The Best Number System

James Philip Rowell

April 1, 2019 (v01)

What might the "best" number system be?

If we could hit the reset-button on the fact that we count using decimal numbers, and were tasked with finding the *best* number system for day-to-day use, would we still end up picking decimal?

I know, I know, until we attempt to define what "best" means, it's impossible to answer such a question, especially for a mathematically inclined reader such as yourself. But please bear with me as we flesh this out a bit.

Let's ignore that the fact that Sesame Street drilled into several generations of kids heads that "10" means ten things. Oh, and that the most of the world has been using decimal for about 600 to 800 years; So ignoring those trivialities ...

So what criteria might we use to select the "best" way to denote individual integers?

I think we can discount esoteric systems, for example mixed radix systems, or base-e, as being impractical for day-to-day use. They are interesting and instructive mathematical curiosities but not suitable for day-to-day use.

We can also discount ancient number systems with their use of different symbols for larger and larger numbers, since there's no way to practically denote arbitrarily large integers. For example Roman Numerals only effectively allowed counting up to 4999.

I believe our choice boils down to the question: "Which value for b should we choose given the following theorem?"

Basis Representation Theorem

Let b be a positive integer greater than 1.

For every positive integer n there is a unique sequence of integers $d_0, d_1, d_2, \ldots, d_k$ such that:

$$n = d_k b^k + d_{k-1} b^{k-1} + \cdots + d_2 b^2 + d_1 b^1 + d_0 b^0,$$

where $0 \leq d_i < b$ for all i in $\{0, 1, 2, \ldots, k\}$ and $d_k \neq 0$.

Definition: n is represented in base-b by the string of base-b-digits $(d_k d_{k-1} \cdots d_2 d_1 d_0)_b$

The theorem doesn't say anything about the practicalities behind using a given base. For example, it's convenient to have simple symbols to represent each integer from $0, 1, \ldots, (b-1)$ so that any number can be written in way that's easy for us to read. Needless to say, in decimal we use these symbols for the integers zero through nine: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Computer Scientists add the letters A, B, C, D, E, F to represent the integers ten through fifteen when writing numbers in base-sixteen. For example the number $(10\mathrm{F}2)_{16}$ is the decimal integer 4,338.

So whatever b we pick, then it better be small enough that we can remember all the digits from zero to $(b-1)$. For example, $b = 4096$ is probably a poor choice.

Also we possibly need new words for the numbers that go along with our choice for b. For example the words “fourteen” and “twenty-seven” are intimately tied with their meaning in decimal. It’s unclear how you’d even say “14” or “27” in a different base. For example “14” in base-5 is really the integer nine.

Anyway, whatever we pick, we need to consider those things for practical use. We need symbols, and words to describe the numbers and a small enough base such that we can fairly easily remember all the digits from zero to $(b - 1)$.