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Homework 4

Method

The goal for this problem was to minimize the weight of the truss system which is described with this formula:

$$\text{Weight} = 2\sqrt{2}x_1 + x_2.$$

There are a number of inequality constraints that needed to be met. All nine inequality constraints were converted to equality constraints with the inclusion of respective slack variables:

```
P    = 20;
f    = @(x1,x2)    2*sqrt(2)*x1 + x2;
h1   = @(x1,x2,s1) P*(x2+sqrt(2)*x1)/(sqrt(2)*x1^2 + 2*x1*x2) - 20 + s1^2;
h2   = @(x1,x2,s2) P/(x1 + sqrt(2)*x2) - 20 + s2^2;
h3   = @(x1,x2,s3) P*x2/(sqrt(2)*x1^2 + 2*x1*x2) - 15 + s3^2;
h4   = @(A1,s4)    A1 - 5 + s4^2;
h5   = @(A2,s5)    A2 - 5 + s5^2;
h6   = @(A3,s6)    A3 - 5 + s6^2;
h7   = @(A1,s7)    A1 - 0.1 + s7^2;
h8   = @(A2,s8)    A2 - 0.1 + s8^2;
h9   = @(A3,s9)    A3 - 0.1 + s9^2;
```

With these newly formed equality constraints I could then construct the Lagrangian equation which has the following structure:

$$L(\overline{x},\overline{p},\overline{s}) = f(\overline{x}) + \overline{p}^T(\overline{h}(\overline{x},\overline{s}))$$

Like in problem 3 I calculated the gradient of the Lagrangian then solved for all 23 states in Matlab using fsolve(). The derivative of the lagrangian with respect to x_1 and x_2 are very long, so I have attached the full gradient after this write-up. Here are the derivatives of the Lagrangian with respect to the 21 subsequent states:

```
dL_dA1 = p4 + p7;
dL_dA2 = p5 + p8;
dL_dA3 = p6 + p9;
dL_dp1 = (20*x2 + 20*2^(1/2)*x1)/(2*x1*x2 + 2^(1/2)*x1^2) + s1^2 - 20;
dL_dp2 = 20/(x1 + 2^(1/2)*x2) + s2^2 - 20;
dL_dp3 = (20*x2)/(2*x1*x2 + 2^(1/2)*x1^2) + s3^2 - 15;
dL_dp4 = A1 + s4^2 - 5;
dL_dp5 = A2 + s5^2 - 5;
dL_dp6 = A3 + s6^2 - 5;
dL_dp7 = A1 + s7^2 - 1/10;
dL_dp8 = A2 + s8^2 - 1/10;
dL_dp9 = A3 + s9^2 - 1/10;
dL_ds1 = 2*p1*s1;
dL_ds2 = 2*p2*s2;
dL_ds3 = 2*p3*s3;
dL_ds4 = 2*p4*s4;
dL_ds5 = 2*p5*s5;
dL_ds6 = 2*p6*s6;
dL_ds7 = 2*p7*s7;
dL_ds8 = 2*p8*s8;
dL_ds9 = 2*p9*s9;
```

Results

By solving for the 23 equations the optimal values of each state are determined:

x1* = 0.789	A1* = 0.1	p1* = 0.132	s1* = 0
x2* = 0.408	A2* = 0.1	p2* = 0	s2* = 2.315
	A3* = 0.1	p3* = 0	s3* = 3.105
		p4* = 0	s4* = 2.214
		p5* = 0	s5* = 2.214
		p6* = 0	s6* = 2.214
		p7* = 0	s7* = 0.006
		p8* = 0	s8* = 0.006
		p9* = 0	s9* = 0.006

It is apparent that the optimal length of the rods are x_1 = 0.789 units, and x_2 = 0.408 units, all with cross sectional areas of 0.1 square units.

Discussion

The constraints had an indirect effect on the beam lengths since the permissible stresses were related to the beam lengths. However it is apparent that the cross sectional areas were “clipped” in that in this case their optimal values were all equal to the minimum permissible area. Had there been no constraints on area it would have minimized to zero. The beam stresses and weight were not functions of area, so the only constraints would be the minimum permissible value, which is why the area will always try to be as small as possible.