AE 5222 – Optimal Control of Dynamical Systems

Homework Submission Cover Page and Statement of Academic Honesty

I, John O'Neill, submit the solution to Homework Problem 5.

My signature below affirms that all of the writing in this submission is my own work. Any reference material that I used to prepare this submission, including text or video resources, but excluding the lecture notes and videos provided on the Canvas site for this course, is properly cited.

To prepare this submission:

■ I verbally collaborated with the following individuals (excluding *Piazza* discussions):

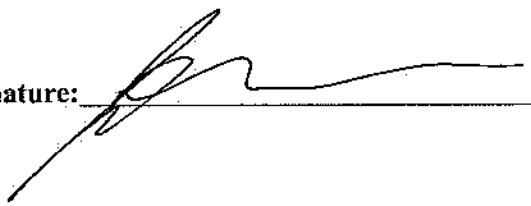
Currently enrolled in AE 5222: Evan Kelly

Not currently enrolled in AE 5222:

☐ I did not verbally collaborate with any other individual.

This submission reflects my individual effort and my own understanding of the course content.

I have read and I understand WPI's Academic Honesty Policy, and my conduct in preparing this submission has been in accordance with this Policy.

Signature:  **Date:** 04/25/2019

Jack O'Neill
AE 5222
Homework 5

Method

The goal of this problem was to minimize the total cost of the structure. The cost is defined as:

$$f(l,h,t,b) = 2.21lh^2 + 0.048\,(bt\,(14+l))$$

There were 12 inequality constraints in this system:

$$b \leq 2$$
$$h \geq 0.125$$
$$h \leq 2$$
$$h \leq b$$
$$t \geq 0.1$$
$$t \leq 10$$
$$l \geq 0.1$$
$$l \leq 10$$
$$P(b,t) \geq 6000$$
$$\tau_{shear}(l,h,t,b) \leq 13600$$
$$\tau_{bending}(b,t) \leq 30000$$
$$d_{end}(b,t) \leq 0.25$$

Where the functions $P(b,t)$, $\tau_{shear}(l,h,t,b)$, $\tau_{bending}(b,t)$, and $d_{end}(b,t)$ are all defined in the problem statement.

The next step was to “convert” these 12 inequality constraints to equality constraints by adding the slack variables. In Matlab, these 12 equations look like this:

```
h1  = @(l,h,t,b,s1 ) b-2      + s1^2;
h2  = @(l,h,t,b,s2 ) 0.125-h  + s2^2;
h3  = @(l,h,t,b,s3 ) h-2      + s3^2;
h4  = @(l,h,t,b,s4 ) h-b      + s4^2;
h5  = @(l,h,t,b,s5 ) 0.1-t    + s5^2;
h6  = @(l,h,t,b,s6 ) t-10     + s6^2;
h7  = @(l,h,t,b,s7 ) 0.1-l    + s7^2;
h8  = @(l,h,t,b,s8 ) l-10     + s8^2;
h9  = @(l,h,t,b,s9 ) 6000-buckling_load(b,t)        + s9^2;
h10 = @(l,h,t,b,s10) shear_stress(l,h,t,b)-13600    + s10^2;
h11 = @(l,h,t,b,s11) bending_stress(l,h,t,b)-30000 + s11^2;
h12 = @(l,h,t,b,s12) end_deflection(l,h,t,b)-0.25   + s12^2;
```

With these newly formed equality constraints I could then construct the Lagrangian equation with the following structure:

$$L(l,h,t,b,\overline{p},\overline{s}) = f(l,h,t,b) + \overline{p}^{T}(\overline{h}(l,h,t,b,\overline{s}))$$

Using Matlab I then determined the 12 partial derivatives of the lagrangian. These are located at the end of this report. I then (attempted to) use fsolve to solve this nonlinear system of equations.

Results

I used an initial guess of {l, h, t, b} = {5, 1, 5, 1.2}.The following values are the result of this mess:

```
l = 6.08 inches
h = 0.94 inches
t = 1.75 inches
b = 1.58 inches
cost = $14.60

buckling load = 523386.08 psi
shear stress = 9177.91 psi
bending stress = 103878.28 psi
end deflection = 0.26 inches
```

Only the physical dimensions met the required constraints as well as the buckling load and shear stress. The bending stress and end deflection however did not satisfy their required constraints.

Discussion

I could not determine the reason what I was doing incorrectly for this problem. I checked over my work many times and fixed any errors I could find, but could never get fsolve() to converge. These results were the best values I could get as an answer since fsolve() could never determine a result within its maximum number of evaluations (2800). I suspect the failure to converge has something to do with the large differences in magnitude between values across the 28 unknown variables. The method I used to set up the system of equations seems to be correct, however I could not get fsolve() to converge despite countless attempts. I have attached the published code to the end of this report and submitted the .m files along with this submission.

```
clc; clear; close all;
E     = 30e6;
G     = 12e6;

cost = @(l,h,t,b) 2.21*l*h^2 + 0.048*(b*t*(14+l));
buckling_load = @(b,t) (4.013*sqrt((E*G*b^2*t^6)/36)/196)*...
                       (1-(t/28)*sqrt(E/(4*G)));
M  = @(l,b,t)   6000*(14 + 0.5*l);
R  = @(l,h,t)   0.5*sqrt(l^2 + (h + t)^2);
J  = @(l,h,t)   sqrt(2)*h*l*(((l^2)/12) + ((h + t)^2)/4);
shear_stress = @(l,h,t,b) sqrt((6000/(sqrt(2)*h*l))^2 + ...
                        (6000/sqrt(2)*h)*(M(l,b,t)/J(l,h,t)) + ...
                        ((M(l,b,t)*R(l,h,t))/J(l,h,t))^2);
bending_stress = @(l,h,t,b) 84*6000/(b*t^2);
end_deflection = @(l,h,t,b) (10976*6000)/(E*b*t^3);

% Inequality Constraints

h1  = @(l,h,t,b,s1 ) b-2     + s1^2;
h2  = @(l,h,t,b,s2 ) 0.125-h + s2^2;
h3  = @(l,h,t,b,s3 ) h-2     + s3^2;
h4  = @(l,h,t,b,s4 ) h-b     + s4^2;
h5  = @(l,h,t,b,s5 ) 0.1-t   + s5^2;
h6  = @(l,h,t,b,s6 ) t-10    + s6^2;
h7  = @(l,h,t,b,s7 ) 0.1-l   + s7^2;
h8  = @(l,h,t,b,s8 ) l-10    + s8^2;
h9  = @(l,h,t,b,s9 ) 6000-buckling_load(b,t)       + s9^2;
h10 = @(l,h,t,b,s10) shear_stress(l,h,t,b)-13600   + s10^2;
h11 = @(l,h,t,b,s11) bending_stress(l,h,t,b)-30000 + s11^2;
h12 = @(l,h,t,b,s12) end_deflection(l,h,t,b)-0.25  + s12^2;

L    = @(l,h,t,b,...
         p1,p2,p3,p4,p5,p6,p7,p8,p9,p10,p11,p12,...
         s1,s2,s3,s4,s5,s6,s7,s8,s9,s10,s11,s12)...
      cost(l,h,t,b)...
    + p1  * h1 (l,h,t,b,s1 ) ...
    + p2  * h2 (l,h,t,b,s2 ) ...
    + p3  * h3 (l,h,t,b,s3 ) ...
    + p4  * h4 (l,h,t,b,s4 ) ...
    + p5  * h5 (l,h,t,b,s5 ) ...
    + p6  * h6 (l,h,t,b,s6 ) ...
    + p7  * h7 (l,h,t,b,s7 ) ...
    + p8  * h8 (l,h,t,b,s8 ) ...
    + p9  * h9 (l,h,t,b,s9 ) ...
    + p10 * h10(l,h,t,b,s10)...
    + p11 * h11(l,h,t,b,s11)...
    + p12 * h12(l,h,t,b,s12);


syms l h t b
syms p1 p2 p3 p4 p5 p6 p7 p8 p9 p10 p11 p12
syms s1 s2 s3 s4 s5 s6 s7 s8 s9 s10 s11 s12
```

```
state_array = [l h t b...
               p1 p2 p3 p4 p5 p6 p7 p8 p9 p10 p11 p12...
               s1 s2 s3 s4 s5 s6 s7 s8 s9 s10 s11 s12];

J = gradient(L,state_array);

for k = 1:length(J)
    fprintf("dL_d" + string(state_array(k)) +...
            " = " + string(J(k)) + ";\n");
end

int_guess = ones(1,28);
int_guess(1:4) = [5 1 5 1.2];


state_out = fsolve(@derivatives_question_5,int_guess);

fprintf("\nl = %0.2f inches\n",state_out(1));
fprintf("h = %0.2f inches\n"  ,state_out(2));
fprintf("t = %0.2f inches\n"  ,state_out(3));
fprintf("b = %0.2f inches\n"  ,state_out(4));
fprintf("cost = $%0.2f\n",cost(state_out(1),state_out(2),...
                               state_out(3),state_out(4)));


l = state_out(1);
h = state_out(2);
t = state_out(3);
b = state_out(4);

fprintf("\nbuckling load = %0.2f psi\n",buckling_load(b,t));
fprintf("shear stress = %0.2f psi\n",shear_stress(l,h,t,b));
fprintf("bending stress = %0.2f psi\n",bending_stress(l,h,t,b));
fprintf("end deflection = %0.2f inches\n",end_deflection(l,h,t,b));

dL_dl = p8 - p7 + (6*b*t)/125 + (221*h^2)/100 - (p10*(36000000/
(h^2*l^3) - (437328071996551125*2^(1/2))/(68719476736*l*((h +
 t)^2/4 + l^2/12)) + (1166208191990803*2^(1/2)*(3000*l + 84000))/
(3298534883328*((h + t)^2/4 + l^2/12)^2) - (3000*l + 84000)^2/
(4*h^2*l*((h + t)^2/4 + l^2/12)^2) + (1166208191990803*2^(1/2)*(3000*l
 + 84000))/(549755813888*l^2*((h + t)^2/4 + l^2/12)) + ((3000*l +
 84000)^2*((h + t)^2 + l^2))/(24*h^2*l*((h + t)^2/4 + l^2/12)^3)
 + ((3000*l + 84000)^2*((h + t)^2 + l^2))/(4*h^2*l^3*((h + t)^2/4
 + l^2/12)^2) - ((18000000*l + 504000000)*((h + t)^2 + l^2))/
(8*h^2*l^2*((h + t)^2/4 + l^2/12)^2)))/(2*(18000000/(h^2*l^2) +
 (1166208191990803*2^(1/2)*(3000*l + 84000))/(549755813888*l*((h
 + t)^2/4 + l^2/12)) + ((3000*l + 84000)^2*((h + t)^2 + l^2))/
(8*h^2*l^2*((h + t)^2/4 + l^2/12)^2))^(1/2));
dL_dh = p3 - p2 + p4 + (221*h*l)/50 - (p10*(36000000/(h^3*l^2) +
 ((3000*l + 84000)^2*((h + t)^2 + l^2))/(4*h^3*l^2*((h + t)^2/4
 + l^2/12)^2) - ((3000*l + 84000)^2*(2*h + 2*t))/(8*h^2*l^2*((h
 + t)^2/4 + l^2/12)^2) + (1166208191990803*2^(1/2)*(3000*l +
 84000)*(h/2 + t/2))/(549755813888*l*((h + t)^2/4 + l^2/12)^2)
```

```
 + ((3000*l + 84000)^2*((h + t)^2 + l^2)*(h/2 + t/2))/
(4*h^2*l^2*((h + t)^2/4 + l^2/12)^3)))/(2*(18000000/(h^2*l^2) +
 (1166208191990803*2^(1/2)*(3000*l + 84000))/(549755813888*l*((h
 + t)^2/4 + l^2/12)) + ((3000*l + 84000)^2*((h + t)^2 + l^2))/
(8*h^2*l^2*((h + t)^2/4 + l^2/12)^2))^(1/2));
dL_dt = p6 - p5 + (6*b*(l + 14))/125 +
 p9*((501625*2^(1/2)*5^(1/2)*10^(1/2)*(b^2*t^6)^(1/2))/2744
 + (3009750*10^(1/2)*b^2*t^5*((2^(1/2)*5^(1/2)*t)/112 - 1))/
(49*(b^2*t^6)^(1/2))) - (1008000*p11)/(b*t^3) - (4116*p12)/(625*b*t^4)
 - (p10*((1166208191990803*2^(1/2)*(3000*l + 84000)*(h/2 + t/2))/
(549755813888*l*((h + t)^2/4 + l^2/12)^2) - ((3000*l + 84000)^2*(2*h +
 2*t))/(8*h^2*l^2*((h + t)^2/4 + l^2/12)^2) + ((3000*l + 84000)^2*((h
 + t)^2 + l^2)*(h/2 + t/2))/(4*h^2*l^2*((h + t)^2/4 + l^2/12)^3)))/
(2*(18000000/(h^2*l^2) + (1166208191990803*2^(1/2)*(3000*l + 84000))/
(549755813888*l*((h + t)^2/4 + l^2/12)) + ((3000*l + 84000)^2*((h +
 t)^2 + l^2))/(8*h^2*l^2*((h + t)^2/4 + l^2/12)^2))^(1/2));
dL_db = p1 - p4 + (6*t*(l + 14))/125 -
 (504000*p11)/(b^2*t^2) - (1372*p12)/(625*b^2*t^3) +
 (1003250*10^(1/2)*b*p9*t^6*((2^(1/2)*5^(1/2)*t)/112 - 1))/
(49*(b^2*t^6)^(1/2));
dL_dp1 = b + s1^2 - 2;
dL_dp2 = s2^2 - h + 1/8;
dL_dp3 = h + s3^2 - 2;
dL_dp4 = h - b + s4^2;
dL_dp5 = s5^2 - t + 1/10;
dL_dp6 = t + s6^2 - 10;
dL_dp7 = s7^2 - l + 1/10;
dL_dp8 = l + s8^2 - 10;
dL_dp9 = s9^2 + (1003250*10^(1/2)*((2^(1/2)*5^(1/2)*t)/112 -
 1)*(b^2*t^6)^(1/2))/49 + 6000;
dL_dp10 = (18000000/(h^2*l^2) + (1166208191990803*2^(1/2)*(3000*l
 + 84000))/(549755813888*l*((h + t)^2/4 + l^2/12)) + ((3000*l
 + 84000)^2*((h + t)^2 + l^2))/(8*h^2*l^2*((h + t)^2/4 +
 l^2/12)^2))^(1/2) + s10^2 - 13600;
dL_dp11 = s11^2 + 504000/(b*t^2) - 30000;
dL_dp12 = s12^2 + 1372/(625*b*t^3) - 1/4;
dL_ds1 = 2*p1*s1;
dL_ds2 = 2*p2*s2;
dL_ds3 = 2*p3*s3;
dL_ds4 = 2*p4*s4;
dL_ds5 = 2*p5*s5;
dL_ds6 = 2*p6*s6;
dL_ds7 = 2*p7*s7;
dL_ds8 = 2*p8*s8;
dL_ds9 = 2*p9*s9;
dL_ds10 = 2*p10*s10;
dL_ds11 = 2*p11*s11;
dL_ds12 = 2*p12*s12;

Solver stopped prematurely.

fsolve stopped because it exceeded the function evaluation limit,
options.MaxFunctionEvaluations = 2800 (the default value).
```

```
l = 6.08 inches
h = 0.94 inches
t = 1.75 inches
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cost = $14.60

buckling load = 523386.08 psi
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