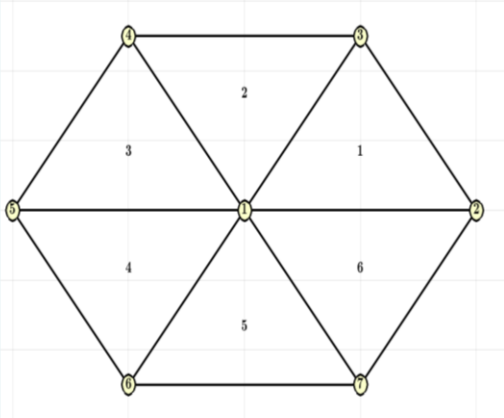
P1

Consider the hexagonal domain shown in the figure,with nodes (0,0), (1,0), (1/2,1/2), (-1/2,1/2), (-1,0), (-1/2,-1/2), (1/2,-1/2). We want to solve the Poisson equation $-c\Delta u = 5$

with $c = 11$ using 6 triangular elements given by the right isosceles triangles of the figure.

(a) (2 points) Let us choose a local numbering in element Ω^1 given by the global nodes 2,3 and 1. Let K^1 be the local stiffness matrix at this element. Which is the value of K^1_{11}?



- -1.10e+01
- 5.50e+00✔
- Empty answer (no penalty)
- 1.10e+01
- -5.50e+00

(b) (2 points) Give the value K_{11} of the assembled stiffness matrix K.

- Empty answer (no penalty)
- 1.10e+01
- -1.10e+01
- 1.65e+01
- 4.40e+01✔

Hint: The sum of any row or column is zero.

(c) (3 points) In this section we consider the essential boundary conditions given by $u = 7$ at the boundary of the hexagon, that is, at the edges that join the vertices 2,3,4,5,6 and 7. Which is the value of u at node 1?

- Empty answer (no penalty)
- 7.3432e+00
- 7.2789e+00
- 6.7936e+00
- 7.0568e+00✔

(d) (3 points) In this section, we replace the essential boundary conditions only on edges 2-3 and 3-4 by the following natural conditions. First, assume that on the edge 2-3 we take an insulating condition $q_n = 0$. Second, on edge 3-4 we consider the following condition: $\frac{\partial u}{\partial y} = 1 + x$

Which is the value of Q_3 in the assembled system?

- Empty answer (no penalty)
- 6.4167e+00✔
- 4.5833e+00
- 8.2500e+00
- 6.6819e+00

P2

We consider a domain D with two circular holes of radius one centered at the points $(2, 2)$ and $(6, 6)$.
We consider its triangular mesh **meshEscaire2foratsTriang**.
(a) (4 points) Let $H \subset \partial D$ and $R = \partial D \setminus H$ be the parts of the boundary of the domain D that correspond to both holes and to the rest of the boundary, respectively. Let $g(x, y) = ax + y$ and $h(x, y) = x + ay$ with $a = 1.9$. Let g_H be the sum of the values of g over all boundary nodes in H. Let h_R be the sum of the values of h over all boundary nodes in R. The value of g_H is:

- 649.38✔
- Empty answer (no penalty)
- 648.92
- 649.05
- 649.83

Hint: $h_R = 763.25$

(b) (3 points) Let $f(x, y) = \sin(x-2)\sin(y-6)$. Let $P = (x_*, 1.9)$ and $Q = (x_*, 2.1)$ with $x_* = 5.8$. Considering the nodal values $f(x_i, y_i)$ at node i, the approximated value of f at the point P obtained by linear interpolation is:

- Empty answer (no penalty)
- -0.470525
- -0.475278✔
- -0.477655
- -0.469100

Hint: The approximated value of f at the point Q is -0.392519

(c) (3 points) How many nodes are vertices of exactly 3 triangular elements?

- ○ 93
- ○ 96
- ○ Empty answer (no penalty)
- ○ 94
- ◉ 95 ✔

Hint 1: Just one node is the vertex of eight triangular elements
Hint 2: If Z is a vector or a matrix with integer entries and m in an integer, then the Matlab command **length(find(Z==m))** gives the number of times that m appears in Z.

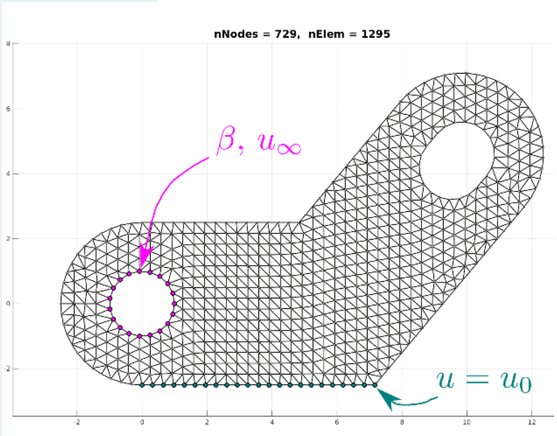
P3

Solve the thermal 2D linear Boundary Value Problem given by the Laplace equation $-k_c \Delta u = 0$ with $k_c = 1$, the mesh given above in file meshPletina.m, and BCs given in the picture, with the following values:
$u_0 = 200.00$, $\beta = 10.00$, $u_\infty = 54.00$.
(a) (4 points) The approximate solution $u_{apr}(x, y)$ at the point $(x, y) = (5, 0)$ is:

- ○ 141.6610
- ○ 146.4904
- ◉ 160.9784 ✔
- ○ 144.8806
- ○ Empty answer (no penalty)

(b) (2 points) Compute the partial derivative $\frac{\partial u_{apr}}{\partial x}(x, y)$ at the point $(x, y) = (5, 0)$:
Hint: Find the coefficients a, b and c such that $u_{apr}(x, y) = a + bx + cy$ in the triangular element that contains the point $(x, y) = (5, 0)$.

- ○ 7.1328
- ○ Empty answer (no penalty)
- ○ 7.2949
- ○ 7.3760
- ◉ 8.1055 ✔



(c) (2 points) Find the minimum temperature m among all the boundary nodes:

- ◉ 38.1813 ✔
- ○ 33.5995
- ○ Empty answer (no penalty)
- ○ 34.3632
- ○ 41.6176

(d) (2 points) Compute the number of nodes where the temperature is in the interval $[50, 75]$:

- ○ 105
- ○ 105
- ○ 103
- ○ Empty answer (no penalty)
- ◉ 95 ✔

P4

We exert an horizontal traction force of 4.0e+03 N/mm on the right boundary of the mesh given by the file `meshAleta2DQuad`. This corresponds to a piece of an alien material of thickness 0.59 mm, Young Modulus 1.1e+07 N/mm^2 and Poisson ratio 0.41. Assume that the left boundary of the piece is fixed.

(a) (1 point) Let K be the global stiff matrix. The entry $K(10,10)$ is :

- Empty answer (no penalty)
- 7.7829e+06
- 7.7982e+06 ✔
- 7.8002e+06
- 7.7749e+06

Hint: The entry $K(20,20)$ is: 7.6618e+06

(b) (3 points) The maximal horizontal displacement (in absolute value) is

- 2.056666e-02
- Empty answer (no penalty)
- 2.061499e-02
- 2.072278e-02 ✔
- 2.054704e-02

Hint: The maximal vertical displacement (in absolute value): 2.2775e-02

(c) (3 points) The maximal value of Von Misses stress is

- 8.8749e+04 ✔
- 8.8287e+04
- 8.8188e+04
- 8.8602e+04
- Empty answer (no penalty)

Hint: The average is: 1.2261e+04

(d) (3 points) Suppose that, now, a compression force of 6.0e+03 N/mm acts on the right boundary. The maximal value of Von Misses stress is

- 1.341958e+05
- 1.343078e+05
- 1.331235e+05 ✔
- Empty answer (no penalty)
- 1.319245e+05

Hint: The maximal vertical displacement (in absolute value): 3.4163e-02