AE 5222 – Optimal Control of Dynamical Systems

Homework Submission Cover Page and Statement of Academic Honesty

I, John O'Neill, submit the solution to Homework Problem 8.

My signature below affirms that all of the writing in this submission is my own work. Any reference material that I used to prepare this submission, including text or video resources, but excluding the lecture notes and videos provided on the Canvas site for this course, is properly cited.

To prepare this submission:

■ I verbally collaborated with the following individuals (excluding *Piazza* discussions):

Currently enrolled in AE 5222: Evan Kelly

Not currently enrolled in AE 5222: ______

☐ I did not verbally collaborate with any other individual.

This submission reflects my individual effort and my own understanding of the course content.

I have read and I understand WPI's Academic Honesty Policy, and my conduct in preparing this submission has been in accordance with this Policy.

Signature: ______ **Date:** 04/23/2019

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AE 5222
Homework 8

Method

To minimize the path cost I needed to first derive the set of necessary conditions for the discretized vehicle kinematics problem. The kinematics and cost function are shown below:

$$x(k+1) = x(k) + V\cos\psi(k), \quad y(k+1) = y(k) + V\sin\psi(k), \quad k = 0, \ldots, N-1.$$

$$\mathcal{J} = \tfrac{1}{2}(x(N)-1)^2 + \tfrac{1}{2}(y(N)-1)^2 + \sum_{k=1}^{N} c(x(k), y(k)).$$

Where N = 10. The first step in this problem is to determine the discrete equivalent to the Hamiltonian equation which is of the following form:

$$H(k) \;=\; l(\overline{x}(k), \overline{u}(k)) + \overline{p}^T(k+1)\, f(\overline{x}(k), \overline{u}(k))$$

There are three necessary conditions for optimality for this "Hamiltonian":

$$\frac{\partial H}{\partial \overline{u}(k)} = 0 \quad (for\ all\ k = 0, 1, ..., N-1)$$
$$\frac{\partial H}{\partial \overline{x}(k)} = \overline{p}^T(k) \quad (for\ all\ k = 0, 1, ..., N-1)$$
$$\frac{\partial H}{\partial \overline{x}(N)} = \overline{p}^T(N)$$

After defining these necessary conditions it is time to begin constructing them for this problem. Since N=10 there will be 11 equations for each state and costate variables (N = 0,...,10) plus ten more for each heading angle. This leaves a total of 54 total equality equations to solve for this problem.

The equations for solving for the costate equations (p_1 and p_2) are shown below:

$$p_1^*(k) = p^*(k+1) + \sum_{i=1}^{25} \frac{\partial c}{\partial x(i)}$$
$$p_2^*(k) = p_2^*(k+1) + \sum_{i=1}^{25} \frac{\partial c}{\partial y(i)}$$

$$p_1^*(N) = x(N) + \sum_{i=1}^{25} \frac{\partial c}{\partial x(i)} - 1$$

$$p_2^*(N) = y(N) + \sum_{i=1}^{25} \frac{\partial c}{\partial y(i)} - 1$$

The derived equation for the heading angle ψ is:

$$\psi^*(k) = arctan(\frac{p_2^*(k+1)}{p_1^*(k+1)})$$

I set these 54 equations equal to zero and solved the resulting nonlinear system of equations in Matlab using the fsolve() command.

<u>Results</u>

After attempting to adjust both the initial guess as well as the weighting coefficients in the cost function, the closest I was able to get to a valid result looked like this:

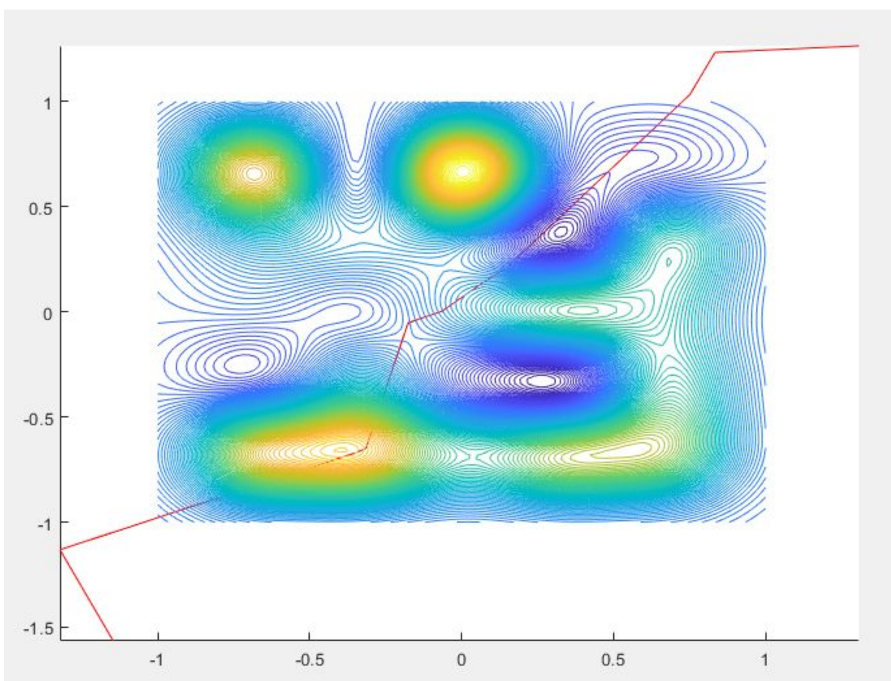

Figure 1: Best result

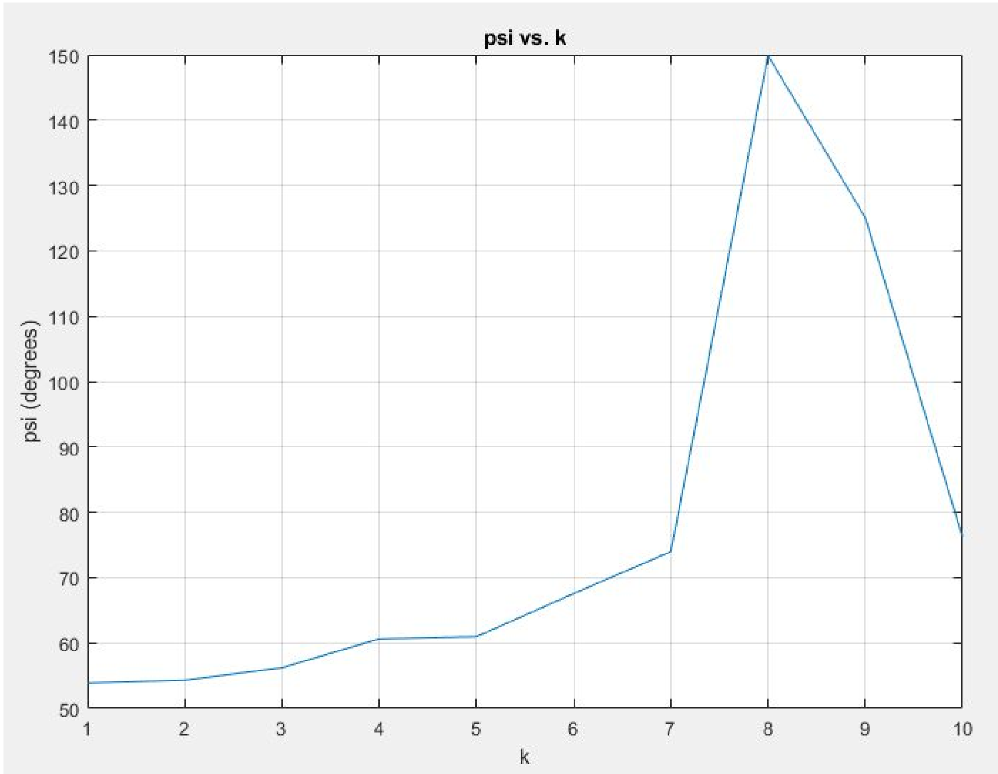

Figure 2: Sequence of heading angles

This was a result of the following initial conditions:

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x0(1)  = -0;       x0(12)= -1;
x0(2)  = -0.9;     x0(13)= -0.9;
x0(3)  = -0.7;     x0(14)= -0.7;
x0(4)  = -0.5;     x0(15)= -0.5;
x0(5)  = -0.2;     x0(16)= -0.2;
x0(6)  = 0.001;    x0(17)= 0.001;
x0(7)  = 0.2;      x0(18)= 0.2;
x0(8)  = 0.5;      x0(19)= 0.5;
x0(9)  = 0.7;      x0(20)= 0.7;
x0(10)= 0.9;       x0(21)= 0.9;
x0(11)= 1;         x0(22)= 1;
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Figure 2: Initial conditions which gave the most realistic results

It is apparent that fsolve did not actually find a solution to this system of equations. However this path was the closest I could get to having the path start and end at the required coordinates.

<u>Discussion</u>

In the end I was unable to find a valid optimal path for this problem. However I know that the method I used to determine the necessary conditions and the equation for heading angle were correct. I think that with more trial and error I could eventually find a combination of the correct initial guess and sufficient weighting coefficients in the cost function such that I would actually obtain a valid result but it definitely would have taken a very long time to do so.