Dynamics of the Kirschner-Panetta Model

Breaking down the functional

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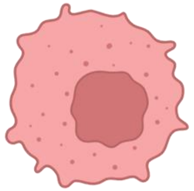
Defining the model

$$\frac{dT}{dt} = r_2(T)T - \frac{\alpha ET}{g_2 + T}$$

$$\frac{dE}{dt} = cT - \mu_2 E + \frac{p_1 E I_{L2}}{g_1 + I_{L2}} + s_1$$

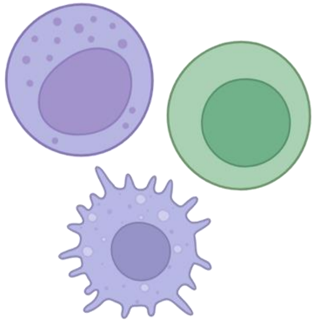
$$\frac{dI_{L2}}{dt} = \frac{p_2 ET}{g_3 + T} - \mu_3 I_{L2} + s_2$$

Tumor Cells

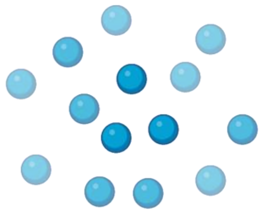


Effector Cells

NK Cells,
T Cells,
Macrophages



Interleukin-2
(IL-2)



Non-dimensionalization

- The model is non-dimensionalized for computation.
- This handles large changes more smoothly.

$$\frac{dy}{d\tau} = r_2(1 - by)y - \frac{\alpha xy}{g_2 + y}$$

$$\frac{dx}{d\tau} = cy - \mu_2 x + \frac{p_1 xz}{g_1 + z} + s_1$$

$$\frac{dz}{d\tau} = \frac{p_2 xy}{g_3 + y} - \mu_3 z + s_2$$

Hamiltonian of the model

Functional:

$$F(s_1, s_2) = \int_0^{t_f} Ax(t) - By(t) - C_1 s_1(t) - C_2 s_2(t) + Dx(t)z(t) - \gamma z(t)^2 dt$$

Lagrangian:

$$\mathcal{L} = -Ax(t) + By(t) + C_1 s_1(t) + C_2 s_2(t) - Dx(t)z(t) + \gamma z(t)^2$$

Hamiltonian:

$$\mathcal{H}(x, y, z, \lambda_1, \lambda_2, \lambda_3) = -\,\mathcal{L} + \lambda_1 \frac{dx}{dt} + \lambda_2 \frac{dy}{dt} + \lambda_3 \frac{dz}{dt}$$

Functional integrand components

Functional:

$$F(s_1, s_2) = \int_0^{t_f} Ax(t) - By(t) - C_1 s_1(t) - C_2 s_2(t) + Dx(t)z(t) - \gamma z(t)^2 dt$$

$Ax(t): Immune\ cell\ population.$

$By(t): Tumor\ cell\ population.$

$C_1 s_1(t): Immune\ cell\ treatment\ dose, C_2 s_2(t): IL - 2\ treatment\ dose.$

$Dx(t)z(t): Immune\ cell\ population\ * IL - 2, IL - 2\ is\ crucial\ to\ immune\ cell\ activation, and\ immune\ cell\ presence\ is\ essential\ for\ IL - 2\ treatment\ effectiveness.\ This\ term\ reflects\ this\ dependent\ relationship.$

$\gamma z(t)^2: IL - 2\ dosage\ is\ also\ comes\ with\ toxicity\ to\ some\ extent.\ Too\ high\ dosage\ can\ lead\ to\ immune\ cell\ exhaustion, serious\ auto - immune\ reaction, or\ a\ phenomenon\ known\ as\ cytokine\ storm.\ This\ term\ reflects\ these\ detrimental\ effects\ on\ the\ system.$