

Considering K-12 Learners' Use of Bayesian Methods

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Abstract: Bayesian statistical methods have become increasingly widely-used by statisticians, scientists, and engineers, yet their use has remained within the purview of professionals—and not learners, especially those at the K-12 level. At the same time, decades of developmental research indicates that children learn about the world in ways that align with Bayesian models of cognition. In this paper, we review prior research on Bayesian methods and learning, and offer specific, targeted areas where we think that Bayesian methods could have make an impact, and then discuss future directions for Bayesian methods as a conceptual and statistical tool for learners.

Introduction

From a Bayesian perspective, probability and degrees of certainty are central. From this perspective, probability is used to describe the state of our knowledge about the world. Bayes Theorem, then, provides a rigorous and coherent mathematical framework to update our knowledge about the world, that is, learn, based on data and what we already know. Results from a Bayesian analysis quantify our (un)certainly about parameters.

Bayesian methods allow us to make statements based on our prior knowledge as well as the data; statements such as “Given much earlier studies and the data we collected in our study, the credible range of the length of *Ambystoma maculatom* is from 5.85-6.42 cm., with 6.17 as the most likely length.” In contrast, a frequentist perspective conceptualizes probability in terms of infinite repeat sampling: Frequentist methods aim at estimating the value of the one true parameter based on models about the sampling process and sampling distribution. In consequence, results from a frequentist analysis would phrase the result about salamander lengths in the following terms: “Over many samples of *Ambystoma maculatom*, 95% of length measurements will fall in the confidence interval of 5.85-6.43 cm. ($\beta = 6.20$, $SE = .44$, $p < .05$)” In sum, Bayesian methods readily allow analysts (at any age) to incorporate existing subject-matter knowledge into their analysis and to interpret results in a more intuitive way that aligns better with how people think about the world.

Bayesian methods have not only been used by statisticians, although that is their provenance: Recent research has translated Bayesian ideas beyond their statistical provenance, particularly into the domain of developmental science. This work, broadly, highlights how children intuitively view the world in a Bayesian way (Gopnik, 2012). This paper is predicated on the possibility that the utility of Bayesian ideas to questions about how children *develop* might also have a bearing on how children and youth *learn*, especially in the context of interest on the part of learning scientists in data science and statistics education (Wilkerson & Polman, 2020).

Our goal, then, is to introduce Bayesian methods as an underutilized tool for learners. To do so, we offer two examples of the potential for Bayesian ideas and methods in learning sciences-related areas of research.

The Potential for Bayes in Learning Sciences-Related Areas of Research

Bayesian methods have the potential to bring a new perspective to questions that learning sciences researchers presently ask. In the following, we outline two examples—one from science education and one on the topic of conceptual change—of how a Bayesian perspective may help to redress existing learning-related challenges.

Probability and chance events play an important role in the life sciences, particularly in evolutionary processes and learning about them (Tibell & Harms, 2017). Recently, Fiedler et al. (2019) found that statistical reasoning accounted for nearly a third of the variance in students' knowledge about evolution. Thus, supporting students' statistical reasoning could greatly improve their learning about evolution. Given that research on students' statistical reasoning in the context of evolution has been predominantly approached from a frequentist paradigm (e.g., Fiedler et al., 2019), the potential of a Bayesian perspective remains largely unexplored. Bayesian methods could be useful because a Bayesian view of probability may align better with the ideas about statistics that learners already possess (Gigerenzer & Hoffrage, 1995) or amassing evidence suggesting that human cognition generally follows Bayesian principles (Gopnik, 2012). Thus, we think Bayesian methods could have particular utility in learning about evolutionary processes, and, more generally, probabilistic processes in scientific phenomena.

Bayesian models of cognition may also help to solve a riddle that has vexed the conceptual change literature for some time – the refutation text effect (Broughton et al., 2010). In contrast to traditional expository texts, refutation texts a) state a misconception, b) explicitly show how the misconception does not fully address

the issue at hand, then c) present the accepted scientific position as an alternative. Numerous studies have demonstrated that such texts are very effective in facilitating conceptual change across a range of topics and domains; however, the nature of these effects remains to be little understood and an active area of research (e.g., Mason et al., 2019). Bayesian models of cognition offer a straightforward explanation of many different findings concerning the effects of refutation texts: Students' misconceptions are ideas about concepts acquired through everyday observations which prove to be effective in the contexts in which they are acquired. As an example, consider how children often think that only living things have energy (Watts, 1983). From a Bayesian perspective, the evaluation of the question can be framed as follows: the misconception that only living things have energy reflects a very strong prior for the association between being alive and having energy. A refutation text could help the reader to reevaluate their prior knowledge, and, thus, to weight new information (e.g., information about airplanes in flight having energy) more relative to their prior ideas. Scholars have begun to successfully model students' conceptual development in a Bayesian framework (Bonawitz et al., 2019). This and the above example illustrate how Bayesian methods can be applied to already-active areas of research. In the future, Bayesian methods may also invite new research questions for learning scientists.

Future Directions for Research

Advances in technological tools and statistical software may make the conceptual accessibility of Bayesian methods more technically accessible; one encouraging example of such a technological tool is the JASP statistical software (JASP Team, 2020), which allows for Bayesian models to be estimated as easily as frequentist models (even for models as foundational as *t*-tests and linear models); JASP also has features that are in line with what is known about the development of statistical software that is useable by learners (McNamara, 2019). Technological tools coupled with design-based research may prove to be helpful for understanding whether, when, and how Bayesian methods and tools have a role in K-12 contexts.

Looking ahead, Bayesian methods have proven to be useful to professionals, and Bayesian ideas about how individuals develop (e.g., Gopnik, 2012) suggest that there are opportunities to connect the thinking that children already do with how they learn. Though it may appear far-fetched for tenth-grade students, for example, to apply advanced statistical methods, we are collaboratively working to integrate Bayesian methods into science classrooms—including one of this paper's author's classroom—by shifting the focus away from hypothesis testing and toward understanding variation in data and exploring how arguing from evidence can serve as a context for learners to bring their ideas about how the world works into their scientific explanations.

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