

# Theories of Growth & Development

## EC 390 - Development Economics

Jose Rojas-Fallas

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# Goals

- Look at economic **theories/models** to see what they can say about:
  - What can be done to improve a country's economic growth
  - What changes to expect as a country develops
- We will begin with **growth models** → Factors that influence **economic growth**

# Harrod-Domar Growth Model

# Model 01 - Harrod-Domar

## Savings drive growth

- The **more a country saves**, the more it can **invest in capital**
- To produce output, the only thing we need is **capital**
  - We ignore labor
- Let's start with the **model assumptions**

# Harrod-Domar Assumptions

1. **Net savings** ( $S$ ) is a fixed proportion of **national income** ( $Y$ )
2. A fixed amount of **Capital** ( $K$ ) is need to produce output
3. All new **Investments** ( $I$ ) is used to increase the **Capital Stock** ( $K$ )
4. The **savings-investment** market **clears**

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1. **Net savings** ( $S$ ) is a fixed proportion of **national income** ( $Y$ )

$$S = sY$$

$$0 \leq s \leq 1$$

2. A fixed amount of **Capital** ( $K$ ) is needed to produce output
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# Harrod-Domar Assumptions

1. **Net savings** ( $S$ ) is a fixed proportion of **national income** ( $Y$ )
2. A fixed amount of **Capital** ( $K$ ) is need to produce output
  - **Capital** is a fixed proportion of output where  $c$  is a **capital-output ratio**

$$K = cY$$

$$\Rightarrow c = \frac{K}{Y}$$

$$\Rightarrow \Delta K = c\Delta Y$$

EC390, Lecture 02 | Theories

3. All new **Investments** ( $I$ ) is used to increase the **Capital Stock** ( $K$ )

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$$I = \Delta K$$

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4. The **savings-investment** market **clears**
  - All savings are used as investment

$$S = I$$

# Harrod-Domar Growth Model

We are after the **Growth Rate of Income (or production)**

$$\frac{\Delta Y}{Y}$$

- There are only **4 equations**
- **Algebra** is easy once you know how and where you are going

# Harrod-Domar Growth Model

## Our 4 equations

1.  $S = sY$

2.  $S = I$

3.  $I = \Delta K$

4.  $\Delta K = c\Delta Y$

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Show that

$$sY = c\Delta Y$$

$$sY = S \Rightarrow sY = I \Rightarrow sY = \Delta K \Rightarrow sY = c\Delta Y$$

# Harrod-Domar: Growth Rate of Output

Recall we want to find the **Growth Rate of Income**  $\frac{\Delta Y}{Y}$

We can rearrange  $sY = c\Delta Y$  to get it

$$\frac{\Delta Y}{Y} = \frac{s}{c}$$

- This states that the **Rate of Growth of GDP** is determined by the **net national savings ratio** ( $s$ ) and the **national capital-output ratio** ( $c$ ), at the same time
  - **Positively** related to the savings ratio
  - **Negatively** related to the economy's capital-output ratio

# Harrod-Domar: Growth Rate of Output

$$\frac{\Delta Y}{Y} = \frac{s}{c}$$

Let's break it down a bit

$\frac{1}{c}$  measures the **efficiency of capital use**

- The lower the value of  $c$  (the more efficient) that an economy runs at, the greater the output that can be gained from additional investment
  - A higher  $c$  means less output of  $Y$
  - If  $c$  increases, then the **growth rate decreases**

# Harrod-Domar: Growth Rate of Output

$$\frac{\Delta Y}{Y} = \frac{s}{c}$$

$s$  is the economy's **saving rate**, which influences the level of investment

- A higher  $s$  implies a higher investment level
- If  $s$  increases, then the **growth rate increases**

# Harrod-Domar Lessons

In other words, the **Rate of Growth** depends as much on the **efficiency of capital investments** as the **amount of capital invested**

In its simplest form, a country that wants to **speed up development**:

1. **Save more**
2. Build more **efficient capital**

# Harrod-Domar Lessons

In other words, the **Rate of Growth** depends as much on the **efficiency of capital investments** as the **amount of capital invested**

In its simplest form, a country that wants to **speed up development**:

## 1. Save more

- Difficult for individuals in **developing countries** → Why?
- Can be helped by **Foreign Aid/Investment**

## 2. Build more **efficient capital**



# Harrod-Domar Lessons

In other words, the **Rate of Growth** depends as much on the **efficiency of capital investments** as the **amount of capital invested**

In its simplest form, a country that wants to **speed up development**:

1. Save more

2. Build more **efficient capital**

- **Technology** helps with making capital more efficient

# Harrod-Domar Simple Example

- In 2011, Indonesia had a **capital-output** ratio of 4

$$\frac{K}{Y} = c = 4$$

If we want a **growth rate of 6%**, **Harrod-Domar** tells us that Indonesia needs a **savings rate** of?

$$\frac{\Delta Y}{Y} = \frac{s}{c} \Rightarrow 6 = \frac{s}{4} \Rightarrow 24 = s$$

# Harrod-Domar Growth Model

With theoretical models, you should always ask yourself:

**What? Why? Huh?**

- **Why** this model?
  - After World War II, much of Europe was destroyed
  - There was a lack of **capital** ( $K$ )
  - The **Marshall Plan** was a large **foreign aid package** from the US to Western Europe
  - The aid package greatly sped up recovery of Europe and boosted economic growth
- **What** does it do?
  - Explains how savings, capital, and output are potentially linked together
- **Huh? That's not realistic**

# Criticisms of Harrod-Domar

**No model is perfect**

**But good models help explain a small part of life**

**But these are not without proper criticism:**

- Overly simplified
- No **population growth**
- No **technology change**
- The  $K = cY$  assumption is concerning
  - Assumes that turning capital into output is easy
  - Assumes **constant returns to capital**
- Policy enacted based on the model did not increase economic growth significantly

# Solow Growth Model

# Model 02 - Solow Growth Model

Now we can add some **important features**

- Population growth
- Technological change
- Emphasizes looking at outcomes in **per worker** terms

# Solow Growth Model Assumptions

1. **Output per worker**  $y$  depends only on the amount of **capital per worker**  $k$
2. Every worker **saves a proportion**  $s$  of their income
3. **Population grows** at rate  $n$
4. **Capital depreciates** at rate  $\delta$
5. **Capital stock** depends on new **investment**

# Solow Growth Model Assumptions

**1. Output per worker  $y$**  depends only on the amount of **capital per worker  $k$**

$$y = f(k)$$

- $f()$  is **increasing** (more capital leads to more output)
- $f()$  is **concave** (decreasing returns to capital)

**2.** Every worker **saves a proportion  $s$**  of their income

**3. Population grows** at rate  $n$

**4. Capital depreciates** at rate  $\delta$

**5. Capital stock** depends on new **investment**



# Solow Growth Model Assumptions

1. **Output per worker**  $y$  depends only on the amount of **capital per worker**  $k$

2. Every worker **saves a proportion**  $s$  of their income

$$0 \leq s \leq 1$$

3. **Population grows** at rate  $n$

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4. **Capital depreciates** at rate  $\delta$
5. **Capital stock** depends on new **investment**
  - Every time period workers save some of their income  $sf(k)$
  - **However**, capital **depreciates** so we lose  $\delta k$
  - The **populations grows** at rate  $n$  so the **capital per worker** gets smaller by  $nk$

# Solow - Change in Capital per Worker

$$\Delta k = sf(k) - nk - \delta k$$

- $\Delta k$ : Growth of capital per worker
- $sf(k)$ : Savings
- $nk$ : Net new workers
- $\delta k$ : Capital depreciation

# Solow - Change in Capital per Worker

$$\Delta k = sf(k) - nk - \delta k$$

- $\Delta k$ : **Growth of capital per worker**
  - The more capital a worker has to work with, the more output that they can produce
  - The change in **capital per worker** depends on the other components
- $sf(k)$ : Savings
- $nk$ : Net new workers
- $\delta k$ : Capital depreciation

# Solow - Change in Capital per Worker

$$\Delta k = sf(k) - nk - \delta k$$

- $\Delta k$ : Growth of capital per worker
- $sf(k)$ : **Savings** → **(Positive)**
  - Each worker saves a proportion of their income and is reinvested into “**capital in the future**”
- $nk$ : Net new workers
- $\delta k$ : Capital depreciation

# Solow - Change in Capital per Worker

$$\Delta k = sf(k) - nk - \delta k$$

- $\Delta k$ : Growth of capital per worker
- $sf(k)$ : Savings
- **$nk$ : Net new workers  $\rightarrow$  (Negative)**
  - Population (workers) grow at a **rate**  $n \geq 0$  (usually very small)
  - As there are more people, there is **less capital per worker** determined by  $nk$
- $\delta k$ : Capital depreciation

# Solow - Change in Capital per Worker

$$\Delta k = sf(k) - nk - \delta k$$

- $\Delta k$ : Growth of capital per worker
- $sf(k)$ : Savings
- $nk$ : Net new workers
- **$\delta k$ : Capital depreciation  $\rightarrow$  (Negative)**
  - Capital requires service (repairs)  $\rightarrow$  We lose  $\delta k$  every period
  - $0 < \delta < 1$



# Solow Growth Model - Steady State

The **Solow Model** allows us to consider a **steady state** level of capital

- **Steady State:** When the economy has fully adjusted and there is no change in some variable
  - The economy is in an **equilibrium** that is **stable**
  - **Output** and **capital per worker** are no longer changing

We want  $k$  to be in **steady state**

- This means that  $\Delta k = 0 \rightarrow$  Because  $k$  is no longer changing
- We call this level of **capital**  $k^*$

# Solow Growth Model - Steady State

When the economy is in **steady state**  $k^*$  we have:

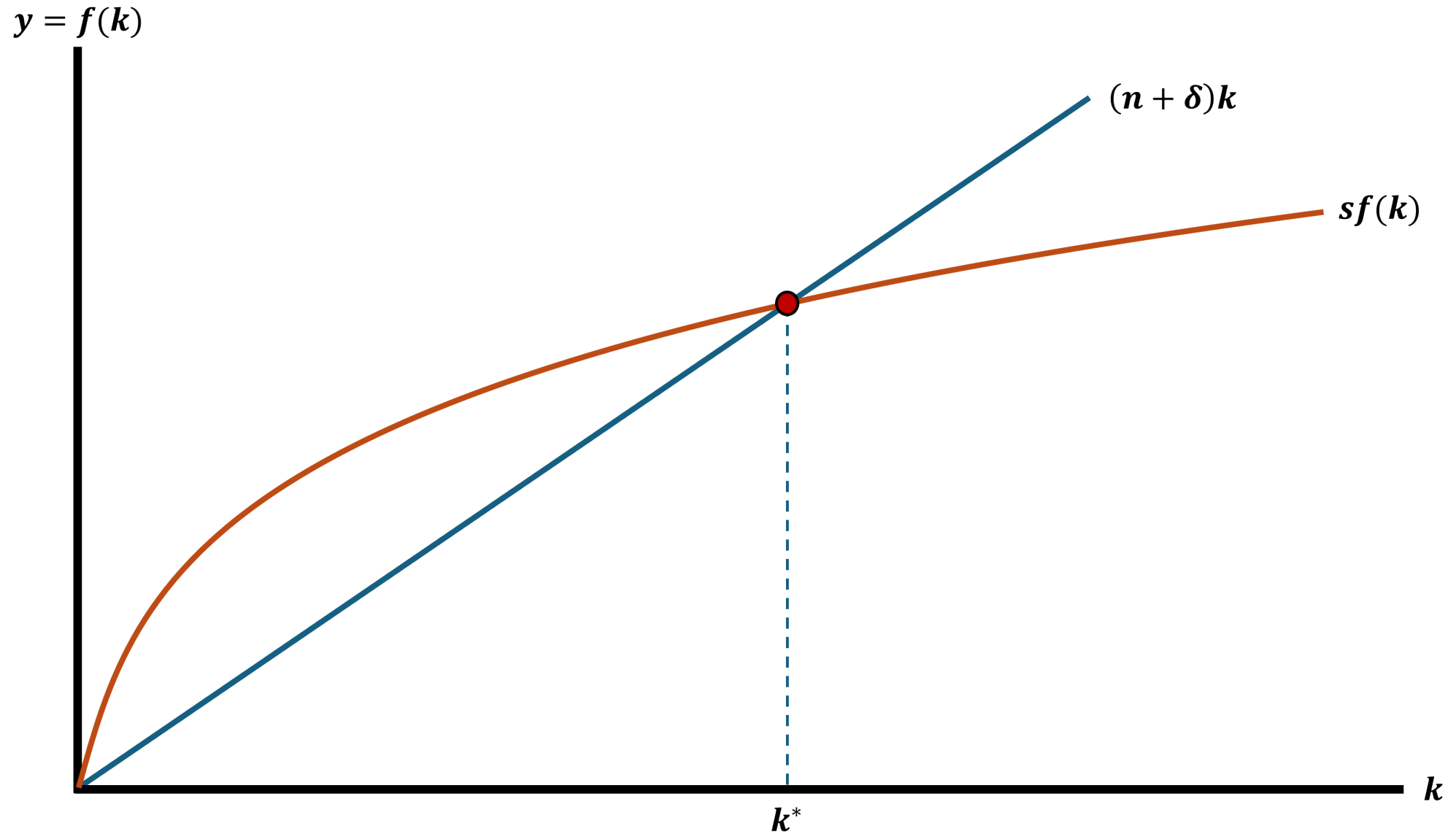
$$\Delta k = 0 = sf(k^*) - (n + \delta)k^*$$

$$sf(k^*) = (n + \delta)k^*$$

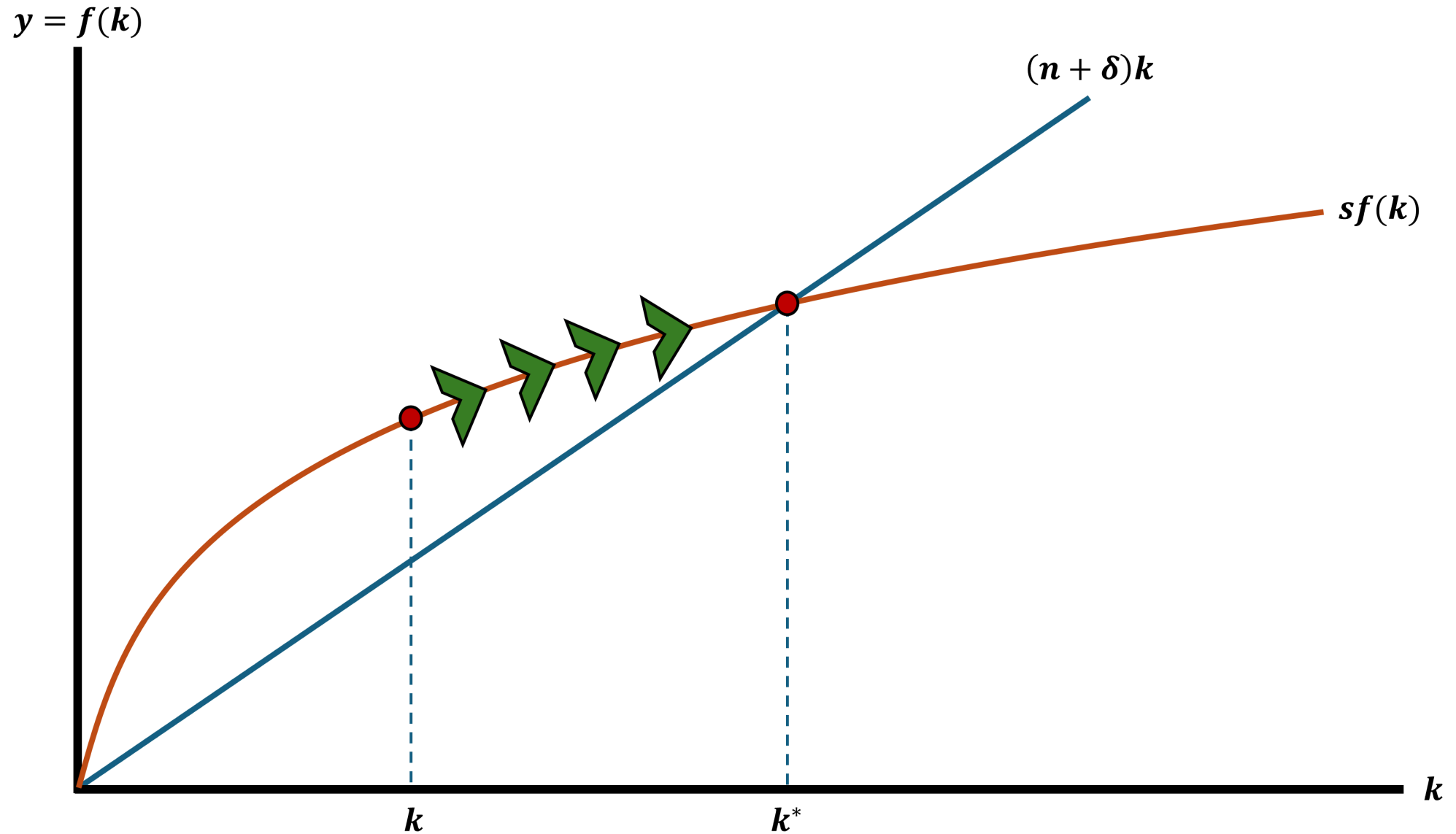
More than the math, I want you to understand this intuitively

- Graphs are great to be able to talk about this

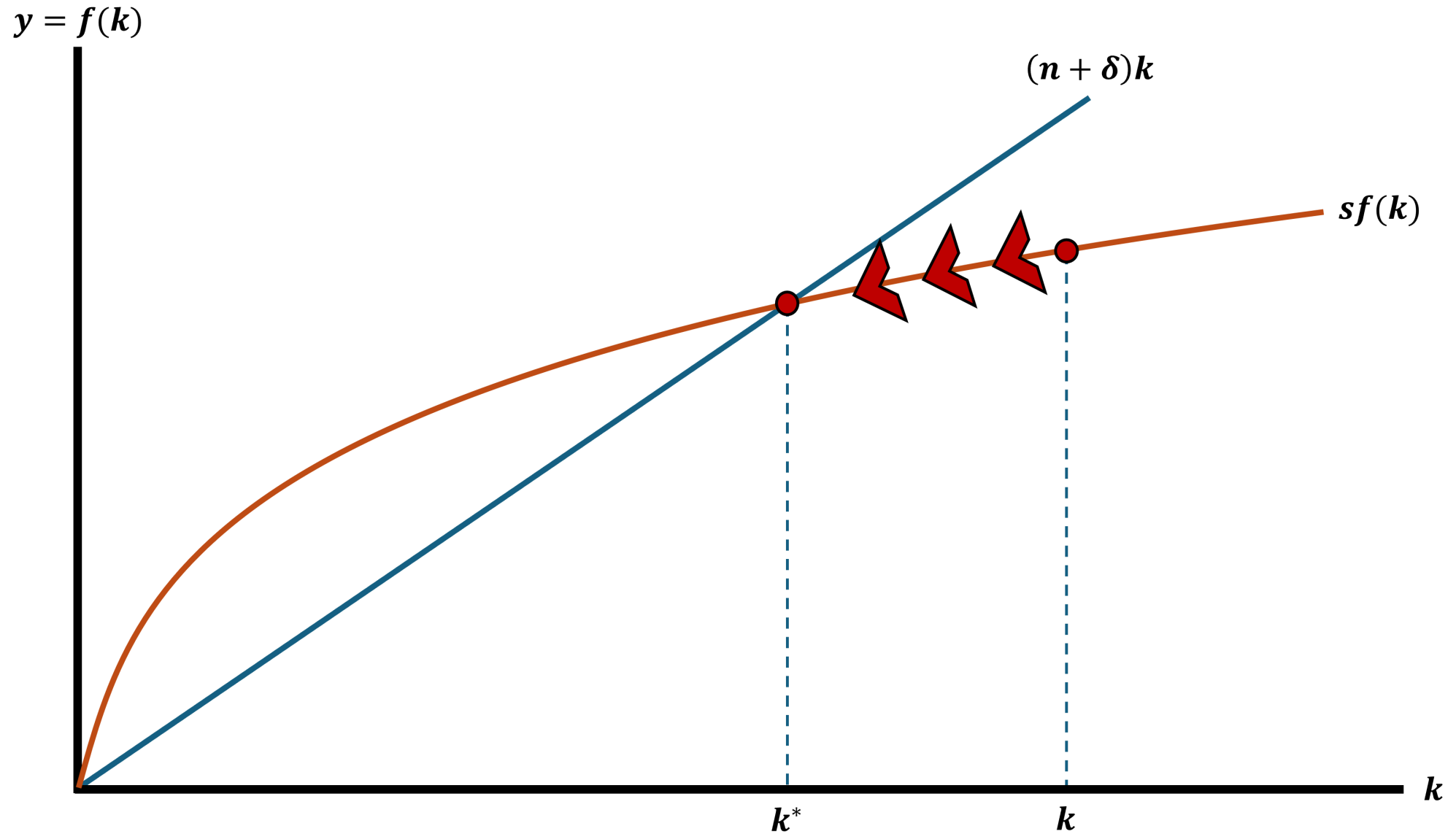
# Steady State



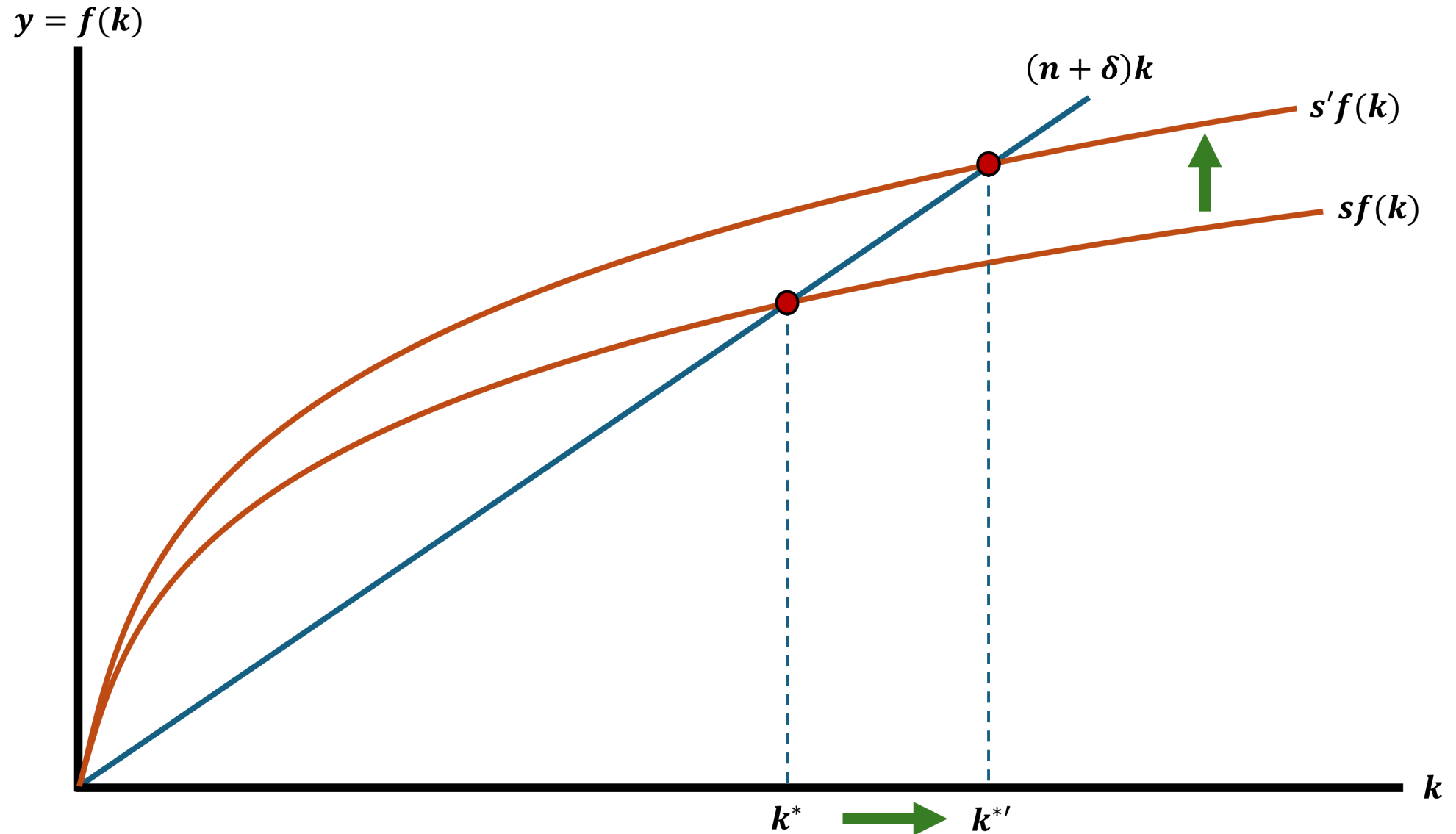
# Out of Steady State



# Out of Steady State



# Increase in Savings Rate



# Solow Growth Model Dynamics

Be aware of all the possible moving pieces in this model

It allows us to **explore the effects of changes in:**

- $s$ : Savings Rates
- $\delta$ : Depreciation Rate
- $n$ : Population Growth Rate

# Solow Growth Model

## What? Why? Huh?

- **Why** this model?
  - It is a simple and intuitive way of understanding how **savings, population growth**, and technology change impacts **long-run economic growth**
- **What** does it do? → Decomposes growth into **two key forces**
  - 1. Capital accumulation** through savings and investment
  - 2. Population growth** as the labor force expands
- **Huh?** Again with this fantasy stuff?



# Criticisms of Solow

- There is a constant savings rate that is **exogenously given**
- Ignores **human capital**
- Predicts too much convergence
  - Economies with the same rates  $(s, n, \delta)$  **should** reach the same

# Structural Change Models

# Lewis Two-Sector Model

Was the general theory of the development process in **surplus-labor developing nations** in the 60s and 70s

- Focuses on **structural transformation** of a primarily **subsistence economy**
  - **Definition:** A major alteration in the industrial composition of any economy
  - Usually we talk about moving from subsistence agricultural sector to manufacturing sector

# Model Set Up

There is an **underdeveloped economy** with:

- Two Sectors
  1. A traditional, overpopulated, rural subsistence sector
  2. A high-productivity, modern, urban industrial sector
- Primary Focus
  - Process of labor transfer/movement between sectors
  - Growth of output and employment in the **industrial** sector

The transfer of labor between sectors and employment growth in the **industrial** sector is due to **expansion in the industrial sector**

# Industrial Sector Expansion

The speed at which the **industrial sector** grows is determined by **investment and capital accumulation** in that sector

- Capitalists make profits in the **industrial sector** above paid **wages**
- Assume that they invest it **all** back into the sector (more capital enters)
- **Capital stock** increases and they hire more labor

## Some questions/doubts arise

- 1.** Where is all the labor coming from? I thought the other side was a **subsistence agricultural sector**?
- 2.** I thought economists didn't believe in positive profits
- 3. Capital** only increases in the industrial sector?

# Subsistence Agricultural Sector

There are two primary assumptions to consider

## 1. There is a **surplus of labor**

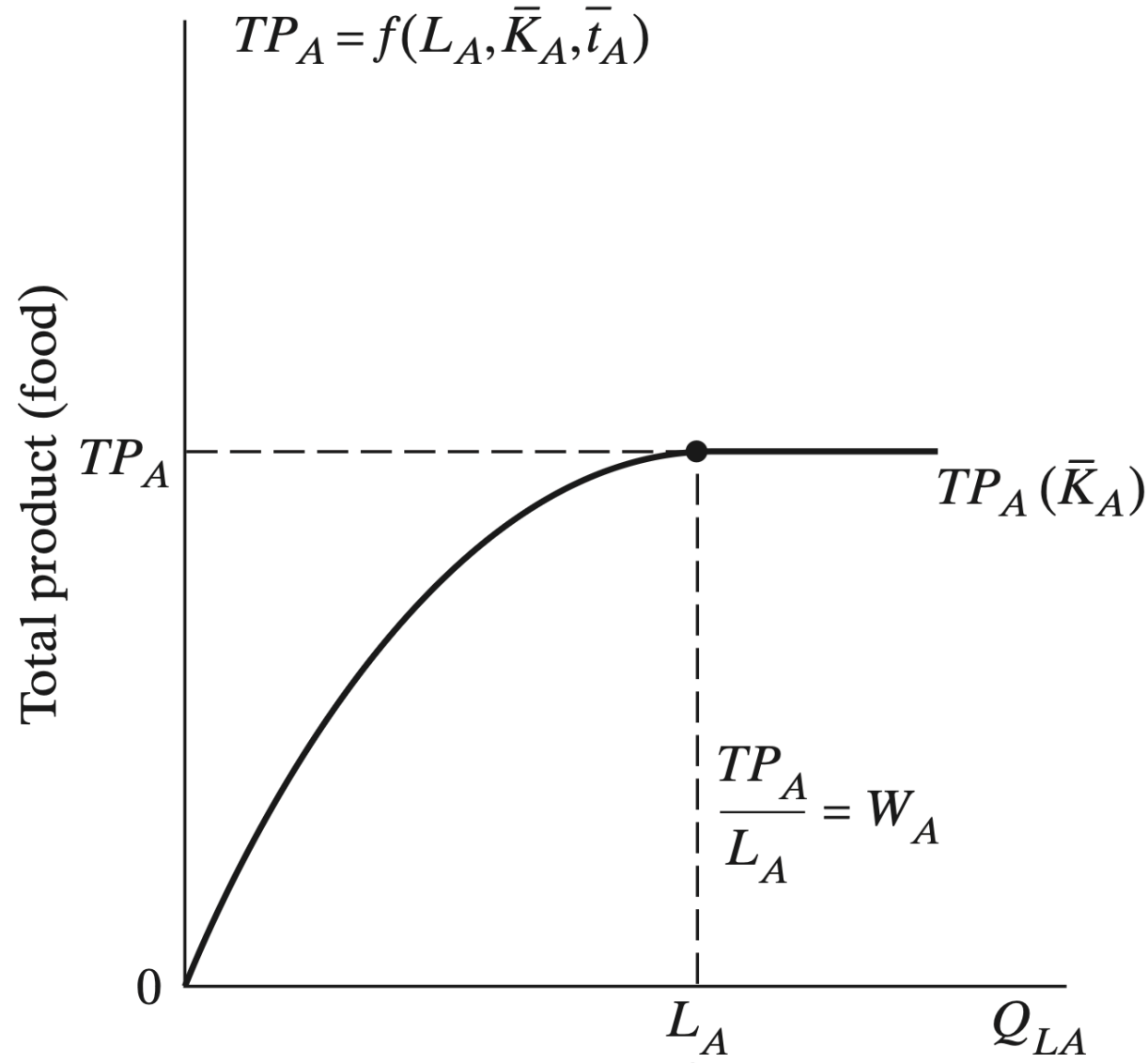
- Marginal productivity of labor ( $MP_{LA}$ ) is zero

## 2. All **rural workers** share their output equally

- Rural wages are determined by the average product of labor

- There are  $L_A$  agricultural workers that produce  $TP_A$  food

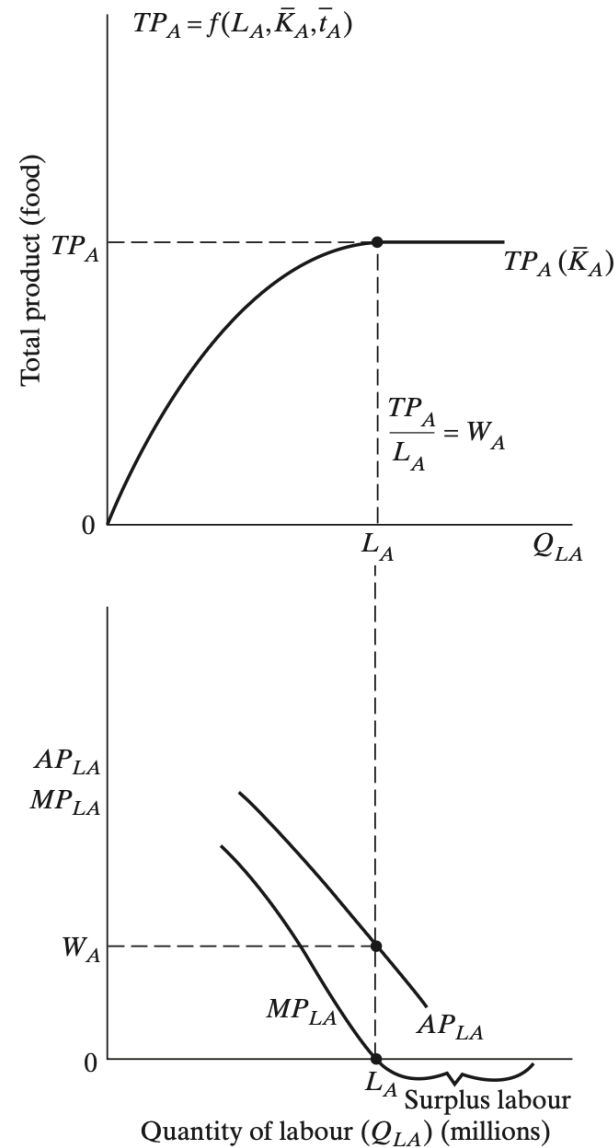
# Subsistence Agricultural Sector (Production)



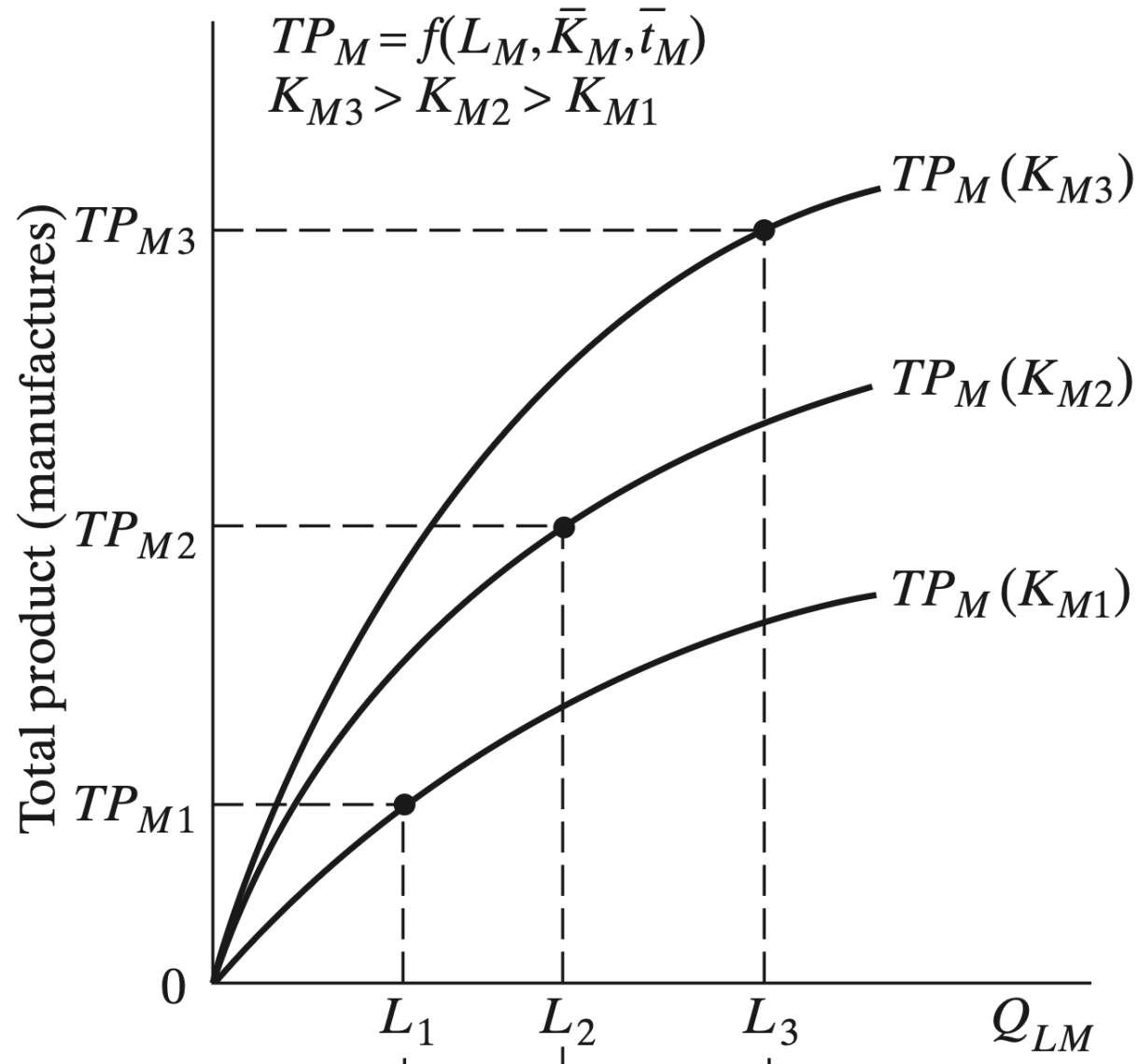
# Subsistence Agricultural Sector (Labor Demand)



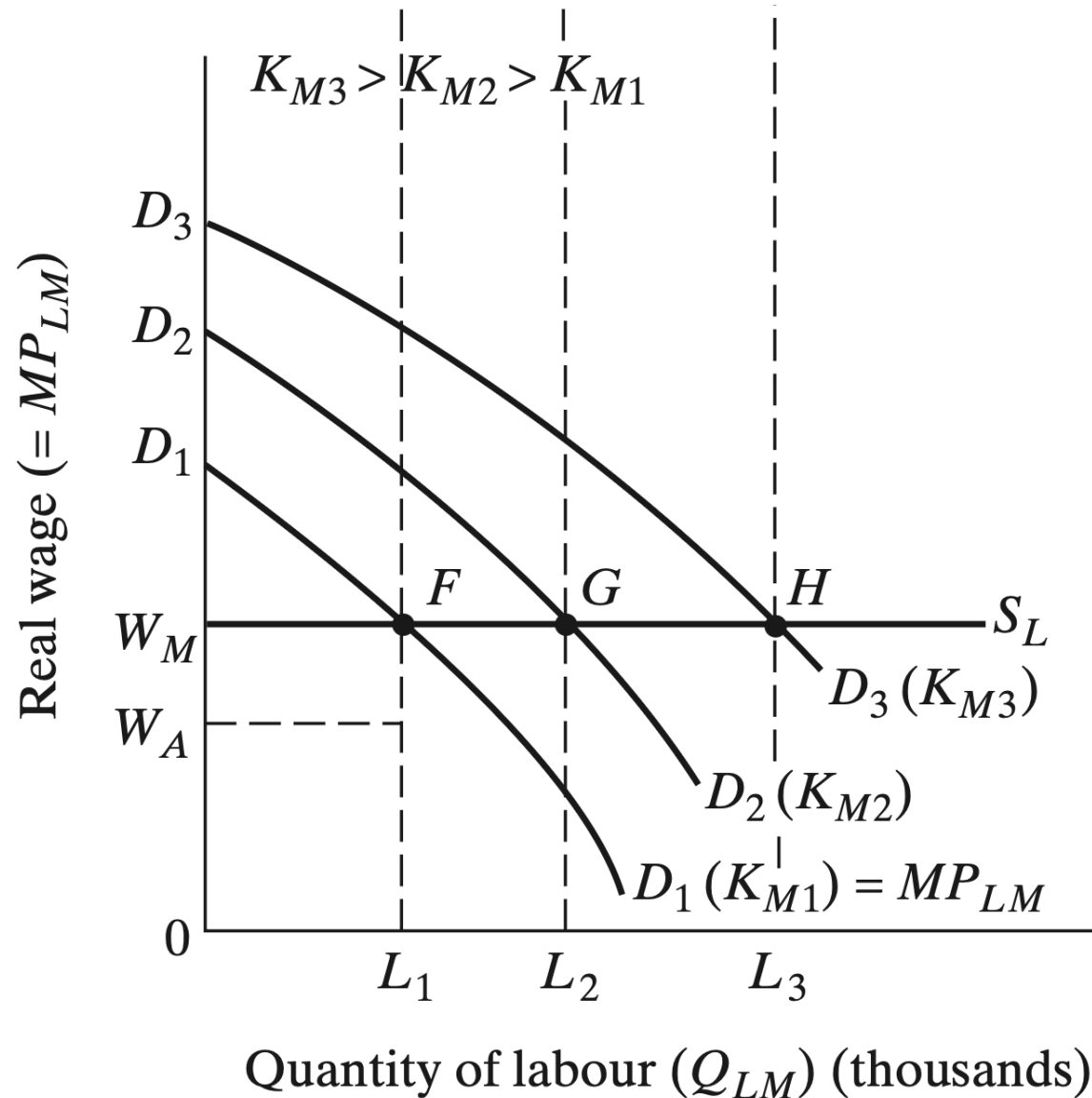
# Subsistence Agricultural Sector (Both)



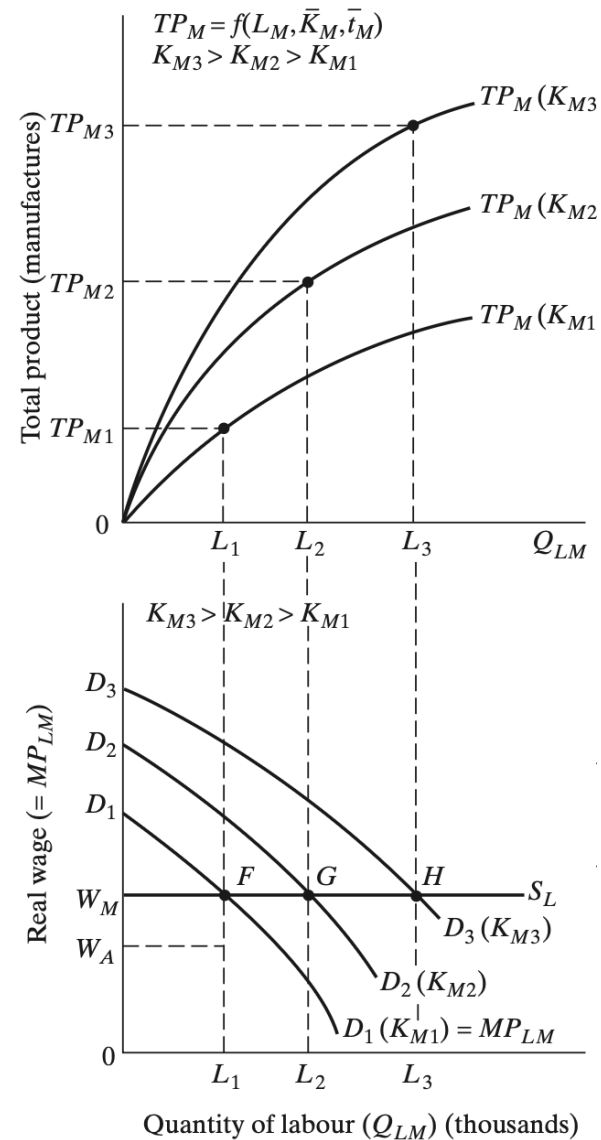
# Industrial Sector (Production)



# Industrial Sector (Labor Demand)



# Industrial Sector (Both)

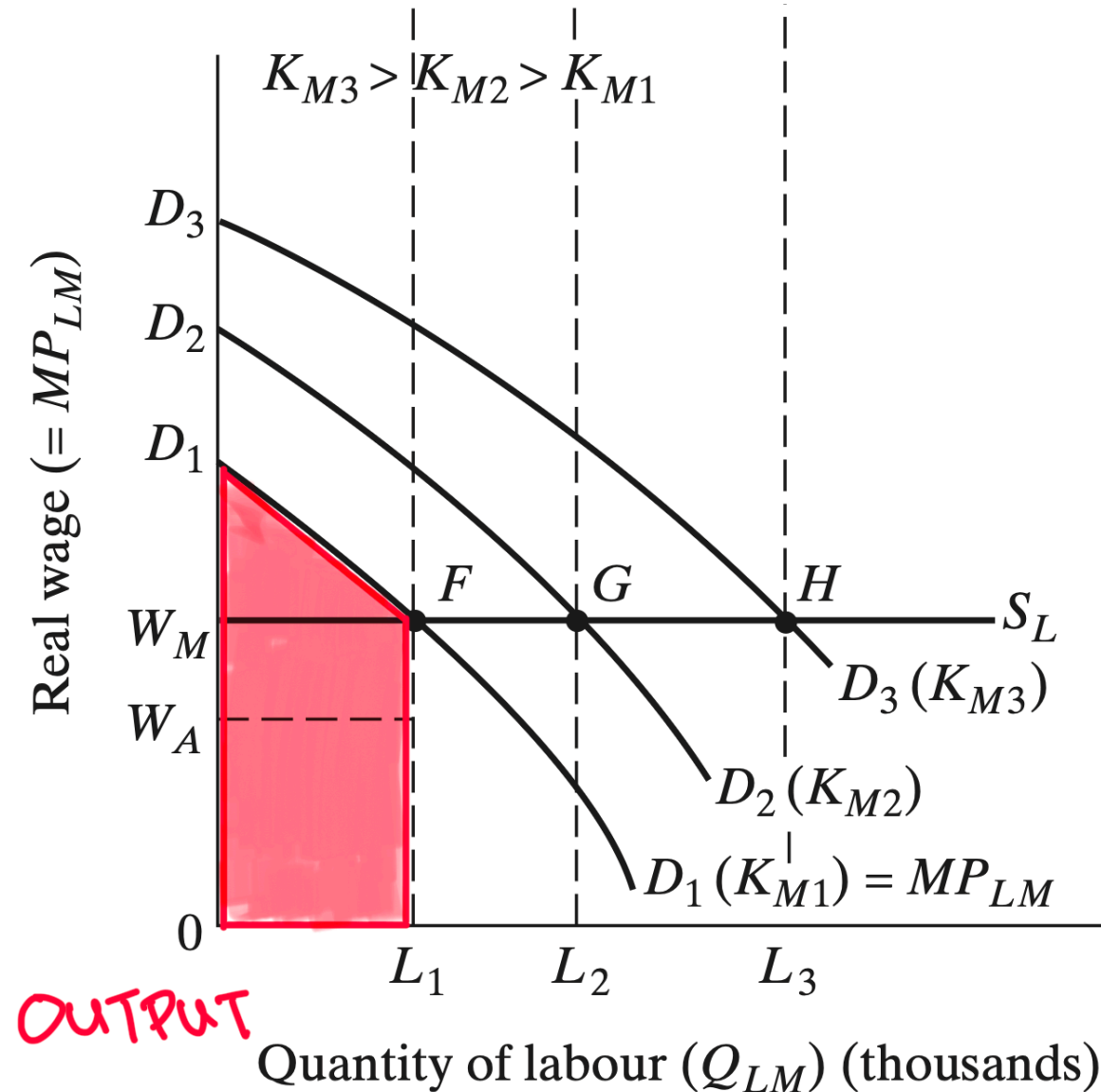


# Wage Differences

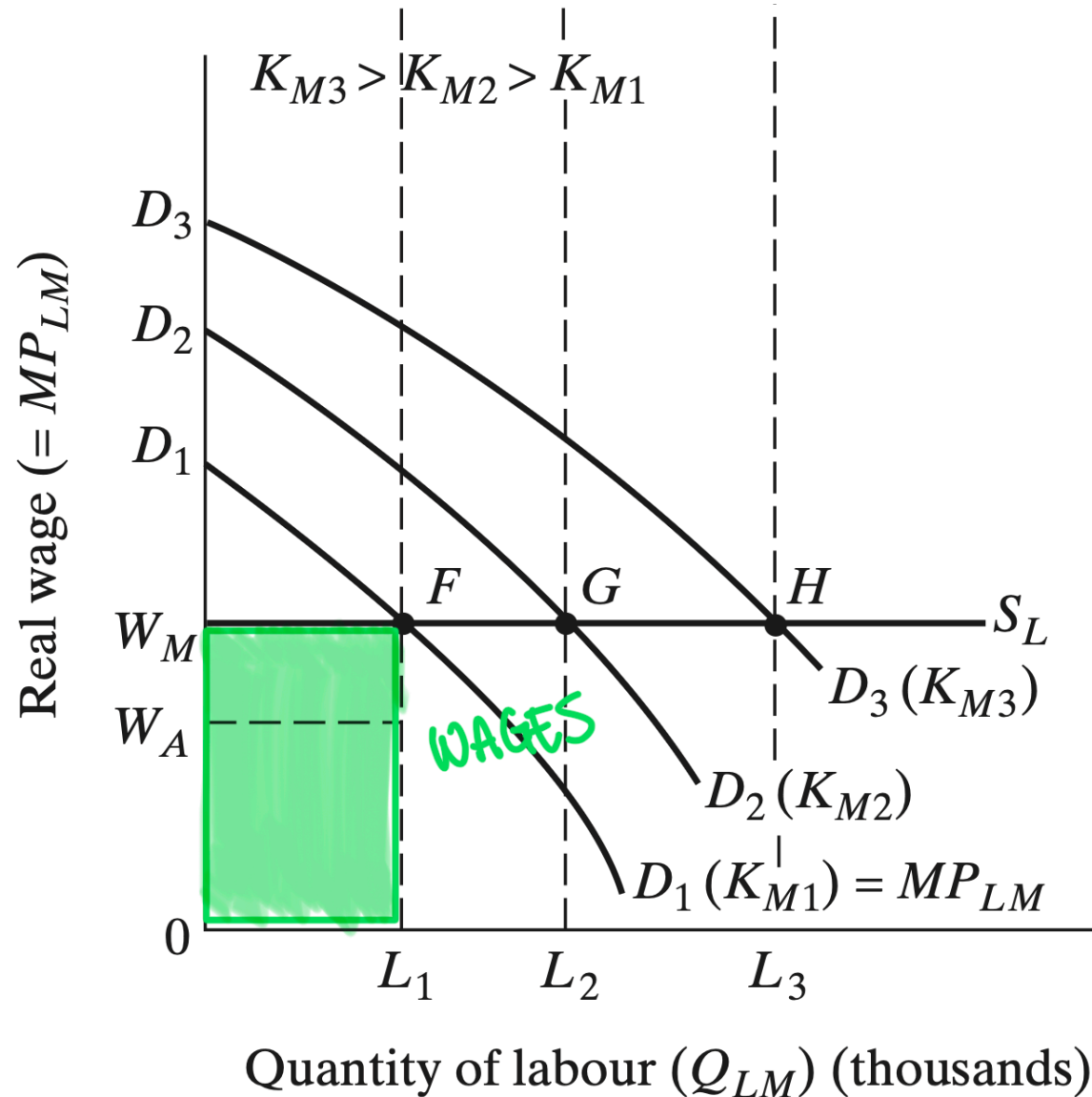
**Agricultural wages** are lower than **wages in the industrial sector**

- Supply of agricultural labor is assumed to be **perfectly elastic/unlimited**
  - This means that at industrial wages ( $W_M$ ) above agricultural wages ( $W_A$ ), the industrial sector can hire **as many surplus workers as they want without fear of raising wages**
- We assume labor to have diminishing returns (1 more labor  $\neq$  1 more output)
- The profit-maximizing industrial sector hires labor until their **marginal product** is equal to **the real wage**

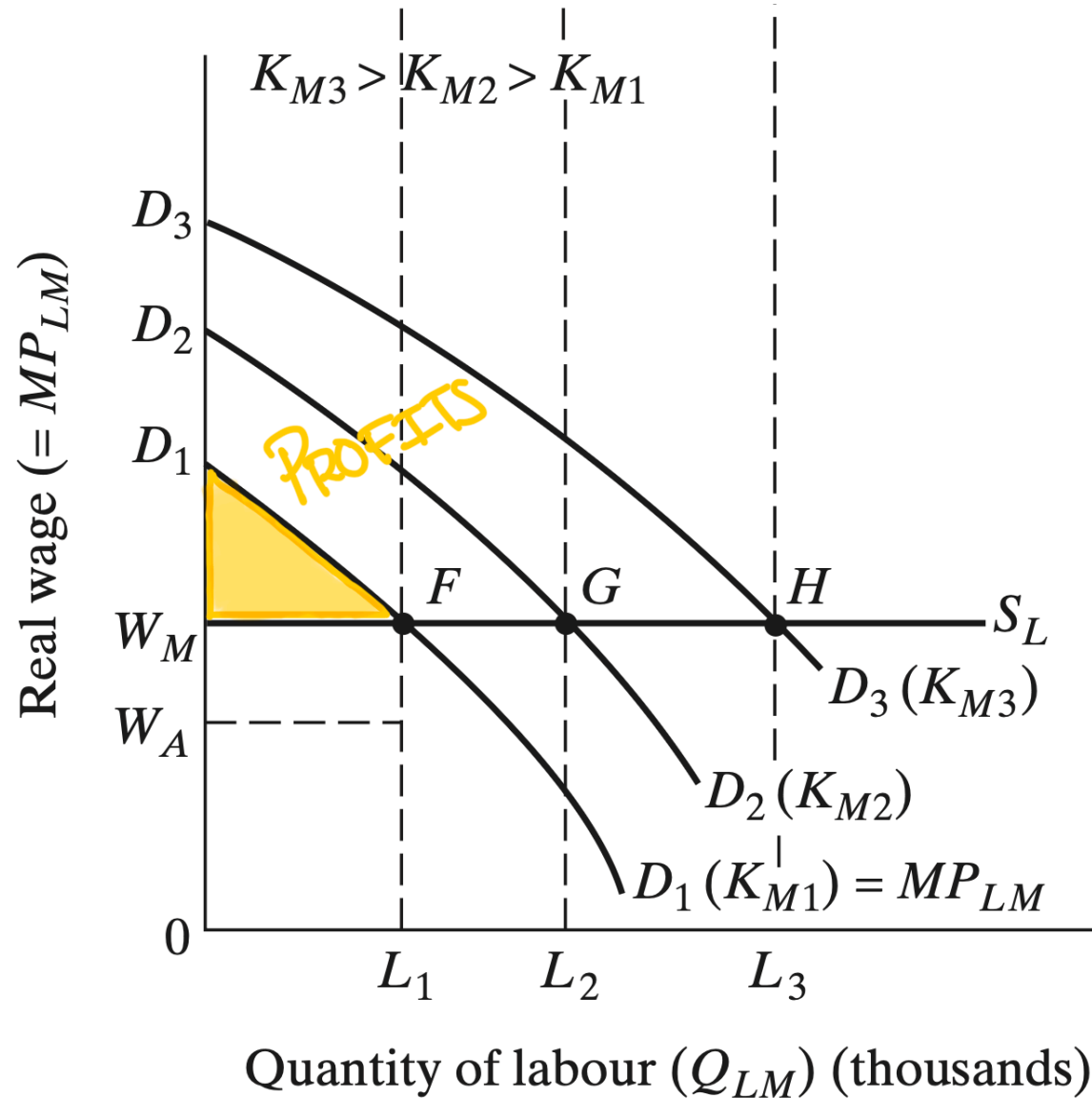
# Industrial Sector Output



# Industrial Sector Wages

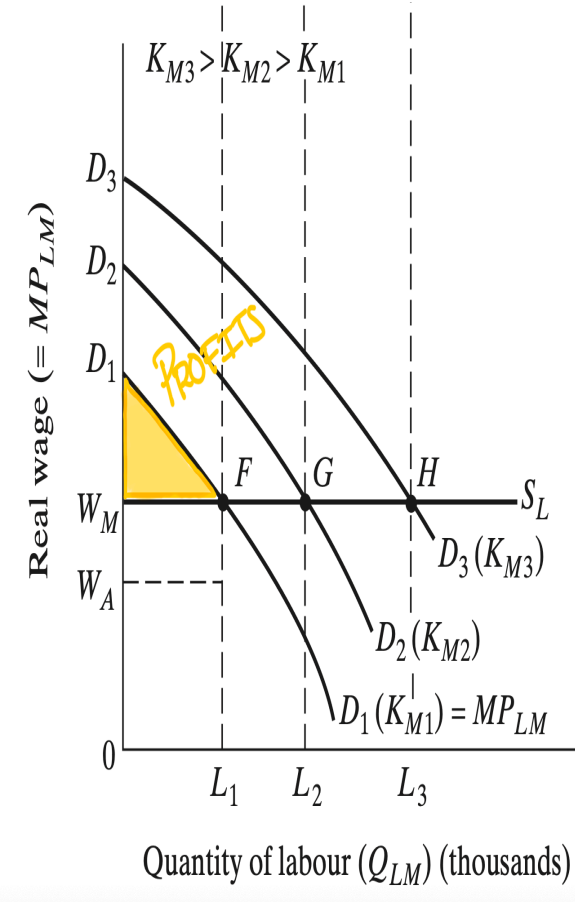
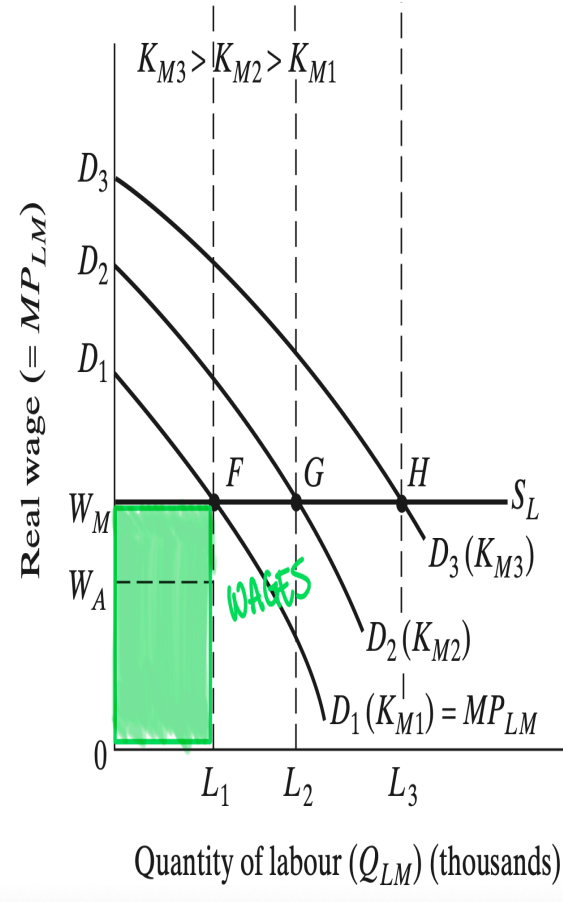
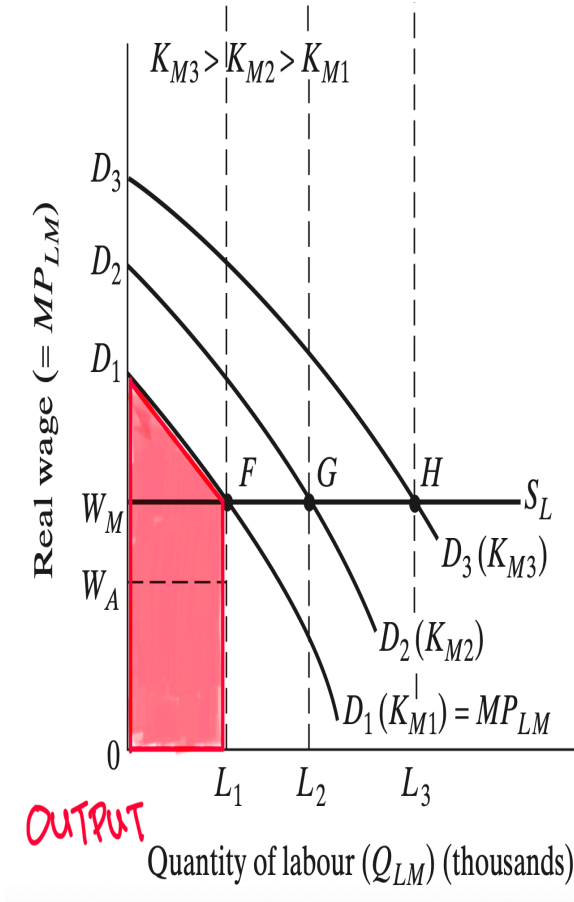


# Industrial Sector Profits

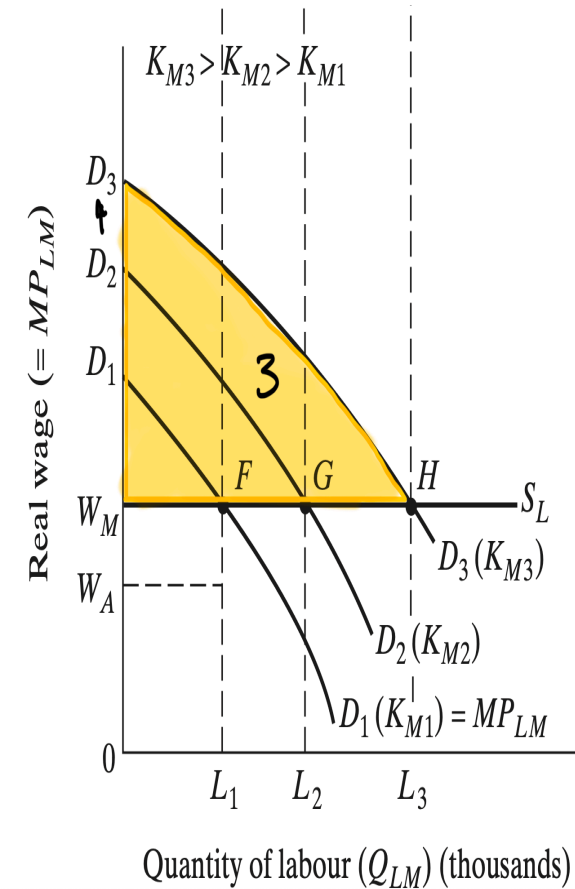
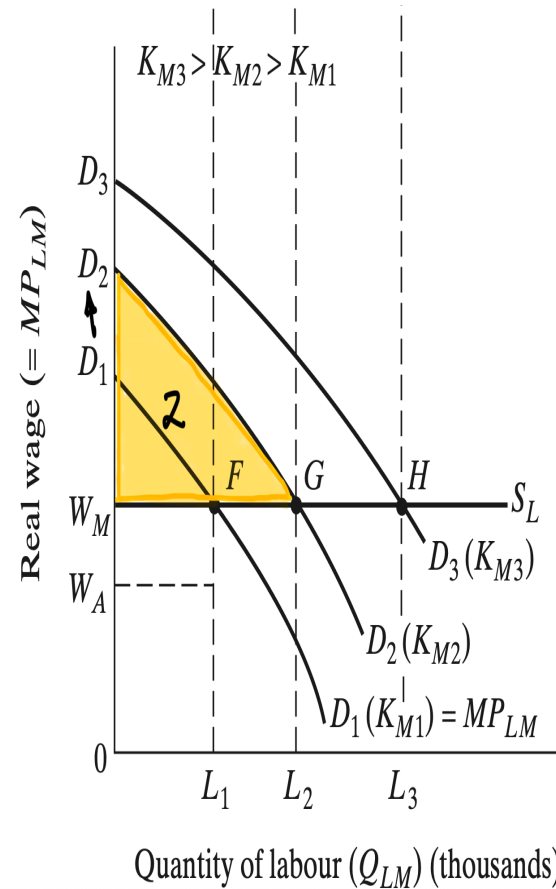
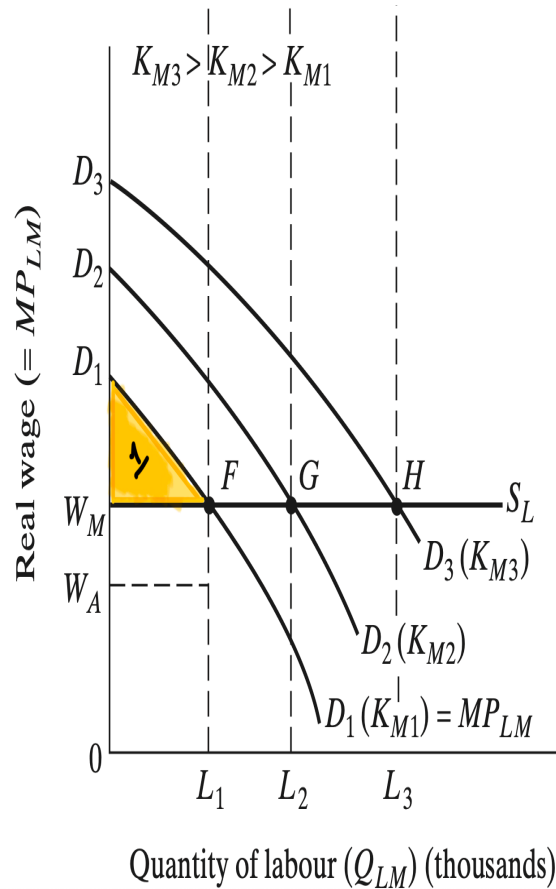




# All Together



# Investments



# Investments

Because the industrial sector owners make a **positive profit** and they fully reinvest those back into **capital**

## **We get self-sustaining growth**

- This will happen until all **surplus rural labor** is absorbed into the industrial sector
- The **industrial sector** can extract additional workers **at a higher cost** since it would now imply a loss of food production
- This is the **Lewis Turning Point**
  - The labor supply curve becomes **positively sloped** in the industrial sector and wages increase in the economy
  - Eventually the market goes through a **structural transformation** where it shifted from a traditional rural agricultural to a modern urban industrial setting