Jack O'Neill
AE 5222
Homework 2

Method
To find the dimensions of the rectangular box which would minimize the surface area I needed to set up a lagrangian which took into account the following constraint:

$$x_1 x_2 x_3 - V = 0$$

Where x_1 , x_2 , x_3 are the length, width, and height, respectively. My goal was to minimize the following equation:

$$f(x_1, x_2, x_3) = 2x_3(x_1 + x_2) + x_1 x_2$$

The resulting Lagrangian is as follows:

$$L(x_1, x_2, x_3, p) = 2x_3(x_1 + x_2) + x_1 x_2 + p(x_1 x_2 x_3 - V)$$

To find the minimum values of these four states the derivative of the Lagrangian with respect to each of the four states must be equal to zero. These equations are shown below:

$$\frac{\partial L}{\partial x_1} = 2x_3^* + x_2^* + p x_2^* x_3^* = 0$$
$$\frac{\partial L}{\partial x_2} = 2x_3^* + x_1^* + p x_1^* x_3^* = 0$$
$$\frac{\partial L}{\partial x_3} = 2x_1^* + 2x_2^* + p x_1^* x_2^* = 0$$
$$\frac{\partial L}{\partial p} = x_1^* x_2^* x_3^* - V = 0$$

Results
With four equations and four unknowns I could then solve for each of the four unknowns which would allow me to determine the dimensions that minimize the surface area:

$$x_1^* = \sqrt[3]{2V}$$
$$x_2^* = \sqrt[3]{2V}$$
$$x_3^* = \frac{V}{(2V)^{\frac{2}{3}}}$$

Discussion

Intuitively it makes sense that the length and width of the box would be equal when aiming to minimize the surface area of a box. The following plot illustrates the minimization of the surface area given the constraint on volume. It minimizes surface area assuming volume equals 1 unit:

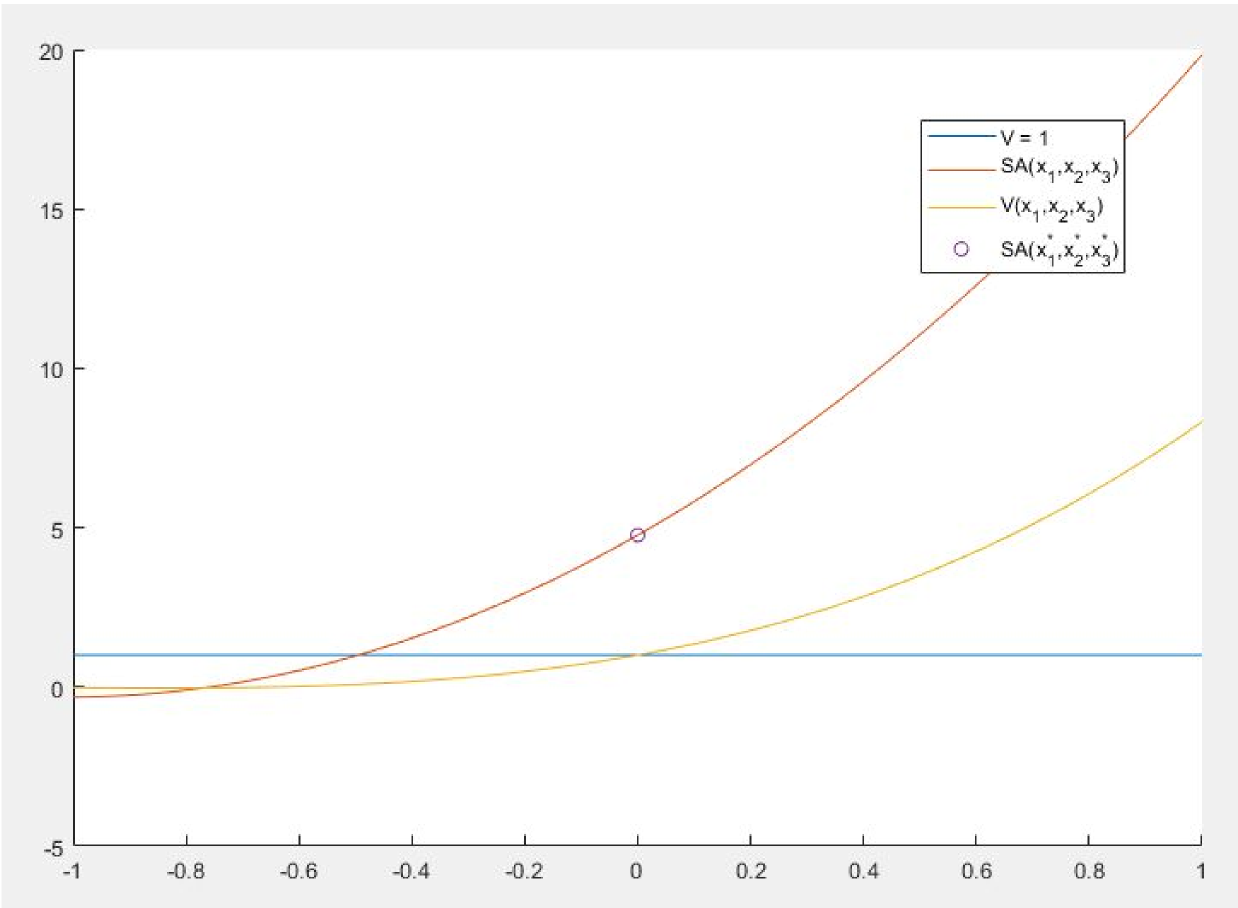

Figure 1: Visualization of surface area minimization with volume constraint