MÈTODES NUMÈRICS: Ex-Parcial **(a) Q1-2017-18**

Name and surnames:

(**Obs.** On answering you can use Catalan or Spanish. Use `format short e` for the results)

1. Using the mesh file **Circlemesh02** we define a temperature for each node according to the function

$$f(x, y) = \frac{20(x^2 + y^2)}{\sqrt{x^2 + y^2 + 0.01}}$$

(a) Compute the interpolated temperature associated to point $p = (-0.6, 0.6)$ and the triangle it belongs. Which is the relative error when comparing the interpolated temperature and the one assigned by the function?

numTriang	T(p)	RelError
25	1.7013e+01	9.4262e-03

(*Hint:* For node 35 the temperature value is $3.8437e-01$.)

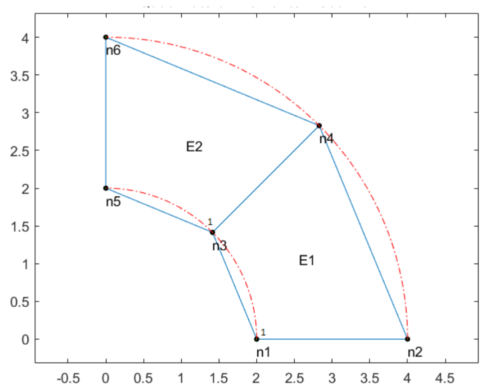
(b) If we assign to each triangle the temperature value at its barycenter. The triangle with maximum temperature and its value are

numTriang	Temp
27	1.7985e+01

(*Hint:* For element 37 the temperature value is $1.2465e+01$.)

(3 points)

2. Consider the ring area shown in the figure, with radius 2 and 4 respectively and angle $\theta \in [0, 90]$. We approximate its geometry using two quadrilateral elements defined with angle $45^{\underline{o}}$ as it is shown.



(a) Compute the approximated area and compare with the true value $A = (R^2 - r^2)\frac{\pi}{4}$

AreaApprox	RelError
8.4853e+00	9.9684e-02

(b) Use a similar approximation to compute the integral of $f(x, y) = x^2 + 5y^2$ on this ring area, which real value is 90π.

IntegApprox	RelError
2.2971e+02	1.8758e-01

(*Hint:* The value of the integral for $f(x, y)$ at the first element is $6.6569e+01$.) **(3 points)**

(3) Let Ω^R be the rectangle defined by the vertices $v_1 = (-1, -1); v_2 = (1, -1); v_3 = (1, 1); v_4 = (-1, 1)$.

(a) If ψ_i^R denote its shape functions and $p = (\frac{1}{2}, -\frac{1}{4})$ is an interior point, then:

$$\boxed{\psi_1^R(p) = \frac{5}{32}}$$

(*Hint:* It must be a value among the following ones: $\frac{1}{32}; \frac{3}{32}; \frac{5}{32}; \frac{7}{32}; \frac{9}{32}; \frac{11}{32}; \frac{13}{32}; \frac{15}{32}; \frac{17}{32}; \frac{19}{32}$)

(b) If f is a function with measured values at the vertices $f(v_1) = -1; f(v_2) = 2; f(v_3) = 3; f(v_4) = 4$, then the approximated/interpolated value at the point p is:

$$\boxed{f(p) = 2}$$

(*Hint:* It must be a value among the following ones: $0, \frac{1}{8}; \frac{7}{16}; \frac{1}{2}; \frac{5}{8}; 1; \frac{25}{16}; \frac{27}{16}; 2; \frac{17}{8}; \frac{9}{2}; 3$)

(2 points)

(4) Let Ω^K be the rectangle defined by the vertices $\bar{v}_1 = (0, 0); \bar{v}_2 = (2, 0); \bar{v}_3 = (2, 2); \bar{v}_4 = (0, 2)$.

(a) Compute the coordinates of point $\tilde{p} \in \Omega^K$, corresponding to the point $p = (\frac{1}{2}, \frac{1}{3})$ in the reference quadrilateral Ω^R:

$\tilde{p}_x =$	$\frac{3}{2}$
$\tilde{p}_y =$	$\frac{4}{3}$

(*Hint:* The x coordinate must be a value among the following ones: $\frac{3}{2}; \frac{2}{3}; \frac{5}{4}; \frac{1}{4}; \frac{4}{3}; \frac{3}{4}; \frac{5}{3}; \frac{4}{5}; \frac{2}{5}; \frac{1}{2}$)

(b) If ψ_i^K denote its shape functions and $\bar{p} = (\frac{1}{2}, \frac{5}{4})$ an interior point, then:

$\psi_1^K(\bar{p}) = \frac{9}{32}$

(*Hint:* It must be a value among the following ones: $\frac{1}{32}; \frac{3}{32}; \frac{5}{32}; \frac{7}{32}; \frac{9}{32}; \frac{11}{32}; \frac{13}{32}; \frac{15}{32}; \frac{17}{32}; \frac{19}{32}$)

(2 points)