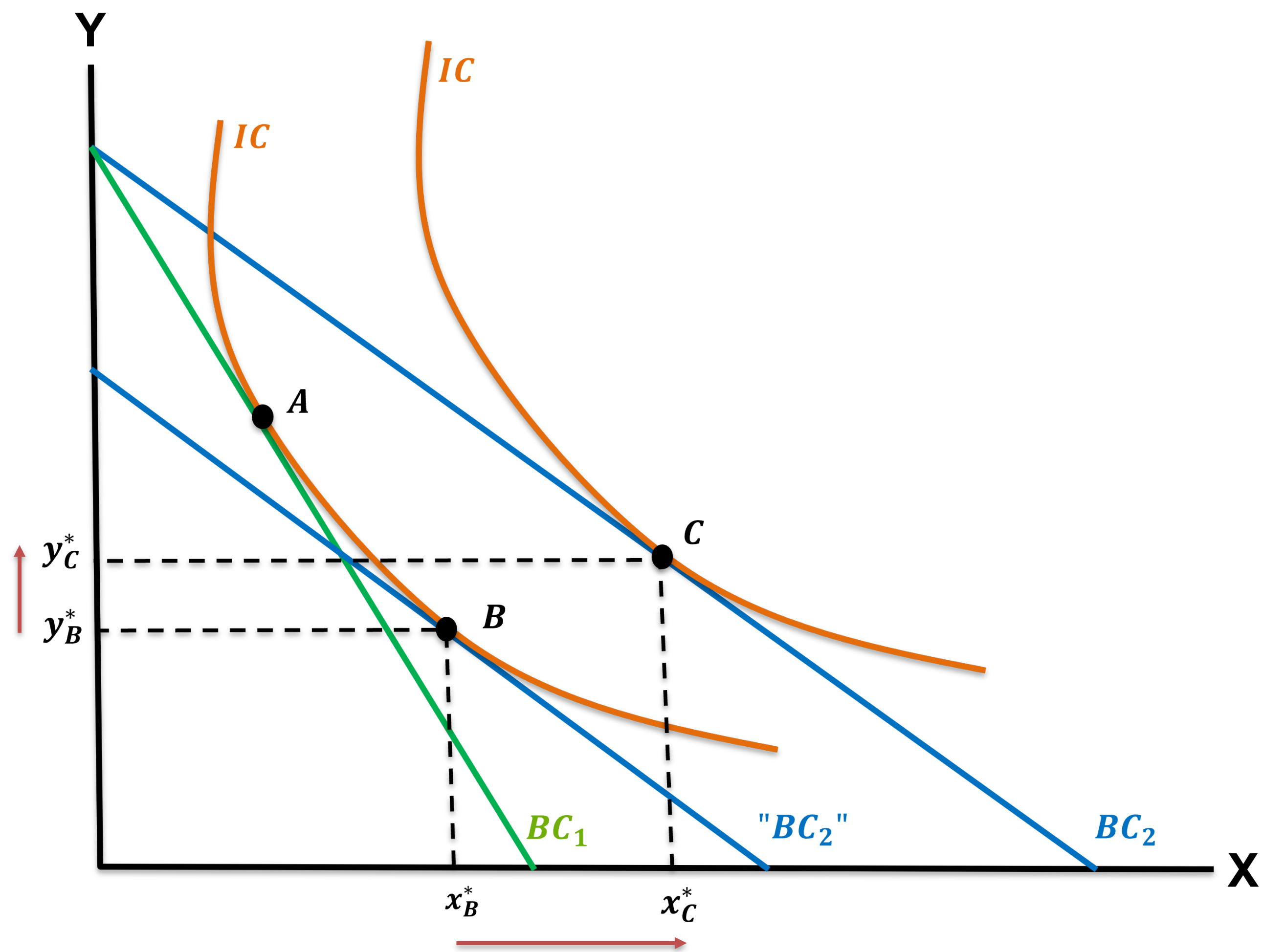
Finding the Income Effect - Step 3

Find the Optimal Bundle under Original Utility and Hypothetical Budget Constraint



Finding the Income Effect - Step 4

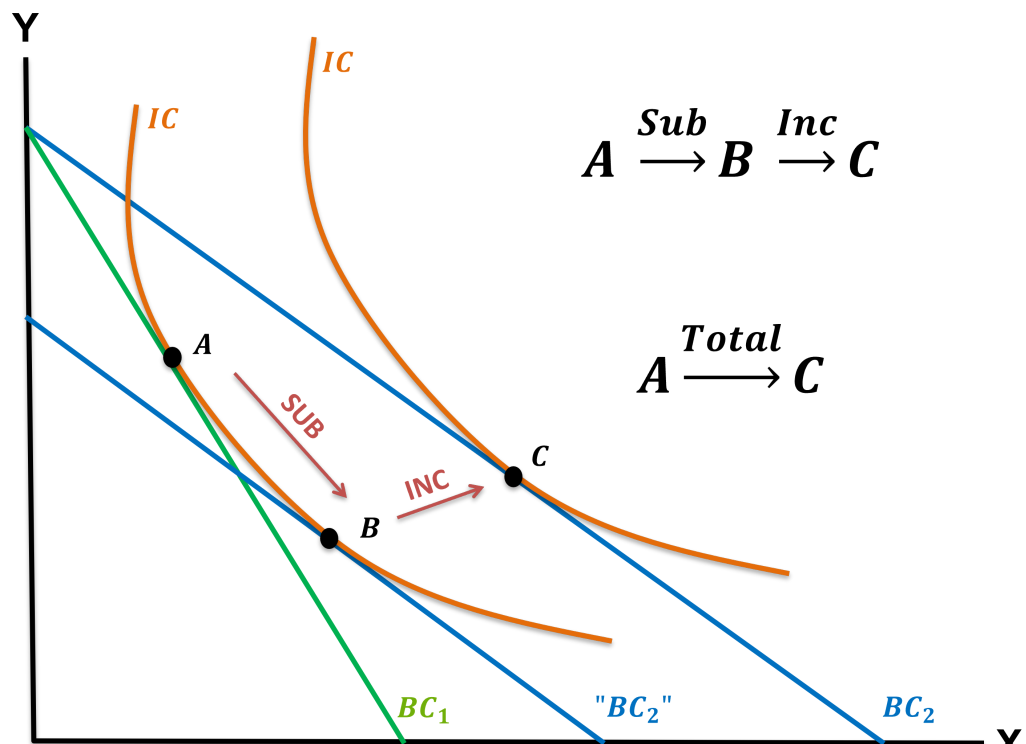
Find the Difference Between Hypothetical and New Bundle



Substitution and Income Effect Summary

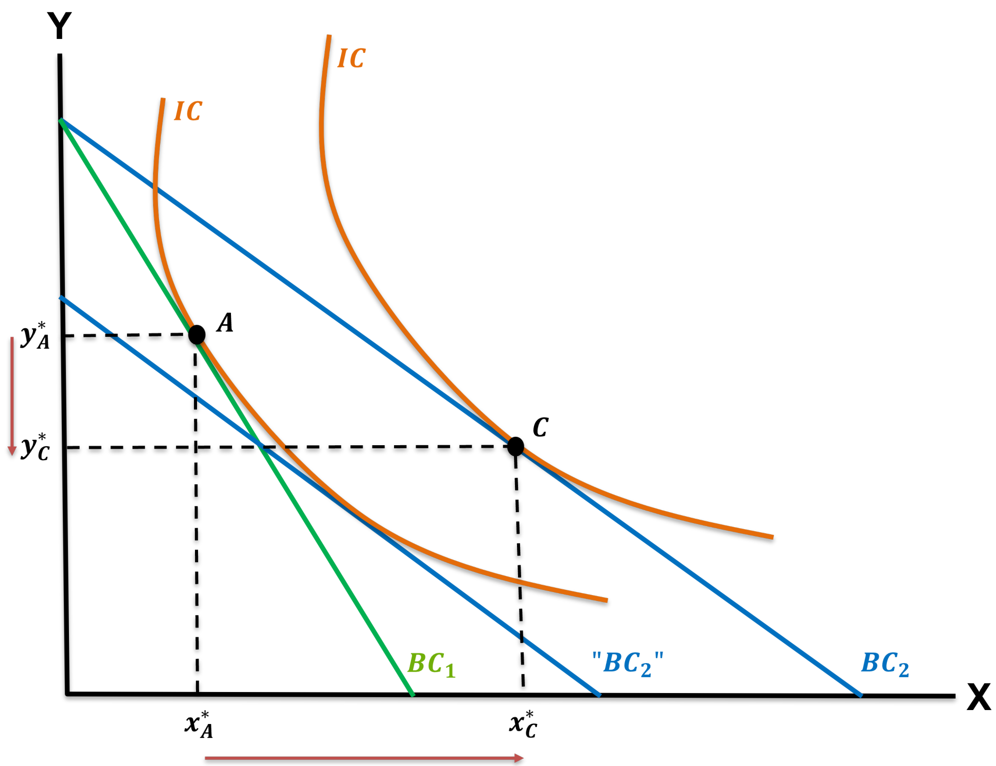
The **Substitution Effect** is the effect of changing the slope of the budget line **without changing the indifference curve you started on**

The **Income Effect** is the difference between the **Substitution Effect** and the **new utility maximizing point**



Total Effect

> **Remember: In the real world we can only observe the Total Effect**



We try to decompose what drives people decisions and that's how we arrive at the Sub & Inc Effects

Changes in Another Good's Price

Other Price Changes

Let's begin by recalling some important points

- We started by noting that **Demand Functions** can be functions of own price, other price, and income

$$y^* = f(P_x, P_y, M)$$

- We have covered own price and income, so now we deal with what happens when the price of the other good changes
  - To do this, we will consider goods that are **Substitutes** and **Complements**

Substitutes

Let's review

- An example of substitute goods are **Coke & Pepsi**
- If the price of a substitute increases, quantity demanded of the other good will ???
  - **Increase**
- Now let's think about derivatives. We say that x is a substitute for y if:

$$\frac{\partial y^*}{\partial P_x} \gtreqless 0 \qquad\qquad \frac{\partial y^*}{\partial P_x} > 0$$

Complements

Let's review

- An example of complement goods are **Coffee Beans & Coffee Creamer**
- If the price of a complement increases, quantity demanded of the other good will ???
  - **Decrease**
- Now let's think about derivatives. We say that x is a complement for y if:

$$\frac{\partial y^*}{\partial P_x} \gtreqless 0 \qquad\qquad \frac{\partial y^*}{\partial P_x} < 0$$

Do All Goods Need to be Perfect Subs. or Complements?

Goods need not be **perfect complements/substitutes**

Their demands can be independent of each other's prices

- Cobb-Douglas goods are an example

$$x^* = \frac{\alpha}{\alpha + \beta} \cdot \frac{M}{P_x} \quad \rightarrow \quad \frac{\partial x*}{\partial P_y} = 0$$

- We will not be doing much with other price changes beyond finding out if goods are substitutes or complements and their **Elasticity of Substitution**

Elasticity of Substitution

$$E_{x^*,P_y} = \frac{\partial x^*}{\partial P_y} \cdot \frac{P_y}{x^*} \qquad \text{and let } \; x^* = \frac{M}{P_y} - P_x$$

Find the Elasticity and interpret it

$$E_{x^*,P_y} = \frac{-M}{P_y^2} \cdot \frac{P_y}{\frac{M}{P_y} - P_x} = \frac{-M}{P_y} \cdot \frac{1}{\frac{M}{P_y} - P_x} = \frac{-M}{P_y \cdot \frac{M}{P_y} - P_y P_x}$$

$$= \frac{-M}{M - P_y P_x} = \left| \frac{-M}{M - P_y P_x} \right| > 1 \Rightarrow \text{Elastic}$$

Individual Demand $\Rightarrow$ Market Demand

Up to Now

All the work we have done until now has been to understand and characterize where demand for a good comes from at an **individual level**

But now, we want to talk about how **markets behave and where prices come from**

To do so, we need a measure of **Aggregate or Market Demand**

Market Demand

We will begin by assuming the following demand function for **ONE** consumer in a market

$$x^* = f(P_x, M) = M - P_x$$

If I said that this market was made up of 2 consumers with **identical utility functions** and incomes how can I get market demand?

- Add them up
- Market Demand (Q_D) is the **sum** of all individual demands

$$Q_D = x^* + x^* = 2x^* \;\rightarrow\; 2(M - P_x) = 2M - 2P_x$$

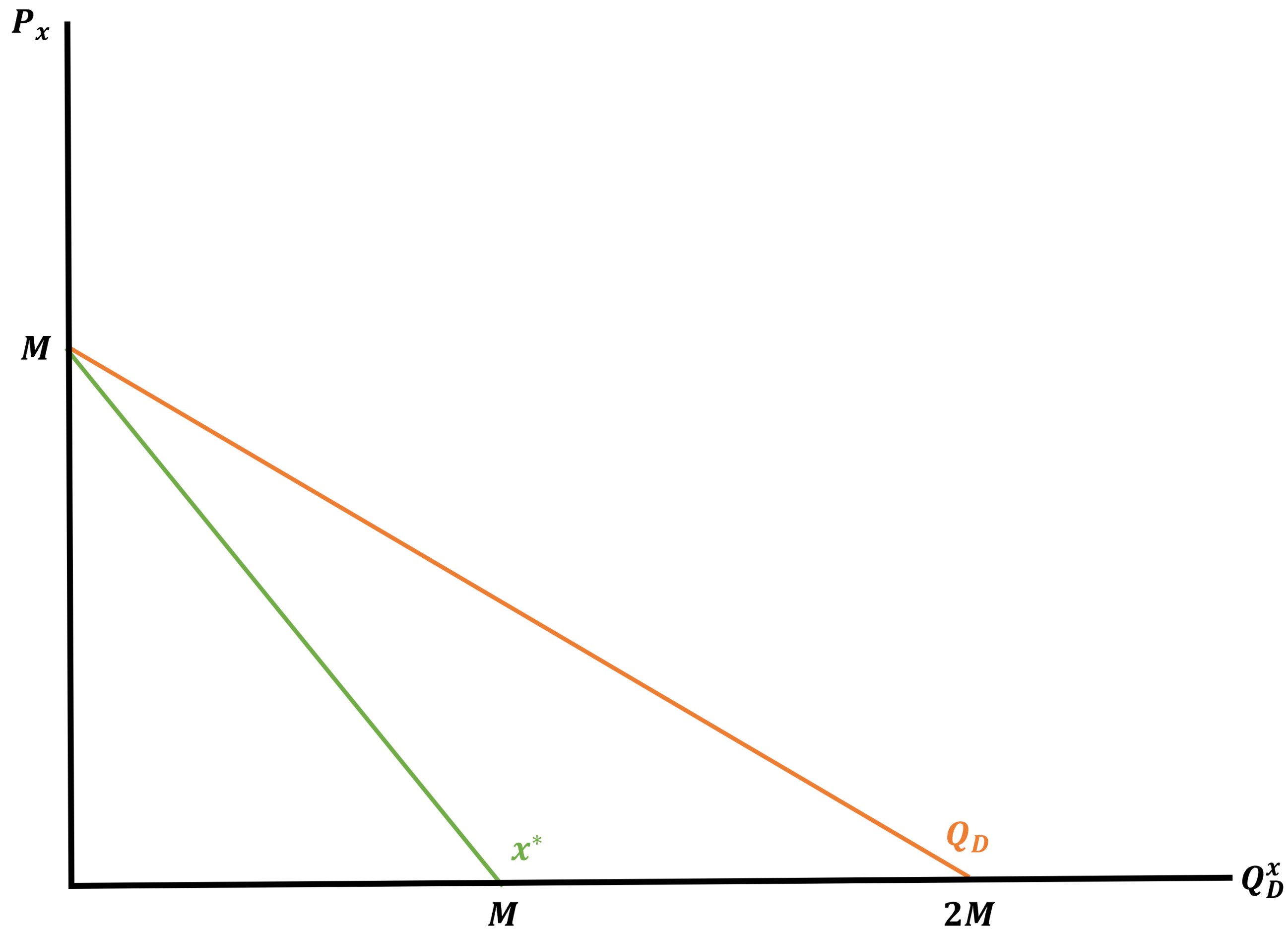
Market Demand - Graphically

Individual Demand

$$x^* = M - P_x$$
$$P_x = M - x^*$$

Market Demand

$$Q_D = 2M - 2P_x$$
$$2P_x = 2M - Q_D$$
$$P_x = M - \frac{Q_D}{2}$$



Let's Make It More Complicated

We will call our 2 agents Jose and Maria

Now we will assume the same demand $x^* = M - P_x$ but they have different incomes (M):

- Jose has $M = 6$
- Maria has $M = 10$

It is still the case that market demand is the sum of both individuals

$$Q_D = x^*_J + x^*_M = 10 - P_x + 6 - P_x = 16 - 2P_x$$

But there is an issue here!

Complicated Market Demand

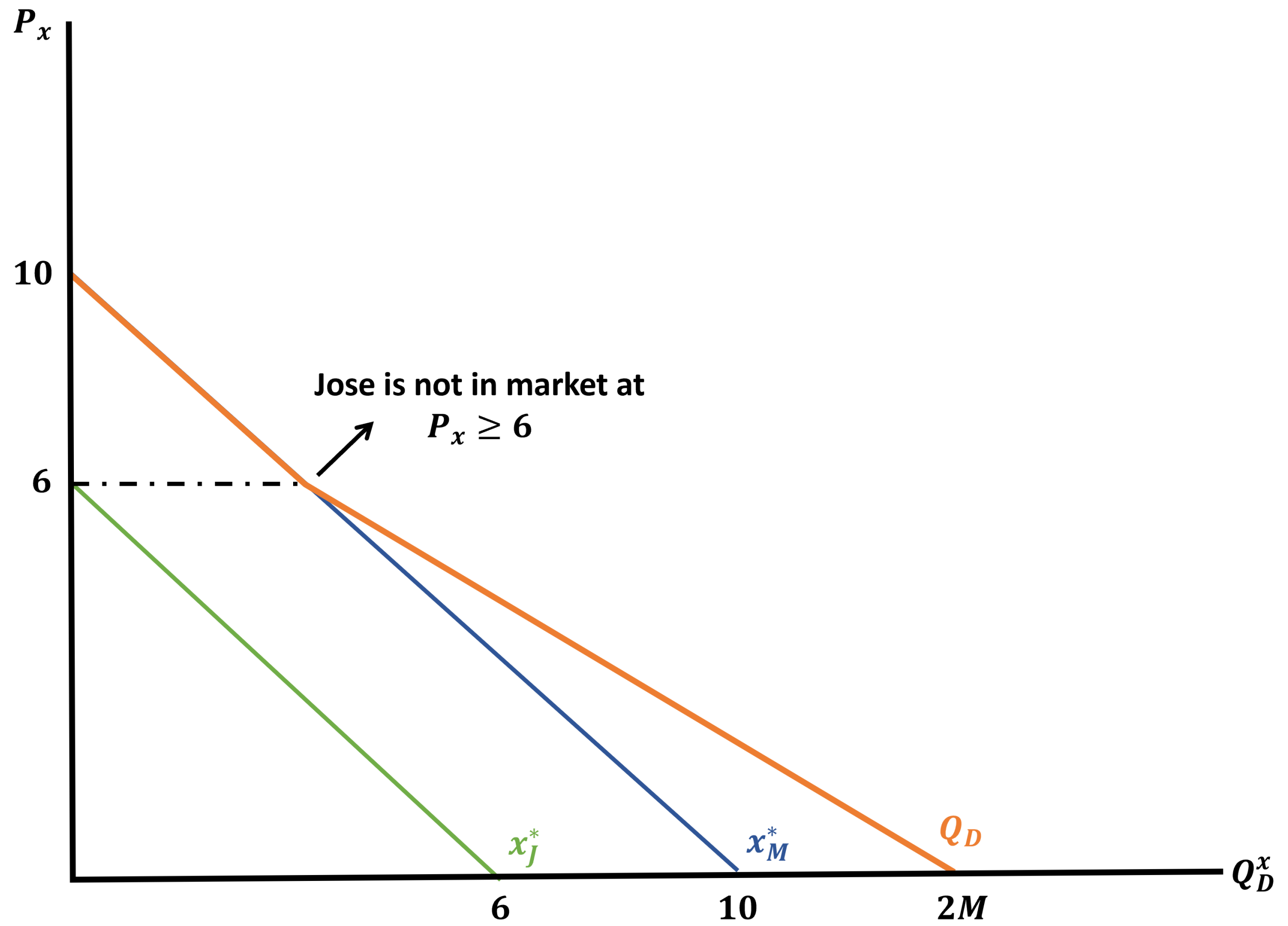
Individual Demands

$$x_J^* = 6 - P_x$$
$$P_x = 6 - x_J^*$$

$$x_M^* = 10 - P_x$$
$$P_x = 10 - x_M^*$$

Market Demand

$$Q_D = 16 - 2P_x$$
$$2P_x = 16 - Q_D$$
$$P_x = 8 - \frac{Q_D}{2}$$



Market Demand

Whenever you add up **linear demand curves** across individuals with **different P_x intercepts** in the demand curves, you will get a small kink in the market demand function

> **Note: The only linear demand curves we have are Perfect Substitutes and Quasi-linear so you should be able to identify them**

Market Demand - Example 2

Let there be 2 agents with the following utilities

$$U_a = ln(x) + y \quad \& \quad U_b = 2ln(x) + y \quad ; \quad P_y = 1$$

> **Find the market demand curve (Hint: Find x_a^* and x_b^*)**

$$x_a^* \;:\; MRS = \frac{P_x}{P_y} \rightarrow \frac{MU_x}{MU_y} = \frac{P_x}{1} \rightarrow \frac{1}{x_a^*} = P_x \rightarrow x_a^* = \frac{1}{P_x}$$

$$x_b^* \;:\; MRS = \frac{P_x}{P_y} \rightarrow \frac{MU_x}{MU_y} = \frac{P_x}{1} \rightarrow \frac{2}{x_b^*} = P_x \rightarrow x_b^* = \frac{2}{P_x}$$

$$Q_D \;:\; x_a^* + x_b^* = \frac{1}{P_x} + \frac{2}{P_x} = \frac{3}{P_x} \rightarrow Q_D = \frac{3}{P_x} \rightarrow P_x = \frac{3}{Q_D}$$

Market Demand - Example 2 Graph

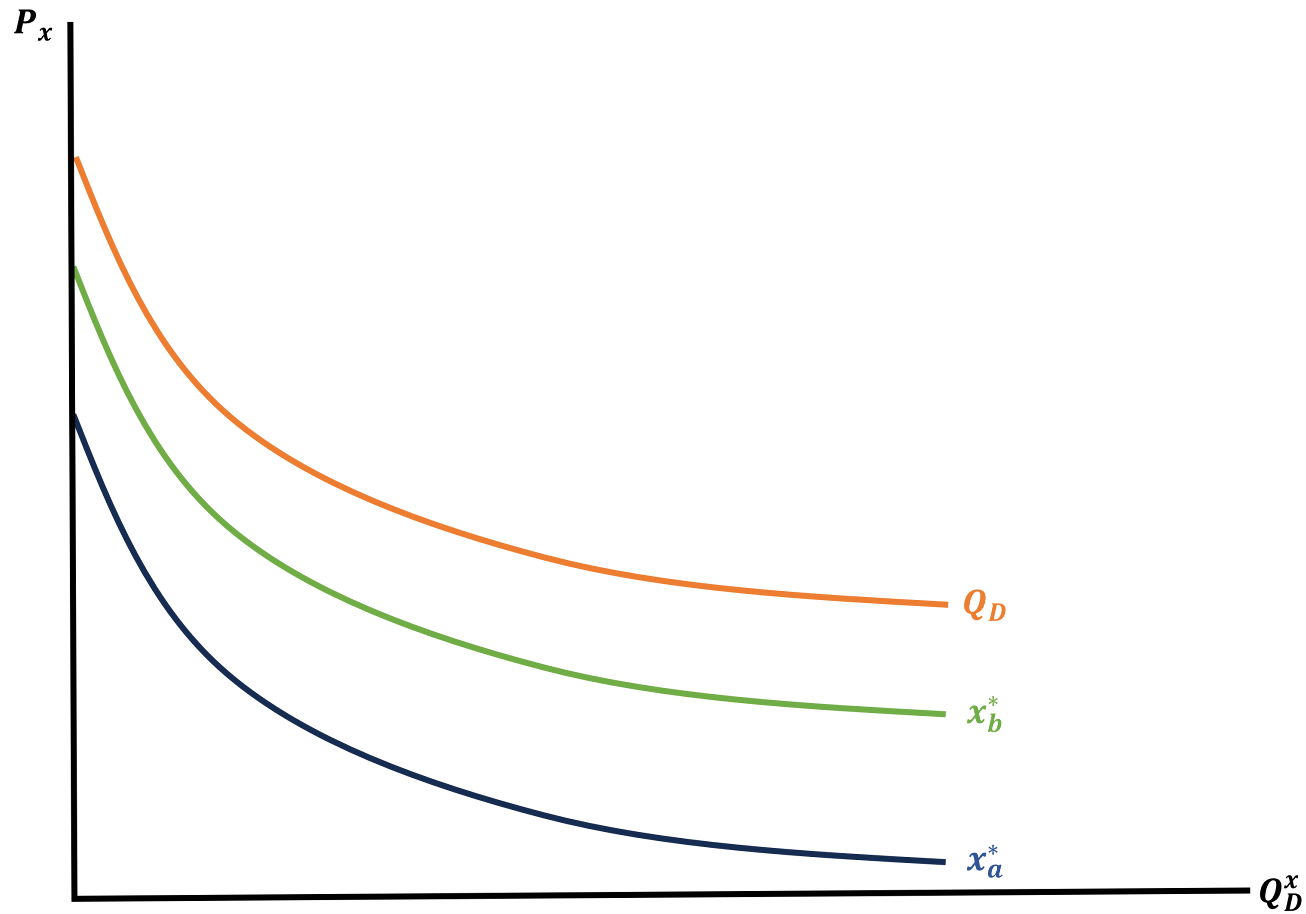
Individual Demand Functions

$$P_x = \frac{1}{x_a^*}$$

$$P_x = \frac{2}{x_b^*}$$

Market Demand Function

$$P_x = \frac{3}{Q_D}$$



Consumer Theory

All we have seen up to now is the basics of **Consumer Theory**

A quick example of an application can be:

- Let's say you are interested in the demand for energy drinks
- You build utility functions for a couple different groups in the market that you believe represent their preferences for energy drinks and other consumption
- You solve the maximization problem and understand how the optimal solutions change when inputs change
- Then you add up the solutions across groups in the market to measure the relationsihp between price and demand