# AE 5222 – Optimal Control of Dynamical Systems

## Homework Submission Cover Page and Statement of Academic Honesty

I, John O'Neill, submit the solution to Homework Problem 3.

My signature below affirms that all of the writing in this submission is my own work. Any reference material that I used to prepare this submission, including text or video resources, but excluding the lecture notes and videos provided on the Canvas site for this course, is properly cited.

To prepare this submission:

☐ I verbally collaborated with the following individuals (excluding *Piazza* discussions):

Currently enrolled in AE 5222: ______________________________

______________________________

Not currently enrolled in AE 5222: ______________________________

______________________________

■ I did not verbally collaborate with any other individual.

This submission reflects my individual effort and my own understanding of the course content.

I have read and I understand WPI's Academic Honesty Policy, and my conduct in preparing this submission has been in accordance with this Policy.

**Signature:** ______________________________ **Date:** 04/20/19

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AE 5222
Homework 3

Method

In order to minimize the function given the inequality constraints I needed to incorporate the constraint equations with respective slack variables. The following equations show the conversion of the given inequality constraints into equality constraints:

$$h_1 = x_1^2 + x_2^2 - x_3^2 + s_1^2$$
$$h_2 = 2 - \sqrt{x_1^2 + x_2^2 + x_3^2} + s_2^2$$
$$h_3 = x_3 - 5 + s_3^2$$

With these three equality constraints the Lagrangian can be constructed:

$$L(\overline{x}, \overline{p}, \overline{s}) = f(\overline{x}) + \overline{p}^T(\overline{h}(\overline{x}, \overline{s}))$$

To minimize the function, the gradient of L must be set to zero, and all states must be solved. The gradients of L are shown below:

```
dL_dx1 = 2*p1*x1 - 12*x1 + 3*x1^2 - (p2*x1)/(x1^2 + x2^2 + x3^2)^(1/2) + 11;
dL_dx2 = 2*p1*x2 - (p2*x2)/(x1^2 + x2^2 + x3^2)^(1/2);
dL_dx3 = p3 - 2*p1*x3 - (p2*x3)/(x1^2 + x2^2 + x3^2)^(1/2) + 1;
dL_dp1 = s1^2 + x1^2 + x2^2 - x3^2;
dL_dp2 = s2^2 - (x1^2 + x2^2 + x3^2)^(1/2) + 2;
dL_dp3 = x3 + s3^2 - 5;
dL_ds1 = 2*p1*s1;
dL_ds2 = 2*p2*s2;
dL_ds3 = 2*p3*s3;
```

Using Matlab fsolve() function I could easily calculate the optimal state matrix.

Results

The states which minimize the function f with the inequality constraints are shown as follows:

```
x1* = 2
x2* = 0
x3* = 2
p1* = 0.25
p2* = 0
p3* = 0
s1* = 0
s2* = 0.91
s3* = 1.732
```

Discussion

The results from this homework problem were realistic and unless there were any errors typing in equations in Matlab I am confident in these values. The three inequality states along with $\mathbf{x} \in \mathbb{R}^3_{\geqslant 0}$ can all quickly be confirmed to be satisfied with these output values.