Supporting K-12 Student Development of Mathematical Argumentation

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Comprehensive Exam Question

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Original Prompt: It is the aim of this literature review to summarize major tenets and research to support K-12 students’ development of mathematical argumentation, emphasizing the social practice of student collaboration, largely through interpersonal discourse.

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The ability for students to develop their own mathematical arguments at the K-12 level remains a priority in mathematics education. Scholars agree that developing mathematical arguments ensures true understanding when one can convince oneself and others of one’s mathematical explanation (Ellis, 2007; Cáceres, Nussbaum, Marroquín, Gleisner, & Marquínez, 2017). Additionally, as evidenced by the National Council of Teachers of Mathematics principles and standards for school mathematics (2000) and the Common Core State Standards Initiative (2010), the eight mathematical practice standards in which students should consistently engage include the development of mathematical arguments and the critique of others’ arguments. A major factor in creating the space for students to develop and critique mathematical arguments with one another is the dynamic of social interaction in the mathematics classroom. Student participation in a mathematical learning community in a classroom is dependent on the culture. Civil and Hunter (2015) found it was necessary to have an open atmosphere that allows social talk and humor, so that students feel comfortable to share, make mistake, and engage in dialogue about the mathematical learning. To further the mathematical learning, Rojas-Drummond and Zapata (2004) state that this opened the door for students to engage in exploratory talk, where they felt free to examine their own opinions, observations, and explanations of the mathematics. In emphasizing the use of students’ interpersonal discourse with one another, researchers have begun to study how students engage in healthy and open dialogue focused on the mathematics to create opportunities that advance student understanding and thinking towards mathematics through the lens of mathematical argumentation.

Background of the Study

There remain connections, then, between student and educator understanding of a mathematical argument and the ways in which classrooms are positioned to support student development of these arguments. The mathematics education community seems to agree upon and have substantial research on the importance of mathematical argument development at the K-12 level (Brown, 2017; Byrne, 2013; Yee, Boyle, Ko, & Bleiler-Baxter, 2017). However, a dearth of research remains concerning the means in which students support the building of their knowledge of mathematical arguments through interpersonal communication. This study analyzes both the conceptualization of mathematical arguments and how K-12 students use their own discussions to advance in critical thinking and deeper cognitive mathematical arguments. Further, the project will not only discover research-based facts for the elements of argumentation at the K-12 level, particularly for mathematics classrooms, but also the pedagogical approaches that K-12 mathematics teachers can take towards regular implementation of argumentation in their classrooms. Finally, as a result of collaborative discourse, students will begin to grow in their autonomous thoughts about the mathematics and thus develop their own valid mathematical arguments.

Theoretical Framework

The situated learning lens is used in this study to understand how students learn. Using this perspective, we seek to understand learning in the context in which it happens. Particularly, this research study emphasizes social participation of students in the small group discussions, so they consider social norms and the culture of the classroom as they participate in their argument development. All of these factors impact their learning in the K-12 setting (Anderson, Greeno, Reder, & Simon, 2000). Learning mathematics in the context of the standards of mathematical practice is a collaborative process, and it should be studied within the contexts in which it is occurring, particularly for K-12 education. This project study of small group discourse in collective argumentation requires participation in the mathematical learning community. The community can influence the discourse that supports student learning.

Further, with the emphasis on discourse of the students, this research takes on a discursive framework, narrowing in on the elements of discussion that students use with one another in their collaborative small groups as they discuss, argue, and critique mathematical concepts with their own ideas of mathematical facts and evidence.

Finally, both the situated learning and discursive framework fall within a greater idea of social constructivism, in which student learning is formed through the interactions that students have with the content in the classroom settings. The foundational learning theory of constructivism provides the necessary lens to look at a deeper, more refined look of learning in this research in the overlapping ideas of the situated learning lens and discursive learning framework.

**Purpose**

The purpose of this review, then, is to summarize published research on major tenets and practices that support both K-12 students’ understanding and development of mathematical arguments, emphasizing the social interaction of student collaboration through interpersonal discourse. The goal of this review is the begin to dissect the research base on K-12 mathematics instruction that supports the conceptualization of arguments for K-12 mathematics as well as the student development of mathematical argumentation, with the hopes of making sense of and identifying the gaps in the research concerning the ideas of mathematical argumentation for K-12 education and the impact of the social practice of interpersonal discourse among students. Therefore, implications for future areas of research will also be provided. Particular attention will be given to contexts and classroom settings in which research studies about student argumentation and mathematical argumentation were conducted, student collaboration and the interpersonal discourse among students, and general effects or outcomes of argument development for K-12 students on their overall mathematical understanding and learning of mathematics.

Review of the Literature

Student creation of mathematical arguments has been established as a primary focus in K-12 mathematics classrooms by national organizations and the recent creation of national standards for mathematical practices. The National Council of Teachers of Mathematics (1989) recommended that reasoning and proof should regularly be incorporated in K-12 classrooms, which was emphasized again in 2010 by the Common Core State Standards (CCSS) publication of eight mathematical practice standards in which students should consistently engage (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The Common Core State Standards specifically define an argument as the ability to make both claims and counterclaims, develop them, and link them to reasons and evidence in a cohesive format that ends with a logical conclusion (Shivers, Levenson, & Tan, 2017). Although argumentation has been established as a recommendation for K-12 mathematics, because several mathematics curricula and teaching practices have associated the concept of proof and argumentation with upper levels of schooling (Stylianides & Stylianides, 2009), classroom teachers still lack in fostering student development of valid mathematical arguments in a way that satisfies these standards. The review of the literature considers the definition of argumentation and its application at the K-12 level, including the components of an argument and an analysis of the purpose of arguments in K-12 education. Specifically, the review emphasizes mathematics classrooms and the environments that support student learning and understanding of mathematics through proof and argumentation.

**Argumentation in K-12 Education**

The ability for students to develop arguments in general has consistently become a regular mode of learning for K-12 classrooms. The ability to create, defend, and challenge others’ arguments points to the idea of discovering the truth (Joshi, 2016), which lies foundational to academic learning for K-12 students. Joshi (2016) expounds on this idea of truth as the cornerstone of education in arguments by stating that beliefs come from arguments, and the students’ belief directly influences student knowledge and their learning. Further, the act of argumentation develops student conceptual understanding and critical thinking, both skills that are critical in all academic realms, but particularly STEM fields such as mathematics education (Nussbaum, 2011). In looking at how argumentation has developed throughout various disciplines of K-12 education, Nussbaum (2011) points generally to the idea that argumentation, psychologically, is more focused on the process and the social interaction that students engage in to both construct and critique arguments. The emphasis on a social structure and the social interaction of students with one another lays foundation to the value of student discussion with one another to develop arguments for learning at the K-12 level. To go deeper into this idea, O’Keefe (1982) distinguishes between two different senses of the word “argument,” where one considers the idea of producing a final decision or statement as an argument and the other refers to the social processes in which students (at minimum of two) must engage in a dialogue to both construct and critique their arguments. Considering both of these definitions of the word “argument,” mathematics education researchers and scholars recognize the value of the process of the formation of an argument through the dialogue students have to construct and critique their final decisions or considerations about mathematical truths. Different types of arguments have been defined and determined throughout educational psychological research, but the importance of recognizing the role that dialogue plays in the process of development an argument is crucial to K-12 education and mathematics and STEM education.

Much argumentation research refers back to the purpose of argumentation at the K-12 level. Since one major purpose of argumentation is analytical understanding, one of the most widely accepted models of argumentation is Toulmin’s model, which breaks down arguments according to an analytical framework while also helping judge the strength of an argument and describing the claims and explanations that qualify the strength of an argument (Nussbaum, 2011). The philosopher Stephen Toulmin first introduced his model in his book *The Uses of Argument*, where he provides the six major components of an argument: claim, grounds, warrants (linking grounds to claims), backing for warrants, rebuttals, and modal qualifiers (Nussbaum, 2011). These components of Toulmin’s model are the driving forces behind future research in mathematics education’s implementation of argumentation in several different manners, as the research emphasizes students socially interacting with one another in dialogue with one another to create and critique arguments according to these six major components of an argument of Toulmin’s model.

To elaborate on Toulmin’s model, Brockriede and Ehninger (1960) identify the major components of Toulmin’s model and the role that they play in advancing student critical thinking and thus understanding. Firstly, Brockriede and Ehninger (1960) recognize that Toulmin defines an argument as movement from accepted data to a claim through a warrant. The necessity of believing accepted data and having to make a claim through evidence gained in the warrant encourages students to consider the truth as they certify assumptions and approach their understanding from the viewpoint of having to be able to justify and defend what is known as truth. At the heart of this process, students are beginning to learn how they must think on their own, carefully analyzing various claims of truth and making connections between different types of information to create a logical connection from data to a claim. Particularly concerning mathematics education, students approach making mathematical proofs as arguments as they consider the truths that believe and know in geometry, number skills, and algebra, to name a few (Metaxas, Potari, & Zachariades, 2016). By developing the skills of justifying, challenging, counterchallenging, and conceding, students are able to improve the quality of an argument in that they are offering their own warrants and backings by engaging in the process of argumentation (Metaxas et al, 2016). The idea of using counterarguments or rebuttals, which is the fifth of the six components of Toulmin’s model of arguments, challenges students to find exceptions to what they have already considered and the data that they have accepted and utilized for their claims, requiring students to more deeply analyze the truths and data that they have considered and other ways to conceptualize this information. The different elements of Toulmin’s model directly incorporate into the critical thinking that lies at the heart of mathematics education.

Further, research in terms of winning oral arguments in dialogue and speech has elaborated on the credibility of Toulmin’s model for argumentation. Brockriede and Ehninger (1960) make specific claims to the value of Toulmin’s model because the components require backing of arguments and rationale, which are the major components that students engage in to develop the critical thinking and the conceptual understanding that lay at the heart of K-12 education, particularly in mathematics education. As students must back their arguments with logic and consider other opinions and viewpoints through proposed rebuttals, students must engage in conversation with another to hear another opinion. Nussbaum (2011) states that Toulmin’s model was first used in the formation of arguments in speech and communication courses, but the same principles of development of claims and then defense of rebuttals applies in mathematics classrooms.

In looking at other disciplines as they approach an appropriate model for argumentation in K-12 education, the main tenets of teaching an argument still lie in a student’s interaction with both the content material and his or her classmates. The Perception, Interpretation, Expression model, or PIE model, reflects how students’ change their minds as they develop, critique, and modify an argument after the intervention of working with one another (Shivers et al, 2017). This process recognizes that students initially conceive and develop an argument in phases. In the first phase, students perceive and describe what is presented before them (Shivers et al., 2017), which in mathematics argumentation, is the known and accepted data or facts. The second phase is when the students build the argument based on their own perceptions and descriptions, which is reflected in Toulmin’s necessity of warrant to back the arguments that students develop in mathematics classes (Shivers et al., 2017). Finally, the final phase occurs when students critique, revise, and structure their arguments differently as a result of their creative thinking and interactions with the content and with one another, particularly as they justify their ideas so that their argument is considered successful, or grounded in clear reason and supported claims (Shivers et al., 2017). This model recognizes that argument do not all have to look the same, even if students do progress through this model of perception, interpretation, and expression equitably. This is important to note in the field of mathematics education because a broader understanding of the phases of argumentation that allows students to meet the criteria of Toulmin’s model creates the opportunity for mathematics education to occur at all levels in K-12 education.

Underlying Toulmin’s model and the PIE model of argument development, collaboration of students as they utilize their own discourse to perceive information, interpret different accepted information and data in mathematics, and express their own thoughts and understanding via mathematical arguments. Collaboration, then, remains a necessary process that allows students to view their own beliefs and actions differently and, according to Toulmin’s model, modify their arguments as necessary as they develop a deeper understanding via critical thinking and collaborative work.Combining Toulmin’s model with the PIE model, this model of argumentation was used for the purposes of visual critiques in artistic expression through art and digital media classes, but the social process of argumentation remained vital to the student development of critical thinking and deeper conceptual understanding of the content material.

As a final characteristic of teaching argumentation in K-12 schooling in general throughout all disciplines, Shivers and colleagues (2017) recognize the power of argumentation development across different modes of student expression to best benefit the students’ understanding and learning. Firstly, Shivers recognizes that writing is one mode that helps students develop argumentation skills, but it reiterates that writing should point to the overall goal of student thinking, which is what the student collaborative oral arguments does to support their writing. Secondly, Shivers (2017) explains that argumentation through art education required students to persuade others of their points of view and their knowledge of artistic qualities, which lies at the heart of the definition of argumentation in the context of mathematics education. Finally, looking at the more practical approach of how to incorporate argumentation skills in the K-12 setting, Shivers (2017) agrees that a more open-ended task without a finite correct single answer for which the teacher is looking is best to promote student discovery and the dialogue necessary for rich dialogue among K-12 students. These tenets of argumentation of art education ring true for STEM education, as all education at this level aims to produce critical thinkers and deep conceptual understanding of their discipline and content material.

**Arguments in the Mathematics Classroom**

With a foundation in recognizing what are the values and the teaching strategies appropriate for K-12 argumentation, research has identified certain characteristics or aspects of arguments specifically for mathematics classrooms that drive pedagogical practices and student learning in today’s K-12 mathematics classrooms. In regard to mathematics arguments, many researchers, scholars, and teachers agree that math arguments largely also lie in the field of mathematics proofs. Proof is one of the core aspects of mathematical activity, which is partially why math argumentation and proof has become such a priority in national standards and expectations today (Ball, Hoyles, Jahnke, & Moshovitz-Hadar, 2002; National Governors Association/Council of Chielf State School Officers, 2010; Sowder & Harrel, 1998). As an example, argumentation schemes have been studied with the use of the components of Toulmin’s model to study how students have moved from a simple example in a proof to a more rigorous, procedural algorithm to make a more advanced proof or mathematical argument (Metaxas, Potari, & Zachariades, 2016). Even further, Harel and Sowder (1998) developed a framework for proofs to help qualify the cognitive level of student mathematical proofs, which reflects the characteristics of an advanced mathematics argument according to Toulmin’s model. Stylianides & Stylianides (2009) explain the analytical framework for reasoning-and-proving that supports student understanding of the development of a viable mathematical argument through the process of reasoning through a mathematical problem and proving its validity via various components of a mathematical argument. The value of this framework is the clear distinction of the qualities of a good mathematical argument that meets the criteria of an argument according to Toulmin’s model within the context of K-12 mathematical learning.

In this framework, Harel and Sowder (1998) recognizes that students will often rely on specific examples as an entry point into thinking about the field of mathematics and the content proposed in this claim. Known as an empirical argument, Stylianides & Stylianides (2009) acknowledges the weakness in a solely empirical argument as it does not meet the criteria of an argument being true for all cases mathematically, which elicits the necessity of backing of an argument in Toulmin’s model. They continue in acknowledging that K-12 mathematics teachers, particularly at the elementary level, must also consider how best to teach students to move beyond an empirical argument. A secondary type of argument that lacks in the evidence required in a strong mathematical argument is one that appeals to authority, where students rely on authority of someone who is considered higher or more knowledgeable in the field of mathematics or upon accepted data itself as the warrant or evidence behind the claims in their arguments (Harel & Sowder, 1998). Although the students are seeking to make a conclusive statement about the mathematical claim, the lack of strong evidence prevents the argument from fully satisfying the components of a strong mathematical argument according to Toulmin’s model.

Ascending the hierarchy of mathematical arguments, Harel & Sowder recognize other types of arguments that more robustly satisfy the components of a vaid mathematical argument, but are lacking only in some of the validity of the evidence. In this next level, students either inaccurately use accepted mathematical information to create a generalized mathematical argument or proof, or they use information or data that is largely inaccessible to the appropriate mathematics for their content and level, such as relying on calculations via technology or graphical representations that do not prove the algebraic reasoning or thinking appropriate for the class (Harel & Sowder, 1998). Arguments that use appropriate mathematics are considered arguments but not “proofs,” which defines the highest level of an argument according to Harel and Sowder (1998). The most advanced level of an argument is one that correctly and accurately uses the evidence that is appropriate to the class and level of the student (Harel & Sowder, 1998). These types of arguments described in this framework have been hierarchized in recent research (King & Campbell, in press). Descriptions of the levels and types of arguments are seen in Figure 1 below.

**Figure 1**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Level 1** | | **Level 2** | | **Level 3** |
| Empirical | Appeal to Authority | Unsuccessful Valid Argument | Valid Argument, Not a Proof | Proof |

Further research has been completed to better understand the students’ conceptual understandings and misunderstandings so that practitioners can address these understandings to better pinpoint appropriate pedagogical tools to teach these skills, as well as to pinpoint misunderstandings that students can address (Healy & Hoyles, 2000). The study by Healy and Hoyles is unique but powerful in its application to mathematics education to better address how to support both K-12 educators and students in the development of student mathematical arguments.

Because Healy and Hoyles (2000) researched student perceptions of mathematical arguments, they were able to dive deeper into the reasoning for and power behind different types of arguments that make argumentation more accessible to the K-12 student population. They recognized formal, algebraic, and narrative proofs (Healy & Hoyles, 2000) all as valid forms of mathematical representation of student learning when developing their own arguments, helping underline the necessity of both evidence according to Toulmin’s model and student social interaction as they engage in narrative dialogue to develop their own thoughts to provide warrant and evidence to their arguments. As evidenced by this research and the concept of mathematics arguments, the social practice of student discourse is necessary for K-12 mathematics argumentation.

**Looking Forward to the Future: Implications for Future Research and Practice**

Although much research has been completed on the frameworks for argumentation in K-12 mathematics and the usage of mathematical arguments to support student learning, there are still more implications for future research and practice that remain. Because argumentation has only been widely accepted and nationally encouraged recently in the past few decades, the mathematics education research realm still seeks answers to the questions about what causes students to develop deeper conceptual understanding and critical thinking via argumentation. Nardi and Knuth (2017) only recently questioned how to change classroom cultures to support mathematical proofs and arguments that produce long-lasting effects in student understanding. For the purposes of future research, longitudinal studies and studies of student motivation and engagement to create and critique viable mathematical arguments could contribute to practitioners’ and researchers’ understanding of what makes mathematical argumentation successful in K-12 classrooms.

Future research that emphasizes the use of student discourse with one another also stands to narrow in what aspects of the dialogue that students have with one another support student advancement of mathematical arguments. In referring to Toulmin’s model’s components of argumentation, future research could emphasize the actions, interactions, and reactions that students have with one another in oral dialogue as they initially create, listen to and critique, and modify arguments in open-ended mathematical problems that push students to critically think and analyze various mathematical concepts across all K-12 levels. It is important to note that argumentation can be encouraged and use through all levels of K-12 mathematics when scaffolded correctly. As an example, elementary school mathematics teachers and researchers such as Rumsey & and Langrall (2016) have promoted mathematical argumentation even at the elementary level with language supports, specified conditions, and manipulation of familiar content to the unfamiliar, to name a few concrete examples of teaching practices. Continued research studies on the insights of teacher practices to support K-12 student development of mathematical arguments can focus on the student learning, the elements of the discourse in the arguments, the elements of the arguments, and the teacher practices to promote student learning through mathematical argumentation.

**Conclusion**

This literature reviewed served the purposes of understanding student development of mathematical arguments and the research that supports current findings for both the understanding of mathematical argumentation at the K-12 level and the teaching practices and pedagogical implications of mathematical arguments for students in K-12 mathematics. It is important to lay a foundation to the understanding of what are the value and the components of an argument in K-12 education, as seen by the extensive research and backing for Toulmin’s model of argumentation. By studying various disciplines, Toulmin’s model and the elements of his model of argumentation help guide the framework for research studies on mathematical argumentation that lead to better understanding of how students interact and thus gain deeper conceptual understanding and a greater ability to critical think through mathematical tasks. The Toulmin’s model and other similar studies of argumentation models and frameworks for K-1 education point to the need for students to engage in discourse with one another to meet the different aspects of a viable argument. Future research will help better understand how students utilize their discussions with one another to create a better mathematical argument and how teachers can support their teachers in creating and critiquing more viable mathematical arguments.

References

Anderson, J.R., Greeno, J.G., Reder, L.M., & Simon, H.A. (2000). Perspectives on learning, thinking, and activity. *Educational Researcher, 29*(4), 11-13. Retrieved from https://utk-almaprimo.hosted.exlibrisgroup.com/primo-explore/fulldisplay?docid=TN\_ericEJ617280&context=PC&vid=01UTK&search\_scope=OneSearch&tab=default\_tab&lang=en\_US

Ball, D., Hoyles, C., Jahnke, H., & Movshovitz-Hadar, N. (2002). The teaching of proof. *Paper presented at the international congress of mathematicians.*

Brockriede, W., & Ehninger, D. (1960). Toulmin on argument: An interpretation and application. *Quarterly Journal of Speech, 46*(1), 44-53. doi:10.1080/00335636009382390

Brown, R. (2017). Using collective argumentation to engage students in a primary mathematics classroom. Mathematics Education Research Journal, 29, 183-199. Retrieved from https://doi.org/10.1007/s13394-017-0198-2

Byrne, M. (2013). Cooperative learning and traversing the continuum of proof expertise: Preliminary results. In *Proceedings of the 16th Annual Conference on Research in Undergraduate Mathematics Education*.

Cáceres, M., Nussbaum, M., Marroquín, M., Gleisner, S., & Marquínez, J. (2017). Building arguments: Key to collaborative scaffolding. *Interactive Learning Environments, 26*(3), 355-371. doi:10.1080/10494820.2017.1333010

Civil, M. & Hunter, R. (2015). Participation of non-dominant students in argumentation in the mathematics classroom. Intercultural Education, 26(4), 296-312. doi:10.1080/14675986.2015.1071755

Common Core State Standards Initiative (CCSSI). 2010 Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math-Standards.pdf

Ellis, A. (2007). Connections between generalizing and justifying students’ reasoning with linear relationships. Journal for Research in Mathematics Education, 38(3), 194-229. Retrieved from https://www.jstor.org/stable/30034866

Harel, G & Sowder, L. (1998). Students’ proof schemes: Results from exploratory studies. In A. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.). *Research in collegiate mathematics education, III* (pp. 234–283). Washington, DC: Mathematical Association of America.

Healy, L., & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for Research in Mathematics Education, 31*(4), 396-428. Retrieved from http://www.jstor.org/stable/749651

Howe, C., Tolmie, A., Thurston, A., Topping, K., Christie, D., Livingston, K., Jessiman, E. and Donaldson, C. (2007). Group work in elementary science: Organizational principles for classroom teaching. *Learning and Instruction, 17*, 549-563. Retrieved from https://doi.org/10.1016/j.learninstruc.2007.09.004

Joshi, P. (2016). Argumentation in democratic education: The crucial role of values. *Theory Into Practice, 55*, 279-286. doi:10.1080/00405841.2016.1208066

King, S. & Campbell (in press). Using interpersonal discourse in small group development of mathematical arguments. In NNN (Eds.) *Proceedings of the 41st annual meeting of the International Group for the Psychology of Mathematics Education,* (Vol. 1, pp. XX-YY). St. Louis, MO: University of Missouri at Columbia & University of Missouri at St. Louis.

Metaxas, N., Potari, D., & Zachariades, T. (2016). Analysis of a teacher’s pedagogical arguments using Toulmin’s model and argumentation schemes. *Educational Studies in Mathematics, 93*, 383-397. doi:10.1007/s10649-016-9701-z

Nardi, E., & Knuth, E. (2017). Changing classroom culture, curricula, and instruction for proof and proving: how amenable to scaling up, practicable for curricular integration, and capable of producing long-lasting effects are current interventions? *Educational Studies in Mathematics, 96*, 267-274. doi:10.1007/s10649-017-9785-0

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: The Council.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center/Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics*. Washington, DC: Council of Chief State School Officers.

Nussbaum, E. (2011). Argumentation, dialogue theory, and probability modeling: Alternative frameworks for argumentation research in education. *Educational Psychologist, 46*(2), 84-106. doi:10.1080/00461520.2011.558816

O’Keefe, D. J. (1982). The concepts of argument and arguing. In J. R. Cox & C. A. Willard (Eds.), Advances in argumentation theory and research (pp. 3–23). Carbondale, IL: Southern Illinois University Press.

Rojas-Drummond, S. & Zapata, M. (2004). Exploratory talk, argumentation, and reasoning in Mexican primary school children. *Language and Education, 18*(6), 539-557. doi:10.1080/09500780408666900

Rumsey, C., & Langrall, C. (2016). Promoting mathematical argumentation. *Teaching Children Mathematics, 22*(7), 412-419. Retrieved from http://www.nctm.org/Publications/Teaching-Children-Mathematics/2016/Vol22/Issue7/Promoting-Mathematical-Argumentation/

Shivers, J., Levenson, C., & Tan, M. (2017). Visual literacy, creativity, and the teaching of argument. *Learning Disabilities: A Contemporary Journal, 15*(1), 67-84. Retrieved from https://eric.ed.gov/contentdelivery/servlet/ERICServlet?accno=EJ1141995

Stylianides, A., & Stylianides, G. (2009). Proof constructions and evaluations. *Educational Studies in Mathematics, 72*, 237-253. doi:10.1007/s10649-009-9191-3

Stylianides, G. (2008). An Analytic Framework of Reasoning-and-Proving. *For the Learning of Mathematics,* *28*(1), 9-16. Retrieved from https://www.jstor.org/stable//40248592

Yee, S. P., Boyle, J. D., Ko, Y., & Bleiler-Baxter, S. K. (2017). Effects of constructing critiquing, and revising arguments within university classrooms. Journal of Mathematical Behavior, 49, 145-162. doi:10.1016/j.jmathb.2017.11.009