problema_2.pdf

2. Suposem que un problema, que volem resoldre pel mètode dels elements dels elements finits, té per equació,

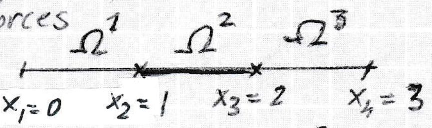
$$-\frac{d}{dx}\left(x^2\frac{du}{dx}\right)+u=x, \qquad x\in(0,3)$$

Considerem un element finit lineal, Ω^k, definit a $[1,2]$ i el seu sistema associat, $[K^k]u=F^k+Q^k$.

(a) Doneu la funció de forma $\psi_1^k(x)$.

(b) Calculeu els valors de K_{11}^k, K_{12}^k de la matriu de rigidesa elemental.

(c) Calculeu el valor F_1^k del vector de forces



◁ Solució.

$x_1^1=x_1=0,\ x_2^1=x_1^2=x_2=1,\ x_2^2=x_1^3=2,\ x_2^3=3$

$k=2$: $\psi_1^k(x)=\dfrac{x-x_2^k}{x_1^k-x_2^k}=\dfrac{x-2}{1-2}=2-x,$

$\psi_2^k(x)=\dfrac{x-x_1^k}{x_2^k-x_1^k}=\dfrac{x-1}{2-1}=x-1.$

(a) $\boxed{\psi_1^k(x)=\dfrac{x-2}{1-2}=2-x}$

(b) $\psi_2^k(x)=\dfrac{x-1}{2-1}=x-1$; $\quad\dfrac{d\psi_1^k}{dx}(x)=-1,\quad \dfrac{d\psi_2^k}{dx}(x)=1.$

$a_1(x)=x^2$,

$$K_{11}^{k,1}=\int_1^2 x^2\underbrace{\frac{d\psi_1^k}{dx}(x)}_{=-1}\frac{d\psi_1^k}{dx}(x)\,dx=\int_1^2 x^2dx=\left[\frac{x^3}{3}\right]_1^2=\frac{8}{3}-\frac{1}{3}=\frac{7}{3}=\frac{7}{3}=K_{22}^{k,1}$$

$$K_{12}^{k,1}=\int_1^2 x^2\underbrace{\frac{d\psi_1^k}{dx}(x)}\frac{d\psi_2^k}{dx}(x)\,dx=-\int_1^2 x^2dx=-\left[\frac{x^3}{3}\right]_1^2=-\left(\frac{8}{3}-\frac{1}{3}\right)=-\frac{7}{3}=K_{21}^{k,1}$$

$a_0(x)=1$,

$$K_{11}^{k,0}=\int_1^2 1\cdot\psi_1^k(x)\psi_1^k(x)\,dx=\int_1^2(2-x)^2dx=-\frac{1}{3}(2-x)^3=\frac{1}{3}$$

$$K_{12}^{k,0}=\int_1^2 1\cdot\psi_1^k(x)\psi_2^k(x)\,dx=\int_1^2(x-1)(2-x)\,dx=\overset{\text{c. de v}}{\left\{\begin{array}{l}s=x-1,\\ x=s+1\end{array}\right\}}=\int_0^1 s(1-s)\,ds$$

$$=\left(\frac{s^2}{2}-\frac{s^3}{3}\right)\Big|_0^1=\frac{1}{6}$$

Llavors:

$$K_{11}^k=K_{11}^{k,1}+K_{11}^{k,0}=\frac{7}{3}+\frac{1}{3}=\boxed{\frac{8}{3}};\quad K_{12}^k=K_{12}^{k,1}+K_{12}^{k,0}=-\frac{14}{6}+\frac{1}{6}=\boxed{-\frac{13}{6}}$$

(c) $$F_1^k=\int_1^2 x\,\psi_1^k(x)\,dx=\int_1^2 x(2-x)\,dx=\left\{\begin{array}{l}s=x-1\\ x=s+1\end{array}\right\}=\int_0^1(s+1)(1-s)\,ds=\int_0^1(1-s^2)\,ds$$

$$=\left(s-\frac{s^3}{3}\right)\Big|_0^1=1-\frac{1}{3}=\boxed{\frac{2}{3}}.\ ▷$$

Notebloc