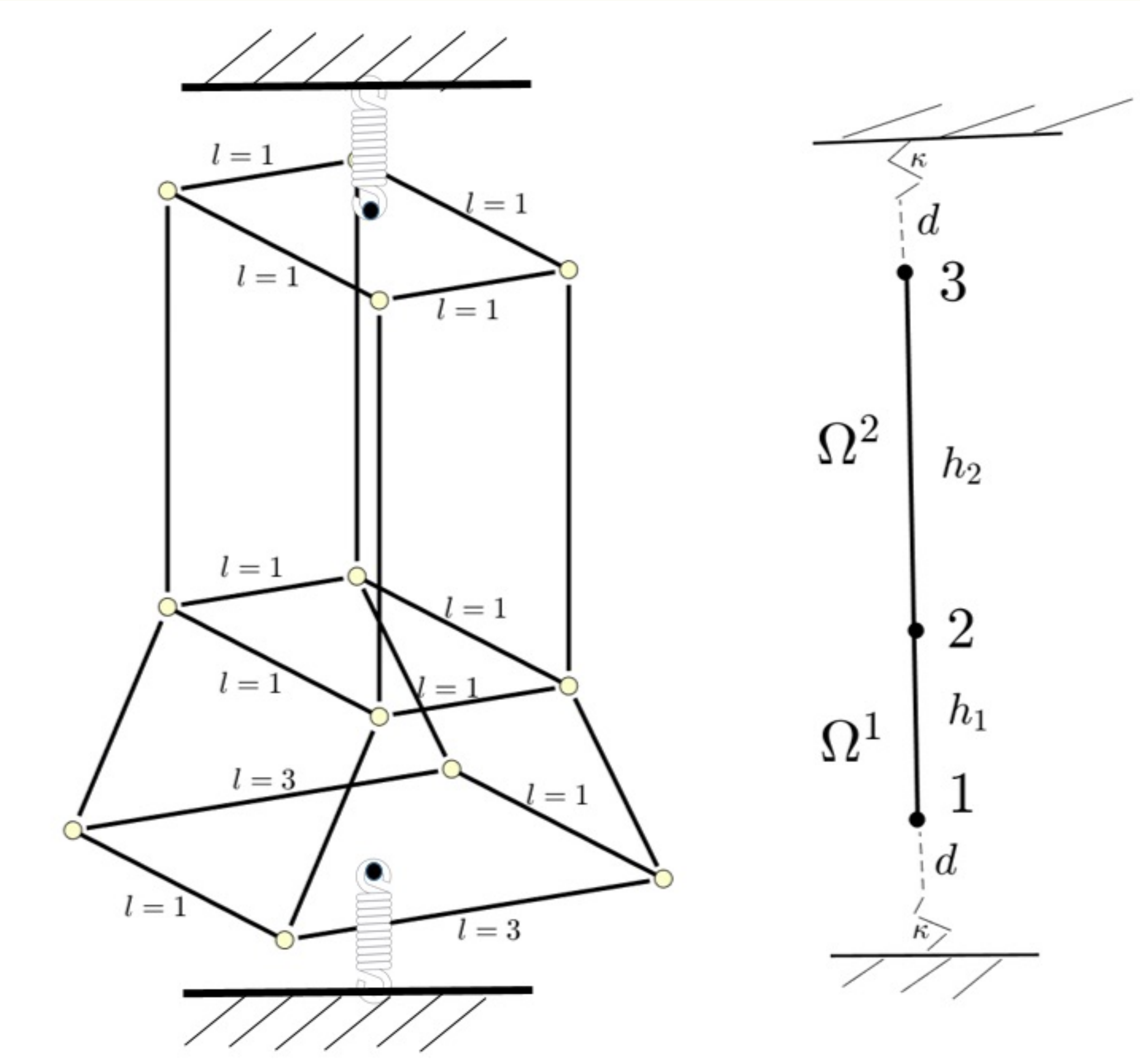
Problem 1



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κ
d
3
Ω^2
h_2
2
h_1
Ω^1
1
d
κ

Pregunta 1
Correcte
Puntuació 10,00 sobre 10,00

Consider a pillar made of two block pieces with the dimensions shown in the figure and vertical lengths $h_1 = 2$, $h_2 = 4$, in a certain unit system. Only for academic purposes we can consider that the Young modulus of the material is $E = 2$ and the specific weight is $\omega = 0.01$.
We want study the elastic deformation of the pillar through the Finite Element Method with two linear elements as symbolically are shown in the figure
Also, as it is seen, we assume that the top and the bottom of the column are both initially placed at a distance $d = 1$ of a two anti-vibration springs system of recovering constant $\kappa = 1$. Then we attach the pillar to the two anti-vibration spring system. We want to study the deformation of the pillar caused by the weigth and the spring recovering forces
In order to avoid unnecessary calculations you can use the formulas of K and F when they depend linearly on x. You have to answer the following questions:
(a) (2 points) The entry $K(2,2)$ of the stiff matrix of the global assembled system is

- 2.8000e+00
- Leave it empty (no penalty)
- 2.2500e+00
- 2.5000e+00 ✔
- 2.7500e+00
- 2.0000e+00

Puntuació 3,00 sobre 3,00

La resposta correcta és: 2.5000e+00

Hint1: The value of $K(2,3)$ is -5.0000e-01
(b) (3 points) The second component of the global assembled force $F(2)$ is

- -3.0667e-02
- -3.6667e-02 ✔
- -3.4667e-02
- Leave it empty (no penalty)
- -3.2667e-02
- -4.1667e-02

Puntuació 4,00 sobre 4,00

La resposta correcta és: -3.6667e-02

Hint2: The value of $F(3)$ is -2.0000e-02
(c) (5 points) You can realize that, in the 1-node, the secondary variable is $Q_1 = -\kappa(d + U_1)$ instead. Then the displacement U_2 of the global node 2 is (you can use Hint3)

- -3.9022e-01
- -3.9222e-01 ✔
- -3.9422e-01
- -3.8722e-01
- Leave it empty (no penalty)
- -3.8822e-01

Puntuació 3,00 sobre 3,00

La resposta correcta és: -3.9222e-01

Hint3: The value of $U(1)$ is -6.0259e-01

Torna a començar | Desa | Emplena amb les respostes correctes | Envia i acaba | Tanca la previsualització

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Opcions de visualització

Solution

$l_1 = 3,\ l_2 = 1;\ h_1 = 2,\ h_2 = 4;\ E = 2;\ W = 0.01$

$A_1 = l_1 l_2 = 3,\ A_2 = l_2^2 = 1;$

$$A(x) = \begin{cases} A_1 + \dfrac{A_2 - A_1}{h_1} x = 3 - x, & 0 \leq x \leq h_1 = 2 \\ A_2 = 1, & h_1 = 2 \leq x \leq L = h_1 + h_2 = 6 \end{cases}$$

Then,

$$a_1(x) = EA(x) = \begin{cases} 6 - 2x, & 0 \leq x \leq h_1 = 2 \\ 2, & h_1 = 2 \leq x \leq L = h_1 + h_2 = 6 \end{cases}$$

$$f(x) = -WA(x) = \begin{cases} -0.03 + 0.01x, & 0 \leq x \leq h_1 = 2 \\ 0.01, & h_1 = 2 \leq x \leq L = h_1 + h_2 = 6 \end{cases}$$

We got formulas for $K^{e,1}$ and F^e in the case of linear elements and affine dependence of a_1 and f on x, so

- If $a_1(x) = \alpha x + \beta$, α & β constants on the element Ω^e, then,

$$K^{e,1} = \frac{\alpha}{2h_e}(x_1^e + x_2^e)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \frac{\beta}{h_e}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad (1)$$

• If $f(x) = ax + b$, a & b constants on the element Ω^e, then

$$F^e = \frac{a h_e}{6} \begin{pmatrix} 2x_1^e + x_2^e \\ x_1^e + 2x_2^e \end{pmatrix} + \frac{b h_e}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (2)$$

where h_e is the length of the element, and x_1^e, x_2^e are the positions of the first ant the second nodes respectively of the element Ω^e.

(a) For Ω^1, we apply (1) with $\alpha = -2$, $\beta = 6$, $h_1 = 2$, $x_1^1 = 0$, $x_2^1 = h_1 = 2$:

$$K^1 = K^{1,1} = -\frac{2}{2 \cdot 2}(0+2)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \frac{6}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

Remark: note that, as $a_0(x) = 0$, so $K^{e,0} = 0$, for $e = 1, 2$.

For Ω^2, we apply (1) with $\alpha=0$, $\beta=2$, and $h_2=4$. Therefore,

$$K^2 = K^{2,1} = \frac{2}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

The coupled stiff matrix is then:

$$K = \begin{pmatrix} 2 & -2 & \\ -2 & 5/2 & -1/2 \\ & -1/2 & 1/2 \end{pmatrix}$$

Hence: $\boxed{K_{22} = 2.5}$

$$\left(\text{Hint } 1.\ K_{23} = -1/2 = -0.5\right).$$

(b) For Ω^1 we apply (2) with $a=0.01$ & $b=-0.03$

$h_1=2$, $x_1^e=0$, $x_2^e=h_1=2$,

$$F^1 = \frac{0.02}{6}\begin{pmatrix} 2 \\ 4 \end{pmatrix} - \frac{0.03\times 2}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= 10^{-2}\begin{pmatrix} 4/6 \\ 8/6 \end{pmatrix} - 10^{-2}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = 10^{-2}\begin{pmatrix} -7/3 \\ -5/3 \end{pmatrix}$$

For Ω^2 we apply (2) with $a=0$ & $b=-0.03$,

$h_2 = 4$,

$$F_2 = -\frac{0.01\times 4}{2}\begin{pmatrix}1\\1\end{pmatrix} = 10^{-2}\begin{pmatrix}-2\\-2\end{pmatrix}.$$

and the coupled vector F is then,

$$F = 10^{-2}\begin{pmatrix}-7/3\\-2-5/3\\-2\end{pmatrix} = 10^{-2}\begin{pmatrix}-7/3\\-11/3\\-2\end{pmatrix}$$

$$\begin{array}{l|l} 11 & 3 \\ \hline 20 & 3.66 \\ 20 & \dots \\ 2\dots & \end{array}$$

Hence, $\boxed{F_2 = -11/3 \times 10^{-2} = -3.\widehat{6}\times 10^{-2}}$

(Hint 2. $F_3 = -2.0\times 10^{-2}$)

(c) $K=1$, $d=1$, Q

Natural Boundary Conditions:

$$Q_1 = -K(d+U_1) = -1-U_1$$

$$Q_2 = 0$$

$$Q_3 = K(d-U_3) = 1-U_3$$

Note that there are not essential B.C., since the problem is completely determined by the two natural B.C. already fixed. The coupled system is then

$$\begin{pmatrix} 4 & -4 & 0 \\ -4 & 5 & -1 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} -14/3 \times 10^{-2} \\ -22/3 \times 10^{-2} \\ -4 \times 10^{-2} \end{pmatrix} + \begin{pmatrix} -2-2U_1 \\ 0 \\ 2-2U_3 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 6 & -4 & 0 \\ -4 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} -2 - 14/3 \times 10^{-2} \\ -22/3 \times 10^{-2} \\ 2 - 4 \times 10^{-2} \end{pmatrix} \qquad (3)$$

To solve (3) we can use hint 3., $U_1 = -6.0259 \times 10^{-1}$:

$$\begin{pmatrix} 5 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 0.22/3 + 4U_1 \\ 1.96 \end{pmatrix}$$

and Cramer's rule:

$$U_2 = \frac{1}{14} \begin{vmatrix} -0.22/3 + 4U_1 & -1 \\ 1.96 & 3 \end{vmatrix}$$

$$= \frac{1}{14}\left(-0.22 + 12U_1 + 1.96\right)$$

$$= \frac{1}{14}\left(-0.22 + 12 \times (-0.60259) + 1.96\right)$$ 6

$$= \boxed{-3.9222 \times 10^{-1}}$$

$$\begin{array}{r} -0.60259 \\ \times\ 12 \\ \hline 120518 \\ 60259 \\ \hline -7.23108 \\ -0.22 \\ \hline -7.45108 \\ 1.96 \\ \hline -5.49108 \end{array}$$

$$\begin{array}{l|l} -5.49108 & 14 \\ \cline{2-2} 129 & -0.39222\ldots \\ 031 & \\ 30 & \\ 28 & \\ 0\ldots & \end{array}$$