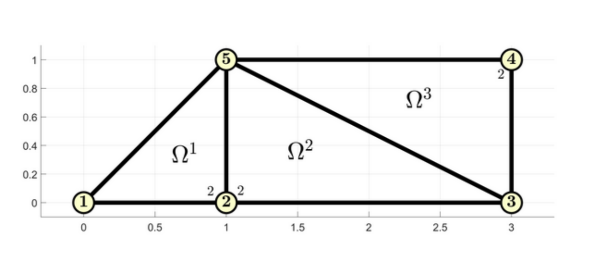
P1

Consider the Poisson heat diffusion on the domain shown above with the three triangular finite elements, nodes and local and global numbering plotted there. We consider that the thermal conductivity is $k_c = 4$ and the density of internal heating is $f = 3$. Also a fixed temperature $T = T_0$ is applied on all the bottom boundary, while a convection of coefficient $\beta = 6.00$ and bulk temperature T_∞ is present on the right boundary

(a) (1 point) The second shape function of the third element $\psi_2^3(x, y)$ is



- x/2 + y - 3/2
- 1 - y
- 3/2 - x/2
- x + y/2 - 3/2
- Leave it empty (no penalty) ✖

La resposta correcta és: x/2 + y - 3/2

(b) (1 points) The value of K_{52}, the $(5, 2)$ entry of the global stiff matrix K is

- -6
- 2
- -4
- -2
- Leave it empty (no penalty) ✖

La resposta correcta és: -6

Hint2:the value of K_{34} is -4.0000e+00

(c) (2 points) The value of F_5, the global internal heating of global node 5 is

- 2.5e+00
- 2.6e+00
- 2.3e+00
- 3.0e+00
- Leave it empty (no penalty) ✖

La resposta correcta és: 2.5e+00

Hint3:the value of F_1 is 5.0000e-01

(d) (2 points) Now suppose that a linear flow q_n on the edge between nodes1 and 5 of values $q_0 = 30$ and 0 respectively is applied. Then the value of Q^1_{33} is

- 7.0711e+00
- 1.2294e+01
- 8.1983e+00
- 7.7762e+00
- Leave it empty (no penalty) ✖

La resposta correcta és: 7.0711e+00

(e) (2 points) When computing Q_4 as part of a convective edge, the expression relating T_3, T_4, T_∞ is

- $(-1)T_3 + (-2)T_4 + (3)T_\infty$
- $(-2)T_3 + (-1)T_4 + (1)T_\infty$
- $(1)T_3 + (-1)T_4 + (4)T_\infty$
- $(-3)T_3 + (0)T_4 + (2)T_\infty$
- Leave it empty (no penalty)

La resposta correcta és: $(-1)T_3 + (-2)T_4 + (3)T_\infty$

(f) (2 points) Now suppose that $T_0 = 0$ on all the bottom boundary and also $q_n = 0$ on the top edge joining nodes 4 and 5, besides the boundary conditions already done. The value of T_4, when $T_\infty = 0$ and T_5 is 1.4166e+00 is

- 3.4523e-01
- 1.0009e-01
- 5.8898e-01
- 4.2950e-01
- Leave it empty (no penalty) ✖

La resposta correcta és: 3.4523e-01

P2

Taking the air quality measurements of three Barcelona stations located at Parc Vall d'Hebron, coordinates (41.44, 2.14), L'Hospitalet de Llobregat (41.37, 2.12) and Sant Adrià de Besòs (41.43, 2.22), we obtain that for the fine particulate matter air quality index, denoted by $PM_{2.5}$, the levels are 22.67, 38.96, 14.18 respectively.
(a) (4 points) Give the approximate value of $PM_{2.5}$ in the station of Barcelona Gràcia-Sant Gervasi (41.4, 2.15)

- 2.9181e+01
- Leave it empty (no penalty) ✖
- 2.5761e+01
- 2.5270e+01
- 3.0160e+01

La resposta correcta és: 2.9181e+01

(b) (3 points) In the Gracia-Sant Gervasi station, last week the temperatures were 22.64, 21.46, 21.36, 28.69, 25.80^oC from Monday to Friday, and on Sunday a temperature of 27 was reached. Using an approximation polynomial of degree 4, the temperature on Saturday was:

- 2.4870e+01
- 2.9001e+01
- 2.3377e+01
- Leave it empty (no penalty) ✖
- 2.3990e+01

La resposta correcta és: 2.4870e+01

(c) (3 points) Load the mesh **meshTwoHolesQuad.m**. In the element 55 the x-component of the point p with barycentric coordinates (0.25, 0.25, 0.25, 0.25) is:

- 9.0009e+01
- 8.9493e+01
- Leave it empty (no penalty) ✖
- 2.5000e-01
- 9.0270e+01

La resposta correcta és: 9.0270e+01

P3

Consider the rectangle plate meshed using the following matlab sentences

```
x=-2:12;
y=-2:3;
k=0;
for i=1:length(x)
   for j=1:length(y)
      k=k+1;
      nodes(k,:)=[x(i),y(j)];
   end
end
elem = delaunay(nodes(:,1),nodes(:,2));
```

The temperature on the plate is given by the solution of the BVP defined by the Poison's equation $-k_c\Delta T = f$, with $f = 100.00$, $k_c = 2.74$, and the following BC: $\frac{\partial T}{\partial x} = T$ on the left edge and $T = 22.37$°C on the top, bottom, and right edges of the rectangle.

Note. Recall the definition of q_n and use the form $q_n = -\beta(T - T_\infty)$ to express the BC as a convection condition for appropriate β and T_∞.

Hence, use the FEM solution of this BVP to answer the following questions:

(a) (3 points) The maximum nodal temperature on the piece, $\max\limits_{i=1,\dots,N} T_i$, being N the number of nodes, is

- 1.2915e+02
- 1.2913e+02
- 1.2909e+02
- 1.2911e+02
- Leave it empty (no penalty) ✖

La resposta correcta és: 1.2911e+02

Hint. The temperature at node 50 is T_{50} = 9.3176e+01

Now we drill a hole in the plate and **remesh** the initial domain according to data found in **AirFoilmesh01.m**. Consider the the temperature fixed $T = $ -2.72 on the contour of the hole and **preserve** the previous BC. Answer the following questions using this new mesh:

(b) (2 points) The mean of the x-coordinates of the points in the boundary of the hole, is

- Leave it empty (no penalty) ✖
- 7.3770e+00
- 7.7265e+00
- 5.9669e+00
- 5.5339e+00

La resposta correcta és: 5.5339e+00

Hint. Through the appropriate function, you can obtain the number of points on this boundary which is 212

(c) (2 points) The maximum nodal temperature now on the piece, $\max_{i=1,\ldots,N} T_i$, being N the number of nodes, is

- 4.6102e+01
- Leave it empty (no penalty) ✖
- 4.6131e+01
- 4.6146e+01
- 4.6117e+01

La resposta correcta és: 4.6117e+01

Hint. The temperature at node 50 is T_{50} = 7.1634e+00

(d) (1 point) The minimum value of Q on the nodes on the hole boundary is

- -9.0480e+01
- -9.0442e+01
- -9.0467e+01
- Leave it empty (no penalty) ✖
- -9.0455e+01

La resposta correcta és: -9.0455e+01

Hint. The maximum value of Q on the nodes of the left boundary is $Q =$ -2.5721e+01

(e) (2 points) Now, switch the BC on the edge of the internal hole to convection, with coefficient $\beta =$ 0.35 and bulk temperature $T_\infty =$ -3.50, keeping the other BC unchanged. Then, if $i_1, i_2, \ldots, i_M$ are the indices of the nodes on this edge, the corresponding averaged temperature, defined as $\langle T \rangle_{\mathbf{intBd}} = \frac{1}{M}\sum_{k=1}^{M} T_{i_k}$, is

- Leave it empty (no penalty) ✖
- 7.8835e+01
- 7.8851e+01
- 7.8867e+01
- 7.8883e+01

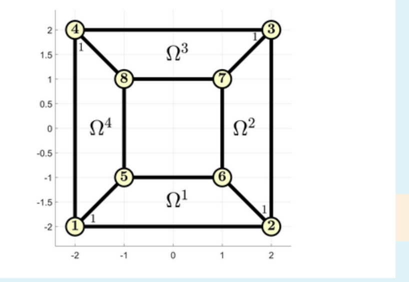
La resposta correcta és: 7.8851e+01

Hint. The global nodal averaged temperature, defined as $\langle T \rangle = \frac{1}{N}\sum_{k=1}^{N} T_k$, being N the number of nodes, is $\langle T \rangle$ =7.4467e+01

P4

Consider the domain shown above with the four quadrilateral finite elements, nodes and local and global numbering plotted there (the side lengths are measured in meters).The domain is made of an elastic material with Young Modulus $E = 10^8 \frac{N}{m^2}$, Poisson ratio $\nu = 0.25$ and thickness $th = 0.01m$. The piece is fixed in the all left exterior boundary while an horizontal force load $[530; 0]\frac{N}{m}$ is applied on the right exterior boundary.

(a) (4 points) The maximum of the absolute value of the vertical values of the displacements is



- 3.7948e-04
- 4.0008e-04
- 6.6437e-04
- 3.4716e-04
- Leave it empty (no penalty)

La resposta correcta és: 3.7948e-04

Hint1: the maximum of the absolute value of the of the horizontal displacements is 3.2981e-03

(b) (3 points) Now suppose that, instead of a constant force load, a **linear force** load is applied on the right exterior boundary, with values of force ranging from $[530; 0]\frac{N}{m}$ on the global node $\mathbf{2}$ to $[0; 0]\frac{N}{m}$ on the global node$\mathbf{3}$. Now the maximum of the absolute value of the vertical values of the displacements is

- 1.4079e-03
- Leave it empty (no penalty)
- 2.0832e-03
- 1.1039e-03
- 1.1437e-03

La resposta correcta és: 1.1039e-03

Hint2:the maximum of the absolute value of the of the horizontal displacements is 2.5632e-03

(c) (3 points) Now supose that no external loads are applied, and just the own weigth of the piece is taken into account. Assume that the density of the material is such that the vector of local element forces F_e can be taken constant for all the elements , with value Fe=[0; -340; 0; -340; 0; -340; 0; -340], the absolute value of the vertical values of the displacements is

- 3.4003e-02
- Leave it empty (no penalty)
- 8.5464e-03
- 2.4006e-02
- 1.7753e-02

La resposta correcta és: 1.7753e-02

Hint3: the maximum of the absolute value of the of the horizontal displacements is 6.3286e-03