The Best Number System

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What might the "best" number system be?

If we hit the reset button on our conventions about how we count, and were tasked with finding the BEST number system for how we denote individual integers, should we pick decimal?

Let's ignore that the fact that Sesame Street drilled into several generations of kids heads that "10" means ten things. Oh, and that the entire world has been using it for about 600 to 800 years or so. So ignoring those insurmountable obstacles...

Then what criteria might we use to select the method by which we count and give names to each individual integer?

I think we can discount many esoteric systems, like mixed radix systems, as being impractical for casual use.

We can also discount systems like more ancient number systems that introduced new symbols for larger and larger numbers (e.g. Roman Numerals) since you can't cover the integers with such a system.

I think the choice simply boils down to which base should we choose given the...

Basis Representation Theorem

Let b be a positive integer greater than 1.

For every positive integer n there is a unique sequence of integers $d_0, d_1, d_2, \ldots, d_k$ such that:

$$n = d_k b^k + d_{k-1} b^{k-1} + \cdots + d_2 b^2 + d_1 b^1 + d_0 b^0,$$

where $0 \leq d_i < b$ for all i in $\{0, 1, 2, \ldots, k\}$ and $d_k \neq 0$.

Definition: n is represented in base-b by the string of base-b-digits $(d_k d_{k-1} \cdots d_2 d_1 d_0)_b$