The Best Number System

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What might the "best" number system be?

If we could hit the reset-button on how we count using decimal numbers, and were tasked with finding the *best* number system for use, would we pick decimal?

I know, I know, until we attempt to define what "best" means, it's impossible to answer such a question, especially for a mathematically inclined reader such as yourself. But please bear with me as we flesh this out a bit.

Let's ignore that the fact that Sesame Street drilled into several generations of kids heads that "10" means ten things. Oh, and that the most of the world has been using decimal for about 600 to 800 years or so; So ignoring those insurmountable obstacles...

Then what criteria might we use to select which number system we'd adopt for counting?

I think we can discount many esoteric systems, for example mixed radix systems, or base-e, as being impractical for day-to-day use. They are interesting and instructive mathematical curiosities but impractical for everyday use.

We can also discount ancient number systems which introduced new symbols for larger and larger numbers, since they don't cover all the integers. For example Roman Numerals only effectively allowed counting upto 4999.

I believe our choice boils down to the question: "Which value for b should we choose given the following theorem?"

Basis Representation Theorem

Let b be a positive integer greater than 1.

For every positive integer n there is a unique sequence of integers $d_0, d_1, d_2, \ldots, d_k$ such that:

$$n = d_k b^k + d_{k-1} b^{k-1} + \cdots + d_2 b^2 + d_1 b^1 + d_0 b^0,$$

where $0 \leq d_i < b$ for all i in $\{0, 1, 2, \ldots, k\}$ and $d_k \neq 0$.

Definition: n is represented in base-b by the string of base-b-digits $(d_k d_{k-1} \cdots d_2 d_1 d_0)_b$

The theorem doesn't say anything about the practicalities behind using a given base. For example, it's convenient to have simple symbols to represent each integer from $0, 1, \ldots, (b-1)$ so that any number can be written in way that's easy for us to read. Needless to say, in decimal we use these symbols for the integers zero through nine: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Computer Scientists add the letters A, B, C, D, E, F to represent the integers ten through fifteen when writing numbers in base-sixteen. For example the number $(10F2)_{16}$ is the decimal integer 4,338.

Also we possibly need new words for the numbers that go along with our choice for b. For example the words "fourteen" and "twenty-seven" are intimately tied with their meaning in decimal. It's

unclear how you'd even say "14" or "27" in a different base. For example "14" in base-5 is really the integer nine.