Final 26-01-2022. Problema 2

Pregunta 1

No s'ha respost encara

Puntuat sobre 10,00

Let us consider the domain Ω, meshed by means of two elements (a quadrilateral and a triangle) as follows.

Nodes: $(0,0),\ (2,0),\ (5,0),\ (0,1),\ (2,1)$.

Connectivity matrix: $\begin{pmatrix} 1 & 2 & 5 & 4 \\ 5 & 2 & 3 & * \end{pmatrix}$.

Using this mesh, we are going to consider the finite element method for the following problem:

$$\begin{cases} -k_c \Delta u = f & \text{on } \Omega, \\ u(x,0) = 3x, & \\ u(0,y) = 2y, & \\ \frac{\partial u}{\partial y}(x,y) = 2, & \text{on the line joining nodes 4 and 5.} \\ \frac{\partial u}{\partial n}(x,y) = 0, & \text{on the line joining nodes 3 and 5.} \end{cases}$$

where $k_c = 12$ and $f \equiv 2$ on Ω^1, $k_c = 6$ and $f \equiv 4$ on Ω^2.

(a) (2 points) What is the value of $\psi_2^1(0.5, 0.5)$?

- 3/16
- 5/36
- Leave it empty (no penalty)
- 3/8
- 1/8

(b) (2 points) Let $[K]$ be the assembled matrix of the system. What is the value of $K(5,2)$?

Hint: You don't need the full $[K]$ matrix. On the other hand, the manual assembly of a rectangle and a triangle is done the same way of the assembly of two triangles.

- Leave it empty (no penalty)
- $-13/4$
- -1
- $-3/2$
- -16

(c) (2 points) Let F be the assembled force vector of the system. $[K]U = F + Q$, what is the value of $F(5)$?

- 4
- Leave it empty (no penalty)
- 2
- 3
- 5

(d) (2 points) What is the value of $Q_{ij}^k = Q_{33}^1$?

- 18
- 12
- 36

- ● 24
- ○ Leave it empty (no penalty)

(e) (2 points) What is the value of U_5, the approximated solution at node 5?

- ○ 12.78
- ○ Leave it empty (no penalty)
- ○ 7.28
- ● 6.26
- ○ 3.69

Torna a començar | Desa | Emplena amb les respostes correctes | Envia i acaba | Tanca la previsualització

Informació tècnica

Comportament que s'està utilitzant: Retroalimentació diferida

Fracció mínima: -0.25

Fracció màxima: 1

Variant de pregunta: 1

Resum de la pregunta: Let us consider the domain \(\Omega \), meshed by means of two elements (a quadrilateral and a triangle) as follows. Nodes: \((0,0), (2,0), (5,0), (0,1), (2,1). \) Connectivity matrix: \(\left (\begin {array}{cccc} 1&2&5&4 \\ 5&2&3&*\end {array}\right)\). Using this mesh, we are going to consider the finite element method for the following problem: \(\left \{\begin {array}{ll} -k_c\Delta u=f & \hbox { on} \Omega , \\ u(x,0)=3x, & \\ u(0,y)=2y, & \\ \frac {\partial u}{\partial y}(x,y)=2, & \hbox { on the line joining nodes 4 and 5 } \\ \frac {\partial u}{\partial n}(x,y)=0, & \hbox { on the line joining nodes 3 and 5 } \\ \end {array}\right . \) where \(k_c=12\) and \(f\equiv 2\) on \(\Omega ^1\), \(k_c=6\) and \(f\equiv 4\) on \(\Omega ^2\). (a) (2 points) What is the value of \(\psi ^1_2(0.5,0.5)\)? {3/16; 5/36; Leave it empty (no penalty); 3/8; 1/8} (b) (2 points) Let \([K]\) be the assembled matrix of the system. What is the value of \(K(5,2)\)? _Hint:_ You don't need the full \([K]\) matrix. On the other hand, the manual assembly of a rectangle and a triangle is done the same way of the assembly of two triangles. {Leave it empty (no penalty); \(-13/4\); \(-1\); \(-3/2\); \(-16\)} (c) (2 points) Let \(F\) be the assembled force vector of the system. \([K]U=F+Q\), what is the value of \(F(5)\)? {4; Leave it empty (no penalty); 2; 3; 5} (d) (2 points) What is the value of \(Q^k_{ij}=Q^1_{33}\)? {18; 12; 36; 24; Leave it empty (no penalty)} (e) (2 points) What is the value of \(U_5\), the approximated solution at node 5? {12.78; Leave it empty (no penalty); 7.28; 6.26; 3.69}

Resum de la resposta correcta: part 1: 1/8; part 2: \(-16\); part 3: 3; part 4: 24; part 5: 6.26

Resum de respostes:

Estat de la pregunta: todo

Download this question in Moodle XML format

Contreu-ho tot

Opcions de l'intent

Com es comporten les preguntes

Retroalimentació diferida

Puntuat sobre

10

Torna a començar amb aquestes opcions

Opcions de visualització

Si és correcte

Mostrat

Puntuacions

Mostra la puntuació i el màxim

Problema 2

Let us consider the domain Ω, meshed by means of two elements (a quadrilateral and a triangle) as follows. Nodes $(0,0), (2,0), (5,0), (0,1), (2,1)$. Connectivity matrix $\begin{pmatrix} 1 & 2 & 5 & 4 \\ 5 & 2 & 3 & * \end{pmatrix}$

Using this mesh, we are going to consider the finite element method for the following problem:

$$\begin{cases} -k_c \, \Delta u = f \\ u(x,0) = 3x \\ u(0,y) = 2y \\ \dfrac{\partial u}{\partial y}(x,y) = 2, \text{ on the line joining nodes 4 and 5.} \\ \dfrac{\partial u}{\partial n}(x,y) = 0, \text{ on the line joining nodes 3 and 5.} \end{cases}$$

Where $k_c = 12$ and $f \equiv 2$ on Ω^1, $k_c = 6$ and $f \equiv 4$ on Ω^2.

(a) [2 points] What is the value of $\psi_2^1(0.5, 0.5)$?

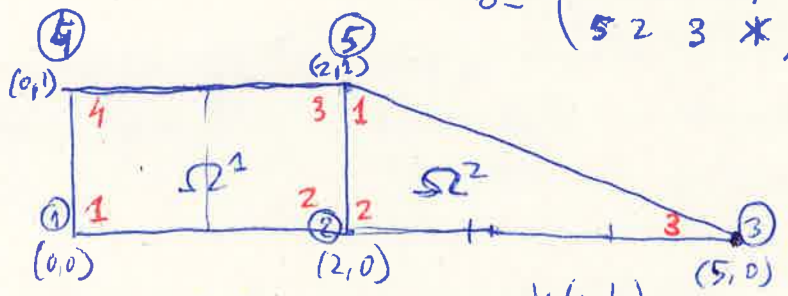
(b) [2 points] Let $[K]$ be the assembled matrix of the system. What is the value of $K(5,2)$?
Hint: You don't need the full $[K]$ matrix. On the other hand, the manual assembly of a rectangle and a triangle is done the same way of the assembly of two triangles.

(c) [2 points] Let F be the assembled force vector of the system $[K]U = F+Q$, what is the value of $F(5)$?

(d) [2 points] What is the value of $Q_{ij}^k = Q_{33}^1$?

(e) [2 points] What is the value of U_5, the approximated solution at node 5?

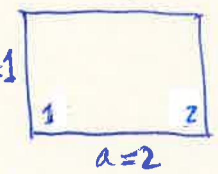
$$B = \begin{pmatrix} 1 & 2 & 5 & 4 \\ 5 & 2 & 3 & * \end{pmatrix}; \quad \Omega^1: \begin{cases} k_c = k_c^1 = 12 \\ f = f^1 = 2 \end{cases}, \quad \Omega^2: \begin{cases} k_c = k_c^2 = 6 \\ f = f^2 = 4 \end{cases}$$



(a) $\psi_2^1(x,y) = \frac{x(1-y)}{2}, \quad \psi_2^1(\tfrac{1}{2},\tfrac{1}{2}) = \frac{\frac{1}{2}(1-\frac{1}{2})}{2} = \boxed{1/8}$

(b) $K_{52} = K_{32}^1 + K_{12}^2 = -7-9 = \boxed{-16}$

$\boxed{K_{32}^1}$: $b=1$



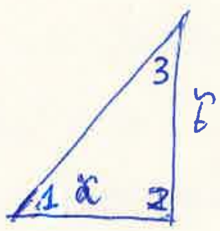
$a_{11}^1 = a_{22}^1 = k_c^1 = 12$

$$K^{k,11} = \frac{b\, a_{11}^k}{6a} \begin{pmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{pmatrix}$$

$$K^{k,22} = \frac{a\, a_{22}^k}{6b} \begin{pmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{pmatrix}$$

$$K_{32}^{k=1} = \frac{b\, a_{11}^1}{6a}(1) + \frac{a \cdot a_{22}^1}{6b}(-2) = \frac{12}{12}(1) + \frac{2 \times 12}{6}(-2) = 1-8 = \underline{-7}$$

$\boxed{K_{12}^2}$:



$$K^k = \frac{\hat{c}}{2\hat{a}\hat{b}} \begin{pmatrix} \hat{b}^2 & -\hat{b}^2 & 0 \\ -\hat{b}^2 & \hat{a}^2+\hat{b}^2 & -\hat{a}^2 \\ 0 & -\hat{a}^2 & \hat{a}^2 \end{pmatrix}$$

$\hat{a} = 1, \ \hat{b} = 5-2 = 3$

$\hat{c} = a_{11}^2 = a_{22}^2 = k_c^2 = 6$

$$K_{12}^{k=2} = \frac{\hat{c}}{2\hat{a}\hat{b}}(-\hat{b}^2) = \frac{6}{2 \times 1 \times 3}(-3^2) = \underline{-9}$$

(c) $F_5 = F_3^1 + F_1^2 = 1+2 = \boxed{3}$

$F^k = \frac{f^k A_k}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, so $F_3^2 = \frac{4 \cdot 3/2}{2} = \underline{3}$

$$F_3^1 = \iint_{\Omega^1} f^1 \cdot \psi_3^1(x,y)\, dx\,dy \overset{(*)}{=} \frac{2}{2}\int_0^2 x\, dx \int_0^1 y\, dy = \left[\frac{x^2}{2}\right]_0^2 \cdot \left[\frac{y^2}{2}\right]_0^1 = \frac{4}{2} \cdot \frac{1}{2} = \underline{1}$$

(*) $\psi_3^1(x,y) = \frac{xy}{2}$

Note: we use formulas in T2-MN-FEM2D.pdf, page 24.

(d) $Q_{33}^1 = \frac{q_3^1 h_3^1}{2} \overset{(*)}{=} \frac{24\times 2}{2} = \boxed{24}$

$q_3^1 = k_c^1 \frac{\partial u}{\partial y}(x,2) = 12\cdot 2 = 24, \quad 0\leq x\leq 2$

$h_3^1 = 2$

Alternatively:

$$Q_{33}^1 = \int_0^{h_3^1=2} q_3^1(2-s,1)\cdot \psi_{33}^1(s)\,ds = \int_0^2 q_3^1(2-s,1)\cdot\psi_3^1(2-s,1)\,ds$$

$$= \int_0^2 24\cdot\frac{(2-s)\cdot 1}{2}\,ds = 12\int_0^2 (2-s)\,ds = 12\left(2s-\frac{s^2}{2}\right)\Big]_{s=0}^{s=2}$$

$$= 12\cdot(4-2) = \boxed{24}$$

(e) $K_{52} = -16$ (from (b))

$K_{53} = K_{13}^2 = \underline{0}$.

$K_{54} = K_{34}^1 = \frac{b a_{11}^1}{6a}(-2) + \frac{a\cdot a_{22}^1}{6b}(1) = \frac{12}{12}(-2) + \frac{2\times 12}{6\times 1}(1) = -2+4 = \underline{2}$

$K_{55} = K_{33}^1 + K_{11}^2 = \frac{b a_{11}^1}{6a}(2) + \frac{a\cdot a_{22}^1}{6b}(2) + \frac{c}{2ab}(b^2) = \frac{12}{12}(2) + \frac{2\times 12}{6}(2) + \frac{6}{6}(9)$

$= 2+8+9 = \underline{19}$

Natural B.C.: $Q_5 = Q_{33}^1 + Q_{13}^2 \overset{(*)}{=} 24$

(*) $Q_{33}^1 = 24$ (from (d))

$Q_{13}^2 = 0$ ($q_3^2 \equiv 0$, since $\frac{\partial u}{\partial n}\equiv 0$ on the line joining nodes 3 and 5)

Essential B.C.: $U_1 = 0$, $U_2 = 6$, $U_3 = 15$, $U_4 = 2$. $\quad$ $\overset{3}{''}$ (from (c))

$K_{51}U_1 + K_{52}U_2 + K_{53}U_3 + K_{54}U_4 + K_{55}U_5 = F_5 + Q_5$

$\Leftrightarrow K_{51}\times 0 - 16\times 6 + 0\times U_3 + 2\times 2 + 19\,U_5 = 3+24$

$\Leftrightarrow 19\,U_5 = 27+96-4 = 27+92 = 119$

$\Leftrightarrow U_5 = \frac{119}{19} \simeq \boxed{6.26}$