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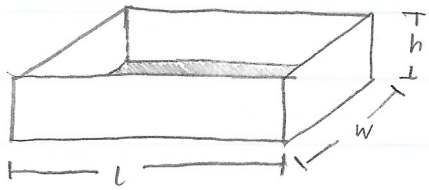
HW#2

2)



minimize: $f(l,w,h) = 2(lh) + 2(wh) + lw$

$= 2h(l+w) + lw$

$$\underline{f(x_1,x_2,x_3) = 2x_3(x_1+x_2) + x_1x_2}$$

Constraint $\quad x_1x_2x_3 = V \longrightarrow \quad h(x_1,x_2,x_3) = x_1x_2x_3 - V = 0$

$2x_3x_1 + 2x_3x_2 \qquad P\,x_1x_2x_3 - V$

$$L(\underline{x},p) = 2x_3(x_1+x_2) + x_1x_2 + p\,(x_1x_2x_3 - V)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x_1} &= 0 = 2x_3 + x_2 + Px_2x_3 \\ \frac{\partial L}{\partial x_2} &= 0 = 2x_3 + x_1 + Px_1x_3 \\ \frac{\partial L}{\partial x_3} &= 0 = 2x_1 + 2x_2 + Px_1x_2 \\ 0 &= x_1x_2x_3 - V \end{aligned}\right\} \text{Solve for } x_1^*\ x_2^*,\ x_3^*,\ P^*$$

$$P = -\frac{(2x_3+x_2)}{x_2x_3}$$

$$\frac{2x_3+x_2}{x_2\cancel{x_3}} = \frac{2x_3+x_1}{x_1\cancel{x_3}} \longrightarrow x_1(2x_3+x_2) = x_2(2x_3+x_1)$$

$$P = -\frac{(2x_3+x_1)}{x_1x_3}$$

$$\cancel{2}x_1x_3 + \cancel{x_1x_2} = \cancel{2}x_2x_3 + \cancel{x_1x_2} \longrightarrow x_1\cancel{x_3} = x_2\cancel{x_3}$$

$$\underline{\underline{x_1 = x_2}}$$

$$P = -\frac{(2x_1+2x_2)}{x_1x_2} = -\frac{4x_1}{x_1^2}$$

$$V = x_1^2x_3 \longrightarrow \underline{\underline{x_3 = \frac{V}{x_1^2}}}$$

$$0 = 2\frac{V}{x_1^2} + x_1 - \frac{4\cancel{x_1}^2}{\cancel{x_1^2}}\cdot\frac{V}{x_1^2} = 2\frac{V}{x_1^2} + x_1 - \frac{4V}{x_1^2} \longrightarrow -\frac{2V}{x_1^2} = -x_1$$

$$2V = x_1^3$$

$$\therefore \boxed{x_1^* = \sqrt[3]{2V}} \quad \boxed{x_2^* = \sqrt[3]{2V}} \quad \boxed{x_3^* = \frac{V}{(2V)^{2/3}}}$$