Mètodes Numèrics (240032)
Plane elasticity. Weak Formulation

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Definiton 1 (Model equation for plane elasticity problems [Red06])

For the plane elasticity problems, the the normal stress in the x direction, $\sigma_{xx} = \sigma_{xx}(x,y)$, the normal stress in the y direction, $\sigma_{yy} = \sigma_{yy}(x,y)$, and the shear stress, $\sigma_{xy} = \sigma_{xy}(x,y)$, satisfy the BVP given by the two coupled system of PDE,

$$\left.\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + f_x(x,y) = 0 \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + f_y(x,y) = 0 \end{aligned}\right\} \quad \text{on } \Omega \subset R^2. \tag{1}$$

Being,

- f_x: the component of the body force vector (per unit volume) along the x direction.
- f_y: the component of the body force vector (per unit volume) along the y direction.

the *natural* B.C.

$$\left.\begin{aligned} t_x \equiv \sigma_{xx} n_x + \sigma_{xy} n_y = \hat{t}_x \\ t_y \equiv \sigma_{xy} n_x + \sigma_{yy} n_y = \hat{t}_y \end{aligned}\right\} \quad \text{on } \Gamma_\sigma, \tag{2}$$

and the *essential* B.C.

$$u = \hat{u}, \quad v = \hat{v} \quad \text{on } \Gamma_u \tag{3}$$

where n_x, n_y denote the components (or the direction cosines) of the unit normal vector, $\mathbf{n}^\top = (n_x, n_y)$, on the boundary of Ω, $\Gamma = \partial\Omega$; Γ_σ and Γ_u are two disjoint pieces of Γ; $\hat{t}_x$ and $\hat{t}_y$ denote the components of the specified traction vector, $\mathbf{t}^\top = (t_x, t_y)$; $\hat{u}$, and $\hat{v}$ are the components of the specified displacement vector $\mathbf{w}^\top = (u, v)$.

Example: an algorithm with caption

Data: $n \geq 0$
Result: $y = x^n$
1 $y \leftarrow 1$
2 $X \leftarrow x$
3 $N \leftarrow n$
4 **while** $N \neq 0$ **do**
5 **if** N *is even* **then**
6 $X \leftarrow X \times X$
7 $N \leftarrow \frac{N}{2}$ `# This is a comment`
8 **else**
9 **if** N *is odd* **then**
10 $y \leftarrow y \times X$
11 $N \leftarrow N - 1$
12 **end**
13 **end**
14 **end**

Algorithm 1: An algorithm with caption

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