Examen Parcial 05-04-2022. Problema 1.

Pregunta **1**

No s'ha respost encara

Puntuat sobre 10,00

Consider a pillar made of two pieces of the same material (see figure). The first one is a truncated rectangular pyramid of height h=1.00, with length sides a=2 and c=2 of the base and b=1 and also c=2 the length sides of the top. The second part is prismatic of volume of height 2h=2.00. We suppose that the pillar is fixed at the base and only subjected to the gravity force without additional loads. We study the deformation of the pillar as a FEM1D problem using a mesh of two elements, relating to the parts of the pillar The pyramid basis is modelled by one linear element, while the prismatic part by one quadratic element as shown in the figure. In order to simplify the calculations we take $\omega = 3$ as the specific weight of the material and $E(x) = 1$ for the Young modulus.

Answer the following questions:

(a) (2 points) Let ψ_2^2 the second shape function of the quadratic element and p=1.30, a point inside there the value of $\psi_2^k(p)$ is

- ○ 9.8114e-01
- ○ 4.5153e-01
- ○ Leave it empty (no penalty)
- ● 5.1000e-01
- ○ 1.0879e-01

(b) (2 points) The value of K_{21}^1 of the stiff local matrix of the element Ω^1 is

- ○ -1.0000e+00
- ○ -7.5000e-01
- ● -3.0000e+00
- ○ -1.5000e+00
- ○ Leave it empty (no penalty)

(c) (2 points) Knowing that the local vector force of the element Ω^1 is F^1=[-5.00,-4.00] The second component F_2 of the assembled global force vector F is

- ○ -1.2000e+01
- ● -6.0000e+00
- ○ -1.8000e+01
- ○ -2.4000e+01
- ○ Leave it empty (no penalty)

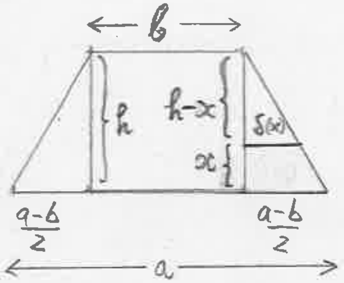
(d) (2 points) Knowing that U_3=-9.833333 and U_4=-11.333333 are solutions of the assembled global system. The value of U_2 is

- ○ -8.5333e+01
- ● -5.3333e+00
- ○ Leave it empty (no penalty)
- ○ -4.8000e+01
- ○ -2.1333e+01

(e) (2 points) The value of the reaction force Q_1 is

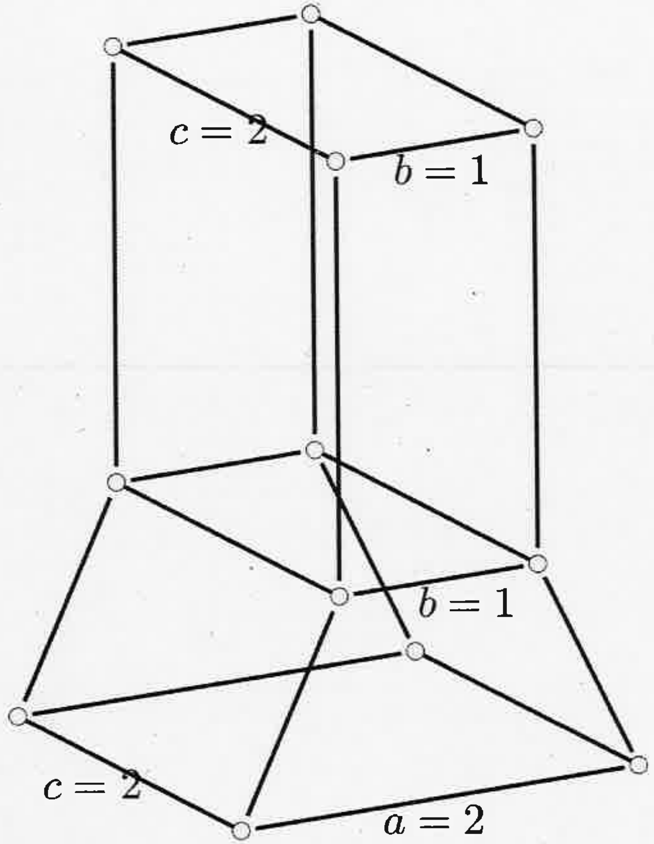
- ○ Leave it empty (no penalty)
- ○ 8.4000e+01
- ● 2.1000e+01
- ○ 4.2000e+01

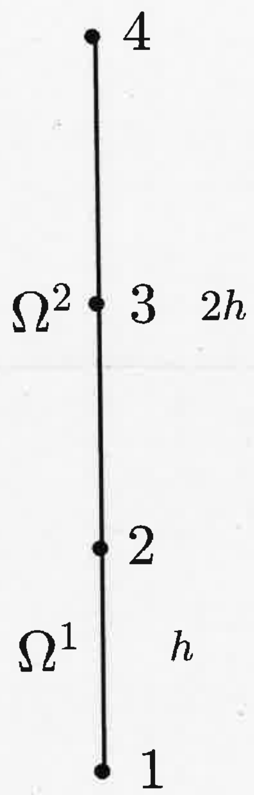
1^{st} **Compute** the base's section area, $A(x)$, $0 \leq x \leq h$



$$\frac{\delta(x)}{h-x} = \frac{\frac{a-b}{2}}{h}$$

$$\delta(x) = \frac{a-b}{2h}(h-x)$$





$$A(x) = c\left(b + 2\delta(x)\right) = c\left(b + \frac{a-b}{h}(h-x)\right)$$

$$= \left\{\begin{matrix} a = 2 = c \\ b = 1 \\ h = 1 \end{matrix}\right\} = 2\left(1+1-x\right) = 2(2-x)$$

ⓐ $h=1: \ \psi_2^2(x) = \dfrac{(x-1)(x-3)}{(2-1)(2-3)} = (3-x)(x-1)$

$p = 1'30: \ \psi_2^2(1'30) = (3-1'30)(1'30-1)$

$= 1'70 \times 0'30 = \boxed{0'51}$

ⓑ $h=1: \ \psi_1^1(x) = \dfrac{x-1}{0-1} = 1-x, \quad \dfrac{d\psi_1^1}{dx} = -1$

$\psi_2^1(x) = \dfrac{x-0}{1-0} = x, \quad \dfrac{d\psi_2^1}{dx} = 1$

$$K_{21}^1 = \int_{x_1^1=0}^{x_2^1=h=1} EA(x)\frac{d\psi_2^1}{dx}\frac{d\psi_1^1}{dx}\,dx = \left\{E=1\right\} = 2\int_0^1 (2-x)(-1)(1)\,dx = -2\int_0^1 (2-x)\,dx = -2\left(2x - \frac{x^2}{2}\right)\Big|_0^1 = -2\left(2-\frac{1}{2}\right) = \boxed{-3}$$

(c) $W(x) = \int_x^{3h} \omega A_2 \, ds$, $h=1 \leq x \leq 3h = 3$ $(h=1) \Rightarrow f(x) = \frac{dW}{dx}(x) = -\omega A_2 =$

$$= -\omega bc = \left\{ \begin{matrix} \omega = 3 \\ b = 1 \\ c = 2 \end{matrix} \right\} = -6, \quad h = 1 \leq x \leq 3h = 3$$

$$\Psi_1^2(x) = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{1}{2}(x-2)(x-3)$$

$$F_1^2 = \int_{x_1^2 = h = 1}^{x_3^2 = 3h = 3} f(x) \Psi_1^2(x) \, dx = -3 \int_1^3 (x-2)(x-2-1) \, dx = -3 \int_1^3 (x-2)^2 dx + 3 \int_1^3 (x-2) dx$$

$$= \left\{ \begin{matrix} \text{c.v.:} \\ t = x-2 : dt = dx \\ x = 3 \Rightarrow t = 1;\ x = 1 \Rightarrow t = -1 \end{matrix} \right\} = -3 \int_{-1}^{1} t^2 dt + 3 \underbrace{\int_{-1}^{1} t \, dt}_{=0} = -6 \int_0^1 t^2 dt$$

$$= -6 \left. \frac{t^3}{3} \right|_0^1 = -2$$

Hence: $F_2 = F_2^1 + F_1^2 = -4 - 2 = \boxed{-6}$

(d) $K^1 = \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 9 & -9 \\ -9 & 9 \end{pmatrix}$, $K^2 = \frac{a_1^2}{3h^2} \begin{pmatrix} 7 & -8 & 1 \\ * & * & * \\ * & * & * \end{pmatrix} = \left\{ \begin{matrix} a_1 = EA_2 = 2 \\ h_2 = 2h = 2 \end{matrix} \right\}$

$$= \frac{1}{3} \begin{pmatrix} 7 & -8 & 1 \\ * & * & * \\ * & * & * \end{pmatrix}$$

We know that $U_3 = -9'8\hat{3} = -\frac{59}{6}$, $U_4 = -11'\hat{3} = -34/3$ and, from the boundary conditions that $Q_2 = 0$ and $U_1 = 0$. Therefore:

$$\frac{1}{3} \begin{pmatrix} 9 & -9 & 0 & 0 \\ -9 & 16 & -8 & 1 \end{pmatrix} \begin{pmatrix} U_1 = 0 \\ U_2 \\ U_3 = -59/6 \\ U_4 = -34/3 \end{pmatrix} = \begin{pmatrix} F_1 = -5 \\ F_2 = -6 \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 = 0 \end{pmatrix} \qquad (\&)$$

and U_2 can be isolated from the 2^{nd} equation. Indeed,

$$-9 \overset{=0}{U_1} + 16 U_2 - 8 \overset{=-59/6}{U_3} + \overset{=-34/3}{U_4} = 3 \overset{=-6}{F_2} + 3 \overset{=0}{Q_2}$$

$$\Leftrightarrow U_2 = \frac{1}{16}\left(-18 - 8\times\frac{59}{6} + \frac{34}{3}\right) = \frac{1}{16}\left(-18 - \frac{236}{3} + \frac{34}{3}\right) = \frac{-290+34}{3\times 16}$$

$$= -\frac{256}{3\times 16} = -\frac{2^8}{3\times 2^4} = -\frac{16}{3} = \boxed{-5'\widehat{3}}$$

(e) Finally, we can get Q_1 from the 1^{st} equation in (&):

$$Q_1 = \frac{1}{3}\left(9\overset{=0}{U_1} - 9U_2 + 0U_3 + 0U_4\right) - \overset{=-5}{F_1} = -3\left(-\frac{16}{3}\right) + 5 = 16+5 = \boxed{21}$$