ODE number: 1

$$dxxy(x) + dx\,(x+1)\,\frac{d}{dx}y(x) = 0$$

Solution:

$$y(x) = C\,(x+1)\,e^{-x}$$

CORRECT
ODE number: 2

$$x\frac{d}{dx}y(x) - 2y(x) = bx^3y^2(x)$$

Solution:

$$y(x) = \frac{5x^2}{C - bx^5}$$

CORRECT
ODE number: 3

$$dxxy(x) + dx\,(x+1)\,\frac{d}{dx}y(x) = 0$$

Solution:

$$y(x) = C\,(x+1)\,e^{-x}$$

CORRECT
ODE number: 4

$$x\frac{d}{dx}y(x) - 2y(x) = bx^3y^2(x)$$

Given solution:

$$y(x) = \frac{5x^2}{C + x^5}$$

is INCORRECT
The correct one is:

$$y(x) = \frac{x^2}{C_1 - \frac{bx^5}{5}}$$

ODE number: 5

$$\frac{d}{dx}y(x) = (9x + y(x) - 1)^2$$

Given solution:

$$Y = -9x + 1 - 3\tan(3C - 3x)$$

is INCORRECT
The correct one is:

$$y(x) = x + x^2 (C_1 - 1) + \frac{x^3}{3} (-9C_1 + 100) + \frac{x^4}{24} (1015C_1 - 972) + \frac{5x^5}{12} (-27C_1 + 320) + C_1 + \mathcal{O}\left(x^6\right)$$

ODE number: 6

$$\frac{d}{dx} y(x) = (9x + y(x) - 1)^2$$

Solution:

$$y(x) = -9x + -\frac{3 \sin (C - 3x)}{\cos (C - 3x)} + 1$$

CORRECT