Assignment ~~#~~ 2

① States

$\mathcal{N} = \{0, \dots, 99\}$ $S = \mathcal{N} \cup \mathcal{T}$
$\mathcal{T} = \{100\}$

Transition prob f'n

First, define the mapping:

```
sl_map = {1:38, 4:14, 9:31, 16:6, 21:42, 28:84,
          36:44, 47:26, 49:11, 51:67, 56:53, 62:19,
          64:60, 71:91, 80:100, 87:24, 93:73,
          95:75, 98:78, 101:99, 102:78, 103:97, 104:96, 105:75}
for i in {0, ..., 105}:
    if i not in sl_map:
        sl_map[i] = i
```

Then:

$$\mathcal{P}(s, s') = \begin{cases} 1/6 & \text{if } s' = \text{sl\_map}[s+i] \text{ for } i \in 1,2,3,4,5,6 \\ 0 & \text{otherwise} \end{cases}$$

Initial state distribution

$$\mu(s) = \begin{cases} 1 & \text{if } s = 0 \\ 0 & \text{otherwise} \end{cases}$$

③ Frog puzzle: we can set this up as an MRP where the states are the lily pads, the transition f'n is uniform over the lily pads in front, & the reward is +1 every step. Then the expected # of steps is just the value f'n, & we can solve it using backward induction:

$V[10] = 0$; $V[9] = 1 + V[10] = 1$; $V[8] = 1 + \frac{1}{2}V[9] + \frac{1}{2}V[10] = \frac{3}{2}$;
$V[7] = 1 + \frac{1}{3}V[10] + \frac{1}{3}V[9] + \frac{1}{3}V[8] = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$; $V[6] = \frac{25}{12}$ ...

~~0, 1, 3/2, 11/6, 25/12, 137/60~~

$$\boxed{V[0] = 2.929}$$