Electronic Noise from Resistors

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Past experiments have documented electronic noise in the output voltage signals of circuits involving resistors and amplifiers [1]. However, concrete results have not been published quantifying the values of these noise signals. An exhaustive experimental set-up and analysis is required to isolate these noise values and generalize them for different resistors. The purpose of this experiment is to quantify resistor-induced noise in a simple circuit consisting of an amplifier and a resistor, as well as eliminate the amplifier-induced noise using Cross-Correlation. In analyzing the results of the resistor induced noise, potential improvements can be realized for decreasing electronic noise in circuits involving resistors.

Quantifying electronic noise in circuits is an important problem in electrical and embedded systems engineering. For example, many technologies use "Low-Noise amplifiers" to amplify low voltage input signals [1]. However, when these low voltages are amplified, there a significant amount of noise created that often corrupts the output signal from the amplifier [1]. When these signals become corrupted, it means there is an amount of noise that renders the information in the signals unreadable. There have also been documented cases of thermal induced noise from resistors that further contributes to the corrupted output signal [2]. Thus, it is important to understand and quantify both the thermal noise from resistors, as well as eliminate the noise from amplifiers in practical circuits. This paper will consider an experimental procedure and analysis aimed at quantifying this thermal noise that will be denoted as "Johnson Noise" in this report.

To help conceptually motivate the experiment, a mathematical derivation of thermal-induced Johnson Noise can be executed. Consider a copper wire with length L with some internal resistance. Standing Electromagnetic waves exist in the copper wire with frequency $f_n = \frac{nc}{2L}$ Where n denotes the nth mode and c denotes the speed of light. Because each photon inside an electromagnetic wave has an energy hf_n at each individual frequency f_n, if we consider l_n photons in each mode, there is total energy in each mode:

$$E_n(l_n) = l_n h f_n \tag{1}$$

The energy modes are independent from each other, meaning that energy in one mode does not transfer to another mode. Because the modes do not interact with each other, the states of the system can be broken down into each individual energy mode. A way to do this is using a partition function – like the partition function used in statistical mechanics – to describe all possible states in a system[3]:

$$Z = \sum_{i=1}^{\infty} e^{\beta E_i} \tag{2}$$

Where E_i is energy in state i, and $\beta = \frac{1}{k_B T}$. So if we are interested in the possible states inside a mode n, then Z_n becomes:

$$Z = \sum_{l_n=0}^{\infty} e^{-\beta l_n h f_n} = \frac{1}{1 - e^{\beta h f_n}} \tag{3}$$

$$\text{Using the relation: } \sum_{j=0}^{\infty} r^j = \frac{1}{1-r} \tag{4}$$

The partition function can be used to find the Johnson noise using the expected energy in a node $\langle E_n \rangle$. The expected energy is the total thermal energy of an isolated system at a given temperature:

$$\langle E_n \rangle = -\frac{1}{Z_n}\frac{\partial Z_n}{\partial \beta} = -\left(1 - e^{-\beta h f_n}\right)\frac{\partial}{\partial \beta} Z_n \tag{5}$$

Substituting the formula for the partition function from (3), Z_n, into (5) the average energy of a mode is:

$$\langle E_n \rangle = \frac{h f_n}{e^{\beta h f_n} - 1} \tag{6}$$

The results of this paper involve energies with very small $\beta h f_n$, so it is appropriate to take the following limit:

$$\lim_{\beta h f_n \to 0} \langle E_n \rangle = \frac{1}{\beta} \lim_{x \to 0} \frac{x}{e^x - 1} = \frac{1}{\beta} k_B T \tag{7}$$

The thermal energy over a frequency range can be expressed as $\Delta f = 2Lk_BT$ [3]. Therefore acknowledging the photons leave the length L wire with speed c the power per unit frequency flowing out of each end of the wire is:

$$\frac{P}{\Delta f} = \frac{2Lk_BT}{c} \cdot \frac{c}{2L} = k_B T \tag{8}$$

Finally using Ohm's Power law for voltage noise $P = \frac{V_{\text{noise}}^2}{4R}$, the voltage variance per frequency spacing Δf from the thermal energy of electrons or V_{noise} moving in the copper wire of Temperature T:

$$\frac{P_{\text{noise}}}{\Delta f} = k_B T \to \frac{V_{\text{noise}}^2}{\Delta f} = 4k_B T R \tag{9}$$

This variation in Voltage per frequency range $\frac{P_{\text{noise}}}{\Delta f}$ is the noise quantified in this experiment. By finding this relation, a percent error can be found comparing the experimentally determined Boltzmann constant k_B with

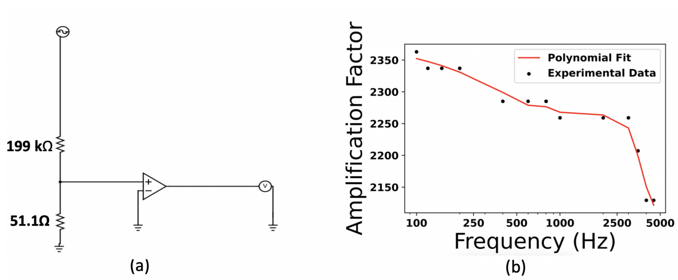

FIG. 1. (a) Schematic of the built voltage divider circuit. Using Resistors 9 $k\Omega$ and 51 Ω in series a voltage amplification value of $\frac{1}{3895}$ is created allows a manageable output voltage for the oscilloscope. (b) The fourth degree polynomial fit of the experimental data of amplification factors vs frequency using the voltage divider circuit.

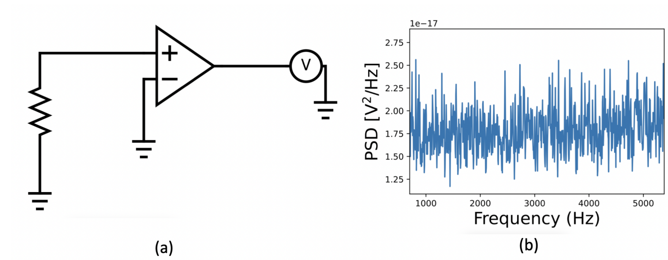

FIG. 2. (a) Basic schematic of the Johnson amplifier circuit that takes an input from a resistor and amplifies the noise using an amplifier. (b) Power Spectral Density graph that is used to calculate the total noise. The shown plot shows the data after dividing by the frequency dependent amplification in the amplifier and constant amplification factor from the sound card.

the theoretical value. However, to find the experimentally determined Boltzmann constant, a series of circuits and measurements must be completed.

As an experimental overview, since the noise measured from the Resistor is related to the input frequency, the Johnson noise can be estimated by taking noise measurements at varying resistors. Resistances from 100 Ohms to $2 \times 10^5\Omega$ were chosen to take noise measurements on. By varying the resistor inside the copper conductor, the value of the output noise changes accordingly, which through analysis can estimate the associated thermal noise for a given resistor. However, the Amplifier itself contributes noise which is frequency dependent. Therefore, to isolate the resistor induced noise, the total noise measured in the circuit needs to be standardized by the gain contributions from the Amplifier as well as the Sound Card components.

The Sound Card amplification factor was found by directly feeding an AC function generator into the Sound Card, and dividing the output voltage by the input voltage. Because the Sound Card amplification is independent of frequency in the range of 1000-10000 kHz, the amplification factor was a constant value. The method used to find the peak to peak voltage for the Sound Card was to fit a sine wave to an output voltage signal, and computationally calculate the amplitude of the sine wave. Then by dividing the peak to peak voltage of the Sound Card output voltage signal by the input voltage signal from the AC function generator, the Sound Card voltage amplification was found to be 8.9.

Quantifying the amplifier gain value was more complex for two reasons. One was because the amplifier increased the input voltage on the order of 10^3, a very small input voltage was required for a reasonable output voltage. The solution was to construct a voltage divider circuit that directs a voltage of a factor of roughly 10^{-3} from the input voltage to the amplifier [Fig. 1a]. The second problem was that the amplifier was frequency dependent, meaning multiple voltage amplification factors were calculated for different frequencies. The solution was to take multiple amplification factor measurements for different frequencies, and fit a fourth order function to the amplifier gain values [Fig. 1b]. The actual data collection method involved feeding a BNC connector from the amplifier to an oscilloscope, and utilizing the machine's auto function and tracers to find the peak to peak voltage of the output amplifier voltage at a given frequency. Due to irregularities in data collection on extreme frequency ends of the amplifier, the Johnson noise analysis was confined to the range of roughly 100Hz to 5000Hz [Fig. 1b].

After the amplification gain factor for the amplifier and Sound card were configured, a second circuit was constructed to take Noise measurements [Fig. 2a] There are 6 main components that makes up the circuit used to measure the total noise: a resistor, a copper conductor, a voltage amplifier, two BNC wires, a sound card, and a computer. The resistor was placed inside the copper conductor that has a BNC connector and the unit acts as a large resistor. The amplifier then amplifies the resistor-conductor unit using a 3 Volt battery connected to the amplifier, and a second wire connects the amplifier to the Sound Card. The Sound Card transcribes the input signal into a readable file that the computer can read and perform analysis on.

This data was initially a WAV file that was cleaned and read as a voltage over time signal. To get the noise over multiple frequencies, a Fourier transform was taken that transformed the data into a Power Spectral Density Graph (PSD). By dividing this PSD graph by the square of the Sound Card amplification gain and the frequency dependent amplifier gain, data was produced that represented the noise for different frequencies [Fig. 2b]. After graphically examining the gain normalized PSD graph for a given resistor, the noise was isolated for frequencies within 1000-5000Hz that had a relatively constant Power Spectral Density value [Fig. 2b]. The average PSD was then taken for those frequencies, resulting in an 'aver-

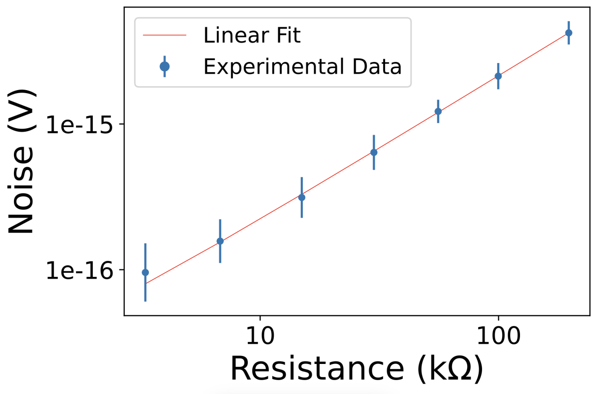

FIG. 3. Graph of the raw experimental data with a cross-correlated linear regression fit. The slope of the regression line was used as an estimate for the Johnson Noise value at room temperature.

age noise', that represents that total noise of the system. This process was repeated over multiple different resistors to produce a relation that represents the total noise for different resistors. But before the Johnson Noise was estimated, it was first isolated from the Amplifier noise.

Cross-Correlation was used to isolate the Resistor Noise from the Amplifier Noise [4]. The voltage noise from the left and right amplifiers was broken into the Johnson noise and the respective side's amplifier noise (10)(11). By performing a Fast Fourier Transform on $V_L(t), V_R(t)$ to get the left and right channel noise: $S_L(f), S_R(f)$, the Johnson noise $S_J(f)$ should be the noise that gets eliminated in the difference of $S_L(f) - S_R(F)$. Therefore, $S_J(f) = S_{\text{total}} - (S_L(f) - S_R(F))$, which allows the calculation of just the Johnson Noise.

$$V_L(t) = V_J(t) + V_{\text{Amp,L}}(t) \tag{10}$$
$$V_R(t) = V_J(t) + V_{\text{Amp,R}}(t) \tag{11}$$

After computationally performing the cross correlation, the Johnson Noise was fit using a linear regression fit [Fig. 3]. The slope of the regression line was calculated to be $2.137 \cdot 10^{-20}$. This denotes Johnson noise at any given Resistance given by (9). Dividing the slope by $4T$ where Temperature T is given by room temperature $(293K)$, the experimentally determined Boltzmann constant k_B^* was found to be $1.79 \cdot 10^{-23}$. The relative error is then 0.297, or a relative accuracy of 0.703.

The relative error of 0.297 is significant enough to warrant concern. If correct, the deviation signals that the experimental noise from resistors is not completely described by the statistical mechanical derivation derived earlier (9). Specifically, the voltage variance V^2_{noise}in a given frequency spacing Δf is not solely $4k_BTR$, but likely another negligible term. However, this suggestion demonstrates an inconsistency with the first law of thermodynamics, because the supposed difference from the expected voltage deviation (9) implies a separate noise source – and thus energy source – from the standing electromagnetic waves in the copper wire. More likely, there has been an experimental step that resulted in an unexpected consequence that affected the final Johnson noise value.

One potential source of experimental error was the cutoff frequency range used to determine the amplification function [Fig. 1b]. The maximum 5000Hz cutoff was chosen for two reasons. One was to minimize any potential errors from the Sound Card in processing high frequencies. Another was due to inconsistencies in higher frequencies Voltage measurements using the oscilloscope. However, the decision to forego measuring higher frequencies may have been erroneous, and resulted in ignoring a larger swath of data that could have improved the derived Boltzmann factor.

Another possible source of error may be from the cross-correlation procedure. In the cross-correlation steps, the procedure assumed the Johnson Voltage variation is identical is each channel, and the Amplifier Noise can be eliminated by separating the left and right channels. Therefore, if the noise on one channel somehow got slightly corrupted it would fail to eliminate the amplifier noise and result in non-Resistor noise reported as Johnson noise. Even a small preservation of amplifier noise could impact the final Johnson Noise significantly enough to cause a percent error of 0.3.

However, there are still clear applications that can be made from the results of this paper. For example, as mentioned earlier there is a persistent problem in electrical engineering of minimizing noise from resistors as well as from low-voltage amplifiers [1] [2]. The results of this experiments strongly suggest that lower resistances in circuits produce lower noise outputs from electronic signals involving resistors. These results could be applied in practice by building circuits with low resistances, which should decrease the amount of noise in the output signal. They also show that at least a portion of electric noise from amplifiers can be removed using cross-correlation. Applied in practice, the problem of limiting noise from amplifiers if not solved, can be mitigated.

Future research into Johnson noise can be focused into reproducing the experiment with different equipment and a larger variance of resistors. Another potential improvement could be in performing Johnson Noise measurements in varying temperatures as the statistical mechanical relation (9) suggests that Johnson noise varies linearly with temperature. Therefore resistors kept at low temperatures should have lower internal Johnson noise than resistors at higher temperatures.

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