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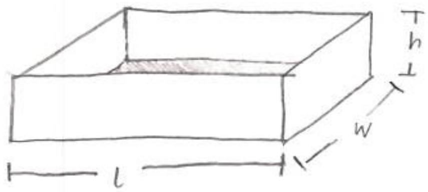
AE5222

HW#2

2)



Minimize: $f(l,w,h) = 2(lh) + 2(wh) + lw$

$$= 2h(l+w) + lw$$

$$\underline{f(x_1, x_2, x_3) = 2x_3(x_1+x_2) + x_1 x_2}$$

Constraint $x_1 x_2 x_3 = V \longrightarrow h(x_1, x_2, x_3) = x_1 x_2 x_3 - V = 0$

$$L(\underline{x}, p) = 2x_3(x_1+x_2) + x_1 x_2 + p(x_1 x_2 x_3 - V)$$

$$\left.\begin{aligned}
\frac{\partial L}{\partial x_1} &= 0 = 2x_3 + x_2 + p x_2 x_3 \\
\frac{\partial L}{\partial x_2} &= 0 = 2x_3 + x_1 + p x_1 x_3 \\
\frac{\partial L}{\partial x_3} &= 0 = 2x_1 + 2x_2 + p x_1 x_2 \\
0 &= x_1 x_2 x_3 - V
\end{aligned}\right\} \text{Solve for } x_1^*\ x_2^*,\ x_3^*,\ p^*$$

$$p = -\frac{(2x_3 + x_2)}{x_2 x_3}$$

$$p = -\frac{(2x_3 + x_1)}{x_1 x_3}$$

$$\frac{2x_3 + x_2}{x_2 \cancel{x_3}} = \frac{2x_3 + x_1}{x_1 \cancel{x_3}} \longrightarrow x_1(2x_3 + x_2) = x_2(2x_3 + x_1)$$

$$\cancel{2} x_1 x_3 + \cancel{x_1 x_2} = \cancel{2} x_2 x_3 + \cancel{x_1 x_2} \longrightarrow x_1 \cancel{x_3} = x_2 \cancel{x_3}$$

$$\underline{\underline{x_1 = x_2}}$$

$$p = -\frac{(2x_1 + 2x_2)}{x_1 x_2} = -\frac{4x_1}{x_1^2}$$

$$V = x_1^2 x_3 \longrightarrow \underline{\underline{x_3 = \frac{V}{x_1^2}}}$$

$$0 = 2\frac{V}{x_1^2} + x_1 - \frac{4\cancel{x_1^2}}{\cancel{x_1^2}} \cdot \frac{V}{x_1^2} = 2\frac{V}{x_1^2} + x_1 - \frac{4V}{x_1^2} \longrightarrow -\frac{2V}{x_1^2} = -x_1$$

$$2V = x_1^3$$

$$\therefore \boxed{x_1 = \sqrt[3]{2V}}$$

$$\boxed{x_2 = \sqrt[3]{2V}}$$

$$\boxed{x_3 = \frac{V}{(2V)^{2/3}}}$$