2023-24-1Q-ExFinal

P1

Consider the following problem on $[0, L]$ given by the equation

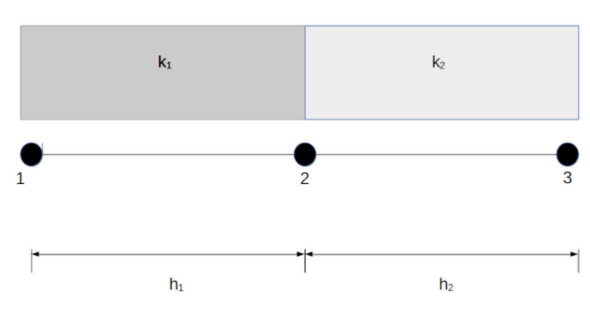
$-\frac{d}{dx}\left(k(x)\frac{du}{dx}\right) = 0, \quad 0 < x < L$

where $L > 0$ and $k = k(x)$ is a function which is constant and equal to k_1 on $\Omega^1 = [0, h_1)$ and constant and equal to k_2 on $\Omega^2 = [h_1, h_1 + h_2]$ with $L = h_1 + h_2$. Assume the following boundary conditions

$u(0) = u_0, \qquad \left[k(x)\frac{du}{dx} + 20\,(u(x) - 50)\right]_{x=L} = 0,$

with $k_1 = 10$, $k_2 = 20$ and $u_0 = 200$.

We will solve the one-dimensional problem by means of a linear elements at Ω^1 and another at Ω^2.



(a) (3 points) Let K be the stiffness matrix of the global problem. Which is the value of $K_{2,2}$ when $h_1 = 0.1$ and $h_2 = 0.2$?

- 3.0000e+02 ✖
- Leave it empty (no penalty)
- 2.5000e+02
- 1.5000e+02
- 2.0000e+02

La resposta correcta és: 2.0000e+02

Hint: if $h_1 = 0.2$ and $h_2 = 0.1$ this value is 2.5000e+02.

(b)(5 points) Assuming $h_1 = 0.1$ and $h_2 = 0.2$, find the approximation U_3 of the solution $u(L)$.

- 1.4375e+02 ✖
- Leave it empty (no penalty)
- 1.6538e+02
- 1.5714e+02
- 1.5000e+02

La resposta correcta és: 1.5714e+02

Hint: Q_3 satisfies the equation $Q_3 + 20(U_3 - 50) = 0$.

(c) (2 points) Now suppose that $h_1 = h_2 = L/2$. Give the value of $L = h_1 + h_2$ so that $U_3 = 120$.

- 7.6190e-01 ✔
- Leave it empty (no penalty)
- 6.6667e-01
- 3.3333e-01
- 1.0000e+00

La resposta correcta és: 7.6190e-01

P2

Consider the equation $-c\ \Delta u = 0$ on the domain $\mathcal{D} = \Omega^1 \cup \Omega^2$ meshed with two elements and connectivity matrix $C = [1\ 2\ 3\ 5;\quad 3\ 4\ 5\ *]$. Ω^1 is a rectangle with node 1 in $(0, 0)$, whose edge 1-2 lies in the OX axis. Edge 1-2 of length 5 and edge 2-3 of length 1. Ω^2 is a right triangle with edge 3-4 of length 3, and edge 4-5 of length 4. The value of c is 30 in Ω^1 and 48 in Ω^2.

Hint: Due to the shape of the elements, there is no need to compute the nodes coordinates

(a) (2 points)The values of K^1_{23} and K^2_{12} are,

- -34 and -9 ✖
- -46 and -16
- Leave it empty (no penalty)
- -41 and -18
- -49 and -32

La resposta correcta és: -49 and -32

(b) (3 points)The value of K_{35} in the assembled matrix is,

- -7 ✖
- Leave it empty (no penalty)
- 7
- 17
- 23

La resposta correcta és: 23

(c) (3 points)Assume that we have boundary conditions $u(x, y) \equiv 1$ on the boundaries 4-5, 5-1 and 1-2, $q^1_n(x, y) \equiv 0$ on 2-3 and $q^2_n(x, y) \equiv 2$ on 3-4. Then, the value we obtain for the approximate solution at node 3 (i.e. U_3) is,

- Leave it empty (no penalty) ✖
- 1.043956
- 1.035714
- 1.046512
- 1.040541

La resposta correcta és: 1.035714

(d) (2 points)Same as (c), but now, $\dfrac{\partial u}{\partial x}(x, y) = 2\ u(x, y)$ on 2-3. Then, the value of U_3 is,

Hint: You can formulate this BC as a convection one for suitable values of β and T_∞

- Leave it empty (no penalty) ✖
- -1.113537
- 1.515625
- -1.914894
- -19.5000

La resposta correcta és: 1.515625

P3

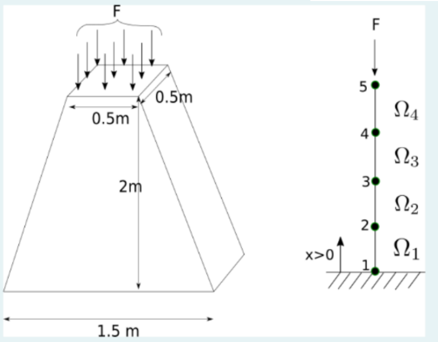
A column is made of a certain material with Young modulus $E = 181 \times 10^6\ kN/m^2$. The geometry of the column is shown in the figure. We wish to analyze the column for displacements using the FEM.

The column is indeed a three-dimensional structure. However, we wish to approximate the column as one-dimensional. To this end, we represent the distributed force at the top of the column as a point force $F = 5$ kN.

The weight of the material is represented by the body force per unit length:

$f(x) = -6.25(3-x) \qquad \mathrm{kN/m}.$ The governing differential equation for the problem is given by: $-\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right) = f(x)$ where $A(x)$ is the cross-sectional area.

Consider a mesh with 4 linear elements of equal length $h = 0.5$ m.



(a) (3 points) The value $K_{2,3}$ of the stiffness matrix is:

- Empty answer (no penalty) ✖
- -2.0362e+08 m
- -1.8326e+08 m
- -2.2806e+08 m
- -2.2602e+08 m

La resposta correcta és: -2.0362e+08 m

(a) (4 points) Compute the approximate solution $u_{apr}(x)$ at $x = 2$.

- Empty answer (no penalty) ✖
- -3.5526e-07 m
- -3.5849e-07 m
- -3.2297e-07 m
- -2.8421e-07 m

La resposta correcta és: -3.2297e-07 m

Hint: The approximate solution at $x = 1$ is $u_{apr}(1) = -1.8998e-07$ m.

(a) (4 points) Compute the approximate solution $u_{apr}(x)$ at $x = 2$.

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La resposta correcta és: -3.2297e-07 m

Hint: The approximate solution at $x = 1$ is $u_{apr}(1) = -1.8998e-07$ m.

(b) (3 points) Find the *elongation* of the last element Ω^4. (Recall that the elongation is the change in length of the element).

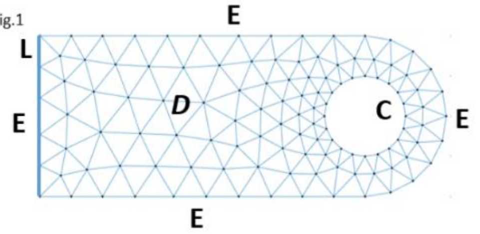
- Empty answer (no penalty) ✖
- -6.0313e-08 m
- -6.7551e-08 m
- -6.5138e-08 m
- -5.4282e-08 m

La resposta correcta és: -6.0313e-08 m

P4

Consider the heat equation $-\Delta T = 0$ on the domain D meshed in file **meshDataHoleS5.m** (see fig.1) Let us define the **interior boundary** C, as the small circumference inside the domain and the **exterior boundary** E as the remaining part of the boundary of D (i.e $E = \partial\Omega \backslash C$). We also consider L, the left part of E, this is,
$L = \{(x, y) \in E,\ x = 0,\ 0 \leq y \leq 2\}$.
Let be P the point inside the element Ω^{82}, with barycentric coordinates $(0.50, 0.25, 0.25)$.
(a) (3 points) Solve the problem when the boundary conditions are: $T \equiv 90$ on C, $T \equiv 50$, on L. The value of $T(P)$ is:
Hint-a1: When no boundary conditions are given, like on $E \backslash L$, it is equivalent to impose the default condition $q_n^k \equiv 0$ which is explicitly impose when you initialize $Q = 0$ in the Matlab code.
Hint-a2: The value of $u(114)$ is 7.1913e+01

Fig.1

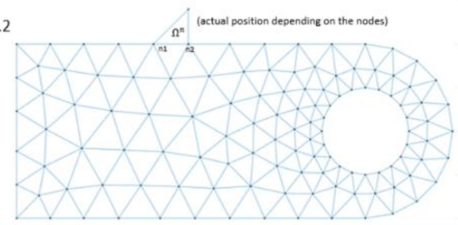


- 7.5778e+01 ✖
- 7.6117e+01
- Leave it empty (no penalty)
- 7.4604e+01
- 7.3212e+01

La resposta correcta és: 7.4604e+01

(b) (2 points) Consider the same problem with boundary conditions in the whole exterior boundary E as in (a), but now the temperature on C be given by $T(x, y) = 25(x - y^2)$. Then, the value of $T(p)$ is:
Hint-b: The value of $u(114)$ is 5.7331e+01

Fig.2



- Leave it empty (no penalty) ✖
- 5.7535e+01
- 5.6370e+01
- 5.5236e+01
- 5.6743e+01

La resposta correcta és: 5.6370e+01

(c) (2 points) Let us define $B = \{(x, y) \in E,\ x > 3.5\}$. Solve the problem with boundary conditions on L and C as in (b), but now, considering on B a convection boundary condition with $\beta = 2$ and $T_\infty = 5$, The value of $T(P)$ is then:
Hint-c1: B contains 13 nodes and the node with the biggest number is 55.
Hint-c2: The value of $u(114)$ is 5.3805e+01

- 5.2499e+01 ✔
- 5.2622e+01
- 5.1918e+01
- Leave it empty (no penalty)
- 5.3700e+01

La resposta correcta és: 5.2499e+01

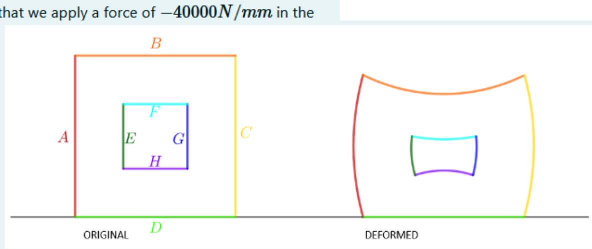
(d) (3 points) By design requirements, we want to enlarge the domain D adding to the mesh a new triangular element, Ω^n, (see fig.2) on the boundary $y = 2$, like an outer spike. Consider Ω^n be an **isosceles right triangle exterior to** D where the nodes 27 and 26 are its respective local nodes 1 and 2 (i.e. they define Γ_1^n).
Therefore, on the initial mesh you have to add a new line both to the elements and nodes tables, with the new element and node respectively.
The new domain $\bar{D} = D \cup \Omega^n$ shares the same boundaries C, L with D. Solve the problem in the new domain $\bar{D}$ with the same boundary conditions: $T \equiv 50$, on L, $T(x, y) = 25(x - y^2)$ on C (like in (b)) and fixing the temperature of the **new node** to 110
The value of $T(P)$ is;
Hint-d: The value of $u(114)$ is 6.3545e+01

- 7.2456e+01 ✖
- 6.9578e+01
- Leave it empty (no penalty)
- 7.2435e+01
- 7.0790e+01

La resposta correcta és: 7.0790e+01

P5

Consider the domain determined by the triangular mesh `chimenea2.m` (the edge lengths are measured in mm). The domain is made of an elastic material with Young Modulus $E = 10^6 N/mm^2$, Poisson ratio $\nu = 0.25$ and thickness $t_h = 0.50\ mm$. Assume that the boundaries of the mesh are defined as shown in the image and that the bottom part of the domain (corresponding to the boundary D) is completely fixed and that we apply a force of $-40000 N/mm$ in the vertical direction on boundary B. Answer the following questions:



(a) (2 points) The number of nodes on the boundary $B \cup E \cup F$ is:

- 76 ✗
- Empty answer (no penalty)
- 74
- 73
- 75

La resposta correcta és: 74

Hint: The number of nodes of the boundary $B \cup E \cup F$ of the mesh `chimenea0.m` is 38.

(b) (2 points) What is the mean horizontal displacement (in mm) of the nodes on boundary C?

- 3.5079e-01 ✓
- Empty answer (no penalty)
- 3.5472e-01
- 3.3866e-01
- 3.4554e-01

La resposta correcta és: 3.5079e-01

Hint: The horizontal displacement of the node 111, using the same parameters and the mesh `chimenea0.m` is 6.7330e-01 mm.

(c) (3 points) What is the maximum displacement (in mm and modulus) of all the nodes?

- 3.3689e+00 ✗
- Empty answer (no penalty)
- 3.2338e+00
- 3.2717e+00
- 3.3244e+00

La resposta correcta és: 3.3244e+00

Hint: The displacement in modulus of the node 60, using the same parameters and the mesh `chimenea0.m` is 3.0387e+00 mm.

(d) (3 points) What is the final area defined by the mesh elements? (This is the sum of the areas of all the elements after applying the force.)

- 3.2310e+02 ✗
- 3.1368e+02
- 3.1917e+02
- 3.0974e+02
- Empty answer (no penalty)

La resposta correcta és: 3.1368e+02

Hint: The area before applying the force is 3.3600e+02 mm^2.