# AE 5222 – Optimal Control of Dynamical Systems

## Homework Submission Cover Page and Statement of Academic Honesty

I, John O'Neill, submit the solution to Homework Problem 2.

My signature below affirms that all of the writing in this submission is my own work. Any reference material that I used to prepare this submission, including text or video resources, but excluding the lecture notes and videos provided on the Canvas site for this course, is properly cited.

To prepare this submission:

☐ I verbally collaborated with the following individuals (excluding *Piazza* discussions):

Currently enrolled in AE 5222: ______________________

______________________

Not currently enrolled in AE 5222: ______________________

______________________

■ I did not verbally collaborate with any other individual.

This submission reflects my individual effort and my own understanding of the course content.

I have read and I understand WPI's Academic Honesty Policy, and my conduct in preparing this submission has been in accordance with this Policy.

**Signature:** ______________________ **Date:** 04/20/2019

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AE 5222
Homework 2

Method

To find the dimensions of the rectangular box which would minimize the surface area I needed to set up a lagrangian which took into account the following constraint:

$$x_1 x_2 x_3 - V = 0$$

Where $x_1$ , $x_2$ , $x_3$ are the length, width, and height, respectively. My goal was to minimize the following equation:

$$f(x_1, x_2, x_3) = 2x_3(x_1 + x_2) + x_1 x_2$$

The resulting Lagrangian is as follows:

$$L(x_1, x_2, x_3, p) = 2x_3(x_1 + x_2) + x_1 x_2 + p(x_1 x_2 x_3 - V)$$

To find the minimum values of these four states the derivative of the Lagrangian with respect to each of the four states must be equal to zero. These equations are shown below:

$$\frac{\partial L}{\partial x_1} = 2x_3^* + x_2^* + p x_2^* x_3^* = 0$$
$$\frac{\partial L}{\partial x_2} = 2x_3^* + x_1^* + p x_1^* x_3^* = 0$$
$$\frac{\partial L}{\partial x_3} = 2x_1^* + 2x_2^* + p x_1^* x_2^* = 0$$
$$\frac{\partial L}{\partial p} = x_1^* x_2^* x_3^* - V = 0$$

Results

With four equations and four unknowns I could then solve for each of the four unknowns which would allow me to determine the dimensions that minimize the surface area:

$$x_1^* = \sqrt[3]{2V}$$
$$x_2^* = \sqrt[3]{2V}$$
$$x_3^* = \frac{V}{(2V)^{\frac{2}{3}}}$$

Discussion

Intuitively it makes sense that the length and width of the box would be equal when aiming to minimize the surface area of a box. The following plot illustrates the minimization of the surface area given the constraint on volume. It minimizes surface area assuming volume equals 1 unit:

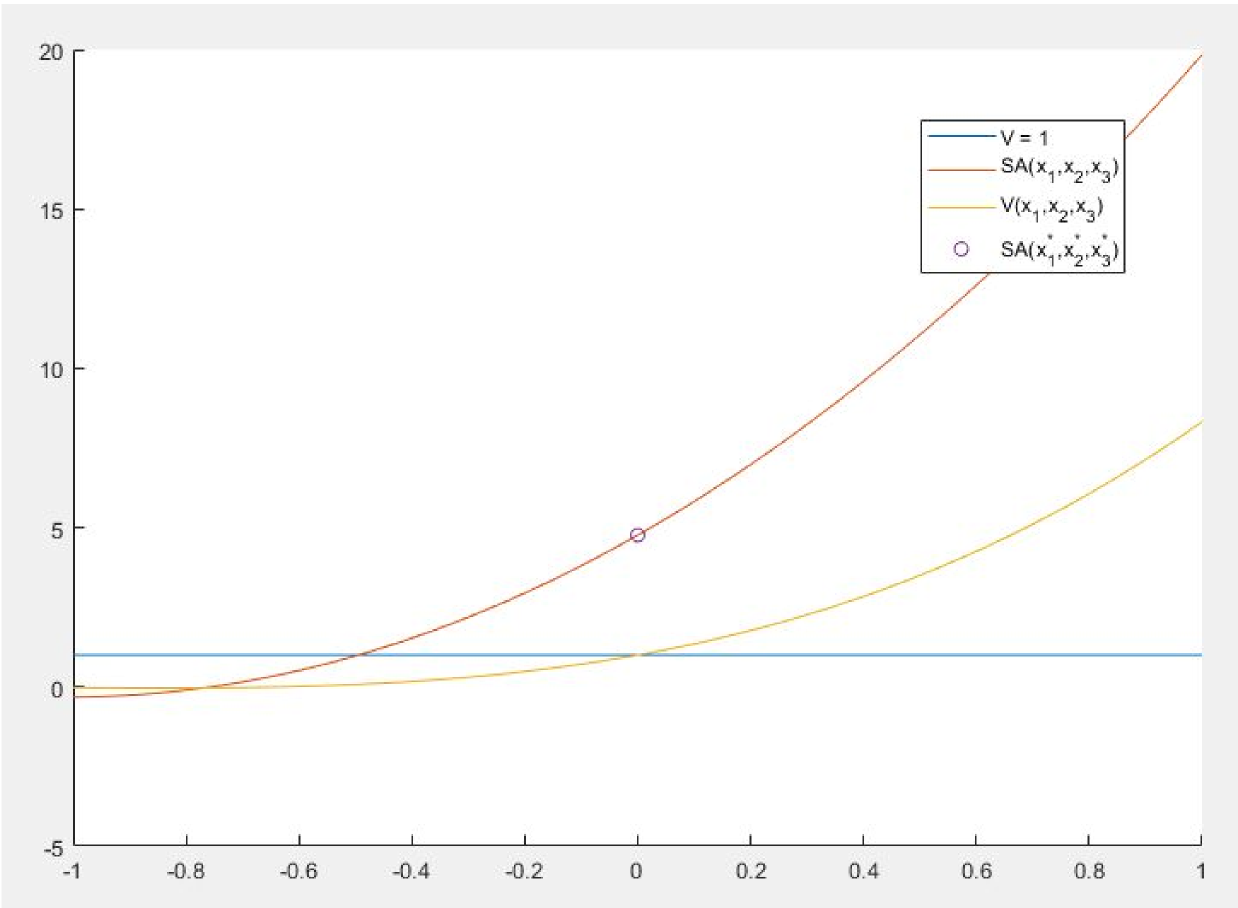


**Figure 1: Visualization of surface area minimization with volume constraint**