The Best Number System

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January 6, 2019 (v01)

What might the "best" number system be?

If we cleared the slate and could hit the reset-button on our conventions for counting in decimal, plus we were tasked with finding the *best* number system for use, would we pick decimal?

I know, I know, until we attempt to define what "best" means, it's impossible to answer such a question, especially for a mathematically inclined reader such as yourself. But please bear with me as we flesh this out a bit.

Let's ignore that the fact that Sesame Street drilled into several generations of kids heads that "10" means ten things. Oh, and that the most of the world has been using decimal for about 600 to 800 years or so; So ignoring those insurmountable obstacles...

Then what criteria might we use to select the method by which we count and give names to each individual integer?

I think we can discount many esoteric systems, for example mixed radix systems, as being impractical for day-to-day use. They are more mathematical curiosities than practical systems for everyday use.

We can also discount more ancient number systems which introduced new symbols for larger and larger numbers (e.g. Roman Numerals) since you can't cover all the integers with such a system.

I believe the choice boils down to which base should we choose given the...

Basis Representation Theorem

Let b be a positive integer greater than 1.

For every positive integer n there is a unique sequence of integers $d_0, d_1, d_2, \ldots, d_k$ such that:

$$n = d_k b^k + d_{k-1} b^{k-1} + \cdots + d_2 b^2 + d_1 b^1 + d_0 b^0,$$

where $0 \leq d_i < b$ for all i in $\{0, 1, 2, \ldots, k\}$ and $d_k \neq 0$.

Definition: n is represented in base-b by the string of base-b-digits $(d_k d_{k-1} \cdots d_2 d_1 d_0)_b$

Our beloved theorem doesn't say anything about the practicalities behind using a given base. For example, it's convenient to have simple symbols to represent each integer from $0, 1, \ldots, (b-1)$ so that any number can be written in way that's easy for us to parse.

We don't live in a world where we routinely deal with numbers up in the nose-bleed heights. Most of us know that 1 billion is equal to 10^9, and probably as many people know that 1 Trillion is 10^12, but it starts getting hazy beyond that. For example, Quadrillion and Quintillion mean what? We also have a few names for some special cases like Googol which is 10^100. Anyway, the point is, we live down near zero, not up in the heights, and we need words to help talk about our day-to-day numbers.

The words we use a intimately tied to our choice of base-ten.