Assignment 8

(1) States: (total asset-liability value, delayed withdrawals, time)
$W_t \in \mathbb{R}$ $D_t \in \mathbb{R}$

Actions: (amt borrowed, portion of money invested)
$y_t \in \mathbb{R}^+$ $\pi_t \in \mathbb{R}^+$

subject to constraints $c_{t+1} \geq \max\left(0, K\cot\left(\frac{\pi c}{2C}\right)\right)$

Rewards: 0 for $t \neq T$, $U(W_T - D_T)$ at time $t = T$, where W_T is wealth at time T & D_T is the remaining withdrawals

Transitions: first borrow, then invest, then regulator fees, then deposits, then withdrawals, then redeem risky asset, then pay back debt

$c_{t+1} = (1-\pi_{t+1})(W_t + y_t)$

if $c_{t+1} < C$: $c_{t+1} -= K\cot\left(\frac{\pi c_{t+1}}{2C}\right)$

then c_{t+1} += deposits (stochastic)

then $c_{t+1} \rightarrow \max(0, c_{t+1} - D_t)$; $D_{t+1} \rightarrow \max(0, D_t - c_{t+1})$

then $c_{t+1} \rightarrow \max(0, c_{t+1} - \text{withdrawals})$; D_{t+1} += $\max(0, \text{withdrawals} - c_{t+1})$ (stochastic)

then $W_{t+1} = c_{t+1} + \pi_t(W_t + y_t)(r_t + 1) - y_t(1+R)$

r_t: return of risky asset (stochastic)

This problem could likely be solved w/ backwards induction using linear function approximation to approximate the value function at each time t (starting at T & moving back).

2 ~~We want to minimize g(S) = g(S), w/ g(S) = p g₁(S) + h g₂(S) and~~

$$\frac{dg}{dS} = p\frac{dg_1}{dS} + h\frac{dg_2}{dS}$$

$$\frac{dg_1}{dS} = \frac{d}{dS}\int_S^{\infty}(x-S)\,f(x)\,dx$$

$$= \frac{d}{dS}\int_S^{\infty} x f(x)\,dx - S\frac{d}{dS}\int_S^{\infty} f(x)\,dx - \int_S^{\infty} f(x)\,dx$$

$$= -Sf(S) + Sf(S) - \int_S^{\infty} f(x)\,dx = -\int_S^{\infty} f(x)\,dx = F(S)-1$$

$$\frac{dg_2}{dS} = \frac{d}{dS}\int_{-\infty}^{S}(S-x)\,f(x)\,dx$$

$$= -\frac{d}{dS}\int_{-\infty}^{S} x f(x)\,dx + S\frac{d}{dS}\int_{-\infty}^{S} f(x)\,dx + \int_{-\infty}^{S} f(x)\,dx$$

$$= -Sf(S) + Sf(S) + \int_{-\infty}^{S} f(x)\,dx = \int_{-\infty}^{S} f(x)\,dx = F(S)$$

$$\Rightarrow \frac{dg}{dS} = p\big(F(S)-1\big) + hF(S) = 0 \Rightarrow \boxed{F(S) = \frac{p}{p+h}}$$

Obviously $\frac{d^2g}{dS^2} > 0$, since F is increasing

In this problem, your holding cost is the same as the payout of h put options on the demand for milk with a strike of S, and the stockout cost is the same as the payout of a call on the same underlying w/ the same strike.

Our minimization is the same as the calculation we would do if someone offered to buy p calls & h puts from us ~~###~~ for some fixed price, and allowed us to choose the strike price (conditioned on all strikes being the same)