Jack O'Neill
AE 5222
Homework 3

Method

In order to minimize the function given the inequality constraints I needed to incorporate the constraint equations with respective slack variables. The following equations show the conversion of the given inequality constraints into equality constraints:

$$h_1 = x_1^2 + x_2^2 - x_3^2 + s_1^2$$
$$h_2 = 2 - \sqrt{x_1^2 + x_2^2 + x_3^2} + s_2^2$$
$$h_3 = x_3 - 5 + s_3^2$$

With these three equality constraints the Lagrangian can be constructed:

$$L(\bar{x}, \bar{p}, \bar{s}) = f(\overline{x}) + \bar{p}^T(\bar{h}(\bar{x}, \bar{s}))$$

To minimize the function, the gradient of L must be set to zero, and all states must be solved. The gradients of L are shown below:

```
dL_dx1 = 2*p1*x1 - 12*x1 + 3*x1^2 - (p2*x1)/(x1^2 + x2^2 + x3^2)^(1/2) + 11;
dL_dx2 = 2*p1*x2 - (p2*x2)/(x1^2 + x2^2 + x3^2)^(1/2);
dL_dx3 = p3 - 2*p1*x3 - (p2*x3)/(x1^2 + x2^2 + x3^2)^(1/2) + 1;
dL_dp1 = s1^2 + x1^2 + x2^2 - x3^2;
dL_dp2 = s2^2 - (x1^2 + x2^2 + x3^2)^(1/2) + 2;
dL_dp3 = x3 + s3^2 - 5;
dL_ds1 = 2*p1*s1;
dL_ds2 = 2*p2*s2;
dL_ds3 = 2*p3*s3;
```

Using Matlab fsolve() function I could easily calculate the optimal state matrix.

Results

The states which minimize the function f with the inequality constraints are shown as follows:

```
x1* = 2
x2* = 0
x3* = 2
p1* = 0.25
p2* = 0
p3* = 0
s1* = 0
s2* = 0.91
s3* = 1.732
```

Discussion

The results from this homework problem were realistic and unless there were any errors typing in equations in Matlab I am confident in these values. The three inequality states along with $\mathbf{x} \in \mathbb{R}^3_{\geqslant 0}$ can all quickly be confirmed to be satisfied with these output values.