AE 5222 – Optimal Control of Dynamical Systems

Homework Submission Cover Page and Statement of Academic Honesty

I, _John O'Neill_ , submit the solution to Homework Problem _4_ .

My signature below affirms that all of the writing in this submission is my own work. Any reference material that I used to prepare this submission, including text or video resources, but excluding the lecture notes and videos provided on the Canvas site for this course, is properly cited.

To prepare this submission:

☐ I verbally collaborated with the following individuals (excluding *Piazza* discussions):

Currently enrolled in AE 5222: _____

_____

Not currently enrolled in AE 5222:_____

_____

■ I did not verbally collaborate with any other individual.

This submission reflects my individual effort and my own understanding of the course content.

I have read and I understand WPI's Academic Honesty Policy, and my conduct in preparing this submission has been in accordance with this Policy.

Signature: _____   Date: _04/20/2019_

Method

The goal for this problem was to minimize the weight of the truss system which is described with this formula:

$$\text{Weight} = 2\sqrt{2}x_1 + x_2.$$

There are a number of inequality constraints that needed to be met. All nine inequality constraints were converted to equality constraints with the inclusion of respective slack variables:

```
P    = 20;
f    = @(x1,x2)      2*sqrt(2)*x1 + x2;
h1   = @(x1,x2,s1)  P*(x2+sqrt(2)*x1)/(sqrt(2)*x1^2 + 2*x1*x2) - 20 + s1^2;
h2   = @(x1,x2,s2)  P/(x1 + sqrt(2)*x2) - 20 + s2^2;
h3   = @(x1,x2,s3)  P*x2/(sqrt(2)*x1^2 + 2*x1*x2) - 15 + s3^2;
h4   = @(A1,s4)      A1 - 5 + s4^2;
h5   = @(A2,s5)      A2 - 5 + s5^2;
h6   = @(A3,s6)      A3 - 5 + s6^2;
h7   = @(A1,s7)      A1 - 0.1 + s7^2;
h8   = @(A2,s8)      A2 - 0.1 + s8^2;
h9   = @(A3,s9)      A3 - 0.1 + s9^2;
```

With these newly formed equality constraints I could then construct the Lagrangian equation which has the following structure:

$$L(\bar{x}, \bar{p}, \bar{s}) = f(\bar{x}) + \bar{p}^T (\bar{h}(\bar{x}, \bar{s}))$$

Like in problem 3 I calculated the gradient of the Lagrangian then solved for all 23 states in Matlab using fsolve(). The derivative of the lagrangian with respect to x_1 and x_2 are very long, so I have attached the full gradient after this write-up. Here are the derivatives of the Lagrangian with respect to the 21 subsequent states:

```
dL_dA1 = p4 + p7;
dL_dA2 = p5 + p8;
dL_dA3 = p6 + p9;
dL_dp1 = (20*x2 + 20*2^(1/2)*x1)/(2*x1*x2 + 2^(1/2)*x1^2) + s1^2 - 20;
dL_dp2 = 20/(x1 + 2^(1/2)*x2) + s2^2 - 20;
dL_dp3 = (20*x2)/(2*x1*x2 + 2^(1/2)*x1^2) + s3^2 - 15;
dL_dp4 = A1 + s4^2 - 5;
dL_dp5 = A2 + s5^2 - 5;
dL_dp6 = A3 + s6^2 - 5;
dL_dp7 = A1 + s7^2 - 1/10;
dL_dp8 = A2 + s8^2 - 1/10;
dL_dp9 = A3 + s9^2 - 1/10;
dL_ds1 = 2*p1*s1;
dL_ds2 = 2*p2*s2;
dL_ds3 = 2*p3*s3;
dL_ds4 = 2*p4*s4;
dL_ds5 = 2*p5*s5;
dL_ds6 = 2*p6*s6;
dL_ds7 = 2*p7*s7;
dL_ds8 = 2*p8*s8;
dL_ds9 = 2*p9*s9;
```

Results

By solving for the 23 equations the optimal values of each state are determined:

```
x1* = 0.789    A1* = 0.1    p1* = 0.132    s1* = 0
x2* = 0.408    A2* = 0.1    p2* = 0        s2* = 2.315
               A3* = 0.1    p3* = 0        s3* = 3.105
                            p4* = 0        s4* = 2.214
                            p5* = 0        s5* = 2.214
                            p6* = 0        s6* = 2.214
                            p7* = 0        s7* = 0.006
                            p8* = 0        s8* = 0.006
                            p9* = 0        s9* = 0.006
```

It is apparent that the optimal length of the rods are x_1 = 0.789 units, and x_2 = 0.408 units, all with cross sectional areas of 0.1 square units.

Discussion

The constraints had an indirect effect on the beam lengths since the permissible stresses were related to the beam lengths. However it is apparent that the cross sectional areas were "clipped" in that in this case their optimal values were all equal to the minimum permissible area. Had there been no constraints on area it would have minimized to zero. The beam stresses and weight were not functions of area, so the only constraints would be the minimum permissible value, which is why the area will always try to be as small as possible.

```
dL_dx1 = 2*2^(1/2) - (20*p2)/(x1 + 2^(1/2)*x2)^2 + p1*((20*2^(1/2))/
(2*x1*x2 + 2^(1/2)*x1^2) - ((2*x2 + 2*2^(1/2)*x1)*(20*x2 +
 20*2^(1/2)*x1))/(2*x1*x2 + 2^(1/2)*x1^2)^2) - (20*p3*x2*(2*x2 +
 2*2^(1/2)*x1))/(2*x1*x2 + 2^(1/2)*x1^2)^2;
dL_dx2 = p1*(20/(2*x1*x2 + 2^(1/2)*x1^2) - (2*x1*(20*x2 +
 20*2^(1/2)*x1))/(2*x1*x2 + 2^(1/2)*x1^2)^2) + p3*(20/(2*x1*x2
 + 2^(1/2)*x1^2) - (40*x1*x2)/(2*x1*x2 + 2^(1/2)*x1^2)^2) -
 (20*2^(1/2)*p2)/(x1 + 2^(1/2)*x2)^2 + 1;
dL_dA1 = p4 + p7;
dL_dA2 = p5 + p8;
dL_dA3 = p6 + p9;
dL_dp1 = (20*x2 + 20*2^(1/2)*x1)/(2*x1*x2 + 2^(1/2)*x1^2) + s1^2 - 20;
dL_dp2 = 20/(x1 + 2^(1/2)*x2) + s2^2 - 20;
dL_dp3 = (20*x2)/(2*x1*x2 + 2^(1/2)*x1^2) + s3^2 - 15;
dL_dp4 = A1 + s4^2 - 5;
dL_dp5 = A2 + s5^2 - 5;
dL_dp6 = A3 + s6^2 - 5;
dL_dp7 = A1 + s7^2 - 1/10;
dL_dp8 = A2 + s8^2 - 1/10;
dL_dp9 = A3 + s9^2 - 1/10;
dL_ds1 = 2*p1*s1;
dL_ds2 = 2*p2*s2;
dL_ds3 = 2*p3*s3;
dL_ds4 = 2*p4*s4;
dL_ds5 = 2*p5*s5;
dL_ds6 = 2*p6*s6;
dL_ds7 = 2*p7*s7;
dL_ds8 = 2*p8*s8;
dL_ds9 = 2*p9*s9;
-------------------
x1* = 0.789
x2* = 0.408
A1* = 0.1
A2* = 0.1
A3* = 0.1
p1* = 0.132
p2* = 0
p3* = 0
p4* = 0
p5* = 0
p6* = 0
p7* = 0
p8* = 0
p9* = 0
s1* = 0
s2* = 2.315
s3* = 3.105
s4* = 2.214
s5* = 2.214
s6* = 2.214
s7* = 0.006
s8* = 0.006
```

```
s9* = 0.006
```

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