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Homework 5

Method

The goal of this problem was to minimize the total cost of the structure. The cost is defined as:

$$f(l, h, t, b) = 2.21lh^2 + 0.048 (bt (14 + l))$$

There were 12 inequality constraints in this system:

$$b \leq 2$$

$$h \geq 0.125$$

$$h \leq 2$$

$$h \leq b$$

$$t \geq 0.1$$

$$t \leq 10$$

$$l \geq 0.1$$

$$l \leq 10$$

$$P(b, t) \geq 6000$$

$$\tau_{shear}(l, h, t, b) \leq 13600$$

$$\tau_{bending}(b, t) \leq 30000$$

$$d_{end}(b, t) \leq 0.25$$

Where the functions  $P(b, t)$ ,  $\tau_{shear}(l, h, t, b)$ ,  $\tau_{bending}(b, t)$ , and  $d_{end}(b, t)$  are all defined in the problem statement.

The next step was to “convert” these 12 inequality constraints to equality constraints by adding the slack variables. In Matlab, these 12 equations look like this:

```
h1 = @(l,h,t,b,s1) b-2 + s1^2;
h2 = @(l,h,t,b,s2) 0.125-h + s2^2;
h3 = @(l,h,t,b,s3) h-2 + s3^2;
h4 = @(l,h,t,b,s4) h-b + s4^2;
h5 = @(l,h,t,b,s5) 0.1-t + s5^2;
h6 = @(l,h,t,b,s6) t-10 + s6^2;
h7 = @(l,h,t,b,s7) 0.1-l + s7^2;
h8 = @(l,h,t,b,s8) l-10 + s8^2;
h9 = @(l,h,t,b,s9) 6000-buckling_load(b,t) + s9^2;
h10 = @(l,h,t,b,s10) shear_stress(l,h,t,b)-13600 + s10^2;
h11 = @(l,h,t,b,s11) bending_stress(l,h,t,b)-30000 + s11^2;
h12 = @(l,h,t,b,s12) end_deflection(l,h,t,b)-0.25 + s12^2;
```

With these newly formed equality constraints I could then construct the Lagrangian equation with the following structure:

$$L(l, h, t, b, \bar{p}, \bar{s}) = f(l, h, t, b) + \bar{p}^T (\bar{h}(l, h, t, b, \bar{s}))$$

Using Matlab I then determined the 12 partial derivatives of the lagrangian. These are located at the end of this report. I then (attempted to) use fsolve to solve this nonlinear system of equations.

## Results

I used an initial guess of  $\{l, h, t, b\} = \{5, 1, 5, 1.2\}$ . The following values are the result of this mess:

```
l = 6.08 inches
h = 0.94 inches
t = 1.75 inches
b = 1.58 inches
cost = $14.60

buckling load = 523386.08 psi
shear stress = 9177.91 psi
bending stress = 103878.28 psi
end deflection = 0.26 inches
```

Only the physical dimensions met the required constraints as well as the buckling load and shear stress. The bending stress and end deflection however did not satisfy their required constraints.

## Discussion

I could not determine the reason what I was doing incorrectly for this problem. I checked over my work many times and fixed any errors I could find, but could never get `fsolve()` to converge. These results were the best values I could get as an answer since `fsolve()` could never determine a result within its maximum number of evaluations (2800). I suspect the failure to converge has something to do with the large differences in magnitude between values across the 28 unknown variables. The method I used to set up the system of equations seems to be correct, however I could not get `fsolve()` to converge despite countless attempts. I have attached the published code to the end of this report and submitted the `.m` files along with this submission.