

Problema - 1

(a) $\psi_2^3(x,y) = a + bx + cy$ s.t.:

$$\psi_2^3(3,0) = a + 3b = 0$$

$$\psi_2^3(3,1) = a + 3b + c = 1$$

$$\psi_2^3(1,1) = a + b + c = 0$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & -1 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 2 & 0 & 0 & -3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Solution:

$$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1$$

So the form function is,

$$\boxed{\psi_2^3(x,y) = \frac{x}{2} + y - \frac{3}{2}}$$

(b) Find the value of K_{52} (the (5,2) entry of the global stiff matrix K)

$$K^1 = \frac{k_c}{2} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}, K^2 = K^3 = \frac{k_c}{4} \begin{pmatrix} 4 & -4 & 0 \\ -4 & 5 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\begin{aligned} K_{52} &= K_{32}^1 + K_{12}^2 = -\frac{k_c}{2} - \frac{4k_c}{4} \\ &= \left(-\frac{1}{2} - 1\right) k_c = -\frac{3}{2} k_c = \boxed{-6} \\ &\quad k_c = 4 \end{aligned}$$

Hint. $K_{34} = K_{12}^3 = \frac{k_c}{4} (-4) = \boxed{-4}$

(c) The value of F_5 (the global internal heating of global node 5),

$$F^1 = \frac{f_1 A_1}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{f}{6} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

$$F^2 = F^3 = \frac{f_{2,3} A_{2,3}}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{f}{6} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix},$$

S_0 ,

$$F_5 = F_3^1 + F_1^2 + F_3^3 = \frac{f}{6} (1+2+2)$$
$$= \frac{5}{6} f = \frac{5}{2} = \boxed{2.5}$$

Hint. $F_1 = F_1^1 = \frac{f}{6} = \frac{3}{6} = \frac{1}{2} = \boxed{0.5}$

(d) Now, suppose that a linear flow q_n on the edge between nodes 1 and 5 of values $q_n = 30$ and 0 respectively is applied. Then the value of Q_{33}^1 is,

$$Q_{33}^1 = \frac{0}{3} \sqrt{2} + \frac{30}{6} \sqrt{2} = 5\sqrt{2} = \boxed{7.0711e+00}$$

(e) When computing Q_4 as part of a convective edge, the expression relating T_3, T_4 and T_∞ is

$$Q_4 = -\frac{\beta T_3}{6} h_1^3 - \frac{\beta T_4}{3} h_1^3 + \frac{\beta T_\infty}{2} h_1^3$$

$$h_1^3 = \boxed{-T_3 - 2T_4 + 3T_\infty}$$

$$\beta = 6.0$$

If) Suppose that $T_0 = 0$ on all the bottom boundary and also $q_n = 0$ on the top edge joining the nodes 4 and 5, besides the boundary conditions already done. The value of T_4 when $T_\infty = 0$ and T_5 is $1.4166e+00$ is,

$$K_{41} \overset{=0}{T_1} + K_{42} \overset{=0}{T_2} + K_{43} \overset{=0}{T_3} + K_{44} T_4 + K_{45} T_5$$

$$(*) = \sum_4 K_C T_4 - \frac{K_C}{4} T_5 = F_4 + Q_4 \quad (\&)$$

$$(*) K_{44} = K_{22}^3 = \frac{5}{4} K_C \quad \left| \begin{array}{l} F_4 = F_2^3 = f_3 = 1 \\ Q_4 = -T_3^3 - 2T_4 + 3T_\infty^3 = 0 \end{array} \right.$$

$$K_{45} = K_{23}^3 = -K_{14}$$

$k_c = 4$, so the system (&) writes,

$$5T_4 - T_S = 1 - 2T_3$$

$$\Leftrightarrow 7T_4 = 1 + T_S$$

$$\Leftrightarrow T_4 = \frac{1 + T_S}{7} = \frac{2.4166}{7}$$

$$= \boxed{3.4523e-01}$$

□

$$\begin{array}{r} 2.4166 \\ \underline{\quad | \quad 7} \\ 31 \qquad 0.345228 \\ 36 \\ 16 \qquad \sqcup 0.34523 \\ 20 \\ 60 \\ 40 \end{array} = 3.4523e-01$$