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Homework 8

Method

To minimize the path cost I needed to first derive the set of necessary conditions for the discretized vehicle kinematics problem. The kinematics and cost function are shown below:

$$x(k+1) = x(k) + V\cos\psi(k), \quad y(k+1) = y(k) + V\sin\psi(k), \quad k = 0, \ldots, N-1.$$

$$\mathcal{J} = \tfrac{1}{2}(x(N)-1)^2 + \tfrac{1}{2}(y(N)-1)^2 + \sum_{k=1}^{N} c(x(k), y(k)).$$

Where N = 10. The first step in this problem is to determine the discrete equivalent to the Hamiltonian equation which is of the following form:

$$H(k) = l(\bar{x}(k), \bar{u}(k)) + \bar{p}^T(k+1)\, f(\bar{x}(k), \bar{u}(k))$$

There are three necessary conditions for optimality for this "Hamiltonian":

$$\frac{\partial H}{\partial \bar{u}(k)} = 0 \quad (for\ all\ k = 0, 1, ..., N-1)$$
$$\frac{\partial H}{\partial \bar{x}(k)} = \bar{p}^T(k) \quad (for\ all\ k = 0, 1, ..., N-1)$$
$$\frac{\partial H}{\partial \bar{x}(N)} = \bar{p}^T(N)$$

After defining these necessary conditions it is time to begin constructing them for this problem. Since N=10 there will be 11 equations for each state and costate variables (N = 0,...,10) plus ten more for each heading angle. This leaves a total of 54 total equality equations to solve for this problem.

The equations for solving for the costate equations (p_1 and p_2) are shown below:

$$p_1^*(k) = p^*(k+1) + \sum_{i=1}^{25} \frac{\partial c}{\partial x(i)}$$
$$p_2^*(k) = p_2^*(k+1) + \sum_{i=1}^{25} \frac{\partial c}{\partial y(i)}$$

$$p_1^*(N) = x(N) + \sum_{i=1}^{25} \frac{\partial c}{\partial x(i)} - 1$$

$$p_2^*(N) = y(N) + \sum_{i=1}^{25} \frac{\partial c}{\partial y(i)} - 1$$

The derived equation for the heading angle ψ is:

$$\psi^*(k) = arctan(\frac{p_2^*(k+1)}{p_1^*(k+1)})$$

I set these 54 equations equal to zero and solved the resulting nonlinear system of equations in Matlab using the fsolve() command.

<u>Results</u>

After attempting to adjust both the initial guess as well as the weighting coefficients in the cost function, the closest I was able to get to a valid result looked like this:

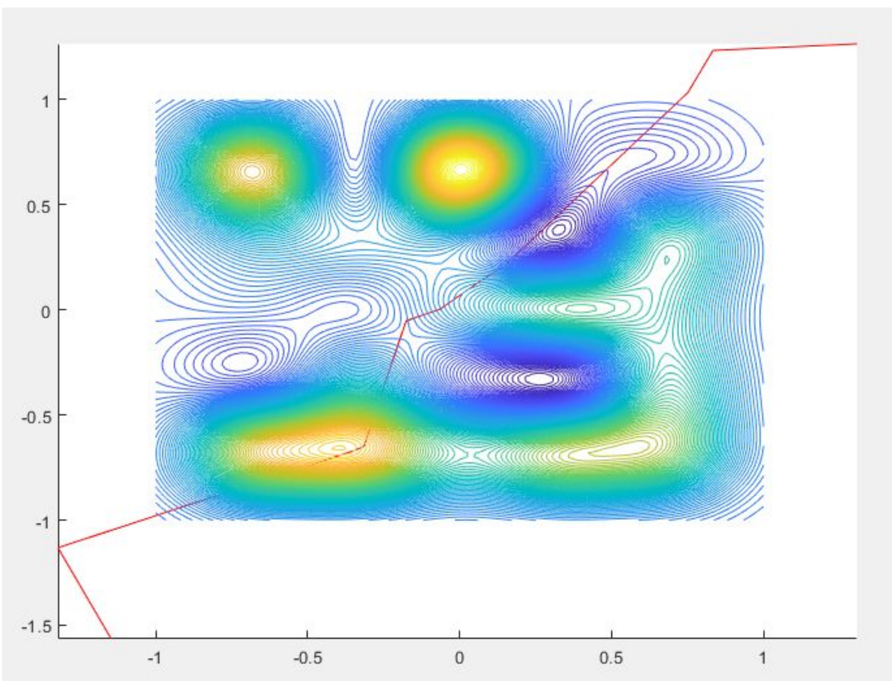

Figure 1: Best result

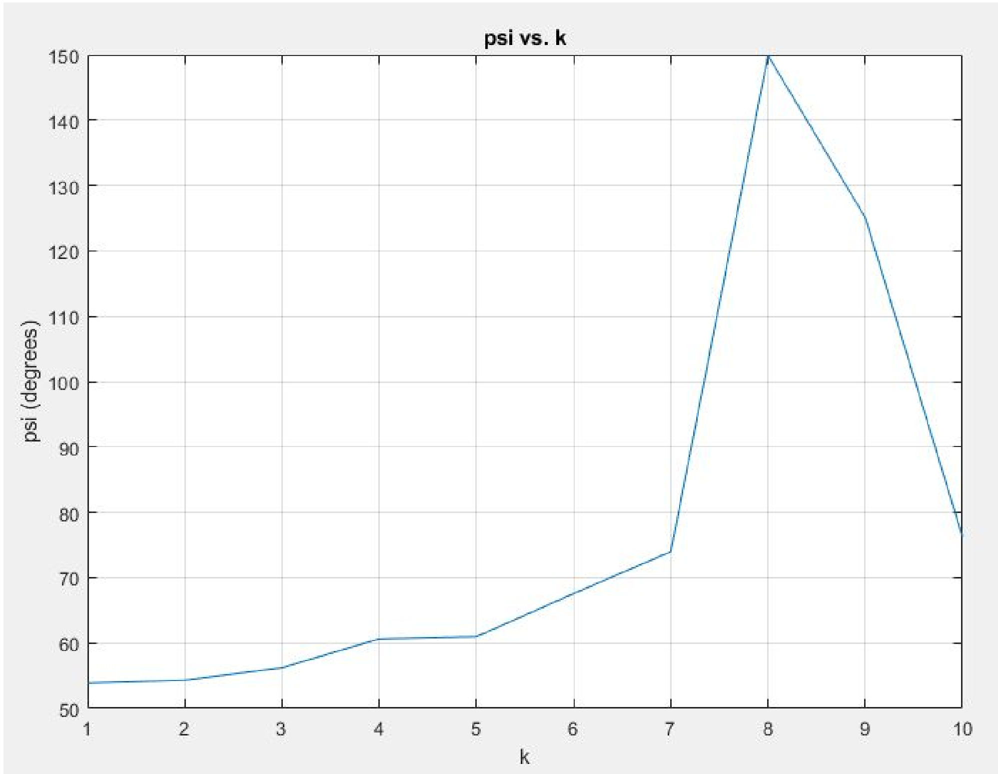

Figure 2: Sequence of heading angles

This was a result of the following initial conditions:

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x0(1)  = -0;       x0(12)= -1;
x0(2)  = -0.9;     x0(13)= -0.9;
x0(3)  = -0.7;     x0(14)= -0.7;
x0(4)  = -0.5;     x0(15)= -0.5;
x0(5)  = -0.2;     x0(16)= -0.2;
x0(6)  = 0.001;    x0(17)= 0.001;
x0(7)  = 0.2;      x0(18)= 0.2;
x0(8)  = 0.5;      x0(19)= 0.5;
x0(9)  = 0.7;      x0(20)= 0.7;
x0(10)= 0.9;       x0(21)= 0.9;
x0(11)= 1;         x0(22)= 1;
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Figure 2: Initial conditions which gave the most realistic results

It is apparent that fsolve did not actually find a solution to this system of equations. However this path was the closest I could get to having the path start and end at the required coordinates.

<u>Discussion</u>

In the end I was unable to find a valid optimal path for this problem. However I know that the method I used to determine the necessary conditions and the equation for heading angle were correct. I think that with more trial and error I could eventually find a combination of the correct initial guess and sufficient weighting coefficients in the cost function such that I would actually obtain a valid result but it definitely would have taken a very long time to do so.