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CME 241: Assignment 12

12.3

We will show that $G_t - V(S_t) = \sum_{u=t}^{T-1} \gamma^{u-t} \cdot (R_{u+1} + \gamma V(S_{u+1}) - V(S_u))$

Assuming that S_T denotes a terminal state, so that $V(S_T) = 0$, the righthand side is equal to

$$\sum_{u=t}^{T-1} \gamma^{u-t}(\gamma V(S_{u+1}) - V(S_u)) = \gamma V(S_{t+1}) - V(S_t) + \gamma^2 V(S_{t+2}) - \gamma V(S_{t+1}) + \cdots + \gamma^{T-t} V(S_T)$$

this sum telescopes into a single remaining term $-V(S_t)$. Therefore,

$$\sum_{u=t}^{T-1} \gamma^{u-t}(R_{u+1} + \gamma V(S_{u+1}) - V(S_u)) = \sum_{u=t}^{T-1} \gamma^{u-t} R_{u+1} + \gamma^{u-t}(\gamma V(S_{u+1}) - V(S_u))$$

$$= G_t - V(S_t)$$